Is the Fundamental Theorem of Arithmetic, The Missing Link Between Solving Arithmetic Fraction Problems and **Rational Expressions** in Algebra?

(Part 1): Reducing, Multiplying and Dividing Fractions

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How many teachers really understand the Fundamental Theorem of Arithmetic? More importantly, **"How many teachers emphasize the Fundamental Theorem of Arithmetic when they teach students to work with fractions in arithmetic?"** This is not a theorem which one will learn in an Advanced Calculus class or in a Nuclear Physics class, but it is a Theorem which has great applications when working with fractions in arithmetic and later when working with all types of fraction type problems, in higher level math classes.

I understand why elementary arithmetic teachers do not emphasize the Fundamental Theorem of Arithmetic and even why they might not even know the Fundamental Theorem of Arithmetic. The Theorem is indirectly mentioned in many arithmetic books when students begin to work with fractions, but the name is seldom mentioned, and the real applications of the Fundamental Theorem of Arithmetic are not emphasized in most arithmetic textbooks. If teachers are never taught about the Theorem when they are in school (k-12) or college and the textbooks only indirectly mention the Theorem, but do not emphasize how to use the Theorem to solve simple and challenging problems in arithmetic, then why would I think that an elementary arithmetic teacher would realize the significance of the Theorem?

Hear are my thoughts about why many, probably over 90% of elementary teachers in the United States never discuss or explain the Fundamental Theorem of Arithmetic to their students.

1. Elementary Teachers were never taught about the importance of the Fundamental Theorem and their teachers in grades k-12 never mentioned the Theorem. I am also disappointed that college professors never mentioned it, but my guess is that college professors assume that the college students already know the Fundamental Theorem of Arithmetic and Factor Trees and why Factor Trees are so important especially when working with fractions.
2. When elementary arithmetic teachers and secondary math teachers were in college preparing to learn how to teach arithmetic and higher-level math classes, probably over 90% of them took a course or courses which helped them illustrate concepts to solve both arithmetic and math problems, BUT they never took a course in college to help them learn how to teach rules and procedures which are necessary to solve challenging

arithmetic problems and many of the problems they are asked to solve in Algebra 1 and higher level math classes.

3. If elementary teachers have no idea what the purpose of the Fundamental Theorem of Arithmetic is, then they have no idea how to teach students how to solve all types of arithmetic fraction problems and present a second method which is different than how they were taught. The Fundamental Theorem of Arithmetic focuses on Composite Numbers, Prime Numbers, and Factor Trees. Using Factor Trees is the key to solve difficult fractions in arithmetic and Rational Expressions (fractions) in Algebra.

Comment: Almost 100% of the fraction problems presented in typical U.S. arithmetic textbooks require the students to add, subtract, reduce, multiply, or divide fraction problems using their mental math skills focused on memorizing the multiplication and division facts for **Positive Integers** which are simply referred to as the **Natural Numbers** from 1-10 or maybe 1-12.

If the previous comment is correct, then "How do elementary arithmetic teachers attempt to teach students the direct method to add and subtract the following problem involving three fractions: "$\frac{1}{60}+\frac{1}{68}-\frac{1}{72}$"? I guess you can allow the students to use trial and error or work in groups or use a calculator to first find a common denominator, but this may take a full class period or more and they may never find the Least Common Denominator (LCD) which is the preferred Common Denominator. The method or methods the students discover and use, may or may not always lead to a correct answer, but more importantly the method or methods that the students think of on their own, will probably NOT help the students be prepared to simplify the following two simple algebra problems "$\frac{1}{x^2y}+\frac{1}{xz}-\frac{1}{xy^2z}$"and "$\frac{1}{(x+2)}+\frac{1}{(x+5)}-\frac{1}{x^2+7x+10}$", which should be the ultimate goal. You may want your students to understand the concepts and allow the students to work in teams for 1 period, 1 week, or 1 month, but without referring to "AI" or allowing the students solve the problems on their cell phones or computers, they may never discover the correct procedures and then give up on solving all math problems now and in the future. I think it is the teacher's job to lead students in the right direction slowly, and illustrate concepts and procedures to solve problems, with the goal to ensure the students understand and memorize the procedures to solve the simple and the most challenging problems, **but only do this if you want your students to be successful in higher level math classes**. The two-algebra addition and subtraction problems indicated are very basic problems which all students who pass an Algebra 2 class must be able to solve/simplify.

Solving the three example fraction problems above using the Fundamental Theorem of Arithmetic along with Factor Trees and knowing how to find equivalent fractions might be a great goal in an **advanced arithmetic** or **advanced prealgebra** class. I will make the following bet with you, "If you ask Algebra 2 teachers the following question, "**Would it be helpful for students to have a solid background involving arithmetic fraction problems similar to the first**

example using Factor Trees, and would students be better prepared for success in Algebra 2 and all higher-level math courses?" I believe that over 90% of Algebra 2 teachers and teachers who teach advanced math classes would agree that it would be helpful.

Comment: Arithmetic is the foundation to build the mathematics' skyscraper. A strong arithmetic foundation regarding a student's understanding of all arithmetic concepts, rules and procedures are essential, if each student is expected to achieve success next year and future years in math. Our goal as educators must be to have every student build a math skyscraper higher than you think they can achieve. If a student does not have a solid arithmetic foundation, then the probability that the student will become discouraged with Algebra and higher-level math classes increases. Success in Algebra and higher-level math classes depends on how much students understand arithmetic concepts and just as importantly how well they know how to apply the rules and procedures taught in arithmetic with the real goal of building the skyscraper high into the sky for all students.

I assume you introduce Factor Trees at some point when you teach fractions, but you probably never illustrated or assigned fraction problems which required students to use Factor Trees. Questions: Why do you think mathematicians named a Theorem the FUNDAMENTAL TEHEOREM of ARITHMETIC if it wasn't really an important Theorem? Why do you introduce FACTOR TREES in arithmetic and never assign a problem where it was necessary to use FACTOR TREES? Are FACTOR TREES just a nice topic to mention with no real purpose?

The Fundamental Theorem of Arithmetic states that every Natural Number greater than "1" is either a composite number or a prime number and every composite number has a unique way (one way) to write the composite number as the product of Prime Factors. Here is the Exact Arithmetic Definition of Prime Natural Numbers = $\{2, 3, 5, 7, 11, 13, 17, 19, \ldots\}$. Here is the Exact Arithmetic Definition of Composite Natural Numbers = $\{4, 6, 8, 9 ,10, 12, 14, 15, 16. \ldots\}$. Here is simple but powerful definition of Factor(s): A factor is a number, term/variable, or expression which is part of a multiplication problem. I will write the following expression " $10\,x^2y(z+1)(z+t)^2$" in factored form " $2 \cdot 5\ \cdot x\ \cdot x\ \cdot y\ \cdot (z+1) \cdot (z+t) \cdot\ (z+t)$". There are eight (8) factors multiplied in this example, two are prime numbers, 3 of the factors are single variables, and three of the factors are quantities and every term is part of the multiplication problem.

Comment: In arithmetic and higher-level math classes, teaching students to add, subtract, and compare magnitudes of fractions in my opinion is much more challenging to teach than teaching students to reduce, multiply, and divide fractions. I am also under the opinion that if it is easier to teach students to reduce, multiply, and divide fractions then it is also easier for the students to learn and remember how to reduce, multiply, and divide fractions, especially if they are going to use the Fundamental Theorem of Arithmetic. Therefore, in this article I will illustrate how to use the Fundamental Theorem of Arithmetic and Factor Trees to solve/simplify fraction problems involving **reducing, multiplying, and dividing fraction problems** in all arithmetic and math classes. I will write part 2 of this article to illustrate how to compare, add, and subtract fractions at all levels. This section is Part 1 and the next section is Part2.

Part 1: Let's begin by solving two simple arithmetic fraction problems involving reducing by using mental math skills first and then using FACTOR TREES.

Reduce/Simplify the following two fractions "$\frac{8}{12}$ and $\frac{35}{45}$". I hope that over 90% or more of your 5th or 6th grade students and 100% of your students who are going to take Algebra 1 next year, immediately recognize when they refer to the fraction "$\frac{8}{12}$" that the two numbers "8" and "12" are both multiples of "4" and your students immediately divide both numbers by "4" to find the equivalent fraction "$\frac{2}{3}$". If they can do this simple problem, then they can use the same logic to simplify "$\frac{35}{45}$" by dividing both numbers by "5" to find the equivalent fraction in the reduced/simplified form "$\frac{7}{9}$". If your students are going to be successful working with basic fractions found in your arithmetic class, then they must know their multiplication facts for the Natural Numbers from 1-10 or 1-12.

Here are the solutions for both fraction problems just mentioned using the Fundamental Theorem of Arithmetic and Factor Trees.

8	12	35	45
2×4	2×6	5×7	3×15
$2 \times 2 \times 2$	$2 \times 2 \times 3$		$3 \times 3 \times 5$

Now: "$\frac{8}{12} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 3} = \frac{2}{3}$" and "$\frac{35}{45} = \frac{5 \cdot 7}{3 \cdot 3 \cdot 5} = \frac{7}{9}$". I intentionally did not write an important step in these examples which teachers must explain. Here are the same two example solutions with the added step in each example "$\frac{8}{12} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 3} = \frac{1 \cdot 1 \cdot 2}{1 \cdot 1 \cdot 2} = \frac{2}{3}$" and "$\frac{35}{45} = \frac{5 \cdot 7}{3 \cdot 3 \cdot 5} = \frac{1 \cdot 7}{3 \cdot 3 \cdot 1} = \frac{7}{9}$". Some teachers may say that I cancelled the two "2's" in the numerator and the denominator and in the second example teachers may say that I cancelled the "5's" in the numerator and denominator. Teachers commonly use the word cancel in these types of example problems, but it is more proper to say we divided each of the "2's" in the numerator with each of the "2's" in the denominator and in both situations we all know that ""$2 \div 2 = 1$" and a teacher can then emphasize the obvious that any number, variable, or expression divided by the identical number, variable, or expression will always equal "1". It is also acceptable to put the number "1" in both the numerator and the denominator. In the second example we also divided the "5" in the numerator by the "5" in the denominator and we must know that "$\frac{5}{5} = 1$". The identical factors do not cancel; they divide and become "1's".

