# PRESENTATION TITLE

**Should students be required to take an Advanced Arithmetic Class or an Advanced Prealgebra Class**

**Which focuses on using the**

**FUNDAMENTAL THEOREM OF ARITHMETIC**

**to solve all types of Fraction Problems prior to entering their first Algebra class?**

**Just something to think about!**

**Question One: If you are a certified mathematics teacher who teaches Algebra:**

**Do you know, teach, and emphasize the**

## FUNDAMENTAL THEOREM OF ALGEBRA?

**Do you think that your students should know this powerful Theorem, and when they might want to reference this Theorem when solving Algebra Polynomial Equations?**

**Question 2: If you are a Certified Math Teacher who teaches Calculus:**

**Do you know, teach, and emphasize the**

## FUNDAMENTAL THEOREM OF CALCULUS, BOTH PART 1 AND PART 2?

**Do you think your students should know this powerful two-part Theorem, and when they might want to reference this Theorem when working with Continuous Functions and finding the Antiderivative and then when Finding the Area under a Curve for the closed interval from [a, b]? Simply amazing and fun to teach.**

**Teaching concepts so students understand how to find the area under a curve always excited me, but I can be a nerd sometimes.**

**Question Three: If you are an Elementary Arithmetic teacher or a Certified Mathematics teacher who teaches Arithmetic when students are starting to work with or simplify fraction problems:**

**Do you know, teach, and emphasize the**

## FUNDAMENTAL THEOREM OF ARITHMETIC?

**Do you think your students should know this powerful Theorem, and when they might want to reference this Theorem when solving Challenging Fraction problems involving all integers from $\{1, 2, 3, \ldots\ 100\}$ not just numbers from $\{1, 2\ , 3, \ldots 10\}$?**

**Follow along, think and study the illustrations, for example problems.** **I tried to list every step in my solutions.**

**Use the Fundamental Theorem of Arithmetic to simplify fractions when reducing, multiplying and dividing fractions.**

**WHY wouldn't I start with Comparing, Adding, and Subtracting Fractions? Doesn't every elementary arithmetic book always begin with problems involving adding and subtracting fractions? YES**

**And every elementary arithmetic book usually focuses on adding and subtracting two fractions with identical denominators. Then they concentrate on two fractions which involve different denominators and all students should be able to determine the Least Common Denominator (LCD) using mental math multiplication skills and lastly convert all the fractions into equivalent fractions with the LCD.**

**This process may sound simple to you, but it is often very confusing to many students. If this is such a simple process, then explain how to simplify this simple fraction addition problem "$\frac{2}{3}+\frac{2}{x}$".**

**There is a lot to think about when you read and study the following slides.**

**Simple fractions which are added and subtracted to begin with:**

1. $\frac{3}{7}+\frac{2}{7}-\frac{1}{7}=\frac{3+2-1}{7}=\frac{4}{7}$ **(Easy, actually very easy.)**
2. $\frac{1}{3}+\frac{1}{2}-\frac{1}{4}=\frac{4}{12}+\frac{6}{12}-\frac{3}{12}=\frac{4+6-3}{12}=\frac{7}{12}$ **(Still supposed to be relatively easy but is it really?)** **Notice how I write one big fraction once all the fractions have the same LCD.**

**The Fundamental Theorem of Arithmetic might not be a focus point for these two example problems. Let's continue with a more challenging ARITHMETIC problem.**

**Consider the next ARITHMETIC addition and subtraction example problem:** $\frac{7}{60}+\frac{5}{27}-\frac{1}{72}=\frac{126+200-15}{1080}=\frac{311}{1080}$.

**CAN YOU SOLVE THIS PROBLEM WITHOUT A CALCULATOR? CAN YOU EXPLAIN TO YOUR STUDENTS HOW TO SOLVE THE PROBLEM WITHOUT using TRIAL AND ERROR AND WITHOUT A CALCULATOR? If not, why not? It is Arithmetic!**

**If I am correct the problem only involves numbers, so it is not an algebra problem. Why is it so difficult to solve without a calculator, and why would I want our students to be able to solve this type of problem, prior to entering their first Algebra class?**

**Most of you will need some guidance if you are going to teach your students how to solve the problem indicated. This is an arithmetic problem with numbers, students have little knowledge of how to solve this arithmetic problem, but you expect the algebra teacher to explain to students how to solve the algebra problem below, based on the way you taught your students in arithmetic:**

**EX:** $$\frac{5}{xy}+\frac{6z}{x^2}-\frac{2}{xy^2}=\frac{5xy+6zy^2-2x}{x^2y^2}$$

**Given the simplified expression, maybe now, you can understand and explain to others how to simplify this simple algebra problem.**

**You expect the algebra teacher to create miracles, but first they will have to go back and teach arithmetic and quite frankly there is little to NO time in the algebra curriculum to teach topics which should have been mastered prior to students entering any high school math class. Suggestion, use interleaving to introduce topics early and to regularly review topics in arithmetic. (Warm up exercises, additional homework problems, etc.).** Students must "Learn and Remember" how to use rules and procedures. I continually used interleaving problems when students entered my algebra class to teach students how to work with fractions.

