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Mathematics and Model Rockets

A Teacher's Guide and Curriculum for Grades 5-12

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TABLE OF CONTENTS

INTRODUCTION

MATHEMATICS AND MODEL ROCKETS

Model rocketry is an extremely useful tool for teaching students in a math classroom. Model rocketry captures the students' interest and involves them in applying math concepts in a real and authentic way. It involves them in experimenting and testing their ideas. Using model rocket projects in a math curriculum aids students in learning to use creative and critical thinking and problem-solving skills. It provides opportunities for them to discover underlying principles of mathematics. Using rocketry also allows curriculum integration by combining math, science, craftsmanship, physical education, prediction, language arts, history, technology and research methods. It provides opportunities for cooperative learning as well as experiences that will have definite personal meaning to the students through the constructing and launching of their own rocket.

Two other curriculum guides from Estes that may be useful are, Science and Model Rockets™ for Grades 5, 6, 7, 8 and Physics and Model Rockets™ for Grades 8, 9, 10 & 11. Science and Model Rockets™ has units that give directions for constructing model rockets, doing simple altitude tracking, simple math and data collection. Physics and Model Rockets™ relates Newton's Laws of Motion to model rocketry. These two guides can be used with the current guide to develop a well-integrated science and math unit.

This curriculum guide is constructed so that teachers of students from fifth through twelfth grade will find it useful. Units include activities that can be used to support basic concepts and to extend students who have the knowledge and ability to work at a more advanced level. The guide can be used as a complete unit or individual projects may be selected to use if the students have the math and science background needed to understand it.

Each group of students is different with a variety of experience and interests. Providing a number of projects allows the teacher and the students to work on activities related to the design of model rockets and the mathematical analysis of the effects of the design on model function that relate to those individual experiences and interests. The teacher will be able to adapt the unit to fit an individual class.

GOALS

Students will understand the connection between math and science.
Students will use math in practical applications related to rocketry.
Students will recognize the importance of careful construction of a model rocket for stability, safety and a successful launch.

STUDENT OUTCOMES

The student will be able to:

- Determine the center of mass experimentally, graphically and/or mathematically.
- Recognize the importance of the center of mass in rocket flight as the point around which an unstable rocket tumbles.
- Recognize the significance of the location of the center of pressure in relation to the location of the center of gravity(center of mass) in providing stability for a model rocket.
- Determine mathematically the centroid and area of a variety of shapes.
- Determine the lateral center of pressure for a model rocket.
- Recognize the importance of stability in model rocket flight and test the stability of their own rocket.
- Track the flight of model rockets to gather data to calculate height, velocities and accelerations.
- Use mathematical equations to determine velocities and accelerations during the rocket flight and describe their relationship to Newton's Second Law.

THINKING SKILLS

- Observing
- Reading and following directions and diagrams
- Problem-solving
- Analysis

GENERAL BACKGROUND FOR THE TEACHER

Stability is the single most important consideration in designing a model rocket. The more stable the rocket, the more it tends to rotate (weather-cock) into the wind during flight. A stable rocket is one that flies in a smooth, uniform direction. An unstable rocket flies along an erratic path, sometimes tumbling or changing direction. Unstable rockets are dangerous because it is not possible to predict where they will go.

All matter, regardless of size, mass or shape, has a point inside called the center of mass. The center of mass is the exact spot where all of the mass of that object is perfectly balanced. The center of mass of an object such as a ruler can be demonstrated by balancing it on your finger. The center of mass can be demonstrated graphically to determine the effect of adding increasing weight to one end of an object such as a rocket body tube.

In addition to the center of mass, the center of pressure inside the rocket affects its flight. When air is flowing past a moving rocket it rubs and pushes against the outer surface of the rocket, causing it to begin moving around the center of mass. Think of a weather vane. The arrow is attached to a vertical rod that acts as a pivot point. The arrow is balanced so that the center of mass is right at the pivot point. When the wind blows, the arrow turns, and the head of the arrow points into the oncoming wind. The tail of the arrow points in the downwind direction. The reasons for this is that the tail of the arrow has a much larger surface area than the arrowhead. The flowing air imparts a greater force to the tail than the head and the tail is pushed away. Similar to the concept of the center of mass, there is a precise point in the arrow where all the aerodynamic forces acting on it are perfectly balanced. This spot is called the center of pressure. It is not the same as the center of mass. The center of pressure is between the center of mass and the tail end of the arrow. The tail end has more surface area than the head end. The lateral center of pressure has to do only with the forces applied to the surface directly by air currents. The larger the surface the greater the forces will be.

Stability is dependent upon the relationship between the center of mass and the center of pressure. The center of pressure in a rocket must be located toward the tail and the center of mass must be located toward the nose. If they are in the same place or very near each other, then the rocket will be unstable in flight.

The center of mass may be moved forward by adding weight to the nose of the rocket. The center of pressure may be moved toward the rear by moving the fins back, increasing their size or by adding fins. The center of pressure can be moved forward by using smaller fins.

Calculating the ratio of length to diameter in a rocket will help determine the potential for stability in a design. An ideal ratio is 10 to 1 (a 1-inch diameter rocket with a length of 10 inches, for example).

Newton's Three Laws of Motion

1. Objects at rest will stay at rest, and objects in motion will stay in motion in a straight line at constant velocity unless acted upon by an unbalanced force.

To understand this law it is necessary to understand the terms: rest, motion and unbalanced force.
Rest and motion can be thought of as opposite. Rest is the state of an object when it is not changing position in relation to its surroundings. Rest cannot be defined as a total absence of motion because it could not exist in nature. All matter in the universe is moving all the time, but in the first law of motion, motion means changing position in relation to surroundings. When an object is at rest, the forces acting upon it are balanced. In order for an object to begin moving, the forces acting upon it must become unbalanced.

A model rocket is at rest when it is on the launch pad. The forces acting upon it are balanced. The force of gravity is pulling the rocket downward and the rocket launch pad is pushing against it holding it up. When the propellant in the engine is ignited, that provides an unbalanced force. The rocket is then set in motion and would stay in a straight line until other unbalanced forces act upon it.

2. Force is equal to mass times acceleration.

This is really a mathematical equation, $f = ma$. This equation applies to launching the rocket off the launch pad. It is essential to understand that there are four basic forces operating on any object moving through the air. These are lift, drag, gravity and thrust. In the context of this unit, the concept of thrust will be emphasized. Thrust is a forward propulsive force that moves an object. In a model rocket, thrust is produced by the rocket engines. Thrust must be greater than the weight of the rocket in order to overcome gravity and lift off from the earth.

As the engine ignites and thrust develops, the forces become unbalanced. The rocket then accelerates skyward with the velocity increasing from its initial state at rest (velocity = 0). When examining how thrust is developed in a rocket engine, force in the equation can be thought of as the thrust of the rocket. Mass in the equation is the amount of rocket fuel being burned and converted into gas that expands and then escapes from the rocket. Acceleration is the rate at which the gas escapes. The gas inside the rocket does not really move. The gas inside the engine picks up speed or velocity as it leaves the engine. The greater the mass of rocket fuel burned and the faster the gas produced can escape the engine, the greater the thrust of the rocket.

3. For every action there is always an opposite and equal reaction.

A rocket can lift off from a launch pad only when it expels gas out of its engine. The rocket pushes on the gas and the gas pushes on the rocket. With rockets, the action is the expelling of gas out of the engine. The reaction is the movement of the rocket in the opposite direction. To enable a rocket to lift off from the launch pad, the action or thrust from the engine must be greater than the weight of the rocket.

NOTES

UNIT PLAN

Lesson 1 - Introduction (1 or 2 days)

Demonstration: Launching a Model Rocket

Objectives of the Demonstration:

The student will be able to:

- Generate individual questions and predictions regarding model rockets and the launch.
- Recognize the need for following the NAR Model Rocketry Safety Code and point out the use of the safety code during the demonstration launch.
- Participate in the demonstration launch as an observer or as a recovery crew member.
- Identify and describe the stages of a model rocket flight.
- Complete an interest survey based on individual knowledge of rockets and plans for the individual construction and testing of a model rocket.
- Discuss the course goals and objectives including the value of building and launching a model rocket to test predictions and what they will need to know to build and launch a model successfully.

BACKGROUND FOR THE TEACHER

Launching a model rocket at the beginning of the unit will capture the interest of the students and lead them to consider questions they may have about how and why rockets function as they do. Launching a rocket and evaluating the flight of the rocket provides a set for the unit and also provides motivation for the students to build their own model.

STRATEGY

Materials needed for each student: A copy of the NAR Safety Code, a copy of the "Focus" sheet and a copy of the interest survey. Students should have a manila envelope or a folder for the materials and sheets that will be accumulated during this unit.   Books, pamphlets, design sheets and catalogs related to model rocketry.

Materials needed for the demonstration launch: Your Estes supplier can provide you with the information you need to acquire an electrical launch system.

Recommended launch area:

ENGINES	SITE DIAMETER Feet/Meters	MAXIMUM ALTITUDE Feet/Meters
1/2 A	50/15	200/61
A	100/30	400/122
B	200/61	800/244
C	400/122	1600/488
D	500/152	1800/549

.

Minimum launch site in dimension for a circular area is diameter in feet/meters and for rectangular area is shortest side in feet/meters.

Choose a large field away from power lines, tall trees and low-flying aircraft. The larger the launch area, the better your chance of recovering the rocket. The chart above shows the smallest recommended launch areas. Football fields, parks and playgrounds are good areas. *Make sure the launch area is free of obstructions, dry weeds, brown grass or highly flammable materials.*

A. Before the demonstration launch, distribute the "focus" sheet or display a transparency on the overhead projector. (Appendix) Ask the students to respond to the questions in the first two columns. After the launch, discuss the questions in the remaining columns. Ask students to fill in the remaining columns. This technique builds support for the unit because it demonstrates value for what the students know and what they want to know.

What do I know about model rockets?	What do I want to learn about model rockets?	After the launch: What is a new idea or knowledge that I have since watching the launch?	After the launch: What did I see that I would like to know more about?

B. Distribute a copy of the NAR model rocket safety code to each student and review the safety code by using the demonstration rocket and going over each step of the demonstration launch and flight.

C. Appoint several people to be the recovery crew - people who follow the flight, recover and return the rocket to the launch pad.

D. Using the Overhead Projector, briefly review the flight sequence of a model rocket. (Appendix A)

Launch the rocket following the directions . . . . . .

Remind the students to watch for the following: Steps of model rocket flight sequence and the performance of the rocket.

Option: Each student could be given a copy of the flight sequence to record their observations.

E. After the rocket flight, process the sequence with the students and discuss their evaluation of the performance of the rocket at each stage.

F. An interest survey may also be used. (Appendix) The interest survey can be useful for the student and for the teacher. It will help the students focus on their own interests and questions so that the project can be meaningful to them individually. It can help guide the teacher's focus within the parameters of the objectives and goals for the course. It is important to provide a number of books, pamphlets, catalogs and design sheets to give students ideas for building their model rockets and ideas for the project types and for the study projects they want to do.

Interest Survey

Name ________________________________

1. Have you ever built a model rocket? Y N
   How many have you built?
2. List three things that were positive about the model rockets you have built.
   List three problems you had with building a model rocket.
3. List at least three things you would like to know more about having to do with space and rocketry.
   As you make a choice of study projects, think about the things you would specifically like to know so that your project fits your interests.
4. You will participate in several study projects with your model rocket.:
   a. Altitude tracking
   b. Data reduction
   c. Speed of a model rocket
   d. Photography from a model rocket (optional)
   e. Egg lofting rocket (optional)

G. Discuss with the students the value of building and launching their own model rockets to test out their ideas. Using their observations of the flight sequence, discuss with the students what they will need to know in order to build and launch a model successfully.

Evaluation:

Observation of student participation and questions. Review student work in notebook on focus sheet, survey and flight sequence observations.

NOTES

Lesson 2 - (1 or 2 days)

Finding the Center of Mass

Objectives of the Lesson:

The student will be able to:

- Find the center of mass of an object experimentally.
- Change the center of mass of a rocket body tube and graph the findings.
- Recognize the role of center of mass in rocket stability.

BACKGROUND FOR THE TEACHER

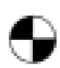
The center of mass (center of gravity) is the exact spot where all of the mass of that object is perfectly balanced. One of the ways that the center of mass can be found is by balancing the object, such as a ruler, on your finger. If the material used to make the ruler is of uniform thickness and density, the center of mass should be at the halfway point between one end of the stick and the other. If a nail were driven into one of the ends of the ruler, then the center of mass would no longer be in the middle. The balance point would be nearer the end with the nail.

The center of mass of an object can be changed by adding weight to one part of the object. For students who are building model rockets, an important concept is the continuing effect on the center of mass that occurs as more weight is added. The center of mass moves less distance as more weight is added. Students can graph this effect.

VOCABULARY

Center of mass: The point at which the mass of an object such as a model rocket is evenly balanced.

Center of gravity: The point in a rocket around which its weight is evenly balanced; the point at which a model rocket will balance on a knife edge; the point at which the mass of the rocket seems to be centered. Abbreviation: CG. Symbol:



STRATEGY

Materials needed for each student: Rocket body tubes. A few wooden dowel sticks about two feet long and small weights that can be attached safely to the sticks (three lead weights weighing about one gram each for each student or group of students).

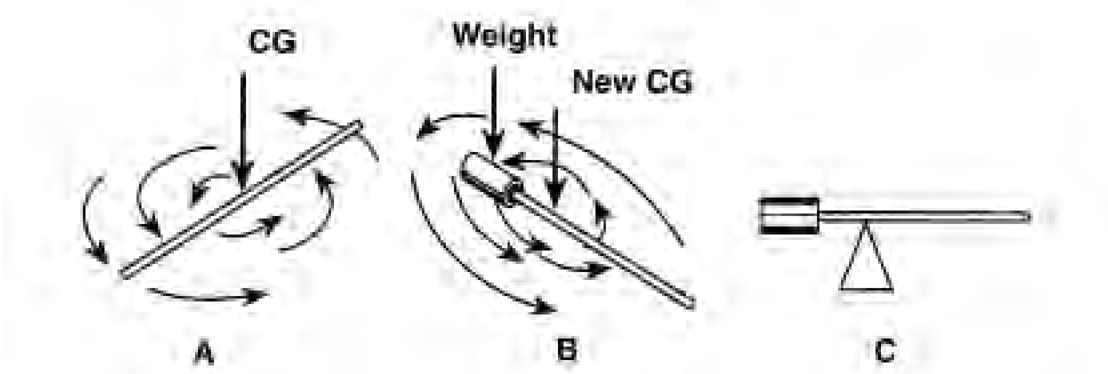
NOTE - Each student will need the supplies necessary to construct a model rocket for this unit.

Estes rocket kits can be purchased individually or Bulk Packs are available which contain the components for twelve model rockets. Teachers can allow class time for students to build a rocket or the students can work on these outside class time.

MOTIVATION:

Briefly discuss with the students the concept of the center of mass and its importance in model rocket flight. Wrap a piece of colored tape around the mid-point of a wooden dowel then toss it vertically and ask the students to observe what the stick does. Ask the students to describe what they observed. Toss it again and ask them to think about the concept of center of mass.

Distribute dowel sticks (each with a midpoint mark) to groups of students. Ask them to toss the stick vertically and horizontally, hard and easy. They should begin to make generalizations about the fact that the stick always rotates around its center. Attach a weight to one end of each stick. Ask the students to observe what changes with the added weight. This time the point about which it rotates will be closer to the weighted end. Show the students that by taking the weighted stick and balancing it across a sharp edge you will find that the new center of gravity has shifted towards the weight and that this is the point about which it now rotates when tossed in the air. Wrap colored tape at this new point and observe again as the dowel is tossed.



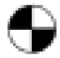
A. Discuss with the students how this experiment relates to model rocket flight. Points to be made:

- The experiment shows how a free body in space rotates around its center of gravity. A model rocket in flight is a free body in “space”. If, for any reason, a force is applied to the flying rocket to cause it to rotate, it will always do so about its center of gravity.
- Rotating forces applied to rockets in flight can result from lateral winds, air drag on nose cones, weights off-center, air drag on launch lugs, crooked fins, engine mounted off-center or at an angle, unbalanced drag on fins or unequal streamlining.
- Since rotating forces will always be present, your rocket must be designed to overcome them. If not, it will loop around and go “everywhere” and end up going nowhere.
- Discuss the demonstration launch in relation to the stability of the rocket.

B. In the following activity the students discover graphically that the movement of the center of mass becomes smaller as more mass is added. This concept will be useful as students begin to understand the importance of the relationship between the center of mass (center of gravity) and the center of pressure in rocket stability.

<u>All age students - elementary students will require more guidance</u>.

Materials needed for each student: A rocket body tube, three lead weights about one gram in mass, graph paper with 1 cm. squares or worksheet in the appendix and a centimeter ruler.

- Distribute a rocket body tube to each student. Review with the students the concept of center of mass. Discuss how the students could determine the center of mass of the rocket body tube. Each student should find the center of mass by balancing the body tube on their finger. Students should mark the point where it balances with a colored pencil. They can use the symbol for center of mass: 

- Students can work in pairs or small groups, but each student should have a rocket body tube and should graph their own findings. Elementary students may need assistance and support as they participate in the graphing activity. The graph (Appendix) is appropriate for elementary and middle school students. Older students can use graph paper. Let each student weigh their own weights to make certain they know the weight.
- Directions:
    1. Place your first weight in one end of the body tube. Find the new center of mass by rebalancing the body tube across your finger. Mark the new center of mass with a different color pencil. Repeat the procedure by adding one weight at a time. Mark each center of mass on the body tube.
    2. Measure the distance between the ends of the rocket tube and the first center of mass you determined without weights and plot that on the graph. Measure the distance the center of mass moved with one weight and plot that on your graph. Continue until you have plotted all the changes.
    3. Discuss the following questions:

       What can you observe about the movement of the center of mass as more weight or mass is added?

       Which way does the center of mass move? Does it move toward or away from the added weights?

       Was the movement smaller or larger?

       Why do you think this happens?

Evaluation: Observation of student participation, review student work in notebook on results of graphing.

NOTES

Lesson 3 (2-4 days)

Finding the Center of Pressure

Objectives of the Lesson:

The student will be able to:

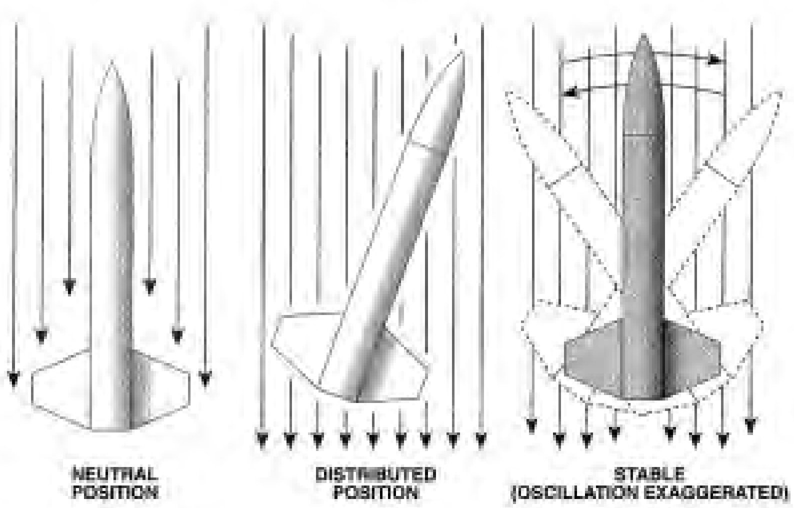
- Recognize the role of center of pressure in rocket stability.
- Determine the approximate center of pressure of geometric shapes experimentally.
- Determine the approximate center of pressure of a model rocket using a cutout of the model.
- Calculate the lateral center of pressure of a model rocket using the math formulas for determining the centroid of a shape and the area of a shape, including a rectangle, a semi-ellipse, a parabola, a triangle, a semi-circle and an ogive to use in calculation of the lateral center of pressure for the entire rocket.

BACKGROUND FOR THE TEACHER

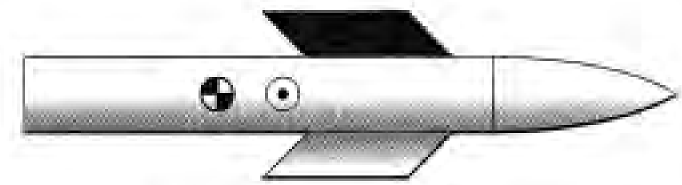
The center of pressure is the point where all the aerodynamic forces are balanced. Aerodynamic forces forward of this point are equal to the aerodynamic forces behind this point. If a model rocket is suspended at its center of pressure in a moving stream of air, the rocket will not attempt to return to “forward flight” should it become “aimed” away from the forward orientation. The lateral center of pressure has to do only with the forces applied to a surface directly by air currents. The larger the surface the greater the forces will be.

To guide itself in a forward direction while moving through the air, the rocket must have its center of gravity forward of its center of pressure. For model rockets, the distance between the center of gravity and the center of pressure should be at least one caliber (the diameter of the body tube). Generally, the greater this distance, the more stable the rocket. If the separation of these two points is less than this distance, the rocket may be unstable. Unstable rockets will not return to their original flight path if disturbed from this path while in motion.

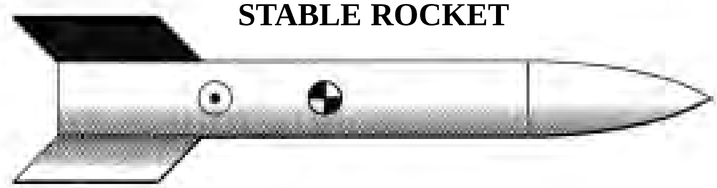
Stability in a model rocket can be defined as the tendency of a rocket to travel in a straight course with the direction of its thrust despite rotating forces caused by outside disturbances. A stable rocket will travel in a relatively straight upward direction, without tumbling and with minimum oscillation. In a later unit, students will test the stability of their own model rocket through the use of a “swing test” or the use of a wind tunnel.



UNSTABLE ROCKET



STABLE ROCKET

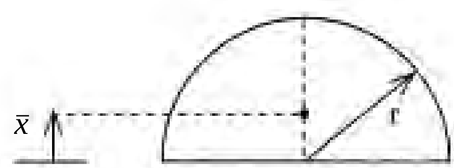


Younger students can be helped to understand the center of pressure by finding the balance point of a number of geographic shapes used in the construction of model rockets. (See Appendix) This balancing point, referred to as the centroid, describes the geometric center of area for a given shape. The term $\bar{x}$ locates the centroid from a reference point.

EQUATIONS:

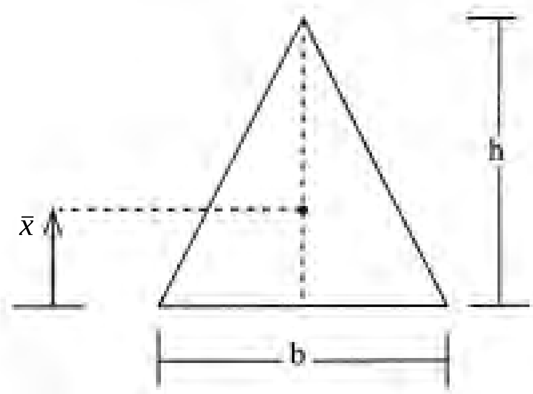
Semi-circle

$$\bar{x} = \frac{4r}{3\pi} \qquad A = \frac{\pi r^2}{2}$$



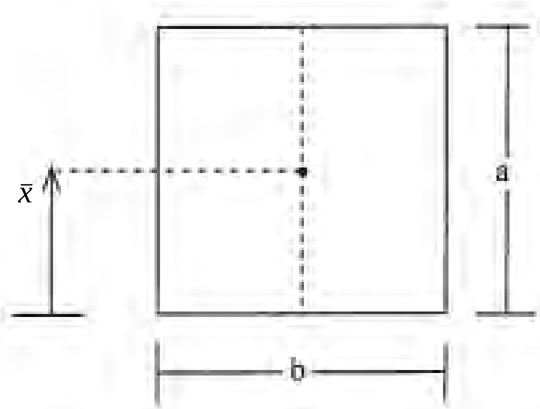
Triangle

$$\bar{x} = \frac{h}{3} \qquad A = \frac{bh}{2}$$



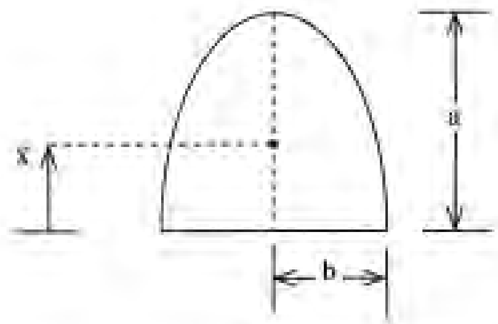
Rectangle or Square

$$\bar{x} = \frac{a}{2} \qquad A = ab$$



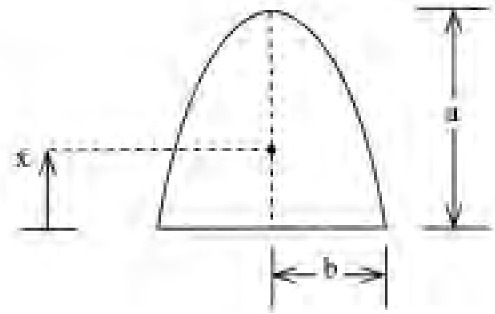
Semi-ellipse

$$\bar{x} = \frac{4a}{3\pi} \qquad A = \frac{\pi ab}{2}$$



Parabola

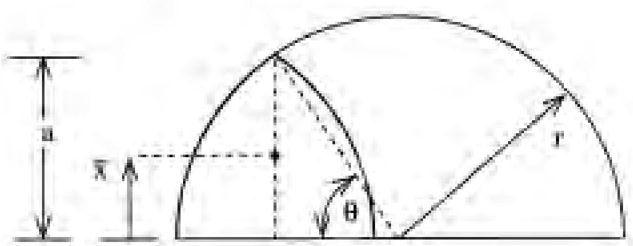
$$\bar{x} = \frac{2a}{5} \qquad A = \frac{4ab}{3}$$



Ogive

$$\bar{x} \approx .38a \qquad A = \frac{r^2(2\theta - \sin 2\theta)}{2}$$

θ in radians
$360^\circ = 2\pi$ radians
a = r sin θ



Calculating the center of pressure of rocket outline

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

A = Area of each section
x = distance from the centroid of each section to the base.

VOCABULARY

Center of pressure: The center for all external aerodynamic forces on the complete rocket including the body and fins. Abbreviation: CP Symbol:

Centroid of a shape: Determined through mathematical calculation or experimentation to provide data for determining the center of pressure. The centroid of a two-dimensional shape defines the geometric center of its area.

STRATEGY

Materials needed for each student: Packet containing geometric shapes, worksheet for determining centroids and area of each shape mathematically, outline of rocket shape and equation for determining lateral center of pressure. Each student will need a centimeter ruler for measuring shapes and rocket outline. For advanced math students, sheets containing center of pressure equations for a variety of rocket configurations.

MOTIVATION: The major concept to achieve in this unit is that the center of pressure is related to the area of a rocket.

Use a low friction pivot to demonstrate lateral center of pressure or the overhead transparency demonstrating the center of pressure (OH - Lesson 3).

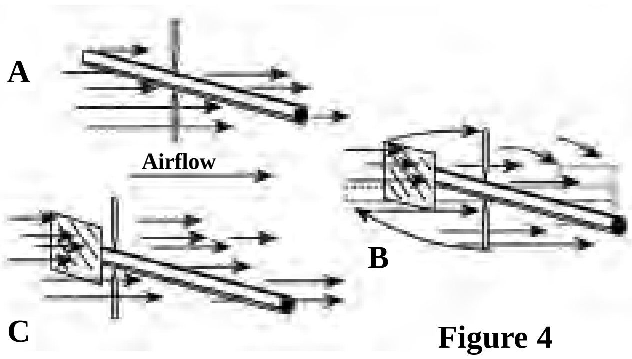


Figure 4

(A low friction pivot consists of two needle points held rigidly in place on opposite sides of the object by a heavy wire or board frame-work. The needle points are placed against the object just tightly enough to hold it, without interfering with its rotating on the axis created between the two points.)

Demonstration: Place a two foot long piece of dowel on a low friction pivot as shown in A of Figure 4.

Hold the dowel in a uniform air current of ten to fifteen miles per hour. If the pivot has been placed in the center of the dowel and if the dowel is uniform in size (area), the forces exerted by the air pressure will be equal on both sides of the pivot and the air current will produce no rotating effect.

If a low friction pivot is not available, the same thinking process can be achieved by using the overhead transparency as a basis for discussion.

Ask the students to observe the effect of the wind on the dowel in this position and allow them to guess why there is no rotating effect.

In this condition, the center of gravity and the center of pressure will be at the same point.

Add a vane of three inch by three inch cardboard by gluing it to one end of the dowel. Put it in the air stream with the pivot in the same position.

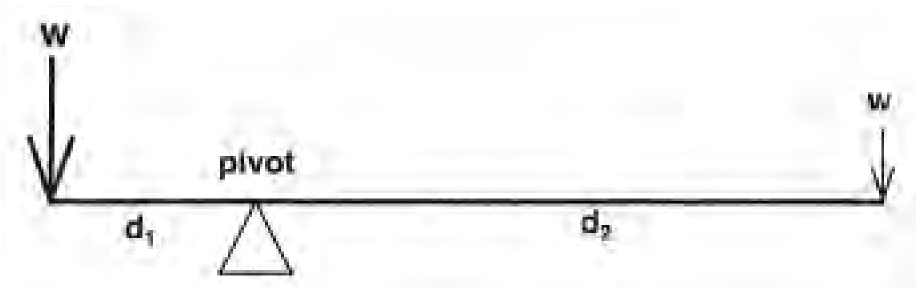
Ask the students to observe the effect of the air current on the dowel and allow them to guess why the effect is different.