Here is a more challenging fraction reducing problem and again I will illustrate the solution by using Factor Trees. Reduce/Simplify - "$\frac{390}{1300} = \frac{2\cdot3\cdot5\cdot13}{2\cdot2\cdot5\cdot5\cdot13} = \frac{1\cdot3\cdot1\cdot1}{1\cdot2\cdot1\cdot5\cdot1} = \frac{3}{10}$". It is an easy problem to simplify/solve once the numerator and denominator are written using Factor Trees. Factoring is the only method students will use in Algebra and higher-level math classes to solve/simplify rational expressions (algebra fractions). Why don't teachers emphasize the Fundamental Theorem of Arithmetic and Factor Trees when working with fraction problems in arithmetic, prior to allowing students to enter and Algebra 1 class?

Here are two algebra reducing (simplification) rational expression problems. Simplify the following expressions by first writing all the factors and then dividing the identical factors in the numerator with the identical matching factors in the denominator which turn into the number 1 in both the numerator and denominator. When the multiplication problem is completed in both the numerator and denominator the numbers equal to "1" have no impact because we know the Identity Property for the number ONE: "$a \times 1 = a \text{ or } 1 \times a = a, etc.$

1. $\frac{24x^2yz^3(x+2)(x+z)}{44x^2z^2(x+z)} = \frac{2\cdot2\cdot2\cdot3\cdot x\cdot x\cdot y\cdot z\cdot z\cdot z\cdot(x+2)(x+z)}{2\cdot2\cdot11\cdot x\cdot x\cdot z\cdot z\cdot(x+z)} = \frac{1\cdot1\cdot2\cdot3\cdot1\cdot1\cdot y\cdot1\cdot1\cdot z\cdot(x+2)\cdot1}{1\cdot1\cdot11\cdot1\cdot1\cdot1\cdot1\cdot1} = \frac{2\cdot3\cdot y\cdot z\cdot(x+2)}{11} = \boldsymbol{\frac{6yz(x+2)}{11}}$

2. **Given**: "$\frac{x^2-25}{x^2+15x+50}$" : One must know some basic algebra factoring rules to simplify.
Now, Factored and simplified: $\frac{(x-5)(x+5)}{(x+5)(x+10)} = \frac{(x-5)\cdot1}{1\cdot(x+10)} = \boldsymbol{\frac{(x-5)}{(x+10)}}$

Here are an Arithmetic multiplication fraction problem and an algebra multiplication problem involving a rational expression, both simplified using Factor Trees.

1. Arithmetic problem - "3/20 X 5/18 X 2/15=1/180" (It is so easy to determine the numbers in the numerator and the numbers in the denominator which reduce when you use slanted lines to write fractions.) **NO IT IS NOT EASY TO WORK WITH FRACTIONS USING SLANTED LINES. I never permitted students to use slanted lines when working with fractions in arithmetic, for two reasons. 1. It is confusing when students want to determine which numbers are in the numerator and in the denominator when they write the problems and must reduce the fractions using mental math skills involving multiplication and division. 2. This type of fraction is never written using slanted lines in algebra or higher-level math classes.**

Here is the same problem written using a vinculum (straight line) for every fraction.
$\boldsymbol{\frac{3}{20}} \times \boldsymbol{\frac{5}{18}} \times \boldsymbol{\frac{2}{15}} = \frac{3\times5\times2}{20\times18\times15} = \boldsymbol{\frac{3 \times 5 \times 2}{2\cdot2\cdot5 \times 2\cdot3\cdot3 \times 3\cdot5}} = \boldsymbol{\frac{1 \times 1 \times 1}{1\cdot2\cdot1 \times 2\cdot1\cdot3 \times 3\cdot5}} = \boldsymbol{\frac{1}{180}}$ Notice, I did not use slanted lines when writing the fractions and I feel it is important to notice that **I also used one big fraction once I realized that all the fractions were multiplied in my second step.** These are two little teaching points which I believe will help students be more successful when they solve a wide variety of fraction problems on a midterm, final test or a standardized test in two or three months after they learned how to simplify/solve fraction problems.

2. Algebra rational expression problem simplified using Factors. 1. " $\frac{5z^3}{6xy} \times \frac{1}{3x} \times \frac{4xy}{15z^2}$ "- I am now going to factor every term and then write the complete multiplication problem as one big fraction. (The one-big fraction idea was something I used when students multiplied and divided two or more fractions and again later in "Part 2", I will use the one big fraction idea when I add and subtract fractions once all the fractions have a least common denominator. I found the one big fraction idea helped students remember the different rules for the two situations.)

$$\frac{5 \cdot z \cdot z \cdot z}{2 \cdot 3 \cdot x \cdot y} \times \frac{1}{3 \cdot x} \times \frac{2 \cdot 2 \cdot x \cdot y}{3 \cdot 5 \cdot z \cdot z} = \frac{5 \cdot z \cdot z \cdot z \times 1 \times 2 \cdot 2 \cdot x \cdot y}{2 \cdot 3 \cdot x \cdot y \times 3 \cdot x \times 3 \cdot 5 \cdot z \cdot z} = \frac{1 \cdot 1 \cdot 1 \cdot z \times 1 \times 1 \cdot 2 \cdot 1 \cdot 1}{1 \cdot 3 \cdot 1 \cdot 1 \times 3 \cdot x \times 3 \cdot 1 \cdot 1 \cdot 1} = \frac{2z}{3 \cdot 3 \cdot 3 \cdot x} = \frac{2z}{27x}$$

3. " $\frac{x^2-1}{x^2-25} \times \frac{x^2-4}{x^2+3x+2} \times \frac{x^2-4x-5}{x^2-x-2}$ " First factor all the numerators and denominators yielding: $\frac{(x+1)(x-1)}{(x+5)(x-5)} \times \frac{(x-2)(x+2)}{(x+1)(x+2)} \times \frac{(x-5)(x+1)}{(x-2)(x+1)}$ (I suggest that students write the entire expression as one big fraction before dividing like terms.) $= \frac{1 \cdot (x-1) \times 1 \cdot 1 \times 1 \cdot 1}{(x+5) \cdot 1 \times 1 \cdot 1 \times 1 \cdot 1} = \frac{\mathbf{(x-1)}}{\mathbf{(x+5)}}\, or\, \frac{\mathbf{x-1}}{\mathbf{x+5}}$ The final expression which is written without parenthesis is the preferred notation but, the answer with parenthesis is acceptable. How would you simplify the original expression without factoring?

Here is an Arithmetic fraction problem and two algebra problems volving both multiplication and division. Notice, I will again factor all the numerators and denominators first and then use the one big fraction notation once every term is multiplied. Factor Trees and Factoring are very important when simplifying these types of problems!!!

1. Arithmetic Example – $\frac{45}{68} \times \frac{72}{75} \div \frac{39}{51} = \frac{45}{68} \times \frac{72}{75} \times \frac{51}{39} = \frac{3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 17} \times \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}{3 \cdot 5 \cdot 5} \times \frac{3 \cdot 17}{3 \cdot 13} =$
   $\frac{3 \cdot 3 \cdot 5 \times 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \times 3 \cdot 17}{2 \cdot 2 \cdot 17 \times 3 \cdot 5 \cdot 5 \times 3 \cdot 13} = \frac{1 \cdot 1 \cdot 1 \times 1 \cdot 1 \cdot 2 \cdot 3 \cdot \cdot 3 \times 3 \cdot 1}{1 \cdot 1 \cdot 1 \times 1 \cdot 1 \cdot 5 \times 1 \cdot 13} = \frac{2 \cdot 3 \cdot 3 \cdot 3}{5 \cdot 13} = \frac{54}{65}$
2. Simple Algebra Example - $\frac{30x^2a^3}{28bz} \div \frac{45y^2a^2}{56bz^2} \times \frac{27ay}{48az} = \frac{30x^2a^3}{28bz} \times \frac{56bz^2}{45y^2a^2} \times \frac{27ay}{48az} =$
   $\frac{30x^2a^3 \times 56bz^2 \times 27ay}{28bz \times 45y^2a^2 \times 48az} = \frac{2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot a \cdot a \cdot a \times 2 \cdot 2 \cdot 2 \cdot 7 \cdot b \cdot z \cdot z \times 3 \cdot 3 \cdot 3 \cdot a \cdot y}{2 \cdot 2 \cdot 7 \cdot b \cdot z \times 3 \cdot 3 \cdot 5 \cdot y \cdot y \cdot a \cdot a \times 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot z} =$
   $\frac{1 \cdot 1 \cdot 1 \cdot x \cdot x \cdot 1 \cdot 1 \cdot 1 \times 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \times 1 \cdot 1 \cdot 3 \cdot a \cdot 1}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \times 1 \cdot 1 \cdot 1 \cdot 1 \cdot y \cdot 1 \cdot 1 \times 1 \cdot 1 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1} = \frac{x \cdot x \times 3 \cdot a}{y \times \cdot 2 \cdot 2} = \frac{3ax^2}{4y}$
3. Slightly Challenging Algebra Expression -$\frac{x^2-x-12}{x^2+5x+6} \times \frac{x^2-4}{x^2-16} \div \frac{x^2-7x+10}{x^2-x-20} =$
   $\frac{x^2-x-12}{x^2+5x+6} \times \frac{x^2-4}{x^2-16} \times \frac{x^2-x-20}{x^2-7x+10} = \frac{(x^2-x-12) \times (x^2-4) \times (x^2-x-20)}{(x^2+5x+6) \times (x^2-16) \times (x^2-7x+10)} =$
   $\frac{(x-4)(x+3) \times (x-2)(x+2) \times (x-5)(x+4)}{(x+3)(x+2) \times (x-4)(x+4) \times (x-5)(x-2)} = \frac{1 \cdot 1 \times 1 \cdot 1 \times 1 \cdot 1}{1 \cdot 1 \times 1 \cdot 1 \times 1 \cdot 1} = 1$

I tried to detail all the solutions and show all the steps possible. When teaching these types of problems, I feel it is important that the teacher writes every step, so all the students understand the process. I also understand that when the students complete these types of problems, they may not write every step, but they should write enough steps, so you know they understand every step well enough to explain their solutions to another student. Do you think students can solve these three problems using only concepts? Maybe rules are important????

