Class –IX Mathematics (Ex. 1.1)

Answers

1. Consider the definition of a rational number.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Zero can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5} \ldots\ldots$.

So, we arrive at the conclusion that 0 can be written in the form of $\frac{p}{q}$, where q is any integer.

Therefore, zero is a rational number.

2. We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 all lie between 3 and 4.

We need to rewrite the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}$ and $\frac{36}{10}$.

We can further convert the rational numbers $\frac{32}{10}, \frac{34}{10}, \frac{35}{10}$ and $\frac{36}{10}$ into lowest fractions.

On converting the fractions into lowest fractions, we get $\frac{16}{5}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.

Therefore, six rational numbers between 3 and 4 are $\frac{31}{10}, \frac{16}{5}, \frac{33}{10}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.

3. We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that the numbers $\frac{3}{5}$ and $\frac{4}{5}$ can also be written as 0.6 and 0.8.

We can conclude that the numbers $0.61, 0.62, 0.63, 0.64$ and 0.65 all lie between 0.6 and 0.8

We need to rewrite the numbers $0.61, 0.62, 0.63, 0.64$ and 0.65 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{61}{100}, \frac{62}{100}, \frac{63}{100}, \frac{64}{100}$ and $\frac{65}{100}$.

We can further convert the rational numbers $\frac{62}{100}, \frac{64}{100}$ and $\frac{65}{100}$ into lowest fractions.

On converting the fractions, we get $\frac{31}{50}, \frac{16}{25}$ and $\frac{13}{20}$.

Therefore, six rational numbers between 3 and 4 are $\frac{61}{100}, \frac{31}{50}, \frac{63}{100}, \frac{16}{25}$ and $\frac{13}{50}$.

4.

(i) Consider the whole numbers and natural numbers separately.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.

We know that natural number series is $1, 2, 3, 4, 5 \ldots\ldots$.

So, we can conclude that every number of the natural number series lie in the whole number series.

Therefore, we conclude that, yes every natural number is a whole number.

(ii) Consider the integers and whole numbers separately.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$, where $q = 1$.

Now, considering the series of integers, we have $\ldots. -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots\ldots$.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.

We can conclude that all the numbers of whole number series lie in the series of integers. But every number of series of integers does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1} \ldots\ldots$

We conclude that every number of the whole number series is a rational number.

But, every rational number does not appear in the whole number series.

Therefore, we conclude that every rational number is not a whole number.

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On converting the fractions, we get $\frac{31}{50}, \frac{16}{25}$ and $\frac{13}{20}$.

Therefore, six rational numbers between 3 and 4 are $\frac{61}{100}, \frac{31}{50}, \frac{63}{100}, \frac{16}{25}$ and $\frac{13}{50}$.

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We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.
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We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$.
We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.
We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1} \ldots\ldots$
We conclude that every number of the whole number series is a rational number.
But, every rational number does not appear in the whole number series.
Therefore, we conclude that every rational number is not a whole number.

Class –IX Mathematics (Ex. 1.2)

Answers

1.

(i) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

Therefore, we conclude that, yes every irrational number is a real number.

(ii) Consider a number line. We know that on a number line, we can represent negative as well as positive numbers.

We know that we cannot get a negative number after taking square root of any number.

Therefore, we conclude that not every number point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

So, we can conclude that every irrational number is a real number. But every real number is not an irrational number.

Therefore, we conclude that, every real number is not a rational number.

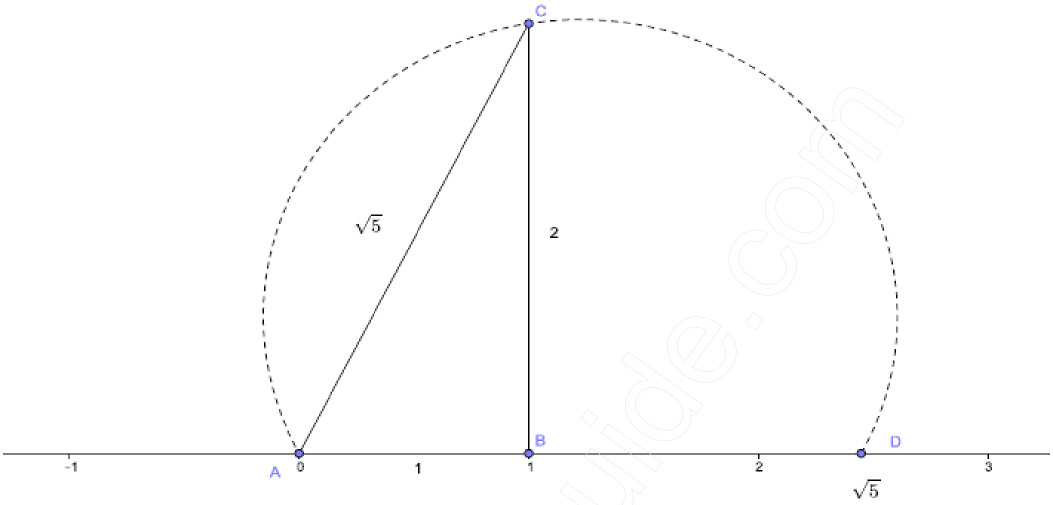
2. We know that square root of every positive integer will not yield an integer.
We know that $\sqrt{4}$ is 2, which is an integer. But, $\sqrt{7}$ or $\sqrt{10}$ will give an irrational number.
Therefore, we conclude that square root of every positive integer is not an irrational number.

3. According to the Pythagoras theorem, we can conclude that

$$\left(\sqrt{5}\right)^2 = (2)^2 + (1)^2.$$

We need to draw a line segment *AB* of 1 unit on the number line. Then draw a straight line segment *BC* of 2 units. Then join the points *C* and *A*, to form a line segment *BC*.

Then draw the arc *ACD*, to get the number $\sqrt{5}$ on the number line.



Class –IX Mathematics (Ex. 1.3)

Answers

1.

(i) $\frac{36}{100}$

On dividing 36 by 100, we get

$$\begin{array}{r} 0.36 \\ 100\overline{)\ 36} \\ \underline{-0} \\ 360 \\ \underline{-300} \\ 600 \\ \underline{-600} \\ \underline{0} \end{array}$$

Therefore, we conclude that $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii) $\frac{1}{11}$

On dividing 1 by 11, we get

$$\begin{array}{r} 0.0909.... \\ 11\overline{)\qquad 1} \\ \underline{-0} \\ 10 \\ \underline{-0} \\ 100 \\ \underline{-99} \\ 10 \\ \underline{-0} \\ 100 \\ \underline{-99} \\ \underline{1} \end{array}$$

We can observe that while dividing 1 by 11, we got the remainder as 1, which will continue to be 1.

Therefore, we conclude that $\frac{1}{11} = 0.0909.....$ or $\frac{1}{11} = 0.\overline{09}$, which is a non-terminating decimal and recurring decimal.

(iii) $4\frac{1}{8} = \frac{33}{8}$

On dividing 33 by 8, we get

$$
\begin{array}{rl}
 & 4.125 \\ \cline{2-2}
8) & 33 \\
 & \underline{-32} \\
 & 10 \\
 & \underline{-8} \\
 & 20 \\
 & \underline{-16} \\
 & 40 \\
 & \underline{-40} \\
 & \underline{0}
\end{array}
$$

We can observe that while dividing 33 by 8, we got the remainder as 0.

Therefore, we conclude that $4\frac{1}{8}=\frac{33}{8}=4.125$, which is a terminating decimal.

(iv) $\frac{3}{13}$

On dividing 3 by 13, we get

$$
\begin{array}{rl}
 & 0.230769.... \\ \cline{2-2}
13) & 3 \\
 & \underline{-0} \\
 & 30 \\
 & \underline{-26} \\
 & 40 \\
 & \underline{-39} \\
 & 10 \\
 & \underline{-0} \\
 & 100 \\
 & \underline{-91} \\
 & 90 \\
 & \underline{-78} \\
 & 120 \\
 & \underline{-117} \\
 & \underline{3}
\end{array}
$$

We can observe that while dividing 3 by 13 we got the remainder as 3, which will continue to be 3 after carrying out 6 continuous divisions.

Therefore, we conclude that $\frac{3}{13}=0.230769.....$ or $\frac{3}{13}=0.\overline{230769}$, which is a non-terminating decimal and recurring decimal.

(v) $\frac{2}{11}$

On dividing 2 by 11, we get

$$\begin{array}{r} 0.1818.... \\ 11\overline{\smash{)}\;\;2} \\ \underline{-0} \\ 20 \\ \underline{-11} \\ 90 \\ \underline{-88} \\ 20 \\ \underline{-11} \\ 90 \\ \underline{-88} \\ \underline{2} \end{array}$$

We can observe that while dividing 2 by 11, first we got the remainder as 2 and then 9, which will continue to be 2 and 9 alternately.

Therefore, we conclude that $\frac{2}{11} = 0.1818.....$ or $\frac{2}{11} = 0.\overline{18}$, which is a non-terminating decimal and recurring decimal.

(vi) $\frac{329}{400}$

On dividing 329 by 400, we get

$$\begin{array}{r} 0.8225 \\ 400\overline{\smash{)}\;\;329} \\ \underline{-0} \\ 3290 \\ \underline{-3200} \\ 900 \\ \underline{-800} \\ 1000 \\ \underline{-800} \\ 2000 \\ \underline{-2000} \\ \underline{0} \end{array}$$

We can observe that while dividing 329 by 400, we got the remainder as 0.

Therefore, we conclude that $\frac{329}{400} = 0.8225$, which is a terminating decimal.

2. We are given that $\frac{1}{7}=0.\overline{142857}$ or $\frac{1}{7}=0.142857......$.

We need to find the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division.

We know that, $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2\times\frac{1}{7},3\times\frac{1}{7},4\times\frac{1}{7},5\times\frac{1}{7}$ and $6\times\frac{1}{7}$.

On substituting value of $\frac{1}{7}$ as $0.142857.....$, we get

$$2\times\frac{1}{7}=2\times0.142857.....=0.285714.........$$

$$3\times\frac{1}{7}=3\times0.142857.....=0.428571$$

$$4\times\frac{1}{7}=4\times0.142857...=0.571428$$

$$5\times\frac{1}{7}=5\times0.142857.....=0.714285$$

$$6\times\frac{1}{7}=6\times0.142857.....=0.857142$$

Therefore, we conclude that, we can predict the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division, to get

$$\frac{2}{7}=0.\overline{285714},\frac{3}{7}=0.\overline{428571},\frac{4}{7}=0.\overline{571428},\frac{5}{7}=0.\overline{714285},\text{ and }\frac{6}{7}=0.\overline{857142}$$

3. **Solution:**

(i) Let $x=0.\overline{6}$ $\Rightarrow$ $x=0.6666.....$ ....(a)

We need to multiply both sides by 10 to get

$10x=6.6666.....$ ....(b)

We need to subtract (a) from (b), to get

$$\begin{array}{r}10x=6.6666.....\\ -x=0.6666.....\\ \hline 9x=6\end{array}$$

We can also write $9x=6$ as $x=\frac{6}{9}$ or $x=\frac{2}{3}$.

Therefore, on converting $0.\overline{6}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{2}{3}$.

(ii) Let $x=0.4\overline{7}$ $\Rightarrow$ $x=0.47777....$ ....(a)

We need to multiply both sides by 10 to get

$10x=4.7777.....$ ....(b)

We need to subtract (a) from (b), to get

$$\begin{aligned}10x &= 4.7777.....\\ -x &= 0.4777....\\ \hline 9x &= 4.3\end{aligned}$$

We can also write $9x = 4.3$ as $x = \frac{4.3}{9}$ or $x = \frac{43}{90}$.

Therefore, on converting $0.4\bar{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

(iii) Let $x = 0.\overline{001}$ $\Rightarrow$ $x = 0.001001....$ ....(a)

We need to multiply both sides by 1000 to get

$1000x = 1.001001.....$ ....(b)

We need to subtract (a) from (b), to get

$$\begin{aligned}1000x &= 1.001001.....\\ -x &= 0.001001....\\ \hline 999x &= 1\end{aligned}$$

We can also write $999x = 1$ as $x = \frac{1}{999}$.

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$.

4. Let $x = 0.99999....$ ....(a)

We need to multiply both sides by 10 to get

$10x = 9.9999....$ ....(b)

We need to subtract (a) from (b), to get

$$\begin{aligned}10x &= 9.99999.....\\ -x &= 0.99999....\\ \hline 9x &= 9\end{aligned}$$

We can also write $9x = 9$ as $x = \frac{9}{9}$ or $x = 1$.

Therefore, on converting $0.99999....$ in the $\frac{p}{q}$ form, we get the answer as 1.

Yes at a glance we are surprised at our answer.
But the answer makes sense when we observe that 0.9999……… goes on forever. SO there is not gap between 1 and 0.9999……. and hence they are equal.

5. We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

$$\begin{array}{r}
0.0588235294117647.... \\
17\overline{)\qquad\qquad\qquad 1} \\
\underline{-0} \\
10 \\
\underline{-0} \\
100 \\
\underline{-85} \\
150 \\
\underline{-136} \\
140 \\
\underline{-136} \\
40 \\
\underline{-34} \\
60 \\
\underline{-51} \\
90 \\
\underline{-85} \\
50 \\
\underline{-34} \\
160 \\
\underline{-153} \\
70 \\
\underline{-68} \\
20 \\
\underline{-17} \\
30 \\
\underline{-17} \\
130 \\
\underline{-119} \\
110 \\
\underline{-102} \\
80 \\
\underline{-68} \\
120 \\
\underline{-119} \\
\underline{1}
\end{array}$$

We can observe that while dividing 1 by 17 we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that $\frac{1}{17} = 0.0588235294117647.....$ or $\frac{1}{17} = 0.\overline{0588235294117647}$, which is a non-terminating decimal and recurring decimal.

6. **Solution:**

Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals.

$$\frac{5}{2} = 2.5$$

$$\frac{5}{4} = 1.25$$

$$\frac{2}{5} = 0.4$$

$$\frac{2}{10} = 0.2$$

$$\frac{5}{16} = 0.3125$$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both.

Therefore, we can conclude that the property, which q must satisfy in $\frac{p}{q}$, so that the rational number $\frac{p}{q}$ is a terminating decimal is that q must have powers of 2, 5 or both.

7. The three numbers that have their expansions as non terminating non recurring decimal are given below.

0.04004000400004....
0.07007000700007....
0.013001300013000013....

8. Let us convert $\frac{5}{7}$ and $\frac{9}{11}$ into decimal form, to get

$\frac{5}{7} = 0.714285....$ and $\frac{9}{11} = 0.818181.....$

Three irrational numbers that lie between 0.714285.... and 0.818181.... are:
0.73073007300073....
0.74074007400074....
0.76076007600076....

9.

(i) $\sqrt{23}$

We know that on finding the square root of 23, we will not get an integer.
Therefore, we conclude that $\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225}$

We know that on finding the square root of 225, we get 15, which is an integer.
Therefore, we conclude that $\sqrt{225}$ is a rational number.

(iii) 0.3796

We know that 0.3796 can be converted into $\frac{p}{q}$.

While, converting 0.3796 into $\frac{p}{q}$ form, we get

$$0.3796=\frac{3796}{10000}.$$

The rational number $\frac{3796}{10000}$ can be converted into lowest fractions, to get $\frac{949}{2500}$.
We can observe that 0.3796 can be converted into a rational number.
Therefore, we conclude that 0.3796 is a rational number.

(iv) 7.478478....

We know that 7.478478.... is a non-terminating recurring decimal, which can be converted into $\frac{p}{q}$ form.

While, converting 7.478478.... into $\frac{p}{q}$ form, we get

$$x=7.478478.... \qquad ....(a)$$

$$1000x=7478.478478.... \qquad ....(b)$$

While, subtracting (a) from (b), we get

$$\begin{array}{r} 1000x=7478.478478.... \\ -x=\quad 7.478478.... \\ \hline 999x=7471 \end{array}$$

We know that $999x=7471$ can also be written as $x=\frac{7471}{999}$.

Therefore, we conclude that 7.478478.... is a rational number.