The moving air current will exert the greatest force against the end of the dowel which has the vane attached to it . This will cause the dowel to rotate until the end away from the vane points into the wind.

Ask the students to make predictions about where the pivot could be moved so that the rotating effect is stopped.

The pivot should be moved closer to the vane end of the dowel until a point is located where equal air pressure will be applied to both ends. The air current will no longer cause any part of the dowel to point into the wind. This point is called the lateral center of pressure. The lateral center of pressure has to do only with the forces applied to the surface directly by air currents and the larger the surface the greater the forces will be.

Aerodynamic forces acting on a rocket are like a teeter-totter (fulcrum and lever).



Not only does the magnitude of the force affect the balance, but also how far away from the pivot point the force acts. The smaller force “w” acting at a distance d_2 has the same effect of a larger force “W” acting at a distance d_1. This is just like the aerodynamic forces acting on a rocket. Forces acting at a distance away from the pivot point (or CG for a rocket) create a “Moment”; force times distance. In the example above, the moments created by W and w are balanced.

A. Distribute packets. Discuss that each one of these shapes has a center of pressure which can be approximately determined by finding the balance point of each one. Point out that these are shapes commonly found in model rockets, the nose cone, the body and the fins. The centroid of these shapes can also be determined mathematically and is a factor in determining the lateral center of pressure of a model rocket. Connect the experiment with the dowel and the cardboard vane to the concept of center of pressure.

For all students from elementary through high school:

• Students may work in pairs or in small groups but each student should have a set of shapes.

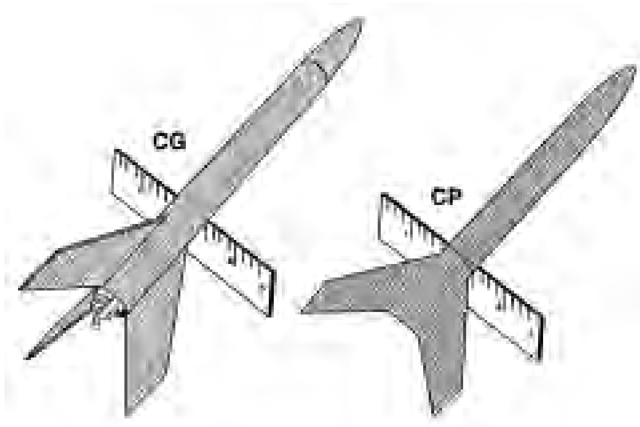
The shapes may be cut out and used as is if a heavy bond paper is used when reproducing the shapes. If the teacher prefers, students may trace the shapes onto tag board and then cut them out. Using a colored pencil, each student should mark on the shape of the square the point where they think the center of pressure is. Next, ask them to balance the square on a finger and mark the point of the actual center of pressure (the point where the shape balances on a finger). Continue the same process with each shape.

For some elementary students and middle school students with guidance and for high school students:

Students should complete the packet for Lesson 3, which includes equations for determining the centroids and areas of the geometric shapes and model rockets. (Appendix) Elementary students may need to be guided through the equations step-by-step. Middle school and high school students can use the equations after some demonstration or review. The computer program, Estes ASTROCAD™: Performance Analysis for Model Rocketry , allows students to check their calculations or to determine the center of pressure given measurements without going through the equations. However, both methods help students thoroughly understand the mathematics involved.

For younger students or those without sufficient math background, a quick way to determine the approximate CP of a model rocket is to make an exact-size cutout of the model (a profile) in cardboard. The point where the cutout will balance on the edge of a ruler will be the approximate lateral center of pressure. Air pressure applied to a surface is proportional to the area of the surface, so the cutout allows the student to approximate the rotating effect of the action of the air pressure.

• Distribute a sheet of cardboard to each student. Students should make a cutout of their own rocket. Lay the rocket over the piece of cardboard and mark around the edges. Cut around the lines and balance the cutout on a knife edge or ruler edge. Mark the center of pressure of the cutout.



These methods determine the lateral center of pressure (the center of pressure with the air currents hitting the rocket broadside). If the rocket is designed so the lateral center of pressure is one body diameter behind the center of gravity it will have ample stability under all reasonable conditions. If, however, the rocket's fins are very crooked, set at opposing angles or if the rocket uses a disc or cone for stabilizing, the lateral center of pressure should be set at least 1 1/2 diameters behind the center of gravity.

A more accurate method for calculating the center of pressure is available in the Appendix, Lesson 7. This method applies the same concept, but uses much more detailed information.

Extension

Using the model outline, students may redraw the fins making larger triangles or making the fins square or rectangular. Using the equations, they can recalculate the center of pressure.

NOTES

Lesson 4 (2-3 days)

Rocket Stability

Objectives of the Lesson:

The student will be able to:

- Describe the relationship between the location of the center of mass and the center of pressure in the stability of a model rocket.
- Discuss the importance of stability in model rocket flight.
- Test the stability of own model rocket using either the swing test or the wind tunnel test.
- Correct the design and construction of own model rocket to provide stability.

For this unit, the student needs to have constructed a model rocket with fins attached and the engine mounted. The launch lug needs to be attached as well.

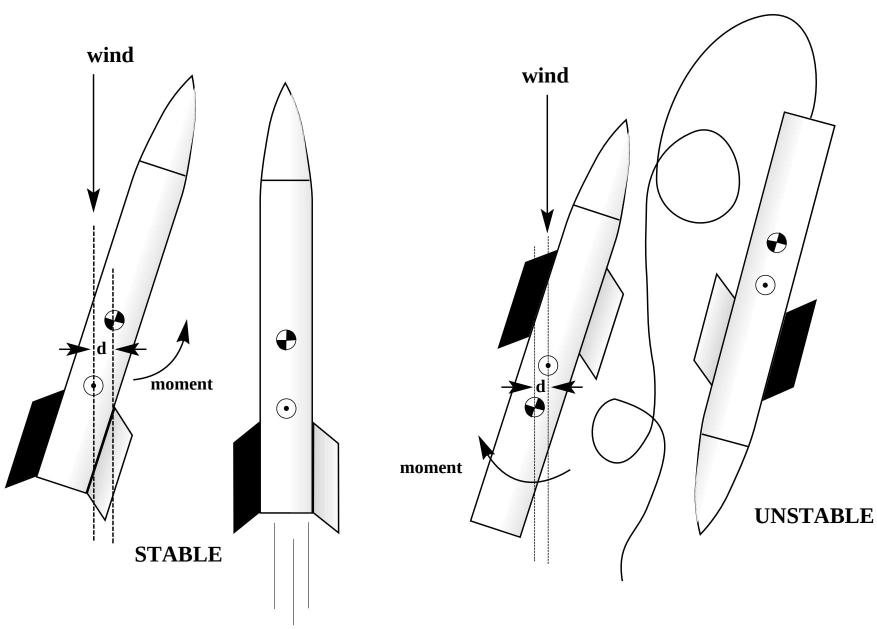
BACKGROUND FOR THE TEACHER

Stability is dependent upon the relationship between the center of mass and the center of pressure. The center of pressure in a rocket must be located toward the tail and the center of mass must be located toward the nose. If they are in the same place or very near each other, the rocket will be unstable in flight.

As the students have seen, the center of mass may be moved forward by adding weight to the nose of the rocket. The center of pressure may be moved toward the rear by moving the fins back, increasing their size or by adding fins. The center of pressure can be moved forward by using smaller fins.

In flight, the rocket will not be traveling sideward, but with its nose pointed into the wind. With the model's nose pointed into the wind, the location of the effective center of pressure will be affected by the shape of the fins, the thickness of the fins, the shape of the nose cone and location of the launch lug. With most designs this shift is to the rear, adding to the stability of the rocket. It is important to emphasize to the students how essential it is to construct the rocket carefully and for the need to have a rocket that is stable.

If a model rocket starts to rotate in flight, it will rotate around its center of gravity. When it turns, the air rushing past it will then hit the rocket at an angle. If the center of pressure is behind the center of gravity on the model, the air pressure will exert the greatest force against the



fins. This will counteract the rotating forces and the model will continue to fly straight. If the center of pressure is ahead of the center of gravity, the air currents will exert a greater force against the nose end of the rocket. This will cause it to rotate even farther. Once it has begun rotating it will become unstable. Remember, the lateral center of pressure is the one point where the aerodynamics forces act. In the illustrations, all the wind can then be treated as a single force acting through the center of pressure. This force acting at a distance away (d) from the CG creates a moment that either stabilizes or destabilizes the rocket.

It is best to build a rocket with its fins as far as possible to the rear. The farther behind the center of gravity the center of pressure is placed, the stronger and more precise will be the restoring forces on the model and it will fly straighter with less wobbling and side-to-side motion, which robs the rocket of energy. Fins usually should not be placed forward of the center of gravity on a model because this will add to instability. If fins are added forward of the center of gravity, be certain the center of pressure remains behind the center of gravity.

Students can also test the stability of a rocket with precision experimentally through the use of the swing test or the use of a wind tunnel.(Directions for building a wind tunnel may be found in Estes The Classic Collection, Technical Report TR-5, "Building a Wind Tunnel"). The simplest, least expensive method is the swing test.

The rocket to be tested should have its engine in place as it would be in flight. With the engine installed, the center of gravity is shifted further to the rear placing the center of gravity at its most critical position. The rocket is suspended from a string. The string is attached around the rocket body using a loop. Slide the loop to the proper position so the rocket is balanced, hanging perpendicular to the string. Apply a small piece of tape to hold the string in place. If the rocket's center of gravity falls in the fin area, it may be balanced by hooking the string diagonally around the fins and body tube. A straight pin may be necessary at the forward edge of one of the fins to hold the string in place. This string mounting system provides a low friction pivot about which the rocket can rotate freely.

If a wind tunnel is being used, slide a soda straw along the string to a position just above the rocket. Then suspend the rocket in a low velocity air stream with the nose of the rocket pointing into the wind. Then turn the rocket approximately 10˚ out of the wind to see if it recovers. If so, the rocket is stable enough for flight.

The swing test method involves swinging the suspended rocket overhead in a circular path around the individual. If the rocket is stable, it will point forward into the wind created by its own motion. If the center of pressure is extremely close to the center of gravity, the rocket will not point itself into the wind unless it is pointing directly forward at the time the circular motion is started. This is accomplished by holding the rocket in one hand, with the arm extended, and then pivoting the entire body as the rocket is started in the circular path. Sometimes several tries are needed in order to achieve a perfect start. If it is necessary to hold the rocket to start it, additional checks should be made to determine if the rocket is flight-worthy.

Small wind gusts or engine misalignment can cause a rocket that checks out stable when started by hand as described above to be unstable in flight. To be sure that the rocket's stability is sufficient to overcome these problems, the rocket is swung overhead in a state of slight imbalance. Experiments indicate that a single engine rocket will have adequate stability for a safe flight if it remains stable when the above test is made with the rocket rebalanced so the nose drops below the tail with the rocket body at angle of 10˚ from the horizontal.

With cluster powered rockets a greater degree of stability is needed since the engines are mounted off center. The cluster powered rocket should be stable when unbalanced to hang at 15˚ from the horizontal. Heavier rockets which accelerate at a lower rate require a similar margin of stability.

Caution should be exercised when swinging rockets overhead to avoid a collision with objects or people nearby. It is possible to achieve velocities of over 100 miles per hour. This can cause injury.

If the student has constructed a rocket that will not be stable, <u>do not attempt to fly it</u>. Corrections have to be made. Lack of stability will cause the rocket to loop around and around in the air. It will seldom reach over 30 feet in height and can never reach a velocity of more than 20 or 30 miles an hour. Also, it is possible that a rocket could suddenly become stable after making a couple of loops, due to the lessening of the fuel load. This could cause it to fly straight into the ground and could cause serious injury or damage.

If a rocket does not show the degree of stability required for safety it can be easily altered by either moving the center of gravity forward or by moving the center of pressure to the rear. To move the center of gravity forward, a heavier nose cone is used or a weight, such as clay, is added to the nose of the rocket. To move the center of pressure to the back, fins may be made larger or moved farther back on the body tube.

Calculating the ratio of length to diameter in a rocket is another method to determine the potential for stability in a design. An ideal ratio is 10 to 1 (a 1-inch diameter rocket with a length of 10 inches).

The computer program, Estes ASTROCAD™: Performance Analysis for Model Rocketry, allows students to give data about the length, diameter and the location of CG of their rocket to predict the rocket's stability.

VOCABULARY

Stability: The tendency of a rocket with the proper center of gravity/center of pressure relationship to maintain a straight course despite rotating forces caused by variations in design and outside disturbances.

STRATEGY

Materials needed for each student: A completed model rocket, cardboard, ruler, string, tape, worksheet to record data about individual rocket and worksheet to evaluate the stability of each student's rocket and what needs to be done for the rocket to achieve stability.

MOTIVATION: Use the overhead transparencies (stability, CG and CP, stable and unstable flight) to show the relationship between CG and CP in rocket stability. Use the information in the background section of this unit to make the concept clear. A swing test or wind tunnel test using a rocket constructed by the teacher is an excellent way to demonstrate rocket stability.

A. As the students ready their rockets for the swing test, they can observe the relationship of the center of gravity, which is where the string will be, and the lateral center of pressure of the rocket.

For students who understand the equations for calculating the CP:

B. If the computer program is to be used to predict stability of the rocket, the students should enter the data about their rocket on the data sheet. This program calculates the CP and gives the CG for student rockets. The calculations for CP use the same equations that the students used in the unit on center of pressure.

C. Allow each student to do the swing test on their rocket, complete the evaluation sheet and make any alterations necessary.

Evaluation: Observation of each student's swing test and evaluation of worksheets.

NOTES

Lesson 5 (2 to 3 days)

Math and Rocket Flight - Launching a Rocket

Objectives of the Lesson:

The student will be able to:

- Participate appropriately in the launching of each student's rocket.
- Demonstrate proper safety procedures during a launch.
- Record flight data on a class chart and on an individual chart.
- Track the flight of model rockets using an altitude measuring device to determine the angular distance the rocket traveled from launch to apogee.
- Use mathematical equations to determine the altitude or height reached by the model rocket flight using the data collected.
- Graphically determine the altitude reached by the model rocket.
- Use mathematical equations to determine velocities and accelerations during the rocket flight and describe their relationship to Newton's Second Law.

BACKGROUND FOR THE TEACHER

The launch area should be large enough, clear of people and clear of any easy to burn materials. On the day of launch, the wind speed should not be more than 20 mph. Early morning or early evening when there is little wind is the best time of the day to launch model rockets.

The launch pad and the launch wire should be anchored down by bricks or something similar.

The safety cap should be on the launch rod at all times except during the launch.

The teacher should be in possession of the safety key at all times.

DETERMINING ALTITUDE:

Students will be interested in how high their rockets went. Accurate determination of heights reached requires care and precision in measuring, recording and calculating.

Tracking:

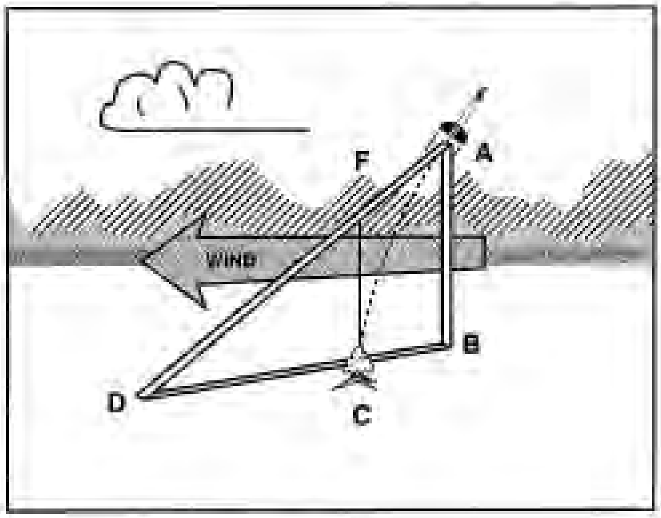
First, determine the length of the baseline. The baseline is the distance between the launcher and the observer or tracker with an altitude measuring device. Accuracy in measuring the baseline is very important in determining altitude. The baseline must be level with the tracking stations on the same level with the launcher.

Next, determine the angular distance the rocket travels from launch to apogee (maximum altitude). The angular distance is determined by measuring the change in elevation angle, as seen by the tracker, between the rocket's position on the launch pad and apogee reached by the rocket in flight. A measuring device, such as the Estes AltiTrak™, is used to find angular distance. The use of the device involves tracking the rocket from the launch pad to apogee, noting and recording the angular distance and then determining the actual height reached by the rocket by the use of a mathematical formula or plotting the information on graph paper.

Set up a tracking system that suits the needs of your group. Accuracy in making and recording all measurements is very important. The simplest is one station tracking. The results are generally reliable. In one station, there is one baseline and one observer using an altitude measuring device such as the Estes AltiTrak™. One station tracking assumes that the flight will be almost vertical.

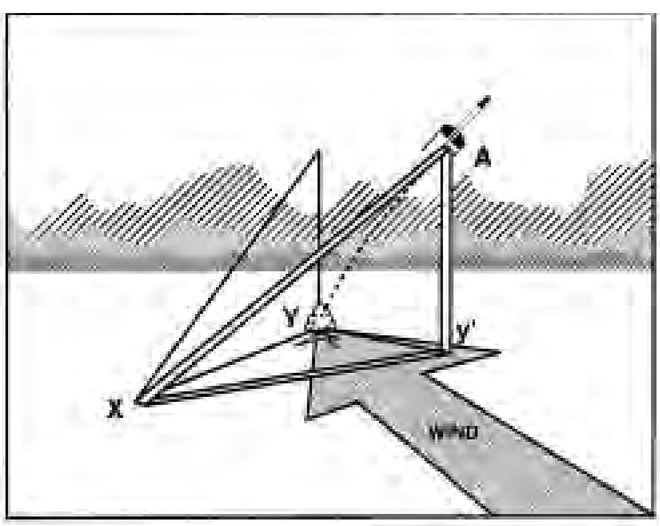
If only one elevation tracker is used, it is a good idea to station it at a right angle to the wind flow. For example, if the wind is blowing to the west, the tracker should be either north or south of the launcher

(Be careful not to stare into the sun). In this way we will keep the angle at C as close to a right angle as possible. By experimenting with a protractor and a straight edge, the rocketeer can demonstrate why the error would be less if the tracker is on a line at a right angle to the flow of the wind.



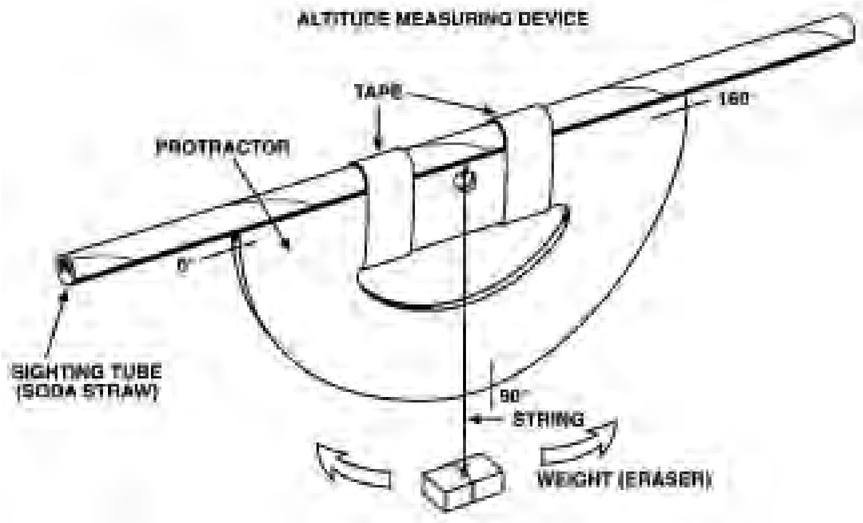
In the figure above, the wind is blowing from B to D. The rocket is launched at point C, weathercocks into the wind following approximately line CA and at its maximum altitude is at point A. If the tracker is downwind from the launcher, the rocket will be seen at point F, and compute the altitude as the distance from C to F. The computed altitudes will be considerably lower than the true altitudes. On the other hand, if the rocket drifts toward the tracker, the computed altitude will be considerably higher than the true altitude.

However, if the tracker is at point X in the following figure and the launcher at Y, then the rocket will appear to be at point A. Although the distance from the tracker to point y[1] will be slightly greater than the baseline used in computing the altitude, the error will not be nearly as great. Also, the small additional distance will serve to make altitude readings more conservative, as the baseline is increased.



By observing the proper relation between wind direction and the position of the tracker, we can generally determine with 90% or better accuracy the altitude the rocket reaches from data given by only one elevation tracker. Thus, on a calm day with a good model, almost perfect accuracy can be approached.

Students can make an altitude tracking device using a straw, a protractor, string and an eraser. (Worksheet in Appendix)

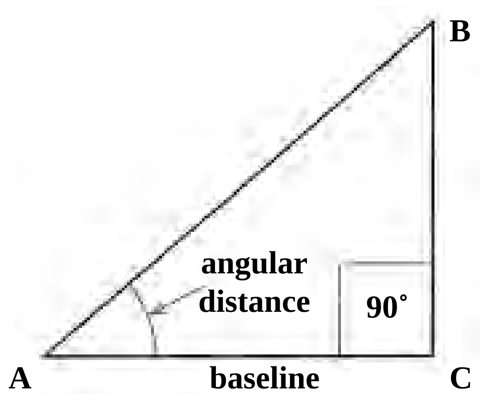


CALCULATIONS

One Station Tracking - Elementary and Early Middle School

The formula for determining the height reached by a model rocket flight using one station tracking is:

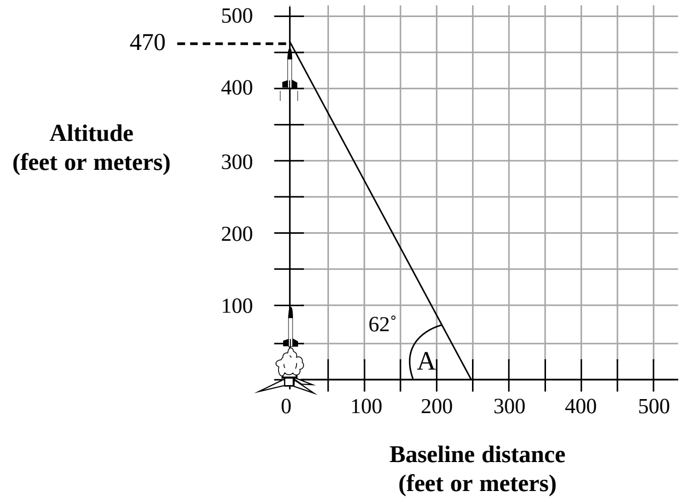
Height = Baseline x Tangent of Angular distance



The tangent is used to determine altitude because the tangent of an angle in a right triangle is the ratio of the opposite side to the adjacent side. In this example, the adjacent side is the distance along the baseline. The opposite side is the distance from the launcher to the rocket's maximum altitude. Tangents can be found in the Table of Tangents in the Appendix.

For younger students that may not be able to comprehend the idea of a tangent, a graphical approach can be used to calculate the altitude. To use this method, a student must be able to plot angles with a protractor and layout scaled distances on graph paper.

Referring to the graph in the figure, use the horizontal axis to plot the baseline distance (distance from launcher to tracker). With the same scale, use the vertical axis to plot the rocket's altitude. The rocket is launched at the origin on the graph paper and climbs vertically up the vertical axis. Mark the tracker's position on the horizontal axis (250 feet) and plot the elevation angle (62˚). Extend the angle (line of sight) until it intersects the vertical axis. The intersected point on the vertical axis is the rocket's altitude. (Blank graph paper for plotting actual flight in Appendix).



If we assume that the rocket flight is vertical, we can call angle C (in the previous figure) a right angle, 90˚. B is equal to 90˚ minus A because the sum of the angles in a triangle is 180˚. By definition:

$\tan\angle = \dfrac{opp}{adj}$ $\tan A = \dfrac{BC}{AC}$ so to find the distance from C to B or the height the rocket reached, take the tangent of Angle A times the distance along the baseline, side AC.

Example:
Baseline = 250 ft.
Angle observed by tracker = 62˚
Tangent of 62˚ = 1.88
H = 250 ft. x 1.88
H = 470 ft.

Two Station Tracking - Middle School and High School:

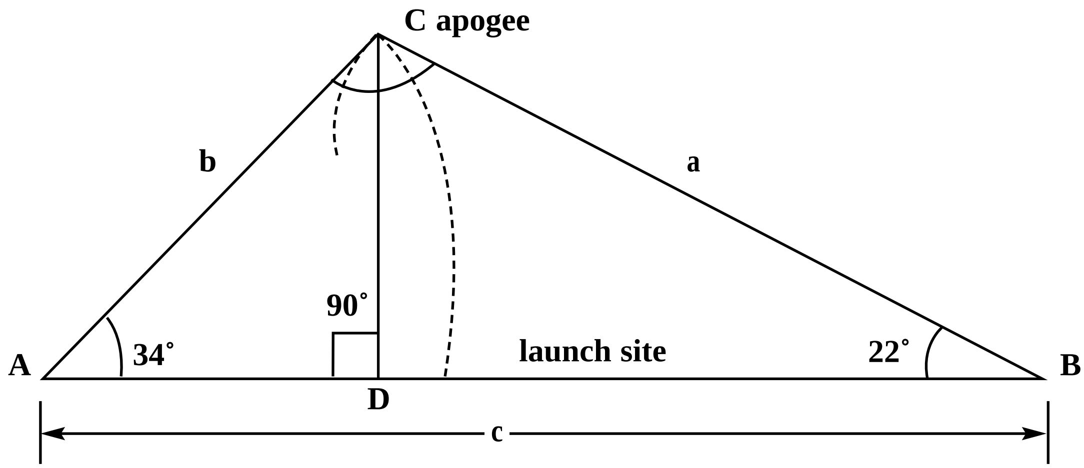
A. When using two trackers without azimuth readings, the tracking stations are set up on opposite sides of the launcher. Preferably, to obtain the greatest accuracy, the stations should be in line with the wind, unlike the system used in single station tracking. If the wind is blowing to the south, one station will be north and the other south of the launch area.

B. In the simplest method of two station tracking, the angles from each station are used together to get one value of height.

C. This more accurate system of two station tracking uses two tracking stations placed on opposite sides of the launch pad in line with the wind. It uses sines instead of tangents. The formula for this method is:

$$CD = \frac{c \times \sin A \times \sin B}{\sin C} \; where\ C = 180^{o} - (A + B)$$

For example, stations A and B are located on a 1000 ft. baseline with the launcher between them. Station A calls in an angular distance of 34˚ and station B calls in an angular distance of 22˚. The total of these two angles is 56˚. Therefore angle C, located at the apogee of the rocket's flight is 124˚ , or 180˚ - 56˚. We know the measurements of three angles and one side.



First, list the angle and their sines.

Angle A = 34˚ Sine A = 0.5592 (Taken from the table of sines and tangents)
Angle B = 22˚ Sine B = 0.3746
Angle C = 124˚ Sine C = 0.8290

c = 1000 ft.

Sin C = 0.8290

The "law of sines" states that $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

Using the formula with the sines and distances known:

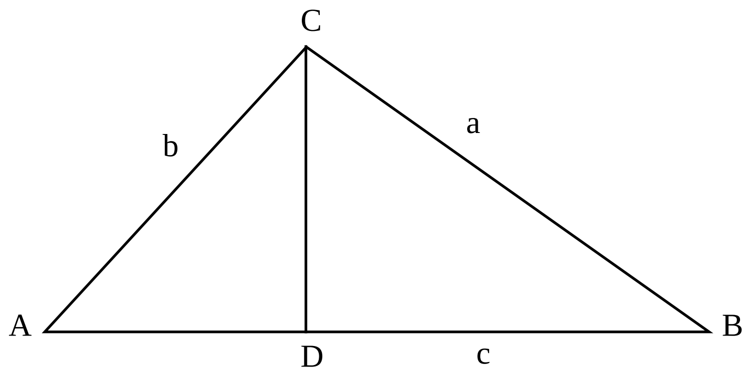
$$\frac{a}{0.5592} = \frac{b}{0.3746} = \frac{1000}{0.8290} \qquad 1206 = \frac{b}{0.3746} = \frac{a}{.5592}$$

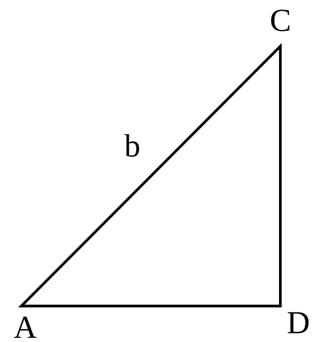
The quotient needed to find dividends, to solve for b and a is 1206.

b = 1206 x 0.3746
b = 451.77 feet

a = 1206 x 0.5592
a = 674.40 feet

Three sides of the triangle are now known.





The altitude of the rocket is the distance from D to C in the diagram. The angle formed by the meeting of AB and CD is a right angle. The sine of an angle in a right triangle is the relation of the opposite side to the hypotenuse.