**I hope everyone emphasizes concepts first, but understanding concepts is only the first step when solving math problems, NOT the final step when learning how to solve mathematics problems!!!!!**

**If students do not "LEARN AND REMEMBER" mathematics "RULES AND PROCEDURES" then solving advanced mathematics problems in high school and beyond, IS IMPOSSIBLE.**

# WHAT IS THE FUNDAMENTAL THEOREM OF ARITHMETIC?

**The Fundamental Theorem of Arithmetic stated in its simplest form is:**

**"Every composite number greater than 1 has a unique set (exactly one set) of PRIME FACTORS." usually referred to as Factor Trees in Arithmetic.**

**This seems simple enough, but the theorem becomes very powerful when one wants to add, subtract, reduce, multiply, and/or divide fractions with numbers greater than 10.**

**Basic rules for simplifying/solving BOTH fractions in arithmetic using Factor Trees (The Fundamental Theorem of Arithmetic) and Rational Expressions in Algebra!**

1. **A. When Reducing, Multiplying, and Dividing Fractions: COMPLETELY FACTOR all the numerators and denominators.** **It is assumed that you know how to invert the fraction to the right of the division operation symbol (÷) and change the division operation symbol into a multiplication operation symbol.** **(I EXPECT THAT ALL ARITHMETIC TEACHERS AND ALL MATH CERTIFIED TEACHERS CAN EXPLAIN TO ALL STUDENTS WHY THIS RULE WORKS. I WILL ILLUSTRATE WHY THIS RULE WORKS AT THE END OF THIS SLIDE PRESENTATION BUT I DOUBT THAT I WILL HAVE TIME TO EXPLAIN THE PROCEDURE IN THE TIME ALLOTTED.)**

   **B. Suggestion, once all the numerators and denominators are completely factored, write all the fractions as one fraction, not two fractions, or three fractions, etc.; write them all as <u>ONE (1) BIG FRACTION</u>, then divide identical factors in both the numerator and denominator and realize they all become ONE's (1's).**

   **C. You can now ignore all the ONE's because when you multiply by (1) the terms in both the numerator and denominator will not be affected (except if the only term remaining in the numerator is one then in the final answer, the numerator will equal ONE (1)).**

   **D. You may now multiply all the numbers in the numerator and in the denominator and you will have the final reduced answer unless you need to convert the answer into a mixed number. If you are working with Rational Expressions, you will probably NOT multiply the expressions in the numerator and denominator. The only terms you would be expected to multiply in this situation would be the factors which are numbers.**

**<u>I am making a major assumption that everyone knows the definition of the word "FACTOR" but, I also know from experience I am probably wrong.</u>**

**A Factor is any Number, Variable (letter), Expression, Variable with an Exponent, or an Expression with an Exponent, which is part of a multiplication problem.**

**Example multiplication problem with 5 Factors:**

$$5x^3z(2x+5)(x^3-1)^2$$

**Factor 1 is: "5".**

**Factor 2 is: "$x^3$".**

**Factor 3 is: "$z$".**

**Factor 4 is: "$(2x+5)$".**

**Factor 5 is: "$(x^3-1)^2$".**

**2. A. When Comparing, Adding, and/or Subtracting fractions in arithmetic and Rational Expressions in Algebra, you must make sure all the fractions have identical denominators, referred to as the Least Common Denominator (LCD).**

**B. 1. In step one if you need to find the LCD, COMPLETELY FACTOR ALL THE DENOMINATORS, list the factors for each denominator to the side for further reference.**

**B. 2. Write a multiplication problem involving all the different factors.**

**B. 3. Now, the important step required to find the LCD is, go back and reference each set of factors for all the denominators of all the fractions involved in the problem: To find the LCD using the multiplication problem mentioned in step 2 above and when looking at the sets of factors, write the multiplication problem in step 2 using the factors which appear the most amount of times in anyone set of factors to find the LCD for the entire problem. Example find the LCD for the following 3 denominators: 6, 10, 24.**

**Step 1: The factor trees for these three numbers: 6=2X3, 10=2X5, 24=2X2X2X3.**

**Step 2: Start by writing a multiplication problem with all the different factors: This maybe the LCD but you need to check one last thing which is step 3. LCD???=2 X 3 X 5? In this problem this is not the LCD, YET.**

**Step 3: What is the most times that the factor "2" occurred in one set of factors. In the denominator "24" the factor "2" occurred 3 times but when looking at the number "6"and "10" the factor "2" only occurred 1 time therefore, the most times that the number "2" occurs in the sets of factors is 3 times. Therefore, in step three to find the LCD you must multiply by "2" 3 times. The factors "3" and "5" occurred 1 time respectfully for the numbers"6" and "10".**

**Now, we can find the LCM= 2 x 2 x 2 x 3 x 5 = $\mathbf{2^3 \times 3 \times 5 = 120.}$ Now you know the rules and procedures to find LCDs in arithmetic with**

**fractions and in algebra with rational expressions. There is one more important step to complete the addition and subtraction process.**
**C. 1. Convert all the fractions into equivalent fractions with the LCD, by multiplying by a special form of the number "1". There is only one number contained in the infinite sets of numbers with the quality to multiply by this number and divide by this number and get an equivalent number or expression. You guessed it, that number is ONE "1".**

**After reviewing the two sets of rules for working with fractions in Arithmetic and rational expressions in Algebra, it appears that the list of rules and procedures for adding and subtracting fractions is more work and more complicated than the rules and procedures required to work with multiplying and dividing fractions and rational expressions.**

**I consider reducing, multiplying and dividing fractions to be easier to work with when solving challenging arithmetic fraction problems and when simplifying algebra expressions. Therefore, the next section concentrates on using rules and procedures to reduce, multiply and divide fractions and rational expressions, then in the last section I will concentrate on the rules and procedures required to compare the magnitudes, add, and subtract fractions.**

**I will begin with example Problems involving reducing, multiplying and dividing factions and rational expressions. I will continually refer to the steps indicated previously to complete these three types of problems. Follow the**

**rules listed above Rule 1. A. Factor all the numerators and denominators and Rule 1. B. Divide identical factors contained in both the numerators and denominators.**

**You need to remember the Definition of a FACTOR:**

1. **Simplify Reduce: "$\frac{8}{12}$"** **Factor Trees 8 12**

| | 8 | 12 |
|---|---|---|
| | **2 X 4** | **2 X 6** |
| | **2 X 2 X 2** | **2 X 2 X 3** |
| | $\mathbf{2^3}$ | $\mathbf{2^2}$ **X 3** |

