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Solving Arithmetic Fraction Problems Prior to Entering an Algebra Class and Solving Algebra Fraction Problems (Rational Expressions) in the Years Which Follow.

DO WE, CAN WE, SHOULD WE

TEACH STUDENTS THE SAME RULES FOR BOTH SITUATIONS?

IF NOT,

THEN ANSWER THE OBVIOUS QUESTION,

WHY NOT?

The following was posted on the NCTM Media website on May 1, 2025, and asked readers to find the solution.

$\frac{40}{100} = \frac{?}{10}$ We all know the answer is "4" but how would you explain to a student in 5th grade the **rules** to solve this problem?

What would you say to a 5th grader, "Would you suggest that they reduce the given fraction into the simplest form "$\frac{2}{5}$"?

How would you use the Fundamental Theorem of Arithmetic to complete the conversion $\frac{40}{100} = \frac{4}{10}\ or\ \frac{2}{5}$?

Question 1: Why would I even mention the Fundamental Theorem of Arithmetic when reducing/simplifying the fraction $\frac{40}{100}$, everyone knows you can use your mental math skills and divide each number by "20" and then we know that "40 "divided by "20" equals "2" and "100" divided by "20" equals "5".

Question 2: Now, go back again and think about why you might have chosen "20" and not "2, or 4, or 5, or **10**, or what about the number "8"?

We all know that ten (10) is the best number to use to find the answer to the original problem.

What are the simplified forms of the following two basic rational expressions (algebra fractions)?

1. $\frac{68x^4z^2(2z+1)^2}{120x^2z^2(2z+1)^3} = \frac{17x^2}{30(2z+1)}$ 2. $\frac{x^2-9}{x^2-6x+9} = \frac{x+3}{x-3}$

There is only one number which a person may multiply a given **number**, **term**, or **expression** by, with the goal to determine an equivalent (equal) **number**, **term**, or **expression**. The following basic math Identity property illustrates this fact:
"$a\times 1 = a \quad or \quad 1\times a = a \quad or \quad a = a\times 1 \quad or \quad a = 1\times a$".

You guessed it, the number we are thinking about is "ONE" (1).

Find four equivalent fractions for the fraction: $\frac{7}{15}$

$$\frac{7}{15} = \frac{7}{15} \times 1 = \frac{7}{15} \times \frac{2}{2} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$$

$$\frac{7}{15} = \frac{7}{15} \times 1 = \frac{7}{15} \times \frac{11}{11} = \frac{7 \times 11}{15 \times 11} = \frac{77}{165}$$

$$\frac{7}{15} = \frac{7}{15} \times 1 = \frac{7}{15} \times \frac{2.2}{2.2} = \frac{7 \times 2.2}{15 \times 2.2} = \frac{15.4}{33}$$

$$\frac{7}{15} = \frac{7}{15} \times 1 = \frac{7}{15} \times \frac{x}{x} = \frac{7\times z}{15\times z} = \frac{7z}{15z}$$

Notice how we use **one big fraction** prior to multiplying to get the final fraction equal to the starting fraction.

$Add\ and\ Subtract$ the following fractions and find the simplified answer or expression. The point of interest for these examples is to emphasize that if fractions have **identical denominators**, then and only then can a person focus on the numerators and add and subtract the numerators.

Notice the one big fraction used prior to adding and subtracting the numerators. Why Not?

$$\frac{3}{17}+\frac{4}{17}+\frac{9}{17}=\frac{3+4+9}{17}=\frac{16}{17}$$

$$\frac{25}{27}+\frac{1}{27}-\frac{16}{27}-\frac{5}{27}=\frac{25+1-16-5}{27}=\frac{5}{27}$$

$$\frac{24}{47}+\frac{1}{47}+\frac{26}{47}-\frac{4}{47}=\frac{24+1+26-4}{47}=\frac{47}{47}=1$$

$$\frac{25.8}{27}+\frac{5.3}{3}+\frac{1.2}{27}-\frac{2.3}{3}=\frac{25.8+1.2}{27}+\frac{5.3-2.3}{3}=$$

$$\frac{27}{27}+\frac{3}{3}=1+1=2$$

$$\frac{25}{x}+\frac{1}{x}-\frac{16}{x}-\frac{5}{x}=\frac{25+1-16-5}{x}=\frac{5}{x}$$

Notice how we use **one big fraction** prior to **adding and subtracting** the numerators to get the final fraction answer. Below, note how we use **one big fraction** prior to multiplying fractions.

Simplify/Solve the following fraction problems involving reducing, multiplying, and dividing fractions.

Reduce: $\frac{12}{15}$ Divide both numbers by the GCF "3" $\therefore \frac{12}{15} = \frac{4}{5}$

Reduce: $\frac{18}{30}$ Divide both numbers by the GCF "6" $\therefore \frac{18}{30} = \frac{3}{5}$

Multiply: $\frac{3}{5} \times \frac{7}{15} \times \frac{10}{49} = \frac{3 \times 7 \times 10}{5 \times 15 \times 49} = \frac{1 \times 1 \times 2}{1 \times 5 \times 7} = \frac{2}{35}$

Multiply: I want students to be able to do mental math in their head, quickly.

$$\frac{9}{14} \times \frac{25}{81} \times \frac{7}{20} \times \frac{11}{1} = \frac{9 \times 25 \times 7 \times 11}{14 \times 81 \times 20 \times 1} = \frac{1 \times 5 \times 1 \times 11}{2 \times 9 \times 4 \times 1} = \frac{55}{72}$$

Divide: $\frac{3}{5} \div \frac{9}{20}$ which $= \frac{3}{5} \times \frac{20}{9} = \frac{3 \times 20}{5 \times 9} = \frac{1 \times 4}{1 \times 3} = \frac{4}{3} = 1\frac{1}{3}$

Multiply and Divide: $\frac{3}{5} \times \frac{7}{15} \div \frac{14}{25} = \frac{3}{5} \times \frac{7}{15} \times \frac{25}{14} =$

$$\frac{3 \times 7 \times 25}{5 \times 15 \times 14} = \frac{1 \times 1 \times 5}{1 \times 5 \times 2} = \frac{1 \times 1 \times 1}{1 \times 1 \times 2} = \frac{1}{2}$$

We assume you used your knowledge of the multiplication and division facts to simplify these simple arithmetic fraction problems, but the rules you used to simplify/solve these problems are not the same rules you must use to reduce, add, and subtract rational expressions (algebra fractions). Students must learn a new set of rules in algebra which should be taught in arithmetic, if only teachers understood the importance of the Fundamental Theorem of Arithmetic.

