CLASS – X Mathematics

Chapter-01 Real Numbers (Exercise 1.1)

Answers:

1. (i) 135 and 225

We have 225 > 135,

So, we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Here remainder 90 ≠ 0, we apply the division lemma again to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder 45 ≠ 0, and apply the division lemma to obtain

$$90 = 2 \times 45 + 0$$

Since that time the remainder is zero, the process get stops.

The divisor at this stage is 45

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

We have 38220 > 196,

So, we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

Since we get the remainder is zero, the process stops.

The divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

We have 867 > 255,

So, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Here remainder 102 ≠ 0, we apply the division lemma again to 255 and 102 to obtain

$$255 = 102 \times 2 + 51$$

Here remainder 51 ≠ 0, we apply the division lemma again to 102 and 51 to obtain

$$102 = 51 \times 2 + 0$$

Since we get the remainder is zero, the process stops.

The divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

2. Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer
$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer
Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.
Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2. Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

3. We have to find the HCF (616, 32) to find the maximum number of columns in which they can march.

To find the HCF, we can use Euclid's algorithm.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

4. Let a be any positive integer and $b = 3$.
Then $a = 3q + r$ for some integer $q \geq 0$
And $r = 0, 1, 2$ because $0 \leq r < 3$
Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$
Or,

$$a^2 = (3q)^2 or (3q+1)^2 or (3q+2)^2$$
$$a^2 = (9q)^2 or 9q^2 + 6q + 1 \, or \, 9q^2 + 12q + 4$$
$$= 3\times(3q^2) \, or \, 3(3q^2 + 2q) + 1 \, or \, 3(3q^2 + 4q + 1) + 1$$
$$= 3k_1 \, or \, 3k_2 + 1 \, or \, 3k_3 + 1$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

5. Let a be any positive integer and $b = 3$
$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$
$\therefore$ a = 3q or 3q + 1 or 3q + 2
Therefore, every number can be represented as these three forms.
We have three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$$a^3 = (3q + 1)^3$$

$a^3 = 27q^3 + 27q^2 + 9q + 1$
$a^3 = 9(3q^3 + 3q^2 + q) + 1$
$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$
$a^3 = 27q^3 + 54q^2 + 36q + 8$
$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$
$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

CLASS – X Mathematics

Chapter-01 Real Numbers (Exercise 1.2)

Answers:

1. (i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$
   (ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$
   (iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$
   (iv) $5005 = 5 \times 7 \times 11 \times 13$
   (v) $7429 = 17 \times 19 \times 23$

2. (i) 26 and 91
   $26 = 2 \times 13$
   $91 = 7 \times 13$
   HCF = 13
   LCM = $2 \times 7 \times 13 = 182$
   Product of two numbers 26 and 91 = $26 \times 91 = 2366$
   HCF × LCM = $13 \times 182 = 2366$
   Hence, product of two numbers = HCF × LCM
   (ii) 510 and 92
   $510 = 2 \times 3 \times 5 \times 17$
   $92 = 2 \times 2 \times 23$
   HCF = 2
   LCM = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
   Product of two numbers 510 and 92 = $510 \times 92 = 46920$
   HCF × LCM = $2 \times 23460 = 46920$
   Hence, product of two numbers = HCF × LCM
   (iii) 336 and 54
   $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$
   $54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$
   HCF = $2 \times 3 = 6$
   LCM = $2^4 \times 3^3 \times 7 = 3024$
   Product of two numbers 336 and 54 = $336 \times 54 = 18144$
   HCF × LCM = $6 \times 3024 = 18144$
   Hence, product of two numbers = HCF × LCM

3. (i) 12, 15 and 21
   $12 = 2^2 \times 3$
   $15 = 3 \times 5$
   $21 = 3 \times 7$
   HCF = 3
   LCM = $2^2 \times 3 \times 5 \times 7 = 420$
   (ii) 17, 23 and 29
   $17 = 1 \times 17$
   $23 = 1 \times 23$
   $29 = 1 \times 29$

HCF = 1
LCM = 17 × 23 × 29 = 11339

(iii) 8, 9 and 25

$8 = 2 \times 2 \times 2 = 2^3$
$9 = 3 \times 3 = 3^2$
$25 = 5 \times 5 = 5^2$

HCF = 1
LCM = $2^3 \times 3^2 \times 5^2$ = 1800

4. HCF (306, 657) = 9

We know that, LCM × HCF = Product of two numbers

∴ LCM × HCF =306 × 657

$$LCM = \frac{306 \times 657}{HCF} = \frac{306 \times 657}{9}$$

LCM = 22338

5. If any number ends with the digit 0, it should be divisible by 10.

In other words, it will also be divisible by 2 and 5 as 10 = 2 × 5

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

6. Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

7 × 11 × 13 + 13 = 13 × (7 × 11 + 1)
= 13 × (77 + 1) = 13 × 78 = 13 ×13 × 6

The given expression has 6 and 13 as its factors.

Therefore, it is a composite number.

7 × 6 × 5 × 4 × 3 × 2 × 1 + 5
= 5 × (7 × 6 × 4 × 3 × 2 × 1 + 1)
= 5 × (1008 + 1) = 5 ×1009

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors.

Hence, it is a composite number.

7. It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

18 = 2 × 3 × 3 And, 12 = 2 × 2 × 3

LCM of 12 and 18 = 2 × 2 × 3 × 3 = 36

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

CLASS – X Mathematics

Chapter-01 Real Numbers (Exercise 1.3)

Answers:

1. Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

$\Rightarrow \quad b\sqrt{5} = a$

Squaring both sides, we get

$\Rightarrow \quad 5b^2 = a^2$ ... **(1)**

It means that 5 is factor of a^2

Hence, **5 is also factor of *a*** by Theorem. ... **(2)**

If, **5 is factor of *a***, it means that we can write ***a* = 5*c*** for some integer ***c***.

Substituting value of ***a*** in **(1)**,

$5b^2 = 25c^2 \quad \Rightarrow \quad b^2 = 5c^2$

It means that 5 is factor of b^2.

Hence, **5 is also factor of *b*** by Theorem. ... **(3)**

From **(2)** and **(3)**, we can say that **5** is factor of both ***a* and *b***.

But, ***a* and *b*** are **co-prime**.

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

2. We will prove this by contradiction.

Let us suppose that $(3+2\sqrt{5})$ is rational.

It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that

$$\frac{a}{b} = 3 + 2\sqrt{5} \quad \Rightarrow \quad \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\Rightarrow \quad \frac{a-3b}{b} = 2\sqrt{5} \quad \Rightarrow \quad \frac{a-3b}{2b} = \sqrt{5} \qquad \text{... } \mathbf{(1)}$$

a* and *b are integers.

It means **L.H.S** of **(1)** is rational but we know that $\sqrt{5}$ is irrational. It is not possible.

Therefore, our supposition is wrong. $(3+2\sqrt{5})$ cannot be rational.

Hence, $(3+2\sqrt{5})$ is irrational.

3. **(i)** We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = ab$$

$$\Rightarrow \quad \sqrt{2} = \frac{b}{a} \quad \ldots \textbf{(1)}$$

R.H.S of **(1)** is rational but we know that $\sqrt{2}$ is irrational.
It is not possible which means our supposition is wrong.
Therefore, $\frac{1}{\sqrt{2}}$ cannot be rational.
Hence, it is irrational.

(ii) We can prove $7\sqrt{5}$ irrational by contradiction.
Let us suppose that $7\sqrt{5}$ is rational.
It means we have some co-prime integers a and b ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{5} = \frac{a}{7b} \quad \ldots \textbf{(1)}$$

R.H.S of **(1)** is rational but we know that $\sqrt{5}$ is irrational.
It is not possible which means our supposition is wrong.
Therefore, $7\sqrt{5}$ cannot be rational.
Hence, it is irrational.

(iii) We will prove $6+\sqrt{2}$ irrational by contradiction.
Let us suppose that $(6+\sqrt{2})$ is rational.
It means that we have co-prime integers ***a* and *b*** ($b \neq 0$) such that

$$6+\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{a}{b} - 6$$

$$\Rightarrow \quad \sqrt{2} = \frac{a-6b}{b} \quad \ldots \textbf{(1)}$$

a* and *b are integers.
It means **L.H.S** of **(1)** is rational but we know that $\sqrt{2}$ is irrational. It is not possible.
Therefore, our supposition is wrong. $(6+\sqrt{2})$ cannot be rational.
Hence, $(6+\sqrt{2})$ is irrational.

CLASS – X Mathematics

Chapter-01 Real Numbers (Exercise 1.4)

Answers:

1. According to Theorem, any given rational number of the form $\frac{p}{q}$ **where *p* and *q* are co-prime**, has a terminating decimal expansion if q is of the form $2^n \times 5^m$, where m and n are non-negative integers.

(i) $\frac{13}{3125}$

$q = 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

Here, denominator is of the form $2^n \times 5^m$, where m = 5 and n = 0.

It means rational number $\frac{13}{3125}$ has a **terminating** decimal expansion.

(ii) $\frac{17}{8}$

$q = 8 = 2 \times 2 \times 2 = 2^3$

Here, denominator is of the form $2^n \times 5^m$, where m = 0 and n = 3.

It means rational number $\frac{17}{8}$ has a **terminating** decimal expansion.

(iii) $\frac{64}{455}$

$q = 455 = 5 \times 91$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{64}{455}$ has a **non-terminating repeating** decimal expansion.

(iv) $\frac{15}{1600} = \frac{3}{320}$

$q = 320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^6 \times 5$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 6.

It means rational number $\frac{15}{1600}$ has a **terminating** decimal expansion.

(v) $\frac{29}{343}$

$q = 343 = 7 \times 7 \times 7$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{29}{343}$ has **non-terminating repeating** decimal expansion.

(vi) $\frac{23}{2^3 \times 5^2}$

$q = 2^3 \times 5^2$

Here, denominator is of the form $2^n \times 5^m$, where m = 2 and n = 3 are non-negative integers.

It means rational number $\frac{23}{2^3 \times 5^2}$ has **terminating** decimal expansion.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

$q = 2^2 \times 5^7 \times 7^5$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{129}{2^2 \times 5^7 \times 7^5}$ has **non-terminating repeating** decimal expansion.

(viii) $\frac{6}{15} = \frac{2}{5}$

$q = 5 = 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 0.

It means rational number $\frac{6}{15}$ has **terminating** decimal expansion.

(ix) $\frac{35}{50} = \frac{7}{10}$

$q = 10 = 2 \times 5 = 2^1 \times 5^1$

Here, denominator is of the form $2^n \times 5^m$, where m = 1 and n = 1.

It means rational number $\frac{35}{50}$ has **terminating decimal** expansion.

(x) $\frac{77}{210} = \frac{11}{30}$

$q = 30 = 5 \times 3 \times 2$

Here, denominator is not of the form $2^n \times 5^m$, where m and n are non-negative integers.

It means rational number $\frac{77}{210}$ has **non-terminating repeating** decimal expansion.

2. (i) $\frac{13}{3125}=\frac{13}{5^5}=\frac{13\times 2^5}{5^5\times 2^5}=\frac{13\times 2^5}{10^5}\frac{416}{10^5}=0.00416$

(ii) $\frac{17}{8}=\frac{17}{2^3}=\frac{17\times 5^3}{2^3\times 5^3}=\frac{17\times 5^3}{10^3}=\frac{2125}{10^3}=2.215$

(iv) $\frac{15}{1600}=\frac{15}{2^6\times 5^2}=\frac{15\times 5^4}{2^6\times 5^2\times 5^4}=\frac{15\times 5^4}{10^6}=\frac{9375}{10^6}=0.009375$

(vi) $\frac{23}{2^3\times 5^2}=\frac{23\times 5^1}{2^3\times 5^2\times 5^1}=\frac{23\times 5^1}{10^3}=\frac{115}{10^3}=0.115$

(viii) $\frac{6}{15}=\frac{2}{5}=\frac{2\times 2}{5\times 2}=\frac{4}{10}=0.4$

(ix) $\frac{35}{50}=\frac{7}{10}=0.7$

3. (i) 43.123456789
It is rational because decimal expansion is terminating. Therefore, it can be expressed in $\frac{p}{q}$ form where factors of q are of the form $2^n \times 5^m$ where n and m are non-negative integers.

(ii) 0.120120012000120000...
It is irrational because decimal expansion is neither terminating nor non-terminating repeating.

(iii) $43.\overline{123456789}$
It is rational because decimal expansion is non-terminating repeating. Therefore, it can be expressed in $\frac{p}{q}$ form where factors of q **are not** of the form $2^n \times 5^m$ where n and m are non-negative integers.

CLASS – X Mathematics

Chapter-02 Polynomials (Exercise 2.1)

Answers:

1. (i) The graph does not meets x-axis at all. Hence, it does not have any zero.
   (ii) Graph meets x-axis 1 time. It means this polynomial has 1 zero.
   (iii) Graph meets x-axis 3 times. Therefore, it has 3 zeroes.
   (iv) Graph meets x-axis 2 times. Therefore, it has 2 zeroes.
   (v) Graph meets x-axis 4 times. It means it has 4 zeroes.
   (vi) Graph meets x-axis 3 times. It means it has 3 zeroes.

CLASS – X Mathematics

Chapter-02 Polynomials (Exercise 2.2)

Answers:

1. (i) x^2-2x-8

Comparing given polynomial with general form ax^2+bx+c,

We get a = 1, b = -2 and c = -8

We have, x^2-2x-8

$= x^2-4x+2x-8$

$= x(x-4)+2(x-4) = (x-4)(x+2)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$(x-4)(x+2) = 0$

$\Rightarrow$ $x = 4, -2$ are two zeroes.

Sum of zeroes $= 4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes $= 4 \times -2 = -8 = \frac{-8}{1} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1$

Here, a = 4, b = -4 and c = 1

We have, $4s^2-4s+1$

$= 4s^2-2s-2s+1 = 2s(2s-1)-1(2s-1)$

$= (2s-1)(2s-1)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow$ $(2s-1)(2s-1) = 0$

$\Rightarrow s = \frac{1}{2}, \frac{1}{2}$

Therefore, two zeroes of this polynomial are $\frac{1}{2}, \frac{1}{2}$

Sum of zeroes $= \frac{1}{2}+\frac{1}{2} = 1 = \frac{-(-1)}{1}\times\frac{4}{4} = \frac{-(-4)}{4} = \frac{-b}{a} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes $= \frac{1}{2}\times\frac{1}{2} = \frac{1}{4} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iii) $6x^2-3-7x$

Here, a = 6, b = -7 and c = -3

We have, $6x^2-3-7x$

$= 6x^2-7x-3 = 6x^2-9x+2x-3$

$= 3x(2x-3)+1(2x-3) = (2x-3)(3x+1)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow$ $(2x-3)(3x+1) = 0$

$\Rightarrow$ $x = \frac{3}{2}, \frac{-1}{3}$

Therefore, two zeroes of this polynomial are $\frac{3}{2}, \frac{-1}{3}$

Sum of zeroes = $\frac{3}{2}+\frac{-1}{3}=\frac{9-2}{6}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-b}{a}=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes = $\frac{3}{2}\times\frac{-1}{3}=\frac{-1}{2}=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2+8u$

Here, a = 4, b = 8 and c = 0

$4u^2+8u = 4u(u+2)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow$ $4u(u+2) = 0$

$\Rightarrow$ $u = 0, -2$

Therefore, two zeroes of this polynomial are 0, −2

Sum of zeroes = 0−2 = −2 = $\frac{-2}{1}\times\frac{4}{4}=\frac{-8}{4}=\frac{-b}{a}=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes = 0×−2 = 0 = $\frac{0}{4}=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(v) t^2-15

Here, a = 1, b = 0 and c = -15

We have, t^2-15 $\Rightarrow$ $t^2 = 15$ $\Rightarrow$ $t = \pm\sqrt{15}$

Therefore, two zeroes of this polynomial are $\sqrt{15}, -\sqrt{15}$

Sum of zeroes = $\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{0}{1}=\frac{-b}{a}=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes = $\sqrt{15}\times\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2-x-4$

Here, a = 3, b = -1 and c = -4

We have, $3x^2-x-4 = 3x^2-4x+3x-4$

$= x(3x-4)+1(3x-4) = (3x-4)(x+1)$

Equating this equal to 0 will find values of 2 zeroes of this polynomial.

$\Rightarrow$ $(3x-4)(x+1) = 0$

$\Rightarrow$ $x = \frac{4}{3}, -1$

Therefore, two zeroes of this polynomial are $\frac{4}{3}, -1$

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{4-3}{3}=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-b}{a}=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of Zeroes = $\frac{4}{3}\times(-1)=\frac{-4}{3}=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

2. **(i)** $\frac{1}{4}, -1$

Let quadratic polynomial be ax^2+bx+c

Let α and β are two zeroes of above quadratic polynomial.

$\alpha+\beta = \frac{1}{4} = \frac{-b}{a}$

$\alpha \times \beta = -1 = \frac{-1}{1}\times\frac{4}{4} = \frac{-4}{4} = \frac{c}{a}$

$\therefore \quad a=4, b=-1, c=-4$

$\therefore$ Quadratic polynomial which satisfies above conditions = $4x^2-x-4$

(ii) $\sqrt{2}, \frac{1}{3}$

Let quadratic polynomial be ax^2+bx+c

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta = \sqrt{2}\times\frac{3}{3}=\frac{3\sqrt{2}}{3} = \frac{-b}{a}$

$\alpha \times \beta = \frac{1}{3}$ which is equal to $\frac{c}{a}$

$\therefore \quad a=3, b=-3\sqrt{2}, c=1$

$\therefore$ Quadratic polynomial which satisfies above conditions = $3x^2-3\sqrt{2}x+1$

(iii) $0, \sqrt{5}$

Let quadratic polynomial be ax^2+bx+c

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta = 0 = \frac{0}{1} = \frac{-b}{a}$

$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$

$\therefore \quad a=1, b=0, c=\sqrt{5}$

$\therefore$ Quadratic polynomial which satisfies above conditions = $x^2+\sqrt{5}$

(iv) 1, 1

Let quadratic polynomial be ax^2+bx+c

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta = 1 = \frac{-(-1)}{1} = \frac{-b}{a}$

$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\therefore \quad a=1, b=-1, c=1$

$\therefore$ Quadratic polynomial which satisfies above conditions = x^2-x+1

(v) $\frac{-1}{4}, \frac{1}{4}$

Let quadratic polynomial be ax^2+bx+c

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta = \dfrac{-1}{4} = \dfrac{-b}{a}$

$\alpha \times \beta = \dfrac{1}{4} = \dfrac{c}{a}$

$\therefore \quad a = 4, b = 1, c = 1$

$\therefore$ Quadratic polynomial which satisfies above conditions = $4x^2+x+1$

(vi) 4, 1

Let quadratic polynomial be ax^2+bx+c

Let α and β be two zeros of above quadratic polynomial.