(v) 1.101001000100001....

We can observe that the number 1.101001000100001.... is a non terminating non recurring decimal.

We know that non terminating and non recurring decimals cannot be converted into $\frac{p}{q}$ form.

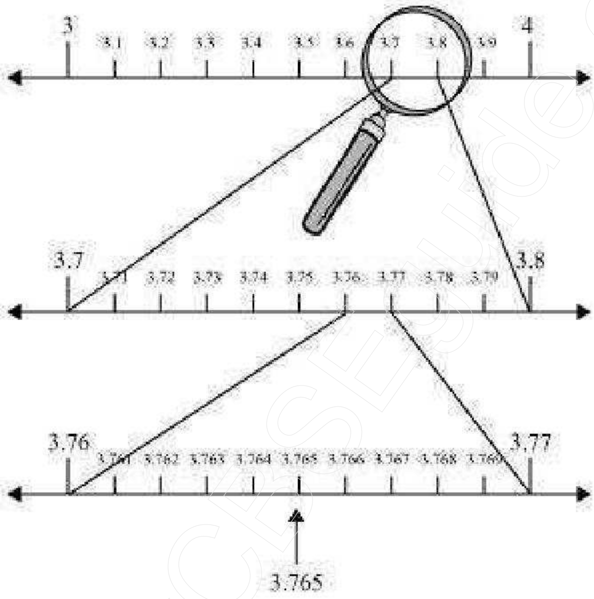
Therefore, we conclude that 1.101001000100001.... is an irrational number.

Class –IX Mathematics (Ex. 1.4)

Answers

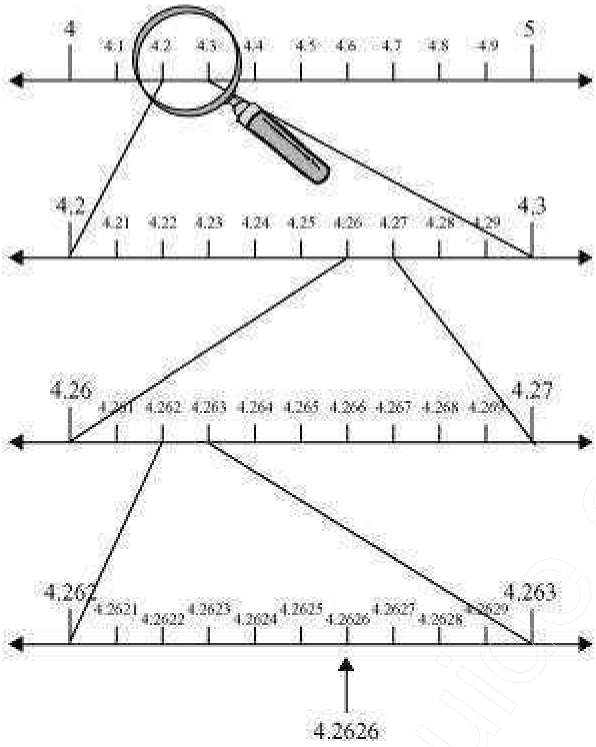
1. We know that the number 3.765 will lie between 3.764 and 3.766.
We know that the numbers 3.764 and 3.766 will lie between 3.76 and 3.77.
We know that the numbers 3.76 and 3.77 will lie between 3.7 and 3.8.
We know that the numbers 3.7 and 3.8 will lie between 3 and 4.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 3 and 4 on the number line.



2. We know that the number $4.\overline{26}$ can also be written as 4.262…. .
We know that the number 4.262…. will lie between 4.261 and 4.263.
We know that the numbers 4.261 and 4.263 will lie between 4.26 and 4.27.
We know that the numbers 4.26 and 4.27 will lie between 4.2 and 4.3.
We know that the numbers 4.2 and 4.3 will lie between 4 and 5.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 4 and 5 on the number line.


4
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5
4.2
4.21
4.22
4.23
4.24
4.25
4.26
4.27
4.28
4.29
4.3
4.26
4.261
4.262
4.263
4.264
4.265
4.266
4.267
4.268
4.269
4.27
4.262
4.2621
4.2622
4.2623
4.2624
4.2625
4.2626
4.2627
4.2628
4.2629
4.263
4.2626

Class –IX Mathematics (Ex. 1.5)

Answers

1.

(i) $2-\sqrt{5}$

We know that $\sqrt{5} = 2.236....,$ which is an irrational number.

$2-\sqrt{5} = 2-2.236....$

$= -0.236....,$ which is also an irrational number.

Therefore, we conclude that $2-\sqrt{5}$ is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

$\left(3+\sqrt{23}\right)-\sqrt{23} = 3+\sqrt{23}-\sqrt{23}$

$= 3.$

Therefore, we conclude that $\left(3+\sqrt{23}\right)-\sqrt{23}$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

We can cancel $\sqrt{7}$ in the numerator and denominator, as $\sqrt{7}$ is the common number in numerator as well as denominator, to get

$\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$

Therefore, we conclude that $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

We know that $\sqrt{2} = 1.414....,$ which is an irrational number.

We can conclude that, when 1 is divided by $\sqrt{2}$, we will get an irrational number.

Therefore, we conclude that $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

We know that $\pi = 3.1415....,$ which is an irrational number.

We can conclude that 2π will also be an irrational number.

Therefore, we conclude that 2π is an irrational number.

2.

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$.

$$\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$$
$$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$$

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$, we get $6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$.

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$.

$$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=3\left(3-\sqrt{3}\right)+\sqrt{3}\left(3-\sqrt{3}\right)$$
$$=9-3\sqrt{3}+3\sqrt{3}-3$$
$$=6$$

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$, we get 6.

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

We need to apply the formula $\left(a+b\right)^2=a^2+2ab+b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$$\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+2\times\sqrt{5}\times\sqrt{2}+\left(\sqrt{2}\right)^2$$
$$=5+2\sqrt{10}+2$$
$$=7+2\sqrt{10}.$$

Therefore, on simplifying $\left(\sqrt{5}+\sqrt{2}\right)^2$, we get $7+2\sqrt{10}$.

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

We need to apply the formula $\left(a-b\right)\left(a+b\right)=a^2-b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$$\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$$
$$=5-2$$
$$=3.$$

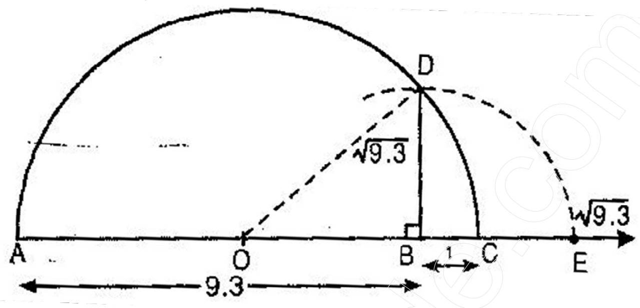
Therefore, on simplifying $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$, we get 3.

3. We know that when we measure the length of a line or a figure by using a scale or any device, we do not get an exact measurement. In fact, we get an approximate

rational value. So, we are not able to realize that either circumference (c) or diameter (d) of a circle is irrational.

Therefore, we can conclude that as such there is not any contradiction regarding the value of π and we realize that the value of π is irrational.

4. Mark the distance 9.3 units from a fixed point A on a given line to obtain a point B such that AB = 9.3 units. From B mark a distance of 1 unit and call the new point as C. Find the mid-point of AC and call that point as O. Draw a semi-circle with centre O and radius OC = 5.15 units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D. Then BD = $\sqrt{9.3}$.



5.

(i) $\frac{1}{\sqrt{7}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}}$ by $\sqrt{7}$, to get

$$\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{7}}{7}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}}$, we get $\frac{\sqrt{7}}{7}$.

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$ by $\sqrt{7}+\sqrt{6}$, to get

$$\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$
$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}$$
$$=\sqrt{7}+\sqrt{6}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$, we get $\sqrt{7}+\sqrt{6}$.

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$, to get

$$\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$
$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$
$$=\frac{\sqrt{5}-\sqrt{2}}{3}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$, we get $\frac{\sqrt{5}-\sqrt{2}}{3}$.

(iv) $\frac{1}{\sqrt{7}-2}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}+2$, to get

$$\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}$$

$$=\frac{\sqrt{7}+2}{3}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-2}$, we get $\frac{\sqrt{7}+2}{3}$.

Class –IX Mathematics (Ex. 1.6)

Answers

1.

(i) $64^{\frac{1}{2}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $64^{\frac{1}{2}}$ can also be written as $\sqrt[2]{64} = \sqrt[2]{8\times 8}$

$\sqrt[2]{64} = \sqrt[2]{8\times 8} = 8.$

Therefore the value of $64^{\frac{1}{2}}$ will be 8.

(ii) $32^{\frac{1}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $32^{\frac{1}{5}}$ can also be written as $\sqrt[5]{32} = \sqrt[2]{2\times 2\times 2\times 2\times 2}$

$\sqrt[5]{32} = \sqrt[2]{2\times 2\times 2\times 2\times 2} = 2$

Therefore the value of $32^{\frac{1}{5}}$ will be 2.

(iii) $125^{\frac{1}{3}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $125^{\frac{1}{3}}$ can also be written as $\sqrt[3]{125} = \sqrt[3]{5\times 5\times 5}$

$\sqrt[3]{125} = \sqrt[3]{5\times 5\times 5} = 5$

Therefore the value of $125^{\frac{1}{3}}$ will be 5.

2.

(i) $9^{\frac{3}{2}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $9^{\frac{3}{2}}$ can also be written as $\sqrt[2]{(9)^3} = \sqrt[2]{9\times 9\times 9} = \sqrt[2]{3\times 3\times 3\times 3\times 3\times 3}$

$$\sqrt[2]{(9)^3} = \sqrt[2]{3\times 3\times 3\times 3\times 3\times 3}$$
$$= 3\times 3\times 3$$
$$= 27.$$

Therefore the value of $9^{\frac{3}{2}}$ will be 27.

(ii) $32^{\frac{2}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $32^{\frac{2}{5}}$ can also be written as

$$\sqrt[5]{(32)^2} = \sqrt[5]{(2\times2\times2\times2\times2)(2\times2\times2\times2\times2)}$$
$$= 2\times2$$
$$= 4.$$

Therefore the value of $32^{\frac{2}{5}}$ will be 4.

(iii) $16^{\frac{3}{4}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $16^{\frac{3}{4}}$ can also be written as

$$\sqrt[4]{(16)^3} = \sqrt[4]{(2\times2\times2\times2)(2\times2\times2\times2)(2\times2\times2\times2)}$$
$$= 2\times2\times2$$
$$= 8.$$

Therefore the value of $16^{\frac{3}{4}}$ will be 8.

(iv) $125^{\frac{-1}{3}}$

We know that $a^{-n} = \frac{1}{a^n}$

We conclude that $125^{\frac{-1}{3}}$ can also be written as $\frac{1}{125^{\frac{1}{3}}}$, or $\left(\frac{1}{125}\right)^{\frac{1}{3}}$.

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We know that $\left(\frac{1}{125}\right)^{\frac{1}{3}}$ can also be written as

$$\sqrt[3]{\left(\frac{1}{125}\right)} = \sqrt[3]{\left(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\right)}$$
$$= \frac{1}{5}.$$

Therefore the value of $125^{\frac{-1}{3}}$ will be $\frac{1}{5}$.

3.

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

We know that $a^m \cdot a^n = a^{(m+n)}$.

We can conclude that $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{2}{3}+\frac{1}{5}}$.

$$2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{10+3}{15}} \quad = (2)^{\frac{13}{15}}$$

Therefore the value of $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ will be $(2)^{\frac{13}{15}}$.

(ii) $\left(3^{\frac{1}{3}}\right)^7$

We know that $a^m \times a^n = a^{m+n}$

We conclude that $\left(3^{\frac{1}{3}}\right)^7$ can also be written as $\left(3^{\frac{7}{3}}\right)$..

(iii) $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

We know that $\dfrac{a^m}{a^n} = a^{m-n}$

We conclude that $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$.

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}}$$

$$= 11^{\frac{1}{4}}$$

Therefore the value of $\dfrac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ will be $11^{\frac{1}{4}}$.

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

We know that $a^m \cdot b^m = (a \times b)^m$.

We can conclude that $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$.

$$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} \quad = (56)^{\frac{1}{2}}$$

Therefore the value of $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$ will be $(56)^{\frac{1}{2}}$.

Class –IX Mathematics (Ex. 2.1)

Answers

1. **Solution:**

(i) $4x^2-3x+7$

We can observe that in the polynomial $4x^2-3x+7$, we have *x* as the only variable and the powers of *x* in each term are a whole number.

Therefore, we conclude that $4x^2-3x+7$ is a polynomial in one variable.

(ii) $y^2+\sqrt{2}$

We can observe that in the polynomial $y^2+\sqrt{2}$, we have *y* as the only variable and the powers of *y* in each term are a whole number.

Therefore, we conclude that $y^2+\sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t}+t\sqrt{2}$

We can observe that in the polynomial $3\sqrt{t}+t\sqrt{2}$, we have *t* as the only variable and the powers of *t* in each term are not a whole number.

Therefore, we conclude that $3\sqrt{t}+t\sqrt{2}$ is not a polynomial in one variable.

(iv) $y+\frac{2}{y}$

We can observe that in the polynomial $y+\frac{2}{y}$, we have *y* as the only variable and the powers of *y* in each term are not a whole number.

Therefore, we conclude that $y+\frac{2}{y}$ is not a polynomial in one variable.

(v) $x^{10}+y^3+t^{50}$

We can observe that in the polynomial $x^{10}+y^3+t^{50}$, we have *x*, *y* and *t* as the variables and the powers of *x*, *y* and *t* in each term is a whole number.

Therefore, we conclude that $x^{10}+y^3+t^{50}$ is a polynomial but not a polynomial in one variable.

2.

(i) $2+x^2+x$

The coefficient of x^2 in the polynomial $2+x^2+x$ is 1.

(ii) $2-x^2+x^3$

The coefficient of x^2 in the polynomial $2-x^2+x^3$ is -1.

(iii) $\frac{\pi}{2}x^2+x$

The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2+x$ is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$

The coefficient of x^2 in the polynomial $\sqrt{2}x-1$ is 0.

3. The binomial of degree 35 can be $x^{35}+9$.
The binomial of degree 100 can be t^{100}.

4.

(i) $5x^3+4x^2+7x$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.
We can observe that in the polynomial $5x^3+4x^2+7x$, the highest power of the variable x is 3.
Therefore, we conclude that the degree of the polynomial $5x^3+4x^2+7x$ is 3.

(ii) $4-y^2$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.
We can observe that in the polynomial $4-y^2$, the highest power of the variable y is 2.
Therefore, we conclude that the degree of the polynomial $4-y^2$ is 2.

(iii) $5t-\sqrt{7}$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.
We observe that in the polynomial $5t-\sqrt{7}$, the highest power of the variable t is 1.
Therefore, we conclude that the degree of the polynomial $5t-\sqrt{7}$ is 1.

(iv) 3

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial 3, the highest power of the assumed variable x is 0.
Therefore, we conclude that the degree of the polynomial 3 is 0.

4.

(i) x^2+x
We can observe that the degree of the polynomial x^2+x is 2.
Therefore, we can conclude that the polynomial x^2+x is a quadratic polynomial.

(ii) $x-x^3$
We can observe that the degree of the polynomial $x-x^3$ is 3.
Therefore, we can conclude that the polynomial $x-x^3$ is a cubic polynomial.

(iii) $y+y^2+4$
We can observe that the degree of the polynomial $y+y^2+4$ is 2.
Therefore, the polynomial $y+y^2+4$ is a quadratic polynomial.

(iv) $1+x$
We can observe that the degree of the polynomial $(1+x)$ is 1.
Therefore, we can conclude that the polynomial $1+x$ is a linear polynomial.

(v) $3t$
We can observe that the degree of the polynomial $(3t)$ is 1.
Therefore, we can conclude that the polynomial $3t$ is a linear polynomial.

(vi) r^2
We can observe that the degree of the polynomial r^2 is 2.
Therefore, we can conclude that the polynomial r^2 is a quadratic polynomial.