I use the **one big fraction notation** for adding and subtracting fractions and then for multiplying fractions to help students distinguish between the two sets of rules. Research indicates that students confuse the rules a couple weeks after they finish working with fractions.

What is the Fundamental Theorem of Arithmetic????

Explanation: Every composite number has a unique set of prime factors. Teachers often use a **factor tree format** to illustrate this **important arithmetic teaching fact**.

36	60	45	124
2×18	2×30	3×15	2×62
$2 \times 2 \times 9$	$2 \times 2 \times 15$	$3 \times 3 \times 5$	$2 \times 2 \times 31$
$2 \times 2 \times 3 \times 3$	$2 \times 2 \times 3 \times 5$		
$\{2^2 \times 3^2\}$	$\{2^2 \times 3 \times 5\}$	$\{3^2 \times 5\}$	$\{2^2 \times 31\}$

Reduce using Factor Trees: $\frac{12}{15} = \frac{2 \times 2 \times 3}{3 \times 5} = \frac{2 \times 2 \times 1}{1 \times 5} = \frac{4}{5}$

Reduce using Factor Trees: $\frac{18}{30} = \frac{2 \times 3 \times 3}{2 \times 3 \times 5} = \frac{1 \times 1 \times 3}{1 \times 1 \times 5} = \frac{3}{5}$

Multiply using Factor Trees: $\frac{3}{5} \times \frac{7}{15} \times \frac{10}{49} =$

$$\frac{3 \times 7 \times 10}{5 \times 15 \times 49} = \frac{3 \times 7 \times 2 \cdot 5}{5 \times 3 \cdot 5 \times 7 \cdot 7} = \frac{1 \cdot 1 \cdot 2 \cdot 1}{1 \cdot 1 \cdot 5 \cdot 1 \cdot 7} = \frac{2}{5 \cdot 7} = \frac{2}{35}$$

Notice how I wrote **one big fraction** in the 2nd fraction to emphasize when a student is multiplying fractions, they should multiply the numerators and then multiply the denominators.

Multiply using Factor Trees:

$$\frac{9}{14} \times \frac{25}{81} \times \frac{7}{20} \times \frac{11}{1} = \frac{9 \times 25 \times 7 \times 11}{14 \times 81 \times 20 \times 1} =$$

$$\frac{3 \cdot 3 \times 5 \cdot 5 \times 7 \times 11}{2 \cdot 7 \times 3 \cdot 3 \cdot 3 \cdot 3 \times 2 \cdot 2 \cdot 5 \times 1} = \frac{1 \cdot 1 \times 1 \cdot 5 \times 1 \times 11}{2 \cdot 1 \cdot 1 \cdot 1 \cdot 3 \times 3 \times 2 \cdot 2 \times 1 \times 1} =$$

$$\frac{5 \times 11}{2 \times 3 \times 3 \times 2 \times 2} = \frac{55}{72}$$

Divide using Factor Trees:

$$\frac{3}{5} \div \frac{9}{20} = \frac{3}{5} \times \frac{20}{9} = \frac{3 \times 20}{5 \times 9} = \frac{3 \times 2 \cdot 2 \cdot 5}{5 \times 3 \cdot 3} = \frac{1 \times 2 \cdot 2 \cdot 1}{1 \times 1 \cdot 3} =$$

$$\frac{2 \cdot 2}{3} = \frac{4}{3} = 1\frac{1}{3}$$

Again, notice the **one big fraction** notation in the third fraction in the expression. **Why do we invert and multiply in this problem?** Notice the fraction we substituted for the number "1" in the third expression below. (Hint: Reciprocal 2nd fraction.)

$$\frac{3}{5} \div \frac{9}{20} = \frac{\frac{3}{5}}{\frac{9}{20}} = \frac{\frac{3}{5}}{\frac{9}{20}} \times 1 = \frac{\frac{3}{5}}{\frac{9}{20}} \times \frac{\frac{20}{9}}{\frac{20}{9}} = \frac{\frac{3}{5} \times \frac{20}{9}}{\frac{9}{20} \times \frac{20}{9}} =$$

$$\frac{\frac{3}{5} \times \frac{20}{9}}{1} = \frac{3}{5} \times \frac{20}{9} = \frac{3 \times 2 \cdot 2 \cdot 5}{5 \times 3 \cdot 3} = \frac{1 \times 2 \cdot 2 \cdot 1}{1 \times 1 \cdot 3} = \frac{2 \cdot 2}{3} = \frac{4}{3} = 1\frac{1}{3}$$

Multiply and Divide using factor trees:

$$\frac{3}{5} \times \frac{7}{15} \div \frac{14}{25} = \frac{3}{5} \times \frac{7}{15} \times \frac{25}{14} = \frac{3 \times 7 \times 5 \cdot 5}{5 \times 3 \cdot 5 \times 2 \cdot 7}$$

$$\frac{1 \times 1 \times 1 \times 1}{1 \times 1 \times 1 \times 2 \times 1} = \frac{1}{2}$$

Reduce the algebra rational expression: Instead of using factor trees we will just write the expressions as factors.