By definition:

$$\text{Sin A} = \frac{\textit{opposite side}}{\textit{hypotenuse}}$$

The value of the opposite, CD, needs to be determined.

Sine A = 0.5592

$$0.5592 = \frac{CD}{451.77}$$

CD = Sin A x b

CD = 0.5592 x 451 feet

CD = 252.63 feet

We know that the altitude reached by the rocket was 252.63 feet.

Combining the two previous steps into one (law of sines and sines) a quicker formula is derived:

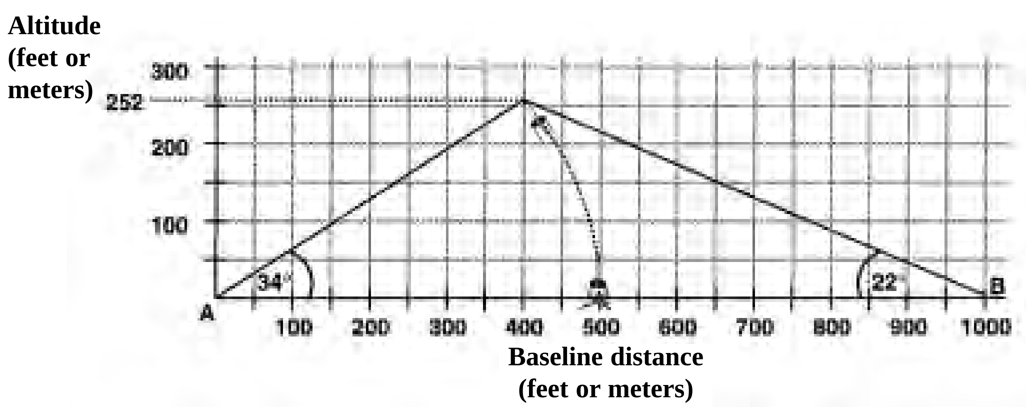
$$CD = \frac{c \times SinA \times SinB}{SinC}$$

$$CD = \frac{1000 \times 0.5592 \times 0.3746}{0.8290}$$

CD = 252.69 *feet*

The second, quicker formula, gives the same answer. The firs formula helps make the relation ship between the angles and sides more clear.

For younger students, this problem can be solved using the graphical approach. Layout baseline distances (horizontal axis) and rocket altitude (vertical axis) on graph paper, as in the figure.



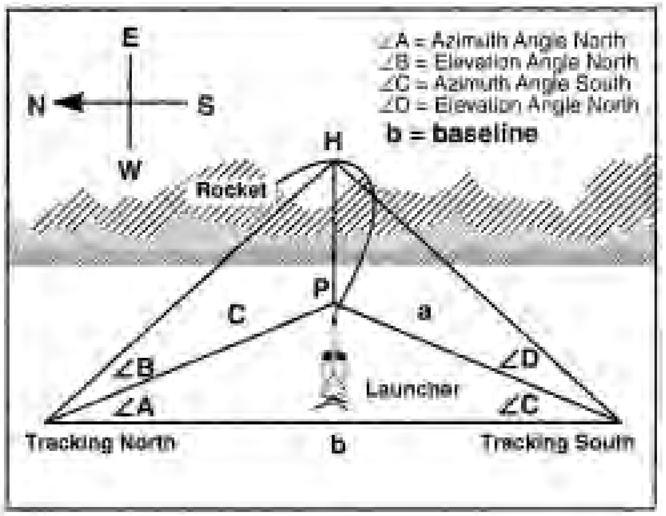
Plot elevation angles A and B, then extend them until they intersect. The point of intersection extended to the vertical axis is the rocket's altitude.

Two Station Tracking Using the Azimuth Angle - High School:

The most accurate system of two station tracking uses the azimuth angle in addition to elevation angles. This system is more complicated but more reliable and compensates for the rocket rarely being vertical over the launch pad at apogee. This system is called the elevation-azimuth system, or the vertical midpoint method.

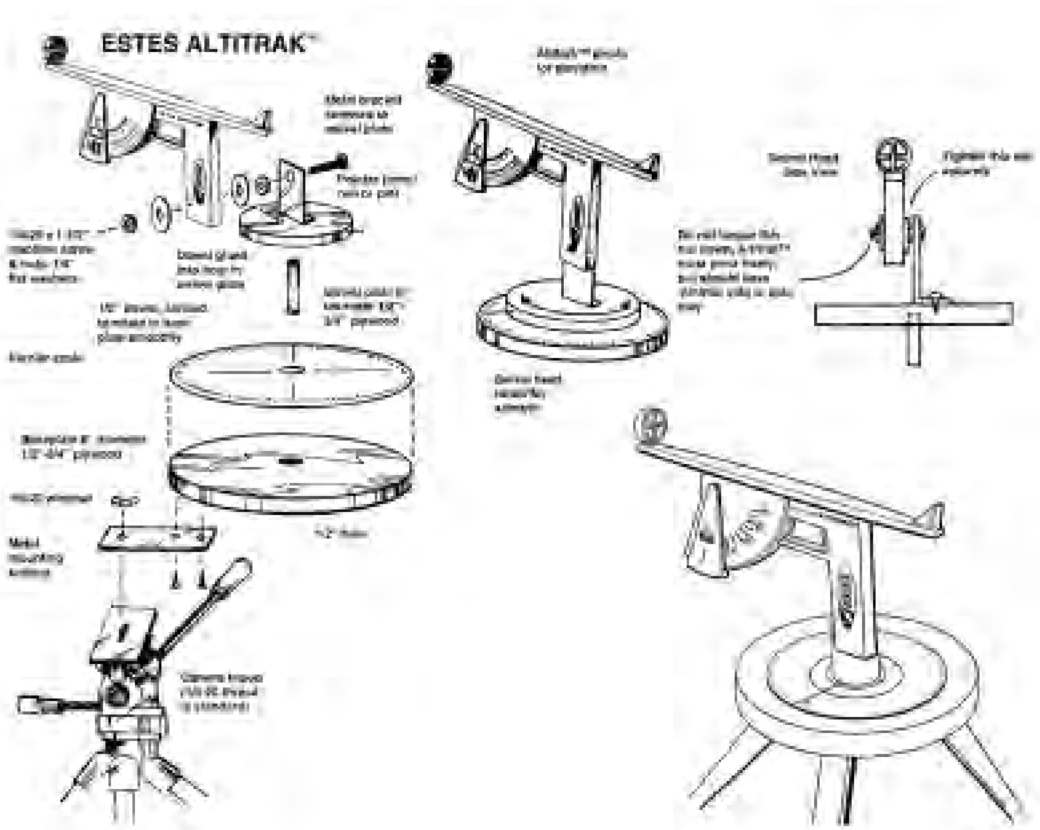
This system requires making five measurements before the height calculations are made. These five measurements are the baseline length, the angular distance the rocket has risen as determined from one tracking station, the azimuth reading of the rocket at apogee from this station, the angular distance the rocket has risen as determined from the other tracking station and the azimuth reading of the rocket at apogee as measured from the other tracking station. An azimuth angle is a direction expressed as a horizontal angle from a given reference point.

Note: Angular distance is often referred to as elevation.



For this system a two station tracking type scope is needed. The two trackers are set up on a base line with the launch pad roughly halfway between them, but off the baseline to one side. Both scopes are then “zeroed” on each other. This means that when looking through the scope toward the other, the azimuth and elevation angles should both read zero degrees. The rocket is then fired and both trackers sight the model until it reaches apogee, at which point they stop and record the angles.

Following is an idea that may be used to construct the tracking station. It uses the Estes AltiTrak® to act as the sighting device and elevation angle recorder (which it is designed to do). The AltiTrak® is mounted to a simple pivoting holder with a pointer to measure azimuth. The holder can be mounted to a camera tripod (it is removable and will not harm the tripod).



The formula for determining the height reached as determined by station “tracking north” is:

$$\text{PH} = \sin \text{C x} \tan\ \angle B \times \frac{(b)}{\sin(180 - [\angle A + \angle C]\ )}$$

The formula for determining the height reached as determined by station “tracking south” is:

$$\text{PH} = \sin\ \angle A \times \tan \angle D \times \frac{(b)}{\sin\left(180 - [\angle A + \angle C\ \ ]\ \right)}$$

Both of these equations are derived by applying the “law of sines” to the azimuth angles to find sides a and c, then using tangents of the vertical angle to find PH.

In this method, heights found by the two tracking stations are averaged. Both heights must be within the range of the average of both heights plus or minus 10 per cent. If they are not, they are considered unreliable and are not used.

Calculate the height reached by a rocket using this method and the following data:

The tracking systems are 200 feet apart.

North

Elevation Angle = 50˚
Azimuth Angle = 30˚
C = 20˚
B = 50˚
A = 30˚
Sine of C = 0.342
Tan of B = 1.192

South

Elevation Angle = 43˚
Azimuth Angle = 20˚
A = 30˚
D = 43˚
C = 20˚
Sine of A = 0.500
Tan of D = 0.933

Tracking North

Height = 0.342 x 1.192 x 200/sin (180-[30˚ + 20˚])
= 0.342 x 1.192 x 200/sin 130˚
= 0.342 x 1.192 x 200/0.776
= 0.342 x 1.192 x 261.1
= 106.4 feet

Tracking South

Height = sin 30˚ x tan 43˚ x 200/sin (180 - [30˚ + 20˚])
= 0.500 x 0.933 x 200/sin 130˚
= 0.500 x 0.933 x 200/0.776
= 0.500 x 0.933 x 261.1
= 121.7 feet

The results from the two tracking stations are averaged:
106.4 + 121.7 = 228 ÷ 2 = 114.1

To see if the results are within 10 percent of the average:
114.1 x 0.10 = 11.4 114.1 + 11.4 = 125.5
The south tracking station with 121.7 is within the 10 percent allowed.
114.1 - 11.4 = 102.7
The north tracking station with 106.4 is within the 10 percent allowed.
In this example 114.1 would be recorded as the average height.

Another formula for calculating the percentage where the results are in respect to one another is the following:

$$C = \frac{h\ north - h\ south}{2h\ avg} x100\ \%$$

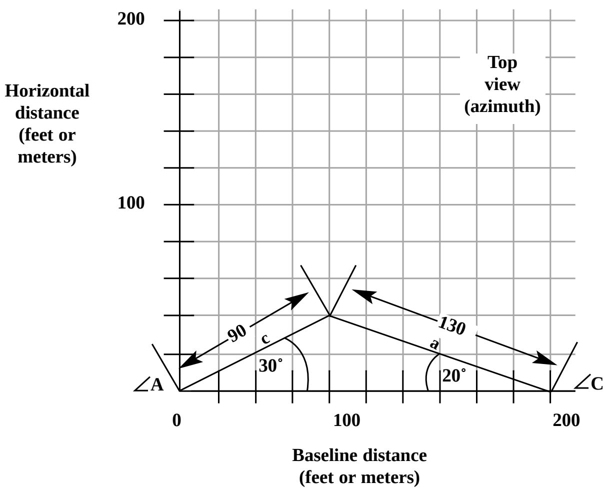
Using the data from the example:

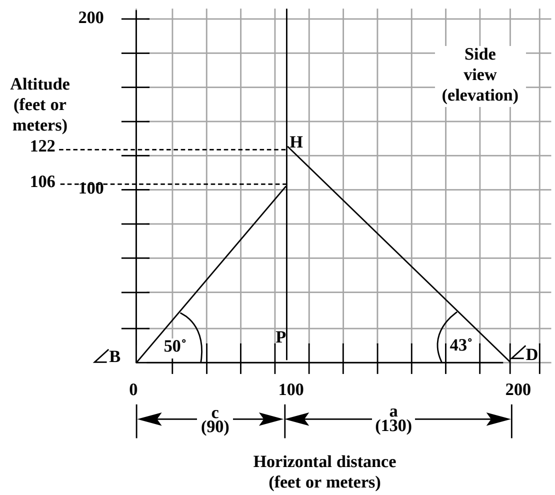
$$C = \frac{106.4-121.7}{2(114.1)} \text{x } 100 = -0.06705\text{x}100$$

$$C = 0.06705 \text{ x } 100$$

$$= 6.705\% \text{ (This is acceptable as it is less than 10\%)}$$

To solve this graphically, plot the baseline distance and azimuth angles as in the top view figure, to find the unknown horizontal distances (a and c). On a separate sheet of graph paper as in the side view figure, plot the elevation angles and distances a and c to graphically depict height PH. When the elevation angles are plotted, there will be two different values of PH so they must be averaged.





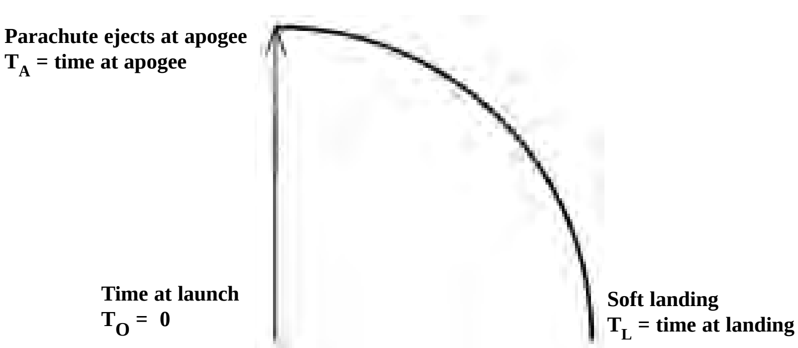
VELOCITIES AND ACCELERATIONS

Calculations

The first method, determining average speed, is an activity for elementary or early middle school students. While it is not as accurate as the more difficult method, it helps them to begin to think about the concept of velocities and accelerations. It is also a method of gathering data about model rocket flight.

Method 1:

The "launch to apogee" average speed and the "apogee to landing" average speed can be calculated. The formula is Average Speed = Distance traveled ÷ Time of travel. Distance traveled on the diagram is the distance between T_o and T_A (launch to apogee).



The students have learned how to determine altitude by the graphical method and by using the following formula:

Height = Tangent of angular distance x baseline.

Use the following example data:

T_o = 0 seconds

T_A = 3.2 seconds (determined by someone with a stopwatch starting at launch and stopping at apogee).

T_L = 4.1 seconds (determined by starting stopwatch at apogee and stopping at landing).

Altitude = 288.7 feet

Average speed ascending = Altitude ÷ (T_A-T_o)
288.7 feet ÷ (3.2 seconds - 0) = 90.21 feet per second.
To determine miles per hour, multiply the answer by 0.682

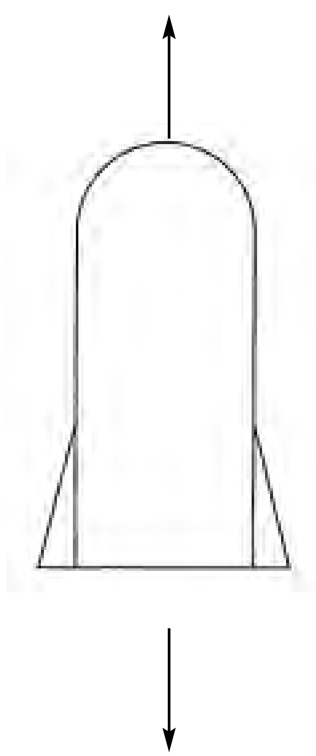
$$\frac{1\ mile}{5280\ ft} \times \frac{60\ sec}{min} \times \frac{60\ min}{hr} = 0.682$$

90.21 x 0.682 = 61.52 miles per hour

Average speed descending, from apogee to landing = Altitude ÷ (T_L - T_A)
288.7 feet ÷ (4.1 - 0 seconds) = 70.41 per second (T_A in this formula = 0 because the stopwatch was restarted at apogee).
70.41 x 0.682 = 48.02 miles per hour.

Method 2:

The second method is for use in middle school and high school. Using the following formulas, values for accelerations and velocities during the acceleration phase (engine burning) for model rockets can be determined. The values given are based on theoretical "no drag" conditions.



These formulas are derived as follows:

$$W_{av} = mg \therefore m = \frac{W_{av}}{g}$$

From Newton's Second Law:

$$F = ma$$

$$T - W_{av} = ma$$

$$T - W_{av} = \frac{W_{av}}{g}\left(\frac{V_m}{t}\right) \quad V_m = at \quad a = \frac{V_m}{t}$$

$$V_m = \frac{(T - W_{av})gt}{W_{av}}$$

$$V_m = \left(\frac{T}{W_{av}} - 1\right)gt$$

V_m = average velocity during engine burn
W_{av} = average weight of rocket
T = thrust (average thrust of rocket engine)
g = acceleration due to gravity (32.2 feet/second^2)
t = engine burn time in seconds

These values may be found in or calculated from information in an Estes catalog. The example involves analysis of an Alpha® rocket using an A8-3 engine. The Alpha® with engine weighs 1.37 ounces (38.8 grams) at lift-off. The weight of propellant in an A8-3 engine is 0.11 ounces (3.12g)giving an average weight of 1.32 ounces (37.2g) during the thrust phase of the flight. The A8-3 thrusts for 0.32 seconds and has a total impulse of .056 pound-seconds (2.50 N-sec). In the metric system, weight is a force whose unit of measurement is the “newton” (N). Since force equals mass times acceleration, weight equals mass times the acceleration of gravity: W = mg. W = weight (N); m = mass (kg); g = acceleration of gravity (9.81 m/s²).

First, find the average thrust. $T = \dfrac{\textit{total impulse}}{\textit{burn time}}$

English

$$T = \frac{0.56 \textit{ pound-seconds}}{0.32 \textit{ sec.}}$$

$$= 1.75 \textit{ pounds}$$

Metric

$$T = \frac{2.50 \textit{ newton-seconds}}{.32 \textit{ sec.}}$$

$$= 7.81 \textit{ newtons}$$

Multiply by the conversion factor of 16 ounces/pound:

$$T = 1.75 \textit{ pounds} \; x \; \frac{16 \textit{ ounces}}{1 \textit{ pound}}$$

$$= 28 \textit{ ounces}$$

Thrust is 28 oz. $V_m = \left(\dfrac{T}{W_{av}} - 1\right) gt$

To convert mass (kg) to weight (newtons, N): W = mg

$$V_m = \left(\frac{28.0 \; oz.}{1.32 \; oz.} - 1\right)(32.2\, ft./\sec^2)(0.32 \sec.)$$

$$= (21.21 - 1)\left[(32.2 \; ft./\sec.^2)(0.32 \sec.)\right]$$

$$= (20.21)(10.24 \; ft./\sec.)$$

$$= 208.27 \; ft./\sec.$$

$$V_m = \left(\frac{7.81 N}{0.365 N} - 1\right) 9.81 \frac{m}{\sec.^2} (.32 \sec.)$$

$$V_m = 64.03 \; m/\sec.$$

The expression $\left(\dfrac{T}{W_{av}} - 1\right)$ gives you an idea of the number of “gravities”, or g’s, the rocket experiences in upward flight during acceleration. “1” is subtracted in this expression to allow for the pull of Earth’s gravity on the rocket. This rocket developed a fairly high velocity by the end of the thrusting phase of the flight.

Velocity developed by the same Alpha® with a C6-5 engine can be determined.

To convert mass (kg) to weight (newtons, N): W = mg

$$\mathrm{T} = \frac{2.25 \mathrm{lb./sec.}}{1.70 \mathrm{sec.}} \; \mathrm{x} \; \frac{16 \mathrm{oz.}}{1 \mathrm{lb.}}$$

$=$

$=$

$$\mathrm{V_m} = \left(\frac{}{1.49 \mathrm{oz.}} - 1\right)(32 \mathrm{ft./sec.^2}) 1.7 \mathrm{sec.}$$

$=$

$=$

$=$

$= \mathrm{ft./sec.}$

$$\mathrm{T} = \frac{10 \text{ N-sec.}}{1.70 \mathrm{sec.}}$$

$=$

$$\mathrm{V_m} = \left(\frac{}{.414} - 1\right)\left(9.81 \frac{\mathrm{m}}{\mathrm{sec.^2}}\right)(1.7 \mathrm{sec.})$$

$=$

$=$

$=$

$= \mathrm{m/sec.}$

When these have been calculated it will be noted that the velocity developed by the Alpha® with the C6-5 engine is more than triple the velocity which was developed by the A8-3 engine. The average thrust and therefore the acceleration in “g’s” produced by the C6-5 engine is less than that produced by the A8-3 engine. However, the maximum velocity is greater when using the C6-5 engine because the burn time for this engine is greater. In reality, these velocities will be lower because of aerodynamic drag. If the earth had no atmosphere (like the moon), these velocities would be accurate. For detailed information about the effects of drag, the following Estes Technical Reports are available: TR-10, “Altitude Predictions Charts” and TR-11, “Aerodynamic Drag of Model Rockets.”

Method 3:

The third method is for use in high school, grades eleven and twelve. In methods one and two previously discussed, the velocities calculated are average velocities. During the time period evaluated, the velocity constantly changes from zero to the peak velocity at engine burnout. To determine the maximum velocity which occurs at burnout an extension of Newton’s Second Law (N2L) must be used. N2L examines a situation at one instant of time, just like a camera snapshot. To evaluate an event over a time period (as a video camera would do) we expand N2L into the concept of Impulse-Momentum.

Starting with N2L:

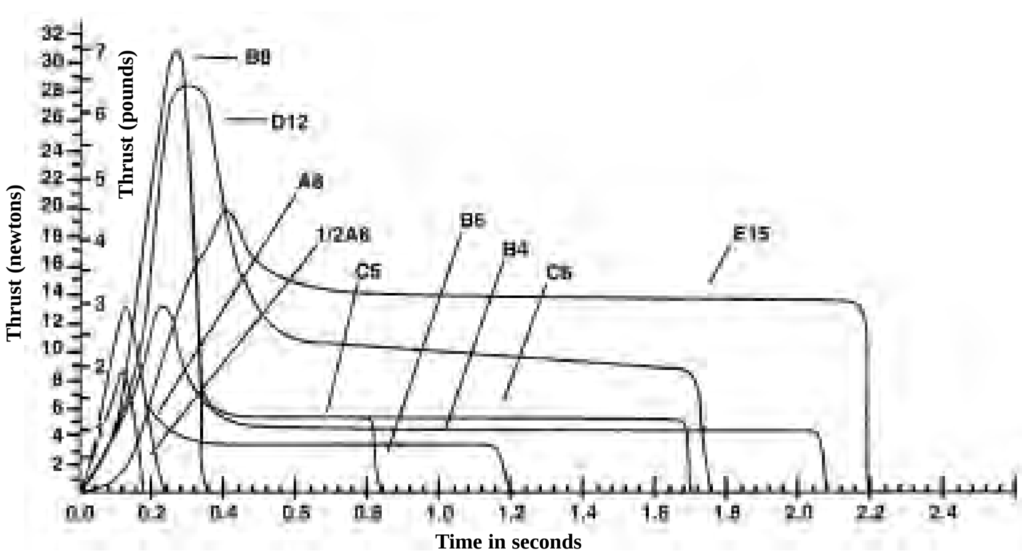
$$F = ma \qquad a = \frac{dv}{dt}$$

$$F = m\frac{dv}{dt}$$

$$Fdt = mdv$$

$$\int_{t_i}^{t_f} Fdt = m\int_{v_i}^{v_f} dv$$

The left side of the equation represents Impulse (force x time). If the force is constant, you multiply it by the change in time (seconds) that the force acts. We can consider the rocket’s weight to be constant even though we know the propellant reduces the weight as it burns, but it is insignificant. Just use average weight as in methods one and two. The thrust developed changes radically as the engine burns. To find the impulse produced by the engine, you need to integrate the thrust functions with respect to time or find the area under the thrust-time curve.



This can become complex, but data about the engine's impulse is readily available in the Estes catalog.

The right side of the equation defines momentum (mass x velocity). Again, we will consider the rocket's mass to be a constant so we multiply it by the difference of initial velocity (zero) and the final velocity.

The equation becomes:

$$\sum \text{Impulse} = \Delta\text{Momentum}$$

(the sum of the linear impulse = the change in linear momentum)

$$\int_{t_i}^{t_f} T\,dt - W\Delta t = m(V_f - V_i)$$

$$10\,NS - .414N(1.70S) = .0422Kg(V_f - 0)$$

$$10NS - .704NS = .0422KgV_f$$

$$9.296NS = .0422KgV_f$$

$$V_f = 220\frac{NS}{Kg} \qquad N = \frac{Kg\,m}{S^2}$$

$$= 220\frac{Kg\,m\,S}{S^2\,Kg}$$

$$V_f = 220\,m/s \qquad \text{velocity at engine burnout}$$

Again, we have neglected to account for aerodynamic drag which will reduce the velocity. Reference Estes Technical Reports TR-10 and TR-11 for detailed information on the effects of drag.

The peak acceleration occurs when the engine's thrust is at a maximum (also listed in the Estes catalog).

$$\#\ g\text{'s} = \frac{T\max}{W_{av}} - 1$$

For the actual numerical value of acceleration, multiply the #g's by 32.2 ft/sec or 9.81 m/sec.

VOCABULARY

Apogee: The point during the rocket's flight (or in orbit of a satellite) at which it is farthest from the surface of the earth.

Tangent of angular distance: In a right triangle, a function of an acute angle equal to the ratio of the side opposite the angle to the side adjacent to the angle.

Sine: In a right-angled triangle ("right triangle"), a function of the acute angle equal to the length of the side opposite the angle to the length of the hypotenuse.

Azimuth: A direction expressed as a horizontal angle from a given direction.

Velocity: A rate of motion in a given direction, measured in terms of distance moved per unit time.

Acceleration: The rate of change in the speed of an object, measured in ft/sec^2 m/sec^2 or "g's".

STRATEGY

Materials for each student: A model rocket ready for launch, worksheets appropriate for the option chosen and a pencil. Also needed will be altitude tracking devices and stop watches.

A. Review the flight sequence of a model rocket flight with the students using the overhead transparency. Call special attention to apogee because that is the height they will try to determine. If their rocket has been constructed carefully, has been tested for stability and has been packed properly, the data they learn from tracking the altitude and from determining the speed of their model rocket will be affirming for them.

As part of the review of the flight sequence a brief discussion of Newton's Three Laws of Motion will have particular meaning.

Use overhead transparencies(Appendix) to project each law of motion. (General Background for the Teacher)

Law #1 - Discuss with the students what they think it means. Use a tennis ball to demonstrate an object at rest. Discuss what would cause it to stay at rest and what would put it into motion. Discuss what balanced forces are holding it at rest. (The force of gravity and floor or the table). What unbalanced force would put it in motion? (Rolling it, tossing it up). Discuss the law in relation to model rocketry.

Law #2 - Emphasize acceleration in relation to the flight sequence.

Law #3 - Discuss the law in relation to the flight sequence.

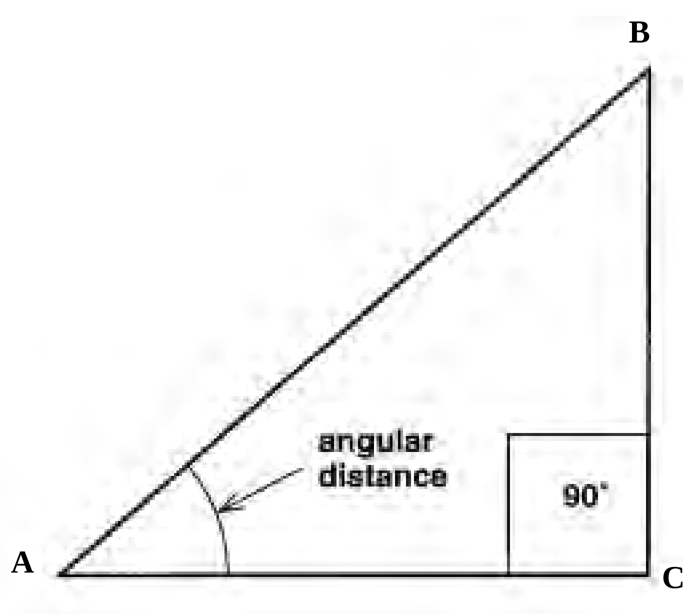
B. Discuss with the students the statistics they could gather, such as altitude of apogee, acceleration and speed.

Option #1:

One station tracking and determining average speed - Elementary level and early middle school level.

A one person tracking station is suitable for less experienced or younger students. The results are generally reliable. In one station, there is one baseline and one observer using an altitude measuring device. One station tracking assumes that the flight will be almost vertical. It is important to master this system before going on to more complex ones.

On the board, draw a diagram like the one in the figure.



Describe the location of the launch pad, C, the tracking station, A, and the rocket at apogee, B. AC is the baseline. Angle A is the angular distance. Angular distance is found by using the altitude tracking device to find a point on the protractor just as the rocket reaches apogee. Remind the students that the sum of the angles of a triangle equals 180˚. The angular distance is determined by subtracting the number on the protractor from 90˚. Demonstrate to the students how to find the tangent of the angular distance on a table of tangents. Explain that the students want to determine the height of the line, CB or the height of apogee of the rocket by using the formula Height = tangent of angular distance x baseline or graphically determine the altitude.

A. Distribute Worksheet (Determining Altitude). Guide the students through using the formula by giving the number of 500 feet for the baseline and write that on the diagram on the board. The degrees found on the protractor at the tracking station is 60˚. Since angular distance is found by subtracting that number from 90˚ (write that at angle C). What is the angular distance found at the tracking station? 30˚. Write that at angle A. Ask, “Do we have enough information to fill in the formula?” No, we need to use the table of tangents to find the tangent of the angular distance of 30˚. Using the table of tangents, that number is found to be .5774.