(vii) $7x^3$
We can observe that the degree of the polynomial $7x^3$ is 3.
Therefore, we can conclude that the polynomial $7x^3$ is a cubic polynomial.

Class –IX Mathematics (Ex. 2.2)

Answers

1.

(vi) Let $f\left(x\right)=5x-4x^2+3$.

We need to substitute 0 in the polynomial $f\left(x\right)=5x-4x^2+3$ to get

$$f\left(0\right)=5\left(0\right)-4\left(0\right)^2+3$$
$$=0-0+3$$
$$=3.$$

Therefore, we conclude that at $x=0$, the value of the polynomial $5x-4x^2+3$ is 3.

(vii) Let $f\left(x\right)=5x-4x^2+3$.

We need to substitute -1 in the polynomial $f\left(x\right)=5x-4x^2+3$ to get

$$f\left(-1\right)=5\left(-1\right)-4\left(-1\right)^2+3$$
$$=-5-4+3$$
$$=-6.$$

Therefore, we conclude that at $x=-1$, the value of the polynomial $5x-4x^2+3$ is -6

(viii) Let $f\left(x\right)=5x-4x^2+3$.

We need to substitute 0 in the polynomial $f\left(x\right)=5x-4x^2+3$ to get

$$f\left(2\right)=5\left(2\right)-4\left(2\right)^2+3$$
$$=10-16+3$$
$$=-3.$$

Therefore, we conclude that at $x=2$, the value of the polynomial $5x-4x^2+3$ is -3.

2.

(v) $p\left(y\right)=y^2-y+1$

At $p\left(0\right)$:

$$p\left(0\right)=\left(0\right)^2-0+1=1$$

At $p\left(1\right)$:

$$p\left(1\right)=\left(1\right)^2-1+1=1-0=1$$

At $p\left(2\right)$:

$$p\left(2\right)=\left(2\right)^2-2+1=4-1=3$$

(vi) $p(t)=2+t+2t^2-t^3$

At $p(0)$:

$p(0)=2+(0)+2(0)^2-(0)^3=2$

At $p(1)$:

$p(1)=2+(1)+2(1)^2-(1)^3=2+1+2-1=4$

At $p(2)$:

$p(2)=2+(2)+2(2)^2-(2)^3=4+8-8=4$

(vii) $p(x)=(x)^3$

At $p(0)$:

$p(0)=(0)^3=0$

At $p(1)$:

$p(1)=(1)^3=1$

At $p(2)$:

$p(2)=(2)^3=8$

(viii) $p(x)=(x-1)(x+1)$

At $p(0)$:

$p(0)=(0-1)(0+1)=(-1)(1)=-1$

At $p(1)$:

$p(1)=(1-1)(2+1)=(0)(3)=0$

At $p(2)$:

$p(2)=(2-1)(2+1)=(1)(3)=3$

3.

(i) $p(x)=3x+1,\quad x=-\frac{1}{3}$

We need to check whether $p(x)=3x+1$ at $x=-\frac{1}{3}$ is equal to zero or not.

$p\left(-\frac{1}{3}\right)=3x+1=3\left(-\frac{1}{3}\right)+1=-1+1=0$

Therefore, we can conclude that $x=-\frac{1}{3}$ is a zero of the polynomial $p(x)=3x+1$.

(ii) $p(x)=5x-\pi,\ x=\frac{4}{5}$

We need to check whether $p(x)=5x-\pi$ at $x=\frac{4}{5}$ is equal to zero or not.

$p\left(\frac{4}{5}\right)=5\left(\frac{4}{5}\right)-\pi=4-\pi$

Therefore, $x=\frac{4}{5}$ is not a zero of the polynomial $p(x)=5x-\pi$.

(iii) $p(x)=x^2-1,\quad x=-1,1$

We need to check whether $p(x)=x^2-1$ at $x=-1,1$ is equal to zero or not.

At $x=-1$

$p(-1)=(-1)^2-1=1-1=0$

At $x=1$

$p(1)=(1)^2-1=1-1=0$

Therefore, $x=-1,1$ are the zeros of the polynomial $p(x)=x^2-1$.

(iv) $p(x)=(x+1)(x-2),\ x=-1,2$

We need to check whether $p(x)=(x+1)(x-2)$ at $x=-1,2$ is equal to zero or not.

At $x=-1$

$p(-1)=(-1+1)(-1-2)=(0)(-3)=0$

At $x=2$

$p(2)=(2+1)(2-2)=(3)(0)=0$

Therefore, $x=-1,2$ are the zeros of the polynomial $p(x)=(x+1)(x-2)$.

(v) $p(x)=x^2,\quad x=0$

We need to check whether $p(x)=x^2$ at $x=0$ is equal to zero or not.

$p(0)=(0)^2=0$

Therefore, we can conclude that $x=0$ is a zero of the polynomial $p(x)=x^2$.

(vi) $p(x)=lx+m,\ x=-\frac{m}{l}$

We need to check whether $p(x)=lx+m$ at $x=-\frac{m}{l}$ is equal to zero or not.

$p\left(-\frac{m}{l}\right)=l\left(-\frac{m}{l}\right)+m=m+m=0$

Therefore, $x=-\frac{m}{l}$ is a zero of the polynomial $p(x)=lx+m$.

(vii) $p(x)=3x^2-1,\quad x=-\frac{1}{\sqrt{3}},\frac{2}{\sqrt{3}}$

We need to check whether $p(x)=3x^2-1$ at $x=-\frac{1}{\sqrt{3}},\frac{2}{\sqrt{3}}$ is equal to zero or not.

At $x=\frac{-1}{\sqrt{3}}$

$$p\left(-\frac{1}{\sqrt{3}}\right)=3\left(-\frac{1}{\sqrt{3}}\right)^2-1=3\left(\frac{1}{3}\right)-1=1-1=0$$

At $x=\frac{2}{\sqrt{3}}$

$$p\left(\frac{2}{\sqrt{3}}\right)=3\left(\frac{2}{\sqrt{3}}\right)^2-1=3\left(\frac{4}{3}\right)-1=4-1=3$$

Therefore, we can conclude that $x=\frac{-1}{\sqrt{3}}$ is a zero of the polynomial $p(x)=3x^2-1$

but $x=\frac{-1}{\sqrt{3}}$ is not a zero of the polynomial $p(x)=3x^2-1$.

(viii) $p(x)=2x+1,\quad x=-\frac{1}{2}$

We need to check whether $p(x)=2x+1$ at $x=-\frac{1}{2}$ is equal to zero or not.

$$p\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)+1=-1+1=0$$

Therefore, $x=-\frac{1}{2}$ is a zero of the polynomial $p(x)=2x+1$

4.

(v) $p(x)=x+5$

$ax+b,$ where $a\neq 0$ and $b\neq 0,$ and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=x+5$ equal to 0, we get

$x+5=0 \quad\Rightarrow\quad x=-5$

Therefore, we conclude that the zero of the polynomial $p(x)=x+5$ is -5.

(vi) $p(x)=x-5$

$ax+b,$ where $a\neq 0$ and $b\neq 0,$ and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=x-5$ equal to 0, we get

$x-5=0 \quad\Rightarrow\quad x=5$

Therefore, we conclude that the zero of the polynomial $p(x)=x-5$ is 5.

(vii) $p(x)=2x+5$

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=2x+5$ equal to 0, we get

$$2x+5=0 \qquad \Rightarrow \qquad x=\frac{-5}{2}$$

Therefore, we conclude that the zero of the polynomial $p(x)=2x+5$ is $\frac{-5}{2}$.

(viii) $p(x)=3x-2$

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=3x-2$ equal to 0, we get

$$3x-2=0 \qquad \Rightarrow \qquad x=\frac{2}{3}$$

Therefore, we conclude that the zero of the polynomial $p(x)=3x-2$ is $\frac{2}{3}$.

(ix) $p(x)=3x$

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=3x$ equal to 0, we get

$$3x=0 \qquad \Rightarrow \qquad x=0$$

Therefore, we conclude that the zero of the polynomial $p(x)=3x$ is 0.

(x) $p(x)=ax, a \neq 0$

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=ax$ equal to 0, we get

$$ax=0 \qquad \Rightarrow \qquad x=0$$

Therefore, we conclude that the zero of the polynomial $p(x)=ax, a \neq 0$ is 0.

(xi) $p(x)=cx+d, c \neq 0,\ c, d$ are real numbers.

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=cx+d$ equal to 0, we get

$$cx+d=0$$

$$\Rightarrow \quad x=-\frac{d}{c}.$$

Therefore, we conclude that the zero of the polynomial

$p(x)=cx+d, c \neq 0$, c, d are real numbers. is $-\frac{d}{c}$.

Class –IX Mathematics (Ex. 2.3)

Answers

1.

(i) $x+1$

We need to find the zero of the polynomial $x+1$.

$x+1=0 \quad \Rightarrow \quad x=-1$

While applying the remainder theorem, we need to put the zero of the polynomial $x+1$ in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p(0)=(-1)^3+3(-1)^2+3(-1)+1$$

$$=-1+3-3+1$$

$$=0.$$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $x+1$, we will get the remainder as 0.

(ii) $x-\frac{1}{2}$

We need to find the zero of the polynomial $x-\frac{1}{2}$.

$x-\frac{1}{2}=0 \quad \Rightarrow \quad x=\frac{1}{2}$

While applying the remainder theorem, we need to put the zero of the polynomial $x-\frac{1}{2}$ in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$$

$$=\frac{1}{8}+3\left(\frac{1}{4}\right)+\frac{3}{2}+1$$

$$=\frac{1+6+12+8}{8}$$

$$=\frac{27}{8}$$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $x-\frac{1}{2}$, we will get the remainder as $\frac{27}{8}$.

(iii) x

We need to find the zero of the polynomial x.

$x = 0$

While applying the remainder theorem, we need to put the zero of the polynomial x in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$\begin{aligned} p(0) &= (0)^3 + 3(0)^2 + 3(0) + 1 \\ &= 0 + 0 + 0 + 1 \\ &= 1. \end{aligned}$$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by x, we will get the remainder as 1.

(iv) $x + \pi$

We need to find the zero of the polynomial $x + \pi$.

$x + \pi = 0 \quad \Rightarrow \quad x = -\pi$

While applying the remainder theorem, we need to put the zero of the polynomial $x + \pi$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$\begin{aligned} p(-\pi) &= (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1 \\ &= -\pi^3 + 3\pi^2 - 3\pi + 1. \end{aligned}$$

Therefore, we conclude that on dividing the polynomial $x^3 + 3x^2 + 3x + 1$ by $x + \pi$, we will get the remainder as $-\pi^3 + 3\pi^2 - 3\pi + 1$.

(v) $5 + 2x$

We need to find the zero of the polynomial $5 + 2x$.

$5 + 2x = 0 \quad \Rightarrow \quad x = -\frac{5}{2}$

While applying the remainder theorem, we need to put the zero of the polynomial $5 + 2x$ in the polynomial $x^3 + 3x^2 + 3x + 1$, to get

$$p(x) = x^3 + 3x^2 + 3x + 1$$

$$p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$$

$$=-\frac{125}{8}+3\left(\frac{25}{4}\right)-\frac{15}{2}+1$$

$$=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$$

$$=\frac{-125+150-60+8}{8}$$

$$=-\frac{27}{4}.$$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $5+2x$, we will get the remainder as $-\frac{27}{4}$.

2. We need to find the zero of the polynomial $x-a$.

$x-a=0 \qquad \Rightarrow \qquad x=a$

While applying the remainder theorem, we need to put the zero of the polynomial $x-a$ in the polynomial x^3-ax^2+6x-a, to get

$$p(x)=x^3-ax^2+6x-a$$

$$p(a)=(a)^3-a(a)^2+6(a)-a$$

$$=a^3-a^3+6a-a$$

$$=5a.$$

Therefore, we conclude that on dividing the polynomial x^3-ax^2+6x-a by $x-a$, we will get the remainder as $5a$.

3. We know that if the polynomial $7+3x$ is a factor of $3x^3+7x$, then on dividing the polynomial $3x^3+7x$ by $7+3x$, we must get the remainder as 0.

We need to find the zero of the polynomial $7+3x$.

$7+3x=0 \qquad \Rightarrow \qquad x=-\frac{7}{3}$

While applying the remainder theorem, we need to put the zero of the polynomial $7+3x$ in the polynomial $3x^3+7x$, to get

$$p(x)=3x^3+7x$$

$$=3\left(-\frac{7}{3}\right)^3+7\left(-\frac{7}{3}\right)=3\left(-\frac{343}{27}\right)-\frac{49}{3}$$

$$=-\frac{343}{9}-\frac{49}{3}=\frac{-343-147}{9}$$

$$=\frac{-490}{9}.$$

We conclude that on dividing the polynomial $3x^3+7x$ by $7+3x$, we will get the remainder as $\frac{-490}{9}$, which is not 0.

Therefore, we conclude that $7+3x$ is not a factor of $3x^3+7x$.

Class –IX Mathematics (Ex. 2.4)

Answers

1.

(i) x^3+x^2+x+1

While applying the factor theorem, we get

$$p(x)=x^3+x^2+x+1$$

$$p(-1)=(-1)^3+(-1)^2+(-1)+1$$

$$=-1+1-1+1$$

$$=0.$$

We conclude that on dividing the polynomial x^3+x^2+x+1 by $(x+1)$, we get the remainder as 0.

Therefore, we conclude that $(x+1)$ is a factor of x^3+x^2+x+1.

(ii) $x^4+x^3+x^2+x+1$

While applying the factor theorem, we get

$$p(x)=x^4+x^3+x^2+x+1$$

$$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$$

$$=1-1+1-1+1$$

$$=1.$$

We conclude that on dividing the polynomial $x^4+x^3+x^2+x+1$ by $(x+1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+x^3+x^2+x+1$.

(iii) $x^4+3x^3+3x^2+x+1$

While applying the factor theorem, we get

$$p(x)=x^4+3x^3+3x^2+x+1$$

$$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$$

$$=1-3+3-1+1$$

$$=1.$$

We conclude that on dividing the polynomial $x^4+3x^3+3x^2+x+1$ by $(x+1)$, we will get the remainder as1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+3x^3+3x^2+x+1$.

(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

While applying the factor theorem, we get

$$p(x)=x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$$

$$p(-1)=(-1)^3-(-1)^2-\left(2+\sqrt{2}\right)(-1)+\sqrt{2}$$

$$=-1-1+2+\sqrt{2}+\sqrt{2}$$

$$=2\sqrt{2}.$$

We conclude that on dividing the polynomial $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$ by $(x+1)$, we will get the remainder as $2\sqrt{2}$, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$.

2.

(i) $p(x)=2x^3+x^2-2x-1, g(x)=x+1$

We know that according to the factor theorem,

$(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(-1)*=0.

$$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$$

$$=-2+1+2-1$$

$$=0.$$

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

(ii) $p(x)=x^3+3x^2+3x+1, g(x)=x+2$

We know that according to the factor theorem,

$(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(-2)*=0.

$$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$$

$$=-8+12-6+1$$

$$=-1.$$

Therefore, we conclude that the *g(x)* is not a factor of *p(x)*.

(iii) $p(x)=x^3-4x^2+x+6, g(x)=x-3$

We know that according to the factor theorem,

$(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(3)*=0.

$$p(3)=(3)^3-4(3)^2+(3)+6$$

$$=27-36+3+6$$

$$=0.$$

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

3.

(i) $p(x) = x^2 + x + k$

We know that according to the factor theorem

$p(a) = 0,$ if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = x^2 + x + k$, then $p(1) = 0$.

$$p(1) = (1)^2 + (1) + k = 0, \text{ or}$$
$$k + 2 = 0$$
$$k = -2$$

Therefore, we can conclude that the value of k is -2.

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

We know that according to the factor theorem

$p(a) = 0,$ if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, then $p(1) = 0$.

$$p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0, \text{ or}$$
$$2 + k + \sqrt{2} = 0$$
$$k = -\left(2 + \sqrt{2}\right).$$

Therefore, we can conclude that the value of k is $-\left(2 + \sqrt{2}\right)$.