Expected Positive Outcomes After Emphasizing the Fundamental Theorem of Arithmetic in an Advanced Arithmetic or Advanced Prealgebra class prior to allowing students to enter an Algebra 1 class. In Part 2 of this paper the goal is to illustrate how to use the Fundamental Theorem of Arithmetic using Factor Trees to simplify/solve adding and subtracting simple and challenging fractions in arithmetic. I believe that it is more difficult for students to add and subtract challenging fractions than it is to reduce, multiply, and divide difficult fractions. Two comments about math research and learning. 1. Research indicates that the number one arithmetic topic learned in 5th grade, which predicts success in algebra, is a strong understanding of all procedures to solve all types of fraction problems. 2. Research also indicates that 1-2 months after students are taught the rules in 5th and 6th grade to solve all types of fraction problems, around 50% of students forget or get confused and they cannot even solve simple problems. If you want all students to "Learn and Remember" fraction rules, giving them two options/methods to solve the basic fraction problems found on standardized tests will greatly increase their probability for success. Option 1 is to have students use mental math skills involving multiplication and division facts, and Option 2 is to have students use The Fundamental Theorem of Arithmetic with Factor Trees.

1. Average and above average students can use two different methods to solve fraction problems in arithmetic, and the second method involving "Factor Trees" will ensure that students are prepared to apply rules and procedures to solve rational expressions in higher-level math classes. These students will not only be prepared for higher-level math classes but one should expect that at least 90% of these students or more will score well on all age-appropriate standardized math tests.
2. When I taught the steps, rules, and procedures focused on the Fundamental Theorem of Arithmetic and Factor Trees to my lowest level achieving pre-algebra classes consisting of 9th, 10th, 11th and 12th graders, many of whom were special needs students, I found that over 90% of the students improved in several areas.
    a. The number of students who previously did not know the multiplication and division facts for the numbers from 1-10 decreased significantly, an "Amazing Outcome". I believe this phenomenon occurred because these students were required to write factor trees for all the numbers and, in the process, they subliminally learned the multiplication facts without just teaching multiplication facts for memorization.
    b. Students learned how to write exponent notation, and they understood the meaning of exponent notation for numbers and variables in algebra.
    c. At the end of the year on the final test, the Special Education Teacher, who co-taught with me was surprised with the results on the final test and made the comments: **"The students learned more math this year than I ever dreamed possible."** Then she added, **"I learned more math than I ever thought possible**."

Next, **Part 2 involves adding and subtracting fractions and rational expressions.**

Is the Fundamental Theorem of Arithmetic, The Missing Link Between Solving Arithmetic Fraction Problems and **Rational Expressions** in Algebra?

(Part 2): Comparing, Adding and Subtracting Fractions

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I always thought that teaching addition, subtraction, and comparing the magnitudes of fractions involved more challenging rules and procedures for students to **master** than the rules and procedures required to reduce, multiply, and divide fractions for two reasons. The first challenging step requires students to find the least common denominator for all the fractions involved. The second challenging step requires the students to know how to find equivalent fractions with the least common denominator using rules and procedures, not a trick which works for simple problems, but the rules and procedures required to find equivalent fractions based on solid math theorems and facts.

In the process of adding and subtracting fractions, I will list all the required steps which a teacher should illustrate to students, but I realize that the students may not write every little step that I indicate, especially after they have solved 5 to 10 problems illustrating all the steps, but the teacher should insist that the students write many of the steps when they do classwork, homework, and solve problems on a test or quiz. I will also incorporate comparing magnitudes of fractions when I find the solutions to the problems indicated below.

First, I will tackle the most difficult step when adding and subtracting fractions with denominators greater than "15". The reason I picked the number "15" is because over 95% of the typical arithmetic addition/subtraction problems written in arithmetic books in the United States use denominators with smaller numbers and the Least Common Denominator can quickly be determined by recalling mental multiplication and division facts.

Second, I will discuss the rules, not a trick, to convert fractions into equivalent fractions once the Least Common Denominator is determined.

Comment: Why do we want **the Least/Smallest Common Denominator** and not simply multiply all the denominators to find a **Common Denominator**?

1. Addition and Subtraction of two simple arithmetic problems, which one can use mental math involving multiplication and division facts. Compare the magnitude of all the fractions involved.
   $a.\ \frac{3}{8}+\frac{7}{12}-\frac{1}{6}=$ Using mental math knowledge we know the LCD is 24. I will set this problem up in vertical form BUT realize in higher level math classes starting with Algebra all fraction problems (Rational Expressions) will always be written in horizontal form like

the remaining problems, and I will solve this problem in both vertical form and horizontal form.

$$\begin{array}{r} \mathbf{\frac{3}{8}} = \frac{?}{24} = +\frac{9}{24} \\ + \mathbf{\frac{7}{12}} = \frac{?}{24} = +\frac{14}{24} \\ - \mathbf{\frac{1}{6}} = \frac{?}{24} = -\frac{4}{24} \\ \hline \end{array}$$

$$Referrence\ the\ numerators: 9 + 14 - 4 = 19$$
$$and\ therefore\ the\ answer\ is\ "\frac{19}{24}".$$

Comment: My guess is, you tell your students, in order to convert the first fraction “$\frac{3}{8}$” into “$\frac{9}{24}$”, divide the denominator “8” into the common denominator “24” get the quotient “3,”and then multiply the numerator of the given fraction “3” by the quotient, “3” and you get the equivalent fraction “$\frac{9}{24}$". This is a common way to explain to students how to find the equivalent fraction, but it is a simple trick which works because of a powerful theorem which will be discussed later. (I refer to it as a trick, but it is based on sound mathematics logic.)

Teachers then tell the students to divide “24” by “12” and get “2” and multiply “7” by “2” and get the second numerator “14”. You then expect the students to complete the conversion trick with the numbers “6”, “24” and”1” to get the third numerator “4”. This procedure is an excellent trick which works for simple arithmetic problems, but few students can apply this trick in higher level math classes to solve more challenging fraction problems.

The divide and multiply trick used to find the new numerator with the correct LCD seems to work for simple arithmetic problems in arithmetic, BUT when students arrive in an Algebra 2 class, they have no idea how to find equivalent fractions with the new LCD. Finding equivalent fractions is the second problem students encounter in Algebra 2 with rational expressions, but the first problem still is, “What are the rules to find the LCD?”

Nothing I did in this first example problem will help students simplify or work with fractions in Algebra, because no one ever taught the students how to solve fraction problems in arithmetic using the Fundamental Theorem of Arithmetic and Factor Trees. Here is the same problem written in horizontal format with explanations using rules indicated and simplified and then the entire problem written as a teacher would write the problem on the whiteboard. $\frac{3}{8} + \frac{7}{12} - \frac{1}{6}$

Step 1 is to find the **Least** Common Denominator for the three denominators “8”, “12”, and “6”. One could multiply all three denominators together and get the common denominator “576” but this is not the **Least/Smallest** Common Denominator. This denominator “576” will work BUT, using this denominator which is the product of all the denominators will create an avalanche of unnecessary problems in arithmetic, and it will make most algebra rational expressions almost impossible to simplify/solve. The Least Common Denominator (LCD) is “24”. Here are the steps/procedures required to find an

LCD using Factor Trees for all addition and subtraction problems in arithmetic and in algebra. **1.** Set up factor trees for all three denominators and the key will be discovered.

8	12	6
2×4	2×6	$\mathbf{2} \times \mathbf{3}$
$\mathbf{2} \times \mathbf{2} \times \mathbf{2}$	$\mathbf{2} \times \mathbf{2} \times \mathbf{3}$	
$\{\mathbf{2^3}\}$	$\{\mathbf{2^2} \times \mathbf{3}\}$	$\{\mathbf{2} \times \mathbf{3}\}$

Once we have all the original denominators written as the product of prime factors, we can determine the LCD for any group of numbers or algebra expressions. **2.** Write all the **different** factors (in all arithmetic example problems the factors will always be prime numbers) as the product of the different prime numbers. Therefore, initially I will write all the different factors as the product "$\mathbf{2} \times \mathbf{3}$". This could be the Least common denominator in some situations, but it is not the Least Common Denominator (LCD) for this problem. If the goal is to find the LCD, go back to the three sets of factor trees indicated above and determine the MOST times each factor (prime number) occurs in anyone set. The prime number "2" occurred **three** times in the first factor tree set, two times in the second factor tree set, and one time in the third factor tree set. The prime number "3" occurred once in the second set of prime factors and once in the third set of prime factors. **3.** The rule is to multiply all the prime numbers as indicated above, but to find the LCD write a multiplication problem with each prime factor as the product using the prime number the **MOST** times it appears in any one set of factors. Therefore, the number "2" will be listed three times in the multiplication process to find the LCD, and the number "3" will only be used once in the multiplication process to find the LCD. The LCD will therefore equal: "$2 \times 2 \times 2 \times 3 = 2^3 \times 3 = 24$".