The students should write the formula as follows:

Height = .5774 x 500 ft.

Height = 288.7 ft.

Allow students to work through the formula using at least two more examples with everyone working together.

Allow students to work the problems on the Worksheet independently and then check them together in class. Students can work in groups of two or three on these problems.

B. Distribute the Worksheet (Determining the Altitude Graphically). Display the OH (Determining the Altitude Graphically). Point out how to plot the baseline distance on the horizontal axis. Demonstrate how to make the vertical axis for altitude. Show how to plot the elevation angle at the baseline distance and extend the line of sight to the vertical axis. Read the rocket’s altitude off the vertical scale at the point of intersection. The students should use the Worksheet to determine altitude graphically using the numbers in the mathematical equation. This worksheet can also be used for a rocket launch.

C. Demonstrate the use of an altitude measuring device and allow students to try it out. Students can learn to use it by measuring the height of objects such as the flagpole, a tall tree and the basketball backboard. Worksheet (How High is the Flagpole?) can give practice in using the measuring device and gathering data for calculation. Remind the students of the information they will need to get and record outdoors: the angular distance and the baseline. (The teacher should have selected some objects to measure. The baseline from the objects to the spot where the tracker will stand should be measured and marked ahead of time.

Allow the students to work in groups of two or three. Point out the objects to be measured, where the tracking stations are and the length of each baseline.

Launch:

D. Review NAR Safety Code before launching. Distribute data sheets to each student.

Students should have their rocket, a pencil and Individual Launch Data Sheet. Review with the group how to use the altitude measuring device - sighting, following the rocket, noting the angle precisely at apogee.

Two other teachers or a student timing team should record the time of each flight to apogee and again to landing using a stopwatch. Make certain each student records their elevation angle, the baseline distance, the time to apogee and the time to recovery on the individual chart.

E. Using the formula for determining altitude, guide the students through the process, if needed. They should record these on their individual launch data sheet. A group launch data sheet can also be completed.

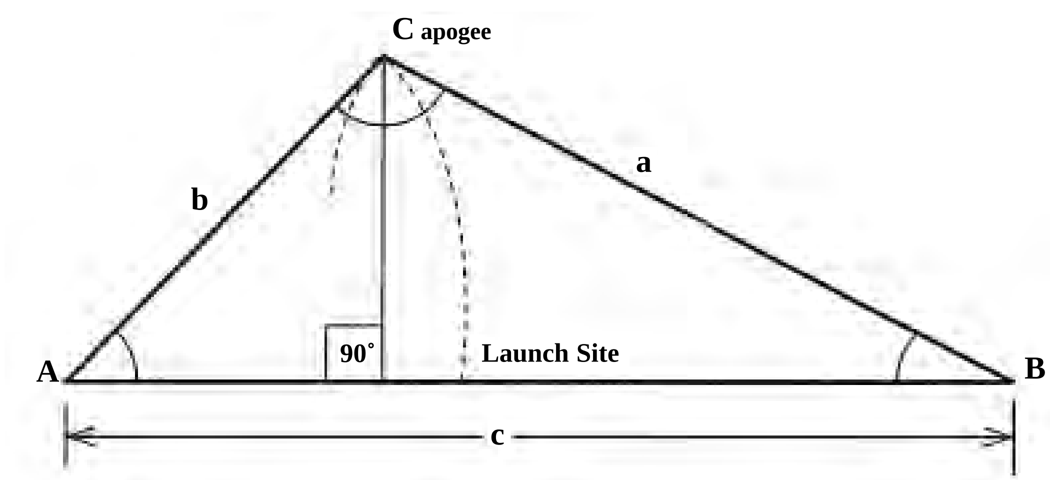
Average Speed:

F. Guide the students through the Worksheet, Determining the Average Speed. Using the formulas, guide the students through the process of determining the average speeds of their rockets ascending and their rockets descending.

Option #2:

Two station tracking - Middle school and high school:

A. Distribute the worksheet on two station tracking. Two station tracking uses two tracking stations placed on opposite sides of the launch pad in line with the wind. Draw figure on the board. Explain that stations A and B are located on a 1000 ft. baseline with the launcher between them. Station A calls in an angular distance of 34˚ and station B calls in an angular distance of 22˚. The total of these two angles is 56˚. Therefore angle C, located at the apogee of the rocket's flight is 124˚, or 180˚ - 56˚. The measurements of three angles and one side are known.



If needed, guide the students through the equations and provide three more examples for practice. Students can work in groups of two or three on these problems. A computer program, AstroCad™, which does the calculations from the student data is available from Estes.

B. Demonstrate the use of an altitude measuring device and allow students to try it out. Students can learn to use it by measuring the height of objects such as the flagpole, a tall tree and the basketball backboard. Worksheet (How High is the Flagpole?) can give practice in using the measuring device and gathering data for calculation. Remind the students of the information they will need to get and record outdoors: the angular distance and the baseline. (The teacher should have selected some objects to measure. The baseline from the objects to the spot where the tracker will stand should be measured and marked ahead of time.)

Allow the students to work in groups of two or three. Point out the objects to be measured, where the tracking stations are and the length of each baseline.

C. Distribute the Worksheet for Determining Altitude Graphically. Display OH, Determining Altitude Graphically for Two Station Tracking. Demonstrate how to plot both elevation angles on graph paper (separated by the baseline distance) with scales as in single station tracking. The intersection of these two lines of sight represent the rocket's altitude as read off the vertical axis. Allow the students to practice using the numbers in the formula to show that it can be demonstrated graphically as well.

Launch:

D. Review NAR Safety Code before launching each rocket. Distribute data sheets to each student.

Each student should have his/her rocket, a pencil and Individual Launch Data Sheet. Review with the group how to use the altitude measuring device - sighting, following the rocket, noting the angle at exact apogee.

Two other teachers or a student timing team should record the time of each flight to apogee and again to landing using a stopwatch. Make certain each student records his/her angle of distance, the baseline distance, the time to apogee and the time to recovery on the individual chart.

E. Using the formula for determining altitude, guide the students through the process, if needed. They should record these on their individual launch data sheet. A group launch data sheet can also be completed.

Velocities and Accelerations:

F. Distribute worksheet on determining velocities and accelerations. Explain the formulas for determining some values for accelerations and velocities for model rockets:

$$Vm = \left(\frac{T}{W_{av}} - 1\right) gt$$

Vm = average velocity during engine burn

Wav = average weight of rocket

T = average thrust of rocket engine

g = acceleration due to gravity (32.2 feet/second^2)

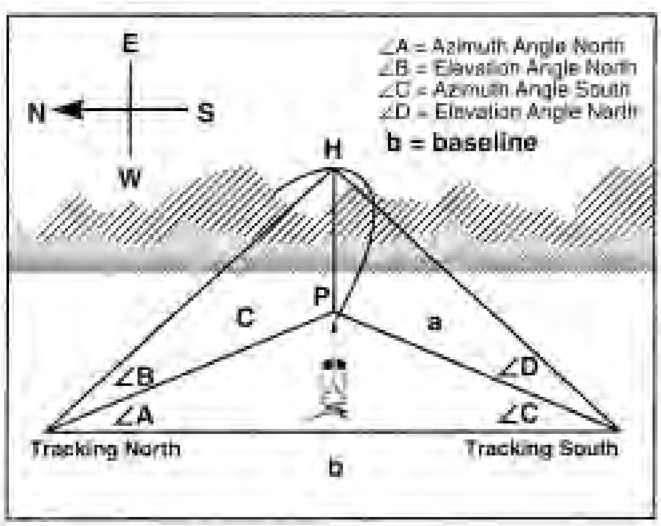
t = engine burn time in seconds

Guide the students through the formulas and equations. Allow them to work on additional problems either independently or in small groups.

Option #3:

Two station tracking using the azimuth angle:

A. Explain the tracking system used in this method (Background for the Teacher). Draw the following figure on the board or display on the overhead projector.



B. Explain the formulas:

The formula for determining the height reached as determined by station "tracking north" is:

$$PH = \sin C \text{ x } \tan \angle B \times \frac{(b)}{\sin(180 - [\angle A + \angle C])}$$

The formula for determining the height reached as determined by station "tracking south" is:

$$PH = \sin \angle A \times \tan \angle D \times \frac{(b)}{\sin\left(180 - [\angle A + \angle C]\right)}$$

Guide the students through formulas and calculations. Provide additional problems for practice in calculating the height using this method.

NOTE: For this system an azimuth/elevation tracking type scope is needed.

Launch:

Same as Option #2.

Velocities and Accelerations:

Use Method 3.

Display the graphs Determining Altitude Graphically for Two Station Tracking Using the Elevation Azimuth System on the overhead projector. Distribute graph worksheets. Demonstrate how to plot the baseline distance and azimuth angles to find the unknown horizontal distances (a and c). On a separate worksheet of graph paper plot the elevation angles and distances a and c to graphically depict height PH. The two different values of PH must then be averaged.

Evaluation: Teacher observation and worksheets for collecting data and making calculations.

NOTES

Lesson 6- Extension (Independent Study) (2 or 3 days)
Aerial Photography

Objectives:

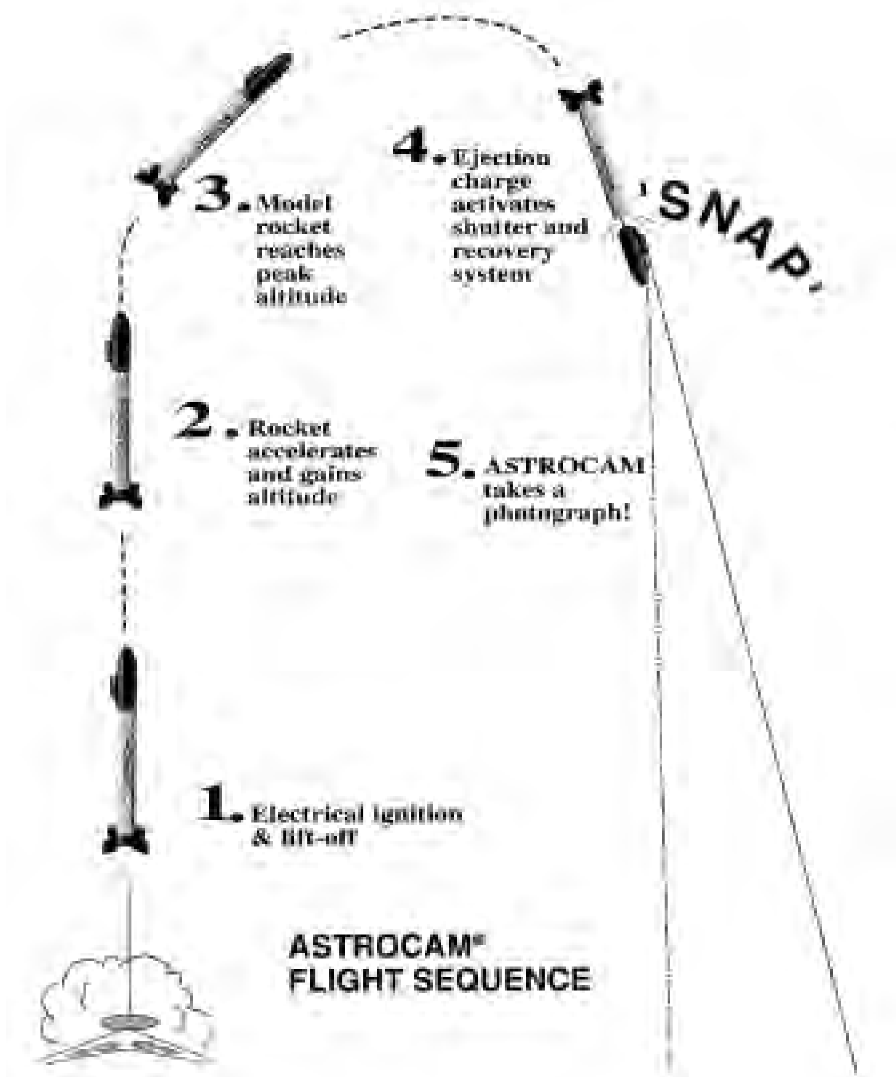
The student will be able to:

- Recognize a practical application for model rocketry.
- Use photo interpretation to calculate the height of a rocket.
- Use photo interpretation to calculate the size of objects on the ground.

BACKGROUND FOR THE TEACHER

All students can participate in and benefit from taking aerial photographs from an aerial camera launched on top of a model rocket. The Astrocam® has a small camera that takes a single photo per flight from an altitude of about 500 feet. The photo is taken at the instant of parachute ejection. The photo is taken "looking forward". To get vertical photos, an extra long delay is used in the rocket engine so the rocket reaches apogee, then turns nose down for a second or so before ejection. When the rocket is recovered, the film is advanced to the next frame, the rocket is prepped with a new engine and recovery wadding and it's ready to fly again.

The camera uses readily available 110 film cartridges, 200 ASA. The film can be processed locally. It is easy to obtain film and processing, so it is a project that is affordable for a classroom.



Students take several shots of the schoolyard or other flying area. They study the "bird's eye view" of prints to learn what they can about the area. They can also make a manual survey of the same area to confirm or analyze their initial findings.

Aerial photos give students a perspective of an area often unobtainable any other way. They may be surprised to see details of a familiar area never before apparent to them. With several in flight shots taken from slightly different vantage points, it is possible to create a montage showing a much larger area than one photo can.

Basic photo interpretation involves calculating the height of the rocket. To do this, compare the size of the image on the negative to the size of a known object. Then, knowing the focal length of the camera lens, you can determine the height. The height of the rocket will equal the ratio of the object size to the image size times the focal length of the lens. This system is only accurate for photos taken vertically or very near vertically.

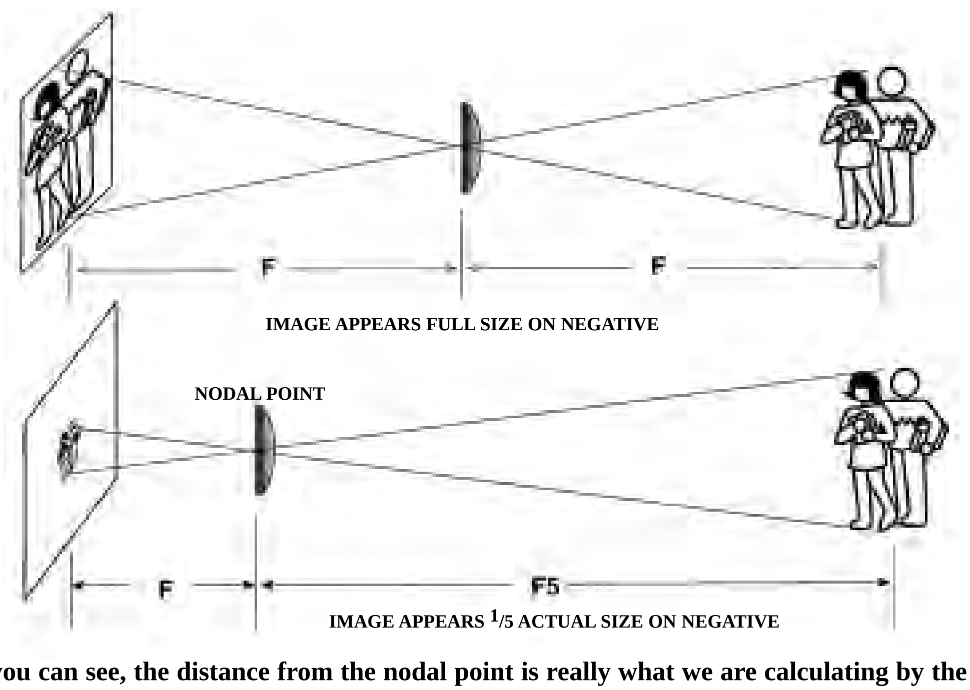
If you are working from a print instead of the negative, the height can still be calculated, but the enlargement factor of the print must be known. Your photo processor can tell you this.

The formula for calculating height:

H = vertical height of camera above object
O = object size
I = image size on negative or print
F = effective focal length of lens
N = enlargement factor of image on print or negative (if applicable)

$$H = \frac{O \times F \times N}{I}$$

The focal length of a lens is the distance between the focused image on the negative and an imaginary point (the nodal point) where the light rays entering the lens cross. Sometimes this point is in the lens, sometimes it is ahead or behind the lens depending on the design. The illustration shows the relationship between image size and object size.



As you can see, the distance from the nodal point is really what we are calculating by the above formula. The Astrocam® lens and its relationship to the negative look like this.

The Astrocam's® flange focal length (1.0155") is measured from the lens flange to the negative. The thickness of the lens is 0.1112". The effective focal length is 1.1966" (30.394mm). This places the nodal point just past the lens as shown.

STRATEGY

A. For those students who have an interest in this unit, allow them to work in a small group to construct the rocket and install the camera. A ready-to-fly Astrocam® is also available.

B. The students should take several shots of the launch area from the ground to learn what they can about the area.

C. Launch the rocket loaded with the camera so that pictures are taken of the area studied from the ground. Students should take several flight shots from slightly different vantage points to show a larger area than one photo can.

D. Students can use the formula for calculating height .

E. Students can track the rocket to calculate height, then apply the formula to calculate the actual size of images on the ground.

F. Student data, work and photographs should be included in their folder for the entire unit.

NOTES

Lesson 7- Extension(Independent Study)

Payloads

Objectives:

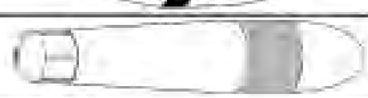
The student will be able to:

- Recognize the importance of careful construction when a payload compartment is added.
- Recognize the changes in CG and CP that will occur and how to adjust for these.

BACKGROUND FOR THE TEACHER

The tables show the weight and size of a variety of payloads and a variety of payload sections.

Specimen	Weight (typical)		Length (typical)		Width (typical)	
	grams	oz.	mm	in	mm	in
Grade A large hen's egg	64.00	2.25	70	2.80	50	2.0
Grade A small hen's egg	58.00	2.04	57	2.25	40	1.6
Grasshopper	2.00	0.07	37	1.50	10	0.4
Fly	0.25	0.01	10	0.40	5	0.2
Spider	0.25	0.01	10	0.40	5	0.2
Earthworm	4.00	0.14	64	2.50	3	0.1
Beetle	0.50	0.02	25	1.00	5	0.2
Cricket	0.30	0.01	25	1.00	7	0.3

Payload Capsule	Fits Tube	Part #'s Required	Inside Diameter		Overall Length	
			mm	in	mm	in
	BT-20	30604[1]	18.0	0.710	49.2	1.938
	BT-50	30608[2]	24.1	0.950	66.7	2.625
	BT-1090	31205[3]	24.8	0.976	100.0	3.938
	BT-60	30614[4]	40.5	1.590	88.9	3.500
	BT-56[5] Omloid™	72640, 72641 & 30156[6]	54.0	2.125	227.0	8.938
	BT-56[5] Scrambler™	30422 & 71050[6]	51.0	2.000	276.0	10.880

[1]You will also need the following parts in addition to the clear capsule: screw eye (EST 2280), tape strips (EST 38412), and NB-20 nose block (EST 70152).

[2]You will also need the following parts in addition to the clear capsule: screw eye (EST 2280), tape strips (EST 38412), and NB-50 nose block (EST 70158).

[3]This clear capsule will fit on the Bandit™ and Delta Clipper™ - do not discard the nose block that is extra with these kits! Can also be substituted for the opaque capsule on the Dagger™, Rampage™, SuperShot™, and Patriot™.

[4]You will need a PNB-60 nose block (EST 71061) with this clear capsule.

[5]This fits the body tube found on the HelioCopter™, AstroCam® 110 and Maniac™. The shoulder may be carefully sanded down to fit a BT-55 body tube.

[6]You will need to order all of these parts to complete the capsule.

The payload capsules listed for BT-20, 50, and 60 can be used for smaller or larger body tubes by using transitional adapters that smoothly taper from one body size to another.

Careful selection of the best engine for the mission, proper construction of the payloader and precise packaging of the payload weights are all important.
It is all right to launch insects, but never launch a hamster or a mouse. The point of payloads is to have a successful return of the payload undamaged. Since that can't be guaranteed, small animals can suffer serious injury or death. The tables give some ideas for payloads.
The formulas for calculating the center of pressure for a rocket with a payload are in the appendix for use by advanced students.

STRATEGY

Distribute copies of the tables and an Estes catalog (can be found at estesrockets.com) so that those students interested can determine the type of rocket, engine, and payload section they need. Remind them of the importance of determining stability and of the relationship of CG to CP.
Launch the rocket with the payload. A successful payload launch involves the payload coming back undamaged.

NOTES

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APPENDIX

Black Line Masters

NAR Model Rocketry Safety Code

The safety code was formulated by experienced rocketeers and has evolved with model rocketry. It should be followed in every model rocketry activity.

1. Materials - My model rocket will be made of lightweight materials such as paper, wood, rubber, and plastic suitable for the power used and the performance of my model rocket. I will not use any metal for the nose cone, body, or fins of a model rocket.

2. Engines/Motors - I will use only commercially-made NAR certified model rocket engines in the manner recommended by the manufacturer. I will not alter the model rocket engine, its parts, or its ingredients in any way.

3. Recovery - I will always use a recovery system in my model rocket that will return it safely to the ground so that it may be flown again. I will use only flame resistant recovery wadding if required.

4. Weight and Power Limits - My model rocket will weigh no more than 1,500 grams (53 ounces) at liftoff, and its rocket engines will produce no more than 320 newton-seconds (4.45 newtons equal 1.0 pound) of total impulse. My model rocket will weigh no more than the engine manufacturer's recommended maximum liftoff weight for the engines used, or I will use engines recommended by the manufacturer for my model rocket.

5. Stability - I will check the stability of my model rocket before its first flight, except when launching a model rocket of already proven stability.

6. Payloads - Except for insects, my model rocket will never carry live animals or a payload that is intended to be flammable, explosive, or harmful.

7. Launch Site - I will launch my model rocket outdoors in a cleared area, free of tall trees, power lines, buildings and dry brush and grass. My launch site will be at least as large as that recommended in the following table:

LAUNCH SITE DIMENSIONS

Engines	**Site Diameter** Feet/Meters	**Maximum Altitude** Feet/Meters
1/2 A	50/15	200/61
A	100/30	400/122
B	200/61	800/244
C	400/122	1600/488
D	500/152	1800/549

Minimum launch site in dimension for a circular area is diameter in feet/meters and for rectangular area is shortest side in feet/meters.

8. Launcher - I will launch my model rocket from a stable launch device that provides rigid guidance until the model rocket has reached a speed adequate to ensure a safe flight path. To prevent accidental eye injury, I will always place the launcher so the end of the rod is above eye level or I will cap the end of the rod when approaching it. I will cap or disassemble my launch rod when not in use, and I will never store it in an upright position. My launcher will have a jet deflector device to prevent the engine exhaust from hitting the ground directly. I will always clear the area round my launch device of brown grass, dry weeds, or other easy-to-burn materials.

9. Ignition System - The system I use to launch my model rocket will be remotely controlled and electrically operated. It will contain a launching switch that will return to “off” when released. The system will contain a removable safety interlock in series with the launch switch. All persons will remain at least 15 feet (5 meters) from the model rocket when I am igniting model rocket engines totaling 30 newton-seconds or less of total impulse and at least 30 feet (9 meters) from the model rocket when I am igniting model rocket engines totaling more than 30 newton-seconds of total impulse. I will use only electrical igniters recommended by the engine manufacturer that will ignite model rocket engine(s) within one second of actuation of the launching switch.

10. Launch Safety - I will ensure that people in the launch area are aware of the pending model rocket launch and can see the model rocket’s liftoff before I begin my audible five-second countdown. I will not launch a model rocket using it as a weapon. If my model rocket suffers a misfire, I will not allow anyone to approach it or the launcher until I have made certain that the safety interlock has been removed or that the battery has been disconnected from the ignition system. I will wait one minute after a misfire before allowing anyone to approach the launcher.

11. Flying Conditions - I will launch my model rocket only when the wind is less than 20 miles (30 kilometers) an hour. I will not launch my model rocket so it flies into clouds, near aircraft in flight, or in a manner that is hazardous to people or property.

12. Pre-Launch Test -When conducting research activities with unproven model rocket designs or methods I will, when possible, determine the reliability of my model rocket by pre-launch tests. I will conduct the launching of an unproven design in complete isolation from persons not participating in the actual launching.

13. Launch Angle - My launch device will be pointed within 30 degrees of vertical. I will never use model rocket engines to propel any device horizontally.

14. Recovery Hazards - If a model rocket becomes entangled in a power line or other dangerous place, I will not attempt to retrieve it.

This is the official Model Rocketry Safety Code of the National Association of Rocketry and the Model Rocket Manufacturers Association.

The largest legal “model” rocket engine, as defined by CPSC, is an “F” (80Ns) engine. To launch rockets weighing over one pound including propellant or rockets containing more than four ounces (net weight) of propellant, a waiver must be obtained from the FAA. Check your telephone directory for the FAA office nearest you.

Focus

Name ____

What do I know about model rockets?	What do I want to learn about model rockets?	After the launch: What is a new idea or knowledge that I have since watching the launch?	After the launch: What did I see that I would like to know more about?

INTEREST SURVEY

Name____

1. Have you ever built a model rocket? Y N

   How many have you built?

2. List three things that were positive about the model rockets you have built.

   List three problems you had with building a model rocket.

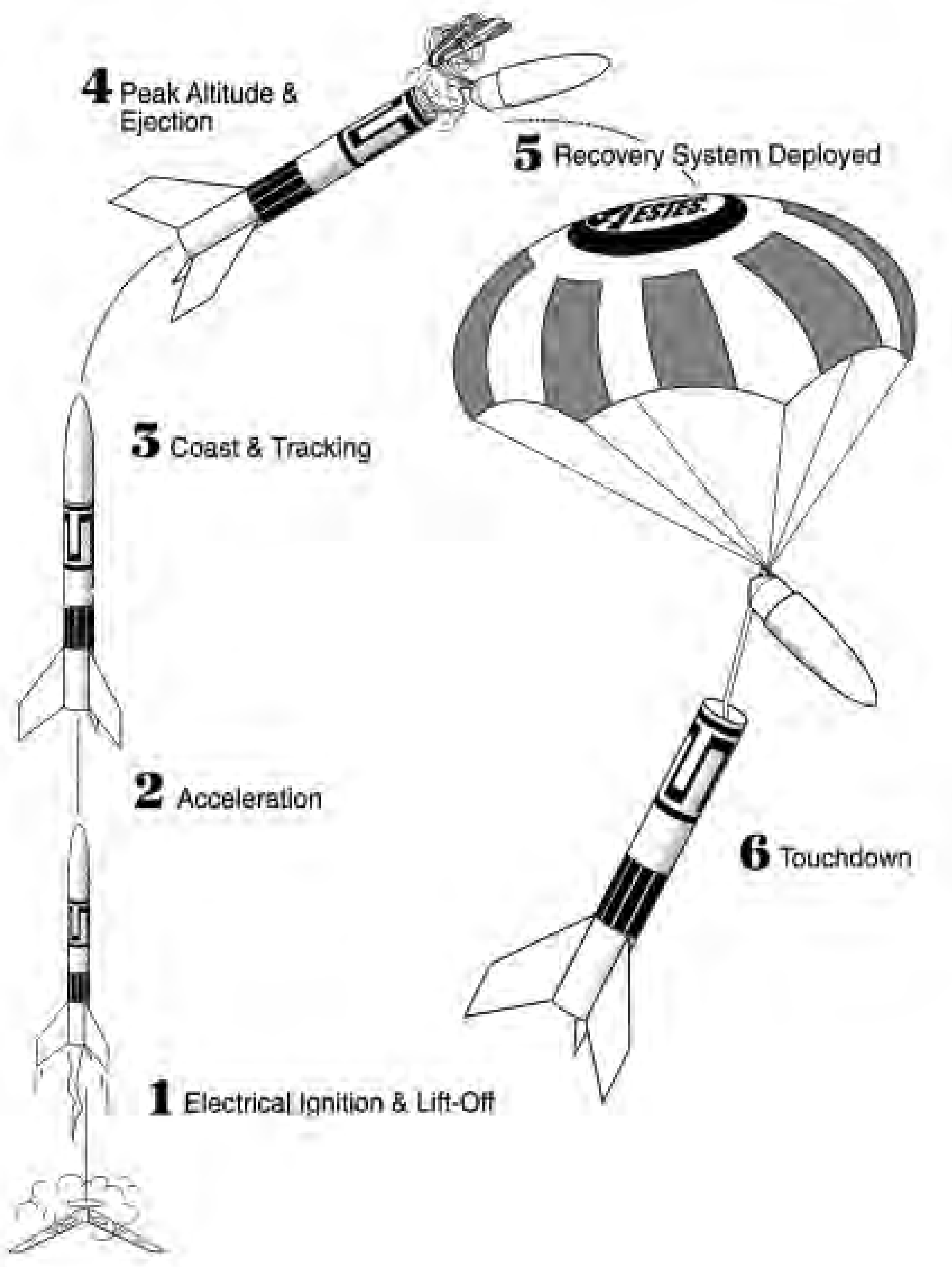
3. List at least three things you would like to know more about having to do with space and rocketry.

   As you make a choice of study projects, think about the things you would specifically like to know so that your project fits your interests.

