Diagnostic Classification Models: Advancement Through Simplicity

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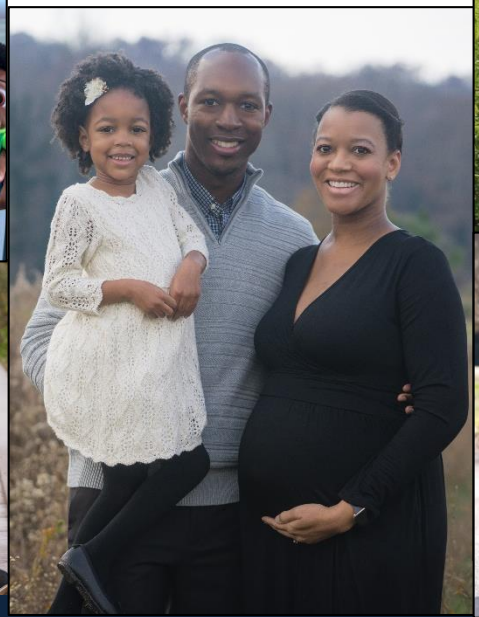
Talk Overview

- A little about me
- Brief intro to diagnostic classification models
- Challenges with implementation
  - » Ideas and potential solutions
  - » Simulated scenarios
- Conclusions

A Little About Me

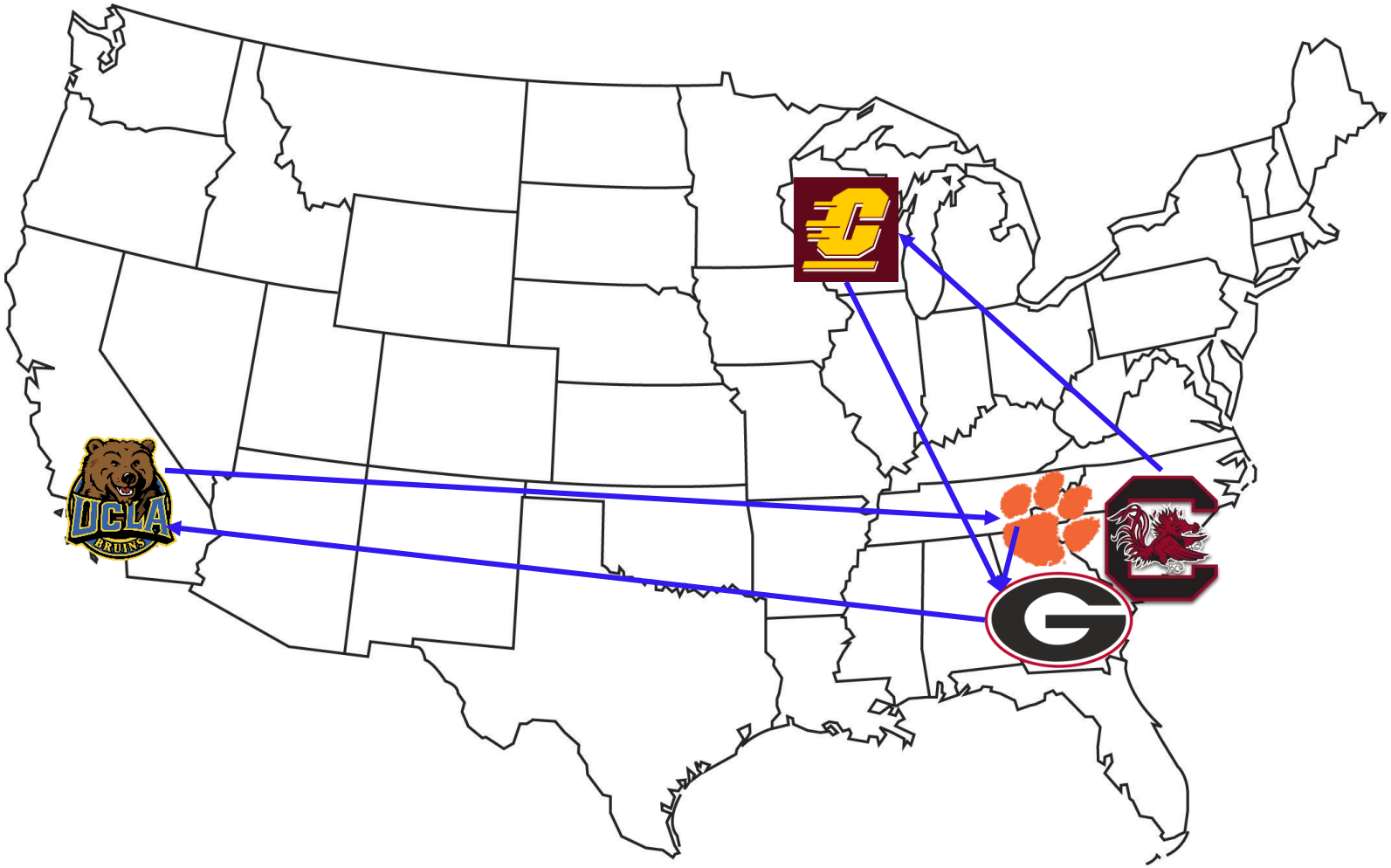
- From Columbia, South Carolina
- Research interests: psychometrics
- Hobbies: sports (basketball, golf), true crime