The process of writing all the "factor trees" for each denominator, then writing a multiplication problem involving all the different factors in the set of factors focusing on the most times each individual factor occurred in any one set of "factor trees" will work for all addition and subtraction problems in arithmetic and algebra. This is a rule, not a trick and when problems become more challenging the rule will always work and the mental math multiplication and division facts will not be efficient when problems involve larger numbers or algebra expressions. Wow.

Once the Least Common Denominator is determined it is time to find the three equivalent fractions using basic arithmetic rules which everyone should know. Everyone should realize that the following algebra expression "$a \times 1 = a$ or $1 \times a = a$" which is referred to as the Multiplication Identity Property indicating, when a number, term, or expression is multiplied by the number "1" the product will always equal the original number, term, or expression. This is not exactly what the expression "$a \times 1 = a$" really means. The expression needs to be expanded in your mind to an advanced level which many people do not consider. The algebra expression “$a \times 1 = a$" means much more than meets the eye. Consider these arithmetic/algebra statements: “$6 \times 1 = 6$ & $(5+1) \times 1 = 6$ & $1 \times (17-10-1) \times 1 = 6$ & $(x+y) \times 1 = (y+x), etc,$" What I am trying to illustrate is, the starting term and the ending term do not have to be identical, but they must be equal. This leads to more advanced mathematics’ facts which everyone must consider. Why do you think “$4\ ft. \times 1 = 48\ in.$” The left-side and right-side terms in the equation might not look identical but we know they are equal. This problem might be a great group discussion for students to think about, “How does the conversion “$4\ ft.$ convert to $48\ in.$" by multiplying by the number ONE”? Students may think about it for a whole period or more before they ask “AI” how this works or maybe it would be better to have a teacher explain the process using the fact that "$1\ ft. = 12\ in.$ and illustrate that “$\frac{12\ in.}{1\ ft.} = 1 = \frac{1\ ft.}{12\ in.}$" I am not going to explain the previous sentence , but it is a powerful statement that few elementary teachers are familiar with but, it is the arithmetic/mathematical reason why “$4\ ft.\ converts\ into\ 48\ in.$" and not only will the correct number appear in the final answer but the correct label will also appear. Solution Remember $\frac{ft.}{ft.} = 1$: $4\ ft. \times 1 = \frac{4\ ft.}{1} \times \frac{12\ in.}{1\ ft.} = \frac{4\ ft. \times 12\ in,}{1 \times 1\ ft.} = \frac{4 \cdot 1 \times 12\ in.}{1 \times 1 \cdot 1} = \frac{4 \times 12\ in.}{1} = \frac{48\ in.}{1} = 48\ in.$ Too often teachers teach tricks to solve arithmetic problems and students never learn the rules which allow the tricks to work.

Now, think about multiplying by the number “1” to find equivalent fractions. We should all agree that the following fractions all equal “1”, $\frac{2}{2} = 1$, $\frac{52}{52} = 1$, $\frac{2.7}{2.7} = 1$, & $\frac{x}{x} = 1$". This also means that the following equations are all facts: $\frac{7}{8} = \frac{7}{8} \times 1 = \frac{7}{8} \times \frac{2}{2} = \frac{7 \times 2}{8 \times 2} = \frac{14}{16}$, $\frac{7}{8} = \frac{7}{8} \times 1 = \frac{7}{8} \times \frac{52}{52} = \frac{7 \times 52}{8 \times 52} = \frac{364}{416}$, $\frac{7}{8} = \frac{7}{8} \times 1 = \frac{7}{8} \times \frac{2.7}{2.7} = \frac{7 \times 2.7}{8 \times 2.7} = \frac{18{,}9}{21.6}$, & $\frac{7}{8} = \frac{7}{8} \times 1 = \frac{7}{8} \times \frac{x}{x} = \frac{7 \times x}{8 \times x} = \frac{7x}{8x}$.

Finally, it is time to convert the three original fractions. “$\frac{3}{8} + \frac{7}{12} - \frac{1}{6}$” into equivalent fractions with the LCD “24”.

Divide the denominator of the first fraction “8” into the common denominator “24” to get the “3” which is the necessary clue, to determine the specific form of the number “$1 = \frac{3}{3}$" to multiply the first fraction “$\frac{3}{8} = \frac{3}{8} \times 1 = \frac{3}{8} \times \frac{3}{3} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$". Using the same process for the second fraction "$\frac{7}{12}$" and dividing the LCD by the denominator “12” we get the answer “2” which is the necessary clue, to determine the specific form of the “$1 = \frac{2}{2}$" to multiply the second fraction “$\frac{7}{12} = \frac{7}{12} \times 1 = \frac{7}{12} \times \frac{2}{2} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$“. Using the same process for the third fraction "$\frac{1}{6}$" and dividing the LCD by the denominator “6” we get the answer “4” which is the necessary clue, to determine the specific form of the

"$1 = \frac{4}{4}$" to multiply the second fraction " "$\frac{1}{6} = \frac{1}{6} \times 1 = \frac{1}{6} \times \frac{4}{4} = \frac{1 \times 4}{6 \times 4} = \frac{4}{24}$".

Now that you have had a chance to think about the procedures just illustrated to convert the three given fractions into three equivalent fractions with the LCD, you should realize why the trick, which many teachers tell students to use when they are finding equivalent fractions with the LCD works. The trick is a good trick when the fractions involve numbers which are multiples of the Natural Numbers from 1-12 but the trick does not seem to help students solve fractions in higher-level math classes.

Comment: Research indicates that two to three months after fractions are discussed in a typical arithmetic class, less than 50% of students remember the tricks/rules commonly illustrated when students are first learning how to work with all types of fractions. **I believe that teaching the rules and procedures using the Fundamental Theorem of Arithmetic with Factor Trees as a second method to work with all types of fractions will help all students understand the purpose of Factor Trees BUT more importantly the students will now have two different ways to simplify fractions in both arithmetic and advanced math classes. The first method is obviously using mental math knowledge of the multiplication and the division facts, and the second method is using Factor Trees and rules. I believe it is beneficial for students to learn two different methods to solve the same problem but realize one method is the only way to simplify/solve rational expressions in algebra.**

Here is the same problem written in the way a teacher may explain the entire process when they illustrate how to solve this problem on the whiteboard. Again, students may not always write all the steps indicated to simplify these problems, but I expect the teacher will always illustrate all the steps when they are explaining to the class how to solve these problems. The original problem written horizontally is written to the right. $\frac{3}{8} + \frac{7}{12} - \frac{1}{6}$

Find the LCD "24" as indicated above using Factor Trees. $\frac{9}{24} + \frac{14}{24} - \frac{4}{24}$

Convert the three given fractions into equivalent fractions with the LCD.

$$\frac{\mathbf{3}}{\mathbf{8}} \times \mathbf{1} = \frac{\mathbf{3}}{\mathbf{8}} \times \frac{\mathbf{3}}{\mathbf{3}} = \frac{\mathbf{9}}{\mathbf{24}} \quad \& \quad \frac{\mathbf{7}}{\mathbf{12}} \times \mathbf{1} = \frac{\mathbf{7}}{\mathbf{12}} \times \frac{\mathbf{2}}{\mathbf{2}} = \frac{\mathbf{14}}{\mathbf{24}} \quad \& \quad \frac{\mathbf{1}}{\mathbf{6}} \times \mathbf{1} = \frac{\mathbf{1}}{\mathbf{6}} \times \frac{\mathbf{4}}{\mathbf{4}} = \frac{\mathbf{4}}{\mathbf{24}}$$

$\frac{\mathbf{9+14-4}}{\mathbf{24}} = \frac{\mathbf{19}}{\mathbf{24}}$ Notice the one big fraction notation once all the fractions were written with the LCD "24". This one big fraction notation is helpful for students to remember the different rules for comparing, adding, and subtracting fractions versus reducing, multiplying, and dividing fractions. This condenses the rules down into two groups of rules not six different sets of rules.

Compare the magnitudes of the three fractions in the problem and the final answer. Arrange the fractions from the largest to smallest and use the correct math notation. Once all the fractions have the same denominator, the numerator is the focus point necessary to determine the magnitude of the fractions. Therefore the 4 fractions arranged from largest to smallest is $\frac{19}{24} > \frac{14}{24} > \frac{9}{24} > \frac{4}{24}$ or $\frac{19}{24} \geq \frac{7}{12} \geq \frac{3}{8} \geq \frac{1}{6}$.

$b.$ $\frac{7}{15} + \frac{9}{20} - \frac{3}{10}$ Using mental math, most students should realize the LCD equals "60".