Set up the original problem using FACTORS $\frac{8}{12} = \frac{2\cdot 2\cdot 2}{2\cdot 2\cdot 3} = \frac{1\cdot 1\cdot 2}{1\cdot 1\cdot 3} = \frac{2}{3}$

2. **Simplify (Multiply and Divide):** $\frac{72}{75} \div \frac{196}{49} \times \frac{35}{36}$ **Simple using Factor Trees**

| **72** | **196** | **35** | **75** | **49** | **36** |
|---|---|---|---|---|---|
| $2 \times 36$ | $2 \times 98$ | | $3 \times 25$ | | $2 \times 18$ |
| $2 \times 2 \times 18$ | $2 \times 2 \times 49$ | | | | $2 \times 2 \times 9$ |
| $2 \times 2 \times 2 \times 9$ | | | | | |
| $\mathbf{2 \times 2 \times 2 \times 3 \times 3}$ | $\mathbf{2 \times 2 \times 7 \times 7}$ | $\mathbf{5 \times 7}$ | $\mathbf{3 \times 5 \times 5}$ | $\mathbf{7 \times 7}$ | $\mathbf{2 \times 2 \times 3 \times 3}$ |
| $\{\mathbf{2^3 \times 3^2}\}$ | $\{\mathbf{2^2 \times 7^2}\}$ | $\{\mathbf{5 \times 7}\}$ | $\{\mathbf{3 \times 5^2}\}$ | $\{\mathbf{7^2}\}$ | $\{\mathbf{2^2 \times 3^2}\}$ |

$$\frac{72}{75} \div \frac{196}{49} \times \frac{35}{36} = \frac{72}{75} \times \frac{49}{196} \times \frac{35}{36} = \frac{2\times2\times2\times3\times3}{3\times5\times5} \times \frac{7\times7}{2\times2\times7\times7} \times \frac{5\times7}{2\times2\times3\times3} =$$

$$\frac{2\times2\times2\times3\times3 \quad \times \quad 7\times7 \quad \times \quad 5\times7}{3\times5\times5 \quad \times \; 2\times2\times7\times7 \; \times \; 2\times2\times3\times3} =$$

$$\frac{1\times1\times1\times1\times1 \; \times \; 1\times1 \quad \times \quad 1\times7}{1\times1\times5 \; \times \; 1\times1\times1\times1 \quad \times \; 1\times2\times1\times3} =$$

Notice, the one big fraction notation!

$$\frac{7}{5 \times 2 \times 3} = \frac{7}{30}$$

**3. Simplify/Reduce:** $\frac{50a^2m^3t^3}{175a^2m^5tx^2} = \frac{2\cdot5\cdot5\cdot a\cdot a\cdot m\cdot m\cdot m\cdot t\cdot t\cdot t}{5\cdot5\cdot7\cdot a\cdot a\cdot m\cdot m\cdot m\cdot m\cdot m\cdot t\cdot x\cdot x} =$

$$\frac{2\cdot1\cdot1\cdot1\cdot1\cdot1\cdot1\cdot1\cdot1\cdot t\cdot t}{1\cdot1\cdot7\cdot1\cdot1\cdot1\cdot1\cdot1\cdot m\cdot m\cdot1\cdot x\cdot x} = \frac{2tt}{7mmxx} = \frac{2t^2}{7m^2x^2}$$

**4. Simplify:** $\frac{60\,y^3\,(2x+1)(q^2-5)}{15y^3(4x+3)} \times \frac{10(4x+1)}{25xy^2w} \div \frac{36x(q^2-5)^2}{45x^2y^2(2x+1)} =$

$$\frac{60\,y^3\,(2x+1)(q^2-5)}{15y^3(4x+3)} \times \frac{10(4x+1)}{25xy^2w} \times \frac{45x^2y^2(2x+1)}{36x(q^2-5)^2} =$$

$$\frac{2\cdot2\cdot3\cdot5\,y\,y\,y\,(2x+1)(q^2-5)}{3\cdot5y\,y\,y(4x+3)} \times \frac{2\cdot5(4x+1)}{5\cdot5x\,y\,y\,w} \times \frac{3\cdot3\cdot5x\,x\,y\,y(2x+1)}{2\cdot2\cdot3\cdot3x(q^2-5)(q^2-5)} =$$

$$\frac{1\cdot1\cdot1\cdot1\cdot1\cdot1\cdot1\,(2x+1)\cdot1\ \times\ 2\cdot1\,(4x+1)\ \times\ 1\cdot1\cdot1\cdot1\cdot1\cdot1\cdot1\,(2x+1)}{1\cdot1\cdot1\cdot1\cdot1\,(4x+3)\ \times\ 1\cdot1\cdot1\cdot1\cdot1\cdot w\ \times\ 1\cdot1\cdot1\cdot1\cdot1\cdot1\cdot1(q^2-5)}$$

$$\frac{(2x+1)\ \times\ 2(4x+1)\ \times\ (2x+1)}{(4x+3)\ \times\ w\ \times\ (q^2-5)} = \frac{2(2x+1)(4x+1)(2x+1)}{w(4x+3)(q^2-5)} = \frac{2(4x+1)(2x+1)^2}{w(4x+3)(q^2-5)}$$