My Journey Summary



Diagnostic Classification Models

- DCMs use item responses to place students into groups according to proficiency or non-proficiency of attributes

Student	Addition	Subtraction	Multiplication	Division
	✓	—	✓	—
	✓	✓	✓	✓

Diagnostic Classification Models

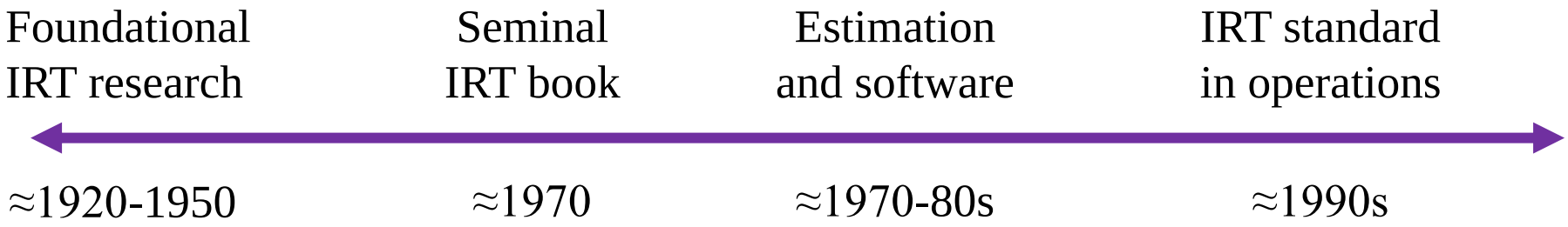
- DCMs are special latent class models
  - » Confirmatory: attribute profiles and Q-matrix
  - » Constrained: measurement model constraints

Student	Addition	Subtraction	Multiplication	Division
	✓	—	✓	—
	✓	✓	✓	✓

- Defining features and benefits
  - » Multidimensionality
  - » Increased classification reliability
  - » Assessment of fine-grained latent traits
  - » Diagnostic interpretations

State of the Field

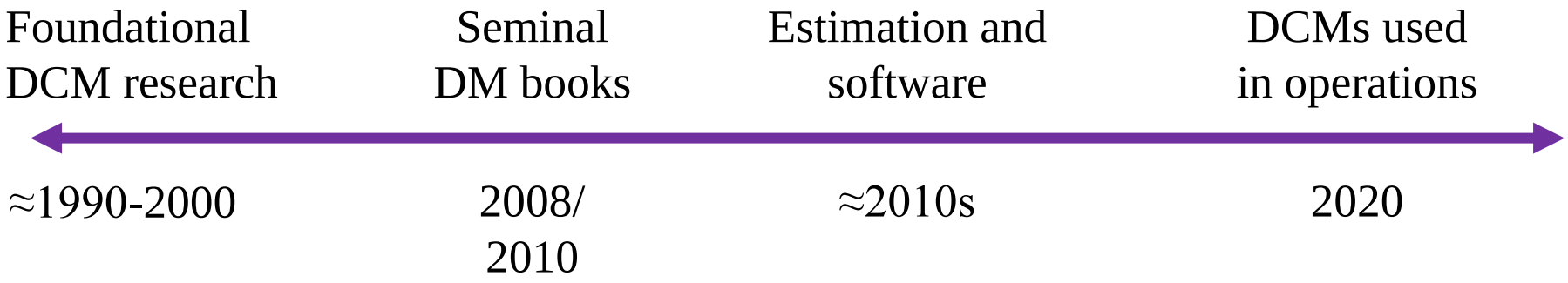
- Item response theory timeline



- Approximately 70 years from foundational research to operations

State of the Field

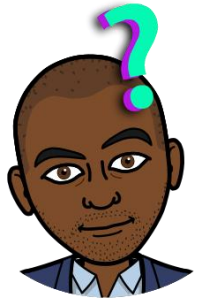
- Diagnostic models timeline



- Still a long way to go to become a standard method

Why not more DCMs?

- **Logistical**
  - » More training and more experts
  - » More time
- **Methodological**
  - » Technical challenges, model fit, etc.
- **Practical**
  - » How do we interpret the model-based classifications?
  - » Statistically? Substantively?



Criterion-referenced?

- Debatable depending on your definition of criterion-referenced
  - » CR because their purpose is not to compare examinees
  - » NR because they classify into two groups

- "…support criterion-referenced interpretations"

Criterion-referenced?