Using Factor Trees 15 20 10

3×5 $2 \times 2 \times 5$ 2×5

These are the Only Prime Factors to Consider: $2 \times 3 \times 5$

But the LCD $= 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5 = 60$

$\frac{28}{60} + \frac{27}{60} - \frac{18}{60}$ Convert the three given fractions into equivalent fractions with the LCD'

$$\frac{\mathbf{7}}{\mathbf{15}} \times \mathbf{1} = \frac{\mathbf{7}}{\mathbf{15}} \times \frac{\mathbf{4}}{\mathbf{4}} = \frac{\mathbf{28}}{\mathbf{60}} \quad \& \quad \frac{\mathbf{9}}{\mathbf{20}} \times \mathbf{1} = \frac{\mathbf{9}}{\mathbf{20}} \times \frac{\mathbf{3}}{\mathbf{3}} = \frac{\mathbf{27}}{\mathbf{60}} \quad \& \quad \frac{\mathbf{3}}{\mathbf{10}} \times \mathbf{1} = \frac{\mathbf{3}}{\mathbf{10}} \times \frac{\mathbf{6}}{\mathbf{6}} = \frac{\mathbf{18}}{\mathbf{60}}$$

$\frac{\mathbf{28+27-18}}{\mathbf{60}} = \frac{\mathbf{37}}{\mathbf{60}}$ Notice, the one big fraction notation is used once all the fractions were written with the LCD "60". I think this is a powerful teaching point for teachers to consider teaching, when multiplying, dividing, and reducing fractions.

Compare the magnitudes of the three fractions in the problem and the final answer. Arrange the factions from the smallest to the largest and use the correct math notation. Once all the fractions have the same denominator the numerators are the only numbers necessary to compare the magnitude of the fractions. Therefore the 4 fractions arranged from smallest to largest is $\frac{18}{60} < \frac{27}{60} < \frac{28}{60} < \frac{37}{60}$ or $\frac{3}{10} \leq \frac{9}{20} \leq \frac{7}{15} \leq \frac{37}{60}$.

Now, Add and Subtract two challenging arithmetic problems using the Fundamental Theorem of Arithmetic and Factor Trees. Compare the magnitudes of all the fractions involved.

$a.$ $\frac{7}{54} + \frac{7}{72} - \frac{3}{88}$ Using Factor Trees 54 72 88

2×27 2×36 2×44

$2 \times 3 \times 9$ $2 \times 2 \times 18$ $2 \times 2 \times 22$

$\mathbf{2 \times 3 \times 3 \times 3}$ $2 \times 2 \times 2 \times 9$ $\mathbf{2 \times 2 \times 2 \times 11}$

$\mathbf{2 \times 2 \times 2 \times 3 \times 3}$

$\{2 \times 3^3\}$ $\{2^3 \times 3^2\}$ $\{2^3 \times 11\}$

These are the Only Prime Factors to Consider: "$2 \times 3 \times 11$"

But the $LCD = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 11 = 2^3 \times 3^3 \times 11 = 2376$

$\frac{308}{2376} + \frac{231}{2376} - \frac{81}{2376}$ Convert the three given fractions into equivalent fractions with the LCD "2376".

$$\frac{\mathbf{7}}{\mathbf{54}} \times \mathbf{1} = \frac{\mathbf{7}}{\mathbf{54}} \times \frac{\mathbf{44}}{\mathbf{44}} = \frac{\mathbf{308}}{\mathbf{2376}} \quad \& \quad \frac{\mathbf{7}}{\mathbf{72}} \times \mathbf{1} = \frac{\mathbf{7}}{\mathbf{72}} \times \frac{\mathbf{33}}{\mathbf{33}} = \frac{\mathbf{231}}{\mathbf{2376}} \quad \& \quad \frac{\mathbf{3}}{\mathbf{88}} \times \mathbf{1} = \frac{\mathbf{3}}{\mathbf{88}} \times \frac{\mathbf{27}}{\mathbf{27}} = \frac{\mathbf{81}}{\mathbf{2376}}$$

$\frac{\mathbf{308+231-81}}{\mathbf{2376}} = \frac{\mathbf{458}}{\mathbf{2376}} = \frac{\mathbf{2 \times 229}}{\mathbf{2 \times 1188}} = \frac{\mathbf{1 \times 229}}{\mathbf{1 \times 1188}} = \frac{\mathbf{229}}{\mathbf{1188}}$ When I reduced the fraction "$\frac{\mathbf{458}}{\mathbf{2376}}$" there were only three prime numbers I needed to consider "2, 3, & 11". Can you explain why I did not have to consider numbers like "5, 7, 13, …" to reduce the fraction?

Arrange the 4 fractions from smallest to largest. The numerators with the same LCD are the keys to arranging the fractions from the smallest to the largest.

$\frac{81}{2376} < \frac{231}{2376} < \frac{308}{2376} < \frac{458}{2376}$ or $\frac{3}{88} \le \frac{7}{72} \le \frac{7}{54} \le \frac{229}{1188}$.

$b.$ $\frac{11}{55} + \frac{17}{75} - \frac{3}{95}$ Factors Trees: "$55 = 5 \times 11$ & $75 = 3 \times 5 \times 5$ & $95 = 5 \times 19$"

This is NOT the LCD, but it might be "$3 \times 5 \times 11 \times 19$".

The LCD $= 3 \times 5 \times 5 \times 11 \times 19 = 3 \times 5^2 \times 11 \times 19 = 15675$

Remember, to find the three equivalent fractions, divide the LCD 15675 by each of the three denominators to determine the special fraction equal to the number "1" to multiply each of the given fractions.

$\mathbf{\frac{11}{55} \times 1 = \frac{11}{55} \times \frac{285}{285} = \frac{3135}{15675}}$ & $\mathbf{\frac{17}{75} \times 1 = \frac{17}{75} \times \frac{209}{209} = \frac{3553}{15675}}$ & $\mathbf{\frac{3}{95} \times 1 = \frac{3}{95} \times \frac{165}{165} = \frac{495}{15675}}$

$\frac{3135}{15675} + \frac{3553}{15675} - \frac{495}{15675} = \frac{3135 + 3553 - 495}{15675} = \frac{6193}{15675} = \frac{11 \times 563}{11 \times 1425} = \frac{1 \times 563}{1 \times 1425} = \frac{563}{1425}$ Why did I think the prime number "11" might be a good number to consider as a factor of both numbers? **Notice**, the extra work required because I did not reduce the fraction "$\mathbf{\frac{11}{55}}$".

Arrange the 4 fractions from largest to smallest. The numerators with the same LCD are the keys to arranging the fractions from the largest to the smallest.

$\frac{6193}{15675} \ge \frac{3553}{15675} \ge \frac{3135}{15675} \ge \frac{495}{15675}$ or $\frac{563}{1425} > \frac{17}{75} > \frac{11}{55} > \frac{3}{95}$ or $\frac{563}{1425} > \frac{17}{75} > \frac{1}{5} > \frac{3}{95}$.

2. Addition and Subtraction of one relatively easy rational expression and two semi-challenging rational expressions. Why am I unable to determine which fractions are larger or smaller?

$a.$ $\frac{6}{x^3yz^2} + \frac{5a}{wxyz} - \frac{4w}{y^2z}$ Write the three denominators written as factors to determine the LCD. $x^3yz^2 = xxxyzz$ & $wxyz = wxyz$ & $y^2z = yyz$

These are the only Factors to consider when finding the LCD $wxyz$ or $w \cdot x \cdot y \cdot z$

And the LCD $= w \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z = wx^3y^2z^2$

Now, convert the three given fractions into equivalent fractions with the LCD. The first step in all three situations is to divide the LCD "$w \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z$" by the denominator of the first fraction and then the denominator of the second fraction and then the denominator of the third fraction. In all three situations the quotient (answer) is the key to determine the value of "1" to multiply each fraction.

$Fraction\ 1 - \frac{w \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z}{x \cdot x \cdot x \cdot y \cdot z \cdot z} = wy$ $\therefore$ $\frac{wy}{wy} = 1$ $Convert$ $\frac{6}{x^3yz^2} \times 1 = \frac{6}{x^3yz^2} \times \frac{wy}{wy}$

$\frac{6 \times wy}{xxxyzz \times wy} = \mathbf{\frac{6wy}{wx^3y^2z^2}}$

$Fraction\ 2 - \frac{w \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z}{wxyz} = xxyz = x^2yz \ \therefore\ \frac{xxyz}{xxyz} = 1\ Convert\ \frac{5a}{wxyz} \times 1 =$
$\frac{5a}{wxyz} \times \frac{xxyz}{xxyz} = \frac{5a\ \times\ xxyz}{wxyz\ \times\ xxyz} = \frac{5ax^2yz}{wx^3y^2z^2}$

$Fraction\ 3 - \frac{w \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z \cdot z}{y^2z} = wxxxz = wx^3z \ \therefore\ \frac{wxxxz}{wxxxz} = 1\ Convert\ \frac{4w}{y^2z} \times 1 =$
$\frac{4w}{yyz} \times \frac{wxxxz}{wxxxz} = \frac{4w\ \times\ wxxxz}{yyz\ \times\ wxxxz} = \boldsymbol{\frac{4w^2x^3z}{wx^3y^2z^2}}$
$\frac{6}{x^3yz^2} + \frac{5a}{wxyz} - \frac{4w}{y^2z} = \frac{6wy}{wx^3y^2z^2} + \frac{5ax^2yz}{wx^3y^2z^2} - \boldsymbol{\frac{4w^2x^3z}{wx^3y^2z^2}} = \frac{6wy + 5ax^2yz - 4w^2x^3z}{wx^3y^2z^2}$ Rules are great!

I am unable to compare the magnitudes of the fractions because I do not know the values of all the variables. This same comment applies to the final two example problems.