5. **Simplify: I will factor all the algebra expressions and then you should be able to determine the simplified expression.**

$$\frac{z^2-10z+9}{21(z^2-4)} \div \frac{6(z^2-1)}{2z^2+8z+8} \times \frac{15(z^2-5z+4)}{30(z^2-81)} \times \frac{1-z^2}{3(z^2+1)} =$$

$$\frac{z^2-10z+9}{21(z^2-4)} \times \frac{2z^2+8z+8}{6(z^2-1)} \times \frac{15(z^2-5z+4)}{30(z^2-81)} \times \frac{1-z^2}{3(z^2+1)} =$$

$$\frac{(z-9)(z-1)}{3\cdot 7(z-2)(z+2)} \times \frac{2(z^2+4z+4)}{3\cdot 2(z-1)(z+1)} \times \frac{3\cdot 5(z-4)(z-1)}{2\cdot 3\cdot 5(z-9)(z+9)} \times \frac{-1(z^2-1)}{3(z^2+1)} =$$

$$\frac{(z-9)(z-1)\ \times\ 2(z+2)(z+2)\ \times\ 3\cdot 5(z-4)(z-1)\ \times\ (-1)(z-1)(z+1)}{3\cdot 7(z-2)(z+2)\ \times\ 3\cdot 2(z-1)(z+1)\ \times\ 2\cdot 3\cdot 5(z-9)(z+9)\ \times\ 3(z^2+1)} =$$

$$\frac{1\cdot 1\ \times\ 1\cdot 1\ (z+2)\times 1\cdot 1\,(z-4)(z-1)\ \times\ (-1)(z-1)\cdot 1}{1\cdot 7\,(z-2)\cdot 1\ \times\ 3\cdot 1\cdot 1\cdot 1\ \times\ 2\cdot 3\cdot 1\cdot 1\,(z+9)\times 3(z^2+1)} =$$

$$\frac{-1\,(z+2)(z-4)(z-1)(z-1)}{7\cdot 3\cdot 2\cdot 3\cdot 3\,(z-2)(z+9)(z^2+1)} =$$

$$\frac{-(z+2)(z-4)(z-1)^2}{378(z-2)(z+9)(z^2+1)}$$

**Notice in the final expressions the numbers are completely multiplied out to find the product, the variables are written with exponents if appropriate, and the expressions with parenthesis are not multiplied and they are written with appropriate** exponents **indicated.**

**Example Problems involving, comparing, adding and subtracting fractions and rational expressions. I will continually refer to the steps indicated previously to complete these three types of problems. Adding and subtracting fractions with larger numbers is very challenging.**

**Here is a Basic sample problem to prove the last statement correct. Add and subtract the following arithmetic problem without a calculator and use the most direct method you know without using trial and error: "$\frac{7}{22}+\frac{9}{80}-\frac{1}{24}$". If I am correct this is an arithmetic problem, but most people have no idea how to find the answer using rules and procedures. Does it help if I tell you that the LCD is "2640"? Wow, how did I find that number? If students know how to solve this problem using Factor Trees, then they will be able to quickly transition their knowledge when simplifying Rational Expressions in Algebra 2.**

**The simplified answer is: "$\frac{1027}{2640}$".** It is all about arithmetic!

FOLLOW ALONG CLOSELY AS I SLOWLY ILLUSTRATE ALL THE STEPS AND PROCEDURES REQUIRED TO SIMIPLIFY THE ADDITION/SUBTRACTION PROBLEM INDICATED ABOVE WHICH ARE THE SAME IDENTICAL STEPS REQUIRED TO SIMPLIFY AN ADDITION OR SUBTRACTION RATIONAL EXPRESSION PROBLEM. **Once you know the rules and procedures which follow, you will be expected to solve the arithmetic problems and the rational expression problems** WITHOUT WRITING ALL THE STEPS INDICATED, **but you will mentally do all the steps and now, you will understand why the steps are necessary.**

**Obviously, solving the arithmetic problems first is an excellent way to begin but it is only the beginning if you want your students to be successful in arithmetic and later be successful in higher level mathematics classes.**

**I always felt that it was extremely important that all arithmetic and math teachers understand how the topics taught this year in their class(s) are going to be applied in future years to solve more exciting problems.**

**Students must "Learn and Remember" the rules and procedures taught this year to be successful next year.**

**It is time to find the answer to the given example problem.**

**1. "$\frac{7}{22}+\frac{9}{80}-\frac{1}{24}$".**

**Step 1: We must find the LCD and the first thing we must do is factor all the denominators. We must use the Fundamental Theorem of Arithmetic using Factor Trees.**

| **22** | **80** | **24** |
|---|---|---|
| $\{2 \times 11\}$ | $2 \times 40$ | $2 \times 12$ |
| | $2 \times 2 \times 20$ | $2 \times 2 \times 6$ |
| | $2 \times 2 \times 2 \times 10$ | $\{2 \times 2 \times 2 \times 3\}$ |
| | $\{2 \times 2 \times 2 \times 2 \times 5\}$ | |
| $\{2 \times 11\}$ | $\{2^4 \times 5\}$ | $\{2^3 \times 3\}$ |

**Step 2. a. Set up a multiplication problem involving all the different PRIME FACTORS: $2 \times 3 \times 5 \times 11$ NOT the LCD yet.**

**Step 2. b. Refer back to the factor trees and use the factors indicated in the previous step AND make sure you write a multiplication problem such that each prime number is part of this multiplication problem BUT, use the prime number the maximum number of times it**

occurs in any one of the set of factor trees, therefore the prime number “2” should be used exactly four times in the multiplication problem because it occurred once in the first set of factors, four times in the second set of prime factors and three times in the third set of factors. The prime number “2” occurred four times in the second set of prime factors which is the most it appeared in any one set and to find the LCD I will multiply by the prime number “2” four times. The prime numbers “3”, “5”, and “11” occurred a maximum of one time in anyone set of prime factors. Therefore, the LCD is found by multiplying the following prime factors:

$$\mathbf{2^4 \times 3 \times 5 \times 11 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 11 = 2640}$$

Since I now have the LCD, you should be able to find the three new fractions with the LCD “2640” and add and subtract the three fractions which are equivalent to the three original fractions with the LCD.