4. You will participate in several study projects with your model rocket.

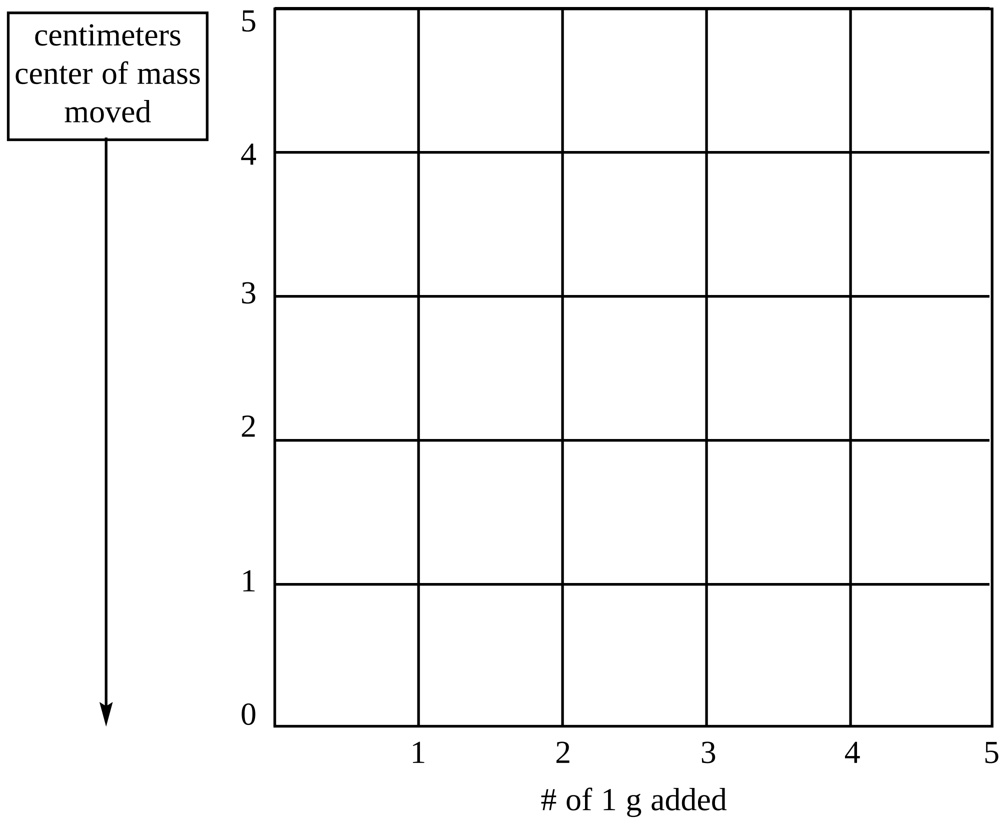
   a. Altitude tracking
   b. Data reduction
   c. Speed of a model rocket
   d. Egg lofting rocket (optional)
   e. Photography from a model rocket (optional)

FLIGHT SEQUENCE OF A MODEL ROCKET



FINDING THE CENTER OF MASS

The effects of added weight on the location of the center of mass.



Directions:

1. Place your first weight in one end of the body tube. Find the new center of mass by rebalancing the body tube across your finger. Mark the new center of mass with a different color pencil. Repeat the procedure by adding one weight at a time. Mark each center of mass on the body tube.
2. Measure the distance between the ends of the rocket tube and the first center of mass you determined without weights. Measure the distance the center of mass moved with one weight and plot that on your graph. Continue until you have plotted all the changes.
3. What can you observe about the movement of the center of mass as more weight or mass is added?

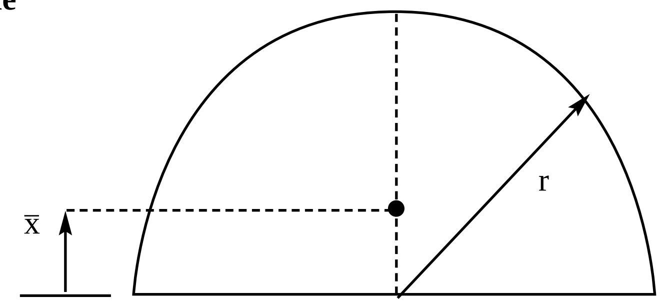
   Which way does the center of mass move? Does it move toward or away from the added weights?

   Was the movement smaller or larger?

   Why do you think this happens?

FINDING CENTROIDS AND AREAS OF SHAPES IN MODEL ROCKETS

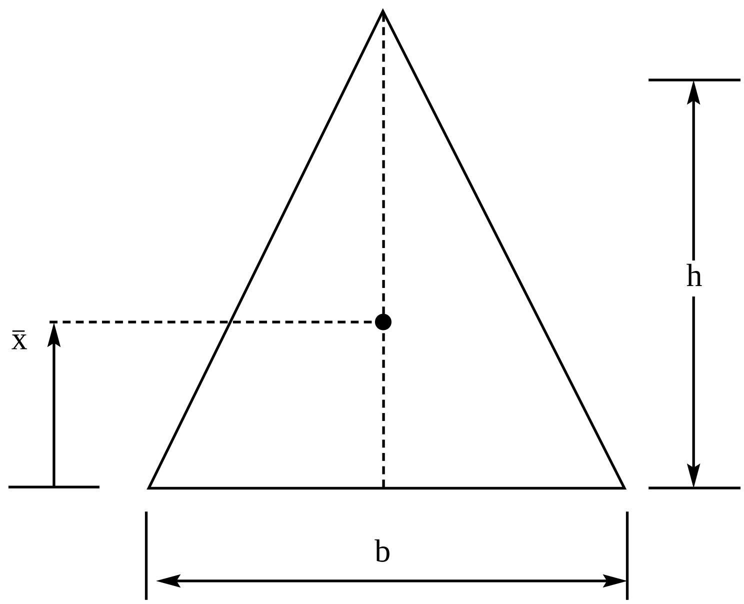
Semi-circle



$$\bar{x} = \frac{4r}{3\pi}$$

$$A = \frac{\pi r^2}{2}$$

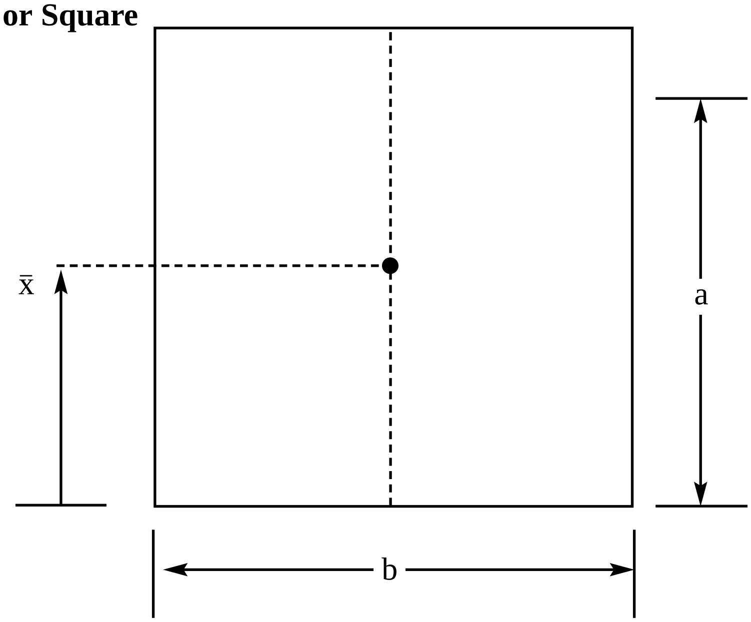
Triangle



$$\bar{x} = \frac{h}{3}$$

$$A = \frac{bh}{2}$$

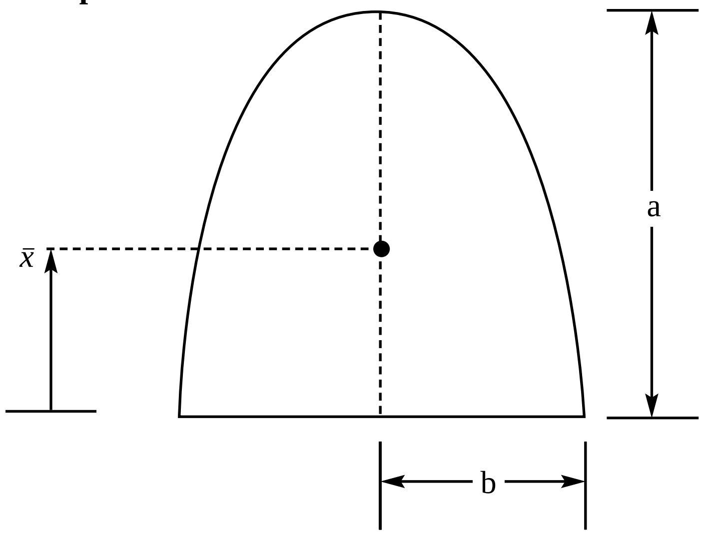
Rectangle or Square



$$\bar{x} = \frac{a}{2}$$

$$A = ab$$

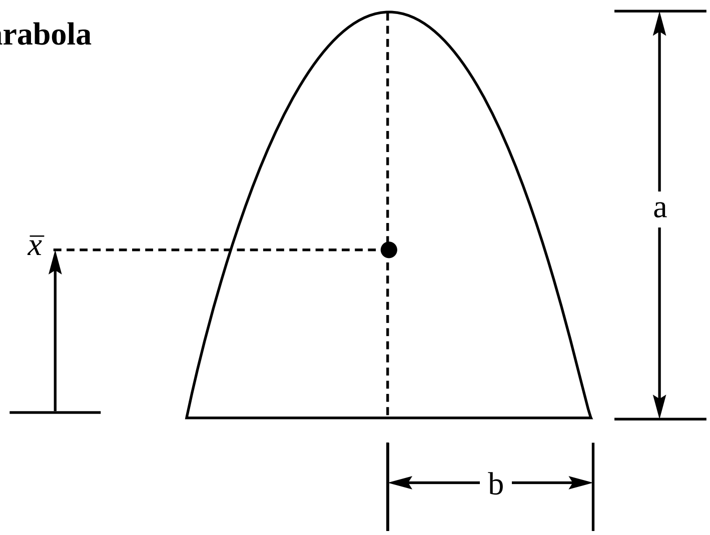
Semi-ellipse



$$\bar{x} = \frac{4a}{3\pi}$$

$$A = \frac{\pi ab}{2}$$

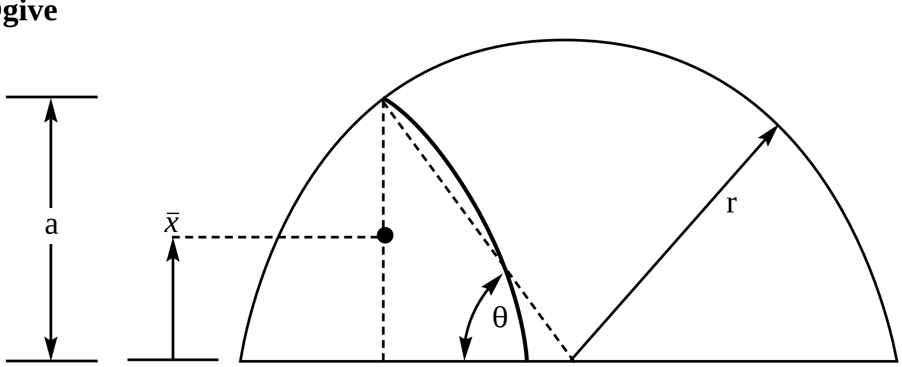
Parabola



$$\bar{x} = \frac{2a}{5}$$

$$A = \frac{4ab}{3}$$

Ogive



$\bar{x} \approx 0.38a$ (for a typical nose cone)

$$A = \frac{r^2 (2\theta - \sin 2\theta)}{2} \quad \theta \text{ in radians}$$

$$360^\circ = 2\pi \, radians$$

$$a = \text{r} \sin \theta$$

WORKSHEET FOR CALCULATING CENTROIDS AND AREAS OF SHAPES

Find the center of pressure of each shape mathematically.

1. Semi-circle

Measurements: $r =$

2. Triangle

Measurements: $h =$

$b =$

3. Rectangle/square

Measurements: $a =$

$b =$

4. Semi-ellipse

Measurements: $a =$

$b =$

5. Parabola

Measurements: $a =$

$b =$

6. Ogive

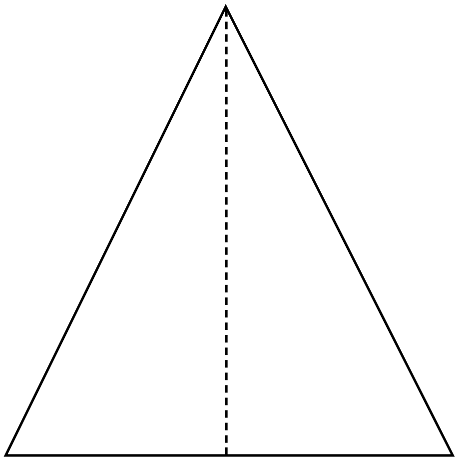
Measurements: $r =$

$\theta =$

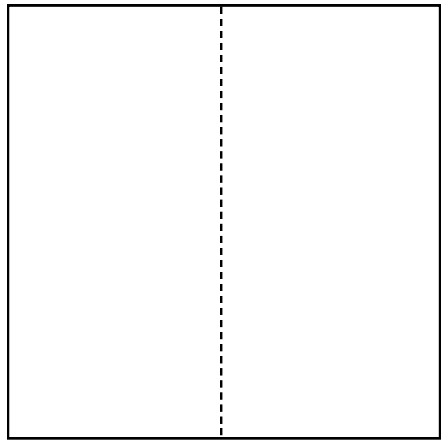
$a =$

You may find the center of pressure of each shape experimentally by cutting them out and balancing each one on your finger. Mark the point where each balances with a pencil. Did the math calculation match the experimental mark?

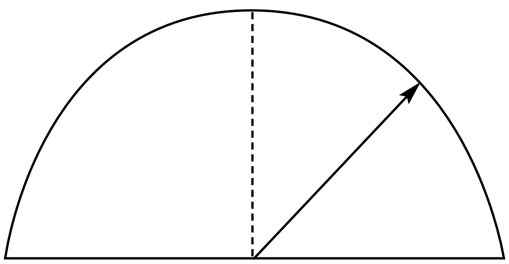
TRIANGLE



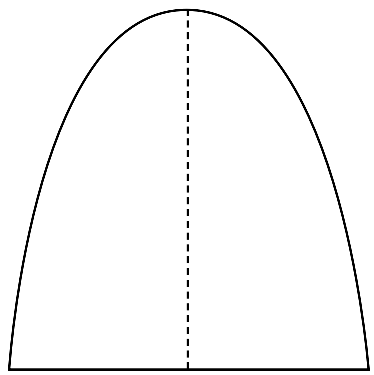
RECTANGLE/SQUARE



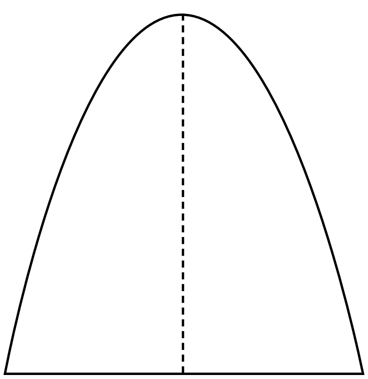
SEMI-CIRCLE



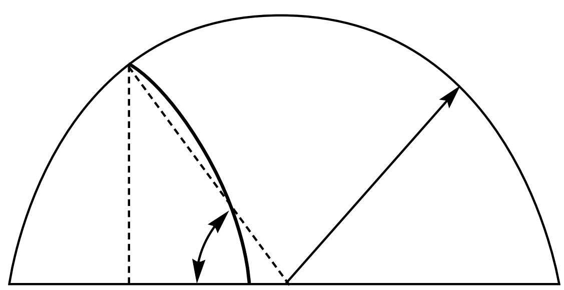
SEMI-ELLIPSE



PARABOLA

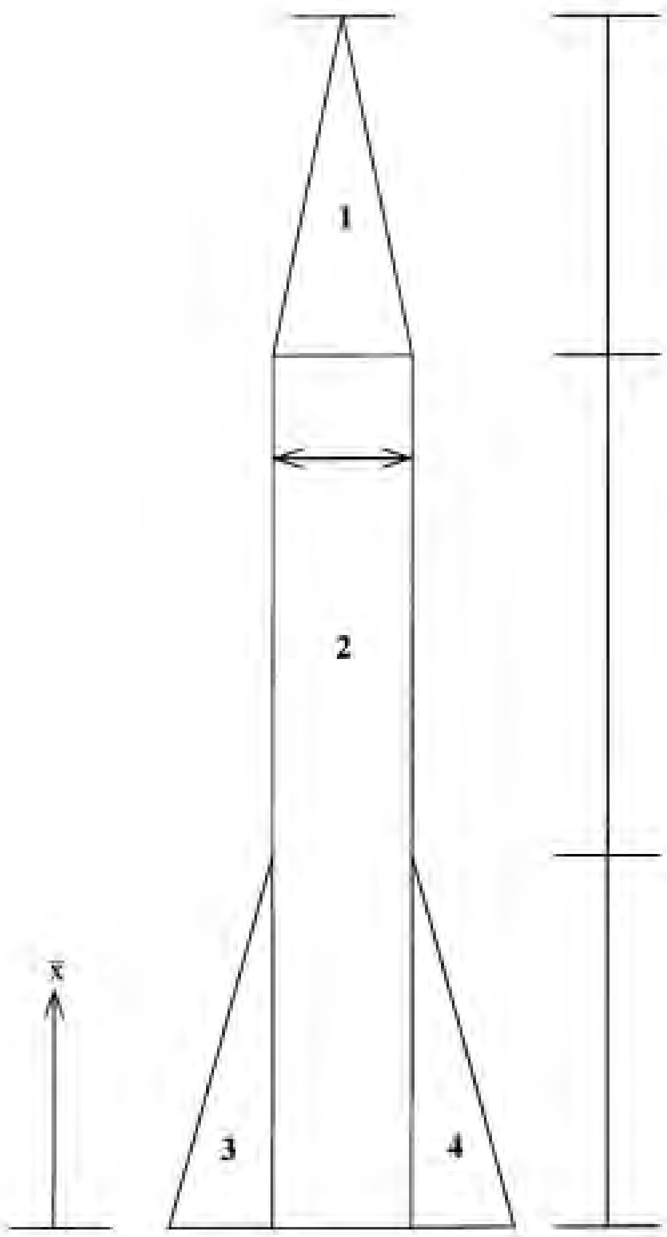


OGIVE



CALCULATING THE CENTER OF PRESSURE OF ROCKET OUTLINE

Use the following formula to calculate the center of pressure of this rocket outline:



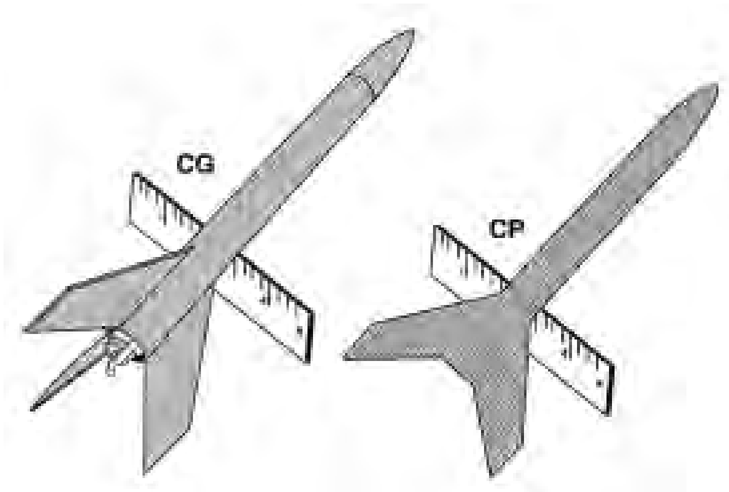
$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

A_1 = Area of each section

x_1 = distance from the centroid of each section to the base

CALCULATING THE CENTER OF PRESSURE OF YOUR OWN MODEL ROCKET

On the cardboard the teacher has given you make a cutout of your rocket by laying your rocket over the piece of cardboard and mark around the edges. Cut around the lines and balance the cutout on a knife edge or ruler. Mark the center of pressure of the cutout.



Use the formula to calculate the center of pressure of your own rocket outline, just as you did on page 3-4.

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

A = area of each section
x = distance from the centroid of each section to the base

WORK SHEET
EVALUATION OF ROCKET STABILITY USING SWING TEST

FIRST TEST: Rocket balanced horizontally

Was rocket stable? ☐ Yes ☐ No

My rocket did not achieve stability. I need to do one of the following:

- ☐ Move the center of gravity forward.
  - ☐ Add weight to the nose.
  - ☐ Use a heavier nose cone.
- ☐ Move the center of pressure to the rear.
  - ☐ Use larger fins.
  - ☐ Move the fins farther back on the body tube.

SECOND TEST: Rocket balanced 10˚ nose down

Was rocket stable? ☐ Yes ☐ No

My rocket did not achieve stability. I need to do one of the following:

- ☐ Move the center of gravity forward.
  - ☐ Add weight to the nose.
  - ☐ Use a heavier nose cone.
- ☐ Move the center of pressure to the rear.
  - ☐ Use larger fins.
  - ☐ Move the fins farther back on the body tube.

WORKSHEET
COMPUTER INPUT
(to be used with computer program, if available)

Rocket name ____

Type of guide used for launch:

Number of fins on rocket____

Type of fin ____

Type of nose cone ____

Determine the distance in centimeters between the CG and the CP on your rocket_____

Mass of rocket in grams ____

Mass of rocket in grams with engine to be used ______________________________

Measurements in centimeters:

Diameter of rocket body ____

Entire length of rocket, end of nose to base ___________________________________

Length of nose cone____

Style of nose cone ____

Length of rocket without the nose cone ______________________________________

Length of fins ____

Width of fins ____

Style of fins ____

Number of fins____

Feed this data into the computer program that will predict the stability of your rocket. It will also calculate the CP and give you the CM (CG).

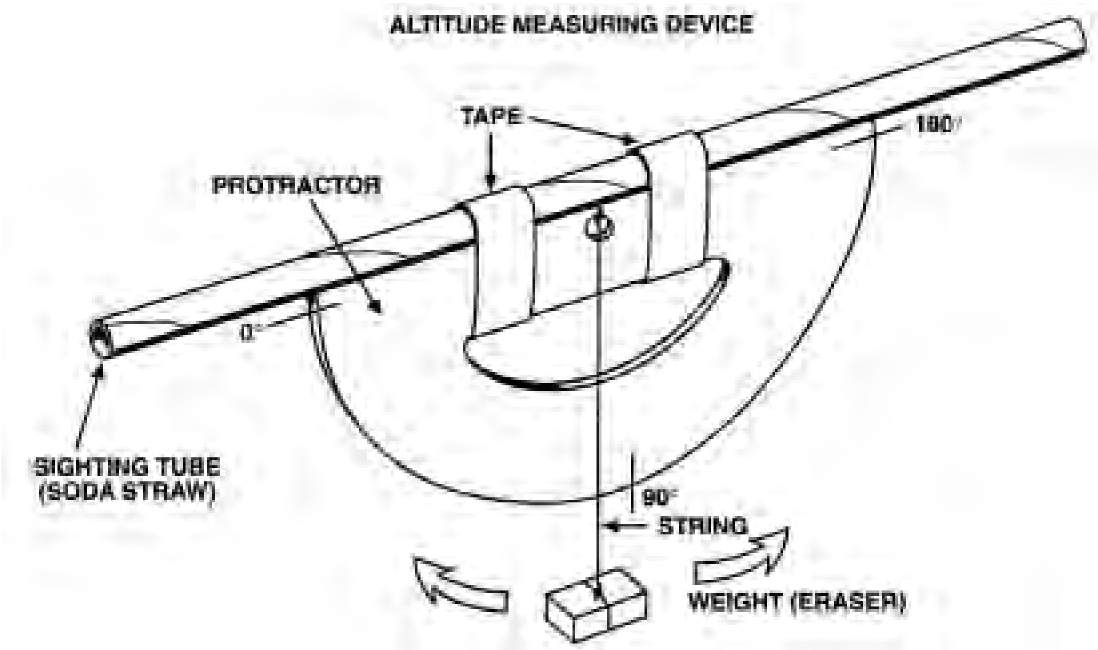
CP of my rocket

CM of my rocket

DETERMINING ALTITUDE
Making your Own Altitude Measuring Device

You will need the following things:

- A large diameter soda straw
- A 20 cm length of string
- A protractor
- A weight (an eraser)
- Tape



You will be constructing a device that looks like the one in the diagram. Tape the straw across the top of the protractor as shown. The straw will act as a sighting tube. Secure the string to the protractor by slipping it under the straw and around. Tie it to itself and tape it to the back of the protractor. Tie the eraser at the opposite end of the string so that it can act as a weight.

The way an altitude tracking device is used:

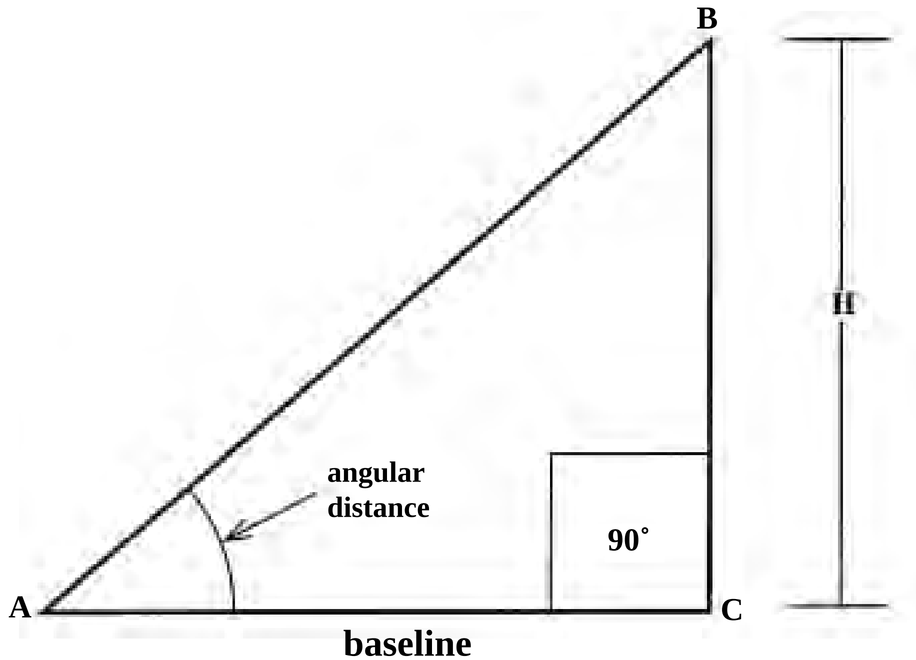
Hold the straw up to your eye. You will focus on the rocket as it is being launched. Move the device up as the rocket ascends. When you see the parachute on your rocket pop out, you will know your rocket has reached apogee. At that instant, hold the string with your finger exactly where it is on the protractor. Read the number on your protractor and record it on a pad of paper. That number will help you determine how high your rocket went.

Try the procedure several times so you can get the feel of it before the rocket launch. Your teacher has selected some objects, such as a flagpole, on which to practice measuring altitude.

Stand at the place your teacher has marked for each object. Hold the straw up to your eye. Move the other end of the device up until you can see the top of the object. At that point, hold your finger on the string against the protractor. Record the angle.

Your teacher will give you the baseline measurement. Use the formula and the table of tangents to determine the height or altitude of each object.

CALCULATING ALTITUDE



The rocket is being launched at C. You are standing at A with your altitude tracking device. You are trying to determine the angle at A by tracking your rocket as it travels from C to B. B is apogee and that is where you need to note where the string is on the protractor. Remember, if you are using the homemade tracker made with the protractor, you have to subtract that number from 90˚ in order to get the angular distance. If you are using an Estes Altitrak™ you can read the angular distance directly off of the Altitrak™.

The sum of the angles of a triangle is 180˚. The angle at C is a right angle and is 90˚.

$$\tan A = \frac{BC}{AC} = \frac{H}{Baseline}$$

so H = Tangent of angular distance x baseline.
Angular distance = 25˚
Tangent of angular distance = ? (You will need your table of tangents)
Baseline = 150 feet
H = ______________________________
Angular distance = 40˚
Tangent of angular distance =_________
Baseline = 300 feet
H = ______________________________

Make up problems for your group to solve.

PRACTICE DETERMINING ALTITUDE

Flagpole

Angular distance = ___________________

Tangent of angular distance =__________

Baseline = ___________________________

H = _________________________________

Tall Tree

Angular distance = ___________________

Tangent of angular distance =__________

Baseline = ___________________________

H = _________________________________

Basketball backboard

Angular distance = ___________________

Tangent of angular distance =__________

Baseline = ___________________________

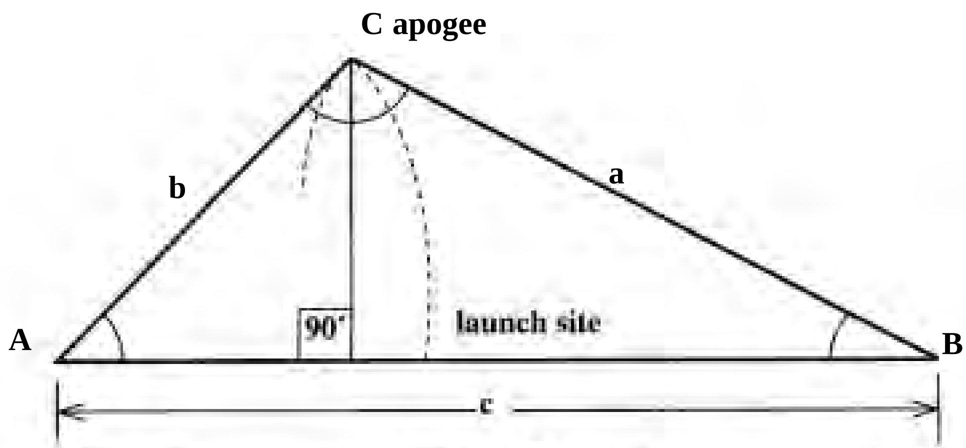
H = _________________________________

Make up problems for your partner to solve.