$$\frac{4a^3y^2(b+5)^2}{6a^2y^2(b+5)} = \frac{2\cdot 2\times a\cdot a\cdot a\times y\cdot y\times (b+5)(b+5)}{2\cdot 3\times a\cdot a\times y\cdot y\times (b+5)} =$$

$$\frac{1\cdot 2\times 1\cdot 1\cdot a\times 1\cdot 1\times 1\cdot (b+5)}{1\cdot 3\times 1\cdot 1\times 1\cdot 1\times 1} = \frac{2\cdot a\cdot (b+5)}{3} = \frac{2a(b+5)}{3}$$

Multiply the algebra expression: Instead of using factor trees we just wrote the expressions as factors.

$$\frac{z^2(x^2-25)}{(x^2+8x+12)} \times \frac{(x^2+7x+10)}{z(x+5)^2} \times \frac{(x^2-x-30)}{z(x^2+9x+20)} =$$

$$\frac{z^2(x-5)(x+5)}{(x+6)(x+2)} \times \frac{(x+2)(x+5)}{z(x+5)(x+5)} \times \frac{(x-6)(x+5)}{z(x+4)(x+5)} =$$

$$\frac{z\,z\,(x-5)(x+5)\ \cdot\ (x+2)(x+5)\cdot(x-6)(x+5)}{(x+6)(x+2)\cdot z\cdot (x+5)(x+5)\cdot z\cdot (x+4)(x+5)} =$$

$$\frac{1\cdot 1\cdot (x-5)\cdot 1\ \cdot\ 1\cdot 1\ \cdot\ (x-6)\cdot 1}{(x+6)\cdot 1\ \cdot\ 1\cdot 1\ \cdot 1\ \cdot\ 1\cdot (x+4)\cdot 1} = \frac{(x-5))(x-6)}{(x+6)(x+4)}$$

Divide & $Multiply$ the algebra expressions: Instead of using factor trees we will just write the expressions as factors.

$$\frac{r^2-5r+6}{r^2+9r+20} \div \frac{r^2+r-12}{r^2-25} \times \frac{r^2-9}{r^2-7r+10} =$$

$$\frac{r^2-5r+6}{r^2+9r+20} \times \frac{r^2-25}{r^2+r-12} \times \frac{r^2-9}{r^2-7r+10} =$$

$$\frac{(r-3)(r-2)}{(r+4)(r+5)} \times \frac{(r-5)(r+5)}{(r+4)(r-3)} \times \frac{(r-3)(r+3)}{(r-2)(r-5)} =$$

$$\frac{(r-3)(r-2)\times (r-5)(r+5)\times (r-3)(r+3)}{(r+4)(r+5)\times\ (r+4)(r-3)\times (r-2)(r-5)} =$$

$$\frac{1\ \cdot\ 1\ \times\ 1\ \cdot\ 1\ \times\ (r-3)(r+3)}{(r+4)\cdot\ 1\ \times\ (r+4)\cdot\ 1\ \times\ 1\ \cdot\ 1} =$$

$$\frac{(r-3)(r+3)}{(r+4)(r+4)} = \frac{r^2-9}{(r+4)^2}$$

Notice, when I reduced identical expressions in the numerators and denominators and changed them to the number "1", I did not reduce the individual terms inside the parenthesis, I reduced the entire quantity. **The terms inside the quantities are not factors.**

When reducing any fraction written using factors, one may only reduce **identical factors** because of the rule that a number divided by the identical number will always equal **ONE (1). One last comment, what is a factor in arithmetic or algebra? A factor is any number, term, or expression which is a multiplied term when finding the product of several terms.**

Examples: $2 \cdot x \cdot w \cdot (5 - y)$ **contains four distinct factors: "** $2"$ $"x"$ $"w"$ & $"(5 - y)"$. **The number "5" and the variable** $"y"$ **are not factors individually because they go together to form a factor. The entire expression** $(5 - y)$ **is a factor but the individual terms inside the parenthesis are part of a subtraction expression.**

The first two examples on the next slide involve basic 5th and 6th grade fraction problems, and one can find the LCD if they have memorized the basic multiplication and division facts for the numbers from 0-12.

I will illustrate all the procedures to simplify these problems later.

1. $\frac{3}{5} + \frac{1}{10} + \frac{3}{4} - \frac{1}{2} =$

$$\frac{12}{20} + \frac{2}{20} + \frac{15}{20} - \frac{10}{20} = \frac{19}{20}$$

2. $\frac{5}{8} + \frac{3}{6} + \frac{1}{12} - \frac{3}{4} =$

$$\frac{15}{24} + \frac{12}{24} + \frac{2}{24} - \frac{18}{24} = \frac{11}{24}$$

Problem 1: The LCD is 20. Can you explain all the steps which I did not illustrate to determine the final solution?

Problem 2: The LCD is 24. Can you explain all the steps which I did not illustrate to determine the final solution?

Simplify/Solve:

$$\frac{7}{36} - \frac{7}{63} + \frac{5}{18} = \frac{49}{252} - \frac{28}{252} + \frac{70}{252} = \frac{91}{252}$$

You might already know how to find the LCD by focusing on the **denominators** and using The Fundamental Theorem of Arithmetic with Factor Trees.

	36	63	18
Step 1: Factor Trees	2×18	3×21	2×9
for all the	$2\times2\times9$	$3\times3\times7$	$2\times3\times3$
denominators.	$2\times2\times3\times3$		
Now, be careful!	$2^2 \times 3^2$	$3^2\times7$	2×3^2

Step 2: Find the LCD - The procedure is to multiply all the different prime numbers in all the sets of factor trees. In this situation multiply the following prime numbers: "$2\times3\times7$" **this is not the LCD**.

Step 3: Go back to the three sets of prime numbers and determine the maximum times each of the prime numbers occurred in any one set of prime factors.

The LCD $= 2\times2\times3\times3\times7 = 2^2\times3^2\times7 = 252$

Now, we need to convert the three given fractions into equivalent fractions with the LCD we just determined. Here is the original problem with the 3 original fractions indicated: $\frac{7}{36} - \frac{7}{63} + \frac{5}{18}$.