**How do you tell students to change the fraction "$\frac{2}{5}$" into the equivalent fraction with the denominator of "15"? Do you say divide the denominator "5" into the new denominator "15" get the number "3" and multiply the numerator "2" by "3" and get the new equivalent fraction, "$\frac{6}{15}$"?** $\frac{2}{5} = \frac{?}{15}$

**YOU TELL YOUR STUDENTS TO PREFORM THIS PROCEDURE IN THEIR HEAD AND GET THE EQUIVALENT FRACTION. NOW, WE REALLY NEED TO THINK ABOUT THIS PROCEDURE AND EMPHASIZE THE RULES TO FULLY INGRAIN THE PROCESS IN THE MINDS OF YOUR STUDENTS FOR ETERNITY TO SOLVE CHALLENGING PROBLEMS.**

**Here is the direct way to mathematically complete the process discussed in the previous paragraph.**

**Focus on each denominator of all the fractions and the LCD just determined. DIVIDE THE LCD BY EACH DENOMINATOR WITH THE GOAL TO MULTIPLY EACH ORIGINAL FRACTION BY A NEW FRACTION EQUAL TO ONE (1), BECAUSE WE KNOW THE ONLY NUMBER WHICH WE CAN MULTIPLY OR DIVIDE BY AND GET AN EQUIVALENT ENDING NUMBER OR EXPRESSION TO THE STARTING EXPRESSION IS TO MULTIPLY OR DIVIDE BY THE NUMBER ONE (1) OR AN *EXPRESSION* EQUAL TO ONE (1).**

**First Fraction Conversion:** $\frac{7}{22} \quad \therefore \frac{LCD}{1st\ Denominator} = \frac{2640}{22} = \mathbf{120}$

**Multiply** $\frac{7}{22} \times \mathbf{1} = \frac{7}{22} \times \frac{120}{120} = \frac{7 \times 120}{22 \times 120} = \frac{840}{2640}$

**Second Fraction Conversion:** $\frac{9}{80} \therefore \frac{LCD}{2nd\ Denominator} = \frac{2640}{80} = \mathbf{33}$

**Multiply** $\frac{9}{80} \times \mathbf{1} = \frac{9}{80} \times \frac{33}{33} = \frac{9 \times 33}{80 \times 33} = \frac{297}{2640}$

**Third Fraction Conversion:** $\frac{1}{24} \therefore \frac{LCD}{3nd\ Denominator} = \frac{2640}{24} = \mathbf{110}$

**Multiply** $\frac{1}{24} \times \mathbf{1} = \frac{1}{24} \times \frac{110}{110} = \frac{1 \times 110}{24 \times 110} = \frac{110}{2640}$

**Same Problem with Equivalent Fractions with the LCD:**

$$\frac{7}{22} + \frac{9}{80} - \frac{1}{24} = \frac{840}{2640} + \frac{297}{2640} - \frac{110}{2640} = \frac{840+297-110}{2640} = \frac{1027}{2640}$$

**If you want to try and reduce this final answer, you only must test the four prime numbers used to calculate the LCD:**

**Test: 2, 3, 5, & 11 They should all divide perfectly into the LCD but in this situation, none will divide perfectly into 1027.**

**NOTICE THE ONE BIG FRACTION NOTATION!**

**2.** $\frac{2}{11}+\frac{6}{11}-\frac{5}{11}=\frac{2+6-5}{11}=\frac{3}{11}$ **Very easy problem.**

**3.** $\frac{77.3}{91}+\frac{16.9}{91}-\frac{21.6}{91}+\frac{13.4}{91}=\frac{77.3+16.9-21.6+13.4}{91}=\frac{86}{91}$ **Still an easy problem. If you think it might reduce, write 91 using prime factors. I INTERLEAVED decimals into my fraction example problem. Interleaving in arithmetic and mathematics problems is a great way to review concepts and research indicates that it helps students "Learn and Remember" how to solve other math problems involving rules and procedures.**

**4.** $\frac{57}{5xy}+\frac{72}{5xy}-\frac{24}{5xy}=\frac{57+72-24}{5xy}=\frac{105}{5xy}=\frac{5\times 21}{5\cdot x\cdot y}=\frac{1\times 21}{1\cdot x\cdot y}=\frac{21}{xy}$

**Another example problem and in this problem, I INTERLEAVED a little algebra into the problem along with reducing the final answer. INTERLEAVING IS A GREAT WAY TO INTRODUCE NEW CONCEPTS AND REVIEW PREVIOUSLY LEARNED CONCEPTS.**

5. $\frac{2}{3} + \frac{1}{4} + \mathbf{1} =$ **Many arithmetic teachers and even students can do most of the work in their head and write very little work. We know the LCD between "3" and "4" is "12", and you can mentally visualize that $\frac{2}{3}$ must be written with the LCD 12 so visualize that "$\frac{2}{3} = \frac{}{12}$" and we mentally divide 3 into 12 and get 4 then multiply the numerator 2 times 4 and get "$\frac{8}{12}$". Then do the same thing with the fraction "$\frac{1}{4} = \frac{}{12}$", preform the same mental mathematics procedures with 12 divided by 4 and get 3 then multiply 3 times the numerator 1 and get the new equivalent fraction, "$\frac{1}{4} = \frac{3}{12}$". I expect all arithmetic teachers and many students over 95% of students can and will do this type of conversion in their head, but now you should understand the rules that prove why these steps really work??**

$$\frac{2}{3} + \frac{1}{4} + \mathbf{1} = \frac{8}{12} + \frac{3}{12} + \mathbf{1} = \frac{8+3}{12} + \mathbf{1} = \frac{11}{12} + \mathbf{1} = \mathbf{1}\,\frac{11}{12}$$