$\alpha+\beta = 4 \dfrac{-(-4)}{1} = \dfrac{-b}{a}$

$\alpha \times \beta = 1 = \dfrac{1}{1} = \dfrac{c}{a}$

$\therefore \quad a = 1, b = -4, c = 1$

$\therefore$ Quadratic polynomial which satisfies above conditions = x^2-4x+1

CLASS – X Mathematics

Chapter-02 Polynomials (Exercise 2.3)

Answers:

1. **(i)**

$$
\begin{array}{l}
\qquad\qquad\qquad x-3 \\
x^2-2)\ x^3-3x^2+5x-3 \\
\qquad \pm x^3 \qquad \pm 2x \\
\hline
\qquad\quad -3x^2+7x-3 \\
\qquad\quad \mp 3x^2 \qquad \pm 6 \\
\hline
\qquad\qquad\quad 7x-9
\end{array}
$$

Therefore, quotient = x – 3 and Remainder = 7x – 9

(ii)

$$
\begin{array}{l}
\qquad\qquad\qquad x\ \ +x-3 \\
x^2-x+1)\ x^4-3x^2+4x+5 \\
\qquad \pm x^4 \pm x^2 \qquad \mp x^3 \\
\hline
\qquad\quad -4x^2+4x+5+x^3 \\
\qquad\quad \mp x^2 \pm x \qquad \pm x^3 \\
\hline
\qquad\quad -3x^2+3x+5 \\
\qquad\quad \mp 3x^2 \pm 3x \mp 3 \\
\hline
\qquad\qquad\qquad 8
\end{array}
$$

Therefore, quotient = $x^2 + x - 3$ and, Remainder = 8

(iii)

$$
\begin{array}{l}
\qquad\qquad\qquad -x^2-2 \\
-x^2+2)\ x^4-5x+6 \\
\qquad \pm x^4 \qquad \mp 2x^2 \\
\hline
\qquad\quad -5x+6+2x^2 \\
\qquad\qquad \mp 4 \pm 2x^2 \\
\hline
\qquad\quad -5x+10
\end{array}
$$

Therefore, quotient = $-x^2 - 2$ and, Remainder = $-5x + 10$

2. (i)

$$2t^2+3t+4$$
$$x^2-3)\ 2t^4+3x^3-2t^2-9t-12$$
$$\pm 2t^4 \qquad \mp 6t^2$$
$$+3t^3+4t^2-9t-12$$
$$\pm 3t^3 \qquad \mp 9t$$
$$+4t^2+0-12$$
$$\pm 4t^2 \qquad \mp 12$$
$$0$$

$\because$ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(ii)

$$3x^2-4x+2$$
$$x^2+3x+1)\ 3x^4+5x^3-7x^2+2x+2$$
$$\pm 3x^4 \pm 9x^3 \pm 3x^2$$
$$-4x^3-10x^2+2x+2$$
$$\mp 4x^3 \mp 12x^2 \mp 4x$$
$$+2x^2+6x+2$$
$$\pm 2x^2 \pm 6x \pm 2$$
$$0$$

$\because$ Remainder = 0

Hence first polynomial is a factor of second polynomial.

(iii)

$$x^2-1$$
$$x^3-3x+1)\ x^5-4x^3+x^2+3x+1$$
$$\pm x^5 \mp 3x^3 \mp x^2$$
$$-x^3+3x+1$$
$$\mp x^3+3x \mp 1$$
$$2$$

$\because$ Remainder $\neq 0$

Hence first polynomial is not factor of second polynomial.

3. Two zeroes of ($3x^4+6x^3-2x^2-10x-5$) are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ which means that $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=x^2-\frac{5}{3}$ is a factor of ($3x^4+6x^3-2x^2-10x-5$).

Applying Division Algorithm to find more factors we get:

$$\begin{array}{r|l} & 3x^2+6x+3 \\ \hline x^2-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\ & \pm 3x^4 \quad \pm 5x^2 \\ \hline & +6x^3+3x^2-10x-5 \\ & \pm 6x^3 \quad \mp 10x \\ \hline & +3x^2 \quad -5 \\ & \pm 3x^2 \quad \mp 5 \\ \hline & 0 \end{array}$$

We have p(x) = g(x) × q(x).

$\Rightarrow$ ($3x^4+6x^3-2x^2-10x-5$) = ($x^2-\frac{5}{3}$)($3x^2+6x+3$)

= ($x^2-\frac{5}{3}$)$3(x^2+2x+1)$ = 3($x^2-\frac{5}{3}$)($x^2+x+x+1$)

= 3($x^2-\frac{5}{3}$)($x+1$)($x+1$)

Therefore, other two zeroes of ($3x^4+6x^3-2x^2-10x-5$) are −1 and −1.

4. Let p(x) = x^3-3x^2+x+2, q(x) = (x – 2) and r(x) = (–2x+4)

According to Polynomial Division Algorithm, we have

p(x) = g(x).q(x) + r(x) $\Rightarrow x^3-3x^2+x+2 = g(x).(x-2)-2x+4$

$\Rightarrow x^3-3x^2+x+2+2x-4 = g(x).(x-2)$ $\Rightarrow x^3-3x^2+3x-2 = g(x).(x-2)$

$\Rightarrow g(x) = \frac{x^3-3x^2+3x-2}{x-2}$

So, Dividing (x^3-3x^2+3x-2) by ($x-2$), we get

$$\begin{array}{r|l} & x^2-x+1 \\ \hline x-2 & x^3-3x^2+3x-2 \\ & \pm x^3 \mp 2x^2 \\ \hline & -x^2+3x-2 \\ & \mp x^2 \pm 2x \\ \hline & x-2 \\ & \pm x \mp 2 \\ \hline & 0 \end{array}$$

Therefore, we have $g(x) = \dfrac{x^3-3x^2+3x-2}{x-2} = x^2-x+1$

5. (i) Let p(x) = $3x^2+3x+6$, g(x) = 3

$$\begin{array}{r|l} & x^2+x+2 \\ \hline 3) & 3x^2+3x+6 \\ & \pm 3x^2 \\ \hline & +3x+6 \\ & \pm 3x \\ \hline & +6 \\ & \pm 6 \\ \hline & 0 \end{array}$$

So, we can see in this example that deg p(x) = deg q(x) = 2

(ii) Let p(x) = x^3+5 and g(x) = x^2-1

$$\begin{array}{r|l} & x \\ \hline x^2-1) & x^3+5 \\ & \pm x^3 \quad \mp x \\ \hline & x+5 \end{array}$$

We can see in this example that deg q(x) = deg r(x) = 1

(iii) Let p(x) = x^2+5x-3, g(x) = $x+3$

$$\begin{array}{r|l} & x+2 \\ \hline x+3) & x^2+5x-3 \\ & \pm x^2 \pm 3x \\ \hline & +2x-3 \\ & \pm 2x \pm 6 \\ \hline & -9 \end{array}$$

We can see in this example that deg r(x) = 0

CLASS – X Mathematics

Chapter-02 Polynomials (Exercise 2.4)

Answers:

1. (i) Comparing the given polynomial with ax^3+bx^2+cx+d, we get $a=2, b=1, c=-5$ and $d=2$.

$$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2 = \frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2 = \frac{1+1-10+8}{0} = 0$$

$$p(1)=2(1)^3+(1)^2-5(1)+2 = 2+1-5+2 = 0$$

$$p(-2)=2(-2)^3+(-2)^2-5(-2)+2 = 2(-8)+4+10+2 = -16+16 = 0$$

$\therefore$ $\frac{1}{2}$, 1 and -2 are the zeroes of $2x^3+x^2-5x+2$.

Now, $\alpha+\beta+\gamma = \frac{1}{2}+1+(-2)=\frac{1+2-4}{2}=\frac{-1}{2}=\frac{-b}{a}$

And $\alpha\beta+\beta\gamma+\gamma\alpha = \left(\frac{1}{2}\right)(1)+(1)(-2)+(-2)\left(\frac{1}{2}\right) = \frac{1}{2}-2-1 = \frac{-5}{2}=\frac{c}{a}$

And $\alpha\beta\gamma = \frac{1}{2}\times1\times(-2)=-1=\frac{-2}{2}=\frac{-d}{a}$

(ii) Comparing the given polynomial with ax^3+bx^2+cx+d, we get $a=1, b=-4, c=5$ and $d=-2$.

$$p(2)=2(2)^3-4(2)^2+5(2)-2 = 8-16+10-2 = 0$$

$$p(1)=(1)^3-4(1)^2+5(1)-2 = 1-4+5-2 = 0$$

$\therefore$ 2,1 and 1 are the zeroes of x^3-4x^2+5x-2.

Now, $\alpha+\beta+\gamma = 2+1+1=4=\frac{-(-4)}{1}=\frac{-b}{a}$

And $\alpha\beta+\beta\gamma+\gamma\alpha = (2)(1)+(1)(1)+(1)(2) = 2+1+2 = \frac{5}{1}=\frac{c}{a}$

And $\alpha\beta\gamma = 2\times1\times1=2=\frac{-(-2)}{1}=\frac{-d}{a}$

2. Let the cubic polynomial be ax^3+bx^2+cx+d and its zeroes be α, β and γ.

Then $\alpha+\beta+\gamma = 2 = \frac{-(-2)}{1}=\frac{-b}{a}$ and $\alpha\beta+\beta\gamma+\gamma\alpha = -7=\frac{-7}{1}=\frac{c}{a}$

And $\alpha\beta\gamma = -14=\frac{-14}{1}=\frac{d}{a}$

Here, $a=1, b=-2, c=-7$ and $d=14$

Hence, cubic polynomial will be $x^3-2x^2-7x+14$.

3. Since $(a-b), a, (a+b)$ are the zeroes of the polynomial x^3-3x^2+3x+1.

$\therefore \quad \alpha+\beta+\gamma = a-b+b+a+b = \dfrac{-(-3)}{1} = 3$

$\Rightarrow \quad 3a = 3 \quad \Rightarrow \quad a = 1$

And $\quad \alpha\beta+\beta\gamma+\gamma\alpha = (a-b)a + a(a+b) + (a+b)(a-b) = \dfrac{1}{1} = 1$

$\Rightarrow \quad a^2 - ab + a^2 + ab + a^2 - b^2 = 1$

$\Rightarrow \quad 3a^2 - b^2 = 1 \quad \Rightarrow \quad 3(1)^2 - b^2 = 1 \qquad [\because a = 1]$

$\Rightarrow \quad 3 - b^2 = 1 \quad \Rightarrow \quad b = \pm 2$

Hence $a = 1$ and $b = \pm 2$.

4. Since $2 \pm \sqrt{3}$ are two zeroes of the polynomial $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$.

Let $x = 2 \pm \sqrt{3} \quad \Rightarrow \quad x - 2 = \pm\sqrt{3}$

Squaring both sides, $\quad x^2 - 4x + 4 = 3 \quad \Rightarrow \quad x^2 - 4x + 1 = 0$

Now we divide $p(x)$ by $x^2 - 4x + 1$ to obtain other zeroes.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & \pm x^4 \mp 4x^3 \pm x^2 \\
\hline
 & -2x^3 - 27x^2 + 138x \\
 & \mp 2x^3 \pm 8x^2 \mp 2x \\
\hline
 & -35x^2 + 140x - 35 \\
 & \mp 35x^2 \pm 140x \mp 35 \\
\hline
 & 0
\end{array}
$$

$\therefore \quad p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35) = (x^2 - 4x + 1)(x^2 - 7x + 5x - 35)$

$= (x^2 - 4x + 1)[x(x-7) + 5(x-7)] = (x^2 - 4x + 1)(x+5)(x-7)$

$\Rightarrow \quad (x+5)$ and $(x-7)$ are the other factors of $p(x)$.

$\therefore \quad -5$ and 7 are other zeroes of the given polynomial.

5. Let us divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$.

$$x^2 - 4x + (8-k)$$

$$x^2 - 2x + k\,\big)\; x^4 - 6x^3 + 16x^2 - 25x + 10$$

$$\pm x^4 \mp 2x^3 \pm kx^2$$

$$-4x^3 + (16-k)x^2 - 25x + 10$$

$$\mp 4x^3 \pm 8x^2 \mp 4kx$$

$$(8-k)x^2 + (5k-25)x + 10$$

$$\mp (8-k)x^2 \mp 2(8-k)x \pm (8-k)k$$

$$(2k-9)x - (8-k)k + 10$$

$\therefore$ Remainder = $(2k-9)x - (8-k)k + 10$

On comparing this remainder with given remainder, i.e. $x + a$,

$2k - 9 = 1 \quad \Rightarrow \quad 2k = 10 \quad \Rightarrow \quad k = 5$

And $\quad -(8-k)k + 10 = a \quad \Rightarrow \quad a = -(8-5)5 + 10 = -5$

CLASS – X Mathematics

Chapter-02 Pair of Linear Equations in two Variables (Exercise 3.1)

Answers:

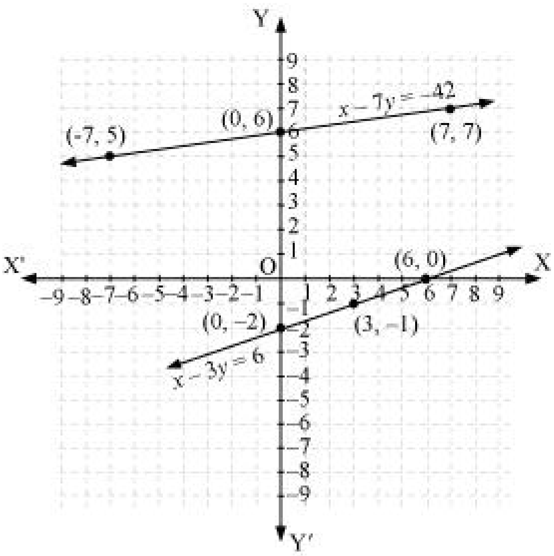
1. Let the present age of Aftab and his daughter be x and y respectively.
Seven years ago, Age of Aftab = x - 7 and Age of his daughter = y - 7
According to the given condition,
$(x-7)=7(y-7)\quad\Rightarrow\quad x-7=7y-49\quad\Rightarrow\quad x-7y=-42$
Again, Three years hence, Age of Aftab = x + 3 and Age of his daughter = y + 3
According to the given condition,
$(x+3)=3(y+3)\quad\Rightarrow\quad x+3=3y+9\quad\Rightarrow\quad x-3y=6$
Thus, the given conditions can be algebraically represented as:
x - 7y = –42 $\Rightarrow$ x = –42 + 7y
Three solutions of this equation can be written in a table as follows:

x	–7	0	7
y	5	6	7

And x – 3y = 6 $\Rightarrow$ x = 6 + 3y
Three solutions of this equation can be written in a table as follows:

x	6	3	0
y	0	–1	–2

The graphical representation is as follows:



Concept insight: In order to represent the algebraic equations graphically the solution set of equations must be taken as whole numbers only for the accuracy. Graph of the two linear equations will be represented by a straight line.

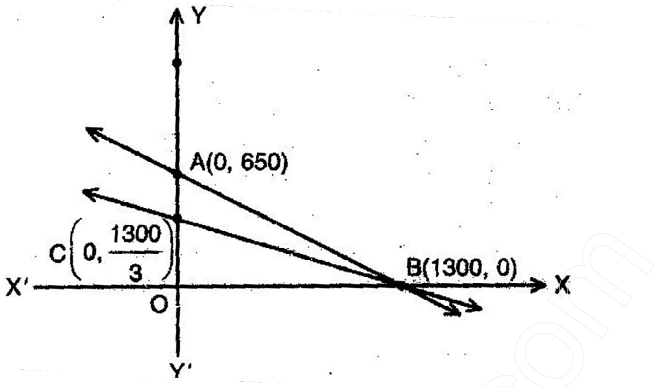
2. Let cost of 1 cricket bat = Rs x and let cost of 1 cricket ball= Rs y
According to given conditions, we have
$3x + 6y = 3900 \quad\Rightarrow\quad x + 2y = 1300$ ... **(1)**
And x + 3y = 1300 ... **(2)**
To represent them graphically, we will find 3 sets of points which lie on the lines.
For equation x + 2y = 1300, we have following points which lie on the line

x	0	1300
y	650	0

For equation x + 3y = 1300, we have following points which lie on the line

x	0	1300
y	$\frac{1300}{3}$	0

We plot the points for both of the equations and it is the graphical representation of the given situation.



It is clear that these lines intersect at B (1300, 0).

3. Let cost of 1 kg of apples = Rs x and let cost of 1 kg of grapes= Rs y

According to given conditions, we have

$2x + y = 160$ ... **(1)**

$4x + 2y = 300$ $\Rightarrow$ $2x + y = 150$ ... **(2)**

So, we have equations **(1)** and **(2)**, $2x + y = 160$ and $2x + y = 150$ which represent given situation algebraically.