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

We know that according to the factor theorem

$p(a) = 0,$ if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$, then $p(1) = 0$.

$$p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0, \text{ or}$$
$$k - \sqrt{2} + 1 = 0$$
$$k = \sqrt{2} - 1.$$

Therefore, we can conclude that the value of k is $\sqrt{2} - 1$.

(iv) $p(x) = kx^2 - 3x + k$

We know that according to the factor theorem

$p(a) = 0,$ if $x - a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x) = kx^2 - 3x + k$, then $p(1) = 0$.

$$p(1) = k(1)^2 - 3(1) + k, \text{ or } 2k - 3 = 0 \quad \Rightarrow \quad k = \frac{3}{2}$$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

4.

(xii) $12x^2-7x+1$

$$12x^2-7x+1=12x^2-3x-4x+1$$
$$=3x(4x-1)-1(4x-1)$$
$$=(3x-1)(4x-1).$$

Therefore, we conclude that on factorizing the polynomial $12x^2-7x+1$, we get $(3x-1)(4x-1)$.

(xiii) $2x^2+7x+3$

$$2x^2+7x+3=2x^2+6x+x+3$$
$$=2x(x+3)+1(x+3)$$
$$=(2x+1)(x+3).$$

Therefore, we conclude that on factorizing the polynomial $2x^2+7x+3$, we get $(2x+1)(x+3)$.

(xiv) $6x^2+5x-6$

$$6x^2+5x-6=6x^2+9x-4x-6$$
$$=3x(2x+3)-2(2x+3)$$
$$=(3x-2)(2x+3).$$

Therefore, we conclude that on factorizing the polynomial $6x^2+5x-6$, we get $(3x-2)(2x+3)$.

(xv) $3x^2-x-4$

$$3x^2-x-4=3x^2+3x-4x-4$$
$$=3x(x+1)-4(x+1)$$
$$=(3x-4)(x+1).$$

Therefore, we conclude that on factorizing the polynomial $3x^2-x-4$, we get $(3x-4)(x+1)$.

5.

(viii) x^3-2x^2-x+2

We need to consider the factors of 2, which are $\pm1,\pm2$.

Let us substitute 1 in the polynomial x^3-2x^2-x+2, to get

$$(1)^3-2(1)^2-(1)+2=1-2-1+2=0$$

Thus, according to factor theorem, we can conclude that $(x-1)$ is a factor of the polynomial x^3-2x^2-x+2.

Let us divide the polynomial x^3-2x^2-x+2 by $(x-1)$, to get

$$
\begin{array}{r|l}
 & x^2-x-2 \\
\hline
x-1 & x^3-2x^2-x+2 \\
 & \underset{-}{x^3}\underset{+}{-x^2} \\
\cline{2-2}
 & -x^2-x \\
 & \underset{+}{-x^2}\underset{-}{+x} \\
\cline{2-2}
 & -2x+2 \\
 & \underset{+}{-2x}\underset{-}{+2} \\
\cline{2-2}
 & 0 \\
\cline{2-2}
\end{array}
$$

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$.

$$
\begin{aligned}
x^3-2x^2-x+2 &= (x-1)(x^2-x-2) \\
&= (x-1)(x^2+x-2x-2) \\
&= (x-1)\left[x(x+1)-2(x+1)\right] \\
&= (x-1)(x-2)(x+1).
\end{aligned}
$$

Therefore, we can conclude that on factorizing the polynomial x^3-2x^2-x+2, we get $(x-1)(x-2)(x+1)$.

(ix) x^3-3x^2-9x-5

We need to consider the factors of -5, which are $\pm1,\pm5$.

Let us substitute 1 in the polynomial x^3-3x^2-9x-5, to get

$$(-1)^3-3(-1)^2-9(-1)-5=-1-3+9-5=0$$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial x^3-3x^2-9x-5.

Let us divide the polynomial x^3-3x^2-9x-5 by $(x+1)$, to get

$$
\begin{array}{rl}
 & x^2-4x-5 \\ \cline{2-2}
x+1) & x^3-3x^2-9x-5 \\
 & \underset{-\quad\;\; -}{x^3+x^2} \\ \cline{2-2}
 & -4x^2-9x \\
 & \underset{+\qquad\;\; +}{-4x^2-4x} \\ \cline{2-2}
 & -5x-5 \\
 & \underset{+\quad\;\; +}{-5x-5} \\ \cline{2-2}
 & \quad 0 \\ \cline{2-2}
\end{array}
$$

$$
\begin{aligned}
x^3-3x^2-9x-5 &= (x+1)(x^2-4x-5) \\
&= (x+1)(x^2+x-5x-5) \\
&= (x+1)\left[x(x+1)-5(x+1)\right] \\
&= (x+1)(x-5)(x+1).
\end{aligned}
$$

Therefore, we can conclude that on factorizing the polynomial x^3-3x^2-9x-5, we get $(x+1)(x-5)(x+1)$.

(x) $x^3+13x^2+32x+20$

We need to consider the factors of 20, which are $\pm5, \pm4, \pm2, \pm1$.

Let us substitute -1 in the polynomial $x^3+13x^2+32x+20$, to get

$(-1)^3+13(-1)^2+32(-1)+20 = -1+13-32+20 = -20+20 = 0$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3+13x^2+32x+20$.

Let us divide the polynomial $x^3+13x^2+32x+20$ by $(x+1)$, to get

$$
\begin{array}{rl}
 & x^2+12x+20 \\ \cline{2-2}
x+1) & x^3+13x^2+32x+20 \\
 & \underset{-\quad\;\; -}{x^3+x^2} \\ \cline{2-2}
 & 12x^2+32x \\
 & \underset{-\qquad\;\; -}{12x^2+12x} \\ \cline{2-2}
 & 20x+20 \\
 & \underset{-\qquad -}{20x+20} \\ \cline{2-2}
 & \quad 0 \\ \cline{2-2}
\end{array}
$$

$$
\begin{aligned}
x^3+13x^2+32x+20 &= (x+1)(x^2+12x+20) \\
&= (x+1)(x^2+2x+10x+20) \\
&= (x+1)\left[x(x+2)+10(x+2)\right] \\
&= (x+1)(x+10)(x+2).
\end{aligned}
$$

Therefore, we can conclude that on factorizing the polynomial $x^3+13x^2+32x+20$, we get $(x+1)(x-10)(x+2)$.

(xi) $2y^3+y^2-2y-1$

We need to consider the factors of -1, which are ±1.

Let us substitute 1 in the polynomial $2y^3+y^2-2y-1$, to get

$2(1)^3+(1)^2-2(1)-1=2+1-2-1=3-3=0$

Thus, according to factor theorem, we can conclude that $(y-1)$ is a factor of the polynomial $2y^3+y^2-2y-1$.

Let us divide the polynomial $2y^3+y^2-2y-1$ by $(y-1)$, to get

$$
\begin{array}{r|l}
 & 2y^2+3y+1 \\ \hline
y-1 & 2y^3+y^2-2y-1 \\
 & 2y^3-2y^2 \\
 & \underline{-\qquad\;\; +} \\
 & 3y^2-2y \\
 & 3y^2-3y \\
 & \underline{-\qquad +} \\
 & y-1 \\
 & y-1 \\
 & \underline{-\quad -} \\
 & \underline{\quad 0}
\end{array}
$$

$$
\begin{aligned}
2y^3+y^2-2y-1 &= (y-1)(2y^2+3y+1) \\
&= (y-1)(2y^2+2y+y+1) \\
&= (y-1)\left[2y(y+1)+1(y+1)\right] \\
&= (y-1)(2y+1)(y+1).
\end{aligned}
$$

Therefore, we can conclude that on factorizing the polynomial $2y^3+y^2-2y-1$, we get $(y-1)(2y+1)(y+1)$.

Class –IX Mathematics (Ex. 2.5)

Answers

1.

(ix) $(x+4)(x+10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(x+4)(x+10)$

$$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$$
$$=x^2+14x+40.$$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2+14x+40$.

(x) $(x+8)(x-10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(x+8)(x-10)$

$$(x+8)(x-10)=x^2+\left[8+(-10)\right]x+\left[8\times(-10)\right]$$
$$=x^2-2x-80.$$

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2-2x-80$.

(xi) $(3x+4)(3x-5)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(3x+4)(3x-5)$

$$(3x+4)(3x-5)=(3x)^2+\left[4+(-5)\right]3x+\left[4\times(-5)\right]$$
$$=9x^2-3x-20.$$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2-3x-20$.

(xii) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2$$
$$=y^4-\frac{9}{4}.$$

Therefore, we conclude that the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ is $\left(y^4-\frac{9}{4}\right)$.

(xiii) $(3+2x)(3-2x)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $(3+2x)(3-2x)$

$$(3+2x)(3-2x)=(3)^2-(2x)^2$$
$$=9-4x^2.$$

Therefore, we conclude that the product $(3+2x)(3-2x)$ is $(9-4x^2)$.

2.

(ix) 103×107

103×107 can also be written as $(100+3)(100+7)$.

We can observe that, we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$$(100+3)(100+7)=(100)^2+(3+7)(100)+3\times7$$
$$=10000+1000+21$$
$$=11021.$$

Therefore, we conclude that the value of the product 103×107 is 11021.

(x) 95×96

95×96 can also be written as $(100-5)(100-4)$

We can observe that, we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$$(100-5)(100-4)=(100)^2+[(-5)+(-4)](100)+(-5)\times(-4)$$
$$=10000-900+20$$
$$=9120$$

Therefore, we conclude that the value of the product 95×96 is 9120.

(xi) 104×96

104×96 can also be written as $(100+4)(100-4)$.

We can observe that, we can apply the identity $(x+y)(x-y)=x^2-y^2$ with respect to the expression $(100+4)(100-4)$, to get

$$(100+4)(100-4)=(100)^2-(4)^2$$
$$=10000-16$$
$$=9984.$$

Therefore, we conclude that the value of the product 104×96 is 9984.

3.

(i) $9x^2+6xy+y^2$

$9x^2+6xy+y^2=(3x)^2+2\times3x\times y+(y)^2$

We can observe that, we can apply the identity $(x+y)^2=x^2+2xy+y^2$

$\Rightarrow \quad (3x)^2+2\times3x\times y+(y)^2=(3x+y)^2$.

(ii) $4y^2-4y+1$

$4y^2-4y+1=(2y)^2-2\times2y\times1+(1)^2$

We can observe that, we can apply the identity $(x-y)^2=x^2-2xy+y^2$

$\Rightarrow \quad (2y)^2-2\times2y\times1+(1)^2=(2y-1)^2$.

(iii) $x^2-\frac{y^2}{100}$

We can observe that, we can apply the identity $(x)^2-(y)^2=(x+y)(x-y)$

$\Rightarrow \quad (x)^2-\left(\frac{y}{10}\right)^2=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$.

4.

(xvi) $(x+2y+4z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(x+2y+4z)^2$.

$$\begin{aligned}(x+2y+4z)^2&=(x)^2+(2y)^2+(4z)^2+2\times x\times2y+2\times2y\times4z+2\times4z\times x\\&=x^2+4y^2+16z^2+4xy+16yz+8zx.\end{aligned}$$

(xvii) $(2x-y+z)^2$

We know that $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(2x-y+z)^2$.

$$\begin{aligned}(2x-y+z)^2&=\left[2x+(-y)+z\right]^2\\&=(2x)^2+(-y)^2+(z)^2+2\times2x\times(-y)+2\times(-y)\times z+2\times z\times2x\\&=4x^2+y^2+z^2-4xy-2yz+4zx.\end{aligned}$$

(xviii) $(-2x+3y+2z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+3y+2z)^2$.

$$\begin{aligned}(-2x+3y+2z)^2 &= \left[(-2x)+3y+2z\right]^2\\ &= (-2x)^2+(3y)^2+(2z)^2+2\times(-2x)\times 3y+2\times 3y\times 2z+2\times 2z\times(-2x)\\ &= 4x^2+9y^2+4z^2-12xy+12yz-8zx.\end{aligned}$$

(xix) $(3a-7b-c)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(3a-7b-c)^2$.

$$\begin{aligned}(3a-7b-c)^2 &= \left[3a+(-7b)+(-c)\right]^2\\ &= (3a)^2+(-7b)^2+(-c)^2+2\times 3a\times(-7b)+2\times(-7b)\times(-c)+2\times(-c)\times 3a\\ &= 9a^2+49b^2+c^2-42ab+14bc-6ac.\end{aligned}$$

(xx) $(-2x+5y-3z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+5y-3z)^2$.

$$\begin{aligned}(-2x+5y-3z)^2 &= \left[(-2x)+5y+(-3z)\right]^2\\ &= (-2x)^2+(5y)^2+(-3z)^2+2\times(-2x)\times 5y+2\times 5y\times(-3z)+2\times(-3z)\times(-2x)\\ &= 4x^2+25y^2+9z^2-20xy-30yz+12zx.\end{aligned}$$

(xxi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

$$\begin{aligned}\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 &= \left[\frac{a}{4}+\left(-\frac{b}{2}\right)+1\right]^2\\ &= \left(\frac{a}{4}\right)^2+\left(-\frac{b}{2}\right)^2+(1)^2+2\times\frac{a}{4}\times\left(-\frac{b}{2}\right)+2\times\left(-\frac{b}{2}\right)\times 1+2\times 1\times\frac{a}{4}\\ &= \frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}.\end{aligned}$$

5.

(xii) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

The expression $4x^2+9y^2+16z^2+12xy-24yz-16xz$ can also be written as $(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x.$

We can observe that, we can apply the identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression $(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x$, to get

$(2x+3y-4z)^2$

Therefore, we conclude that after factorizing the expression $4x^2+9y^2+16z^2+12xy-24yz-16xz$, we get $(2x+3y-4z)^2$.

(xiii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

We need to factorize the expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$.

The expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$ can also be written as $\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right).$

We can observe that, we can apply the identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression $\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\times\left(-\sqrt{2}x\right)\times y+2\times y\times\left(2\sqrt{2}z\right)+2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$, to get

$\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$

Therefore, we conclude that after factorizing the expression $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$, we get $\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$.

6.

(i) $(2x+1)^3$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.

$$\therefore (2x+1)^3 = (2x)^3 + (1)^3 + 3\times 2x \times 1(2x+1)$$
$$= 8x^3 + 1 + 6x(2x+1)$$
$$= 8x^3 + 12x^2 + 6x + 1.$$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3 + 12x^2 + 6x + 1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\therefore (2a-3b)^3 = (2a)^3 - (3b)^3 - 3\times 2a \times 3b(2a-3b)$$
$$= 8a^3 - 27b^3 - 18ab(2a-3b)$$
$$= 8a^3 - 36a^2b + 54ab^2 - 27b^3.$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3 - 36a^2b + 54ab^2 - 27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times 1\left(\frac{3}{2}x+1\right)\therefore$$
$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$$
$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\therefore \left(x-\frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3\times x\times\frac{2}{3}y\left(x-\frac{2}{3}y\right) = x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$$
$$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3.$$

Therefore, the expansion of the expression $\left(x-\frac{2}{3}y\right)^3$ is $x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3$.

7.

(i) $(99)^3$

$(99)^3$ can also be written as $(100-1)^3$.

Using identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

$$(100-1)^3=(100)^3-(1)^3-3\times100\times1(100-1)$$
$$=1000000-1-300(99)$$
$$=999999-29700$$
$$=970299.$$

(ii) $(102)^3$

$(102)^3$ can also be written as $(100+2)^3$.

Using identity $(x+y)^3=x^3+y^3+3xy(x+y)$

$$(100+2)^3=(100)^3+(2)^3+3\times100\times2(100+2)$$
$$=1000000+8+600(102)$$
$$=1000008+61200$$
$$=1061208.$$

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$

$$(1000-2)^3=(1000)^3-(2)^3-3\times1000\times2(1000-2)$$
$$=1000000000-8-6000(998)$$
$$=999999992-5988000$$
$$=994011992.$$

8.