- Brief simulation

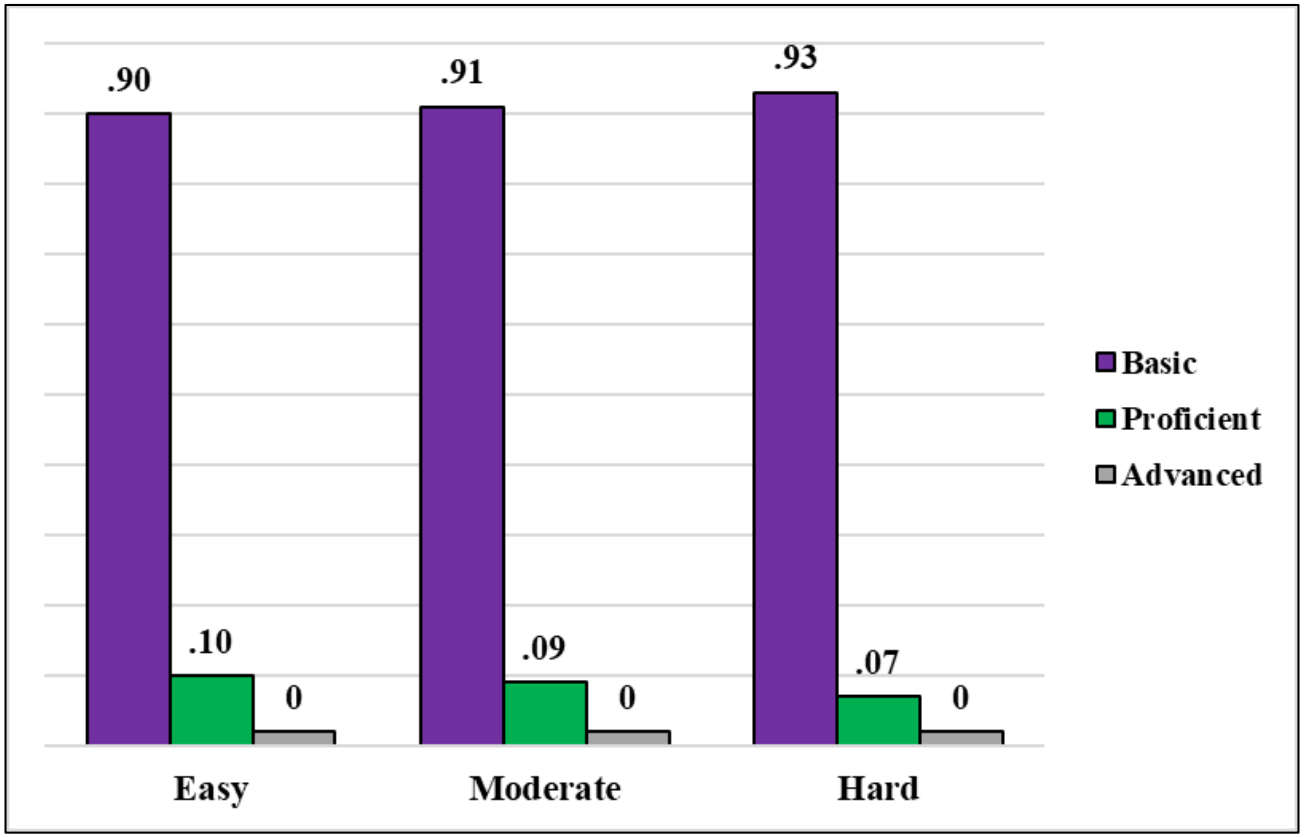
- Consider three examinees and a trichotomous attribute
  - » Jordan: basic (proficiency level 1)
  - » Taylor: proficient (proficiency level 2)
  - » Maya: advanced (proficiency level 3)

- I simulated them taking 100 easy, moderate, hard tests
  - » Test difficulty defined by item response probabilities

- Does the criterion change depending on the test?