$b.\ \frac{(x-1)}{xy} + \frac{(2x-3)}{xy} - \frac{x}{xy}$ An Easy Problem Ans. $\frac{(x-1)+(2x-3)-x}{xy} = \frac{x-1+2x-3-x}{xy} = \frac{2x-4}{xy} = \frac{2(x-2)}{xy}$
The denominators are identical so simply add and subtract the numerators.

$c.\ \frac{x}{x^2-5x+6} + \frac{2x}{x^2-6x+9} - \frac{2}{x^2-9} = \frac{x}{(x-2)(x-3)} + \frac{2x}{(x-3)(x-3)} - \frac{2}{(x-3)(x+3)}$ Normally, in an algebra 2 class you would factor all the numerators and all the denominators in the second step. I only had to factor the denominators in the second step because there is nothing to factor in the numerators. Also, algebra rational expressions (Algebra Fractions) are solved by writing the solutions from left to right as indicated in this step and as indicated in the last step in problems "a." and "b" above.

Now, we must determine the LCD for the three fractions and multiply all the factors already indicated in the three factored denominators.
This might be the LCD "$(x-2)(x-3)(x+3)$". I multiplied all the different factors But, it is **Not** the LCD.
This is the correct LCD: $(x-3)(x-3)(x-2)(x+3) = (x-2)(x-3)^2(x+3)$.

Next, we must divide the LCD by each denominator to determine the fraction equal to "1" required to multiply each of the three fractions by a special fraction equal to "1".
$Fraction\ 1 - \frac{LCD}{Denom} = \frac{(x-3)(x-3)(x-2)(x+3)}{(x-2)(x-3)} = (x-3)(x+3) \therefore \frac{(x-3)(x+3)}{(x-3)(x+3)} = 1.$
Find the equivalent fraction with the LCD - $\frac{x}{x^2-5x+6} \times 1 = \frac{x}{x^2-5x+6} \times \frac{(x-3)(x+3)}{(x-3)(x+3)} =$
$\frac{x\ \times\ (x-3)(x+3)}{(x-2)(x-3) \times (x-3)(x+3)} = \frac{x \times (x^2-9)}{(x-3)(x-3)(x-2)(x+3)} = \boldsymbol{\frac{x^3-9x}{(x-3)(x-3)(x-2)(x+3)}}.$
$Fraction\ 2 - \frac{LCD}{Denom} = \frac{(x-3)(x-3)(x-2)(x+3)}{(x-3)(x-3)} = (x-2)(x+3) \therefore \frac{(x-2)(x+3)}{(x-2)(x+3)} = 1.$
Find the equivalent fraction with the LCD $-\ \frac{2x}{x^2-6x+9} \times 1 = \frac{2x}{x^2-6x+9} \times \frac{(x-2)(x+3)}{(x-2)(x+3)} =$
$\frac{2x\ \times\ (x-2)(x+3)}{(x-3)(x-3) \times (x-2)(x+3)} = \frac{2x \times (x^2+x-6)}{(x-3)(x-3)(x-2)(x+3)} = \frac{2x^3+2x^2-12x}{(x-3)(x-3)(x-2)(x+3)}.$

$Fraction\ 3 - \frac{LCD}{Denom} = \frac{(x-3)(x-3)(x-2)(x+3)}{(x-3)(x+3)} = (x-3)(x-2) \therefore \frac{(x-3)(x-2)}{(x-3)(x-2)} = 1.$

Find the equivalent fraction with the LCD $\frac{2}{x^2-9} \times 1 = \frac{2}{x^2-9} \times \frac{(x-3)(x-2)}{(x-3)(x-2)} =$

$\frac{2\ \times\ (x-3)(x-2)}{(x-3)(x+3)\times(x-3)(x-2)} = \frac{2\times(x^2-5x+6)}{(x-3)(x-3)(x-2)(x+3)} = \frac{\mathbf{2x^2-10x+12}}{\mathbf{(x-3)(x-3)(x-2)(x+3)}}.$

$\frac{x}{x^2-5x+6} + \frac{2x}{x^2-6x+9} - \frac{2}{x^2-9} = \frac{\mathbf{x^3-9x}}{\mathbf{(x-3)(x-3)(x-2)(x+3)}} + \frac{2x^3+2x^2-12x}{(x-3)(x-3)(x-2)(x+3)} - \frac{2x^2-10x+12}{(x-3)(x-3)(x-2)(x+3)} =$

$\frac{(x^3-9x)\ +(2x^3+2x^2-12x)-\ (2x^2-10x+12)}{(x-3)(x-3)(x-2)(x+3)} = \frac{x^3-9x\ +2x^3+2x^2-12x\ -\ 2x^2+10x-12}{(x-3)(x-3)(x-2)(x+3)} =$

$\frac{\mathbf{3x^3-11x-12}}{\mathbf{(x-3)(x-3)(x-2)((x+3)}}$ The final expression is normally written with the numerator and the denominator in factored form to determine if there are any identical factors in both the numerator and the denominator. The numerator in this answer is not factored because there is no simple way to factor the polynomial "$x^3 - 11x - 12$".

NOW THAT IS A DOUBLE $WOW\ WOW$.

I hope you agree that adding and subtracting fractions is much more challenging than reducing, multiplying, and dividing fractions. It might not seem as obvious when students are required to solve simple two term fraction problems in arithmetic which only require the students to master the multiplication and division facts for the numbers from 1-10 or 1-12, but when students are required to solve more challenging fractions and then rational expressions in algebra, it becomes obvious that the addition and subtraction fraction problems are much more challenging than the multiplication and division fraction problems.

Algebra 2 is a challenging class for many students to experience success. The topics, in Algebra 2 not only consist of rational expressions as illustrated, but other topics could involve solving systems of 3 or more simultaneous equations, mastering all facets of quadratic equations, being able to graph and interpret a wide variety of graphs, deriving the quadratic formula and mastering how to use the quadratic formula, understanding the Fundamental Theorem of Algebra, and sometimes working with logarithms and conic sections.

Going from an Arithmetic Class to an Algebra 1 class is like going from a hot air balloon to a World War 2 Mustang, but going from an Algebra 1 class to an Algebra 2 class is like going from the Mustang to a modern fighter Jet. In Algebra 2 there are so many different topics which students must master and be able to apply all the rules and procedures which were taught in all previous math classes. Once a student fully understands all the topics mentioned in an Advanced Algebra 2 class, they are ready to take off in a Rocket to continue with higher level math classes. You can only ride on the Rocket into higher level math classes if you understand the concepts then "Learn and Remember" all the rules and procedures from all your previous math classes and then the height of your math skyscraper may grow to infinity. Higher & Higher

Conclusions/Results: Memorandum Regarding the link between arithmetic fractions and algebra rational expressions, is it The Fundamental Theorem of Arithmetic? I THINK IT MIGHT BE A BIG LINK TO HELP ALL STUDENTS.

I was a high school principal for a total of 10 years. In October 1999 I wrecked my motorcycle and spent 28 days in the hospital, four months in a wheelchair, and over two years in therapy. I went back to school at Chartiers Valley High School in Bridgeville PA in a wheelchair, less than three months after the accident and then crutches and a cane for almost two years. The Chartiers Valley District is located less than 8 miles southwest of downtown Pittsburgh. When I was the Principal, I considered myself to be the school leader, and I did not sit in my office all day and wait for teachers, parents, school administrators and problems to come to my office. I would walk through the halls for at least 10 minutes twice in the morning and twice in the afternoon and I visited the cafeteria at least three times a week.

Since I was still in therapy and still struggling to get around the hallways and attending therapy after school, I asked the school board if I could return to the math classroom when one of my experienced math teachers decided to retire at the end of the school year. The school board owed me nothing and there was nothing in my contract which guaranteed that my request should be honored. The school board allowed me to return to the classroom, but according to the union contract I was considered a six-year employee at Chartiers Valley but for pay purposes I was given credit for all my years of experience in Pennsylvania by the school board, 21 years of teaching at Tyrone Are School District and 4 years as a principal at Glendale School District, therefore I was placed at the top step of the salary scale. The significance of the union issue was, any teacher with at least 16 years in the district had the right to select their schedule for the next year in late May based on the schedules that the principle had planned for the following year. For pay purposes I was a senior teacher, but I was not permitted to select my teaching schedule.

When I returned to the classroom the school board allowed me to have 5 extra duties which upped my salary significantly, so I was making slightly less as a teacher than I was as the principal. One of my extra duties was, I would be the math department head. This was significant because one of my duties as the department head was to assist the principal make up teaching schedules for all the classes for all the 11 teachers in the district. This meant in late May, 5 of the 11 senior math teachers in the high school would pick their schedule out of the 11 schedules which were posted for the next school year. When I worked with the principal to make up the schedules with only the classes indicated, not times or periods, I frequently made one schedule which had 2 of the lowest level 9-12 grade students, mostly special needs students, 2 average level Geometry classes, and 2 of the highest level Precalculus classes. The 5 teachers who were permitted to select their schedules always wanted the two Precalculus classes, BUT they did not want the two low level prealgebra classes. Since I was the department head, I was allowed to select the remaining schedule after the first 5 teachers picked their schedule and I always picked the schedule with the two underachieving math students and the two honors Precalculus classes.

I had been a teacher for 21 years prior to becoming a principal and I was the department head in my first district for 16 of my first 21 years. As the department head for the 16 years I always felt if I was going to be a math leader I needed to teach all the math classes offered at Tyrone Junior Senior High

School, so I made sure I taught every class from the lowest level 7th graders who still needed to take a basic arithmetic course up through the advanced seniors in Calculus.

I had an idea what I was getting myself into at Chartiers Valley High School when I returned to the classroom and I looked forward to working with the best students who were exceptional math students and wanted to get 800 on their math SAT's and the low achieving students in Pre-Algebra grades 9-12 who would tell you on day 1, "We do not do any homework!" which I also realized they meant "We do not do any classwork!"