The first Denominator equals 36. The second Denominator equals 63. The third Denominator equals 18. Formula to determine the specific value of "1" to multiply each fraction: $\frac{LCD}{Denominator\ of\ Interest} = key\ to\ the\ number$ "1".

1st denominator: $252 \div 36 = 7$ ∴ **multiply using the fraction** $\frac{7}{7} = 1$

$$\frac{7}{36} = \frac{7}{36}\times 1 = \frac{7}{36} \times \frac{7}{7} = \frac{49}{252}$$

2nd denominator: $252 \div 63 = 4$ ∴ **multiply using the fraction** $\frac{4}{4} = 1$

$$\frac{7}{63} = \frac{7}{63}\times 1 = \frac{7}{63} \times \frac{4}{4} = \frac{28}{252}$$

3rd denominator: $\frac{252}{18} = 14$ ∴ **multiply using the fraction** $\frac{14}{14} = 1$

$$\frac{5}{18} = \frac{5}{18}\times 1 = \frac{5}{18} \times \frac{14}{14} = \frac{70}{252}$$

The final step is the easy step, set up the original problem with the three new fractions and the denominator "252" then add and subtract the numerators. $\frac{7}{36} - \frac{7}{63} + \frac{5}{18} = \frac{49}{252} - \frac{28}{252} + \frac{70}{252} = \frac{49-28+70}{252} = \frac{91}{252}$

We will use the same procedures in the next arithmetic addition and subtraction problem and in the two rational expressions involving addition and subtraction.

Simplify: $\frac{5}{24} - \frac{5}{16} - \frac{5}{12} + \frac{27}{28}$

Step 1 factor all denominators:

24	16	12	28
$2\times2\times2\times3$	$2\times2\times2\times2$	$2\times2\times3$	$2\times2\times7$
$2^3\times3$	2^4	$2^2\times3$	$2^2\times7$

Not the LCD: $2 \times 3 \times 7$

This is the LCD: $2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7 = 336$

1st fraction denominator: $\frac{336}{24} = 14 \therefore \frac{14}{14} = 1$

$$\frac{5}{24} = \frac{5}{24} \times 1 = \frac{5}{24} \times \frac{14}{14} = \frac{70}{336}$$

2nd fraction denominator: $\frac{336}{16} = 21 \therefore \frac{21}{21} = 1$

$$\frac{5}{16} = \frac{5}{16} \times 1 = \frac{5}{16} \times \frac{21}{21} = \frac{105}{336}$$

3rd fraction denominator: $\frac{336}{12} = 28 \therefore \frac{28}{28} = 1$

$$\frac{5}{12} = \frac{5}{12} \times 1 = \frac{5}{12} \times \frac{28}{28} = \frac{140}{336}$$

4th fraction denominator: $\frac{336}{28} = 12 \ therefore \ \frac{12}{12} = 1$

$$\frac{27}{28} = \frac{27}{28} \times 1 = \frac{27}{28} \times \frac{12}{12} = \frac{324}{336}$$

Ans: $\frac{5}{24} - \frac{5}{16} - \frac{5}{12} + \frac{27}{28} = \frac{70}{336} - \frac{105}{336} - \frac{140}{336} + \frac{324}{336} = \frac{70-105-140+324}{336} = \frac{149}{336}$

Simplify the following rational expression. I will write the factors when necessary for algebra expressions, but you should still be able to follow the steps based on the previous two arithmetic examples. **So exciting, right?**

Simplify/Add: "$\frac{2}{5} + \frac{3}{5}$" The denominators are "$x$" & "5" therefore the only factors must be "x" & "5" and consequently the $LCD = 5x$.

1st fraction denominator:

$$\frac{5x}{x} = 5 \therefore \frac{5}{5} = 1 \quad \frac{2}{x} = \frac{2}{x} \times 1 = \frac{2}{x} \times \frac{5}{5} = \frac{2 \times 5}{x \times 5} = \frac{10}{5x}$$

2nd fraction denominator:

$$\frac{5x}{5} = x \therefore \frac{x}{x} = 1 \quad \frac{3}{5} = \frac{3}{5} \times 1 = \frac{3}{5} \times \frac{x}{x} = \frac{3 \cdot x}{5 \cdot x} = \frac{3x}{5x}$$

Ans: $\frac{2}{x} + \frac{3}{5} = \frac{10}{5x} + \frac{3x}{5x} = \frac{10+3x}{5x}$

This example is a simple rational expression addition problem.

Simplify/Add: "$\frac{(x+1)}{x^2-9} + \frac{x}{x^2-x-6}$"

A relatively simple algebra addition problem.

Step 1 factor both denominators: $x^2 - 9 = (x+3)(x-3)$ and

$$x^2 - x - 6 = (x-3)(x+2)$$

All the different factors multiplied: $(x+3)(x-3)(x+2)$

The expression above is the LCD. Why?

1st fraction denominator: $\frac{(x+3)(x-3)(x+2)}{(x+3)(x-3)} = (x+2) \therefore \frac{(x+2)}{(x+2)} = 1$

$$\frac{(x+1)}{x^2-9} = \frac{(x+1)}{x^2-9} \times 1 = \frac{(x+1)}{x^2-9} \times \frac{(x+2)}{(x+2)} = \frac{(x+1)}{(x+3)(x-3)} \times \frac{(x+2)}{(x+2)}$$

$$\frac{(x+1)(x+2)}{(x+3)(x-3)(x+2)} = \frac{x^2+3x+2}{(x+3)(x-3)(x+2)}$$

2nd fraction denominator:

$\frac{(x+3)(x-3)(x+2)}{(x-3)(x+2)} = (x+3)\ therefore\ \frac{(x+3)}{(x+3)} = 1$

$$\frac{(x+3)(x-3)(x+2)}{(x-3)(x+2)} \frac{2(x^2+3x+1)}{(x-3)(x+2)(x+3)} = (x+3) \therefore \frac{(x+3)}{(x+3)} = 1$$

$$\frac{x}{x^2-x-6} = \frac{x}{x^2-x-6} \times 1 = \frac{x}{(x-3)(x+2)} \times \frac{(x+3)}{(x+3)} =$$

$$\frac{x\,(x+3)}{(x-3)(x+2)(x+3)} = \frac{x^2+3x}{(x-3)(x+2)(x+3)}$$

Ans: $\frac{(x+1)}{x^2-9} + \frac{x}{x^2-x-6} = \frac{x^2+3x+2}{(x+3)(x-3)(x+2)} + \frac{x^2+3x}{(x+3)(x-3)(x+2)} =$

$$\frac{(x^2+3x+2)+(x^2+3x)}{(x-3)(x+2)(x+3)} = \frac{x^2+3x+2+x^2+3x}{(x-3)(x+2)(x+3)} = \frac{2x^2+6x+2}{(x-3)(x+2)(x+3)} =$$

$\frac{x(x^2+3+1)}{(x-3)(x+3)(x+2)}$ WOW, and this is an easy problem.

More thoughts to think about regarding the Power of the number ONE.

We all agree that ONE (1) is the only number with the magical power that if you **MULTIPLY** or **DIVIDE** by the number ONE (1) you will end with the identical **number**, term, or expression that you started with **which is the common understanding of the following Property:** "$a \times 1 = a$**" and other forms of this equation.** There is more to this **basic fact** which everyone needs to understand.

NOW, think about this fact which I named so everyone will remember,

"$If\ a = b$ and neither **"a" or "b" equals ZERO (0)**, then

"$\frac{a}{b}$ $or\ a \div b\ will\ always\ equal\ ONE\ (1)$".

This theorem/fact is so simple but powerful, **"If two numbers, terms, or expressions are equal, not necessarily identical, if one writes a fraction or ratio using either side as a numerator and the other side as the denominator the fraction/ratio will always equal ONE (1)."**

There is an exception???

This leads to a second just as powerful theorem which is actually an extension of the Multiplication Identity Property mentioned above:
If one multiplies or divides a number, term, or expression by another fraction, ratio, term, or expression which equals ONE (1) then the product will always be equal to the starting number, term, or expression even though the starting and ending expressions may not **look identical.**

Examples for the first theorem mentioned above:

a. $11.2 = 11.2 \therefore \frac{11.2}{11.2} = 1$ b. $6 + 2 = 11 - 3 \therefore \frac{6+2}{11-3} = 1 = \frac{11-3}{6+2}$

c. $a + a = 2a \therefore \frac{a+a}{2a} = 1 = \frac{2a}{a+a}$

Recommendation: When working with fractions/ratios NEVER allow students to use slanted lines.

d. $7.6\times 2 = 15.3 - 0.1 \therefore \frac{7.6\times 2}{15.3-0.1} = 1 = \frac{15.3-0.1}{7.6\times 2}$

e. $12\ in. = 1\ ft. \therefore \frac{12\ in.}{1\ ft.} = 1 = \frac{1\ ft.}{12\ in.}$ "Amazing fact!"

f. $60\ min. = 1\ hr. \therefore \frac{60\ min.}{1\ hr.} = 1 = \frac{1\ hr.}{60\ min.}$

g. $\frac{8}{7} = \frac{8}{7} \therefore \frac{\frac{8}{7}}{\frac{8}{7}} = 1$ h1. $10 = 10 \therefore \frac{10}{10} = 1$

h2. $1000 = 1000 \therefore \frac{1000}{1000} = 1$

We know that the percent symbol "%" is a mathematical symbol which means to divide by "100". If we further analyze this thought, then If these two terms are equal and we apply the theorem we just illustrated in examples a. thru h2. above, then . So how can we use this fact? Even the bigger question is, **"How can we use all the facts above from examples a. through to help students in 5th, 6th and 7th grade be more successful in arithmetic and higher-level math and science classes?"**

Example problems which use both the first and second theorems indicated above to solve basic conversion problems in 4th, 5th, 6th, 7th grade arithmetic and higher-level math conversion problems in algebra through calculus and all science courses.

The importance of the number, ONE (1), should be evident when I explain the solutions which follow. I refer to the number ONE (1) as the most powerful number, but that is a topic which may be debated.

I named the first theorem mentioned above as "**Young's Equation/Ratio Theorem**" and the second theorem mentioned above as "**Young's Conversion Theorem.**" I named the theorems not because I am conceited, but because I believe if a theorem is given a name, it is more likely individuals will remember the name of the theorem and consequently remember the purpose and meaning of the theorem.

Problems:

1. Convert $\frac{7}{8} = \frac{x}{28}$.
2. Convert $4\ ft.$ into $x\ in.$
3. Convert $7\ m\ into\ ?\ cm$
4. Convert $1.78\ in.\ into\ mi.$
5. Convert $3\ yds.^3\ into\ ?\ in.^3$
6. Convert 32.32 into a percentage.
7. Convert 72 mph into x fps.
8. Convert 34% into a decimal.
9. Convert the division problem $\frac{3}{4} \div \frac{7}{8}$ into a multiplication problem.
10. Solve the following division problem using long division. $2.8512 \div 0.32$. Illustrate why you can write the same problem as $285.12 \div 32$.
11. Convert the following leaking spigot/pipe problem $5.8\ gals.\ per\ minute\ into\ x\ cups\ per\ second.$ (This must be a large pipe.)

Use rules and procedures NOT Tricks, to solve the problems listed above.