(i) $8a^3+b^3+12a^2b+6ab^2$

The expression $8a^3+b^3+12a^2b+6ab^2$ can also be written as

$$=(2a)^3+(b)^3+3\times2a\times2a\times b+3\times2a\times b\times b$$
$$=(2a)^3+(b)^3+3\times2a\times b(2a+b).$$

Using identity $(x+y)^3=x^3+y^3+3xy(x+y)$ with respect to the expression $(2a)^3+(b)^3+3\times 2a\times b(2a+b)$, we get $(2a+b)^3$.

Therefore, after factorizing the expression $8a^3+b^3+12a^2b+6ab^2$, we get $(2a+b)^3$.

(ii) $8a^3-b^3-12a^2b+6ab^2$

The expression $8a^3-b^3-12a^2b+6ab^2$ can also be written as

$=(2a)^3-(b)^3-3\times 2a\times 2a\times b+3\times 2a\times b\times b$

$=(2a)^3-(b)^3-3\times 2a\times b(2a-b)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(2a)^3-(b)^3-3\times 2a\times b(2a-b)$, we get $(2a-b)^3$.

Therefore, after factorizing the expression $8a^3-b^3-12a^2b+6ab^2$, we get $(2a-b)^3$.

(iii) $27-125a^3-135a+225a^2$

The expression $27-125a^3-135a+225a^2$ can also be written as

$=(3)^3-(5a)^3-3\times 3\times 3\times 5a+3\times 3\times 5a\times 5a$

$=(3)^3-(5a)^3+3\times 3\times 5a(3-5a)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(3)^3-(5a)^3+3\times 3\times 5a(3-5a)$, we get $(3-5a)^3$.

Therefore, after factorizing the expression $27-125a^3-135a+225a^2$, we get $(3-5a)^3$.

(iv) $64a^3-27b^3-144a^2b+108ab^2$

The expression $64a^3-27b^3-144a^2b+108ab^2$ can also be written as

$=(4a)^3-(3b)^3-3\times 4a\times 4a\times 3b+3\times 4a\times 3b\times 3b$

$=(4a)^3-(3b)^3-3\times 4a\times 3b(4a-3b)$.

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(4a)^3-(3b)^3-3\times 4a\times 3b(4a-3b)$, we get $(4a-3b)^3$.

Therefore, after factorizing the expression $64a^3-27b^3-144a^2b+108ab^2$, we get $(4a-3b)^3$.

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

The expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ can also be written as

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times3p\times\frac{1}{6}+3\times3p\times\frac{1}{6}\times\frac{1}{6}$$

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right).$$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right)$, to get $\left(3p-\frac{1}{6}\right)^3$.

Therefore, after factorizing the expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$, we get $\left(3p-\frac{1}{6}\right)^3$.

9.

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.

$$\Rightarrow \quad x^3+y^3=(x+y)^3-3xy(x+y)$$

$$=(x+y)\left[(x+y)^2-3xy\right]$$

$\because$ We know that $(x+y)^2=x^2+2xy+y^2$

$$\therefore \quad x^3+y^3=(x+y)(x^2+2xy+y^2-3xy)$$

$$=(x+y)(x^2-xy+y^2)$$

Therefore, the desired result has been verified.

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

We know that $(x-y)^3=x^3-y^3-3xy(x-y)$.

$$\Rightarrow \quad x^3-y^3=(x-y)^3+3xy(x-y)$$

$$=(x-y)\left[(x-y)^2+3xy\right]$$

$\because$ We know that $(x-y)^2=x^2-2xy+y^2$

$$\therefore \quad x^3-y^3=(x-y)(x^2-2xy+y^2+3xy)$$

$$=(x-y)(x^2+xy+y^2)$$

Therefore, the desired result has been verified.

10.

(i) $27y^3+125z^3$

The expression $27y^3+125z^3$ can also be written as $(3y)^3+(5z)^3$.

We know that $x^3+y^3=(x+y)(x^2-xy+y^2)$.

$$\begin{aligned}(3y)^3+(5z)^3&=(3y+5z)\left[(3y)^2-3y\times5z+(5z)^2\right]\\&=(3y+5z)(9y^2-15yz+25z^2).\end{aligned}$$

(ii) $64m^3-343n^3$

The expression $64m^3-343n^3$ can also be written as $(4m)^3-(7n)^3$.

We know that $x^3-y^3=(x-y)(x^2+xy+y^2)$.

$$\begin{aligned}(4m)^3-(7n)^3&=(4m-7n)\left[(4m)^2+4m\times7n+(7n)^2\right]\\&=(4m-7n)(16m^2+28mn+49n^2)\end{aligned}$$

Therefore, we conclude that after factorizing the expression $64m^3-343n^3$, we get $(4m-7n)(16m^2+28mn+49n^2)$.

11. The expression $27x^3+y^3+z^3-9xyz$ can also be written as $(3x)^3+(y)^3+(z)^3-3\times3x\times y\times z$.

We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

$$\begin{aligned}\therefore(3x)^3+(y)^3+(z)^3-3\times3x\times y\times z&=(3x+y+z)\left[(3x)^2+(y)^2+(z)^2-3x\times y-y\times z-z\times3x\right]\\&=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz).\end{aligned}$$

Therefore, we conclude that after factorizing the expression $27x^3+y^3+z^3-9xyz$, we get $(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$.

12. LHS is $x^3+y^3+z^3-3xyz$ and RHS is $\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$.

We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

And also, we know that $(x-y)^2=x^2-2xy+y^2$.

$$\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

$$\frac{1}{2}(x+y+z)\left[(x^2-2xy+y^2)+(y^2-2yz+z^2)+(z^2-2xz+x^2)\right]$$

$$\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$$

$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

Therefore, we can conclude that the desired result is verified.

13. We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

We need to substitute $x^3+y^3+z^3=0$ in

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$, to get

$$x^3+y^3+z^3-3xyz=(0)(x^2+y^2+z^2-xy-yz-zx), \text{ or}$$

$$x^3+y^3+z^3-3xyz=0$$

$$\Rightarrow x^3+y^3+z^3=3xyz.$$

Therefore, the desired result is verified.

14.

(i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12, b=7$ and $c=5$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=-12+7+5=0$

$$\therefore \quad (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$$

$$= -1260$$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $a=28, b=-15$ and $c=-13$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=28-15-13=0$

$$\therefore \quad (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$$

$$= 16380$$

15.

(i) Area : $25a^2-35a+12$

The expression $25a^2-35a+12$ can also be written as $25a^2-15a-20a+12$.

$$25a^2-15a-20a+12=5a(5a-3)-4(5a-3)$$

$$=(5a-4)(5a-3).$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2-35a+12$ is Length $=(5a-4)$ and Breadth $=(5a-3)$.

(ii) Area : $35y^2+13y-12$

The expression $35y^2+13y-12$ can also be written as $35y^2+28y-15y-12$.

$$35y^2+28y-15y-12=7y(5y+4)-3(5y+4)$$

$$=(7y-3)(5y+4).$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2+13y-12$ is Length $=(7y-3)$ and Breadth $=(5y+4)$.

16.

(i) Volume : $3x^2-12x$

The expression $3x^2-12x$ can also be written as $3\times x\times(x-4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2-12x$ is $3, x$ and $(x-4)$.

(ii) Volume : $12ky^2+8ky-20k$

The expression $12ky^2+8ky-20k$ can also be written as $k\left(12y^2+8y-20\right)$.

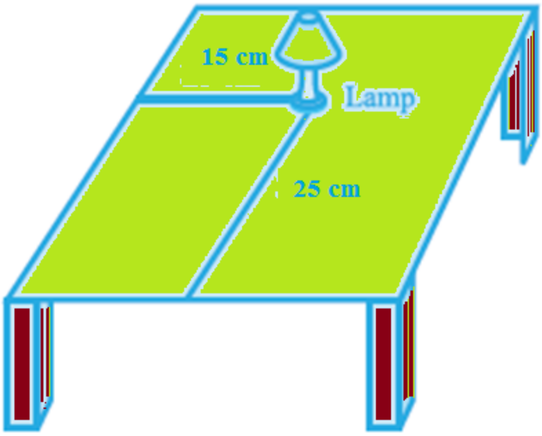
$$\begin{aligned}k\left(12y^2+8y-20\right)&=k\left(12y^2-12y+20y-20\right)\\&=k\left[12y(y-1)+20(y-1)\right]\\&=k(12y+20)(y-1)\\&=4k\times(3y+5)\times(y-1).\end{aligned}$$

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2+8ky-20k$ is $4k,(3y+5)$ and $(y-1)$.

Class –IX Mathematics (Ex. 3.1)

Answers

1. Let us consider the given below figure of a study stable, on which a study lamp is placed.



Let us consider the lamp on the table as a point and the table as a plane. From the figure, we can conclude that the table is rectangular in shape, when observed from the top. The table has a short edge and a long edge.

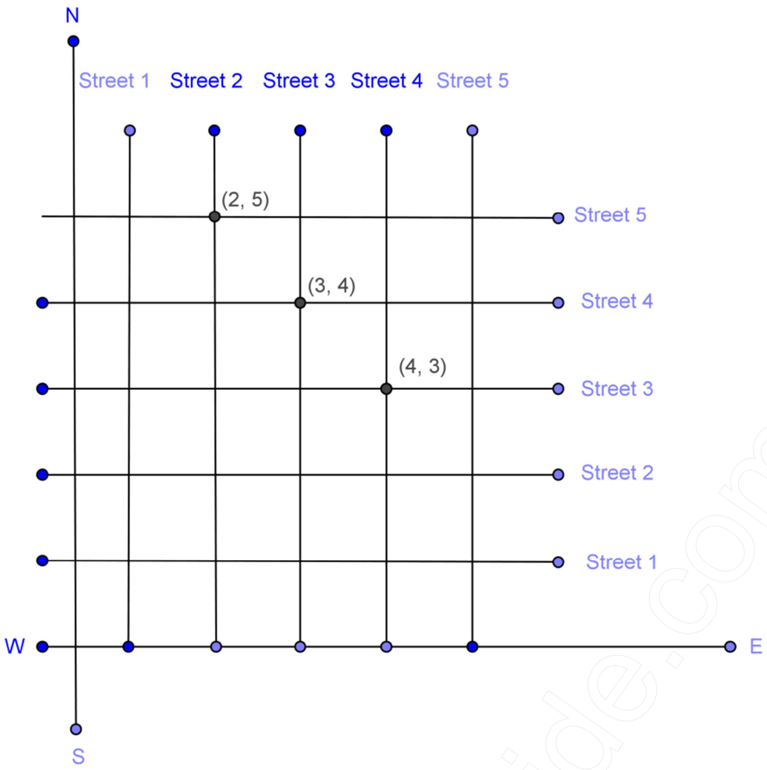
Let us measure the distance of the lamp from the shorter edge and the longer edge. Let us assume that the distance of the lamp from the shorter edge is 15 cm and from the longer edge, its 25 cm.

Therefore, we can conclude that the position of the lamp on the table can be described in two ways depending on the order of the axes as $(15, 25)$ or $(25,15)$.

2. We need to draw two perpendicular lines as the two main roads of the city that cross each other at the center and let us mark it as N-S and E-W.

Let us take the scale as 1 cm = 200m.

We need to draw five streets that are parallel to both the main roads, to get the given below figure.



(i) From the figure, we can conclude that only one point have the coordinates as $(4,3)$.

Therefore, we can conclude that only one cross - street can be referred to as (4, 3).

(ii) From the figure, we can conclude that only one point have the coordinates as $(3,4)$.

Therefore, we can conclude that only one cross - street can be referred to as (3, 4).

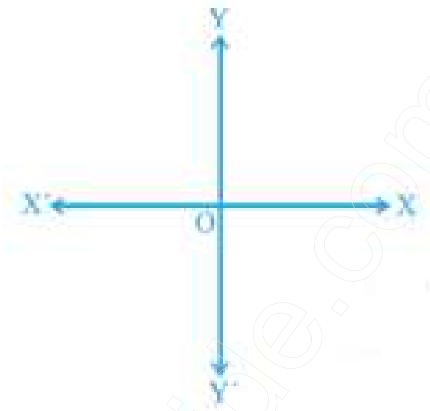
Class –IX Mathematics (Ex. 3.2)

Answers

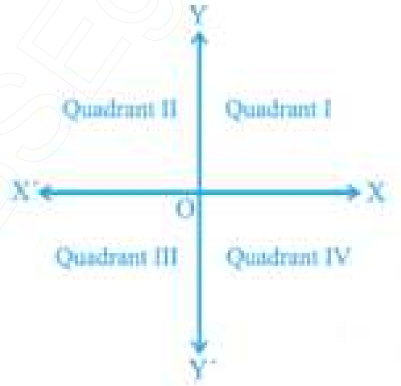
1.

(i) The horizontal line that is drawn to determine the position of any point in the Cartesian plane is called as ***x*-axis**.

The vertical line that is drawn to determine the position of any point in the Cartesian plane is called as ***y*-axis**.



(ii) The name of each part of the plane that is formed by *x*-axis and *y*-axis is called as **quadrant**.



(iii) The point, where the *x*-axis and the *y*-axis intersect is called as **origin**.

2. We need to consider the given below figure to answer the following questions.

(i) The coordinates of point *B* in the above figure is the distance of point *B* from *x*-axis and *y*-axis. Therefore, we can conclude that the coordinates of point *B* are $(-5, 2)$.

(ii) The coordinates of point C in the above figure is the distance of point C from x-axis and y-axis. Therefore, we can conclude that the coordinates of point C are $(5, -5)$.

(iii) The point that represents the coordinates $(-3, -5)$ is E.

(iv) The point that represents the coordinates $(2, -4)$ is G.

(v) The abscissa of point D in the above figure is the distance of point D from the y-axis. Therefore, we can conclude that the abscissa of point D is 6.

(vi) The ordinate of point H in the above figure is the distance of point H from the x-axis. Therefore, we can conclude that the abscissa of point H is -3.

(vii) The coordinates of point L in the above figure is the distance of point L from x-axis and y-axis. Therefore, we can conclude that the coordinates of point L are $(0, 5)$.

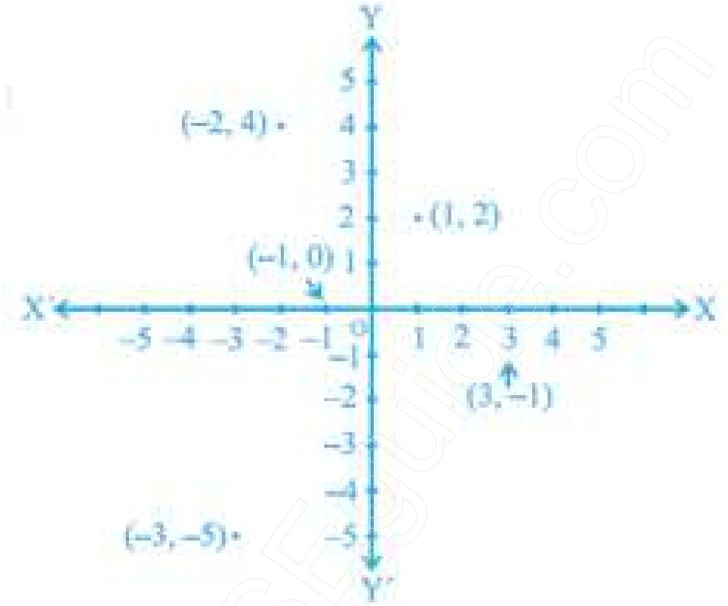
(viii) The coordinates of point M in the above figure is the distance of point M from x-axis and y-axis. Therefore, we can conclude that the coordinates of point M are $(-3, 0)$.

Class –IX Mathematics (Ex. 3.3)

Answers

1. We need to determine the quadrant or axis of the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5).

First, we need to plot the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5) on the graph, to get



We need to determine the quadrant, in which the points (– 2, 4), (3, – 1), (– 1, 0), (1, 2) and (– 3, – 5) lie.

From the figure, we can conclude that the point (– 2, 4) lie in IInd quadrant.

From the figure, we can conclude that the point (3, – 1) lie in IVth quadrant.

