The Most Powerful Number 1

This is an interesting title, but is the number 1 really that important in the world of arithmetic and mathematics? This textbook focuses on how to teach fractions in elementary school starting in 3rd, 4th, or 5th grade, so students use the same identical rules and procedures to work with rational expressions in algebra, identities in trigonometry, limits in calculus, etc. The rules and procedures which should be taught and learned in arithmetic are the same rules and procedures which are required to solve problems in higher level math classes. The rules do not change, the problems just become more exciting.

If the book focuses on teaching fractions, "Why is the title of the textbook: "*One The Most Powerful Number*"? **What is the importance of the number "1" when working with fractions and rational expressions?**

I met Dr. Robert Siegler, and he invited me to attend his math research meetings at Carnegie Mellon University. During one of the meetings, the research team was talking about equivalent fractions and Dr. Siegler wrote a simple conversion problem on the whiteboard. Why does the fraction "$\frac{3}{4} = \frac{9}{12}$" and how do you convert the fraction with the smaller numbers into the equivalent fraction with larger numbers? He wrote the conversion process like this: $\frac{3}{4} = \frac{3}{4} \times \mathbf{1} = \frac{\mathbf{3}}{\mathbf{4}} \times \frac{\mathbf{3}}{\mathbf{3}} = \frac{\mathbf{9}}{\mathbf{12}}$**".** I was amazed to find a professor who emphasized the fact that multiplying by the number **1 is the only number** which can be used in a multiplication problem, to find an equivalent number, term, or expression.

This simple process may sound so unnecessary, but ask someone how to convert "4 ft." into "48 in." My assumption is 9 out of 10 educators will tell you to simply multiply the number "4" by the number "12" to find the answer "48 in." Think about this conversion explanation for a moment: I cannot find any mathematical property which states or even implies that if you multiply a number, term, or expression by the number "12" you will get an equivalent number, term, or expression. **The only number which one may multiply by, to compute an equivalent expression is the NUMBER, 1**.

If you are questioning this last comment in bold print, then you must read and study this textbook when you are preparing to become an elementary arithmetic or secondary mathematics teacher.

Now, think about the importance of the number **1** when you are required to reduce a rational expression in algebra or a fraction in arithmetic. Simplify the following rational expression, "$\frac{x^2-9}{x+3}$" into the simplified expression "$x + 3$". What is the first step which you should have learned in arithmetic to reduce any fraction? The answer is - the first step should always be to factor both the numerator and the denominator if possible. Consider the fraction "$\frac{9}{12}$" which everyone should immediately recognize equals "$\frac{3}{4}$", how is the number **1** important in this reducing process? If one refers to both the numerator "9" and the denominator "12" in the given fraction and they are taught to factor both the number "9" into the prime factors "3×3" and the number "12" into the set of prime factors "$2 \times 2 \times 3$", then the original fraction, $\frac{9}{12} = \frac{3 \times 3}{2 \times 2 \times 3} = \frac{3}{2 \times 2} \times \frac{3}{3} = \frac{3}{2 \times 2} \times \frac{1}{1} = \frac{3}{2 \times 2} = \frac{3}{4}$". The terms in the numerator and denominator of a fraction only reduce because **identical factors** in the numerator and in the denominator represent a simple division problem. In the example problem one must recognize the division problem "$\frac{3}{3} = \frac{1}{1} = 1$". "Factor Trees" are important and beneficial.

If the first step when one reduces a fraction in arithmetic is to factor the numerator and the denominator with the goal to reduce the fraction, then the rule to reduce a rational expression in algebra must be to factor the numerator and the denominator. When the numerator and the denominator in the rational expression written above are factored the rational expression is written like this, "$\frac{x^2-9}{x+3} = \frac{(x+3)(x-3)}{x+3} = \frac{(x+3)}{(x+3)} \times (x-3) = \frac{1}{1} \times (x-3) = (x-3)$". Did you recognize that the division problem "$\frac{(x+3)}{(x+3)} = 1$". Maybe, the "**Fundamental Theorem of Arithmetic**" is important!

Without arithmetic rules it is impossible to solve algebra and higher-level math problems. This is only the beginning.