1. Convert $\frac{7}{9} = \frac{?}{99}$ Step 1: $\frac{99}{9}\ or\ 99 \div 9 = 11$

Fact - $11 = 11 \therefore \frac{11}{11} = 1$, because of the first theorem $listed\ above.$

Step 2: Using the second theorem listed above we know that

$$\frac{7}{9} = \frac{7}{9} \times 1 = \frac{7}{9} \times \frac{11}{11} = \frac{77}{99}$$

2. Convert $4\ ft.$ into $x\ in.$ Step 1: Fact - $12\ in. = 1\ ft.\ \therefore$
$\frac{12\ in.}{1\ ft.} = 1 = \frac{1\ ft.}{12\ in.}$ because of the first theorem listed above.

Step 2: Using the second theorem listed above we know that

$$4\ ft. = \frac{4\ ft.}{1} = \frac{4\ ft.}{1} \times 1 = \frac{4\ ft.}{1} \times \frac{12\ in.}{1\ ft.} = \frac{4\ ft. \times 12\ in.}{1 \times 1\ ft.} =$$

$$\frac{4 \times 12\ in.}{1 \times 1} = 4 \times 12\ in. = 48\ in.$$

3. Convert $7\ m\ into\ ?\ cm$

Step 1: Fact – $100\ cm = 1\ m\ \therefore \frac{100\ cm}{1\ m} = 1 = \frac{1\ m}{100\ cm}$ because of the first theorem listed above, "**Young's Equation/Ratio Theorem**".

Step 2: Using the second theorem I mentioned above and named "**Young's Conversion Theorem**" we know that

$$7\ m = \frac{7\ m}{1} = \frac{7\ m}{1} \times 1 = \frac{7\ m}{1} \times \frac{100\ cm}{1\ m} = \frac{7\ m \times 100\ cm}{1 \times 1\ m} =$$

$$\frac{7 \times 100\ cm}{1} = 7 \times 100\ cm = 700\ cm$$

"Young's Conversion Theorem" is an extension/**corollary** of the Identity Property for the number "1" involving multiplication and division. "$a \times 1 = a$"

4. Convert $1.78\ in.\ into\ mi.$ This problem involves a lot of arithmetic skills.

Step 1: Fact – $1\ mi. = 5280\ ft.\ \therefore \frac{1\ mi.}{5280\ ft.} = 1 = \frac{5280\ ft.}{1\ mi.}$ &

$$12\ in. = 1\ ft.\ \therefore \frac{12\ in.}{1\ ft.} = 1 = \frac{1\ ft.}{12\ in.}$$

The previous ratios all equal ONE (1), because of "**Young's Equation/Ratio Theorem**".

Step 2: Using "**Young's Conversion Theorem**" we know that

$$1.78\ in. = \frac{1.78\ in}{1} = \frac{1.78\ in.}{1} \times 1 \times 1 = \frac{1.78\ in.}{1} \times \frac{1\ ft.}{12\ in.} \times \frac{1\ mi.}{5280\ ft.} =$$

$$\frac{1.78\ in. \times 1\ ft. \times 1\ mi.}{1 \times 12\ in. \times 5280\ ft.} = \frac{1.78\ mi.}{12 \times 5280} = \frac{1.78\ mi.}{63360} = 0.0000281\ mi.$$

Convert 3 $yds.^3$ $into\ ?\ in.^3$ Do you understand that $6^3 = 6\times6\times6$? If you do, then $yds.^3 = yd. \times yd. \times yd.$ & $in.^3 = in. \times in. \times in.$

Step 1: Fact: $36\ in. = 1\ yd.\ \therefore \frac{36\ in.}{1\ yd.} = 1 = \frac{1\ yd.}{36\ in.}$ based on applying "**Young's Equation/Ratio Theorem**".

Step 2: Using "**Young's Conversion Theorem**" we know that

$$3\ yds.^3 = 3\times yd. \times yd. \times yd. = \frac{3\ yd. \times yd. \times yd.}{1} =$$

$$\frac{3\ yd. \times yd. \times yd.}{1}\times1\times1\times1 =$$

$$\frac{3\ yd. \times yd. \times yd.}{1} \times \frac{36\ in.}{1\ yd.} \times \frac{36\ in.}{1\ yd.} \times \frac{36\ in.}{1\ yd.} =$$

$$\frac{3\ yd. \times yd. \times yd. \times 36\ in. \times 36\ in. \times 36\ in.}{1 \times 1\ yd. \times 1\ yd. \times 1\ yd.} =$$

$$\frac{3 \times 36\ in. \times 36\ in. \times 36\ in.}{1 \times 1 \times 1 \times 1}$$

$$3\times36\times36\times36\times in. \times in. \times in. = 139968\ in.^3$$

Convert 32.32 into a percentage. Interesting facts: The mathematical symbol "%" means to divide by 100 or $\frac{1}{100}$.

Step 1: Facts: $100\% = \frac{100}{100} = 1 \therefore 100\% = 1$ – Using "**Young's Equation/Ratio Theorem**" we know that $\frac{100\%}{1} = 1 = \frac{1}{100\%}$.

Step 2: Use "**Young's Conversion Theorem**".

$$32.32 = \frac{32.32}{1} = \frac{32.32}{1}\times1 = \frac{32.32}{1} \times \frac{100\%}{1} =$$

$$\frac{32.32 \times 100\%}{1 \times 1} = 3232\%$$

Convert 72 mph into x fps. The abbreviations "mph" and "fps" stand for "miles per hour ($\frac{mi.}{hr.}$)" and "feet per second ($\frac{ft.}{sec.}$). The word "per" means to divide. Use "**Young's Equation/Ratio Theorem**" to determine helpful ratios.