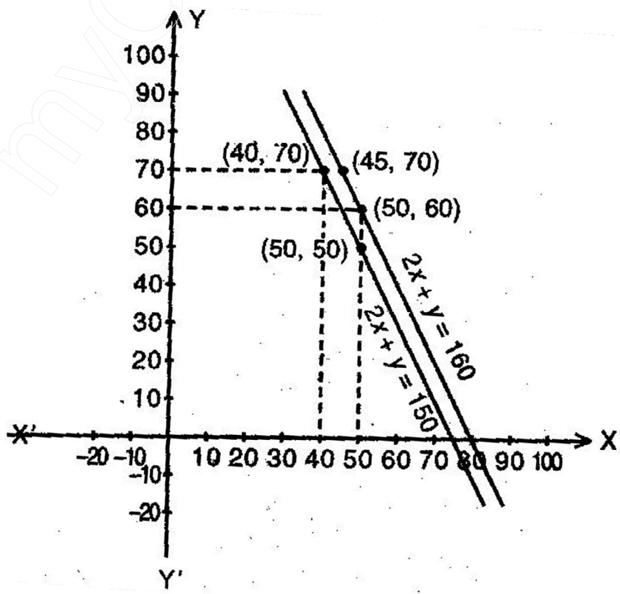
For equation 2x + y = 160, we have following points which lie on the line

x	50	45
y	60	70

For equation 2x + y = 150, we have following points which lie on the line

x	50	40
y	50	70

We plot the points for both of the equations and it is the graphical representation of the given situation.



CLASS – X Mathematics

Chapter-02 Pair of Linear Equations in two Variables (Exercise 3.2)

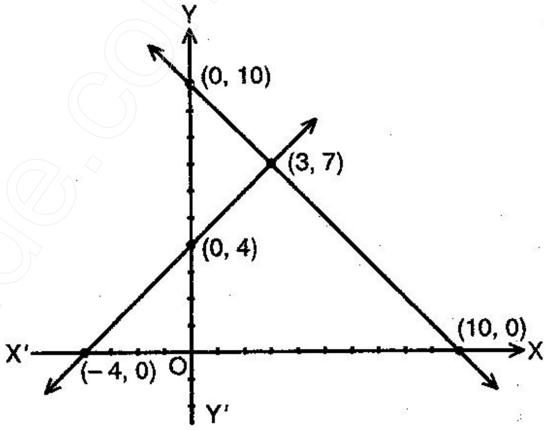
Answers:

1. (i) Let number of boys who took part in the quiz = x
Let number of girls who took part in the quiz = y
According to given conditions, we have
$x + y = 10$ ... **(1)**
And, $y = x + 4$ ⇒ $x - y = -4$ ... **(2)**
For equation x + y = 10, we have following points which lie on the line

x	0	10
y	10	0

For equation x – y = –4, we have following points which lie on the line

x	0	– 4
y	4	0



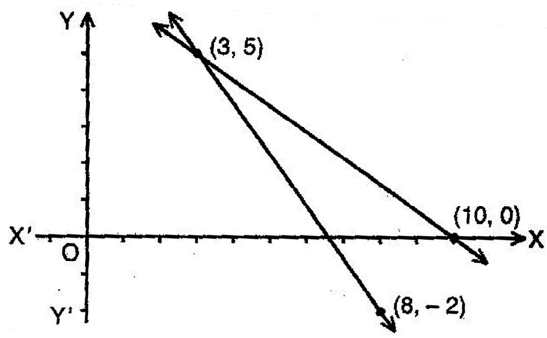
We plot the points for both of the equations to find the solution.
We can clearly see that the intersection point of two lines is **(3, 7).**
Therefore, number of boys who took park in the quiz = 3 and, number of girls who took part in the quiz = 7.

(ii) Let cost of one pencil = Rs x and Let cost of one pen = Rs y
According to given conditions, we have
$5x + 7y = 50$ ... **(1)**
$7x + 5y = 46$ ... **(2)**
For equation 5x + 7y = 50, we have following points which lie on the line

x	10	3
y	0	5

For equation 7x + 5y = 46, we have following points which lie on the line

x	8	3
y	–2	5



We can clearly see that the intersection point of two lines is **(3, 5).**
Therefore, cost of pencil = Rs 3 and, cost of pen = Rs 5

2. (i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$
Comparing equation $5x - 4y + 8 = 0$ with $a_1x + b_1y + c_1 = 0$ and $7x + 6y - 9 = 0$ with $a_2x + b_2y + c_2 = 0$,
We get, $a_1 = 5$, $b_1 = -4$, $c_1 = 8$, $a_2 = 7$, $b_2 = 6$, $c_2 = -9$
We have $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ because $\frac{5}{7} \neq \frac{-4}{6}$

Hence, lines have unique solution which means they intersect at one point.

(ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$

Comparing equation $9x + 3y + 12 = 0$ with $a_1x + b_1y + c_1 = 0$ and $18x + 6y + 24 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 9$, $b_1 = 3$, $c_1 = 12$, $a_2 = 18$, $b_2 = 6$, $c_2 = 24$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$

Hence, lines are coincident.

(iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$

Comparing equation $6x - 3y + 10 = 0$ with $a_1x + b_1y + c_1 = 0$ and $2x - y + 9 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 6$, $b_1 = -3$, $c_1 = 10$, $a_2 = 2$, $b_2 = -1$, $c_2 = 9$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{6}{2} = \frac{-3}{-1} \neq \frac{10}{9}$

Hence, lines are parallel to each other.

3. (i) $3x + 2y = 5$, $2x - 3y = 7$

Comparing equation $3x + 2y = 5$ with $a_1x + b_1y + c_1 = 0$ and $2x - 3y = 7$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 3$, $b_1 = 2$, $c_1 = 5$, $a_2 = 2$, $b_2 = -3$, $c_2 = -7$

$\frac{a_1}{a_2} = \frac{3}{2}$ and $\frac{b_1}{b_2} = \frac{2}{3}$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ which means equations have unique solution.

Hence they are consistent.

(ii) $2x - 3y = 8$, $4x - 6y = 9$

Comparing equation $2x - 3y = 8$ with $a_1x + b_1y + c_1 = 0$ and $4x - 6y = 9$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 2$, $b_1 = -3$, $c_1 = -8$, $a_2 = 4$, $b_2 = -6$, $c_2 = -9$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{1}{2} = \frac{1}{2} \neq \frac{-8}{-9}$

Therefore, equations have no solution because they are parallel.

Hence, they are inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$, $9x - 10y = 14$

Comparing equation $\frac{3}{2}x + \frac{5}{3}y = 7$ with $a_1x + b_1y + c_1 = 0$ and $9x - 10y = 14$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = \frac{3}{2}$, $b_1 = \frac{5}{3}$, $c_1 = -14$, $a_2 = 9$, $b_2 = -10$, $c_2 = -14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}$ and $\frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, equations have unique solution.
Hence, they are consistent.

(iv) $5x - 3y = 11, -10x + 6y = -22$

Comparing equation $5x - 3y = 11$ with $a_1x + b_1y + c_1 = 0$ and $-10x + 6y = -22$ with $a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 5, b_1 = -3, c_1 = -11, a_2 = -10, b_2 = 6, c_2 = 22$

$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$ and $\frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the lines have infinite many solutions.
Hence, they are consistent.

4. (i) $x + y = 5, 2x + 2y = 10$

For equation x + y – 5 = 0, we have following points which lie on the line

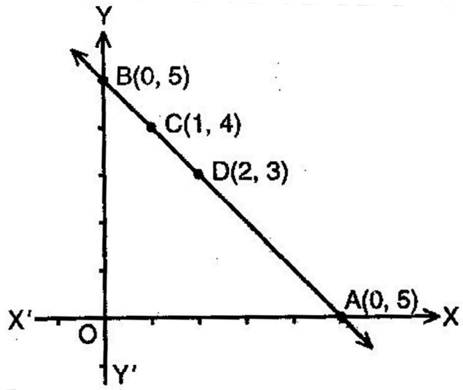
x	0	5
y	5	0

For equation 2x + 2y – 10 = 0, we have following points which lie on the line

x	1	2
y	4	3

We can see that both of the lines coincide. Hence, there are infinite many solutions. Any point which lies on one line also lies on the other. Hence, by using equation ($x + y - 5 = 0$), we can say that $x = 5 - y$

We can assume any random values for y and can find the corresponding value of x using the above equation. All such points will lie on both lines and there will be infinite number of such points.



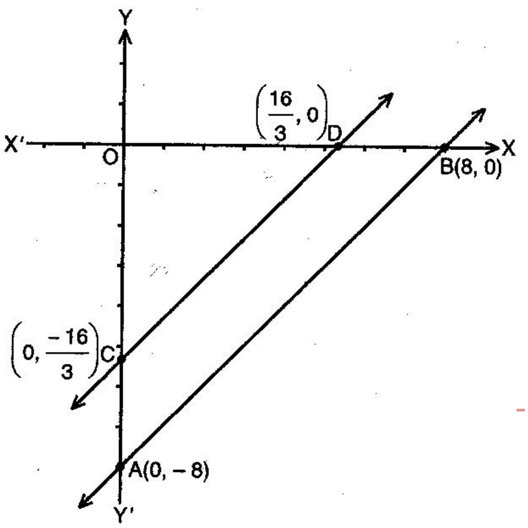
(ii) $x - y = 8, 3x - 3y = 16$

For x – y = 8, the coordinates are:

x	0	8
y	– 8	0

And for 3x – 3y = 16, the coordinates

x	0	$\frac{16}{3}$
y	$\frac{-16}{3}$	0



Plotting these points on the graph, it is clear that both lines are parallel. So the two lines have no common point. Hence the given equations have no solution and lines are inconsistent.

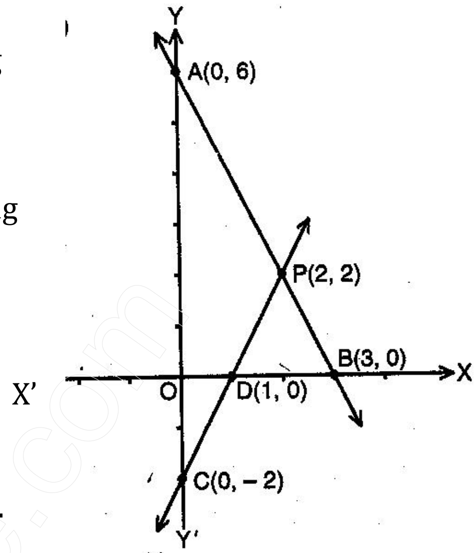
(iii) $2x + y = 6$, $4x - 2y = 4$

For equation $2x + y - 6 = 0$, we have following points which lie on the line

x	0	3
y	6	0

For equation $4x - 2y - 4 = 0$, we have following points which lie on the line

x	0	1
y	–2	0



We can clearly see that lines are intersecting at (2, 2) which is the solution.
Hence x = 2 and y = 2 and lines are consistent.

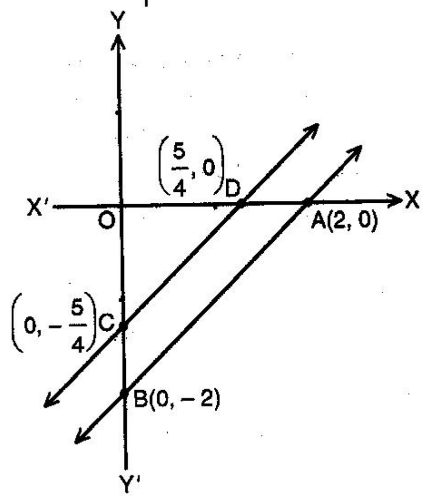
(iv) $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

For $2x - 2y - 2 = 0$, the coordinates are:

x	2	0
y	0	–2

And for $4x - 4y - 5 = 0$, the coordinates

x	0	$\frac{5}{4}$
y	$\frac{-5}{4}$	0



Plotting these points on the graph, it is clear that both lines are parallel. So the two lines have no common point. Hence the given equations have no solution and lines are inconsistent.

5. Let length of rectangular garden = x metres
Let width of rectangular garden = y metres
According to given conditions, perimeter = 36 m $\Rightarrow$ $x + y = 36$ ......(i)
And $x = y + 4$ $\Rightarrow$ $x - y = 4$ ....(ii)
Adding eq. (i) and (ii),
$2x = 40$ $\Rightarrow$ $x = 20$ m
Subtracting eq. (ii) from eq. (i),
$2y = 32$ $\Rightarrow$ $y = 16$ m
Hence, length = 20 m and width = 16 m

6. (i) Let the second line be equal to $a_2x + b_2y + c_2 = 0$
Comparing given line **$2x + 3y - 8 = 0$** with $a_1x + b_1y + c_1 = 0$,
We get $a_1 = 2$, $b_1 = 3$ and $c_1 = -8$

Two lines intersect with each other if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, second equation can be **x + 2y = 3** because $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) Let the second line be equal to $a_2x + b_2y + c_2 = 0$
Comparing given line **2x + 3y – 8 = 0 w**ith $a_1x + b_1y + c_1 = 0$,
We get $a_1 = 2$, $b_1 = 3$ and $c_1 = -8$

Two lines are parallel to each other if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, second equation can be **2x + 3y – 2 = 0** because $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) Let the second line be equal to $a_2x + b_2y + c_2 = 0$
Comparing given line **2x + 3y – 8 = 0 w**ith $a_1x + b_1y + c_1 = 0$,
We get $a_1 = 2$, $b_1 = 3$ and $c_1 = -8$

Two lines are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

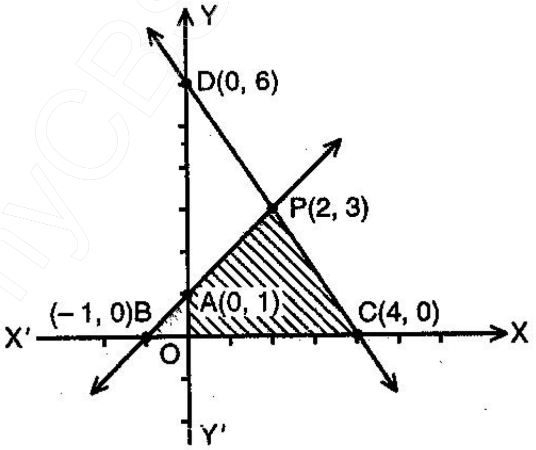
So, second equation can be **4x + 6y – 16 = 0** because $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

7. For equation x – y + 1 = 0, we have following points which lie on the line

x	0	–1
y	1	0

For equation 3x + 2y – 12 = 0, we have following points which lie on the line

x	4	0
y	0	6



We can see from the graphs that points of intersection of the lines with the x–axis are (–1, 0), (2, 3) and (4, 0).

CLASS – X Mathematics

Chapter-02 Pair of Linear Equations in two Variables (Exercise 3.3)

Answers:

1. (i) $x + y = 14$ ... (1)

$x - y = 4$ ... (2)

$x = 4 + y$ from equation (2)

Putting this in equation (1), we get

$4 + y + y = 14 \Rightarrow 2y = 10 \Rightarrow y = 5$

Putting value of y in equation (1), we get

$x + 5 = 14 \Rightarrow x = 14 - 5 = 9$

Therefore, $x = 9$ and $y = 5$

(ii) $s - t = 3$ ... (1)

$\frac{s}{3} + \frac{t}{2} = 6$ ... (2)

Using equation (1), we can say that $s = 3 + t$

Putting this in equation (2), we get

$\frac{3+t}{3} + \frac{t}{2} = 6 \Rightarrow \frac{6+2t+3}{6} = 6 \Rightarrow 5t + 6 = 36$

$\Rightarrow 5t = 30 \Rightarrow t = 6$

Putting value of t in equation (1), we get

$s - 6 = 3 \Rightarrow s = 3 + 6 = 9$

Therefore, $t = 6$ and $s = 9$

(iii) $3x - y = 3$ ... (1)

$9x - 3y = 9$ ... (2)

Comparing equation $3x - y = 3$ with $a_1x + b_1y + c_1 = 0$ and equation $9x - 3y = 9$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3, b_1 = -1, c_1 = -3, a_2 = 9, b_2 = -3$ and $c_2 = -9$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, we have infinite many solutions for x and y

(iv) $0.2x + 0.3y = 1.3$ ... (1)

$0.4x + 0.5y = 2.3$ ... (2)

Using equation (1), we can say that

$0.2x = 1.3 - 0.3y \Rightarrow x = \frac{1.3-0.3y}{0.2}$

Putting this in equation (2), we get

$0.4\left(\frac{1.3-0.3y}{0.2}\right) + 0.5y = 2.3 \Rightarrow 2.6 - 0.6y + 0.5y = 2.3$

$\Rightarrow -0.1y = -0.3 \Rightarrow y = 3$

Putting value of y in (1), we get

$0.2x + 0.3(3) = 1.3 \Rightarrow 0.2x + 0.9 = 1.3$

$\Rightarrow 0.2x = 0.4 \Rightarrow x = 2$

Therefore, $x = 2$ and $y = 3$

(v) $\sqrt{2}x+\sqrt{3}y=0$ ....(1)

$\sqrt{3}x-\sqrt{8}y=0$ ....(2)

Using equation (1), we can say that

$x=\frac{-\sqrt{3}y}{\sqrt{2}}$

Putting this in equation (2), we get

$\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$ $\Rightarrow$ $\frac{-3y}{\sqrt{2}}-\sqrt{8}y=0$

$\Rightarrow$ $y\left(\frac{-3}{\sqrt{2}}-\sqrt{8}\right)=0$ $\Rightarrow$ $y=0$

Putting value of y in (1), we get $x = 0$

Therefore, $x = 0$ and $y = 0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$ ... (1)

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$ ... (2)

Using equation (2), we can say that

$x=\left(\frac{13}{6}-\frac{y}{2}\right)\times 3$ $\Rightarrow$ $x=\frac{13}{2}-\frac{3y}{2}$

Putting this in equation (1), we get

$\frac{3}{2}\left(\frac{13}{2}-\frac{3y}{2}\right)-\frac{5y}{3}=\frac{-2}{1}$ $\Rightarrow$ $\frac{39}{4}-\frac{9y}{4}-\frac{5y}{3}=-2$

$\Rightarrow$ $\frac{-27y-20y}{12}=-2-\frac{39}{4}$ $\Rightarrow$ $\frac{-47y}{12}=\frac{-8-39}{4}$

$\Rightarrow$ $\frac{-47y}{12}=\frac{-47}{4}$ $\Rightarrow$ $y=3$

Putting value of y in equation (2), we get

$\frac{x}{3}+\frac{3}{2}=\frac{13}{6}$ $\Rightarrow$ $\frac{x}{3}=\frac{13}{6}-\frac{3}{2}=\frac{13-9}{6}=\frac{4}{6}=\frac{2}{3}$

$\Rightarrow$ $\frac{x}{3}=\frac{2}{3}$ $\Rightarrow$ $x = 2$

Therefore, $x = 2$ and $y = 3$

2. $2x + 3y = 11$ ... (1)

$2x - 4y = -24$ ... (2)

Using equation (2), we can say that

$2x = -24 + 4y$ $\Rightarrow$ $x = -12 + 2y$

Putting this in equation (1), we get

$2(-12 + 2y) + 3y = 11$ $\Rightarrow$ $-24 + 4y + 3y = 11$

$\Rightarrow$ $7y = 35$ $\Rightarrow$ $y = 5$

Putting value of y in equation (1), we get

$2x + 3(5) = 11 \Rightarrow 2x + 15 = 11$

$\Rightarrow 2x = 11 - 15 = -4 \Rightarrow x = -2$

Therefore, $x = -2$ and $y = 5$

Putting values of x and y in $y = mx + 3$, we get

$5 = m(-2) + 3 \Rightarrow 5 = -2m + 3$

$\Rightarrow -2m = 2 \Rightarrow m = -1$

3. (i) Let first number be x and second number be y.