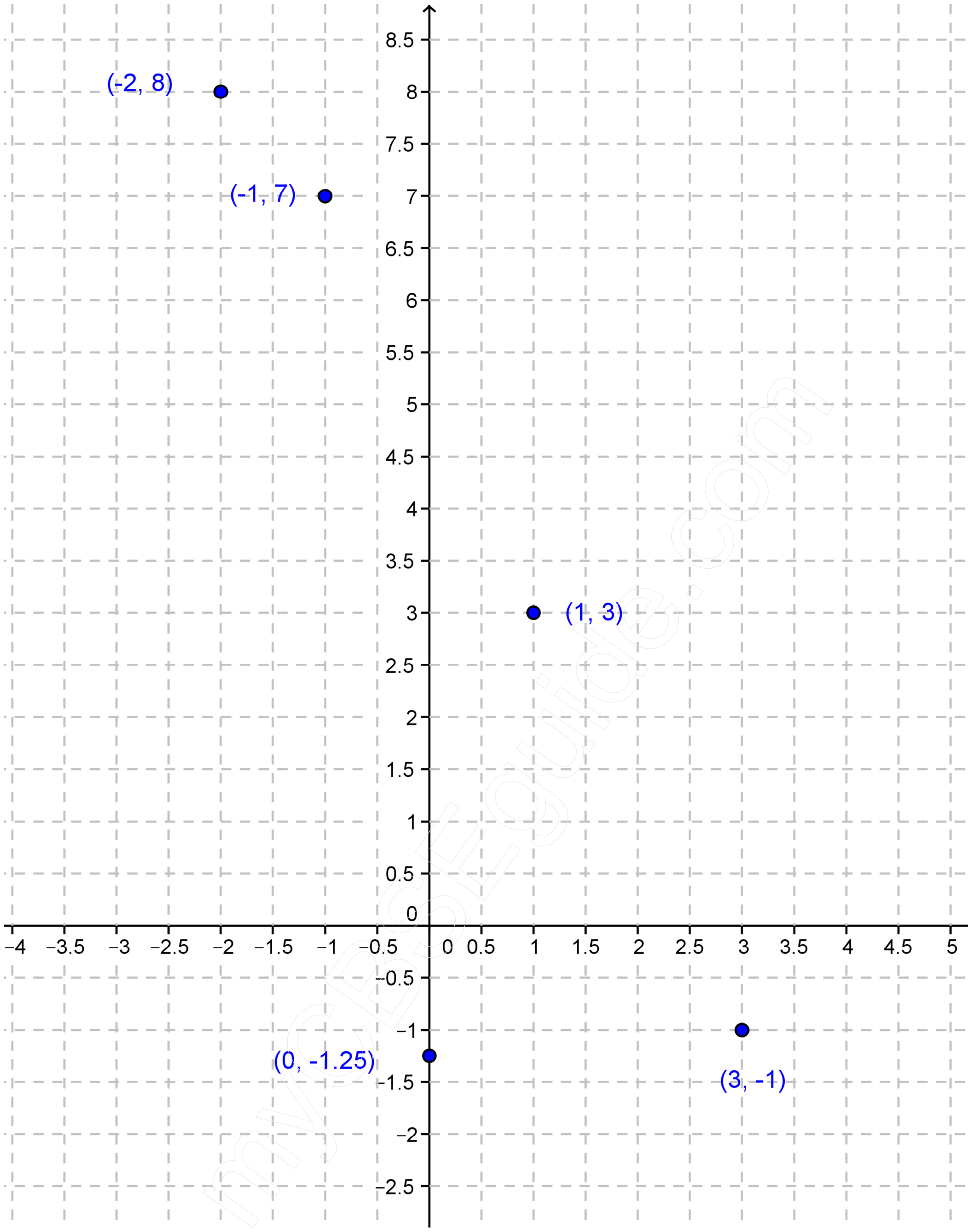
From the figure, we can conclude that the point (–1, 0) lie on *x*-axis.

From the figure, we can conclude that the point (1, 2) lie in Ist quadrant.

From the figure, we can conclude that the point (– 3, – 5) lie in IIIrd quadrant.

2. We need to plot the given below points on the graph by using a suitable scale.

X	–2	–1	0	1	3
y	8	7	–1.25	3	–1


8.5
8
7.5
7
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
(-2, 8)
(-1, 7)
(1, 3)
(0, -1.25)
(3, -1)

Class –IX Mathematics (Ex. 4.1)

Answers

1. Let the cost of a notebook be $Rs.\ x$.

Let the cost of a pen be $Rs.\ y$.

We need to write a linear equation in two variables to represent the statement, “Cost of a notebook is twice the cost of a pen”.

Therefore, we can conclude that the required statement will be $x = 2y$.

2.

(i) $2x + 3y = 9.3\overline{5}$

We need to express the linear equation $2x + 3y = 9.3\overline{5}$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$2x + 3y = 9.3\overline{5}$ can also be written as $2x + 3y - 9.3\overline{5} = 0$.

We need to compare the equation $2x + 3y - 9.3\overline{5} = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = 2, b = 3$ and $c = -9.3\overline{5}$.

(ii) $x - \frac{y}{5} - 10 = 0$

We need to express the linear equation $x - \frac{y}{5} - 10 = 0$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$x - \frac{y}{5} - 10 = 0$ can also be written as $1 \cdot x - \frac{y}{5} - 10 = 0$.

We need to compare the equation $1 \cdot x - \frac{y}{5} - 10 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = 1, b = -\frac{1}{5}$ and $c = -10$.

(iii) $-2x + 3y = 6$

We need to express the linear equation $-2x + 3y = 6$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$-2x + 3y = 6$ can also be written as $-2x + 3y - 6 = 0$.

We need to compare the equation $-2x + 3y - 6 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = -2, b = 3$ and $c = -6$.

(iv) $x = 3y$

We need to express the linear equation $x = 3y$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$x = 3y$ can also be written as $x - 3y + 0 = 0$.

We need to compare the equation $x - 3y + 0 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = 1, b = -3$ and $c = 0$.

(v) $2x = -5y$

We need to express the linear equation $2x = -5y$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$2x = -5y$ can also be written as $2x + 5y + 0 = 0$.

We need to compare the equation $2x + 5y + 0 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = 2, b = 5$ and $c = 0$.

(vi) $3x + 2 = 0$

We need to express the linear equation $3x + 2 = 0$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$3x + 2 = 0$ can also be written as $3x + 0 \cdot y + 2 = 0$.

We need to compare the equation $3x + 0 \cdot y + 2 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = 3, b = 0$ and $c = 2$.

(vii) $y - 2 = 0$

We need to express the linear equation $y - 2 = 0$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$y - 2 = 0$ can also be written as $0 \cdot x + 1 \cdot y - 2 = 0$.

We need to compare the equation $0 \cdot x + 1 \cdot y - 2 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = 0, b = 1$ and $c = -2$.

(viii) $5 = 2x$

We need to express the linear equation $5 = 2x$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$5 = 2x$ can also be written as $-2x + 0 \cdot y + 5 = 0$.

We need to compare the equation $-2x + 0 \cdot y + 5 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = -2, b = 0$ and $c = 5$.

Class –IX Mathematics (Ex. 4.2)

Answers

1. We need to the number of solutions of the linear equation $y = 3x + 5$.

We know that any linear equation has infinitely many solutions.
Justification:
If $x = 0$ then $y = 3 \times 0 + 5 = 5$
If $x = 1$ then $y = 3 \times 1 + 5 = 8$
If $x = -2$ then $y = 3 \times (-2) + 5 = -1$
Similarly we can find infinite many solutions by putting the values of x.

2.

(i) $2x + y = 7$

We know that any linear equation has infinitely many solutions.
Let us put $x = 0$ in the linear equation $2x + y = 7$, to get

$$2(0) + y = 7 \quad \Rightarrow \quad y = 7.$$

Thus, we get first pair of solution as $(0, 7)$.

Let us put $x = 2$ in the linear equation $2x + y = 7$, to get

$$2(2) + y = 7 \quad \Rightarrow \quad y + 4 = 7 \quad \Rightarrow \quad y = 3.$$

Thus, we get second pair of solution as $(2, 3)$.

Let us put $x = 4$ in the linear equation $2x + y = 7$, to get

$$2(4) + y = 7 \quad \Rightarrow \quad y + 8 = 7 \quad \Rightarrow \quad y = -1.$$

Thus, we get third pair of solution as $(4, -1)$.

Let us put $x = 6$ in the linear equation $2x + y = 7$, to get

$$2(6) + y = 7 \quad \Rightarrow \quad y + 12 = 7 \quad \Rightarrow \quad y = -5.$$

Thus, we get fourth pair of solution as $(6, -5)$.

Therefore, we can conclude that four solutions for the linear equation $2x + y = 7$ are $(0, 7), (2, 3), (4, -1)$ and $(6, -5)$.

(ii) $\pi x + y = 9$

We know that any linear equation has infinitely many solutions.
Let us put $x = 0$ in the linear equation $\pi x + y = 9$, to get

$$\pi(0) + y = 9 \quad \Rightarrow \quad y = 9$$

Thus, we get first pair of solution as $(0, 9)$.

Let us put $y=0$ in the linear equation $\pi x+y=9$, to get

$\pi x+(0)=9 \qquad \Rightarrow \qquad x=\frac{9}{\pi}.$

Thus, we get second pair of solution as $\left(\frac{9}{\pi},0\right)$.

Let us put $x=1$ in the linear equation $\pi x+y=9$, to get

$\pi(1)+y=9 \qquad \Rightarrow \qquad y=\frac{9}{\pi}$

Thus, we get third pair of solution as $\left(1,\frac{9}{\pi}\right)$.

Let us put $y=2$ in the linear equation $\pi x+y=9$, to get

$\pi x+2=9 \qquad \Rightarrow \qquad \pi x=7 \qquad \Rightarrow \qquad x=\frac{7}{\pi}$

Thus, we get fourth pair of solution as $\left(\frac{7}{\pi},2\right)$.

Therefore, we can conclude that four solutions for the linear equation $\pi x+y=9$ are $(0,9),\left(\frac{9}{\pi},0\right),\left(1,\frac{9}{\pi}\right)$ and $\left(\frac{7}{\pi},2\right)$.

(iii) $x=4y$

We know that any linear equation has infinitely many solutions.
Let us put $y=0$ in the linear equation $x=4y$, to get

$x=4(0) \qquad \Rightarrow \qquad x=0$

Thus, we get first pair of solution as $(0,0)$.

Let us put $y=2$ in the linear equation $x=4y$, to get

$x=4(2) \qquad \Rightarrow \qquad x=8$

Thus, we get second pair of solution as $(8,2)$.

Let us put $y=4$ in the linear equation $x=4y$, to get

$x=4(4) \qquad \Rightarrow \qquad x=16$

Thus, we get third pair of solution as $(16,4)$.

Let us put $y=6$ in the linear equation $x=4y$, to get

$x=4(6) \qquad \Rightarrow \qquad x=24$

Thus, we get fourth pair of solution as $(24,6)$.

Therefore, we can conclude that four solutions for the linear equation $x = 4y$ are $(0,0),(8,2),(16,4)$ and $(24,6)$.

3.

(i) $(0,2)$

We need to put $x = 0$ and $y = 2$ in the L.H.S. of linear equation $x - 2y = 4$, to get

$(0) - 2(2) = -4$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $(0,2)$ is not a solution of the linear equation $x - 2y = 4$.

(ii) $(2,0)$

We need to put $x = 2$ and $y = 0$ in the L.H.S. of linear equation $x - 2y = 4$, to get

$(2) - 2(0) = 2$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $(2,0)$ is not a solution of the linear equation $x - 2y = 4$.

(iii) $(4,0)$

We need to put $x = 4$ and $y = 0$ in the linear equation $x - 2y = 4$, to get

$(4) - 2(0) = 4$

$\therefore$ L.H.S. $=$ R.H.S.

Therefore, we can conclude that $(4,0)$ is a solution of the linear equation $x - 2y = 4$.

(iv) $\left(\sqrt{2}, 4\sqrt{2}\right)$

We need to put $x = \sqrt{2}$ and $y = 4\sqrt{2}$ in the linear equation $x - 2y = 4$, to get

$\left(\sqrt{2}\right) - 2\left(4\sqrt{2}\right) = -7\sqrt{2}$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $\left(\sqrt{2}, 4\sqrt{2}\right)$ is not a solution of the linear equation $x - 2y = 4$.

(v) $(1,1)$

We need to put $x = 1$ and $y = 1$ in the linear equation $x - 2y = 4$, to get

$(1) - 2(1) = -1$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $(1,1)$ is not a solution of the linear equation $x-2y=4$.

4. We know that, if $x=2$ and $y=1$ is a solution of the linear equation $2x+3y=k$, then on substituting the respective values of x and y in the linear equation $2x+3y=k$, the LHS and RHS of the given linear equation will not be effected.

$\therefore \quad 2(2)+3(1)=k \quad \Rightarrow \quad k=4+3 \quad \Rightarrow \quad k=7$

Therefore, we can conclude that the value of k, for which the linear equation $2x+3y=k$ has $x=2$ and $y=1$ as one of its solutions is 7.

Class –IX Mathematics (Ex. 4.3)

Answers

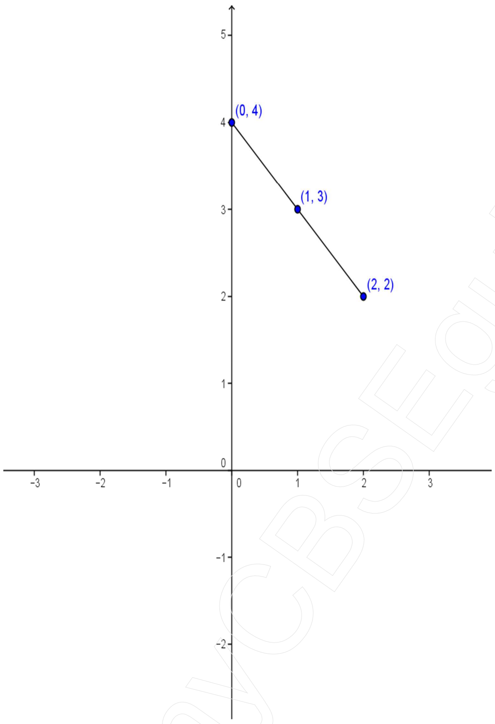
1.

(i) $x+y=4$

We can conclude that $x=0, y=4; x=1, y=3$ and $x=2, y=2$ are the solutions of the linear equation $x+y=4$.

We can optionally consider the given below table for plotting the linear equation $x+y=4$ on the graph.

X	0	1	2
y	4	3	2

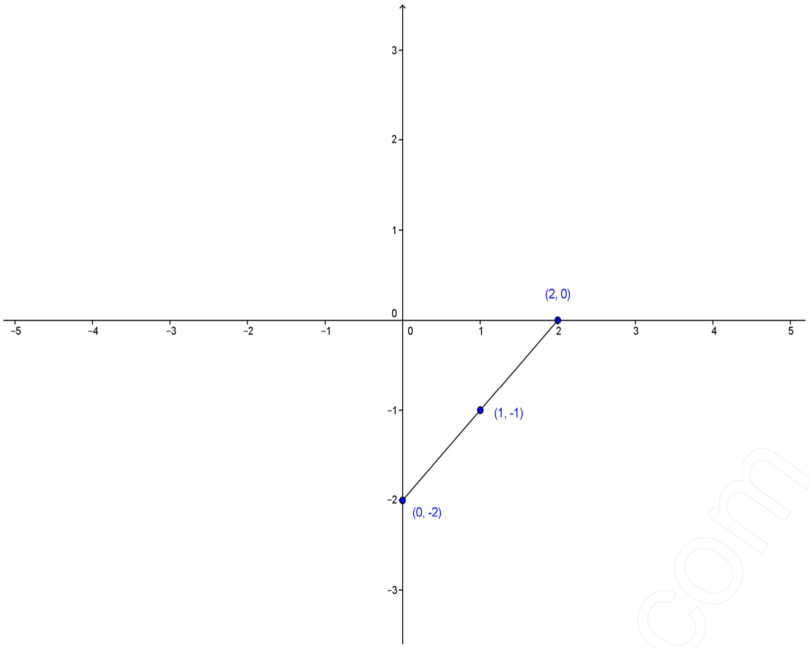


(ii) $x-y=2$

We can conclude that $x=0, y=-2; x=1, y=-1$ and $x=2, y=0$ are the solutions of the linear equation $x-y=2$.