Before I retired 7 years later with 38 years of experience, I was so proud of the improvements that my honors students achieved and many of them did get 800 on their math SAT's or 750 or 700. Wow, that was a great experience getting these students ready for calculus and future advanced math courses. My goal with the honors students was to have them improve by a grade and a half when they were in my honors classes. I was always pleased with the progress these mathletes experienced, and I was pleased with the progress that my academic students made in combined geometry BUT my GREATEST satisfaction came from the improvements demonstrated by over 90% of my low achieving mostly special needs students at the end of the year in my prealgebra classes. My goal when working with these students was to have them grow by at least two grade levels and anything more than this was a bonus. I still get a tear in my eye thinking about the joy the low-achieving students demonstrated and the outward respect and even love they showed me during the year and especially when I saw them after I retired. Too many stories to tell but I will mention three short stories at the end of this memorandum.

What did I do differently in my last couple years to achieve success with my mathematically challenged students?

I will start by saying I emphasized simplifying/solving all types of arithmetic fraction problems from the simplest problems normally found in typical U.S. arithmetic books to very challenging arithmetic fraction problems which can only be solved/simplified if one uses the Fundamental Theorem of Arithmetic. Everyone who teaches arithmetic or high school mathematics should know that according to research the number one (1) arithmetic topic taught in 5th grade which predicts success in higher level math classes is a strong understanding of the methods/procedures to work with fractions, not necessarily hard fractions, just your basic fractions contained in typical arithmetic books in the U.S. Incidentally, the 2nd 5th grade arithmetic topic which predicts success in higher level math classes is a strong understanding of the procedures to solve long division problems. Therefore, my main focus in my prealgebra classes was teaching my students how to solve all types of fraction problems starting on the first day of school and probably 95 percent of the days in the school year at some point I talked about solving/simplifying fraction problems and during the fourth 9 weeks I focused on simplifying some rational expressions which were already written in factored form.

Here are some of the steps I used to teach fractions, and I always emphasized the Fundamental Theorem of Arithmetic with Factor Trees.

1. A. I emphasized learning how to solve all types of fraction problems every day. Sometimes as a 5-minute warm-up or review and sometimes as the main topic which I discussed for 35-40 minutes and many times as part of a quiz.

B. I started on day 2 by discussing how to reduce, multiply, and divide fraction problems using one method to simplify all three types of problems with one additional step when a problem involved dividing a fraction. This additional step for division is obviously, to emphasize when a fraction problem contains a division symbol "÷" then the first step is to convert the division operation symbol "÷ " into a multiplication operation symbol "×" and invert the fraction to the right of the original division symbol "÷" which is now a multiplication "×" symbol. Eventually, usually in the 4th 9 weeks I explained why this switch from a division problem to a multiplication problem worked, but initially I only wanted them to realize that his problem is now treated the same as a reducing or multiplication problem. One basic rule for three different operations when working with fractions. This was the goal for the first 9 weeks. Question, "Can you illustrate/prove why the rule when dividing fractions is to convert the problem into a multiplication problem and invert the fraction immediately to the right of the division symbol "÷"?" You should know why this rule works and explain it to all your high school students.

When working with all three types of fraction problems during the first nine weeks, every problem they simplified required them to use the Fundamental Theorem of Arithmetic and write the "Factor Trees" for every numerator and every denominator and write two or three or four fractions as one big fraction with the numerators and denominators in factored form. They were taught what a factor was and if there was an identical factor in the numerator and denominator then these two identical factors represented a simple division problem because everyone knows that a number or term divided by the same identical number or term always equals ONE (1). When writing the factor trees for every numerator and denominator the student must write the sets of factor trees off to the right side of the problem. I seldom gave example problems which involved only two fractions, usually, my problems involved 3, 4, or 5 fractions except when the one given fraction was supposed to be reduced.

C. 1a. During the 2nd and 3rd nine weeks I started to emphasize the procedures to add and subtract easy and challenging arithmetic problems using the Fundamental Theorem of Arithmetic and Factor Trees. Step one in this challenging process was to have the students learn how to multiply a given fraction by the number one "1", since the goal is to find an equivalent fraction with the LCD. I am talking about multiplying the given fraction by other fractions written in this form: $\frac{3}{3} = 1, \frac{5}{5} = 1, \frac{11}{11} = 1, \frac{7.2}{7.2} = 1, \frac{a}{a} = 1, etc.$

C. 1b. Next, I emphasize how to find Least Common Denominators for all types of fractions using the Fundamental Theorem of Arithmetic and Factor Trees. Students must write the Factor Trees for all the Denominators off to the right side of the paper. Set up a multiplication problem involving all the different factors, prime numbers, then determine if any of the prime numbers ever occurred more than once in any one set of "Factor Trees". If a prime number occurred in more than one set of factor trees and in one situation the prime number occurred 2 times and in another situation the prime number was a factor three different times and in another denominator the prime number was a factor four times then write the multiplication problem involving the prime numbers with the most times the prime factor ever appears in one of the factor trees, so in the situation mentioned above you would list the prime factor four times when multiplying all the prime factors to determine the LCD.

C. 1c. Once you have the least common denominator you must find equivalent fractions using the LCD for all the fractions which are added or subtracted. Once again, most of my problems involved 3, 4, or 5 different fractions which means every fraction needs to be converted into an equivalent fraction with the LCD. This procedure requires the students to write fractions in the following form: “ $\frac{LCD}{Denom.\ of\ the\ first\ fraction} = Quotient, \frac{LCD}{Denom.of\ the\ second\ fraction} = Quotient, \frac{LCD}{Denom.of\ the\ 3rd\ fractioon} = Quotient, etc.$” for every fraction which must be converted into an equivalent fraction with the LCD.

C. 1d. The quotients found in the previous sentence are used to determine the factions equal to ONE “1” which are used to multiply and find the equivalent fractions with the LCD. If the quotient in one of the situations equals “3” then multiply this fraction by “$\frac{3}{3} = 1$” but if the quotient equals “13” then multiply this fraction by "$\frac{13}{13} = 1$”. The quotients found in the step above is the key to determine the special fraction equal to ONE “1” to multiply each fraction. If the procedure is done properly then the denominator of all the equivalent fractions will always be the LCD.

C. 1e. The problem can now be written using all the fractions just determined as an addition or subtraction problem with 2, 3, 4 or more fractions. The combination of all the fractions can now be written as one big fraction with all the numerators added or subtracted and the LCD written once in the denominator.

C. 1f. Now, add and subtract all the numerators and write the answer with the LCD. This may or may not be the final answer because you may have an improper fraction or you may have a fraction which should be reduced.

D. Think about what I did the first nine weeks when I emphasized reducing, multiplying and dividing fractions and then what I did for the 2nd and 3rd nine weeks. I submit to you that the rules and procedures to add and subtract 2, 3, 4, 5 or more fractions are much more challenging for students to master than the rules for multiplying and dividing 2, 3, 4, 5 or more fractions.

2. Now the biggest teaching trick I used to get students to “Learn and Remember” all the steps and procedures to solve all types of fraction simplification/solving problems was - I gave quizzes 2, 3, and sometimes 4 times per week. Remember many of these students will not do homework and they will not do basic classwork, BUT if they know they are taking a quiz for a grade they will take the quiz. In my mind I considered the quizzes to be classwork, but I had to convince the students that these quizzes were going to greatly influence their nine-week grades and the final yearly grade. The quizzes at the beginning of the year usually consisted of 10 relatively simple questions and 5 of the questions were review questions which the students

could quickly solve without much help. I was always available to help the students solve the other 5 questions because my goal was to ensure that the students would earn a grade of 7, 8, 9 or 10 and sometimes 11 if I gave a bonus question on my quizzes. I would write down the steps to solve 2, 3, or 4 of the last 5 questions, but the students had to rewrite the solutions on their quiz paper. I wanted to make sure the students wrote all the steps for all the problems in their handwriting. I would go over the quizzes the next day in class and encouraged the students to make sure they knew how to solve the problems because 2 or 3 of the questions I put on the quiz yesterday may appear in the quiz today or tomorrow.

Eventually, the students started to realize that they were earning a “C”, or “B” or an “A” in many cases. They were getting confident solving the review problems and they also knew if they had a problem that I would help them. A second trick I used when students were talking too much was to simply say, I am not going to help anyone get the correct answers to the problems today because everyone seems to know what they are doing and talking too much. Three fourths of the class would always respond quickly, we need some help “BE QUITE!” The students started to police themselves and sometimes later in the year I would let two students help each other to solve one problem only, but they had to ask me before they could ask someone for help.

The students became so good at solving the first 5 review problems that I increased the number of questions on the quizzes to 15 questions, and the first 10 questions were review questions and the remaining 5 questions focused on what I considered the topic for the week.

The students who entered my classroom in late August and hated math and would not do math, started to feel like they earned a “C” or “B” or an “A”- and they knew if they showed the work and wrote all the steps, they were going to get partial credit even if they made a minor mistake.

3. Since this was a prealgebra class I interleaved many prealgebra problems into the class discussion and quizzes starting the second 9 weeks. I introduced solving equations with a single variable; I introduced the students to graphing on a number line and then graphing on a Cartesian Coordinate System (I made sure the given points formed a straight line or something else recognizable.); I introduced word problems using well known formulas such as standard Area formulas and D=rt, etc. Even when working with these basic **prealgebra topics I included fractions** in my problems on the quizzes.