According to given conditions, we have

$x - y = 26$ (assuming x > y) ... (1)

$x = 3y$ (x > y) ... (2)

Putting equation (2) in (1), we get

$3y - y = 26 \Rightarrow 2y = 26 \Rightarrow y = 13$

Putting value of y in equation (2), we get

$x = 3y = 3 \times 13 = 39$

Therefore, two numbers are 13 and 39.

(ii) Let smaller angle $= x$ and let larger angle $= y$

According to given conditions, we have

$y = x + 18$ ... (1)

Also, $x + y = 180^0$ (Sum of supplementary angles) ... (2)

Putting (1) in equation (2), we get

$x + x + 18 = 180 \Rightarrow 2x = 180 - 18 = 162 \Rightarrow x = 81^0$

Putting value of x in equation (1), we get

$y = x + 18 = 81 + 18 = 99^0$

Therefore, two angles are 81^0 and 99^0.

(iii) Let cost of each bat = Rs x and let cost of each ball = Rs y

According to given conditions, we have

$7x + 6y = 3800$ ... (1)

And, $3x + 5y = 1750$ ... (2)

Using equation (1), we can say that

$7x = 3800 - 6y \Rightarrow x = \frac{3800-6y}{7}$

Putting this in equation (2), we get

$3\left(\frac{3800-6y}{7}\right) + 5y = 1750 \Rightarrow \left(\frac{11400-18y}{7}\right) + 5y = 1750$

$\Rightarrow \frac{5y}{1} - \frac{18y}{7} = \frac{1750}{1} - \frac{11400}{7} \Rightarrow \frac{35y-18y}{7} = \frac{12250-11400}{7}$

$\Rightarrow 17y = 850 \Rightarrow y = 50$

Putting value of y in (2), we get

$3x + 250 = 1750$

$\Rightarrow 3x = 1500 \Rightarrow x = 500$

Therefore, cost of each bat = Rs 500 and cost of each ball = Rs 50

(iv) Let fixed charge = Rs x and let charge for every km = Rs y

According to given conditions, we have

$x + 10y = 105$ ... (1)

$x + 15y = 155$ ... (2)

Using equation (1), we can say that

$x = 105 - 10y$

Putting this in equation (2), we get

$105 - 10y + 15y = 155 \quad \Rightarrow \quad 5y = 50 \quad \Rightarrow \quad y = 10$

Putting value of y in equation (1), we get

$x + 10\,(10) = 105 \quad \Rightarrow \quad x = 105 - 100 = 5$

Therefore, fixed charge = Rs 5 and charge per km = Rs 10

To travel distance of 25 Km, person will have to pay = Rs (x + 25y) = Rs (5 + 25 × 10) = Rs (5 + 250) = Rs 255

(v) Let numerator = x and let denominator = y

According to given conditions, we have

$$\frac{x+2}{y+2} = \frac{9}{11} \quad \dots (1)$$

$$\frac{x+3}{y+3} = \frac{5}{6} \quad \dots (2)$$

Using equation (1), we can say that

$11\,(x + 2) = 9y + 18 \quad \Rightarrow \quad 11x + 22 = 9y + 18$

$\Rightarrow \quad 11x = 9y - 4 \quad \Rightarrow \quad x = \frac{9y-4}{11}$

Putting value of x in equation (2), we get

$6\left(\frac{9y-4}{11}+3\right) = 5\,(y + 3) \quad \Rightarrow \quad \frac{54y}{11}-\frac{24}{11}+18=5y+15$

$\Rightarrow \quad -\frac{24}{11}+\frac{3}{1}=\frac{5y}{1}-\frac{54y}{11} \quad \Rightarrow \quad -\frac{24+33}{11}=\frac{55y-54y}{11}$

$\Rightarrow \quad y = 9$

Putting value of y in (1), we get

$\frac{x+2}{9+2}=\frac{9}{11} \quad \Rightarrow \quad x + 2 = 9 \quad \Rightarrow \quad x = 7$

Therefore, fraction $= \frac{x}{y} = \frac{7}{9}$

(vi) Let present age of Jacob = x years

Let present age of Jacob's son = y years

According to given conditions, we have

$(x + 5) = 3\,(y + 5) \quad \dots (1)$

And, $(x - 5) = 7\,(y - 5) \quad \dots (2)$

From equation (1), we can say that

$x + 5 = 3y + 15 \quad \Rightarrow \quad x = 10 + 3y$

Putting value of x in equation (2) we get

$10 + 3y - 5 = 7y - 35$

$\Rightarrow \quad -4y = -40 \quad \Rightarrow \quad y = 10$ years

Putting value of y in equation (1), we get

$x + 5 = 3\,(10 + 5) = 3 \times 15 = 45$

$\Rightarrow \quad x = 45 - 5 = 40$ years

Therefore, present age of Jacob = 40 years and, present age of Jacob's son = 10 years

CLASS – X Mathematics

Chapter-02 Pair of Linear Equations in two Variables (Exercise 3.4)

Answers:

1. (i) $x + y = 5$ ... (1)

$2x - 3y = 4$ ... (2)

Elimination method:

Multiplying equation (1) by 2, we get equation (3)

$2x + 2y = 10$ ... (3)

$2x - 3y = 4$ ... (2)

Subtracting equation (2) from (3), we get

$5y = 6 \Rightarrow y = \frac{6}{5}$

Putting value of y in (1), we get

$x + \frac{6}{5} = 5 \Rightarrow x = 5 - \frac{6}{5} = \frac{19}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

Substitution method:

$x + y = 5$ ... (1)

$2x - 3y = 4$ ... (2)

From equation (1), we get,

$x = 5 - y$

Putting this in equation (2), we get

$2(5 - y) - 3y = 4 \Rightarrow 10 - 2y - 3y = 4$

$\Rightarrow 5y = 6 \Rightarrow y = \frac{6}{5}$

Putting value of y in (1), we get

$x = 5 - \frac{6}{5} = \frac{19}{5}$

Therefore, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

(ii) $3x + 4y = 10$ ... (1)

$2x - 2y = 2$ ... (2)

Elimination method:

Multiplying equation (2) by 2, we get (3)

$4x - 4y = 4$ ... (3)

$3x + 4y = 10$ ... (1)

Adding (3) and (1), we get

$7x = 14 \Rightarrow x = 2$

Putting value of x in (1), we get

$3(2) + 4y = 10 \Rightarrow 4y = 10 - 6 = 4 \Rightarrow y = 1$

Therefore, $x = 2$ and $y = 1$

Substitution method:

$3x + 4y = 10$ ... (1)
$2x - 2y = 2$ ... (2)

From equation (2), we get

$2x = 2 + 2y \Rightarrow x = 1 + y$ ... (3)

Putting this in equation (1), we get

$3(1 + y) + 4y = 10 \Rightarrow 3 + 3y + 4y = 10$

$\Rightarrow 7y = 7 \Rightarrow y = 1$

Putting value of y in (3), we get $x = 1 + 1 = 2$

Therefore, $x = 2$ and $y = 1$

(iii) $3x - 5y - 4 = 0$ ... (1)
$9x = 2y + 7$ ... (2)

<u>Elimination method:</u>

Multiplying (1) by 3, we get (3)

$9x - 15y - 12 = 0$ ... (3)
$9x - 2y - 7 = 0$ ... (2)

Subtracting (2) from (3), we get

$-13y - 5 = 0 \Rightarrow -13y = 5 \Rightarrow y = \frac{-5}{13}$

Putting value of y in (1), we get

$3x - 5\left(\frac{-5}{13}\right) - 4 = 0 \Rightarrow 3x = 4 - \frac{25}{13} = \frac{52-25}{13} = \frac{27}{13}$

$\Rightarrow x = \frac{27}{13\times 3} = \frac{9}{13}$

Therefore, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

<u>Substitution Method:</u>

$3x - 5y - 4 = 0$ ... (1)
$9x = 2y + 7$ ... (2)

From equation (1), we can say that

$3x = 4 + 5y \Rightarrow x = \frac{4+5y}{3}$

Putting this in equation (2), we get

$9\left(\frac{4+5y}{3}\right) - 2y = 7 \Rightarrow 12 + 15y - 2y = 7$

$\Rightarrow 13y = -5 \Rightarrow y = \frac{-5}{13}$

Putting value of y in (1), we get

$3x - 5\left(\frac{-5}{13}\right) = 4$

$\Rightarrow 3x = 4 - \frac{25}{13} = \frac{52-25}{13} = \frac{27}{13} \Rightarrow x = \frac{27}{13\times 3} = \frac{9}{13}$

Therefore, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

(iv) $\frac{x}{2}+\frac{2y}{3}=-1$ ... (1)

$x-\frac{y}{3}=3$ ... (2)

Elimination method:

Multiplying equation (2) by 2, we get (3)

$2x-\frac{2}{3}y=6$ ... (3)

$\frac{x}{2}+\frac{2y}{3}=-1$ ... (1)

Adding (3) and (1), we get

$\frac{5}{2}x=5 \Rightarrow x=2$

Putting value of x in (2), we get

$2-\frac{y}{3}=3 \Rightarrow y=-3$

Therefore, $x = 2$ and $y = -3$

Substitution method:

$\frac{x}{2}+\frac{2y}{3}=-1$ ... (1)

$x-\frac{y}{3}=3$ ... (2)

From equation (2), we can say that $x=3+\frac{y}{3}=\frac{9+y}{3}$

Putting this in equation (1), we get

$\frac{9+y}{6}+\frac{2}{3}y=-1 \Rightarrow \frac{9+y+4y}{6}=-1$

$\Rightarrow 5y+9=-6 \Rightarrow 5y=-15 \Rightarrow y=-3$

Putting value of y in (1), we get

$\frac{x}{2}+\frac{2}{3}(-3)=-1 \Rightarrow x=2$

Therefore, $x = 2$ and $y = -3$

2. (i) Let numerator $=x$ and let denominator $=y$

According to given condition, we have

$\frac{x+1}{y-1}=1 \text{ and } \frac{x}{y+1}=\frac{1}{2}$

$\Rightarrow x+1=y-1$ and $2x=y+1$

$\Rightarrow x-y=-2$ ... **(1)** and $2x-y=1$ ... **(2)**

So, we have equations **(1)** and **(2)**, multiplying equation **(1)** by 2 we get **(3)**

$2x-2y=-4$ ... **(3)**

$2x-y=1$ ... **(2)**

Subtracting equation **(2)** from **(3)**, we get

$-y=-5 \Rightarrow y=5$

Putting value of y in **(1)**, we get

$x - 5 = -2 \quad \Rightarrow \quad x = -2 + 5 = 3$

Therefore, fraction = $\frac{x}{y} = \frac{3}{5}$

(ii) Let present age of Nuri = x years and let present age of Sonu = y years

5 years ago, age of Nuri = (x – 5) years

5 years ago, age of Sonu = (y – 5) years

According to given condition, we have

$(x - 5) = 3(y - 5) \quad \Rightarrow \quad x - 5 = 3y - 15 \quad \Rightarrow \quad x - 3y = -10$ ... **(1)**

10 years later from present, age of Nuri = $(x + 10)$ years

10 years later from present, age of Sonu = $(y + 10)$ years

According to given condition, we have

$(x + 10) = 2(y + 10) \quad \Rightarrow \quad x + 10 = 2y + 20 \quad \Rightarrow \quad x - 2y = 10$ ... **(2)**

Subtracting equation **(1)** from **(2)**, we get

$y = 10 - (-10) = 20$ years

Putting value of **y** in **(1)**, we get

$x - 3(20) = -10 \quad \Rightarrow \quad x - 60 = -10 \quad \Rightarrow \quad x = 50$ years

Therefore, present age of Nuri = 50 years and present age of Sonu = 20 years

(iii) Let digit at ten's place = x and Let digit at one's place = y

According to given condition, we have

$x + y = 9$ ... **(1)**

And $9(10x + y) = 2(10y + x) \quad \Rightarrow \quad 90x + 9y = 20y + 2x$

$\Rightarrow \quad 88x = 11y \quad \Rightarrow \quad 8x = y \quad \Rightarrow \quad 8x - y = 0$ ... **(2)**

Adding **(1)** and **(2)**, we get

$9x = 9 \quad \Rightarrow \quad x = 1$

Putting value of **x** in **(1)**, we get

$1 + y = 9 \quad \Rightarrow \quad y = 9 - 1 = 8$

Therefore, number = $10x + y = 10(1) + 8 = 10 + 8 = 18$

(iv) Let number of Rs 100 notes = x and let number of Rs 50 notes = y

According to given conditions, we have

$x + y = 25$ ... **(1)**

and $100x + 50y = 2000 \quad \Rightarrow \quad 2x + y = 40$ ... **(2)**

Subtracting **(2)** from **(1)**, we get

$-x = -15 \quad \Rightarrow \quad x = 15$

Putting value of **x** in **(1)**, we get

$15 + y = 25 \quad \Rightarrow \quad y = 25 - 15 = 10$

Therefore, number of Rs 100 notes = 15 and number of Rs 50 notes = 10

(v) Let fixed charge for 3 days = Rs x

Let additional charge for each day thereafter = Rs y

According to given condition, we have

$x + 4y = 27$ ... **(1)**

$x + 2y = 21$ ... **(2)**

Subtracting **(2)** from **(1)**, we get

$2y = 6 \quad \Rightarrow \quad y = 3$

Putting value of **y** in **(1)**, we get

$x + 4(3) = 27 \quad \Rightarrow \quad x = 27 - 12 = 15$

Therefore, fixed charge for 3 days = Rs 15 and additional charge for each day after 3 days = Rs 3

CLASS – X Mathematics

Chapter-02 Pair of Linear Equations in two Variables (Exercise 3.5)

Answers:

1. **(i)** $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Comparing equation $x - 3y - 3 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 9y - 2 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -3, a_2 = 3, b_2 = -9, c_2 = -2$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ this means that the two lines are parallel.

Therefore, there is no solution for the given equations i.e. it is inconsistent.

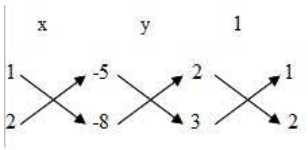
(ii) $2x + y = 5$

$3x + 2y = 8$

Comparing equation $2x + y = 5$ with $a_1x + b_1y + c_1 = 0$ and $3x + 2y = 8$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 2, b_1 = 1, c_1 = -5, a_2 = 3, b_2 = 2, c_2 = -8$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that there is unique solution for the given equations.



$$\frac{x}{(-8)(1)-(2)(-5)} = \frac{y}{(-5)(3)-(-8)(2)} = \frac{1}{(2)2-(3)1}$$

$$\Rightarrow \quad \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3} \qquad \Rightarrow \quad \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow \quad x = 2$ and $y = 1$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

Comparing equation $3x - 5y = 20$ with $a_1x + b_1y + c_1 = 0$ and $6x - 10y = 40$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3, b_1 = -5, c_1 = -20, a_2 = 6, b_2 = -10, c_2 = -40$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

It means lines coincide with each other.

Hence, there are infinite many solutions.