6. $\frac{1}{x}+\frac{2}{3}+\frac{z}{6x^2y}=$ **Step 1: Find the LCD – write all the denominators in factored form - 1st denominator factored is "x", 2nd denominator factored is "3, and the 3rd denominator factored is "$2 \cdot 3 \ \cdot x \ \cdot x \ \cdot y$". Step 2: Multiply all the different factors to begin to find the LCD – "$LCD = 2 \cdot 3 \cdot x \cdot x \cdot y = 6x^2y$" I determined the LCD by multiplying all the different factors and then I determined the most times that each factor appeared in the set of factors for each denominator.**

**Step 2: Convert all three given fractions into equivalent fractions with the LCD, by multiplying each fraction by a fraction equal to ONE (1).**

**First Fraction $\frac{1}{x}$ Conversion:** $\frac{LCD}{Denom\ 1st\ Fraction} = \frac{6xxy}{x} = 6xy \ \therefore$

$Multiply\ first\ fraction\ by\ 1\ and\ 1 = \frac{6xy}{6xy}$

$$\frac{1}{x} = \frac{1}{x} \times 1 = \frac{1}{x} \times \frac{6xy}{6xy} = \frac{1 \times 6xy}{x \times 6xy} = \frac{6xy}{6xxy} = \frac{6xy}{6x^2y}$$

**Second Fraction $\frac{2}{3}$ Conversion:** $\frac{LCD}{Den\ 2nd\ Fraction} = \frac{6xxy}{3} = 2x^2y \ \therefore$

**Multiply the Second fraction by 1 and** $1 = \frac{2x^2y}{2x^2y}$

$$\frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3} \times \frac{2x^2y}{2x^2y} = \frac{2 \times 2x^2y}{3 \times 2x^2y} = \frac{4x^2y}{6x^2y}$$

**Third Fraction $\frac{z}{6x^2y}$ Conversion:** $\frac{LCD}{Den\ 3nd\ Fraction} = \frac{6xxy}{6xxy} = 1 \ \therefore$

**Multiply the Third Fraction by 1 because the Third fraction Denominator is the LCD, so it does not change.** $\frac{z}{6x^2y} \times 1 = \frac{z}{6x^2y}$

**Solution to the problem:**

$$\frac{1}{x}+\frac{2}{3}+\frac{z}{6x^2y} = \frac{6xy}{6x^2y}+\frac{4x^2y}{6x^2y}+\frac{z}{6x^2y} = \frac{6xy+4x^2y+z}{6x^2y}$$

**I will solve problems 7 and 8 by basically only listing the math steps with little written explanations. You may have to refer to the previous problems with the written explanations to understand why I write specific steps.**

**7.** $\frac{2}{x^2y} + \frac{3x}{wy} - \frac{5}{x^2y^3w}$ **To find the LCD you should recognize the factors w, x, y and the most times they appear in any one denominator.** $LCD = x^2y^3w$.

**Convert all three fractions into equivalent fractions with the correct LCD.**

**Fraction 1 :** $\frac{LCD}{Denom\ 1st.\ Frac} = \frac{x^2y^3w}{x^2y} = y^2w\ \therefore\ 1 = \frac{y^2w}{y^2w}$

**Converted Fraction 1:**

$$\frac{2}{x^2y} = \frac{2}{x^2y} \times 1 = \frac{2}{x^2y} \times \frac{y^2w}{y^2w} = \frac{2 \times y^2w}{x^2y \times y^2w} = \frac{2y^2w}{x^2y^3w}$$

**Fraction 2 :** $\frac{LCD}{Denom\ 2nd.\ Frac} = \frac{x^2y^3w}{wy} = x^2y^2\ \therefore\ 1 = \frac{x^2y^2}{x^2y^2}$

**Converted Fraction 2:**

$$\frac{3x}{wy} = \frac{3x}{wy} \times 1 = \frac{3x}{wy} \times \frac{x^2y^2}{x^2y^2} = \frac{3x \times x^2y^2}{wy \times x^2y^2} = \frac{3x^3y^2}{x^2y^3w}$$

**Fraction 3 :** $\frac{LCD}{Denom\ 3rd.\ Frac} = \frac{x^2y^3w}{x^2y^3w} = 1\ \therefore\ 1 = 1$

$$\frac{5}{x^2y^3w} = \frac{5}{x^2y^3w} \times 1 = \frac{5}{x^2y^3w}$$

**Solution :** $\frac{2}{x^2y} + \frac{3x}{wy} - \frac{5}{x^2y^3w} = \frac{2y^2w}{x^2y^3w} + \frac{3x^3y^2}{x^2y^3w} - \frac{5}{x^2y^3w} =$

$$\frac{2y^2w+3x^3y^2-5}{x^2y^3w}$$

8. $\frac{(x+1)}{x^2-16} - \frac{x+2}{x^2-7x+12} = \frac{(x+1)}{(x-4)(x+4)} - \frac{x+2}{(x-3)(x-4)}$

**The three factors in the denominators are:**

**(x-4) & (x+4) & (x-3)   The most times either factor appears in anyone denominator is ONE time therefore, the $LCD = (x-4)(x+4)(x-3)$**

**Fraction 1 :** $\frac{LCD}{Denom\ 1st.\ Frac} = \frac{(x-4)(x+4)(x-3)}{(x-4)(x+4)} = (x-3) \therefore\ 1 = \frac{(x-3)}{(x-3)}$

**Converted Fraction 1:**

$$\frac{(x+1)}{(x^2-16)} = \frac{(x+1)}{(x-4)(x+4)} \times 1 = \frac{(x+1)}{(x-4)(x+4)} \times \frac{(x-3)}{(x-3)} = \frac{(x+1)\times(x-3)}{(x-4)(x+4)\times(x-3)} =$$

$$\frac{(x+1)(x-3)}{(x-4)(x+4)(x-3)} = \frac{x^2-2x-3}{(x-4)(x+4)(x-3)}$$

**Fraction 2 :** $\frac{LCD}{Denom\ 2nd.\ Frac} = \frac{(x-4)(x+4)(x-3)}{(x-3)(x-4)} = (x+4) \therefore\ 1 = \frac{(x+4)}{(x+4)}$