TWO STATION TRACKING

A more accurate system of tracking uses two tracking stations placed on opposite sides of the launch pad in line with the wind. It uses sines instead of tangents. The formula for this method is:

$$CD = \frac{c \times \sin A \times \sin B}{\sin C} \; where \; C = 180^{\circ} - (A + B)$$



Sines and Tangents

∠	Sine	tan	∠	Sine	tan	∠	Sine	tan
1	.0175	.0175	28	.4695	.5317	54	.8090	1.3764
2	.0349	.0349	29	.4848	.5543	55	.8192	1.4281
3	.0523	.0524	30	.5000	.5774	56	.8290	1.4826
4	.0698	.0699	31	.5150	.6009	57	.8387	1.5399
5	.0872	.0875	32	.5299	.6249	58	.8480	1.6003
6	.1045	.1051	33	.5446	.6494	59	.8572	1.6643
7	.1219	.1228	34	.5592	.6745	60	.8660	1.7321
8	.1392	.1405	35	.5736	.7002	61	.8746	1.8040
9	.1564	.1584	36	.5878	.7265	62	.8829	1.8807
10	.1736	.1763	37	.6018	.7536	63	.8910	1.9626
11	.1908	.1944	38	.6157	.7813	64	.8988	2.0503
12	.2079	.2126	39	.6293	.8098	65	.9063	2.1445
13	.2250	.2309	40	.6428	.8391	66	.9135	2.2460
14	.2419	.2493	41	.6561	.8693	67	.9205	2.3559
15	.2588	.2679	42	.6691	.9004	68	.9272	2.4751
16	.2756	.2867	43	.6820	.9325	69	.9336	2.6051
17	.2924	.3057	44	.6947	.9657	70	.9397	2.7475
18	.3090	.3249	45	.7071	1.0000	71	.9455	2.9042
19	.3256	.3443	46	.7193	1.0355	72	.9511	3.0777
20	.3420	.3640	47	.7314	1.0724	73	.9563	3.2709
21	.3584	.3839	48	.7431	1.1106	74	.9613	3.4874
22	.3746	.4040	49	.7547	1.1504	75	.9659	3.7321
23	.3907	.4245	50	.7660	1.1918	76	.9703	4.0108
24	.4067	.4452	51	.7771	1.2349	77	.9744	4.3315
25	.4226	.4663	52	.7880	1.2799	78	.9781	4.7046
26	.4384	.4877	53	.7986	1.3270	79	.9816	5.1446
27	.4540	.5095				80	.9848	5.6713

First, list the angle and their sines.

Angle A = 34˚ Sine A = 0.5592 (Taken from the table of sines and tangents)
Angle B = 22˚ Sine B = 0.3746
Angle C = 124˚ Sine C = 0.8290

c = 1000 ft
Sin C = 0.8290

Using the formula with the sines and distances known:

$$\frac{1000}{0.8290}=\frac{b}{0.3746}=\frac{a}{0.5592}$$

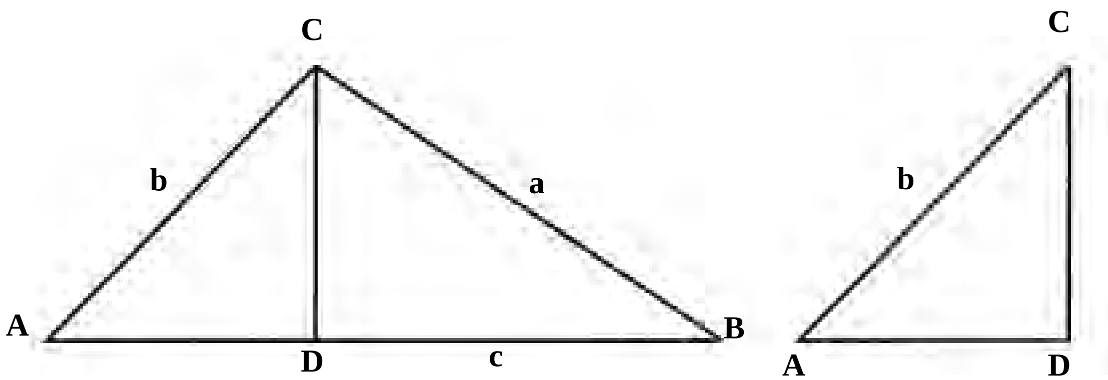
$$\frac{1000}{0.8290}=\frac{b}{0.3746}=\frac{a}{0.5592}$$

The quotient needed to find dividends to solve for b and a is 1205.

b= 1205 x 0.3746
b= 451 feet

a = 1205 x 0.5592
a = 674 feet

Three sides of the triangle are now known.



The altitude of the rocket is the distance from D to C in the diagram. The angle formed by the meeting of AB and CD is a right angle. The sine of an angle in a right triangle is the relation of the opposite side to the hypotenuse. The value of the opposite, CD, needs to be determined.

Sine A = 0.5592

$0.5592 = \frac{CD}{451}$ or Sin a = $\frac{opposite\ side}{hypotenuse}$

CD = Sin A x b
CD = 0.5592 x 451 feet
CD = 252 feet

We know that the altitude reached by the rocket was 252 feet.
Using a quicker formula for this method:

$$CD=\frac{c\times SinA\times SinB}{SinC}$$

$$CD=\frac{1000\times 0.5592\times 0.3746}{0.8290}$$

$$CD=252\ feet$$

Make up some other examples

TWO STATION TRACKING USING AZIMUTH ANGLE

Sines and Tangents

∠	Sine	tan	∠	Sine	tan	∠	Sine	tan
1	.0175	.0175	28	.4695	.5317	54	.8090	1.3764
2	.0349	.0349	29	.4848	.5543	55	.8192	1.4281
3	.0523	.0524	30	.5000	.5774	56	.8290	1.4826
4	.0698	.0699	31	.5150	.6009	57	.8387	1.5399
5	.0872	.0875	32	.5299	.6249	58	.8480	1.6003
6	.1045	.1051	33	.5446	.6494	59	.8572	1.6643
7	.1219	.1228	34	.5592	.6745	60	.8660	1.7321
8	.1392	.1405	35	.5736	.7002	61	.8746	1.8040
9	.1564	.1584	36	.5878	.7265	62	.8829	1.8807
10	.1736	.1763	37	.6018	.7536	63	.8910	1.9626
11	.1908	.1944	38	.6157	.7813	64	.8988	2.0503
12	.2079	.2126	39	.6293	.8098	65	.9063	2.1445
13	.2250	.2309	40	.6428	.8391	66	.9135	2.2460
14	.2419	.2493	41	.6561	.8693	67	.9205	2.3559
15	.2588	.2679	42	.6691	.9004	68	.9272	2.4751
16	.2756	.2867	43	.6820	.9325	69	.9336	2.6051
17	.2924	.3057	44	.6947	.9657	70	.9397	2.7475
18	.3090	.3249	45	.7071	1.0000	71	.9455	2.9042
19	.3256	.3443	46	.7193	1.0355	72	.9511	3.0777
20	.3420	.3640	47	.7314	1.0724	73	.9563	3.2709
21	.3584	.3839	48	.7431	1.1106	74	.9613	3.4874
22	.3746	.4040	49	.7547	1.1504	75	.9659	3.7321
23	.3907	.4245	50	.7660	1.1918	76	.9703	4.0108
24	.4067	.4452	51	.7771	1.2349	77	.9744	4.3315
25	.4226	.4663	52	.7880	1.2799	78	.9781	4.7046
26	.4384	.4877	53	.7986	1.3270	79	.9816	5.1446
27	.4540	.5095				80	.9848	5.6713

The formula for determining the height reached as determined by station "tracking north" is:

$$PH = \sin C \times \tan \angle B \times \frac{(b)}{\sin(180 - [\angle A + \angle C])} \qquad \text{(b = baseline distance)}$$

The formula for determining the height reached as determined by station "tracking south" is:

$$PH = \sin \angle A \times \tan \angle D \times \frac{(b)}{\sin\left(180 - [\angle A + \angle C]\right)} \qquad \text{(b = baseline distance)}$$

In this method, heights found by the two tracking stations are averaged. Both heights must be within the range of the average of both heights plus or minus 10 per cent. If they are not, they are considered unreliable and are not used.

Calculate the height reached by a rocket using this method using the following data:

The tracking systems are 200 feet apart.

<u>North</u>

Elevation Angle = 50˚
Azimuth Angle = 30˚
C = 20˚
B = 50˚
A = 30˚
Sine of C = 0.342
Tan of B = 1.192

<u>South</u>

Elevation Angle = 43˚
Azimuth Angle = 20˚
A = 30˚
D = 43˚
C = 20˚
Sine of A = 0.500
Tan of D = 0.933

Tracking North

Height = 0.342 x 1.192 x 200/sin (180-[30˚ + 20˚])
= 0.342 x 1.192 x 200/sin 130˚
= 0.342 x 1.192 x 200/0.776
= 0.342 x 1.192 x 261.1
= 106.4 feet

Tracking South

Height = sin 30˚ x tan 43˚o x 200/sin (180 - [30˚ + 20˚])
= 0.500 x 0.933 x 200/sin 130˚
= 0.500 x 0.933 x 200/0.776
= 0.500 x 0.933 x 261.1
= 121.7 feet

The results from the two tracking stations are averaged:

106.4 + 121.7 = 228 ÷ 2 = 114.1

To see if the results are within 10 percent of the average:

114.1 x 0.10 = 11.4 114.1 + 11.4 = 125.5
The south tracking station with 121.7 is within the 10 percent allowed.
114.1 - 11.4 = 102.7
The north tracking station with 106.4 is within the 10 percent allowed.
In this example 114.1 would be recorded as the average height.

Another formula for calculating the percentage where the results are in respect to one another is the following:

$$C = \frac{h\ north - h\ south}{2h\ avg} x100\ \%$$

Using the data from the example:

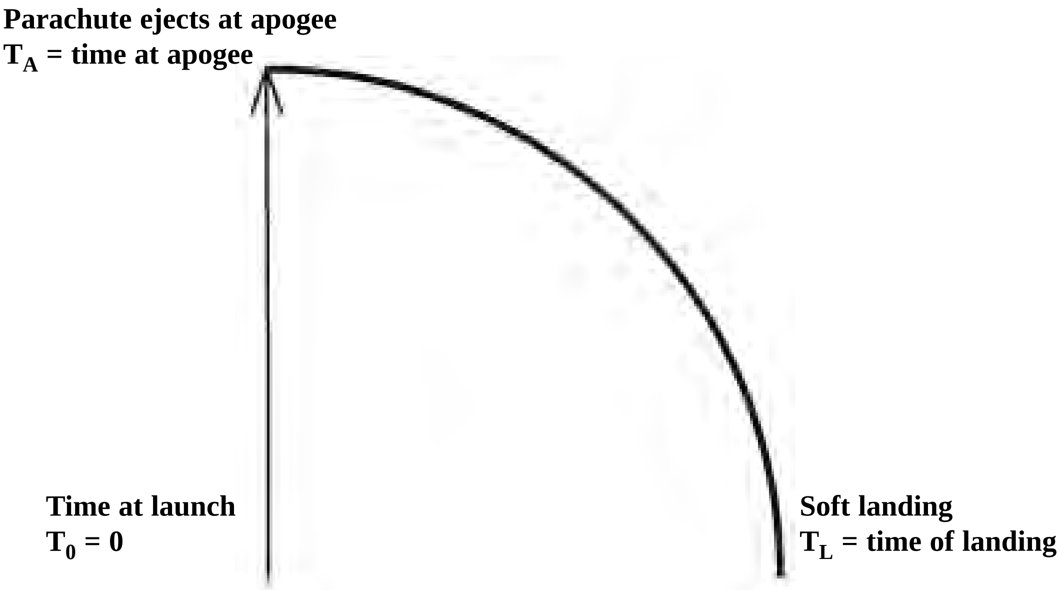
$$C = \frac{106.4 - 121.7}{2(114.1)} \times 100 = -0.06705x100$$

$$C = 0.06705 \times 100$$

$$= 6.705\%$$

Calculate additional problems below using the formulas:

DETERMINING AVERAGE SPEED



We can calculate the "launch to apogee" average speed and the "apogee to landing" average speed.

The formula is Average Speed = Distance traveled ÷ Time of travel. Distance traveled on the diagram is the distance between T_O to T_A (launch to apogee).

You have learned how to determine the altitude or the distance traveled by using your altitude tracking device and using the mathematical formula:

Height = Tangent of angular distance x baseline.

Use the following example data:

T_O = 0 seconds

T_A = 3.2 seconds (this would be determined by someone with a stopwatch starting at launch and stopping at apogee).

T_L = 4.1 seconds (this is determined by someone with a stopwatch starting at apogee and stopping at landing).

Altitude = 288.7 feet

And plug those figures into the following formula:

Average Speed ascending = Altitude ÷ (T_A - T_O)

288.7 feet ÷ (3.2 seconds - 0) = 90.21 feet per second

If you would like to know the miles per hour you can multiply your answer by 0.682.

90.21 feet per second x 0.682 = 61.52 miles per hour.

When you want to know the average speed descending, from apogee to landing, use this formula:

Altitude ÷ T_L - T_A

287.7 feet ÷ (4.1 - 0 seconds) = 70.41 feet per second (T_A in this formula = 0 because the stopwatch was restarted at apogee).

Multiply your answer by 0.682 (70.41 feet per second x 0.682 = 48.02 miles per hour).

VELOCITIES AND ACCELERATIONS

$$V_m = \left(\frac{T}{W_{av}} - 1\right) gt$$

V_m = average velocity during engine burn
W_{av} = average weight of rocket
T = thrust (average thrust of rocket engine)
g = acceleration due to gravity (32.2 feet/second^2)
t = engine burn time in seconds

Example involves analysis of an Alpha® rocket using an A8-3 engine. The Alpha® with engine weighs 1.37 ounces (38.8 grams) at lift-off. The weight of propellant in an A8-3 engine is 0.11 ounces (3.12 grams) giving an average weight of 1.32 ounces (37.2 grams) during the thrust phase of the flight. The A8-3 thrusts for 0.32 seconds and has a total impulse of 0.56 pound-seconds (2.50 N-sec.). These values may be found in or calculated from information in an Estes catalog. In the metric system, weight is a force whose unit of measurement is the "newton" (N). Since forces equals mass times acceleration, weight equals mass times the acceleration of gravity: W = mg. W = weight (N); m = mass (kg); g = acceleration of gravity (9.81 m/s^2)

First, find the average thrust.

$$T = \frac{\textit{total impulse}}{\textit{burn time}}$$

English

$$T = \frac{0.56 \textit{ pound-seconds}}{0.32 \textit{ sec.}} = 1.75 \textit{ pounds}$$

Metric

$$T = \frac{2.50 \textit{ newton-second}}{32 \textit{ sec.}} = 7.81 \textit{ newtons}$$

Multiply by the conversion factor of 16 ounces/pound:

$$T = 1.75 \, pounds \, x \, \frac{16 \, ounces}{1 \, pound}$$

$$= 28 \, ounces$$

Thrust is 28 oz.

$$V_m = \left(\frac{T}{W_{av}} - 1\right) gt$$

w = mg : to convert mass (kg) to weight (newton, N)

$$V_m = \left(\frac{28.0 oz.}{1.32 oz.} - 1\right)(32.2 \, ft./\sec^2)(0.32 \sec.)$$

$$= (21.21 - 1)[(32.2 \, ft./\sec.^2)(0.32 \sec.)]$$

$$= (20.21)(10.24 \, ft./\sec.)$$

$$= 208.27 \, ft./\sec.$$

$$V_m = \left(\frac{7.81 N}{0.365 N} - 1\right) 9.81 \frac{m}{\sec.^2} (.32 \sec.)$$

$$V_m = 64.03 \, m/\sec.$$

The expression $\left(\frac{T}{W_{av}} - 1\right)$ gives you an idea of the number of "gravities", or "g's", the rocket experiences in upward flight during acceleration. "1" is subtracted in this expression to allow for the pull of Earth's gravity on the rocket. This rocket developed a fairly

high velocity by the end of the thrusting phase of the flight.
Velocity developed by the same Alpha® with a C6-5 engine can be determined.
To convert mass (kg) to weight (newtons, N): W = mg

$$T = \frac{2.25 \text{lb./sec.}}{1.70 \text{sec.}} \text{ x } \frac{16 \text{oz.}}{1 \text{lb.}}$$

$=$

$=$

$$V_m = \left(\frac{\quad}{1.49 \text{oz.}} - 1 \right) (32 \text{ft./sec.}^2) 1.7 \text{sec.}$$

$=$

$=$

$=$

$= \text{ft./sec.}$

$$T = \frac{10 \text{ N-sec.}}{1.70 \text{sec.}}$$

$=$

$$V_m = \left(\frac{\quad}{.414} - 1 \right) \left(9.81 \frac{\text{m}}{\text{sec.}^2} \right) (1.7 \text{sec.})$$

$=$

$=$

$=$

$= \text{m / sec.}$

Use the Estes Catalog at estesrockets.com to get some data about various rockets and engines and make up some times in seconds. Develop some additional problems to gain practice in determining values for accelerations and velocities for model rockets below.

LAUNCH DATA - INDIVIDUAL

Student________________________________Date_________________________

BASELINE	ANGULAR DISTANCE (One station tracking)	ANGULAR DISTANCE (Two station tracking) Elev/Azim	ANGULAR DISTANCE (Two station tracking) Elev/Azim	TIME TO APOGEE (Seconds)	TIME TO LANDING (Seconds)

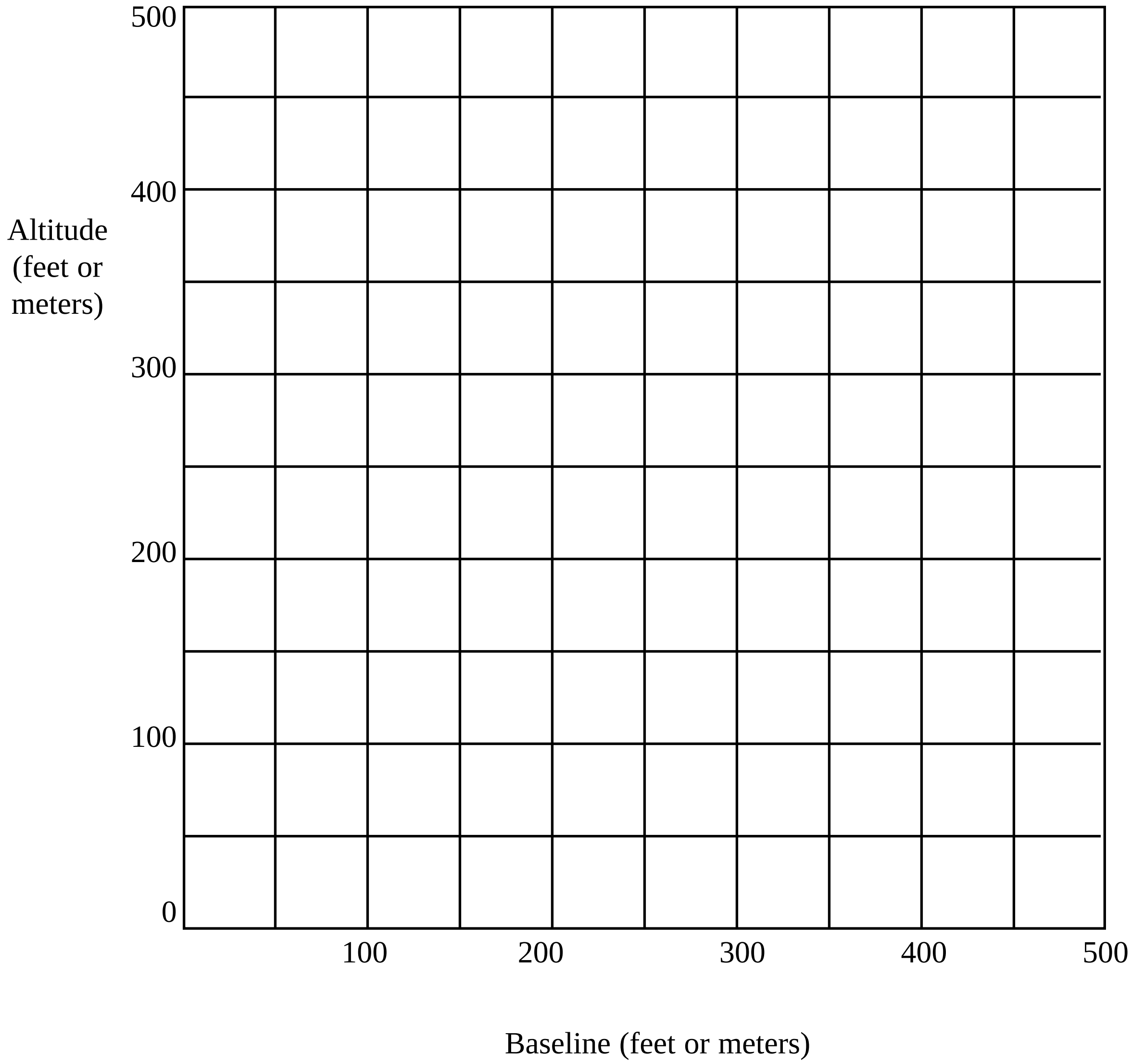
Show your calculations here:

Apogee

Thrust

Velocity

GRAPHING USING ONE STATION TRACKING



Use the horizontal axis to plot the baseline distance (distance from launcher to tracker).

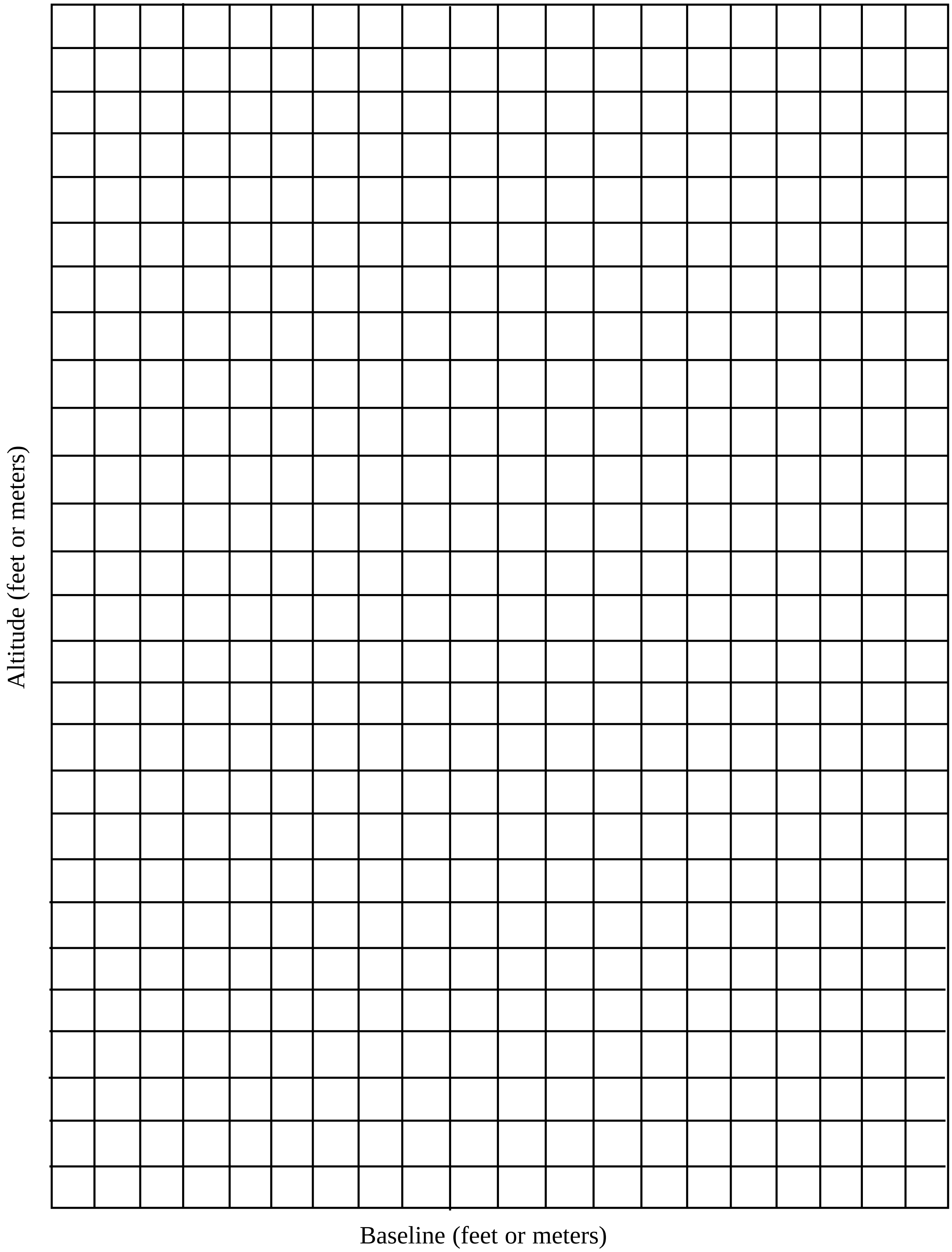
Use the vertical axis to plot the rocket's altitude. The rocket is launched at 0 on the graph paper and climbs vertically up the vertical axis.

Mark the tracker's position on the horizontal axis and plot the elevation angle.

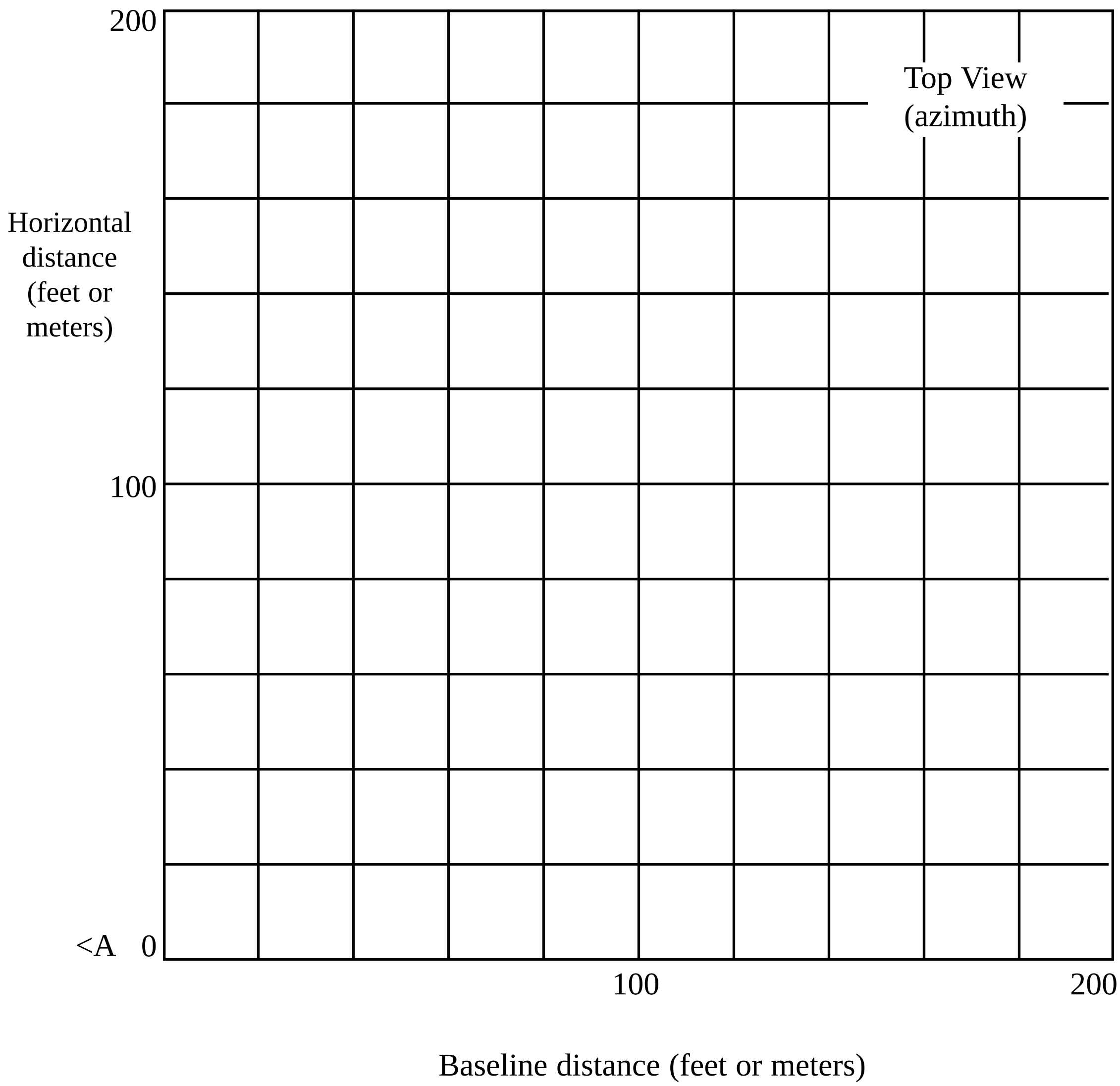
Extend the angle (line of sight) until it intersects the vertical axis. The intersected point on the vertical axis is the rocket's altitude.