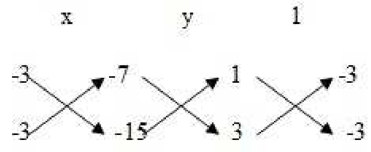
(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

Comparing equation $x - 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $3x - 3y - 15 = 0$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 1, b_1 = -3, c_1 = -7, a_2 = 3, b_2 = -3, c_2 = -15$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ this means that we have unique solution for these equations.



$$\frac{x}{(-3)(-15)-(-3)(-7)} = \frac{y}{(-7)(3)-(-15)(1)} = \frac{1}{(-3)1-(-3)3}$$

$\Rightarrow \quad \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9} \quad \Rightarrow \quad \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$

$\Rightarrow \quad x = 4$ and $y = -1$

2. (i) Comparing equation $2x + 3y - 7 = 0$ with $a_1x + b_1y + c_1 = 0$ and $(a - b)x + (a + b)y - 3a - b + 2 = 0$ with $a_2x + b_2y + c_2 = 0$

We get $a_1 = 2$, $b_1 = 3$ and $c_1 = -7$, $a_2 = (a - b)$, $b_2 = (a + b)$ and $c_2 = 2 - b - 3a$

Linear equations have infinite many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{2-b-3a} \quad \Rightarrow \quad \frac{2}{a-b} = \frac{3}{a+b}$ *and* $\frac{3}{a+b} = \frac{-7}{2-b-3a}$

$\Rightarrow \quad 2a + 2b = 3a - 3b$ and $6 - 3b - 9a = -7a - 7b$

$\Rightarrow \quad a = 5b$ ... (1) and $-2a = -4b - 6$ ... (2)

Putting (1) in (2), we get

$-2(5b) = -4b - 6 \quad \Rightarrow \quad -10b + 4b = -6$

$\Rightarrow \quad -6b = -6 \quad \Rightarrow \quad b = 1$

Putting value of b in (1), we get

$a = 5b = 5(1) = 5$

Therefore, $a = 5$ and $b = 1$

(ii) Comparing $(3x + y - 1 = 0)$ with $a_1x + b_1y + c_1 = 0$ and $(2k - 1)x + (k - 1)y - 2k - 1 = 0)$ with $a_2x + b_2y + c_2 = 0$,

We get $a_1 = 3$, $b_1 = 1$ and $c_1 = -1$, $a_2 = (2k - 1)$, $b_2 = (k - 1)$ and $c_2 = -2k - 1$

Linear equations have no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-2k-1} \quad \Rightarrow \quad \frac{3}{2k-1} = \frac{1}{k-1}$

$\Rightarrow \quad 3(k - 1) = 2k - 1 \quad \Rightarrow \quad 3k - 3 = 2k - 1$

$\Rightarrow \quad k = 2$

3. Substitution Method

$8x + 5y = 9$ ... **(1)**

$3x + 2y = 4$ ... **(2)**

From equation **(1)**,

$5y = 9 - 8x \quad \Rightarrow \quad y = \frac{9-8x}{5}$

Putting this in equation **(2)**, we get

$3x + 2\left(\frac{9-8x}{5}\right) = 4 \quad \Rightarrow \quad 3x + \frac{18-16x}{5} = 4$

$\Rightarrow \quad 3x - \frac{16}{5}x = \frac{4}{1} - \frac{18}{5} \quad \Rightarrow \quad 15x - 16x = 20 - 18 \quad \Rightarrow \quad x = -2$

Putting value of **x** in **(1)**, we get

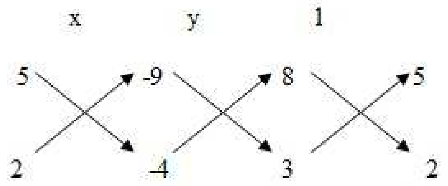
$8(-2) + 5y = 9 \quad \Rightarrow \quad 5y = 9 + 16 = 25 \quad \Rightarrow \quad y = 5$

Therefore, $x = -2$ and $y = 5$

Cross multiplication method

$8x + 5y = 9$ ... **(1)**

$3x + 2y = 4$ ... **(2)**



$$\frac{x}{5(-4)-2(-9)} = \frac{y}{(-9)3-(-4)8} = \frac{1}{8\times2-5\times3}$$

$\Rightarrow \quad \frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15} \quad \Rightarrow \quad \frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$

$\Rightarrow \quad x = -2$ and $y = 5$

4. (i) Let fixed monthly charge = Rs x and let charge of food for one day = Rs y

According to given conditions,

$x + 20y = 1000$ ... (1),

and $x + 26y = 1180$ ... (2)

Subtracting equation (1) from equation (2), we get

$6y = 180 \quad \Rightarrow \quad y = 30$

Putting value of y in (1), we get

$x + 20(30) = 1000 \quad \Rightarrow \quad x = 1000 - 600 = 400$

Therefore, fixed monthly charges = Rs 400 and, charges of food for one day = Rs 30

(ii) Let numerator = x and let denominator = y

According to given conditions,

$\frac{x-1}{y} = \frac{1}{3}$ ...(1) $\quad \frac{x}{y+8} = \frac{1}{4}$ ...(2)

$\Rightarrow \quad 3x - 3 = y$ ... (1) $\quad 4x = y + 8$ ... (1)

$\Rightarrow \quad 3x - y = 3$ ... (1) $\quad 4x - y = 8$ ... (2)

Subtracting equation (1) from (2), we get

$4x - y - (3x - y) = 8 - 3 \quad \Rightarrow \quad x = 5$

Putting value of x in (1), we get

$3(5) - y = 3 \quad \Rightarrow \quad 15 - y = 3 \quad \Rightarrow \quad y = 12$

Therefore, numerator = 5 and, denominator = 12

It means fraction = $\frac{x}{y} = \frac{5}{12}$

(iii) Let number of correct answers = x and let number of wrong answers = y

According to given conditions,

$3x - y = 40$ ... (1)

And, $4x - 2y = 50$ ... (2)

From equation (1), $y = 3x - 40$

Putting this in (2), we get

$4x - 2(3x - 40) = 50 \Rightarrow 4x - 6x + 80 = 50$

$\Rightarrow -2x = -30 \Rightarrow x = 15$

Putting value of x in (1), we get

$3(15) - y = 40 \Rightarrow 45 - y = 40 \Rightarrow y = 45 - 40 = 5$

Therefore, number of correct answers = x = 15and number of wrong answers = y = 5

Total questions = $x + y = 15 + 5 = 20$

(iv) Let speed of car which starts from part A = x km/hr

Let speed of car which starts from part B = y km/hr

According to given conditions,

$\frac{100}{x-y} = 5$ (Assuming x > y)

$\Rightarrow 5x - 5y = 100 \Rightarrow x - y = 20$ ... (1)

And, $\frac{100}{x+y} = 1$

$\Rightarrow x + y = 100$ ... (2)

Adding (1) and (2), we get

$2x = 120 \Rightarrow x = 60$ km/hr

Putting value of x in (1), we get

$60 - y = 20 \Rightarrow y = 60 - 20 = 40$ km/hr

Therefore, speed of car starting from point A = 60 km/hr

And, Speed of car starting from point B = 40 km/hr

(v) Let length of rectangle = x units and Let breadth of rectangle = y units

Area =xy square *units.* According to given conditions,

$xy - 9 = (x - 5)(y + 3)$

$\Rightarrow xy - 9 = xy + 3x - 5y - 15 \Rightarrow 3x - 5y = 6$ ... (1)

And, $xy + 67 = (x + 3)(y + 2)$

$\Rightarrow xy + 67 = xy + 2x + 3y + 6 \Rightarrow 2x + 3y = 61$ ... (2)

From equation (1),

$3x = 6 + 5y \Rightarrow x = \frac{6+5y}{3}$

Putting this in (2), we get

$2\left(\frac{6+5y}{3}\right) + 3y = 61 \Rightarrow 12 + 10y + 9y = 183$

$\Rightarrow 19y = 171 \Rightarrow y = 9$ units

Putting value of y in (2), we get

$2x + 3(9) = 61 \Rightarrow 2x = 61 - 27 = 34 \Rightarrow x = 17$ units

Therefore, length = 17 units and, breadth = 9 units

CLASS – X Mathematics

Chapter-02 Pair of Linear Equations in two Variables (Exercise 3.6)

Answers:

1. (i) $\frac{1}{2x}+\frac{1}{3y}=2$ ... (1)

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$ ... (2)

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

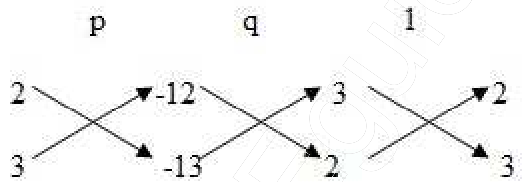
Putting this in equation (1) and (2), we get

$\frac{p}{2}+\frac{q}{3}=2 \text{ and } \frac{p}{3}+\frac{q}{2}=\frac{13}{6}$

$\Rightarrow$ $3p + 2q = 12$ and $6(2p + 3q) = 13(6)$

$\Rightarrow$ $3p + 2q = 12$ and $2p + 3q = 13$

$\Rightarrow$ $3p + 2q - 12 = 0$ ... (3) and $2p + 3q - 13 = 0$ ... (4)



$$\frac{p}{2(-13)-3(-12)}=\frac{q}{(-12)2-(-13)3}=\frac{1}{3\times3-2\times2}$$

$\Rightarrow$ $\frac{p}{-26+36}=\frac{q}{-24+39}=\frac{1}{9-4}$

$\Rightarrow$ $\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$ $\Rightarrow$ $\frac{p}{10}=\frac{1}{5} \text{ and } \frac{q}{15}=\frac{1}{5}$

$\Rightarrow$ $p = 2$ and $q = 3$

But $\frac{1}{x}=p$ and $\frac{1}{y}=q$

Putting value of p and q in this we get

$x=\frac{1}{2}$ and $y=\frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ ... (1)

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$ ... (2)

Let $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$

Putting this in (1) and (2), we get

$2p + 3q = 2$ ... (3)

$4p - 9q = -1$ ... (4)

Multiplying (3) by 2 and subtracting it from (4), we get

$4p - 9q + 1 - 2\,(2p + 3q - 2) = 0$

$\Rightarrow \quad 4p - 9q + 1 - 4p - 6q + 4 = 0$

$\Rightarrow \quad -15q + 5 = 0 \quad \Rightarrow \quad q = \frac{-5}{-15} = \frac{1}{3}$

Putting value of q in (3), we get

$2p + 1 = 2 \quad \Rightarrow \quad 2p = 1 \quad \Rightarrow \quad p = ½$

Putting values of p and q in ($\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$), we get

$\frac{1}{\sqrt{x}} = \frac{1}{2}$ and $\frac{1}{\sqrt{y}} = \frac{1}{3}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{4}$ *and* $\frac{1}{y} = \frac{1}{9} \quad \Rightarrow \quad x = 4$ and $y = 9$

(iii) $\frac{4}{x} + 3y = 14$ ... (1)

$\frac{3}{x} - 4y = 23$ ... (2) and Let $\frac{1}{x} = p$ ... (3)

Putting (3) in (1) and (2), we get

$4p + 3y = 14$ ... (4)

$3p - 4y = 23$ ... (5)

Multiplying (4) by 3 and (5) by 4, we get

$3\,(4p + 3y - 14 = 0)$ and, $4\,(3p - 4y - 23 = 0)$

$\Rightarrow \quad 12p + 9y - 42 = 0$ ... (6) $\quad 12p - 16y - 92 = 0$ ... (7)

Subtracting (7) from (6), we get

$9y - (-16y) - 42 - (-92) = 0$

$\Rightarrow \quad 25y + 50 = 0 \quad \Rightarrow \quad y = 50 - 25 = -2$

Putting value of y in (4), we get

$4p + 3\,(-2) = 14 \quad \Rightarrow \quad 4p - 6 = 14$

$\Rightarrow \quad 4p = 20 \quad \Rightarrow \quad p = 5$

Putting value of p in (3), we get

$\frac{1}{x} = 5 \quad \Rightarrow \quad x = \frac{1}{5}$

Therefore, $x = \frac{1}{5}$ and $y = -2$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$ ... (1)

$\frac{6}{x-1} - \frac{3}{y-2} = 1$ ... (2)

Let $\frac{1}{x-1} = p$ *and* $\frac{1}{y-2} = q$

Putting this in (1) and (2), we get

$5p + q = 2 \Rightarrow 5p + q - 2 = 0$ ... (3)

And, $6p - 3q = 1 \Rightarrow 6p - 3q - 1 = 0$ ... (4)

Multiplying (3) by 3 and adding it to (4), we get

$3(5p + q - 2) + 6p - 3q - 1 = 0$

$\Rightarrow 15p + 3q - 6 + 6p - 3q - 1 = 0$

$\Rightarrow 21p - 7 = 0 \Rightarrow p = \frac{1}{3}$

Putting this in (3), we get

$5\left(\frac{1}{3}\right) + q - 2 = 0 \Rightarrow 5 + 3q = 6$

$\Rightarrow 3q = 6 - 5 = 1 \Rightarrow q = \frac{1}{3}$

Putting values of p and q in ($\frac{1}{x-1} = p$ *and* $\frac{1}{y-2} = q$), we get

$\frac{1}{x-1} = \frac{1}{3}$ *and* $\frac{1}{y-2} = \frac{1}{3}$

$\Rightarrow 3 = x - 1$ and $3 = y - 2 \Rightarrow x = 4$ and $y = 5$

(v) $7x - 2y = 5xy$ ... (1)

$8x + 7y = 15xy$ ... (2)

Dividing both the equations by xy, we get

$\frac{7}{y} - \frac{2}{x} = 5$ ...(3)

$\frac{8}{y} + \frac{7}{x} = 15$ ...(4)

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

Putting these in (3) and (4), we get

$7q - 2p = 5$ ... (5)

$8q + 7p = 15$ ... (6)

From equation (5),

$2p = 7q - 5 \Rightarrow p = \frac{7q-5}{2}$

Putting value of p in (6), we get

$8q + 7\left(\frac{7q-5}{2}\right) = 15 \Rightarrow 16q + 49q - 35 = 30$

$\Rightarrow 65q = 30 + 35 = 65 \Rightarrow q = 1$

Putting value of q in (5), we get

$7(1) - 2p = 5 \Rightarrow 2p = 2 \Rightarrow p = 1$

Putting value of p and q in ($\frac{1}{x} = p$ and $\frac{1}{y} = q$), we get $x = 1$ and $y = 1$

(vi) $6x + 3y - 6xy = 0$ ... (1)

$2x + 4y - 5xy = 0$ ... (2)

Dividing both the equations by xy, we get

$\frac{6}{y}+\frac{3}{x}-6=0$ ...(3)

$\frac{2}{y}+\frac{4}{x}-5=0$ ...(4)

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

Putting these in (3) and (4), we get

$6q + 3p - 6 = 0$ ... (5)

$2q + 4p - 5 = 0$ ... (6)

From (5),

$3p = 6 - 6q \Rightarrow p = 2 - 2q$

Putting this in (6), we get

$2q + 4(2 - 2q) - 5 = 0 \Rightarrow 2q + 8 - 8q - 5 = 0$

$\Rightarrow -6q = -3 \Rightarrow q = \frac{1}{2}$

Putting value of q in (p = 2 – 2q), we get

$p = 2 - 2(\frac{1}{2}) = 2 - 1 = 1$

Putting values of p and q in ($\frac{1}{x}=p$ and $\frac{1}{y}=q$), we get $x = 1$ and $y = 2$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$ ... (1)

$\frac{15}{x+y}-\frac{5}{x-y}=-2$ ... (2)

Let $\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$

Putting this in (1) and (2), we get

$10p + 2q = 4$ ... (3)

$15p - 5q = -2$ ... (4)

From equation (3),

$2q = 4 - 10p \Rightarrow q = 2 - 5p$ ... (5)

Putting this in (4), we get

$15p - 5(2 - 5p) = -2 \Rightarrow 15p - 10 + 25p = -2$

$\Rightarrow 40p = 8 \Rightarrow p = \frac{1}{5}$

Putting value of p in (5), we get

$q = 2 - 5(\frac{1}{5}) = 2 - 1 = 1$

Putting values of p and q in ($\frac{1}{x+y}=p$ and $\frac{1}{x-y}=q$), we get

$\frac{1}{x+y}=\frac{1}{5}$ and $\frac{1}{x-y}=\frac{1}{1}$

$\Rightarrow x + y = 5$ ... (6) and $x - y = 1$ ... (7)

Adding (6) and (7), we get

$2x = 6 \Rightarrow x = 3$

Putting $x = 3$ in (7), we get

$3 - y = 1 \quad \Rightarrow \quad y = 3 - 1 = 2$

Therefore, $x = 3$ and $y = 2$

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$ ... (1)

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$ ... (2)

Let $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$

Putting this in (1) and (2), we get

$p + q = \frac{3}{4}$ and $\frac{p}{2} - \frac{q}{2} = -\frac{1}{8}$

$\Rightarrow \quad 4p + 4q = 3$ ... (3) and $4p - 4q = -1$ ... (4)

Adding (3) and (4), we get

$8p = 2 \quad \Rightarrow \quad p = ¼$

Putting value of p in (3), we get

$4\,(¼) + 4q = 3 \quad \Rightarrow \quad 1 + 4q = 3$

$\Rightarrow \quad 4q = 3 - 1 = 2 \quad \Rightarrow \quad q = ½$

Putting value of p and q in $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$, we get

$\frac{1}{3x+y} = \frac{1}{4}$ and $\frac{1}{3x-y} = \frac{1}{2}$

$\Rightarrow \quad 3x + y = 4$ ... (5) and $3x - y = 2$ ... (6)

Adding (5) and (6), we get

$6x = 6 \quad \Rightarrow \quad x = 1$

Putting $x = 1$ in (5) , we get

$3\,(1) + y = 4 \quad \Rightarrow \quad y = 4 - 3 = 1$

Therefore, $x = 1$ and $y = 1$

2. (i) Let speed of rowing in still water = x km/h

Let speed of current = y km/h

So, speed of rowing downstream = $(x + y)$ km/h

And, speed of rowing upstream = $(x - y)$ km/h

According to given conditions,

$\frac{20}{x+y} = 2$ and $\frac{4}{x-y} = 2$

$\Rightarrow \quad 2x + 2y = 20$ and $2x - 2y = 4$

$\Rightarrow \quad x + y = 10$ ... (1) and $x - y = 2$ ... (2)

Adding (1) and (2), we get

$2x = 12 \quad \Rightarrow \quad x = 6$

Putting $x = 6$ in (1), we get

$6 + y = 10 \quad \Rightarrow \quad y = 10 - 6 = 4$

Therefore, speed of rowing in still water = 6 km/h

Speed of current = 4 km/h

(ii) Let time taken by 1 woman alone to finish the work = x days

Let time taken by 1 man alone to finish the work = y days

So, 1 woman's 1 day work = $(\frac{1}{x})th$ part of the work

And, 1 man's 1 day work = $(\frac{1}{y})th$ part of the work

So, 2 women's 1 day work = $(\frac{2}{x})th$ part of the work

And, 5 men's 1 day work = $(\frac{5}{y})th$ part of the work

Therefore, 2 women and 5 men's 1 day work = $(\frac{2}{x}+\frac{5}{y})th$ part of the work... (1)

It is given that 2 women and 5 men complete work in = 4 days

It means that in 1 day, they will be completing $\frac{1}{4}th$ part of the work ... (2)

Clearly, we can see that (1) = (2)

$\Rightarrow \quad \frac{2}{x}+\frac{5}{y}=\frac{1}{4}$ ... (3)

Similarly, $\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$ ... (4)

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$

Putting this in (3) and (4), we get

$2p + 5q = \frac{1}{4}$ and $3p + 6q = \frac{1}{3}$

$\Rightarrow \quad 8p + 20q = 1$ ... (5) and $9p + 18q = 1$ ... (6)