**Converted Fraction 2:**

$$\frac{x+2}{(x^2-7x+12)} = \frac{(x+2)}{(x-3)(x-4)} \times 1 = \frac{(x+2)}{(x-3)(x-4)} \times \frac{(x+4)}{(x+4)} = \frac{(x+2)\times(x+4)}{(x-3)(x-4)\times(x+4)} =$$

$$\frac{(x+2)(x+4)}{(x-4)(x+4)(x-3)} = \frac{x^2+6x+8}{(x-4)(x+4)(x-3)}$$

**Solution :** $\frac{(x+1)}{x^2-16} - \frac{x+2}{x^2-7x+12} = \frac{(x+1)}{(x-4)(x+4)} - \frac{x+2}{(x-3)(x-4)} =$

$$\frac{x^2-2x-3}{(x-4)(x+4)(x-3)} - \frac{x^2+6x+8}{(x-4)(x+4)(x-3)} = \frac{(x^2-2x-3)-(x^2+6x+8)}{(x-4)(x+4)(x-3)} =$$

$$\frac{x^2-2x-3-x^2-6x-8}{(x-4)(x+4)(x-3)} = \frac{-8x-11}{(x-4)(x+4)(x-3)} = \frac{-(8x+11)}{(x-4)(x+4)(x-3)}$$

**Notice, in the numerator the terms are multiplied but in the denominator the expressions remain in factored form.**

## ADDITIONAL TOPIC TO THINK ABOUT CALLED "Dimensional Analysis"

**Dimensional analysis is a mathematics topic which focuses on labels and treats labels the same way that we treat variables in algebra and higher-level math and science classes. Dimensional Analysis used in its elementary form is an excellent and simple method to convert from one unit type of measurement into an equivalent unit of measurement using a basic concept. This concept is so simple, but very powerful. The method emphasizes a basic fact, when you multiply by the number ONE (1) or a ratio, fraction, or expression which equals ONE (1), you will always get an equivalent ratio, fraction, or expression written in a different form.**

**Before I start to discuss the basics regarding Dimensional Analysis think about these simple arithmetic facts which should be obvious starting in either 5th or 6th grade and then broaden your thinking to other equations and expressions.**

**Arithmetic facts:** $7 = 7 \therefore \frac{7}{7} = 1$

$$16 - 5 = 9 + 2 \therefore \frac{16-5}{9+2} = 1 = \frac{9+2}{16-5}$$

$88 \div 11 = (21 + 3) \div 3 \therefore \frac{88 \div 11}{(21+3) \div 3} = 1 = \frac{(21+3) \div 3}{88 \div 11}$ **There is one major exception to this property: neither side can equal (ZERO) (0), BECAUSE THERE IS NO WAY TO DIVIDE ANY NUMBER OR TERM OR EXPERSSION BY ZERO (0), IT IS IMPOSSIBLE. The theorem is stated like this: IF TWO NUMBERS, EXPRESSIONS, OR QUANTITIES ARE EQUAL, THEN it is possible to WRITE TWO FRACTIONS OR RATIOS BOTH EQUAL TO ONE (1) such that in one of the fractions or ratios, the numerator is the left side of the equation and the right side of the** equation is the denominator and in the second fraction or ratio the numerator and denominator are switched. The one exception to this theorem is neither side my equal zero. The name of this theorem is "Young's Equation/Ratio Theorem." It seems so obvious, but the Theorem can be used to find equivalent expressions in many different situations in math and science.

Some more basic examples before I illustrate how to use this theorem in arithmetic to convert expressions with units into equivalent expressions with different units. Facts we know based on the simple arithmetic examples above:

$\frac{11}{11} = 1, \frac{41.2}{41.2} = 1, \frac{\frac{7}{11}}{\frac{7}{11}} = 1$ *then in algebra* $\frac{x}{x} = 1, \frac{(w^3-3)}{(w^3-3)} = 1$

*Now, I will expand you thinking in other situaions*: 1. "$\frac{hr.}{hr.} = 1$"

*and* "$\frac{km^2}{km^2} = 1$" and "$\frac{\$}{\$} = 1$" 2. 12 in. =1 ft. ∴ "$\frac{12\ in.}{1\ ft.} = 1 = \frac{1\ ft.}{12\ in.}$" and

24 hr. = 1 day ∴ "$\frac{24\ hrs.}{1\ day} = 1 = \frac{1\ day}{24\ hrs.}$" & 2.54 cm=1 in. ∴ "$\frac{2.54\ cm}{1\ in.} = 1 = \frac{1\ in.}{2.54\ cm}$"

3. 12 cars + 4 cars - 3 cars = 13 cars, just like 12x+4x-3x=13x. Dimensional Analysis uses the same rules used in arithmetic with numbers and the same rules used in algebra with variables but now we apply the same rules to labels.

1. How do you tell students to convert 48 inches into feet? Using rules and dimensional analysis: Fact: 12 in.= 1 ft. ∴ $\frac{12\ in.}{1\ ft.} = 1 = \frac{1\ ft.}{12\ in.}$ and

   $48\ inches = 48\ inch. = 48\ in. =$ 48 in. $\times 1 = \frac{48\ in.}{1} \times \frac{1\ ft.}{12\ in.} = \frac{48\ in. \times 1\ ft.}{1 \times 12\ in.} = \frac{48 \times 1\ ft.}{1 \times 12} = \frac{48\ ft.}{12} = 4\ ft.$

2. How do you tell students to convert 5.2 miles to feet?

   Using rules and dimensional analysis: Fact: 5280 ft.= 1 mi. ∴

   $\frac{5280\ ft.}{1\ mi.} = 1 = \frac{1\ mi.}{5280\ ft.}$ and $5.2\ miles = 5.2\ mile. = 5.2\ mi. =$