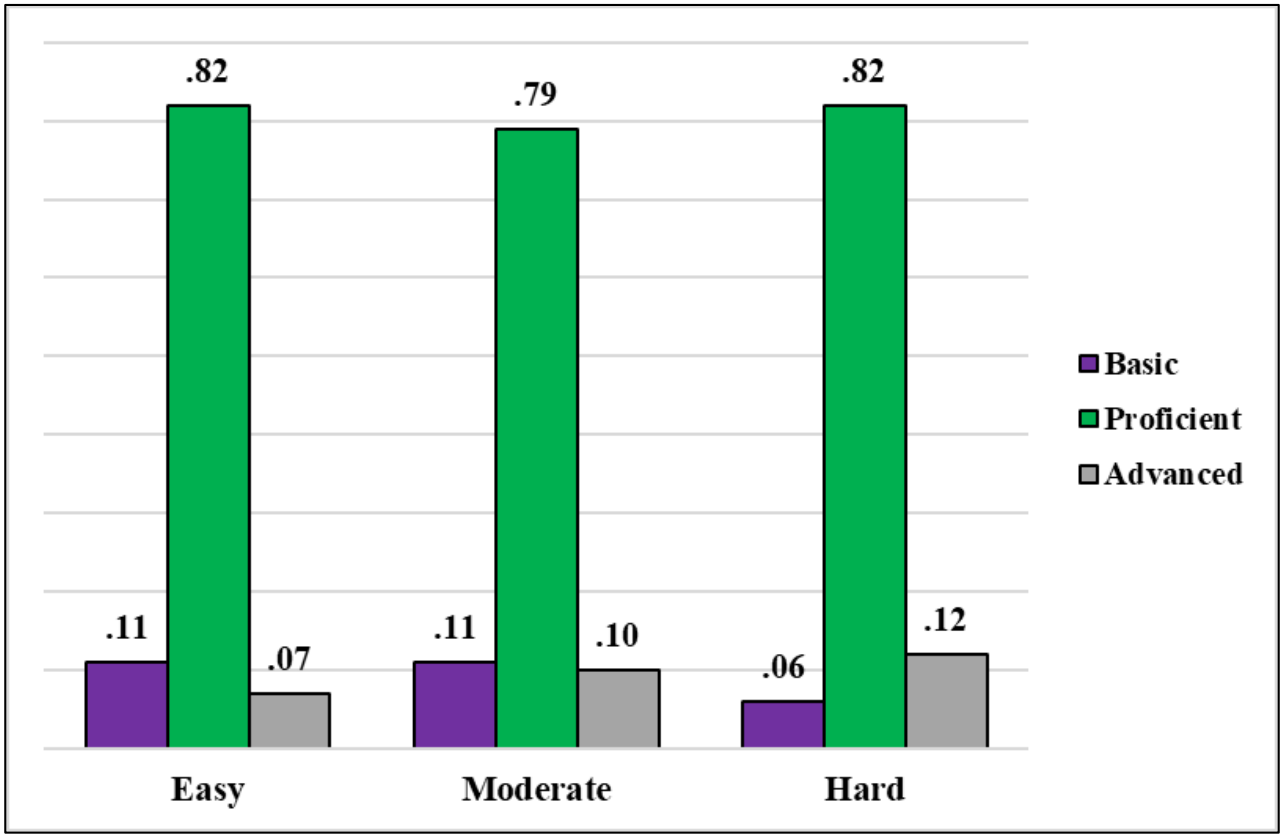
Item Invariance

- Jordan: basic (proficiency level 1)