Multiplying (5) by 9 and (6) by 8, we get

$72p + 180q = 9$ ... (7)

$72p + 144q = 8$ ... (8)

Subtracting (8) from (7), we get

$36q = 1 \quad \Rightarrow \quad q = \frac{1}{36}$

Putting this in (6), we get

$9p + 18(\frac{1}{36}) = 1 \quad \Rightarrow \quad 9p = \frac{1}{2} \quad \Rightarrow \quad p = \frac{1}{18}$

Putting values of p and q in $\frac{1}{x}=p$ and $\frac{1}{y}=q$, we get $x = 18$ and $y = 36$

Therefore, 1 woman completes work in = 18 days

And, 1 man completes work in = 36 days

(iii) Let speed of train = x km/h and let speed of bus = y km/h

According to given conditions,

$\frac{60}{x}+\frac{240}{y}=4 \, and \, \frac{100}{x}+\frac{200}{y}=4+\frac{10}{60}$

Let $\frac{1}{x}=p \, and \, \frac{1}{y}=q$

Putting this in the above equations, we get

$60p + 240q = 4$ ... (1)

And $100p + 200q = \frac{25}{6}$ ... (2)

Multiplying (1) by 5 and (2) by 3, we get

$300p + 1200q = 20$ ... (3)

$300p + 600q = \frac{25}{2}$ ... (4)

Subtracting (4) from (3), we get

$600q = 20 - \frac{25}{2} = 7.5 \Rightarrow q = \frac{7.5}{600}$

Putting value of q in (2), we get

$100p + 200\left(\frac{7.5}{600}\right) = \frac{25}{6} \Rightarrow 100p + 2.5 = \frac{25}{6}$

$\Rightarrow 100p = \frac{25}{6} - 2.5 \Rightarrow p = \frac{10}{600}$

But $\frac{1}{x}=p \, and \, \frac{1}{y}=q$

Therefore, $x = \frac{600}{10} = 60$ km/h and $y = \frac{600}{7.5} = 80$ km/h

Therefore, speed of train = 60 km/h

And, speed of bus = 80 km/h

CLASS – X Mathematics

Chapter-02 Pair of Linear Equations in two Variables (Exercise 3.7)

Answers:

1. Let the age of Ani and Biju be x years and y years respectively.

Age of Dharam = 2x years and Age of Cathy = $\frac{y}{2}$ years

According to question, $x - y = 3$ ... (1)

And $2x - \frac{y}{2} = 30 \Rightarrow 4x - y = 60$ ... (2)

Subtracting (1) from (2), we obtain:

$3x = 60 - 3 = 57 \Rightarrow x =$

Age of Ani = 19 years

Age of Biju = 19 – 3 = 16 years

Again, According to question, $y - x = 3$ ... (3)

And $2x - \frac{y}{2} = 30 \Rightarrow 4x - y = 60$ ... (4)

Adding (3) and (4), we obtain:

$3x = 63 \Rightarrow x = 21$

Age of Ani = 21 years

Age of Biju = 21 + 3 = 24 years

2. Let the money with the first person and second person be Rs x and Rs y respectively.

According to the question,

$x + 100 = 2(y - 100) \Rightarrow x + 100 = 2y - 200 \Rightarrow x - 2y = -300$ ... (1)

Again, $6(x - 10) = (y + 10) \Rightarrow 6x - 60 = y + 10 \Rightarrow 6x - y = 70$ ... (2)

Multiplying equation (2) by 2, we obtain:

$12x - 2y = 140$ ... (3)

Subtracting equation (1) from equation (3), we obtain:

$11x = 140 + 300 \Rightarrow 11x = 440 \Rightarrow x = 40$

Putting the value of x in equation (1), we obtain:

$40 - 2y = -300 \Rightarrow 40 + 300 = 2y \Rightarrow 2y = 340$

$\Rightarrow y = 170$

Thus, the two friends had Rs 40 and Rs 170 with them.

3. Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel be d km.

Since Speed = $\frac{\text{Distance travelled}}{\text{Time taken to travel that distance}} \Rightarrow x = \frac{d}{t} \Rightarrow d = xt$ ... (1)

According to the question

$x + 10 = \frac{d}{t-2} \Rightarrow (x+10)(t-2) = d \Rightarrow xt + 10t - 2x - 20 = d$

$\Rightarrow -2x + 10t = 20$ ......(2) [Using eq. (1)]

Again, $x-10=\frac{d}{t+3}$ $\Rightarrow$ $(x-10)(t+3)=d$ $\Rightarrow$ $xt-10t+3x-30=d$

$\Rightarrow$ $3x-10t=30$ ……(3) [Using eq. (1)]

Adding equations (2) and (3), we obtain:

x = 50

Substituting the value of x in equation (2), we obtain:

(−2) x (50) + 10t = 20 $\Rightarrow$ −100 + 10t = 20

$\Rightarrow$ 10t = 120 t = 12

From equation (1), we obtain:

d = xt = 50 x 12 = 600

Thus, the distance covered by the train is 600 km.

4. Let the number of rows be x and number of students in a row be y.

Total number of students in the class = Number of rows x Number of students in a row = xy

According to the question,

Total number of students = (x – 1) (y + 3)

$\Rightarrow$ xy = (x – 1) (y + 3)

$\Rightarrow$ xy = xy – y + 3x – 3

$\Rightarrow$ 3x – y – 3 = 0

$\Rightarrow$ 3x – y = 3 ... (1)

Total number of students = (x + 2) (y – 3)

$\Rightarrow$ xy = xy + 2y – 3x – 6

$\Rightarrow$ 3x – 2y = −6 ... (2)

Subtracting equation (2) from (1), we obtain:

y = 9

Substituting the value of y in equation (1), we obtain:

3x – 9 = 3

$\Rightarrow$ 3x = 9 + 3 = 12

$\Rightarrow$ x = 4

Number of rows = x = 4

Number of students in a row = y = 9

Hence, Total number of students in a class = xy = 4 x 9 = 36

5. ∠C = 3∠B = 2(∠A + ∠B)

Taking 3∠B = 2(∠A + ∠B)

$\Rightarrow$ ∠B = 2∠A

$\Rightarrow$ 2∠A – ∠B = 0 …….(1)

We know that the sum of the measures of all angles of a triangle is 180°.

∠A + ∠B + ∠C = 180°

$\Rightarrow$ ∠A + ∠B + 3∠B = 180°

$\Rightarrow$ ∠A + 4∠B = 180° …….(2)

Multiplying equation (1) by 4, we obtain:

8∠A – 4∠B = 0 …….(3)

Adding equations (2) and (3), we get

9∠A = 180° $\Rightarrow$ ∠A = 20°

From eq. (2), we get,
20° + 4∠B = 180° ⇒ ∠B = 40°
And ∠C = 3 x 40°= 120°
Hence the measures of ∠A, ∠B and ∠C are 20°, 40° and 120° respectively.

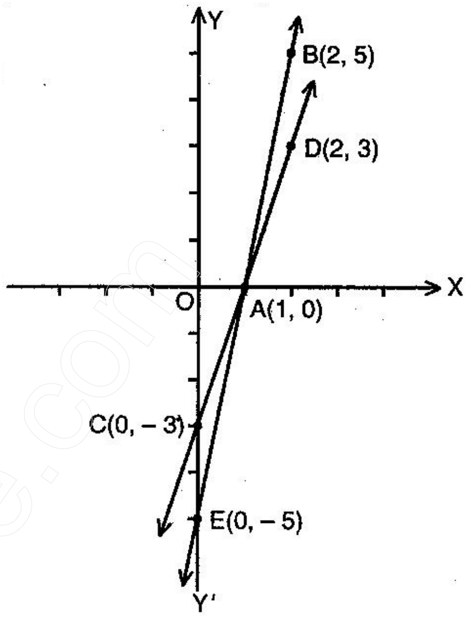
6. $5x - y = 5 \Rightarrow y = 5x - 5$

Three solutions of this equation can be written in a table as follows:

x	0	1	2
y	–5	0	5

$3x - y = 3 \Rightarrow y = 3x - 3$

x	0	1	2
y	–3	0	3



It can be observed that the required triangle is Δ ABC.
The coordinates of its vertices are A (1, 0), B (0, –3), C (0, –5).

7. (i) $px + qy = p - q$ ... (1)

$qx - py = p + q$ ... (2)

Multiplying equation (1) by p and equation (2) by q, we obtain:

$p^2x + pqy = p^2 - pq$ ... (3)

$q^2x - pqy = pq + q^2$ ... (4)

Adding equations (3) and (4), we obtain:

$p^2x + q^2x = p^2 + q^2 \Rightarrow (p^2 + q^2)x = p^2 + q^2$

$\Rightarrow x = \frac{p^2 + q^2}{p^2 + q^2} = 1$

Substituting the value of x in equation (1), we obtain:

$p(1) + qy = p - q \Rightarrow qy = -q \Rightarrow y = -1$

Hence the required solution is x = 1 and y = –1.

(ii) $ax + by = c$ ... (1)

$bx + ay = 1 + c$ ... (2)

Multiplying equation (1) by a and equation (2) by b, we obtain:

$a^2x + aby = ac$ ... (3)

$b^2x + aby = b + bc$ ... (4)

Subtracting equation (4) from equation (3),

$(a^2 - b^2)x = ac - bc - b \Rightarrow x = \frac{c(a - b) - b}{a^2 - b^2}$

Substituting the value of x in equation (1), we obtain:

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\}+by=c \quad\Rightarrow\quad \frac{ac(a-b)-ab}{a^2-b^2}+by=c$$

$$\Rightarrow\quad by=c-\frac{ac(a-b)-ab}{a^2-b^2} \quad\Rightarrow\quad by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$\Rightarrow\quad by=\frac{abc-b^2c+ab}{a^2-b^2} \quad\Rightarrow\quad y=\frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0 \quad\Rightarrow\quad bx-ay=0$ ....(1)

$ax+by=a^2+b^2$ ....(2)

Multiplying equation (1) and (2) by b and a respectively, we obtain:

$b^2x-aby=0$ ....(3)

$a^2x+aby=a^3+ab^2$ ....(4)

Adding equations (3) and (4), we obtain:

$b^2x+a^2x=a^3+ab^2 \quad\Rightarrow\quad x(b^2+a^2)=a(a^2+b^2) \quad\Rightarrow\quad x=a$

Substituting the value of x in equation (1), we obtain:

$b(a)-ay=0 \quad\Rightarrow\quad ab-ay=0 \quad\Rightarrow\quad y=b$

(iv) $(a-b)x+(a+b)y=a^2-2ab-b^2$ ... (1)

$(a+b)(x+y)=a^2+b^2 \quad\Rightarrow\quad (a+b)x+(a+b)y=a^2+b^2$ ....(2)

Subtracting equation (2) from (1), we obtain:

$$(a-b)x-(a+b)x=(a^2-2ab-b^2)-(a^2+b^2)$$

$$\Rightarrow\quad (a-b-a-b)x=-2ab-2b^2 \quad\Rightarrow\quad -2bx=-2b(a+b)$$

$$\Rightarrow\quad x=a+b$$

Substituting the value of x in equation (1), we obtain:

$$(a-b)(a+b)+(a+b)y=a^2-2ab-b^2$$

$$\Rightarrow\quad a^2-b^2+(a+b)y=a^2-2ab-b^2 \quad\Rightarrow\quad (a+b)y=-2ab$$

$$\Rightarrow\quad y=\frac{-2ab}{a+b}$$

(v) $152x-378y=-74$ ... (1)

$-378x+152y=-604$ ... (2)

Adding the equations (1) and (2), we obtain:

$-226x-226y=-678 \quad\Rightarrow\quad x+y=3$ ....(3)

Subtracting the equation (2) from equation (1), we obtain:

$530x-530y=530 \quad\Rightarrow\quad x-y=1$ ....(4)

Adding equations (3) and (4), we obtain:

$2x=4 \quad\Rightarrow\quad x=2$

Substituting the value of x in equation (3), we obtain:

$y=1$

8. We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.

∴ $\angle A + \angle C = 180°$ ⇒ $4y + 20 - 4x = 180°$

⇒ $-4x + 4y = 160°$ ⇒ $x - y = -40°$ ....(1)

Also $\angle B + \angle D = 180°$ ⇒ $3y - 5 - 7x + 5 = 180°$

⇒ $-7x + 3y = 180°$ ....(2)

Multiplying equation (1) by 3, we obtain:

$3x - 3y = -120°$ ....(3)

Adding equations (2) and (3), we obtain:

$-4x = 60°$ ⇒ $x = -15°$

Substituting the value of x in equation (1), we obtain:

$-15 - y = -40°$ ⇒ $y = -15 + 40 = 25$

∴ $\angle A = 4y + 20 = 4 \times 25 + 20 = 120°$

$\angle B = 3y - 5 = 3 \times 25 - 5 = 70°$

$\angle C = -4x = -4 \times (-15) = 60°$

$\angle D = -7x + 5 = -7(-15) + 5 = 110°$

CLASS – X Mathematics

Chapter-04 Quadratic Equations (Exercise 4.1)

Answers:

1. (i) $(x+1)^2 = 2(x-3)$ $\{(a+b)^2 = a^2 + 2ab + b^2\}$

$\Rightarrow x^2 + 1 + 2x = 2x - 6$

$\Rightarrow x^2 + 7 = 0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

(ii) $x^2 - 2x = (-2)(3-x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x - 2x + 6 = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

(iii) $(x-2)(x+1) = (x-1)(x+3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3 = 0$

$\Rightarrow x^2 + x - 2x - 2 - x^2 - 3x + x + 3 = 0$

$\Rightarrow x - 2x - 2 - 3x + x + 3 = 0$

$\Rightarrow -3x + 1 = 0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

(iv) $(x-3)(2x+1) = x(x+5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 + x - 6x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

Here, degree of equation is 2.

Therefore, it is a quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 + x - 5x + 5 = 0$

$\Rightarrow x^2 - 11x + 8 = 0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

(vi) $x^2 + 3x + 1 = (x-2)^2$ $\{(a-b)^2 = a^2 - 2ab + b^2\}$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow 7x - 3 = 0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

(vii) $(x+2)^3 = 2x(x^2-1)$ $\{(a+b)^3 = a^3 + b^3 + 3ab(a+b)\}$

$\Rightarrow x^3 + 2^3 + 3(x)(2)(x+2) = 2x(x^2-1)$

$\Rightarrow x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$\Rightarrow 2x^3 - 2x - x^3 - 8 - 6x^2 - 12x = 0$

$\Rightarrow \quad x^3 - 6x^2 - 14x - 8 = 0$

Here, degree of Equation is 3.

Therefore, it is not a quadratic Equation.

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$ $\quad \{(a-b)^3 = a^3 - b^3 - 3ab\,(a-b)\}$

$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3\,(x)\,(2)\,(x-2)$

$\Rightarrow \quad -4x^2 - x + 1 = -8 - 6x^2 + 12x$

$\Rightarrow \quad 2x^2 - 13x + 9 = 0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

2. (i) We are given that area of a rectangular plot is 528 m^2.

Let breadth of rectangular plot be x metres

Length is one more than twice its breadth.

Therefore length of rectangular plot is $(2x + 1)$ metres

Area of rectangle = length × breadth

$\Rightarrow \quad 528 = x\,(2x+1)$ $\quad \Rightarrow \quad 528 = 2x^2 + x$

$\Rightarrow \quad 2x^2 + x - 528 = 0$

This is a Quadratic Equation.

(ii) Let two consecutive numbers be x and (x + 1).

It is given that $x\,(x+1) = 306$

$\Rightarrow \quad x^2 + x = 306$

$\Rightarrow \quad x^2 + x - 306 = 0$

This is a Quadratic Equation.

(iii) Let present age of Rohan = x years

Let present age of Rohan's mother = (x + 26) years

Age of Rohan after 3 years = (x + 3) years

Age of Rohan's mother after 3 years = x + 26 + 3 = (x + 29) years

According to given condition:

$(x+3)\,(x+29) = 360$

$\Rightarrow \quad x^2 + 29x + 3x + 87 = 360$

$\Rightarrow \quad x^2 + 32x - 273 = 0$

This is a Quadratic Equation.

(iv) Let speed of train be x km/h

Time taken by train to cover 480 km = $480x$ hours

If, speed had been 8km/h less then time taken would be $(480x-8)$ hours

According to given condition, if speed had been 8km/h less then time taken is 3 hours less.

Therefore, $480x - 8 = 480x + 3$

$\Rightarrow \quad 480\,(1x - 8 - 1x) = 3$ $\quad \Rightarrow \quad 480\,(x - x + 8)\,(x)\,(x-8) = 3$

$\Rightarrow \quad 480 \times 8 = 3\,(x)\,(x-8)$ $\quad \Rightarrow \quad 3840 = 3x^2 - 24x$

$\Rightarrow \quad 3x^2 - 24x - 3840 = 0$

Dividing equation by 3, we get

$\Rightarrow \quad x^2 - 8x - 1280 = 0$

This is a Quadratic Equation.