GRAPHING USING TWO STATION TRACKING

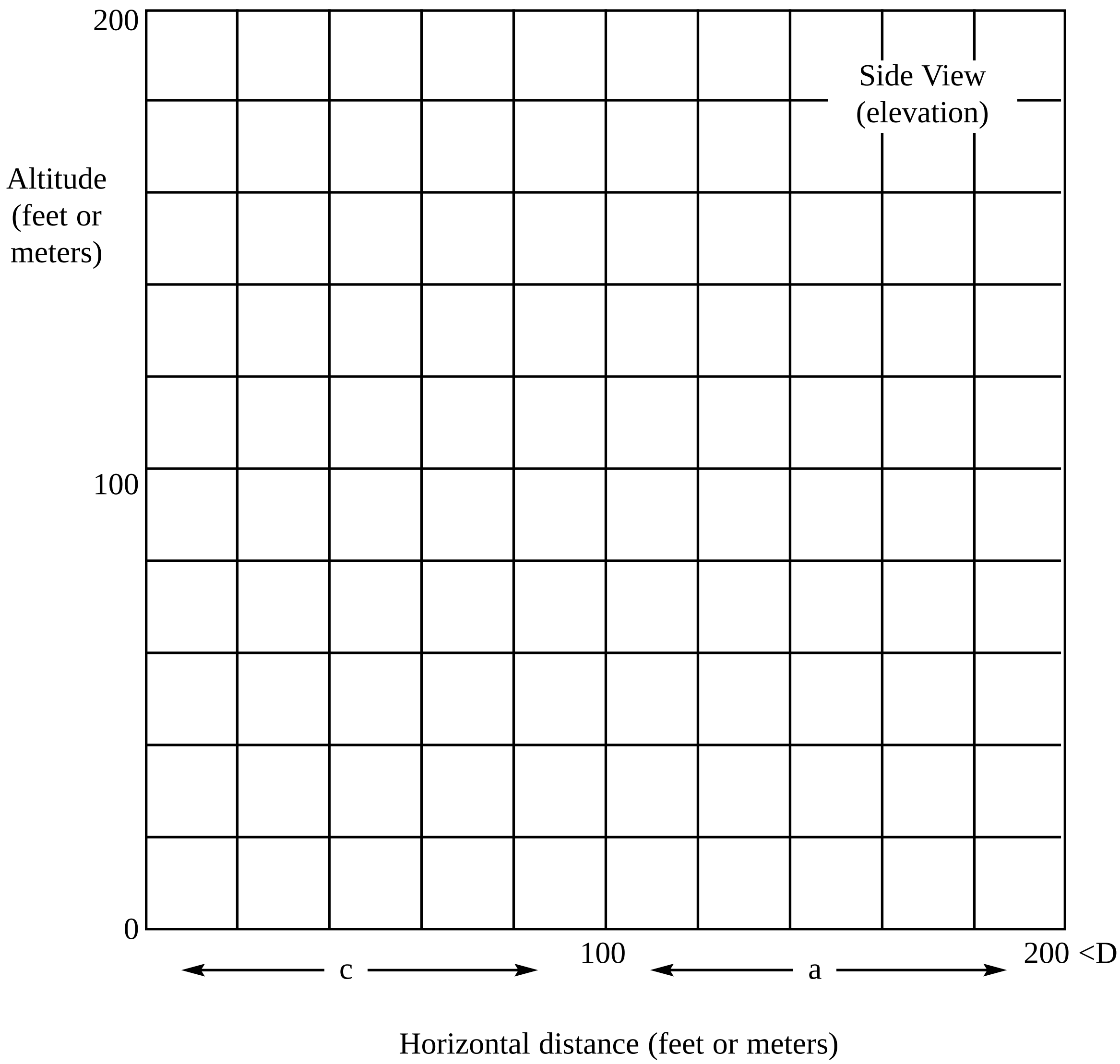
Layout baseline distances (horizontal axis) and rocket altitude (vertical axis) on graph paper. Plot the elevation angles at each end of the graph, then extend them until they intersect. The point of intersection extended to the vertical axis is the rocket's altitude.



GRAPHING FOR TWO STATION TRACKING USING THE AZIMUTH ANGLE



Plot the baseline distance and azimuth angles for the top view graph to find the unknown horizontal distances (a and c).



Plot the elevation angles and distances a and c to graphically depict height PH. When the elevation angles are plotted there will be two different values of PH so they must be averaged.

Calculations

REFINED CENTER OF PRESSURE CALCULATIONS FOR ROCKETS WITH OR WITHOUT PAYLOADS

Up to this point all methods previously discussed calculated the “lateral center of pressure”. The concept and its interaction with center of gravity to explain stability is correct. However, lateral center of pressure only examined the airloads striking the sides of the rocket. In reality, both the nose cone and fins of a rocket produce lift when air flows past it. This lift is perpendicular to the longitudinal axis (perpendicular to the body tube) and is referred to as a “normal” force. Each of these normal forces attempts to rotate the rocket about its center of gravity. Just as was done with lateral center of pressure, we will find the one single point on the rocket where these normal forces and their resulting moments are in equilibrium (balanced, like the teeter-totter). This point is the center of pressure. As a rocket is disturbed from its flight path (i.e. a gust of wind), the relative wind flowing past the rocket is now at an angle. This angle formed between the relative wind and longitudinal axis of the rocket is referred to as the angle of attack. This term is commonly used in aerodynamics when examining how a wing produces lift. With an angle of attack greater than zero degrees, the rocket’s nose cone and fins produce lift (normal force). The center of pressure is the single point where these forces act as a single force creating the same external effect.

The location of the CP in relation to the CG will then determine the rocket’s stability, just as with lateral center of pressure.

As you review the following examples, you will notice there is no mention of the normal force created by the body tube. Wind tunnel testing has shown that a cylindrical tube does not create normal force itself. It does, however, produce interference to the fins and is compensated for.

The final calculation for lateral center of pressure used the equation:

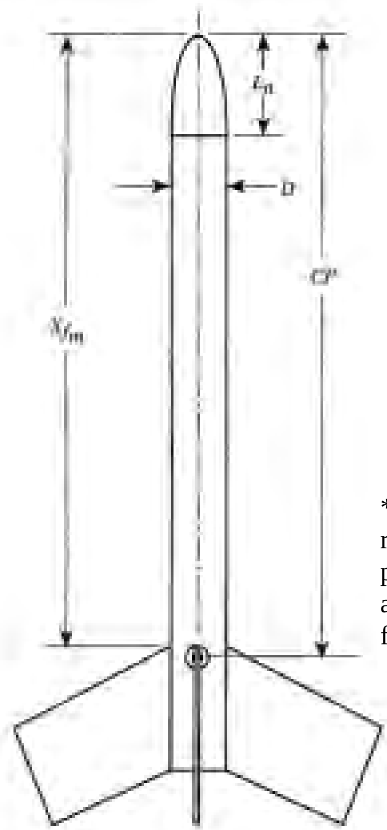
$$\bar{x} = \mathrm{CP} = \frac{A_1 x_1 + A_2 x_2 + \ldots A_n x_n}{A_1 + A_2 + \ldots A_n}$$

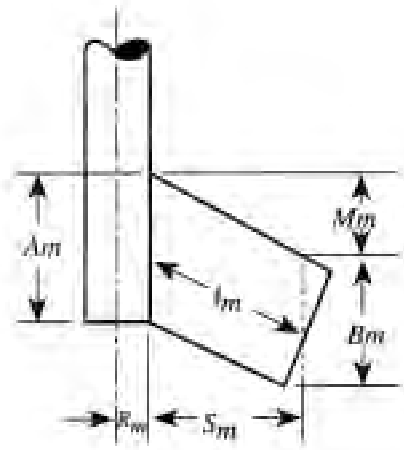
The refined calculation for center of pressure uses a similar equation, substituting normal force (CN) for area (A).

$$\bar{x} = \mathrm{CP} = \frac{C_{N1} x_1 + C_{N2} x_2 + \ldots C_{Nn} x_n}{C_{N1} + C_{N2} + \ldots C_{Nn}}$$

CENTER OF PRESSURE EQUATIONS

Single Stage Without Payload





*NOTE: Dashed line refers to fin tip being parallel to fin root and idealizing actual fin area.

Ln = 31 mm
D = 24.8 mm
X_{f_m} = 298 mm

Main Stage Fin Dimensions
Am = 57 mm
Bm = 42 mm
lm = 56 mm
Mm = 27 mm
Rm = 12.4 mm
Sm = 50 mm
n = 4 (Number of fins)

CENTER OF PRESSURE CALCULATIONS

1. **Nose Cone (Parabolic)**

normal force

$(C_{N\alpha})_n = 2$ (always)

center of pressure

$$\bar{x}_n = \frac{1}{2} L_n = 15.5$$

2. **Main Stage Fins**

interference factor

$$K_{fb_m} = 1 + \frac{Rm}{Sm + Rm} = 1.199$$

normal force on fins in presence of body

$(C_{N\alpha})_{fb_m} = K_{fb_m} (C_{N\alpha})_{f_m} = 31.06$

normal force on four fins

$$(C_{N\alpha})_{fm} = \frac{4n\left(\frac{Sm}{D}\right)^2}{1 + \sqrt{1 + \left(\frac{2lm}{Am + Bm}\right)^2}} = 25.91$$

center of pressure

$$\bar{x}_{fm} = \bar{x}_{fm} + \frac{Mm\,(Am + 2Bm)}{3\,(Am + Bm)} + \frac{1}{6}\left(Am + Bm - \frac{AmBm}{Am + Bm}\right) = 323.29$$

3 **Final CP Calculation**

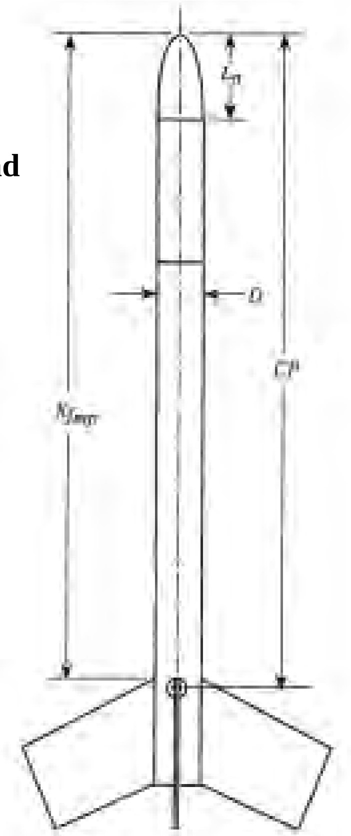
total normal force

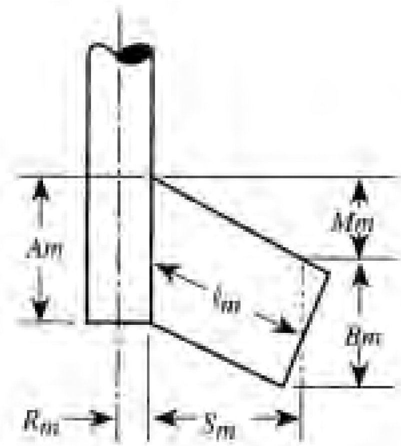
$C_{N\alpha_T} = (C_{N\alpha})_n + (C_{N\alpha})_{fb_m} = 33.06$

center of pressure for entire rocket

$$CP = \frac{(C_{N\alpha})_n \bar{x}_n + (C_{N\alpha})_{fbm} \bar{x}_{fm}}{C_{N\alpha_T}} = 308.22$$

Single Stage With Payload





*NOTE: Dashed line refers to fin tip being parallel to fin root and idealizing actual fin area.

Ln	= 31 mm
D	= 24.8 mm
$X_{f_{mp}}$	= 400 mm

Main Stage Fin Dimensions

Am	= 57 mm
Bm	= 42 mm
lm	= 56 mm
Mm	= 27 mm
Rm	= 12.4 mm
Sm	= 50 mm
n	= 4 (Number of fins)

CENTER OF PRESSURE CALCULATIONS

1. **Nose Cone (Parabolic)**

normal force

$(C_{N\alpha})_n = 2$ (always)

center of pressure

$$\bar{x}_n = \frac{1}{2} L_n = 15.5$$

2. **Main Stage Fins**

interference factor

$$K_{fb_m} = 1 + \frac{Rm}{Sm + Rm} = 1.199$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb_m} = K_{fb_m} (C_{N\alpha})_{b_m} = 31.06$$

normal force on four fins

$$(C_{N\alpha})_{fm} = \frac{4n\left(\frac{Sm}{D}\right)^2}{1 + \sqrt{1 + \left(\frac{2\,lm}{Am + Bm}\right)^2}} = 25.91$$

center of pressure

$$\bar{x}_{fmp} = \bar{x}_{fmp} + \frac{Mm\,(Am + 2Bm)}{3\,(Am + Bm)} + \frac{1}{6}\left(Am + Bm - \frac{AmBm}{Am + Bm}\right) = 425.29$$

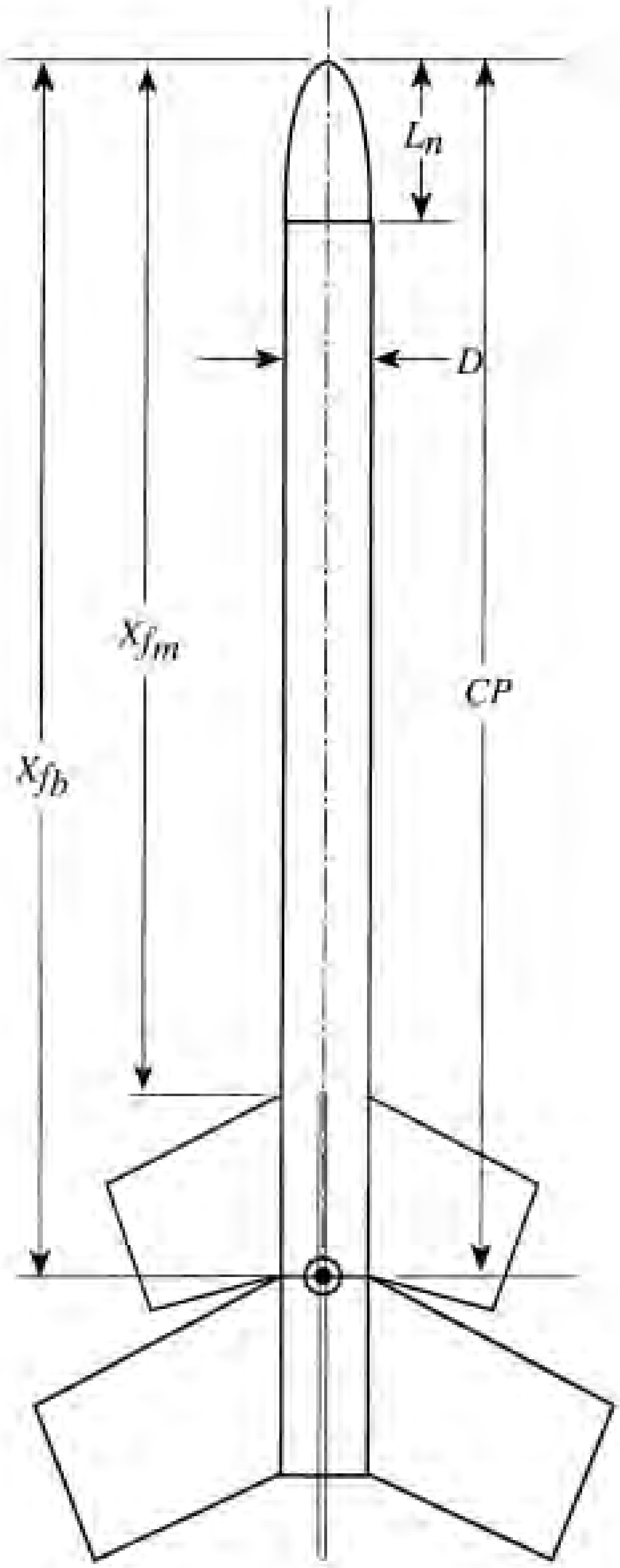
3. **Final CP Calculation**

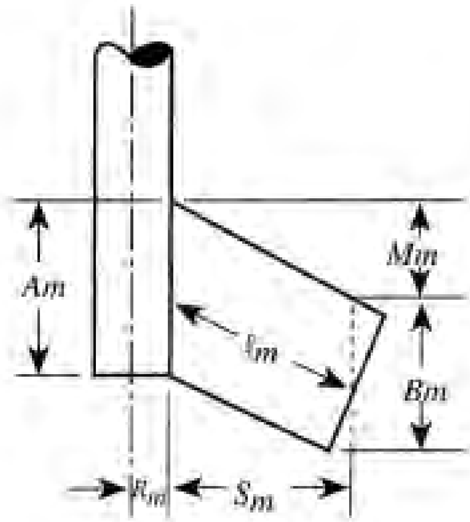
total normal force

$$C_{N\alpha_T} = (C_{N\alpha})_n + (C_{N\alpha})_{fb_m} = 33.06$$

center of pressure for entire rocket

$$CP = \frac{(C_{N\alpha})_n \bar{x}_n + (C_{N\alpha})_{fb_m} \bar{x}_{fmp}}{C_{N\alpha_T}} = 404.05$$

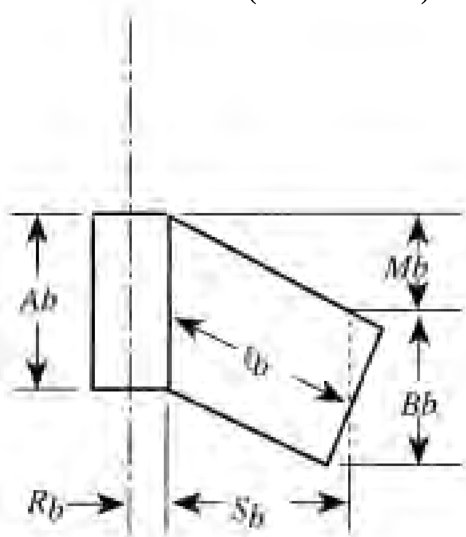




*NOTE: Dashed line refers to fin tip being parallel to fin root and idealizing actual fin area.

Ln = 31 mm
D = 24.8 mm
X_{f_m} = 298 mm
X_{f_b} = 355 mm

Main Stage Fin Dimensions
Am = 57 mm
Bm = 42 mm
lm = 56 mm
Mm = 27 mm
Rm = 12.4 mm
Sm = 50 mm
n = 4 (Number of fins)



Booster Stage Fin Dimensions
Ab = 70 mm
Bb = 53 mm
lb = 70 mm
Mb = 35 mm
Rb = 12.4 mm
Sb = 63 mm
n = 4 (Number of fins)

See next page for Center of Pressure Calculations

CENTER OF PRESSURE CALCULATIONS

1. **Nose Cone (Parabolic)**

normal force

$(C_{N\alpha})_n = 2$ (always)

center of pressure

$$\bar{x}_n = \frac{1}{2} L_n = 15.5$$

2. **Main Stage Fins**

normal force on four fins

$$(C_{N\alpha})_{fm} = \frac{4n\left(\frac{Sm}{D}\right)^2}{1 + \sqrt{1 + \left(\frac{2lm}{Am + Bm}\right)^2}} = 25.91$$

interference factor

$$K_{fbm} = 1 + \frac{Rm}{Sm + Rm} = 1.199$$

normal force on fins in presence of body

$$(C_{N\alpha})_{fb_m} = K_{fb_m}(C_{N\alpha})_{f_m} = 31.06$$

$$\bar{x}_{fm} = \bar{x}_{fm} + \frac{Mm\,(Am + 2Bm)}{3\,(Am + Bm)} + \frac{1}{6}\left(Am + Bm - \frac{AmBm}{Am + Bm}\right) = 323.29$$

3. **Booster Stage Fins**

normal force on four fins

$$(C_{N\alpha})_{fb} = \frac{4n\left(\frac{Sb}{D}\right)^2}{1 + \sqrt{1 + \left(\frac{2lb}{Ab + Bb}\right)^2}} = 41.05$$

interference factor

$$K_{fbb} = 1 + \frac{Rb}{Sb + Rb} = 1.164$$

normal force on fins in presence of body

$(C_{N\alpha})_{fb_b} = K_{fb_b}(C_{N\alpha})_{f_b} = 47.80$

center of pressure

$$\bar{x}_{fb} = \bar{x}_{fb} + \frac{Mm\,(Ab + 2Bb)}{3\,(Ab + Bb)} + \frac{1}{6}\left(Ab + Bb - \frac{AbBb}{Ab + Bb}\right) = 391.94$$

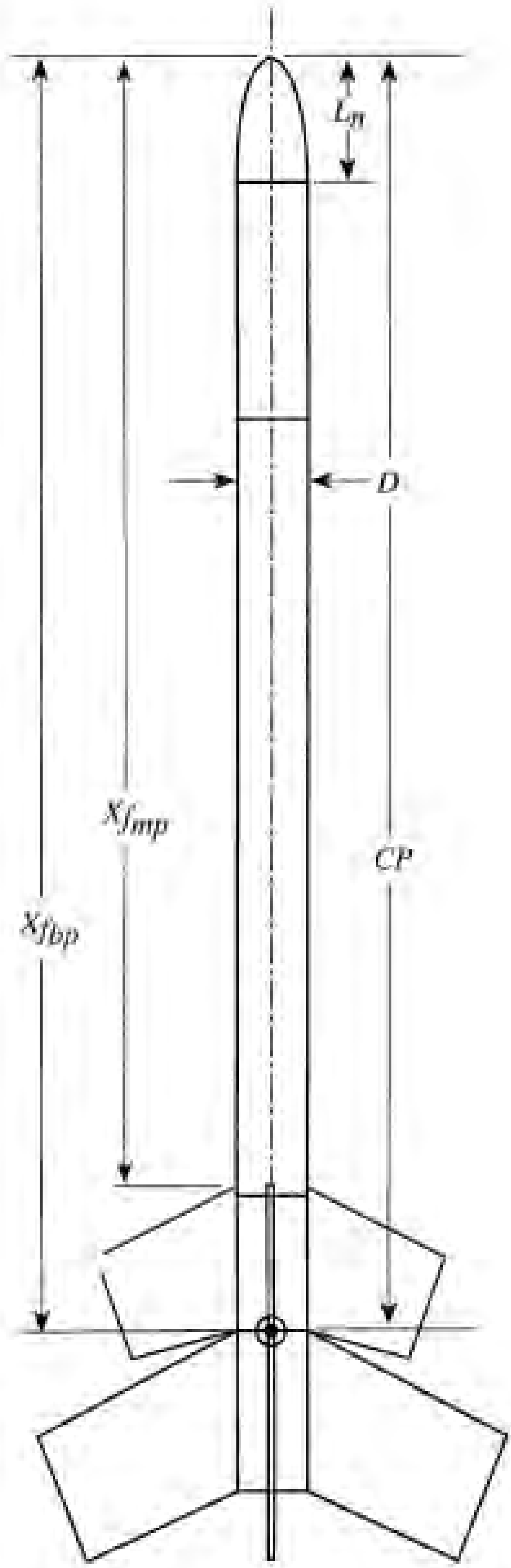
4. **Final CP Calculation**

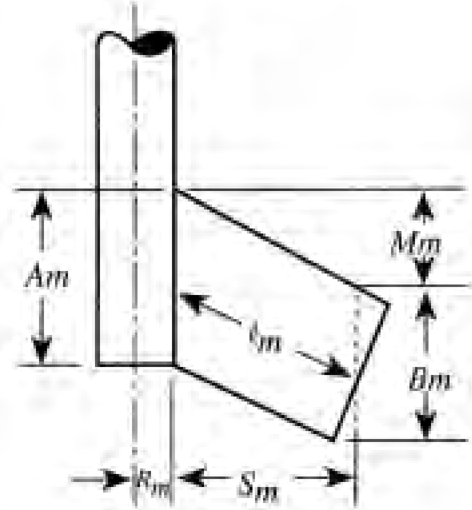
total normal force

$C_{N\alpha_T} = (C_{N\alpha})_n + (C_{N\alpha})_{fb_m} + (C_{N\alpha})_{fb_b} = 80.86$

center of pressure for entire rocket

$$CP = \frac{(C_{N\alpha})_n\,\bar{x}_n + (C_{N\alpha})_{fbm}\,\bar{x}_{fm} + (C_{N\alpha})_{fbb}\,\bar{x}_{fb}}{C_{N\alpha_T}} = 357.71$$

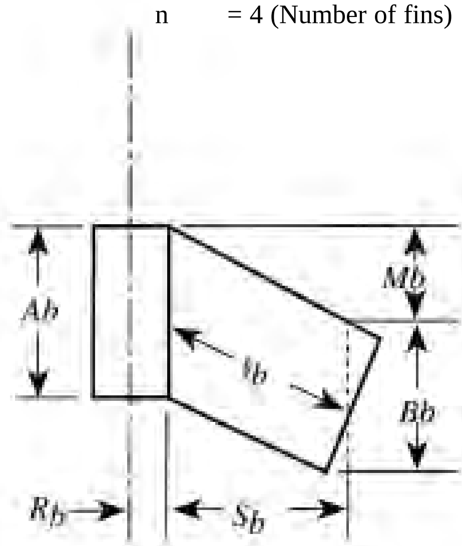




*NOTE: Dashed line refers to fin tip being parallel to fin root and idealizing actual fin area.