Simplify all Examples: a. $\frac{12}{30} = \frac{\mathbf{2\times2\times3}}{\mathbf{2\times3\times5}} = \frac{1\times2\times1}{1\times1\times5} = \frac{1\times2\times1}{1\times1\times5} = \frac{\mathbf{2}}{\mathbf{5}}$

b. $\frac{276}{354} = \frac{\mathbf{2\times2\times3\times23}}{\mathbf{2\times3\times59}} = \frac{1\times2\times1\times23}{1\times1\times59} = \frac{2\times23}{59} = \frac{\mathbf{46}}{\mathbf{59}}$

c. $\frac{1}{4} \div \frac{5}{6} \times \frac{5}{3} \times \frac{2}{3} = \frac{1}{4} \times \frac{6}{5} \times \frac{5}{3} \times \frac{2}{3} = \frac{1}{2\cdot2} \times \frac{2\cdot3}{5} \times \frac{5}{3} \times \frac{2}{3} = \frac{\mathbf{1\cdot\ 2\times3\cdot5\cdot2}}{\mathbf{2\times2\cdot5\cdot3\cdot3}} = \frac{1\cdot1\cdot1\cdot1\cdot1}{1\cdot1\cdot1\cdot1\cdot3} = \frac{1}{3}$

d. $\frac{5}{16} \times \frac{7}{12} \div \frac{25}{36} = \frac{5}{16} \times \frac{7}{12} \times \frac{36}{25} = \frac{5}{2\times2\times2\times2} \times \frac{7}{2\times2\times3} \times \frac{2\times2\times3\times3}{5\times5} = \frac{\mathbf{5\ \cdot\ 7\ \cdot\ 2\times2\times3\times3}}{\mathbf{2\times2\times2\times2\ \cdot\ 2\times2\times3\ \cdot 5\times5}} =$
$\frac{1\times7\times1\times1\times1\times3}{1\times1\times2\times2\times2\times2\times1\times1\times5} = \frac{7\times3}{2\times2\times2\times2\times5} = \frac{\mathbf{21}}{\mathbf{80}}$

e. $\frac{15a^3x^2(x+5y)^3}{25a^2x^2(x+5y)^2} = \frac{\mathbf{3\cdot5\times a\cdot a\cdot a\times x\cdot x\times(x+5y)(x+5y)(x+5y)}}{\mathbf{5\cdot5\times a\cdot a\times x\cdot x\times(x+5y)(x+5y)}} = \frac{3\cdot1\cdot1\cdot1\cdot a\cdot1\cdot1\cdot1\cdot1(x+5y)}{1\cdot5\cdot1\cdot1\cdot1\cdot1\cdot1\cdot1} = \frac{\mathbf{3a(x+5y)}}{\mathbf{5}}$

Notice in all six examples prior to identifying the identical terms in the numerators and the denominators I made one big bold fraction and then I divided identical terms and indicated the number “1” for each term that I technically divided out. I also made my students write one big fraction for every addition and subtraction fraction problem as soon as all the fractions had a common denominator. I really like the one big fraction notation in both situations.

f. $\frac{4m^2}{(x+3)(x-3)} \times \frac{(x+2)(x-3)}{(x+2)^2} \div \frac{6m}{(x+3)} = \frac{4m^2}{(x+3)(x-3)} \times \frac{(x+2)(x-3)}{(x+2)^2} \times \frac{(x+3)}{6m} =$
$\frac{2 \cdot 2 \cdot m \cdot m}{(x+3)(x-3)} \times \frac{(x+2)(x-3)}{(x+2)(x+2)} \times \frac{(x+3)}{2 \cdot 3 \cdot m} = \frac{\mathbf{2 \cdot 2 \cdot m \cdot m} \times +(\boldsymbol{x}+\mathbf{2})(\boldsymbol{x}-\mathbf{3}) \times (\boldsymbol{x}+\mathbf{3})}{(\boldsymbol{x}+\mathbf{3})(\boldsymbol{x}-\mathbf{3}) \times (\boldsymbol{x}+\mathbf{2})(\boldsymbol{x}+\mathbf{2}) \times \mathbf{2 \cdot 3 \cdot m}} = \frac{1 \cdot 2 \cdot 1 \cdot m \times 1 \cdot 1 \times 1}{1 \cdot 1 \times (x+2) \cdot 1 \times 1 \cdot 3 \cdot 1} = \frac{2m}{3(x+2)}$

It was a lot of extra work on my part to make up the quizzes, grade the quizzes, and think up new topics for the next week. I only used the book to help me focus on different topics which the students should be able to work with by the end of the year and topics that the students would be working with when they entered an Algebra 1 class next year.

POSITIVES OUTCOMES WHICH OCCURRED BY THE END OF THE SCHOOL YEAR, SOME ARE OBVIOUS BUT OTHERS WERE NOT INITIALLY SO OBVIOUS.

These are all general comments about the 42/38 students who were in my prealgebra class my last year before my retirement. Four of the students did not finish the year with me because of various reasons.

1. Most of the students who entered my classroom in September did NOT know the multiplication facts for the numbers from 1-10. Every student improved significantly with respect to knowing their multiplication facts for the number from 1-10 by the end of the school year. They were forced every day to use "Factor Trees" and, in the process, they started to be extremely proficient with the multiplication facts involving the prime numbers, 2, 3, 5 and they all demonstrated improvement with the multiplication facts involving the numbers 4, 6, 8, & 10. They all showed improvement with the multiplication and division facts for all the numbers from 1-10 but not as much improvement with the multiplication and division facts for the numbers 7 and 9. I think they improved with their memorization of the multiplication and division facts because they subliminally were learning the facts when they were using the "Factor Trees" every day and not specifically required to memorize multiplication and division facts.
2. Since they were writing factor trees daily for all types of whole numbers, eventually, they were required to write the factored notation using exponents, 95% of the students knew that the number 8 was 2^3 and 25 was 5^2 and 27 was 3^3 and many other basic exponent notations.
3. The students concentrated on the denominator so much when they added and subtracted fractions that they even knew how to quickly simplify the following types of addition and subtraction fraction problems: a. $\frac{73}{x} + \frac{61}{x} - \frac{45}{x} + \frac{11}{x} = \frac{\mathbf{73+61-45+11}}{\boldsymbol{x}} = \frac{100}{x}$ and $\frac{2.1}{3} - \frac{5.2}{3} + \frac{6.1}{3} = \frac{\mathbf{2.1-5.2+6.1}}{\mathbf{3}} = \frac{3}{3} = 1$. I used to interleave/interweave a couple math topics into one problem.
4. The students were required to always make one big fraction whether it involved reducing, multiplying and dividing and then when they were adding and subtracting fractions. These two rules which were emphasized daily greatly simplified the procedures to work with fractions.
5. Students frequently said to me that this year was the first time they thought they knew what they were doing in arithmetic and they learned how to solve problems for the first time ever.
6. There was a special outcome which these students understood well, and many average students do not fully understand when they enter an algebra class. These students understood the definition of the word "factor" and the importance of factors when reducing fractions written with both the numerator(s) and denominator(s) written in factored form. These students knew

that only identical factors in both the numerator and the denominator could be divided and both turned into the number ONE (1) and then the numbers all equaling one “1” could be ignored when writing the final simplified answers.

There is one exception to the rule, ignore the number One (1) mentioned above in the last sentence. If the number “1” is the only number remaining in the numerator, then the NUMBER ONE will remain in the numerator as the only term or number.

To completely understand the last sentence written above the blue rectangle, go back and study the arithmetic fraction examples written in factored form and then the simplified algebra fraction (rational expressions) written in factored form which I included on my quizzes during the fourth nine weeks for students to simplify.

Three specific examples of student improvement.
1. Steve was a senior in my prealgebra class, and he wanted to join the military after graduation. After his junior year he took the ASVAB and scored lower than the 15th percentile on the math ASVAB TEST. After graduation Steve joined the PA National Guard which indicated to me that he had to score above either the 30th or 35th percentile to join the Army
2. Ashley was a 9th grader in my class when she got pregnant in April. I was paying my gas bill at the local Marathon Gas station about 6 years later when I turned around and quickly realized that Ashley was behind me and gave me a big hug. She said, “Because of you I passed Precalculus in College”, it was at Community College of Allegheny County (CCAC) but this is still an amazing accomplishment. When she gave me a hug, I noticed a young boy standing beside her and he seemed confused about why his mom would give an old man a big hug. I just told him he was the luckiest little boy I ever knew, and his mother was an amazing young lady.
3. About two years after I retired, I encountered a young lady who was now about 19. She was in my class for the first semester only because the school sent her to the alternative school since she seemed to always get in trouble. I did not see her for the rest of her junior year in high school, and I was quite shocked when she yelled at me when I parked my car near the bus stop at the end of the block. My wife and I were going to dinner at one of our favorite restaurants and the parking lot was full. She also came over to me as I was walking back to the restaurant and gave me a hug. She told me that I was the only math teacher she ever learned anything from. I was so honored by her comment, but I also had a lot of concern because she was dressed up like she was going to go to work. I have never seen her again, but I have always worried about her.

There are many other individual stories I could talk about regarding these students, but you get the point, I loved those kids and I felt that I had helped these kids improve significantly in that one year.

Every arithmetic and math teacher must at some point in their career realize that students do not and will not learn and remember how to solve math problems without practicing and without experiencing success. A student can sit and watch a teacher illustrate how to solve a challenging arithmetic problem, algebra problem, geometry problem, trigonometry problem, calculus problem - year after year like they are watching a movie or a television program, BUT watching a teacher explain how to solve a math problem will NOT translate into the students “Learning and Remembering” how to solve math problems now and in the future.