CLASS – X Mathematics

Chapter-04 Quadratic Equations (Exercise 4.2)

Answers:

1. (i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0 \Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2) = 0 \Rightarrow x = 5, -2$

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0 \Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (2x-3)(x+2) = 0 \Rightarrow x = \frac{3}{2}, -2$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0 \Rightarrow \sqrt{2}x^2(x+\sqrt{2}) + 5(x+\sqrt{2}) = 0$

$\Rightarrow (\sqrt{2}x+5)(x+\sqrt{2}) = 0 \Rightarrow x = \frac{-5}{\sqrt{2}}, -\sqrt{2}$

$\Rightarrow x = \frac{-5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}, -\sqrt{2} \Rightarrow x = \frac{-5\sqrt{2}}{2}, -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow \frac{16x^2 - 8x + 1}{8} = 0 \Rightarrow 16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0 \Rightarrow 4x(4x-1) - 1(4x-1) = 0$

$\Rightarrow (4x-1)(4x-1) = 0 \Rightarrow x = \frac{1}{4}, \frac{1}{4}$

(v) $100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0 \Rightarrow 10x(10x-1) - 1(10x-1) = 0$

$\Rightarrow (10x-1)(10x-1) = 0 \Rightarrow x = \frac{1}{10}, \frac{1}{10}$

2. (i) $x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0 \Rightarrow x(x-36) - 9(x-36) = 0$

$\Rightarrow (x-9)(x-36) = 0 \Rightarrow x = 9, 36$

(ii) $x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0 \Rightarrow x(x-25) - 30(x-25) = 0$

$\Rightarrow (x-30)(x-25) = 0 \Rightarrow x = 30, 25$

3. Let first number be x and let second number be $(27 - x)$
According to given condition, the product of two numbers is 182.
Therefore,

$x(27 - x) = 182$

$\Rightarrow 27x - x^2 = 182 \Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 14x - 13x + 182 = 0 \Rightarrow x(x-14) - 13(x-14) = 0$

$\Rightarrow (x-14)(x-13) = 0 \Rightarrow x = 14, 13$

Therefore, the first number is equal to 14 or 13

And, second number is = 27 – x = 27 – 14 = 13 or Second number = 27 – 13 = 14
Therefore two numbers are 13 and 14.

4. Let first number be x and let second number be $(x + 1)$
According to given condition,

$x^2 + (x + 1)^2 = 365$ $\{(a + b)^2 = a^2 + b^2 + 2ab\}$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$ $\Rightarrow 2x^2 + 2x - 364 = 0$

Dividing equation by 2

$\Rightarrow x^2 + x - 182 = 0$ $\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$ $\Rightarrow (x + 14)(x - 13) = 0$

$\Rightarrow x = 13, -14$

Therefore first number = 13 {We discard -14 because it is negative number)
Second number = x + 1 = 13 + 1 = 14
Therefore two consecutive positive integers are 13 and 14 whose sum of squares is equal to 365.

5. Let base of triangle be x cm and let altitude of triangle be $(x - 7)$ cm
It is given that hypotenuse of triangle is 13 cm
According to Pythagoras Theorem,

$13^2 = x^2 + (x - 7)^2$ $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow 169 = x^2 + x^2 + 49 - 14x$ $\Rightarrow 169 = 2x^2 - 14x + 49$

$\Rightarrow 2x^2 - 14x - 120 = 0$

Dividing equation by 2

$\Rightarrow x^2 - 7x - 60 = 0$ $\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x - 12) = 0$ $\Rightarrow (x - 12)(x + 5)$

$\Rightarrow x = -5, 12$

We discard $x = -5$ because length of side of triangle cannot be negative.
Therefore, base of triangle = 12 cm
Altitude of triangle = $(x - 7)$ = 12 – 7 = 5 cm

6. Let cost of production of each article be Rs x
We are given total cost of production on that particular day = Rs 90
Therefore, total number of articles produced that day = 90/x
According to the given conditions,

$$x = 2\left(\frac{90}{x}\right) + 3$$

$\Rightarrow x = \frac{180}{x} + 3$ $\Rightarrow x = \frac{180 + 3x}{x}$

$\Rightarrow x^2 = 180 + 3x$ $\Rightarrow x^2 - 3x - 180 = 0$

$\Rightarrow x^2 - 15x + 12x - 180 = 0$ $\Rightarrow x(x - 15) + 12(x - 15) = 0$

$\Rightarrow (x - 15)(x + 12) = 0$ $\Rightarrow x = 15, -12$

Cost cannot be in negative, therefore, we discard $x = -12$
Therefore, $x = Rs$ 15 which is the cost of production of each article.
Number of articles produced on that particular day = $\frac{90}{15} = 6$

CLASS – X Mathematics

Chapter-04 Quadratic Equations (Exercise 4.3)

Answers:

1. (i) $2x^2 - 7x + 3 = 0$

First we divide equation by 2 to make coefficient of x^2 equal to 1,

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

We divide middle term of the equation by $2x$, we get $\frac{7}{2}x \times \frac{1}{2x} = \frac{7}{4}$

We add and subtract square of $\frac{7}{4}$ from the equation $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$,

$$x^2 - \frac{7}{2}x + \frac{3}{2} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 = 0$$

$$\Rightarrow \quad x^2 + \left(\frac{7}{4}\right)^2 - \frac{7}{2}x + \frac{3}{2} + -\left(\frac{7}{4}\right)^2 = 0 \qquad \{(a-b)^2 = a^2 + b^2 - 2ab\}$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 + \frac{24-49}{16} = 0 \qquad \Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \frac{49-24}{16}$$

Taking Square root on both sides,

$$\Rightarrow \quad x - \frac{7}{4} = \pm\frac{5}{4}$$

$$\Rightarrow \quad x = \frac{5}{4} + \frac{7}{4} = \frac{12}{4} = 3 \; and \; x = -\frac{5}{4} + \frac{7}{4} = \frac{2}{4} = \frac{1}{2}$$

Therefore, $x = \frac{1}{2}, 3$

(ii) $2x^2 + x - 4 = 0$

Dividing equation by 2,

$$x^2 + \frac{x}{2} - 2 = 0$$

Following procedure of completing square,

$$x^2 + \frac{x}{2} - 2 + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = 0$$

$$\Rightarrow \quad x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 - 2 - \frac{1}{16} = 0 \qquad \{(a+b)^2 = a^2 + b^2 + 2ab\}$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 - \frac{33}{16} = 0 \qquad \Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

Taking square root on both sides,

$\Rightarrow \quad x+\frac{1}{4}=\pm\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x=\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{\sqrt{33}-1}{4}$ *and* $x=-\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{-\sqrt{33}-1}{4}$

Therefore, $x=\frac{\sqrt{33}-1}{4},\frac{-\sqrt{33}-1}{4}$

(iii) $4x^2+4\sqrt{3}x+3=0$

Dividing equation by 4,

$x^2+\sqrt{3}x+\frac{3}{4}=0$

Following the procedure of completing square,

$\Rightarrow \quad x^2+\sqrt{3}x+\frac{3}{4}+\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2=0$

$\Rightarrow \quad x^2+\left(\frac{\sqrt{3}}{2}\right)^2+\sqrt{3}x+\frac{3}{4}-\frac{3}{4}=0 \qquad \{(a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow \quad \left(x+\frac{\sqrt{3}}{2}\right)^2=0 \qquad \Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=0$

Taking square root on both sides,

$\Rightarrow \quad x+\frac{\sqrt{3}}{2}=0, x+\frac{\sqrt{3}}{2}=0 \qquad \Rightarrow \quad x=-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$

(iv) $2x^2+x+4=0$

Dividing equation by 2,

$x^2+\frac{x}{2}+2=0$

Following the procedure of completing square,

$\Rightarrow \quad x^2+\frac{x}{2}+2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow \quad x^2+\left(\frac{1}{4}\right)^2+\frac{x}{2}+2-\left(\frac{1}{4}\right)^2=0 \qquad \{(a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow \quad \left(x+\frac{1}{4}\right)^2+2-\frac{1}{16}=0 \qquad \Rightarrow \quad \left(x+\frac{1}{4}\right)^2=\frac{1}{16}-2=\frac{1-32}{16}$

Taking square root on both sides

Right hand side does not exist because square root of negative number does not exist.

Therefore, there is no solution for quadratic equation $2x^2+x+4=0$

2. (i) $2x^2-7x+3=0$

Comparing quadratic equation $2x^2-7x+3=0$ with general form $ax^2+bx+c=0$, we get a = 2, b = -7 and c = 3

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(3)}}{2 \times 2} \quad \Rightarrow \quad x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow \quad x = \frac{7 \pm 5}{4} \quad \Rightarrow \quad x = \frac{7+5}{4}, \frac{7-5}{4}$$

$\Rightarrow \quad x = 3, \frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$

Comparing quadratic equation $2x^2 + x - 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = -4$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-4)}}{2 \times 2} \quad \Rightarrow \quad x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\Rightarrow \quad x = \frac{-1 + \sqrt{33}}{4}, \frac{-1 - \sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Comparing quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 4$, $b = 4\sqrt{3}$ and $c = 3$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2 \times 4} \quad \Rightarrow \quad x = \frac{-4\sqrt{3} \pm \sqrt{0}}{8}$$

$$\Rightarrow \quad x = \frac{-\sqrt{3}}{2}$$

A quadratic equation has two roots. Here, both the roots are equal.

Therefore, $x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

Comparing quadratic equation $2x^2 + x + 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = 4$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(4)}}{2 \times 2} \quad \Rightarrow \quad x = \frac{-1 \pm \sqrt{-31}}{4}$$

But, square root of negative number is not defined.

Therefore, Quadratic Equation $2x^2 + x + 4 = 0$ has no solution.

3. (i) $x - \frac{1}{x} = 3$ where $x \neq 0$

$\Rightarrow \frac{x^2 - 1}{x} = 3 \quad \Rightarrow \quad x^2 - 1 = 3x$

$\Rightarrow x^2 - 3x - 1 = 0$

Comparing equation $x^2 - 3x - 1 = 0$ with general form $ax^2 + bx + c = 0$,
We get $a = 1$, $b = -3$ and $c = -1$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$x = \frac{3 \pm \sqrt{(3)^2 - 4(1)(-1)}}{2 \times 1} \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{13}}{2}$

$\Rightarrow x = \frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ where $x \neq -4, 7$

$\Rightarrow \frac{(x-7)-(x+4)}{(x-4)(x-7)} = \frac{11}{30} \quad \Rightarrow \quad \frac{-11}{(x-4)(x-7)} = \frac{11}{30}$

$\Rightarrow -30 = x^2 - 7x + 4x - 28 \quad \Rightarrow \quad x^2 - 3x + 2 = 0$

Comparing equation $x^2 - 3x + 2 = 0$ with general form $ax^2 + bx + c = 0$,
We get $a = 1$, $b = -3$ and $c = 2$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$x = \frac{3 \pm \sqrt{(3)^2 - 4(1)(2)}}{2 \times 1} \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{1}}{2}$

$\Rightarrow x = \frac{3 + \sqrt{1}}{2}, \frac{3 - \sqrt{1}}{2} \quad \Rightarrow \quad x = 2, 1$

4. Let present age of Rehman = x years
Age of Rehman 3 years ago = $(x - 3)$ years.
Age of Rehman after 5 years = $(x + 5)$ years
According to the given condition:

$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3} \quad \Rightarrow \quad \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow 3(2x + 2) = (x - 3)(x + 5) \quad \Rightarrow \quad 6x + 6 = x^2 - 3x + 5x - 15$

$\Rightarrow x^2 - 4x - 15 - 6 = 0 \quad \Rightarrow \quad x^2 - 4x - 21 = 0$

Comparing quadratic equation $x^2 - 4x - 21 = 0$ with general form $ax^2 + bx + c = 0$,
We get $a = 1$, $b = -4$ and $c = -21$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{4 \pm \sqrt{(4)^2 - 4(1)(-21)}}{2 \times 1} \quad \Rightarrow \quad x = \frac{4 \pm \sqrt{16 + 84}}{2}$

$\Rightarrow \quad x=\frac{4\pm\sqrt{100}}{2}=\frac{4\pm10}{2} \qquad \Rightarrow \quad x=\frac{4+10}{2},\frac{4-10}{2}$

$\Rightarrow \quad x = 7, -3$

We discard $x=-3$. Since age cannot be in negative.
Therefore, present age of Rehman is 7 years.

5. Let Shefali's marks in Mathematics = x
Let Shefali's marks in English = $30 - x$
If, she had got 2 marks more in Mathematics, her marks would be = $x + 2$
If, she had got 3 marks less in English, her marks in English would be = $30 - x - 3 = 27 - x$

According to given condition:

$(x + 2)(27 - x) = 210$

$\Rightarrow \quad 27x - x^2 + 54 - 2x = 210 \qquad \Rightarrow \quad x^2 - 25x + 156 = 0$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,
We get $a = 1$, $b = -25$ and $c = 156$

Applying Quadratic Formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{25\pm\sqrt{(25)^2-4(1)(156)}}{2\times1}$

$\Rightarrow \quad x=\frac{25\pm\sqrt{625-624}}{2} \qquad \Rightarrow \quad x=\frac{25\pm\sqrt{1}}{2}$

$\Rightarrow \quad x=\frac{25+1}{2},\frac{25-1}{2} \qquad \Rightarrow \quad x = 13, 12$

Therefore, Shefali's marks in Mathematics = 13 or 12
Shefali's marks in English = $30 - x = 30 - 13 = 17$
Or Shefali's marks in English = $30 - x = 30 - 12 = 18$
Therefore her marks in Mathematics and English are (13, 17) or (12, 18).

6. Let shorter side of rectangle = x metres
Let diagonal of rectangle = $(x + 60)$ metres
Let longer side of rectangle = $(x + 30)$ metres
According to pythagoras theorem,

$(x + 60)^2 = (x + 30)^2 + x^2 \qquad \Rightarrow \quad x^2 + 3600 + 120x = x^2 + 900 + 60x + x^2$

$\Rightarrow \quad x^2 - 60x - 2700 = 0$

Comparing equation $x^2 - 60x - 2700 = 0$ with standard form $ax^2 + bx + c = 0$,
We get $a = 1$, $b = -60$ and $c = -2700$

Applying quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{60\pm\sqrt{(60)^2-4(1)(-2700)}}{2\times1} \qquad \Rightarrow \quad x=\frac{60\pm\sqrt{3600+10800}}{2}$

$\Rightarrow \quad x=\frac{60\pm\sqrt{14400}}{2}=\frac{60\pm120}{2} \qquad \Rightarrow \quad x=\frac{60+120}{2},\frac{60-120}{2}$

$\Rightarrow \quad x = 90, -30$

We ignore –30. Since length cannot be in negative.
Therefore x = 90 which means length of shorter side = 90 metres
And length of longer side = x + 30 = 90 + 30 = 120 metres
Therefore, length of sides are 90 and 120 in metres.

7. Let smaller number = x and let larger number = y
According to condition:
$y^2 - x^2 = 180$ ... (1)
Also, we are given that square of smaller number is 8 times the larger number.
$\Rightarrow$ $x^2 = 8y$ ... (2)
Putting equation (2) in (1), we get
$y^2 - 8y = 180$ $\Rightarrow$ $y^2 - 8y - 180 = 0$

Comparing equation $y^2 - 8y - 180 = 0$ with general form $ay^2 + by + c = 0$,
We get $a = 1$, $b = -8$ and $c = -180$
Using quadratic formula $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \dfrac{8 \pm \sqrt{(-8)^2 - 4(1)(-180)}}{2 \times 1}$ $\Rightarrow$ $y = \dfrac{8 \pm \sqrt{64 + 720}}{2}$

$\Rightarrow$ $y = \dfrac{8 \pm \sqrt{784}}{2} = \dfrac{8 \pm 28}{2}$ $\Rightarrow$ $y = \dfrac{8+28}{2}, \dfrac{8-28}{2}$

$\Rightarrow$ $y = 18, -10$
Using equation (2) to find smaller number:
$x^2 = 8y$
$\Rightarrow$ $x^2 = 8y = 8 \times 18 = 144$ $\Rightarrow$ $x = \pm 12$
And, $x^2 = 8y = 8 \times -10 = -80$ {No real solution for x}
Therefore two numbers are (12, 18) or (–12, 18)

8. Let the speed of the train = x km/hr
If, speed had been 5 km/hr more, train would have taken 1 hour less.
So, according to this condition

$\dfrac{360}{x} = \dfrac{360}{x+5} + 1$ $\Rightarrow$ $360\left(\dfrac{1}{x} - \dfrac{1}{x+5}\right) = 1$

$\Rightarrow$ $360\left(\dfrac{x+5-x}{x(x+5)}\right) = 1$ $\Rightarrow$ $360 \times 5 = x^2 + 5x$

$\Rightarrow$ $x^2 + 5x - 1800 = 0$
Comparing equation $x^2 + 5x - 1800 = 0$ with general equation $ax^2 + bx + c = 0$,
We get $a = 1$, $b = 5$ and $c = -1800$
Applying quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-5 \pm \sqrt{(5)^2 - 4(1)(-1800)}}{2 \times 1}$ $\Rightarrow$ $x = \dfrac{-5 \pm \sqrt{25 + 7200}}{2}$

$\Rightarrow \quad x = \frac{-5 \pm \sqrt{7225}}{2} = \frac{-5 \pm 85}{2} \qquad \Rightarrow \quad x = \frac{-5+85}{2}, \frac{-5-85}{2}$

$\Rightarrow \quad x = 40, -45$

Since speed of train cannot be in negative. Therefore, we discard $x = -45$

Therefore, speed of train = 40 km/hr

9. Let time taken by tap of smaller diameter to fill the tank = x hours

Let time taken by tap of larger diameter to fill the tank = $(x - 10)$ hours

It means that tap of smaller diameter fills $\frac{1}{x}^{th}$ part of tank in 1 hour. ... (1)

And, tap of larger diameter fills $\frac{1}{x-10}^{th}$ part of tank in 1 hour. ... (2)

When two taps are used together, they fill tank in 758 hours.

In 1 hour, they fill $\frac{8}{75}^{th}$ part of tank $\left(\frac{1}{\frac{75}{8}} = \frac{8}{75}\right)$ ... (3)

From (1), (2) and (3),

$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75} \qquad \Rightarrow \quad \frac{x-10+x}{x(x-10)} = \frac{8}{75}$

$\Rightarrow \quad 75(2x - 10) = 8(x^2 - 10x) \qquad \Rightarrow \quad 150x - 750 = 8x^2 - 80x$

$\Rightarrow \quad 8x^2 - 80x - 150x + 750 = 0 \qquad \Rightarrow \quad 4x^2 - 115x + 375 = 0$

Comparing equation $4x^2 - 115x + 375 = 0$ with general equation $ax^2 + bx + c = 0$,

We get $a = 4$, $b = -115$ and $c = 375$

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{115 \pm \sqrt{(-115)^2 - 4(4)(375)}}{2 \times 4} \qquad \Rightarrow \quad x = \frac{115 \pm \sqrt{13225 - 6000}}{8}$

$\Rightarrow \quad x = \frac{115 \pm \sqrt{7225}}{8} \qquad \Rightarrow \quad x = \frac{115 \pm 85}{8}$

$\Rightarrow \quad x = \frac{115+85}{8}, \frac{115-85}{8} \qquad \Rightarrow \quad x = 25, 3.75$

Time taken by larger tap = $x - 10 = 3.75 - 10 = -6.25$ hours

Time cannot be in negative. Therefore, we ignore this value.