Step 1: Facts - $5280\ ft. \ = 1\ mi. \ \therefore \frac{5280\ ft.}{1\ mi.} = 1 = \frac{1\ mi.}{5280\ ft.}$

$$1\ min. \ = 60\ secs. \ \therefore \frac{1\ min.}{60\ secs.} = 1 = \frac{60\ secs.}{1\ min.} \quad \&$$

$$1\ hr. = 60\ mins. \ \therefore \frac{1\ hr.}{60\ mins.} = 1 = \frac{60\ mins.}{1\ hr.}$$

or $1\ hr. = 3600\ secs. \ \therefore \frac{1\ hr.}{3600\ secs.} = 1 = \frac{3600\ secs.}{1\ hr.}$

I will use the last set of ratios indicated above to convert the given problem into seconds rather than the second and third ratios. If I use the 2nd and 3rd ratios I would multiply by "1" twice to convert hours to seconds, but if I use the 4th set of ratios I will only have to multiply by the number "1" one time to convert from hours to seconds.

I must still multiply by the number "1" twice, one time to convert from miles to feet and the second time to convert hours to seconds.

Step 2: Repeated use of "**Young's Conversion Theorem**".

$$72\ mph = 72\frac{mi.}{hr.} = \frac{72}{1} \times \frac{mi.}{hr.} = \frac{72\ mi.}{1\ hr.} = \frac{72\ mi.}{1\ hr.} \times 1 \times 1 =$$

$$\frac{72\ mi.}{1\ hr.} \times \frac{5280\ ft.}{1\ mi.} \times \frac{1\ hr.}{3600\ secs.} = \frac{72\ mi. \times 5280\ ft. \times 1\ hr.}{1\ hr. \times 1\ mi. \times 3600\ secs.} =$$

$$\frac{72 \times 5280\ ft. \times 1}{1 \times 1 \times 3600\ secs.} = \frac{2 \times 5280\ ft. \times 1}{1 \times 1 \times 100\ secs.} = \frac{10560\ ft.}{100\ secs.} =$$

$$\frac{105.6\ ft.}{1\ sec.} = 105.6\ \frac{ft.}{sec.} = 105.6\ fps$$

Convert 34% into a decimal.

Step 1: Fact – Apply “**Young’s Equation/Ratio Theorem**”.

$100\% = 1 \therefore \frac{100\%}{1} = 1 = \frac{1}{100\%}$

Step 2: Apply “**Young’s Conversion Theorem”.**

$$34\% = \frac{34\%}{1} = \frac{34\%}{1} \times 1 = \frac{34\%}{1} \times \frac{1}{100\%} =$$

$$\frac{34\% \times 1}{1 \times 100\%} = \frac{34\%}{100\%} = \frac{34}{100} = .34$$

Convert the division problem $\frac{3}{4} \div \frac{7}{8}$ into a multiplication problem.

Step 1: Fact - $\frac{8}{7} = \frac{8}{7} \therefore \frac{\frac{8}{7}}{\frac{8}{7}} = 1$ I used “**Young’s Equation/Ratio Theorem”**.

Step 2: Rewrite the original problem as a compound fraction.

$$\frac{3}{4} \div \frac{7}{8} = \frac{\frac{3}{4}}{\frac{7}{8}}$$

$$\frac{\frac{3}{4}}{\frac{7}{8}} \times 1 = \frac{\frac{3}{4}}{\frac{7}{8}} \times \frac{\frac{8}{7}}{\frac{8}{7}} = \frac{\frac{3}{4} \times \frac{8}{7}}{\frac{7}{8} \times \frac{8}{7}} = \frac{\frac{3 \times 8}{4 \times 7}}{\frac{7 \times 8}{8 \times 7}} = \frac{\frac{3 \times 8}{4 \times 7}}{\frac{56}{56}} =$$

$$\frac{\frac{3}{4} \times \frac{8}{7}}{1} = \frac{3}{4} \times \frac{8}{7}$$ (At this moment the fraction division problem is now a fraction multiplication problem. Done.) Finish solving the problem. $\frac{3}{4} \times \frac{8}{7} = \frac{3 \times 8}{4 \times 7} = \frac{3 \times 2 \cdot 2 \cdot 2}{2 \cdot 2 \times 7} = \frac{3 \times 2}{7} = \frac{6}{7}$

I used “**Young’s Conversion Theorem**” in the second step.

Solve the following division problem using long division. $2.8512 \div 0.32$. Illustrate why you can write the same problem as $285.12 \div 32$.

Same problem long division box **0. 32** $\overline{)2.8512}$ converted to **32** $\overline{)285.12}$

Step 1: Fact 1 – Given $2.85120 \div .32$ **changed to a fraction** $\frac{2.85120}{0,32}$

Fact 2 - $100 = 100 \therefore \frac{100}{100} = 1$". Based on "**Young's** $Equation/Ratio\ Theorem$".

Step 2.: Use "**Young's Conversion Theorem**" to complete the conversion.

$$2.8512 \div 0.32 = \frac{2.8512}{0.32} = \frac{2.8512}{0.32} \times 1 = \frac{2.8512}{0.32} \times \frac{100}{100} = \frac{2.8512 \times 100}{0.32 \times 100} = \frac{285.12}{32} = 8.91$$

The last step is to change the

fraction division problem into the long division box notation.

These slides represent a short demonstration of the many examples contained in the two books indicated below.

Book 1: Titled "" which is written as a college textbook to help students who are preparing to become an elementary arithmetic teacher or secondary math teacher. (BLACK COVER)

Book 2: Titled " which is written as a professional development book for teachers who have taught elementary arithmetic or higher-level math courses for 3 or more years. (WHITE COVER)

The books are similar, but the assignments are significantly different.