Ln = 31 mm
D = 24.8 mm
$X_{f_{mp}}$ = 400 mm
$X_{f_{bp}}$ = 457 mm

Main Stage Fin Dimensions
Am = 57 mm
Bm = 42 mm
lm = 56 mm
Mm = 27 mm
Rm = 12.4 mm
Sm = 50 mm
n = 4 (Number of fins)



Booster Stage Fin Dimensions
Ab = 70 mm
Bb = 53 mm
lb = 70 mm
Mb = 35 mm
Rb = 12.4 mm
Sb = 63 mm
n = 4 (Number of fins)

See next page for Center of Pressure Calculations

CENTER OF PRESSURE CALCULATIONS

1. **Nose Cone (Parabolic)**

   normal force

   $(C_{N\alpha})_n = 2$ (always)

   center of pressure

   $$\bar{x}_n = \frac{1}{2} L_n = 15.5$$

2. **Main Stage Fins**

   normal force on four fins

   $$(C_{N\alpha})_{fm} = \frac{4n\left(\frac{Sm}{D}\right)^2}{1 + \sqrt{1 + \left(\frac{2lm}{Am + Bm}\right)^2}} = 25.91$$

   interference factor

   $$K_{fb_m} = 1 + \frac{Rm}{Sm + Rm} = 1.199$$

   normal force on fins in presence of body

   $(C_{N\alpha})_{fb_m} = K_{fb_m} (C_{N\alpha})_{f_m} = 31.06$

   center of pressure

   $$\bar{x}_{fmp} = \bar{x}_{fmp} + \frac{Mm\,(Am + 2Bm)}{3\,(Am + Bm)} + \frac{1}{6}\left(Am + Bm - \frac{AmBm}{Am + Bm}\right) = 425.29$$

3. **Booster Stage Fins**

   normal force on four fins

   $$(C_{N\alpha})_{fb} = \frac{4n\left(\frac{Sb}{D}\right)^2}{1 + \sqrt{1 + \left(\frac{2lb}{Ab + Bb}\right)^2}} = 41.05$$

   interference factor

   $$K_{fb_b} = 1 + \frac{Rb}{Sb + Rb} = 1.164$$

   normal force on fins in presence of body

   $(C_{N\alpha})_{fb_b} = K_{fb_b} (C_{N\alpha})_{f_b} = 47.80$

   center of pressure

   $$\bar{x}_{fbp} = \bar{x}_{fbp} + \frac{Mm\,(Ab + 2Bb)}{3\,(Ab + Bb)} + \frac{1}{6}\left(Ab + Bb - \frac{AbBb}{Ab + Bb}\right) = 493.94$$

4. Final CP Calculation

   total normal force

   $C_{N\alpha_T} = (C_{N\alpha})_n + (C_{N\alpha})_{fb_m} + (C_{N\alpha})_{fb_b} = 80.86$

   center of pressure for entire rocket

   $$CP = \frac{(C_{N\alpha})_n \bar{x}_n + (C_{N\alpha})_{fb_m} \bar{x}_{fmp} + (C_{N\alpha})_{fb_b} \bar{x}_{fbp}}{C_{N\alpha_T}} = 457.19$$

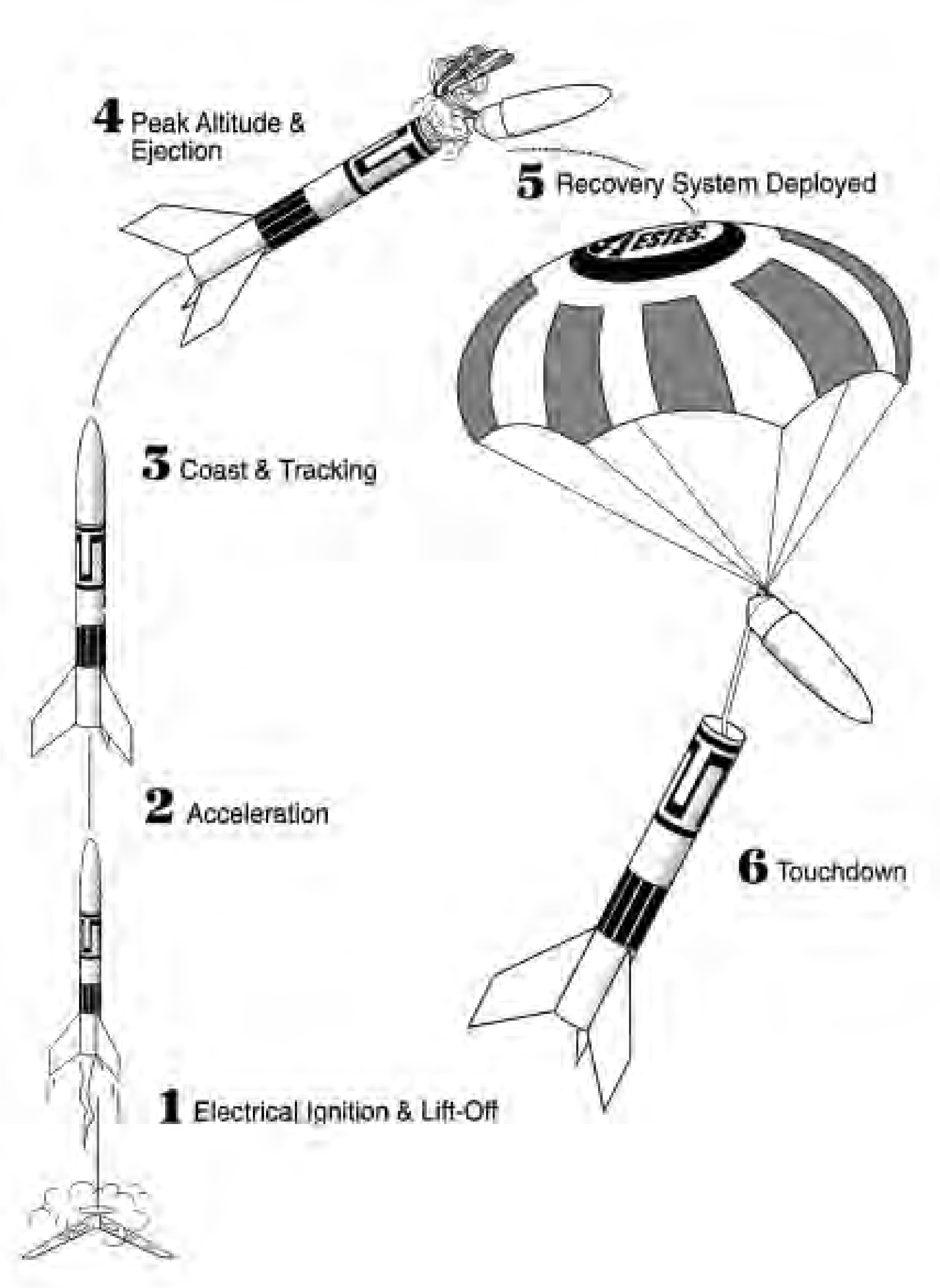
OVERHEAD TRANSPARENCIES

Focus

Name ____

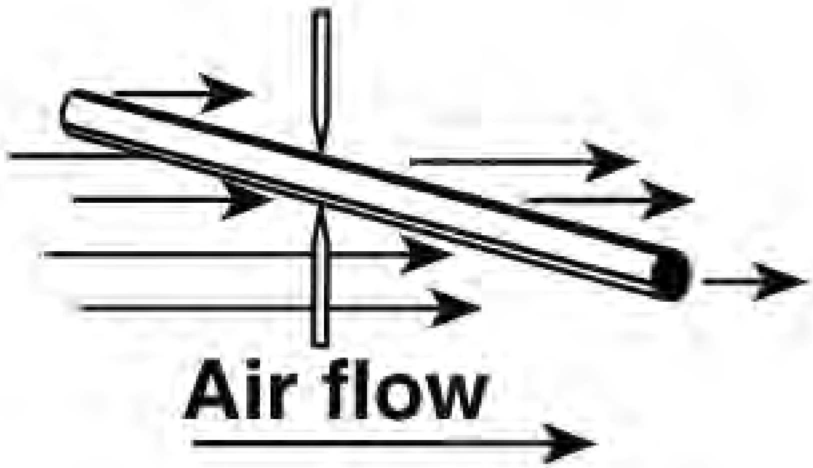
What do I know about model rockets?	What do I want to learn about model rockets?	After the launch: What is a new idea or knowledge that I have since watching the launch?	After the launch: What did I see that I would like to know more about?

FLIGHT SEQUENCE OF A MODEL ROCKET

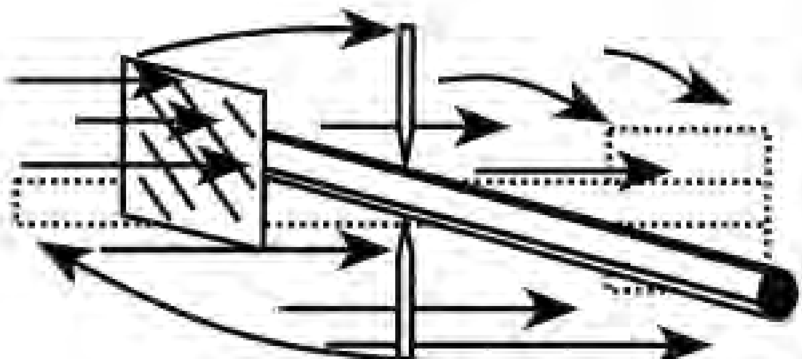


CENTER OF PRESSURE

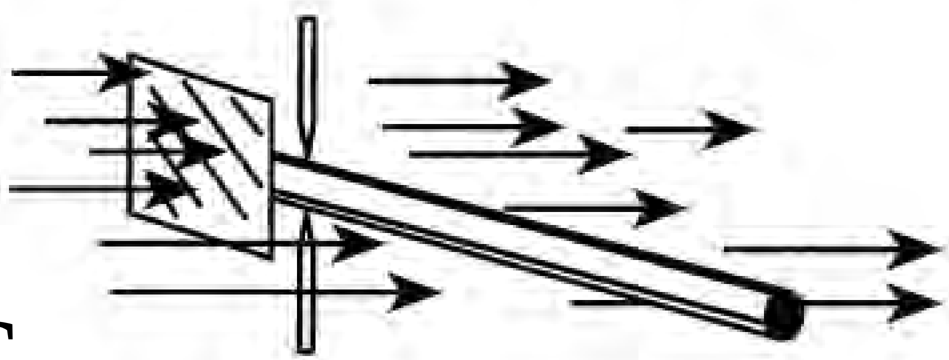
A



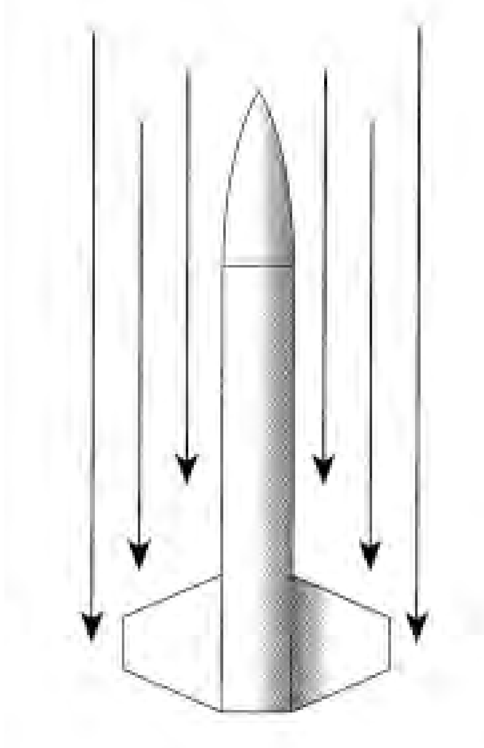
B



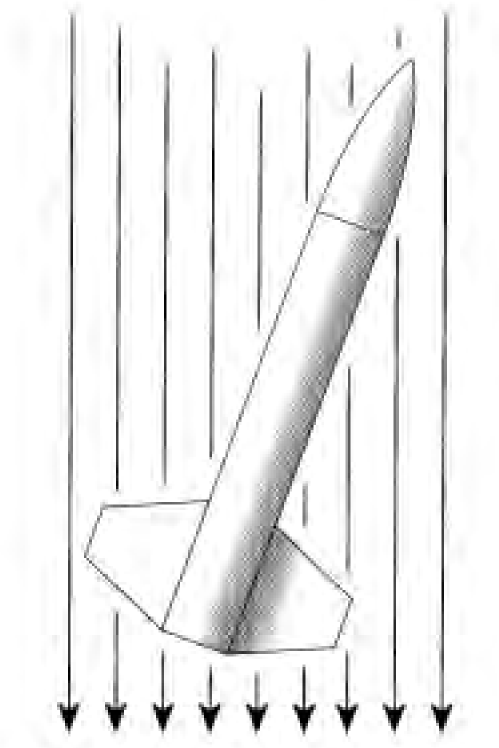
C



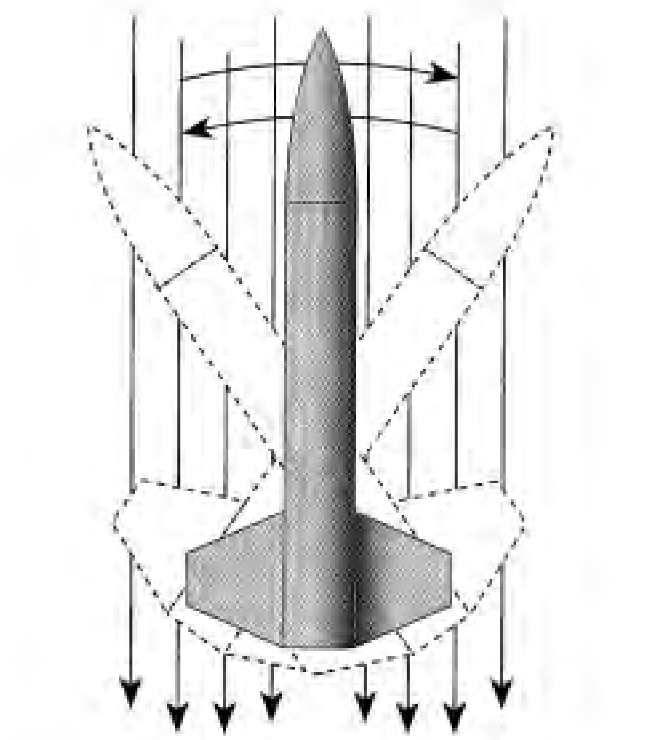
OSCILLATION



NEUTRAL POSITION



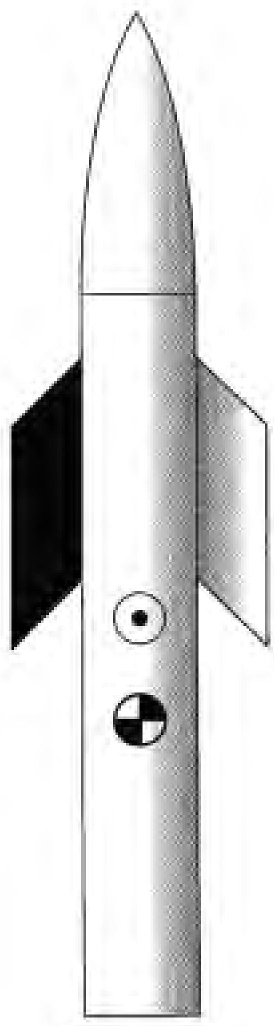
DISTRIBUTED POSITION



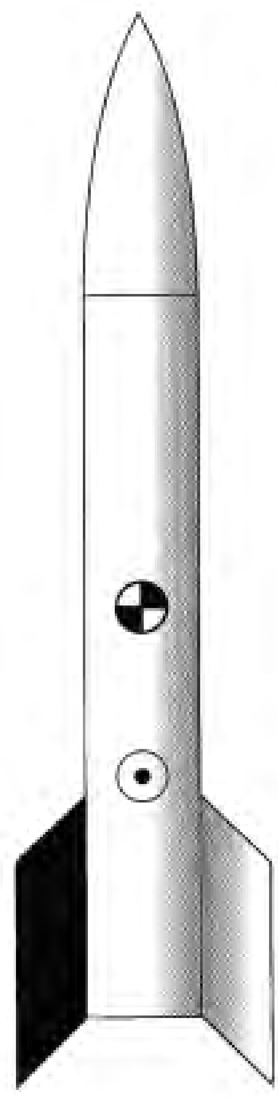
STABLE
(OSCILLATION EXAGGERATED)

STABILITY-FINS

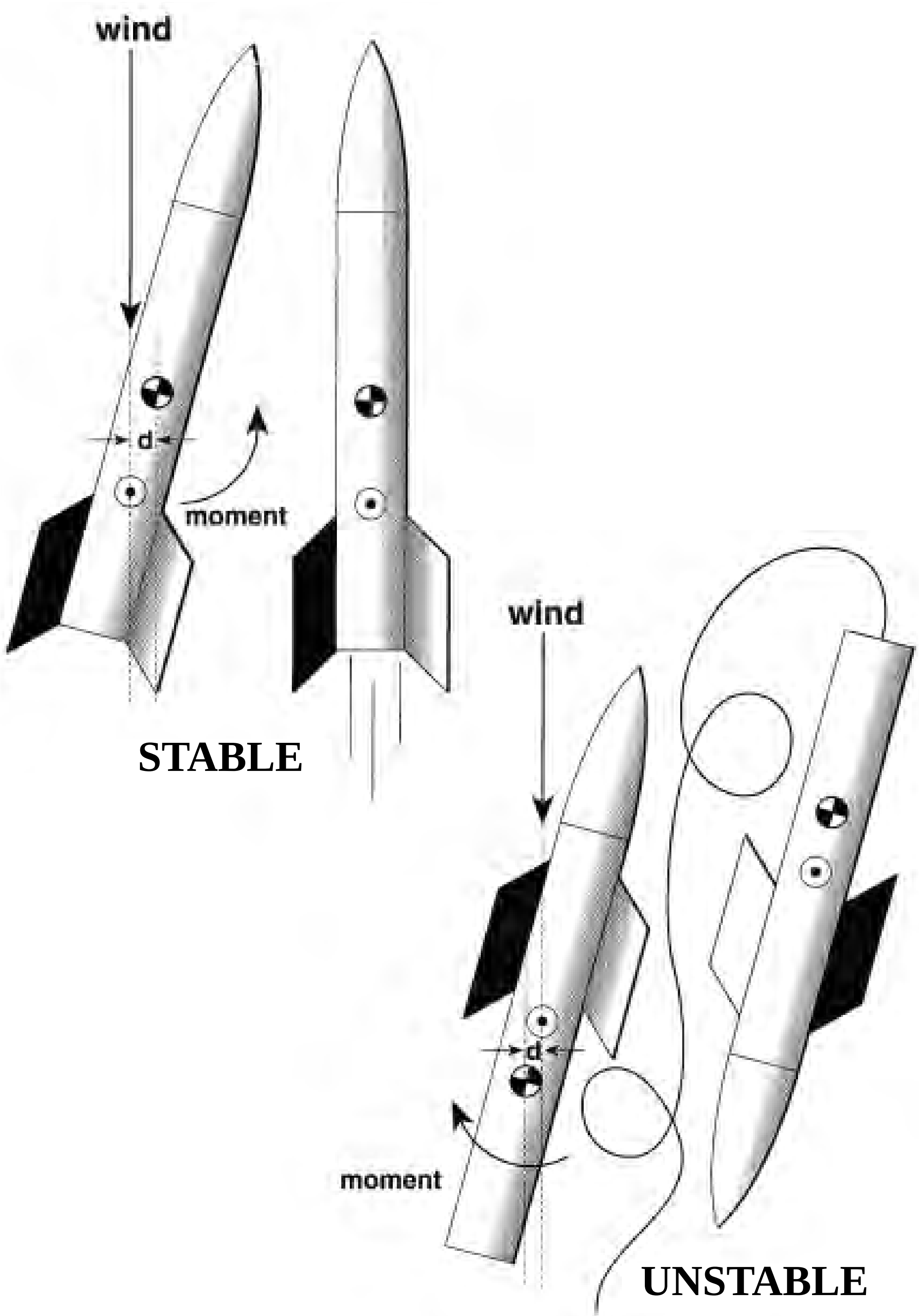
UNSTABLE ROCKET



STABLE ROCKET



STABILITY-CP



1.

NEWTON’S FIRST LAW OF MOTION

- **Objects at rest will stay at rest, and objects in motion will stay in motion in a straight line at constant velocity unless acted upon by an unbalanced force.**

  - **REST**
  - **MOTION**
  - **UNBALANCED FORCE**

2.

NEWTON’S SECOND LAW OF MOTION

- **Force is equal to mass times acceleration.**

  - **MASS**
  - **ACCELERATION**
  - **FORCE**

3.

NEWTON'S THIRD LAW OF MOTION

- **For every action there is an opposite and equal reaction.**

  - **ACTION**

  - **REACTION**

NEWTON'S LAWS OF MOTION

Putting Them Together with Model Rocketry

Law 1:

An unbalanced force must be exerted for a rocket to lift off from a launch pad.

Law 2:

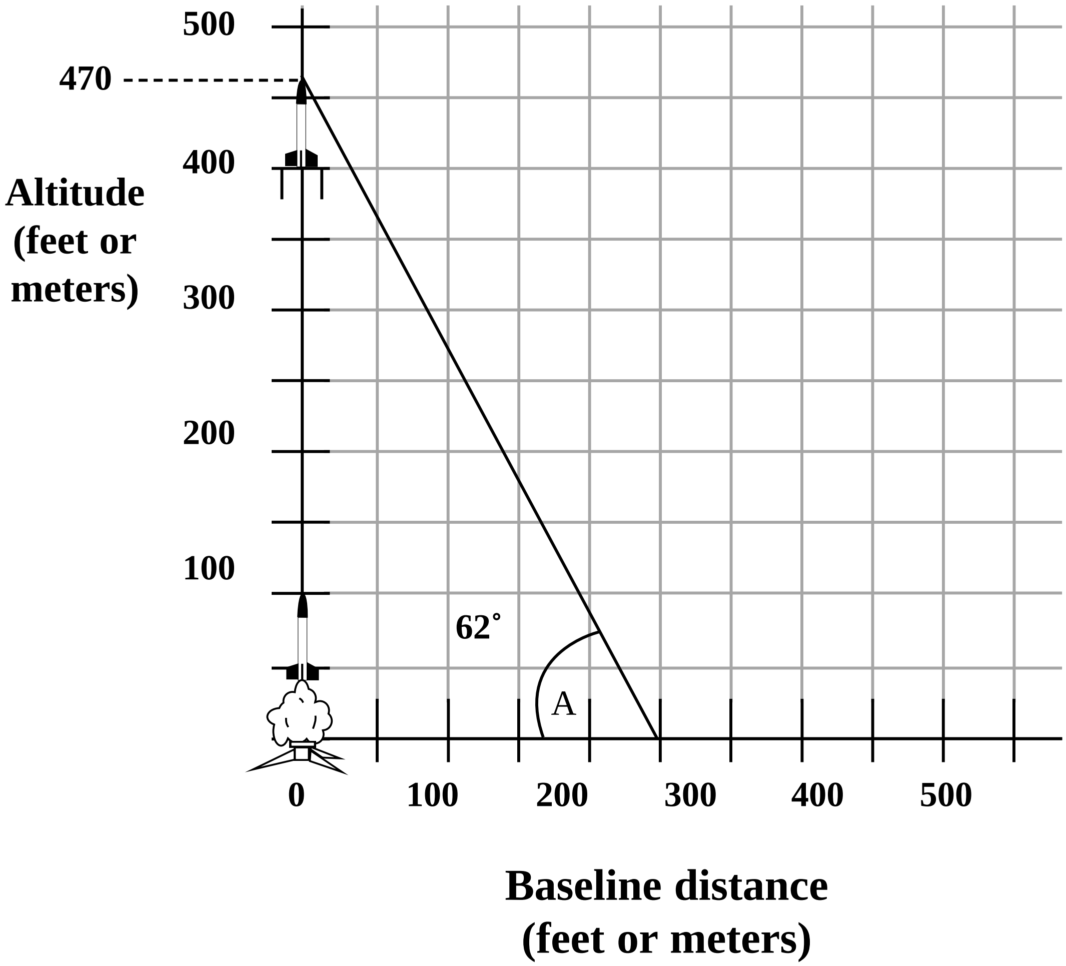
The amount of thrust (force produced by a rocket engine) will be determined by the mass of rocket fuel that is burned and how fast the gas escapes the rocket.

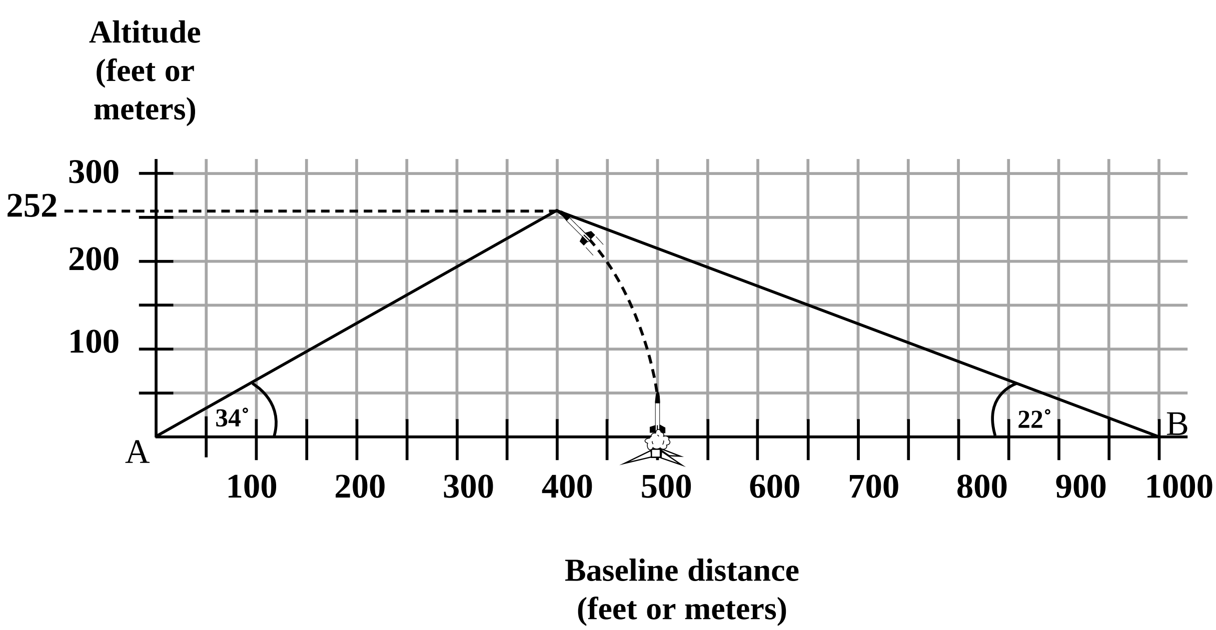
The acceleration of a rocket will be determined by the mass of the rocket and the thrust produced by the engine.

Law 3:

The reaction, or motion, of the rocket is equal to and in an opposite direction from the action, or escaping gas, from the engine.

ALTITUDE TRACKING





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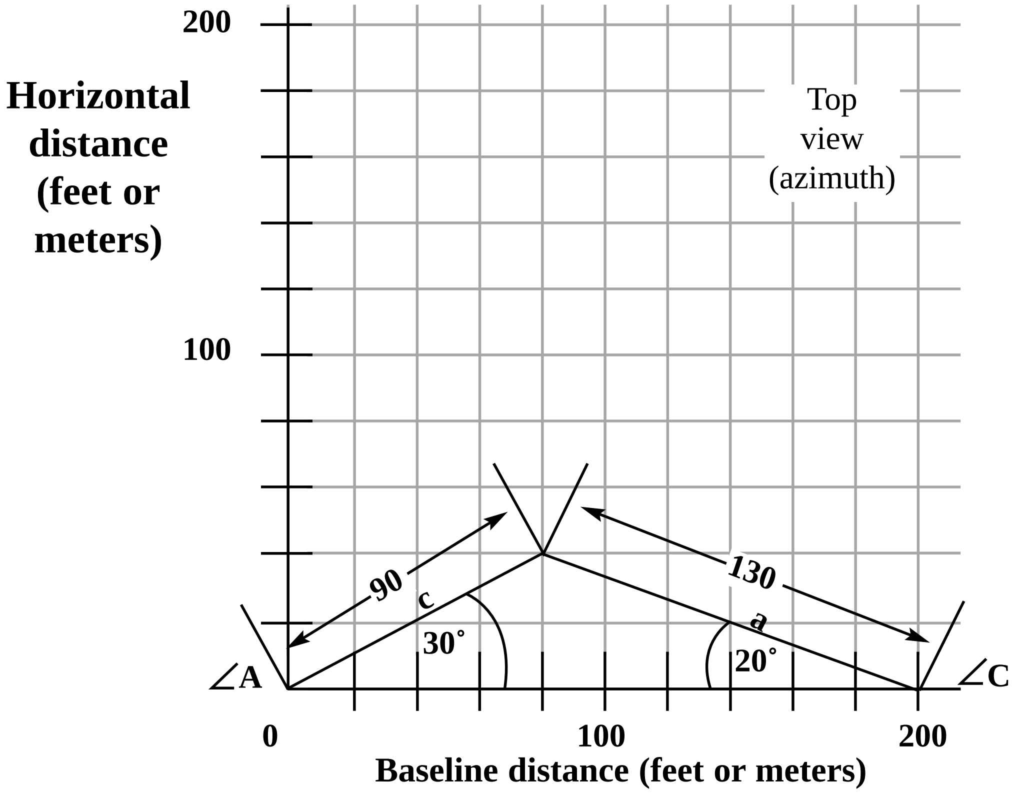

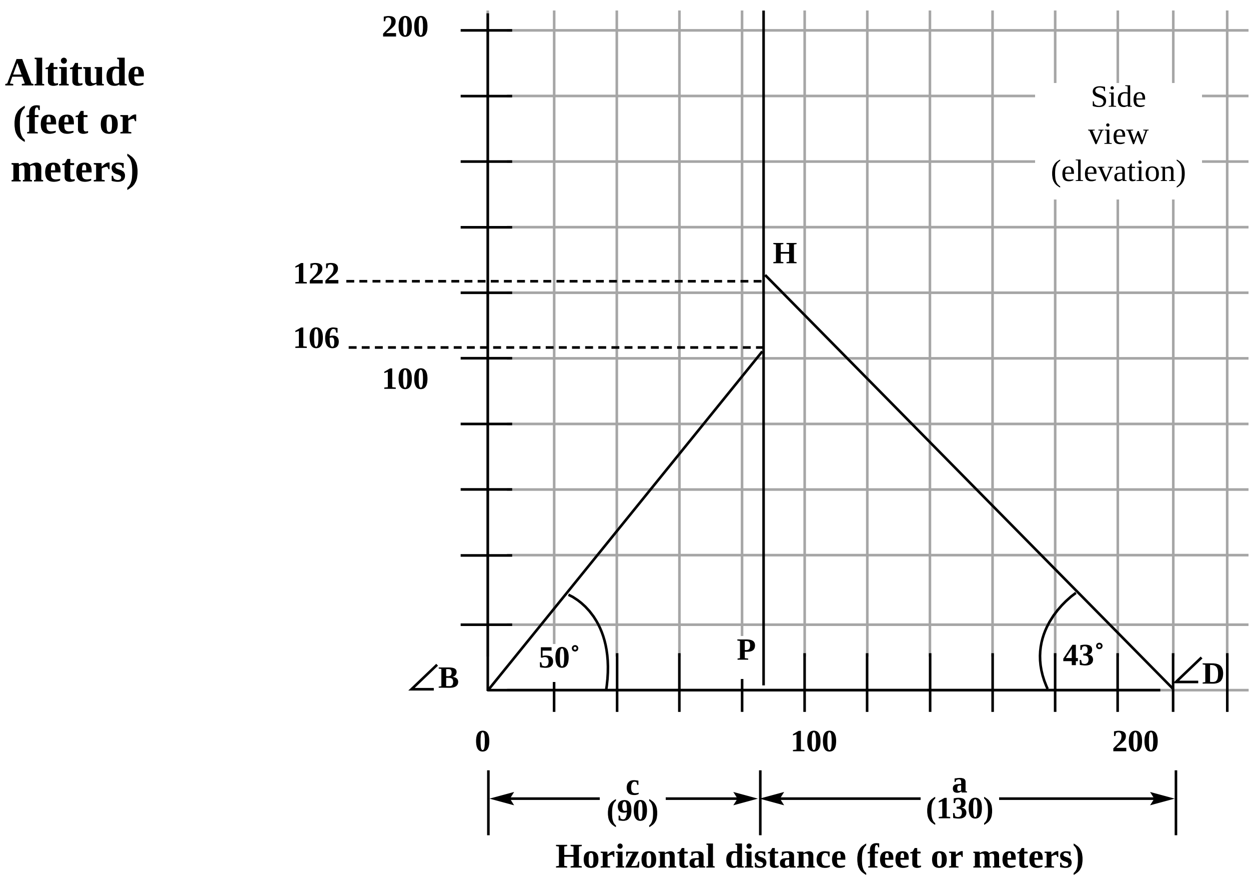

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