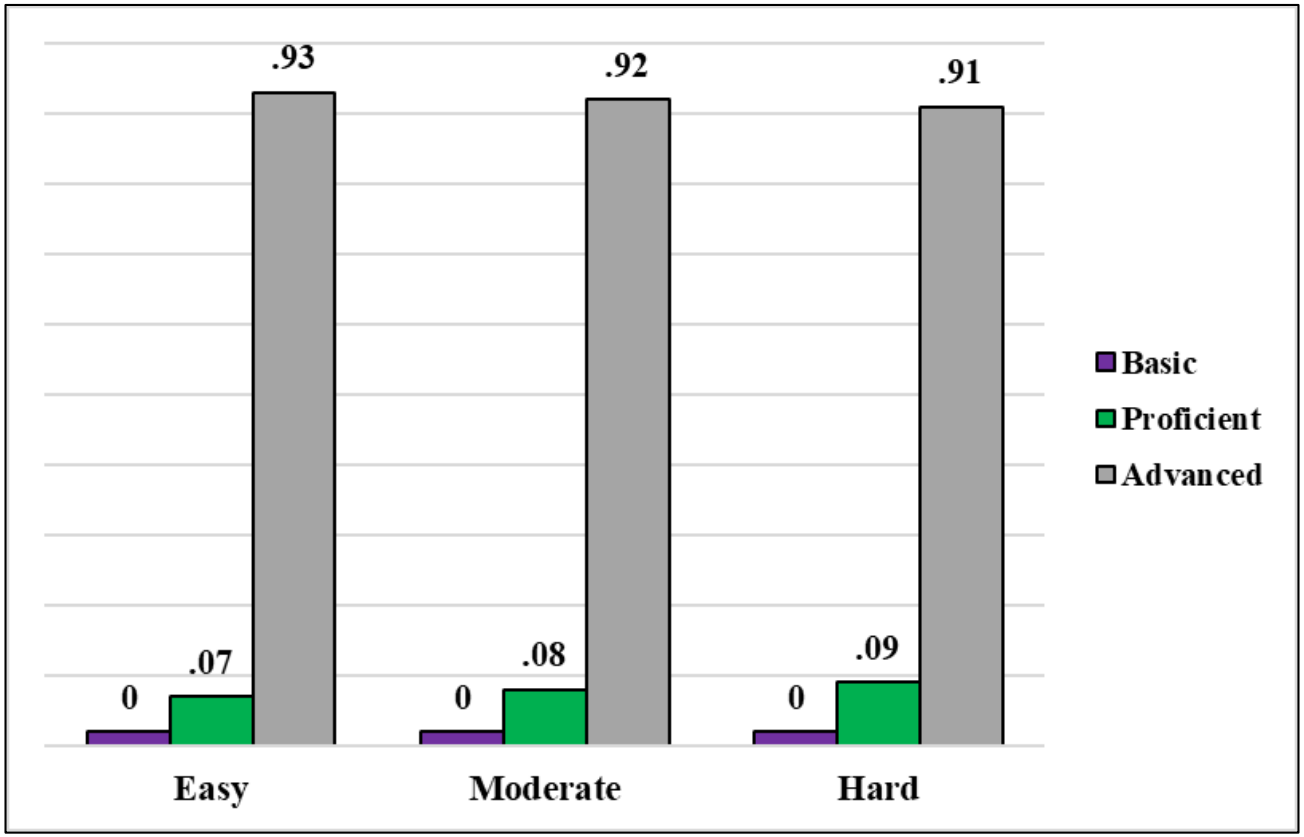
Item Invariance

- Taylor: proficient (proficiency level 2)



Item Invariance

- Maya: advanced (proficiency level 3)



Item Invariance

- Within individual examinees, classifications were consistent across different tests

- Each test also identified the same proportion of examinees in each proficiency level

Test	Basic	Proficient	Advanced
Easy	.346	.308	.346
Moderate	.344	.308	.347
Hard	.345	.308	.346

DCM Invariance

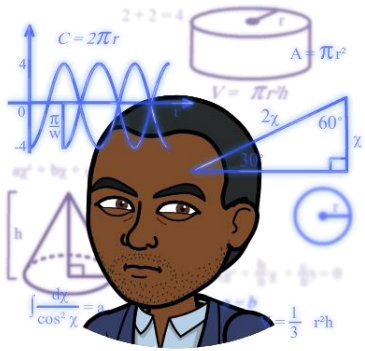
- Item parameters are also invariant under different samples (low vs. moderate vs. high proficiency)
- Artifact of non-arbitrary structural model
  - » Generally, the IRT ability distribution is arbitrarily fixed
  - » DCMs estimate the profile/proficiency proportions

- DCMs sacrifice the continuous latent trait to afford multidimensionality and absolute invariance

Criterion-referenced?

- These invariance properties establish that DCMs operate under a criterion to obtain classifications
  - » Regardless of the items on the test, proficiency level criteria remained the same

- But…the criteria are empirically defined
  - » What if the empirically-defined criteria don't match the pre-specified, practitioner-defined criteria?



Item Influence

- DCM applications use fewer items

- ***Item influence*** – one item (or subset) can have a disproportionate impact on classifications (Jurich & Madison, 2023)
  - » Problematic for construct and content validity

- Item influence metrics:
  - » Item override: how many classifications change if an item is omitted?
  - » **Proportion of attribute information**

Attribute Information

- Item response theory information
  - » Item information function → test information function
- Analogous concept for DCMs
  - » Cognitive diagnostic index (CDI; Henson & Douglas, 2005)
  - » All the items measuring an attribute contribute to the overall information

Item	CDI	Compute	Proportion of Att Info
1	.24		
2	.44		
3	.36		
Total	1.04		

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Attribute Information

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  - » All the items measuring an attribute contribute to the overall information

Item	CDI	Compute	Proportion of Att Info
1	.24	.24 ÷ 1.04	23%
2	.44	.44 ÷ 1.04	42%
3	.36	.36 ÷ 1.04	35%
Total	1.04	1.04 ÷ 1.04	100%

Content Criterion ↔ Blueprint

- Test blueprints guide test development efforts
- Common Core Mathematics Standards
  - » Grade 4 Measurement and Data

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle.
   b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Attribute

Two Subattributes

Blueprint Example

- Suppose in this case, that test developers had prespecified blueprint proportions of 25% and 75%
  - » Then they might allocate 2 and 6 items to the two subattributes

Item	Proportion of Att Information	Empirical Blueprint	Prespecified Blueprint
1			25%
2			
3			75%
4			
5			
6			
7			
8			

Blueprint Example

- Suppose in this case, that test developers had prespecified blueprint proportions of 25% and 75%
  - » Then they might allocate 2 and 6 items to the two subattributes

Item	Proportion of Att Information	Empirical Blueprint	Prespecified Blueprint
1	32		25%
2	28		
3	4		75%
4	7		
5	5		
6	9		
7	8		
8	7		

Blueprint Example

- Suppose in this case, that test developers had prespecified blueprint proportions of 25% and 75%
  - » Then they might allocate 2 and 6 items to the two subattributes

Item	Proportion of Att Information	Empirical Blueprint	Prespecified Blueprint
1	32	60%	25%
2	28		
3	4	40%	75%
4	7		
5	5		
6	9		
7	8		
8	7		

Blueprint Example

- Suppose in this case, that test developers had prespecified blueprint proportions of 25% and 75%
  - » Then they might allocate 2 and 6 items to the two subattributes
  - » This allocation does not guarantee a match

- **Our goal was to examine the ability of DCMs to adhere to prespecified blueprints**

General and Constrained DCMs

- General model: log-linear cognitive diagnosis model (LCDM)
  - » Subsumes many other DCMs
  - » Allows for top-down approach to model building
  - » Blueprint matching not guaranteed

- Constrained model: one-parameter LCDM (1-PLCDM)
  - » Special case of LCDM where attribute main effects are constrained
  - » Analogous to 1-PL IRT model
  - » Nice measurement properties (sufficiency, invariant item ordering)
  - » Assumptions: simple structure Q-matrix and independent attributes

Scenario #1: Summative

- Guiding example: North Carolina end-of-course mathematics tests
  - » Use unidimensional IRT with cutscores to classify examinees into four levels

Table 1. EOC Mathematics domain weight distributions.