Webpage: **www.onethemostpowerfulnumber.com**

Convert the following leaking spigot/pipe problem

$4.5\ gals.\ per\ minute\ into\ x\ cups\ per\ second.$

Step 1: Use the Theorem titled **"Young's Equation/Ratio Theorem" which states that, if two numbers, terms, or expressions are equal then you can form 1 or 2 fractions or ratios which will always equal ONE (1). There is one very big exception which every arithmetic teacher and math teacher must know:** Neither side of the given equation may equal the number **ZERO (0).** The **denominator** of a fraction or ratio can **never equal Zero (0).** I did not say that the number zero may not appear in the denominator, I stated that the denominator cannot equal **Zero (0).** This fraction is okay: $\frac{6}{0+6} = 1$. The denominator contains a zero (0), but the denominator does not equal **Zero (0).**

Facts - $1\ gal. = 4\ qts.\ \therefore \frac{1\ gal.}{4\ qts.} = 1 = \frac{4\ qts.}{1\ gal.}$

$4\ cups = 1\ qt.\ \therefore \frac{4\ cups}{1\ qt.} = 1 = \frac{1\ qt}{4\ cups}$ &

$60\ secs. = 1\ min.\ \therefore \frac{60\ secs.}{1\ min.} = 1 = \frac{1\ min.}{60\ secs.}$

Steo 2: Use **"Young's Conversion Theorem" which states that, "If you multiply or divide any number, term, or expression by the number one (1) or by a fraction, ratio, or expression which equals ONE (1), then the result will always <u>equal</u> the starting number, term, or expression."**

$$4.5\ gals\ per\ minute = 4.5\ \frac{gals.}{min.} = \frac{4.5\ gals.}{1\ min.} =$$

$$\frac{4.5\ gals.}{1\ min.} \times 1 \times 1 \times 1 = \frac{4.5\ gals.}{1\ min.} \times \frac{4\ qts.}{1\ gal.} \times \frac{4\ cups}{1\ qt.} \times \frac{1\ min.}{60\ secs.} =$$

$$\frac{4.5\ gals. \times 4\ qts. \times 4\ cups \times 1\ min.}{1\ min. \times 1\ gal. \times 1\ qt. \times 60\ secs.} = \frac{4.5 \times 4 \times 4\ cups \times 1}{1 \times 1 \times 1 \times 60\ secs.} =$$

$$\frac{4.5 \times 1 \times 4\ cups}{15\ secs.} = \frac{18\ cups}{15\ secs.} = \frac{1.2\ cups}{1\ sec.} = 1.2\ \frac{cups}{sec.} =$$

$$1.2\ cups\ per\ sec.$$

Can you think of other ways to use both "**Young's Equation/Ratio Theorem**" and "**Young's Conversion Theorem**" to find equivalent values for other arithmetic and mathematics expressions? How about these suggestions:

1. Converting from one type of currency to another type of currency based on the daily exchange rate.
2. Converting radians into degrees or degrees into grads, etc.
3. Converting a terminating decimal into a fraction.
4. Converting the given fraction into an equivalent fraction: $\frac{2}{3} = \frac{?}{3x}$.
5. Converting the given fraction into an equivalent fraction:

$$\frac{x}{(x+1)} = \frac{?}{(x+1)(x-1)}$$

6. Solving many types of direct proportion problems. Frequently, we use the cross-multiplication rule when working with direct proportions, which is usually an excellent way to work with direct proportions.

TWO Final Questions

1. Question: **Are you starting to realize why I refer to** *the number* **One (1) as the most powerful number?**

2. After looking at the example solutions where I refer to both "**Young's Equation/Ratio Theorem"** and "**Young's Conversion Theorem**", "**Do you remember the names of the two new theorems?** If you do, that proves a teaching point that I wanted you to think about, "If one names a theorem, people are more likely to remember the name and then hopefully understand the meaning of the theorem."

A goal for all great arithmetic teachers and math teachers: Every summer reflect on what you did well last school year and what you could have done better, then think about what you can change to be a better teacher next year. **None of us are perfect, YOU CAN ALWAYS FIND A WAY TO GET BETTER AND HELP MORE STUDENTS LEARN AND REMEMBER HOW TO SOLVE ARITHMETIC AND MATH PROBLEMS WITH GREATER SUCCESS NEXT YEAR!**

Practice questions:

A. Simplify using the Fundamental Theorem of Arithmetic (factor trees)

1. $\frac{230}{414}$ Hint: $230 = 2\times5\times23$ & $414 = 2\times3\times3\times23$
2. $\frac{63}{75} \times \frac{135}{243} \div \frac{45}{90}$ Hint: $63 = 3\times3\times7$
3. $\frac{7}{60} + \frac{9}{50} - \frac{4}{75}$ Hint: $60 = 2\times2\times3\times5$ & $50 = 2\times5\times5$

   Not the LCD {2·3·5}

B. Simplify rational expressions which are already factored

4. $\frac{2\times2\times5\times z\times z\times z(x-3)(x+11)(x-12)}{2\times3\times3\times5\times5(x-9)(x+11)(x-12)}$ Hint: $\frac{?}{3\cdot3\cdot5\cdot(x-9)}$
5. $\frac{(x-4.5)(z-1)}{(x+3)(y+2.7)} \times \frac{(x+3)}{(x-4.5)} \div \frac{(z-1)(z+1)}{(y+2.7)}$
6. $\frac{2}{(x+1)(x-1)} + \frac{2}{(x+1)(x+1)}$ LCD= $(x+1)^2(x-1)$

C. Convert by using fractions or ratios which equal **1**

7. Convert $\frac{7}{64} = \frac{?}{448}$ Hint: $\frac{7}{7} = 1$
8. Convert 3 mi. into ft. Hint: $\frac{5280\ ft.}{1\ mi.} = 1$
9. Convert 324.71 into a percentage.

   Hint: $\frac{324.71}{1}\times1$ You want a percent symbol in the numerator.

Contact Tim Young for in-service programs at either email address:

twypitts1@gmail.com or twypitts@aol.com Once I receive your email I will respond within two working days.

Website: www.onethemostpowerfulnumber.com

Purchase books on www.Amazon.com/books

Click on Science & Math left hand column

Type in the name of either book in the search box:

One The Most Powerful Number

The Most Powerful Number 1

You are the most important person, if students are going to be successful in your classes and in future arithmetic and math classes.

The two best teachers I ever worked with in my 38-year career had a quality that not enough teachers possess.

The quality I am thinking about is: "Both teachers had a PASSION that all their students would do well in their classes. PASSION! PASSION!