   5.2 mi. $\times 1 = \frac{5.2\ mi.}{1} \times \frac{5280\ ft.}{1\ mi.} = \frac{5.2\ mi. \times 5280\ ft.}{1 \times 1\ mi.} = \frac{5.2 \times 5280\ ft.}{1 \times 1 \times 1} =$

   $\frac{27456\ ft.}{1} = 27456\ ft.$

3. **How do you explain to students how to convert $3.2\ yd.^3$ to $ft.^3$?**

**Fact:** $1\ yd. = 3\ ft.\ \therefore\ \frac{1\ yd.}{3\ ft.} = 1 = \frac{3\ ft.}{1\ yd.}$

$$3.2\ yd.^3 = 3.2\ yd.^3 \times 1 = \frac{3.2\ yd.^3}{1} \times 1 \times 1 \times 1 =$$

$$\frac{3.2\ \cdot\ yd.\ \cdot\ yd.\ \cdot yd.}{1} \times \frac{3\ ft.}{1\ yd.} \times \frac{3\ ft.}{1\ yd.} \times \frac{3ft.}{1\ yd.} =$$

$$\frac{3.2 \cdot yd.\ \cdot\ yd.\ \cdot\ yd.\ \times\ 3\ ft.\ \times\ 3\ ft.\ \times\ 3\ ft.}{1\ \times\ 1\ yd.\ \times\ 1\ yd.\ \times\ 1\ yd.} = \frac{3.2 \times 3 \times 3 \times 3 \cdot ft. \cdot ft. \cdot ft}{1} = 86.4\ ft.^3$$

Minor differences between the notation for variables and labels. Labels can be words, abbreviations, singular, and plural and they are considered identical. When working with variables they must be identical in all situations. There is no space between the number (coefficient) and a variable(s) but there is always a space between a number and a label. Some labels when abbreviated will have a decimal to the right and some will not. Most labels are to the right of the number but there are some labels written to the left of the number.

4. **Fact: the conversion rate between the United States dollar and the Japanese Yen was 1 United States Dollar = 160.35 Japanese Yen on Tuesday June 9, 2026. Using this conversion rate, you landed in the United States on this date with 5000 JPY. How much money in U. S. Currency did you get upon arriving at the airport international exchange bank? Given fact for the day** $1\ U.S.dollar = 160.35\,JPY\ \therefore\ \$1.00 = ¥160.35\ \therefore$

$$\frac{\$1}{¥160.35} = 1 = \frac{¥160.35}{\$1.00} \quad \therefore \quad ¥5000 = \frac{¥5000}{1} \times 1 =$$

$$\frac{¥5000}{1} \times \frac{\$1.00}{¥160.35} = \frac{¥5000\ \times\ \$1.00}{1\ \times\ ¥160.35} = \frac{5000\ \times\ \$1.00}{1\ \times\ 160.35} = \frac{5000 \cdot \$}{160.35} = \$31.18$$

**Be Happy, make sure you always have and demonstrate a passion for teaching arithmetic and mathematics. There is always something new, which you can learn to help you become a better arithmetic and/or mathematics teacher.**

**How can you illustrate to students why you invert and multiply by a fraction to the right of a division operation symbol?**

**Example Problem:** $\frac{3}{4} \div \frac{5}{7} = \frac{\frac{3}{4}}{\frac{5}{7}} = \frac{\frac{3}{4}}{\frac{5}{7}} \times 1 =$ **We know that** $\frac{\frac{7}{5}}{\frac{7}{5}} = 1$. **Now, we will continue solving the problem** $\frac{\frac{3}{4}}{\frac{5}{7}} \times 1 =$

$$\frac{\frac{3}{4}}{\frac{5}{7}} \times \frac{\frac{7}{5}}{\frac{7}{5}} = \frac{\frac{3}{4}\times\frac{7}{5}}{\frac{5}{7}\times\frac{7}{5}} = \frac{\frac{3\times 7}{4\times 5}}{\frac{5\times 7}{7\times 5}} = \frac{\frac{3\times 7}{4\times 5}}{\frac{1\times 1}{1\times 1}} = \frac{\frac{3\times 7}{4\times 5}}{1} = \frac{3\times 7}{4\times 5} = \frac{3}{4}\times\frac{7}{5} =$$

$$\frac{21}{20} = 1\frac{1}{20}$$

**Proof, to explain why you should invert and multiply by a fraction when dividing fractions. This is proof, because I did not use any numbers to explain the process.** $\frac{a}{b} \div \frac{x}{y} = \frac{\frac{a}{b}}{\frac{x}{y}} =$ $\frac{\frac{a}{b}}{\frac{x}{y}} \times 1$ **(Remember that** $\frac{11}{11} = 1 \therefore \frac{m}{m} = 1 \ \& \ \frac{\frac{y}{x}}{\frac{y}{x}} = 1$**)**

**Now:** $\frac{\frac{a}{b}}{\frac{x}{y}} \times 1 = \frac{\frac{a}{b}}{\frac{x}{y}} \times \frac{\frac{y}{x}}{\frac{y}{x}} = \frac{\frac{a}{b}\times\frac{y}{x}}{\frac{x}{y}\times\frac{y}{x}} = \frac{\frac{a\cdot y}{b\cdot x}}{\frac{x\cdot y}{y\cdot x}} = \frac{\frac{ay}{bx}}{\frac{1\cdot 1}{1\cdot 1}} = \frac{\frac{a\times y}{b\times x}}{1} = \frac{a\times y}{b\times x} =$

$$\textit{Therefore} \quad \frac{a}{b}\times\frac{y}{x}$$

**Proof Completed, Mission Accomplished**

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**Books: ONE The Most Powerful Number**

**The Most Powerful Number 1**

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