Time taken by larger tap = $x - 10 = 25 - 10 = 15$ hours

Therefore, time taken by larger tap is 15 hours and time taken by smaller tap is 25 hours.

10. Let average speed of passenger train = x km/h

Let average speed of express train = $(x + 11)$ km/h

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ hours

Time taken by express train to cover 132 km = $\left(\frac{132}{x+11}\right)$ hours

According to the given condition,

$\frac{132}{x}=\frac{132}{x+11}+1$ $\Rightarrow$ $132\left(\frac{1}{x}-\frac{1}{x+11}\right)=1$

$\Rightarrow$ $132\left(\frac{x+11-x}{x(x+11)}\right)=1$ $\Rightarrow$ $132(11)=x(x+11)$

$\Rightarrow$ $1452=x^2+11x$ $\Rightarrow$ $x^2+11x-1452=0$

Comparing equation $x^2+11x-1452=0$ with general quadratic equation $ax^2+bx+c=0$, we get $a=1$, $b=11$ and $c=-1452$

Applying Quadratic Formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-11\pm\sqrt{(11)^2-4(1)(-1452)}}{2\times1}$ $\Rightarrow$ $x=\frac{-11\pm\sqrt{121+5808}}{2}$

$\Rightarrow$ $x=\frac{-11\pm\sqrt{5929}}{2}$ $\Rightarrow$ $x=\frac{-11\pm77}{2}$

$\Rightarrow$ $x=\frac{-11+77}{2},\frac{-11-77}{2}$ $\Rightarrow$ $x=33,-44$

As speed cannot be in negative. Therefore, speed of passenger train = 33 km/h
And, speed of express train = $x+11=33+11=44$ km/h

11. Let perimeter of first square = x metres
Let perimeter of second square = $(x+24)$ metres

Length of side of first square = $\frac{x}{4}$ metres {Perimeter of square = 4 × length of side}

Length of side of second square = $\left(\frac{x+24}{4}\right)$ metres

Area of first square = side × side = $\frac{x}{4}\times\frac{x}{4}=\frac{x^2}{16}m^2$

Area of second square = $\left(\frac{x+24}{4}\right)^2m^2$

According to given condition:

$\frac{x^2}{16}+\left(\frac{x+24}{4}\right)^2=468$ $\Rightarrow$ $\frac{x^2}{16}+\frac{x^2+576+48x}{16}=468$

$\Rightarrow$ $\frac{x^2+x^2+576+48x}{16}=468$ $\Rightarrow$ $2x^2+576+48x=468\times16$

$\Rightarrow$ $2x^2+48x+576=7488$ $\Rightarrow$ $2x^2+48x-6912=0$

$\Rightarrow$ $x^2+24x-3456=0$

Comparing equation $x^2+24x-3456=0$ with standard form $ax^2+bx+c=0$,
We get $a=1$, $b=24$ and $c=-3456$

Applying Quadratic Formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-24\pm\sqrt{(24)^2-4(1)(-3456)}}{2\times1} \quad\Rightarrow\quad x=\frac{-24\pm\sqrt{576+13824}}{2}$$

$$\Rightarrow\quad x=\frac{-24\pm\sqrt{14400}}{2}=\frac{-24\pm120}{2} \quad\Rightarrow\quad x=\frac{-24+120}{2},\frac{-24-120}{2}$$

$\Rightarrow$ $x = 48, -72$

Perimeter of square cannot be in negative. Therefore, we discard $x=-72$.

Therefore, perimeter of first square = 48 metres

And, Perimeter of second square = $x + 24 = 48 + 24 = 72$ metres

$\Rightarrow$ Side of First square = $\frac{Perimeter}{4}=\frac{48}{4}=12\,m$

And, Side of second Square = $\frac{Perimeter}{4}=\frac{72}{4}=18\,m$

CLASS – X Mathematics

Chapter-04 Quadratic Equations (Exercise 4.4)

Answers:

1. (i) $2x^2 - 3x + 5 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,
We get $a = 2$, $b = -3$ and $c = 5$
Discriminant = $b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$
Discriminant is less than 0 which means equation has no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,
We get $a = 3$, $b = -4\sqrt{3}$ and $c = 4$
Discriminant = $b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4) = 48 - 48 = 0$
Discriminant is equal to zero which means equations has equal real roots.
Applying quadratic $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots,

$$x = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{2\sqrt{3}}{3}$$

Because, equation has two equal roots, it means $x = \dfrac{2\sqrt{3}}{3}, \dfrac{2\sqrt{3}}{3}$

(iii) $2x^2 - 6x + 3 = 0$

Comparing equation with general equation $ax^2 + bx + c = 0$,
We get $a = 2$, $b = -6$, and $c = 3$
Discriminant = $b^2 - 4ac = (-6)^2 - 4(2)(3) = 36 - 24 = 12$
Value of discriminant is greater than zero.
Therefore, equation has distinct and real roots.
Applying quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots,

$$x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{3}}{2}$$

$$\Rightarrow \quad x = \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2}$$

2. (i) $2x^2 + kx + 3 = 0$

We know that quadratic equation has two equal roots only when the value of discriminant is equal to zero.
Comparing equation $2x^2 + kx + 3 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 2$, $b = k$ and $c = 3$
Discriminant = $b^2 - 4ac = k^2 - 4(2)(3) = k^2 - 24$
Putting discriminant equal to zero

$$k^2 - 24 = 0 \quad \Rightarrow \quad k^2 = 24$$

⇒ $k = \pm\sqrt{24} = \pm 2\sqrt{6}$ ⇒ $k = 2\sqrt{6}, -2\sqrt{6}$

(ii) $kx(x-2)+6=0$

⇒ $kx^2 - 2kx + 6 = 0$

Comparing quadratic equation $kx^2 - 2kx + 6 = 0$ with general form $ax^2 + bx + c = 0$, we get $a = k$, $b = -2k$ and $c = 6$

Discriminant = $b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

We know that two roots of quadratic equation are equal only if discriminant is equal to zero.

Putting discriminant equal to zero

$4k^2 - 24k = 0$

⇒ $4k(k-6) = 0$ ⇒ $k = 0, 6$

The basic definition of quadratic equation says that quadratic equation is the equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Therefore, in equation $kx^2 - 2kx + 6 = 0$, we cannot have k = 0.

Therefore, we discard k = 0.

Hence the answer is k = 6.

3. Let breadth of rectangular mango grove = x metres

Let length of rectangular mango grove = $2x$ metres

Area of rectangle = length × breadth = $x \times 2x = 2x^2\ m^2$

According to given condition:

$2x^2 = 800$

⇒ $2x^2 - 800 = 0$ ⇒ $x^2 - 400 = 0$

Comparing equation $x^2 - 400 = 0$ with general form of quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = 0$ and $c = -400$

Discriminant = $b^2 - 4ac = (0)^2 - 4(1)(-400) = 1600$

Discriminant is greater than 0 means that equation has two disctinct real roots.

Therefore, it is possible to design a rectangular grove.

Applying quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$x = \frac{0 \pm \sqrt{1600}}{2 \times 1} = \frac{\pm 40}{2} = \pm 20$ ⇒ $x = 20, -20$

We discard negative value of x because breadth of rectangle cannot be in negative.

Therefore, x = breadth of rectangle = 20 metres

Length of rectangle = $2x = 2 \times 20 = 40$ metres

4. Let age of first friend = x years and let age of second friend = $(20 - x)$ years

Four years ago, age of first friend = $(x - 4)$ years

Four years ago, age of second friend = $(20 - x) - 4 = (16 - x)$ years

According to given condition,

$(x-4)(16-x) = 48$ ⇒ $16x - x^2 - 64 + 4x = 48$

⇒ $20x - x^2 - 112 = 0$ ⇒ $x^2 - 20x + 112 = 0$

Comparing equation, $x^2 - 20x + 112 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -20$ and $c = 112$

Discriminant = $b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48 < 0$

Discriminant is less than zero which means we have no real roots for this equation.

Therefore, the give situation is not possible.

5. Let length of park = x metres

We are given area of rectangular park = 400 m^2

Therefore, breadth of park = $\frac{400}{x}$ metres {Area of rectangle = length × *breadth*}

Perimeter of rectangular park = 2 (*length* + *breath*) = $2\left(x+\frac{400}{x}\right)$ metres

We are given perimeter of rectangle = 80 metres

According to condition:

$$2\left(x+\frac{400}{x}\right)=80$$

$$\Rightarrow \quad 2\left(\frac{x^2+400}{x}\right)=80$$

$\Rightarrow$ $2x^2 + 800 = 80x$

$\Rightarrow$ $2x^2 - 80x + 800 = 0$

$\Rightarrow$ $x^2 - 40x + 400 = 0$

Comparing equation, $x^2 - 40x + 400 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -40$ and $c = 400$

Discriminant = $b^2 - 4ac = (-40)^2 - 4\,(1)\,(400) = 1600 - 1600 = 0$

Discriminant is equal to 0.

Therefore, two roots of equation are real and equal which means that it is possible to design a rectangular park of perimeter 80 metres and area 400 m^2.

Using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$$x=\frac{40\pm\sqrt{0}}{2}=\frac{40}{2}=20$$

Here, both the roots are equal to 20.

Therefore, length of rectangular park = 20 metres

Breadth of rectangular park = $\frac{400}{x}=\frac{400}{20}=20\,m$

CLASS – X Mathematics

Chapter-05 Arithmetic Progression (Exercise 5.1)

Answers:

1. (i) Taxi fare for 1st km = Rs 15, Taxi fare after 2 km = 15 + 8 = Rs 23
Taxi fare after 3 km = 23 + 8 = Rs 31
Taxi fare after 4 km = 31 + 8 = Rs 39
Therefore, the sequence is 15, 23, 31, 39...
It is an arithmetic progression because difference between any two consecutive terms is equal which is 8. (23 – 15 = 8, 31 – 23 = 8, 39 – 31 = 8, ...)

(ii) Let amount of air initially present in a cylinder = V

Amount of air left after pumping out air by vacuum pump = $V-\frac{V}{4}=\frac{4V-V}{4}=\frac{3V}{4}$

Amount of air left when vacuum pump again pumps out air

$$=\frac{3}{4}V-\left(\frac{1}{4}\times\frac{3}{4}V\right)=\frac{3}{4}V-\frac{3}{16}V=\frac{12V-3V}{16}=\frac{9}{16}V$$

So, the sequence we get is like $V,\frac{3}{4}V,\frac{9}{16}V...$

Checking for difference between consecutive terms ...

$$\frac{3}{4}V-V=-\frac{V}{4},\frac{9}{16}V-\frac{3}{4}V=\frac{9V-12V}{16}=\frac{-3V}{16}$$

Difference between consecutive terms is not equal.
Therefore, it is not an arithmetic progression.

(iii) Cost of digging 1 meter of well = Rs 150
Cost of digging 2 meters of well = 150 + 50 = Rs 200
Cost of digging 3 meters of well = 200 + 50 = Rs 250
Therefore, we get a sequence of the form 150, 200, 250...
It is an arithmetic progression because difference between any two consecutive terms is equal. (200 – 150 = 250 – 200 = 50...)
Here, difference between any two consecutive terms which is also called common difference is equal to 50.

(iv) Amount in bank after Ist year = $10000\left(1+\frac{8}{100}\right)$ ... (1)

Amount in bank after two years = $10000\left(1+\frac{8}{100}\right)^2$ ... (2)

Amount in bank after three years = $10000\left(1+\frac{8}{100}\right)^3$ ... (3)

Amount in bank after four years = $10000\left(1+\frac{8}{100}\right)^4$ ... (4)

It is not an arithmetic progression because (2) – (1) ≠ (3) – (2)
(Difference between consecutive terms is not equal)
Therefore, it is not an Arithmetic Progression.

2. (i) First term = a = 10, d = 10
Second term = a + d = 10 + 10 = 20
Third term = second term + d = 20 + 10 = 30
Fourth term = third term + d = 30 + 10 = 40
Therefore, first four terms are: 10, 20, 30, 40

(ii) First term = a = –2 , d = 0
Second term = a + d = –2 + 0 = –2
Third term = second term + d = –2 + 0 = –2
Fourth term = third term + d = –2 + 0 = –2
Therefore, first four terms are: –2, –2, –2, –2

(iii) First term = a = 4, d =–3
Second term = a + d = 4 – 3 = 1
Third term = second term + d = 1 – 3 = –2
Fourth term = third term + d = –2 – 3 = –5
Therefore, first four terms are: 4, 1, –2, –5

(iv) First term = a = –1, d = ½
Second term = a + d = –1 + ½ = –½
Third term = second term + d = –½ + ½ = 0
Fourth term = third term + d = 0 + ½ = ½
Therefore, first four terms are: –1, –½, 0, ½

(v) First term = a = –1.25, d = –0.25
Second term = a + d = –1.25 – 0.25 = –1.50
Third term = second term + d = –1.50 – 0.25 = –1.75
Fourth term = third term + d = –1.75 – 0.25 = –2.00
Therefore, first four terms are: –1.25, –1.50, –1.75, –2.00

3. (i) 3, 1, –1, –3...
First term = a = 3,
Common difference (d) = Second term – first term = Third term – second term and so on
Therefore, Common difference (d) = 1 – 3 = –2

(ii) –5, –1, 3, 7...
First term = a = –5
Common difference (d) = Second term – First term
= Third term – Second term and so on
Therefore, Common difference (d) = –1 – (–5) = –1 + 5 = 4

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}...$
First term = a = $\frac{1}{3}$
Common difference (d) = Second term – First term
= Third term – Second term and so on
Therefore, Common difference (d) = $\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9...
First term = a = 0.6
Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = 1.7 – 0.6 = 1.1

4. (i) 2, 4, 8, 16...

It is not an AP because difference between consecutive terms is not equal.

As $4 - 2 \neq 8 - 4$

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad \frac{5}{2} - 2 = 3 - \frac{5}{2} = \frac{1}{2}$

Common difference (d) = ½

Fifth term = $\frac{7}{2} + \frac{1}{2} = 4$ Sixth term = 4 + ½ = $\frac{9}{2}$

Seventh term = $\frac{9}{2} + \frac{1}{2} = 5$

Therefore, next three terms are 4, $\frac{9}{2}$ and 5.

(iii) −1.2, −3.2, −5.2, −7.2...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad -3.2 - (-1.2) = -5.2 - (-3.2) = -7.2 - (-5.2) = -2$

Common difference (d) = −2

Fifth term = −7.2 – 2 = −9.2 Sixth term = −9.2 – 2 = −11.2

Seventh term = −11.2 – 2 = −13.2

Therefore, next three terms are −9.2, −11.2 and −13.2

(iv) −10, −6, −2, 2...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad -6 - (-10) = -2 - (-6) = 2 - (-2) = 4$

Common difference (d) = 4

Fifth term = 2 + 4 = 6 Sixth term = 6 + 4 = 10

Seventh term = 10 + 4 = 14

Therefore, next three terms are 6, 10 and 14

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad 3+\sqrt{2}-3=\sqrt{2}, 3+2\sqrt{2}-(3+\sqrt{2})=3+2\sqrt{2}-3-\sqrt{2}=\sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $3+3\sqrt{2}+\sqrt{2}=3+4\sqrt{2}$

Sixth term = $3+4\sqrt{2}+\sqrt{2}=3+5\sqrt{2}$

Seventh term = $3+5\sqrt{2}+\sqrt{2}=3+6\sqrt{2}$

Therefore, next three terms are $(3+4\sqrt{2}), (3+5\sqrt{2}), (3+6\sqrt{2})$

(vi) 0.2, 0.22, 0.222, 0.2222...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \quad 0.22 - 0.2 \neq 0.222 - 0.22$

(vii) 0, −4, −8, −12...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad -4 - 0 = -8 - (-4) = -12 - (-8) = -4$

Common difference (d) = −4

Fifth term = −12 − 4 = −16 Sixth term = −16 − 4 = −20

Seventh term = −20 − 4 = −24

Therefore, next three terms are −16, −20 and −24

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}...$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

Common difference (d) = 0

Fifth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$ Sixth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Seventh term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Therefore, next three terms are $-\frac{1}{2}, -\frac{1}{2}$ *and* $-\frac{1}{2}$

(ix) 1, 3, 9, 27...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \quad 3 - 1 \neq 9 - 3$

(x) $a, 2a, 3a, 4a$...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad 2a - a = 3a - 2a = 4a - 3a = a$

Common difference (d) = a

Fifth term = $4a + a = 5a$ Sixth term = $5a + a = 6a$

Seventh term = $6a + a = 7a$

Therefore, next three terms are $5a$, $6a$ and $7a$

(xi) a, a^2, a^3, a^4...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \quad a^2 - a \neq a^3 - a^2$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}... \quad \Rightarrow \quad \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \quad 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$ Sixth term = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$

Seventh term = $6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$

Therefore, next three terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}...$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \quad \sqrt{6} - \sqrt{3} \neq \sqrt{9} - \sqrt{6}$

(xiv) $1^2, 3^2, 5^2, 7^2$...
It is not an AP because difference between consecutive terms is not equal.
$\Rightarrow \quad 3^2 - 1^2 \neq 5^2 - 3^2$

(xv) $1^2, 5^2, 7^2, 73$... $\Rightarrow$ 1, 25, 49, 73...
It is an AP because difference between consecutive terms is equal.
$\Rightarrow \quad 5^2 - 1^2 = 7^2 - 5^2 = 73 - 7^2 = 24$
Common difference (d) = 24
Fifth term = 73 + 24 = 97 Sixth term = 97 + 24 = 121
Seventh term = 121 + 24 = 145
Therefore, next three terms are 97, 121 and 145