We can optionally consider the given below table for plotting the linear equation $x-y=2$ on the graph.

X	0	1	2
y	-2	-1	0

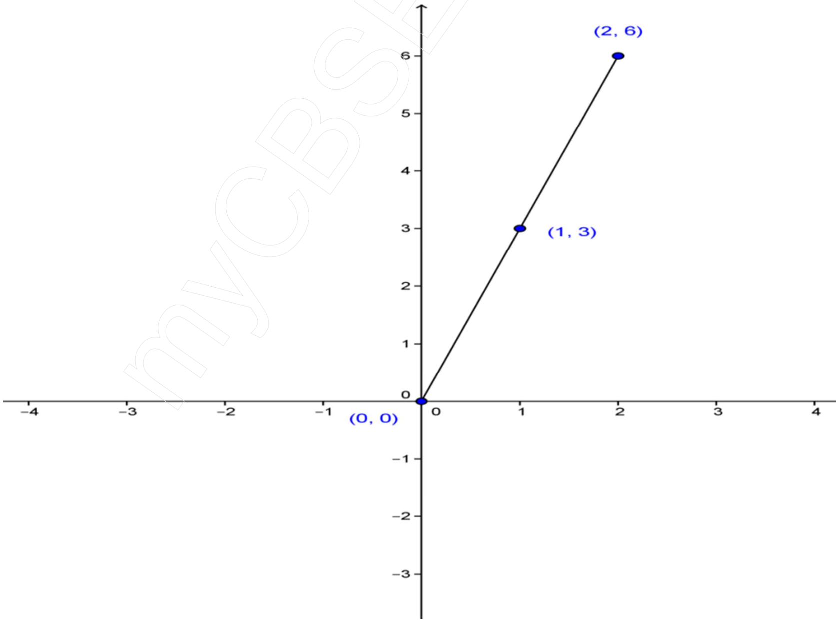


(iii) $y = 3x$

We can conclude that $x = 0, y = 0; x = 1, y = 3$ and $x = 2, y = 6$ are the solutions of the linear equation $y = 3x$.

We can optionally consider the given below table for plotting the linear equation $y = 3x$ on the graph.

X	0	1	2
y	0	3	6

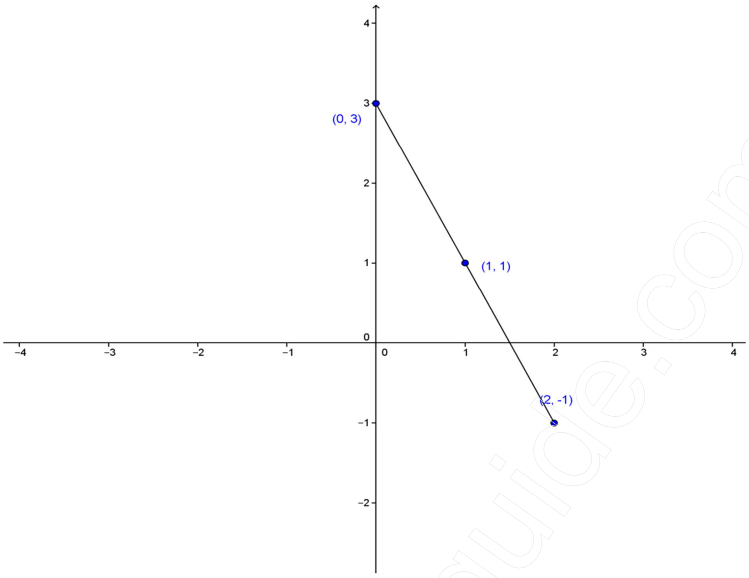


(iv) $3 = 2x + y$

We can conclude that $x = 0,\ y = 3;\ x = 1,\ y = 1$ and $x = 2,\ y = -1$ are the solutions of the linear equation $3 = 2x + y$.

We can optionally consider the given below table for plotting the linear equation $3 = 2x + y$ on the graph.

X	0	1	2
y	3	1	-1



2. We need to give the two equations of the line that passes through the point $(2,14)$.

We know that infinite number of lines can pass through any given point.
We can consider the linear equations $7x - y = 0$ and $2x + y = 18$.
We can conclude that on putting the values $x = 2$ and $y = 14$ in the above mentioned linear equations, we get LHS=RHS.

Therefore, we can conclude that the line of the linear equations $7x - y = 0$ and $28x - 4y = 0$ will pass through the point $(2,14)$.

3. We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.
We can conclude that $(3,4)$ is a solution of the linear equation $3y = ax + 7$.
We need to substitute $x = 3$ and $y = 4$ in the linear equation $3y = ax + 7$, to get

$$3(4) = a(3) + 7 \quad \Rightarrow \quad 12 = 3a + 7$$

$$\Rightarrow \quad 3a = 12 - 7 \quad \Rightarrow \quad 3a = 5 \quad \Rightarrow \quad a = \frac{5}{3}$$

Therefore, we can conclude that the value of a will be $\frac{5}{3}$.

4. From the given situation, we can conclude that the distance covered at the rate Rs 5 per km will be $(x-1)$, as first kilometer is charged at Rs 8 per km.

We can conclude that the linear equation for the given situation will be:

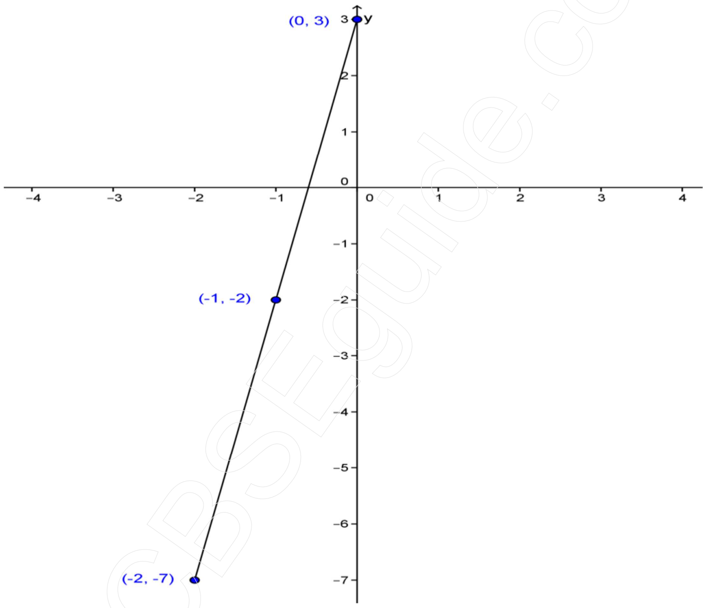
$$8+5(x-1)=y \quad \Rightarrow \quad 8+5x-5=y \quad \Rightarrow \quad 3+5x=y.$$

We need to draw the graph of the linear equation $3+5x=y$.

We can conclude that $x=0, y=3; x=1, y=1$ and $x=2, y=-1$ are the solutions of the linear equation $3+5x=y$.

We can optionally consider the given below table for plotting the linear equation $3+5x=y$ on the graph.

X	0	-1	-2
y	3	-2	-7



5. **For First figure**

(i) $y=x$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

Let us check whether $x=-1, y=1; x=0, y=0$ and $x=1, y=-1$ are the solutions of the linear equation $y=x$.

For $x=-1, y=1$, we get

$$y=x \quad \Rightarrow \quad -1 \neq 1$$

Therefore, the given graph does not belong to the linear equation $y=x$.

(ii) $x+y=0$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=1$, we get

$-1+1=0 \quad \Rightarrow \quad 0=0.$

For $x=0, y=0$, we get

$0+0=0 \quad \Rightarrow \quad 0=0.$

For $x=1, y=-1$, we get

$1+(-1)=0 \quad \Rightarrow \quad 1-1=0 \quad \Rightarrow \quad 0=0.$

Therefore, the given graph belongs to the linear equation $x+y=0$.

(iii) $y=2x$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=1$, we get

$y=2x \quad \Rightarrow \quad -1=2(1) \quad \Rightarrow \quad -1 \neq 2.$

Therefore, the given graph does not belong to the linear equation $y=2x$.

(iv) $2+3y=7x$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=1$, we get

$2+3(1)=7(-1) \quad \Rightarrow \quad 2+3=-7 \quad \Rightarrow \quad 5 \neq -7.$

Therefore, the given graph does not belong to the linear equation $2+3y=7x$.

For Second figure

(i) $y=x+2$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=3$, we get

$3=-1+2 \quad \Rightarrow \quad 3 \neq 1.$

Therefore, the given graph does not belong to the linear equation $y=x+2$.

(ii) $y=x-2$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=3$, we get

$3=-1-2 \quad \Rightarrow \quad 3 \neq -3.$

Therefore, the given graph does not belong to the linear equation $y=x-2$.

(iii) $y=-x+2$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=3$, we get

$3=-(-1)+2 \quad \Rightarrow \quad 3=1+2 \quad \Rightarrow \quad 3=3.$

For $x=0, y=2$, we get

$2=-(0)+2 \quad \Rightarrow \quad 2=2.$

For $x = 2, y = 0$, we get

$$0 = -(2) + 2 \quad \Rightarrow \quad 0 = 0.$$

Therefore, hat the given graph belongs to the linear equation $y = -x + 2$.

(iv) $x + 2y = 6$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x = -1, y = 3$, we get

$$(-1) + 2(3) = 6 \quad \Rightarrow \quad -1 + 6 = 6 \quad \Rightarrow \quad 5 \neq 6.$$

Therefore, the given graph does not belong to the linear equation $x + 2y = 6$.

6. We are given that the work done by a body on application of a constant force is directly proportional to the distance travelled by the body.

Let the work done be W and let constant force be F.

Let distance travelled by the body be D.

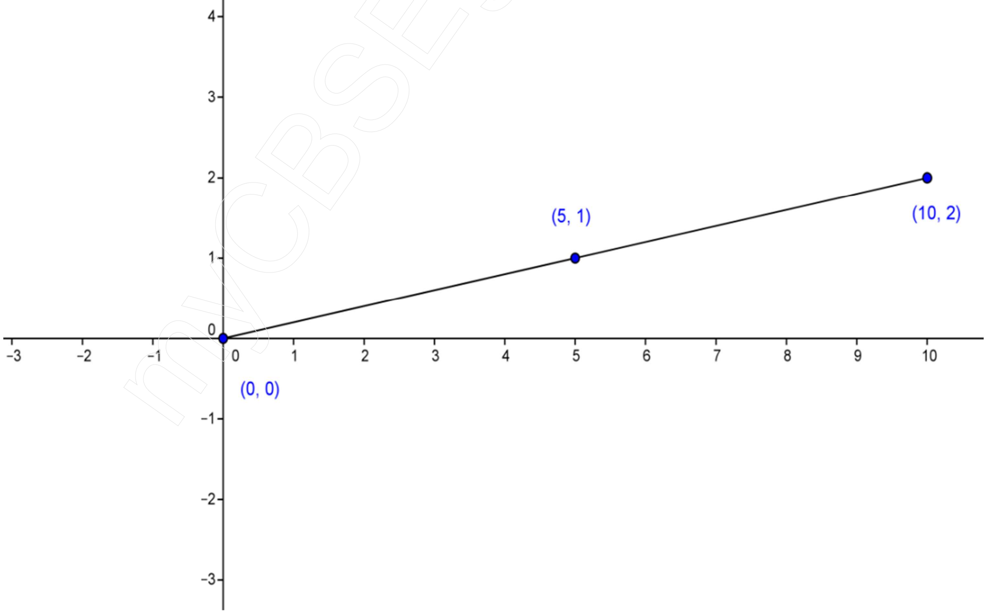
According to the question,

$$W \propto D \quad \Rightarrow \quad W = F \cdot D.$$

We need to draw the graph of the linear equation $W = F \cdot D$, when the force is constant as 5 units, i.e., $W = 5D$.

We can conclude that $x = 0, y = 0; x = 5, y = 1$ and $x = 10, y = 2$ are the solutions of the linear equation $W = 5D$.

W	0	5	10
D	0	1	2



Therefore, we can conclude from the above mentioned graph, the work done by the body, when the distance is 2 units will be 10 units and when the distance is 0 units, the work done will be 0 unit.

7. The contribution made by Yamini is Rs *x* and the contribution made by Fatime is Rs *y*.

We are given that together they both contributed Rs 100.

We get the given below linear equation from the given situation.

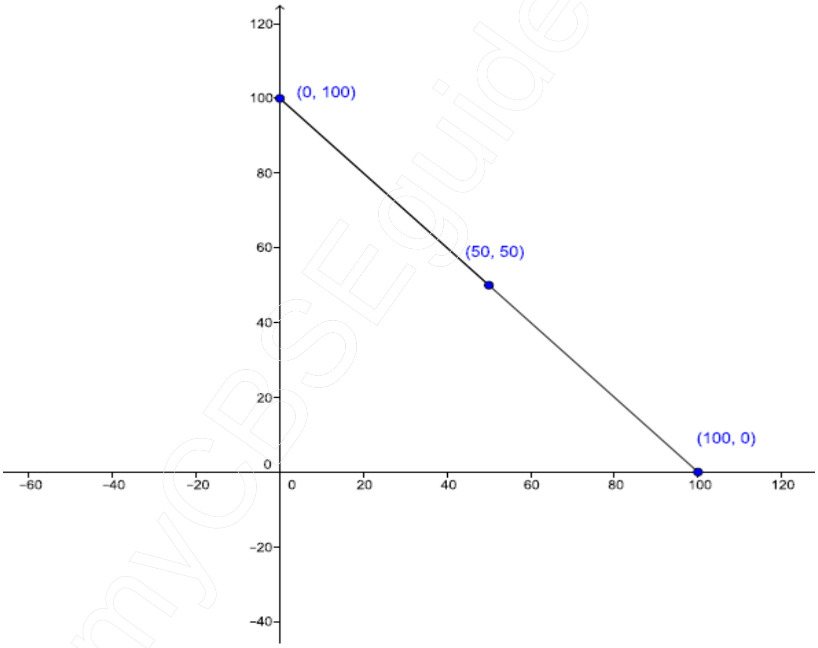
$x + y = 100$.

We need to consider any 3 solutions of the linear equation $x + y = 100$, to plot the graph of the linear equation $x + y = 100$.

We can conclude that $x = 0, y = 3; x = 1, y = 1$ and $x = 2, y = -1$ are the solutions of the linear equation $x + y = 100$.

We can optionally consider the given below table for plotting the linear equation $x + y = 100$ on the graph.