Domain	NC Math 1
Number and Quantity and Algebra	36 - 40%
Functions	32 - 36%
Geometry	8 - 12%
Statistics and Probability	18 - 20%
Total	100%

Scenario #1: Summative

- One polytomous attribute with four subattributes
  - » Prespecified blueprint: 40 / 28 / 12 / 20
  - » Item allocation (25 items): 10 / 7 / 3 / 5
- N = 2000
- Item parameters:
  - » Level 1: probability correct uniform on $(0, .25)$
  - » Each subsequent level increased by $(.05, .30)$

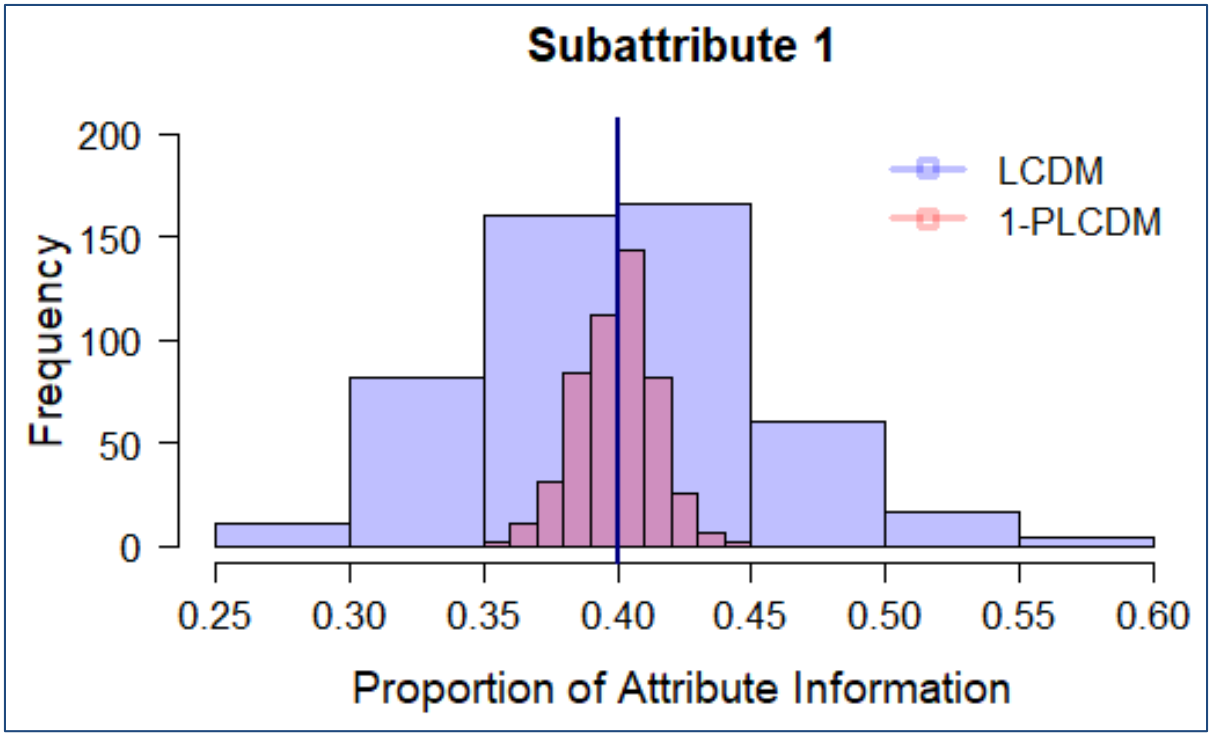
Scenario #1: Summative

- Estimated the polytomous LCDM and 1-PLCDM
  - » *mirt* package (Chalmers, 2012)

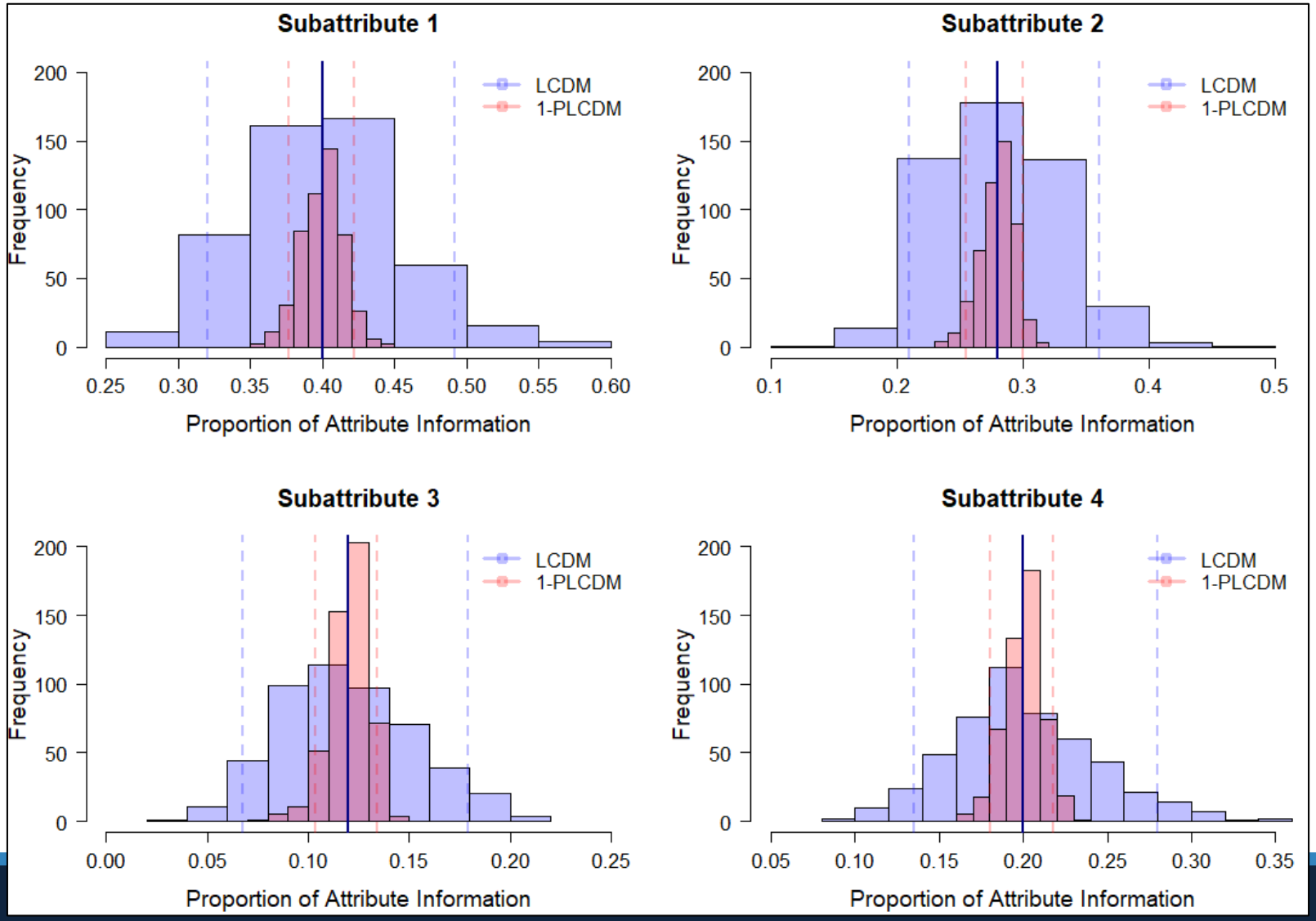
- Compared the empirical and prespecified blueprints
  - » 500 replications

Scenario #1 Results

- On *average*, both models approximated the prespecified blueprint
  - » Precision was much better for 1-PLCDM



Scenario #1 Results



Scenario #1 Results

- On *average*, both models approximated the prespecified blueprint
  - » Precision was much better for 1-PLCDM
  - » LCDM ranged from $(25, 57)$
  - » 1-PLCDM ranged from $(35,45)$

- Combined absolute error much lower for 1-PLCDM
  - » LCDM had mean blueprint error of 14%, max of 37%
  - » 1-PLCDM had mean blueprint error of 4%, max of 10%

Scenario #2: Intermediate

- Guiding example: Common Core State Standards

Cluster

Attribute #1

Attribute #2

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Scenario #2: Intermediate

- Three dichotomous attributes with subattributes
  - Attribute 1 blueprint: 50 / 50 (3 / 3)
  - Attribute 2 blueprint: 40 / 40 / 20 (4 / 4 / 2)
  - Attribute 3 blueprint: 60 / 40 (6 / 4)

- Similar design to Scenario #1
  - N = 2000
  - Simple structure Q-matrix
  - Attribute correlations uniform $(.25, .75)$

Scenario #2 Results

- Similar trend to Scenario #1
  - » LCDM error exacerbated in the multiattribute settings
- On *average*, both models approximated the prespecified blueprint
  - » Precision was much better for 1-PLCDM
  - » Attribute #1 (50/50)
    - LCDM 90% interval was (41, 59); worst replication was 0% / 100%
    - 1-PLCDM 90% interval was (49, 51); worst replication was 48% / 52%
- 1-PLCDM had much less total error
  - » LCDM mean error of 16%, max of 255%
  - » 1-PLCDM mean error of 1%, max of 5%