X	0	50	100
y	100	50	0



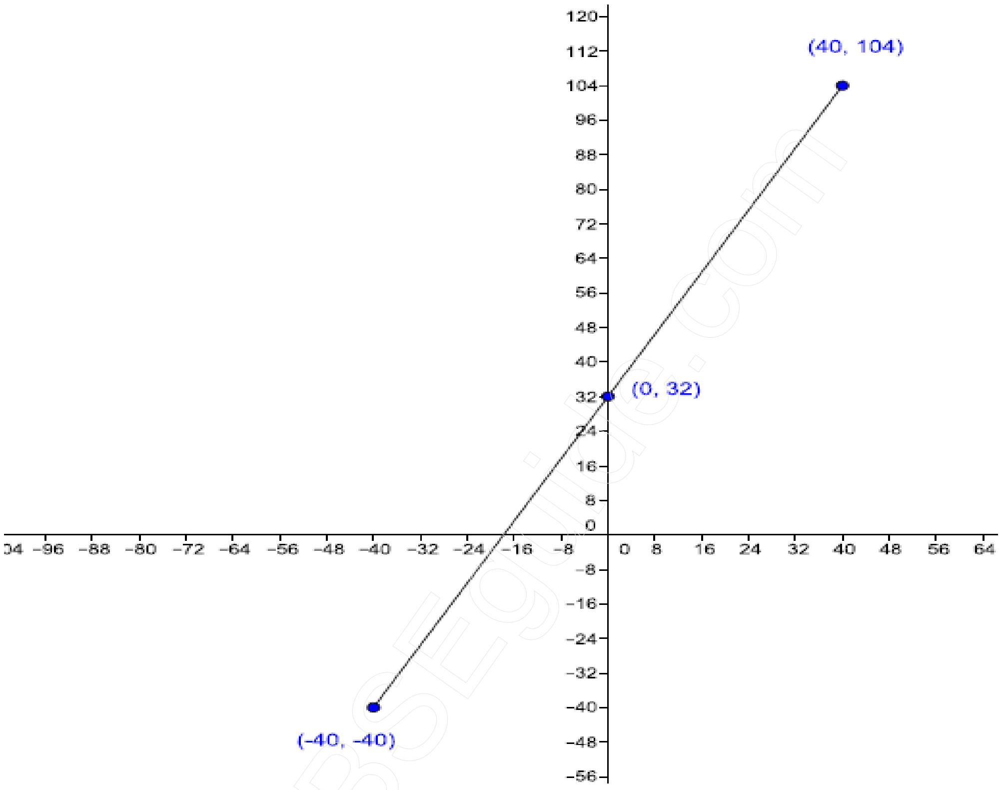
8. We are given a linear equation that converts the temperature in Fahrenheit into degree Celsius.

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) We need to consider any 3 solutions of the linear equation $F = \left(\frac{9}{5}\right)C + 32$, to plot the graph of the linear equation $F = \left(\frac{9}{5}\right)C + 32$.

We can conclude that $x=0, y=3; x=1, y=1$ and $x=2, y=-1$ are the solutions of the linear equation $F=\left(\frac{9}{5}\right)C+32$.

C	-40	0	40
F	-40	32	104



(ii) We need to find the temperature in Fahrenheit, when the temperature in degree Celsius is 30°.

$$F=\left(\frac{9}{5}\right)(30)+32=9\times6+32=86^{\circ}$$

Therefore, we can conclude that the temperature in Fahrenheit will be $86^{\circ}F$.

(iii) We need to find the temperature in degree Celsius, when the temperature in Fahrenheit is 95°.

$$95=\left(\frac{9}{5}\right)C+32 \quad \Rightarrow \quad \frac{9}{5}C=95-32 \quad \Rightarrow \quad C=63\times\frac{5}{9}=35^{\circ}.$$

Therefore, we can conclude that the temperature in degree Celsius will be 35°.

(iv) We need to find the temperature in Fahrenheit, when the temperature in degree Celsius is 0°.

$F = \left(\frac{9}{5}\right)(0) + 32 = 32°$

Therefore, we can conclude that the temperature in Fahrenheit will be 32°.

We need to find the temperature in degree Celsius, when the temperature in Fahrenheit is 0°.

$0 = \left(\frac{9}{5}\right)C + 32 \quad \Rightarrow \quad \frac{9}{5}C = 0 - 32 \quad \Rightarrow \quad C = -32 \times \frac{5}{9} = -17.77^{\circ}.$

Therefore, we can conclude that the temperature in degree Celsius will be -17.77°

(v) We need to find a temperature that is numerically same in both Fahrenheit and degree Celsius.

$F = \left(\frac{9}{5}\right)F + 32 \quad \Rightarrow \quad F - \frac{9F}{5} = 32 \quad \Rightarrow \quad -\frac{4F}{5} = 32$

$\Rightarrow \quad F = -40^{\circ}.$

Therefore, we can conclude that the temperature that is numerically same in Fahrenheit and degree Celsius will be -40°.

Class –IX Mathematics (Ex. 4.4)

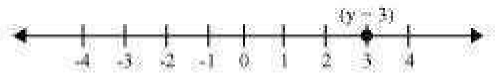
Answers

1.

(i) We need to represent the linear equation $y = 3$ geometrically in one variable.

We can conclude that in one variable, the geometric representation of the linear equation $y = 3$ will be same as representing the number 3 on a number line.

Given below is the representation of number 3 on the number line.



(ii) We need to represent the linear equation $y = 3$ geometrically in two variables.

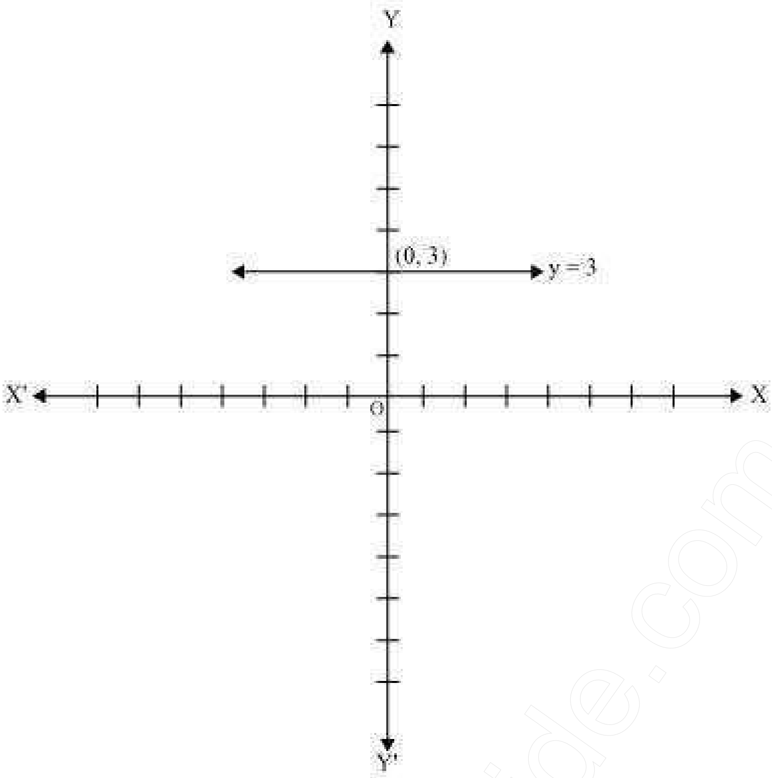
We know that the linear equation $y = 3$ can also be written as $0 \cdot x + y = 3$.

We can conclude that in two variables, the geometric representation of the linear equation $y = 3$ will be same as representing the graph of linear equation $0 \cdot x + y = 3$.

Given below is the representation of the linear equation $0 \cdot x + y = 3$ on a graph.

We can optionally consider the given below table for plotting the linear equation $0 \cdot x + y = 3$ on the graph.

X	1	0
y	3	3



2.

(i) We need to represent the linear equation $2x+9=0$ geometrically in one variable.

We know that the linear equation $2x+9=0$ can also be written as $x=-\frac{9}{2}$ or $x=-4.5$.

We can conclude that in one variable, the geometric representation of the linear equation $2x+9=0$ will be same as representing the number -4.5 on a number line.

Given below is the representation of number 3 on the number line.

(ii) We need to represent the linear equation $2x+9=0$ geometrically in two variables.

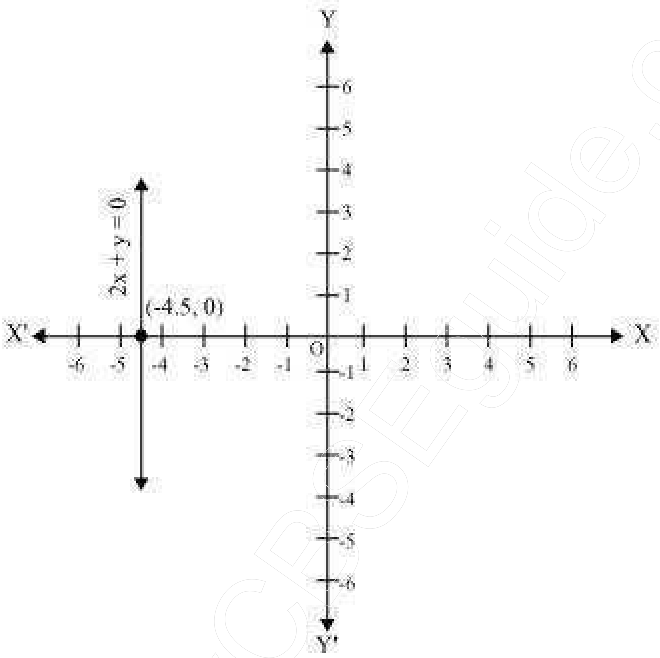
We know that the linear equation $2x+9=0$ can also be written as $2x+0\cdot y=9$.

We can conclude that in two variables, the geometric representation of the linear equation $2x+9=0$ will be same as representing the graph of linear equation $2x+0\cdot y=9$.

Given below is the representation of the linear equation $2x+0\cdot y=9$ on a graph.

We can optionally consider the given below table for plotting the linear equation $2x+0\cdot y=9$ on the graph.

X	1	0
y	4.5	4.5



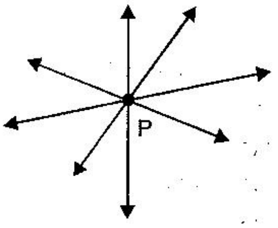
Class –IX Mathematics (Ex. 5.1)

Answers

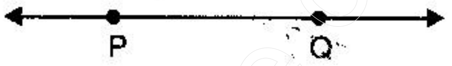
1.

(i) **False**

Correct statement: Infinite many lines can pass through a single point.
This is self evident and can be seen visually by the student given below:



(ii) **False** because the given statement contradicts the postulate I of the Euclid that assures that there is a unique line that passes through two distinct points.



Through two points P and Q a unique line can be drawn.

(iii) **True**

Reason:
We need to consider Euclid’s Postulate 2: “A terminated line can be produced indefinitely.

”

(iv) **True**

Reason:
Let us consider two circles with same radii.
We can conclude that, when we make the two circles overlap with each other, we will get a superimposed figure of the two circles.
Therefore, we can conclude that the radii of both the circles will also coincide and will be same.

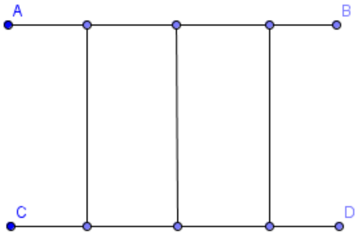
(v) **True**

Reason:
We are given that AB = PQ and PQ = XY.
We need to consider the axiom: “Given two distinct points, there is a unique line that passes through them.”
Therefore, we can conclude that AB, PQ and XY are the lines with same dimensions, and hence if AB = PQ and PQ = XY, then AB = XY.

2.

(i) Parallel lines

Two lines are said to be parallel, when the perpendicular distance between these lines is always constant or we can say that the lines that never intersect each other are called as parallel lines.



We need to define line first, in order to define parallel lines.

(ii) Perpendicular lines

Two lines are said to be perpendicular lines, when angle between these two lines is 90°.

We need to define line and angle, in order to define perpendicular lines.

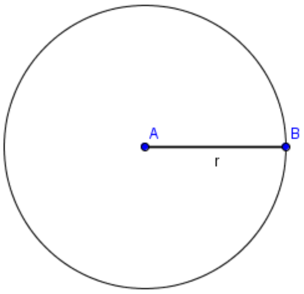
(iii) Line segment

A line of a fixed dimension between two given points is called as a line segment.

We need to define line and point, in order to define a line segment.

(iv) Radius of a circle

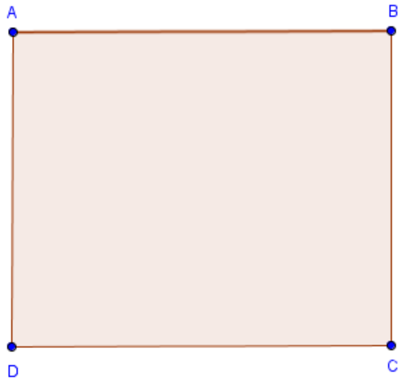
The distance of any point lying on the boundary of a circle from the center of the circle is called as radius of a circle.



We need to define circle and center of a circle, in order to define radius of a circle.

(v) Square

A quadrilateral with all four sides equal and all four angles of 90° is called as a square.



We need to define quadrilateral and angle, in order to define a square.

3. We are given with following two postulates

(i) Given any two distinct points A and B, there exists a third point C, which is between A and B.

(ii) There exists at least three points that are not on the same line.

The undefined terms in the given postulates are point and line.

The two given postulates are consistent, as they do not refer to similar situations and they refer to two different situations.

We can also conclude that, it is impossible to derive at any conclusion or any statement that contradicts any well known axiom and postulate.

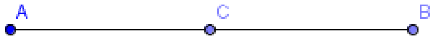
The two given postulates do not follow from the postulates given by Euclid.

The two given postulates can be observed following from the axiom, “Given two distinct points, there is a unique line that passes through them”.

4. We are given that a point C lies between two points B and C, such that $AC = BC$.

We need to prove that $AC = \frac{1}{2}AB$.

Let us consider the given below figure.



We are given that $AC = BC$. ...(*i*)

An axiom of the Euclid says that "If equals are added to equals, the wholes are equal."

Let us add AC to both sides of equation (*i*).

$AC + AC = BC + AC$.

An axiom of the Euclid says that "Things which coincide with one another are equal to one another."

We can conclude that $BC + AC$ coincide with AB, or

$AB = BC + AC$. ...(*ii*)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

From equations (*i*) and (*ii*), we can conclude that

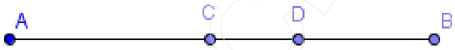
$AC + AC = AB$, or $2AC = AB$.

An axiom of the Euclid says that "Things which are halves of the same things are equal to one another."

Therefore, we can conclude that $AC = \frac{1}{2}AB$.

5. We need to prove that every line segment has one and only one mid-point.

Let us consider the given below line segment AB and assume that C and D are the mid-points of the line segment AB.



If C is the mid-point of line segment AB, then

$AC = CB$.

An axiom of the Euclid says that "If equals are added to equals, the wholes are equal."

$AC + AC = CB + AC$. (i)

From the figure, we can conclude that $CB + AC$ will coincide with AB.

An axiom of the Euclid says that "Things which coincide with one another are equal to one another."

$AC + AC = AB$. (ii)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another.”

Let us compare equations (i) and (ii), to get

$AC + AC = AB$, or $2AC = AB$. (iii)

If D is the mid-point of line segment AB, then

$AD = DB$.

An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.”

$AD + AD = DB + AD$. (iv)

From the figure, we can conclude that $DB + AD$ will coincide with AB.

An axiom of the Euclid says that “Things which coincide with one another are equal to one another.”

$AD + AD = AB$. (v)

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (iv) and (v), to get

$AD + AD = AB$, or

$2AD = AB$. (vi)

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (iii) and (vi), to get

$2AC = 2AD$.

An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.”

$AC = AD$.

Therefore, we can conclude that the assumption that we made previously is false and a line segment has one and only one mid-point.

6. We are given that $AC = BD$.

We need to prove that $AB = CD$ in the figure given below.

From the figure, we can conclude that

$AC = AB + BC$, and

$BD = CD + BC$.

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

$AB + BC = CD + BC$. (i)

An axiom of the Euclid says that "when equals are subtracted from equals, the remainders are also equal."

We need to subtract BC from equation (i), to get

$AB + BC - BC = CD + BC - BC$

$AB = CD$.

Therefore, we can conclude that the desired result is proved.

7. We need to prove that Euclid's fifth axiom is considered as a universal truth.

Euclid's fifth axiom states that "the whole is greater than the part."

The above given axiom is a universal truth. We can apply the fifth axiom not only mathematically but also universally in daily life.

Mathematical proof:

Let us consider a quantity z, which has different parts as a, b, x and y.

$z = a + b + x + y$.

Therefore, we can conclude that z will always be greater than its corresponding parts a, b, x and y.

Universal proof:

We know that Mumbai is located in Maharashtra and Maharashtra is located in India.

In other words, we can conclude that Mumbai is a part of Maharashtra and Maharashtra is a part of India.

Therefore, we can conclude that whole India will be greater than Mumbai or Maharashtra or both.

Therefore, we can conclude that Euclid's fifth axiom is considered as a 'Universal truth'.