Summary

- Explored DCMs' ability to adhere to prespecified blueprints
  - » Critical for classification validity and interpretation
- Developed a framework for estimating empirical blueprints
  - » Proportion of attribute information
- Two simulated scenarios (summative and intermediate)
  - » 1-PLCDM was able to approximate prespecified blueprints
  - » Confirmed that general models struggle to match blueprints

Summary

- Not a criticism of general models
  - » Need general models to support use of constrained models
  - » 1-PLCDM limited to simple structure and makes strong assumptions

- DCMs can be used to ensure that practitioner and empirical blueprints are aligned
  - » Increases validity and interpretability of classifications
  - » Expands the settings in which DCMs can be applied

Bonus Scenario

- Can we scale and classify using an IRT model and a DCM?
  - » Typically, one priority will suffer in validity/reliability

- What would it look like if we **1-PL + 1-PLCDM**?

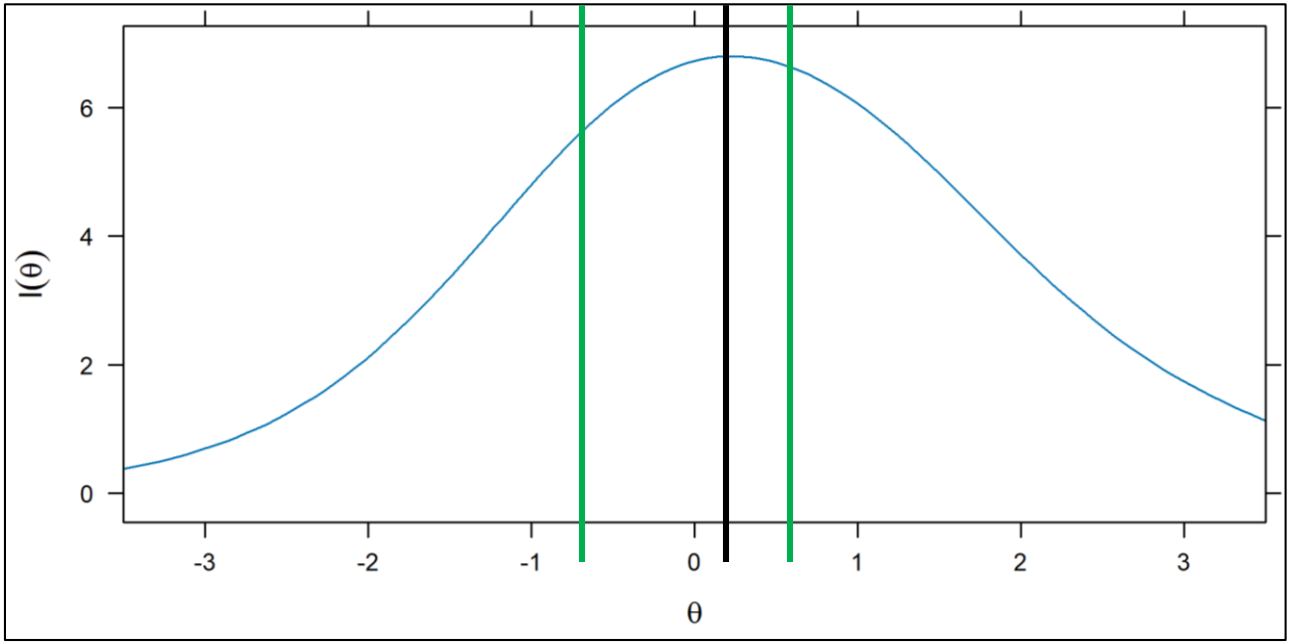
1-PL + 1-PLCDM

- Data: mathematics problem-solving test
  - » 21 items, single attribute
  - » 879 students in 6^{th} - 8^{th} grade

- Analysis:
  - » Calibrated 1-PL and 1-PLCDM separately

1-PL + 1-PLCDM

Sum Score	Class2	Class3	Ability
0	1	1	-2.05
1	1	1	-1.67
2	1	1	-1.37
3	1	1	-1.11
4	1	1	-0.88
5	**1**	**2**	**-0.67**
6	1	2	-0.48
7	1	2	-0.3
8	1	2	-0.12
9	1	2	0.05
10	**2**	**2**	**0.22**
11	2	2	0.39
12	**2**	**3**	**0.56**
13	2	3	0.74
14	2	3	0.92
15	2	3	1.11
16	2	3	1.31
17	2	3	1.52
18	2	3	1.75
19	2	3	2.02
20	2	3	2.32



Why not more DCMs?

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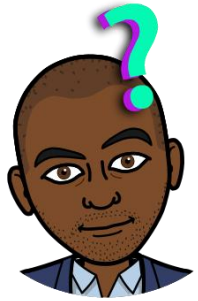

If you have questions or comments, feel free to contact me:

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