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ORIGINAL ARTICLE

A Paradox Involving Representational States and Activities

Blake Myers

University of Wisconsin-Madison

In this paper, I present a novel paradox that pertains to a variety of representational states and activities. I begin by proving that there are certain contingently true propositions that no one can occurrently believe. Then, I use this to develop a further proof by which I derive a contradiction, thus giving us the paradox. Next, I differentiate the paradox from the Liar Paradox, and I show how a common response to the different variations of the Liar Paradox fails to avoid the type of paradox provided in this paper. Finally, I demonstrate how the general ideas behind the paradox regarding occurrent belief can be extended to a wide range of other representational states and activities.

Keywords paradox; logic; truth; contradiction; representation

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1 Introduction

One strategy that is commonly used in response to certain paradoxes is to argue that at least some key claim used to generate the paradox has no truth value. Consider, for instance, the Liar Paradox; that is, the paradox resulting from the sentence, "This sentence is false." Given the assumption that the sentence is either true or false, along with other principles in logic, some suggest that the sentence is both true and false (e.g., see Priest 1979, 1984, 2006). Roughly, the reason is that if the sentence is false, then it has the truth value that it says it has, and therefore, it is true. On the other hand, if the sentence is true, then it does not have the truth value that it says it has, and therefore, it is false.

In attempt to avoid the conclusion that the sentence is both true and false, one common strategy is to argue that such sentences have no truth value (Kripke 1975; Martin and Woodruff 1975; Quine 1976; Russell 1908; Tarski 1944). In this paper, I generate a paradox that is resistant to this type of response. In Sections 2 and 3, I show that there are certain contingently true propositions that no one can occurrently believe. In Section 4, I use the conclusions from Sections 2 and 3 as lemmas in a further proof to derive a contradiction, thus giving us the paradox. In Section 5, I consider how the aforementioned strategy used in response to the Liar Paradox faces problems when responding to the type of paradox provided in this paper. Finally, in Section 6, I show how the general ideas behind the paradox presented in Sections 2–4 can be extended to a wide range of representational states and activities.

Correspondence to: E-mail: bamyers2@wisc.edu

2 A proposition that no one can truly occurrently believe

To begin, let W be the proposition, <Some narwhal weighs more than 1.5 t>, and let R be the proposition, <No one at any time *truly occurrently believes* a proposition that entails W but is not entailed by W>.

As a point of clarification, I am *not* saying that R is the modal proposition, <It is impossible that someone truly occurrently believes a proposition that entails W but is not entailed by W>. Rather, I take it that for any possible world ν, the proposition R is true at ν if and only if no one ever truly occurrently believes at ν a proposition that entails W but is not entailed by W. Thus, R is true at some worlds and false at others. Moreover, I take it that (W & R) is likewise true at some worlds and false at others. Specifically, it is true at any world at which W is true and at which no one ever truly occurrently believes a proposition that entails W but is not entailed by W. As I now show, however, it is *not* possible that someone truly occurrently believes that (W & R).

Suppose, for the purpose of reductio ad absurdum, that it is possible that someone truly occurrently believes that (W & R). Thus, there is a possible world μ, and a subject S, such that S truly occurrently believes at μ some proposition that entails W but is not entailed by W; for, (W & R) entails W but is not entailed by W. So:

(a) someone truly occurrently believes at μ a proposition that entails W but is not entailed by W.

Furthermore, given our initial assumption, we know that S truly occurrently believes at μ that (W & R). Thus, both of these claims also hold (since S's belief is a *true* belief):

(b) W is true at μ; and

(c) R is true at μ.

However, given (a), it is clearly false at μ that no one at any time truly occurrently believes a proposition that entails W but is not entailed by W, which is to say:

(d) $\sim R$ is true at μ (since R is the proposition, <No one at any time truly occurrently believes a proposition that entails W but is not entailed by W>).

And given (c) and (d), it follows that (R & $\sim R$) is true at μ. So, given our initial assumption that it is possible that someone truly occurrently believes that (W & R), it follows that it is possible that (R & $\sim R$). But since it is not possible that (R & $\sim R$), we know that our initial assumption is false. That is:

Lemma 1 *It is not possible that someone truly occurrently believes that (*W *&* R*).*

3 A second proposition that no one can truly occurrently believe

Now, again let W be the proposition, <Some narwhal weighs more than 1.5 t>. And let N be the proposition, <No one at any time truly occurrently believes a proposition that entails $\sim W$ but is not entailed by $\sim W$>. Then, consider how we can run an argument that is similar to the one provided in Section 2, but where the new argument shows that it is impossible that someone truly occurrently believes that ($\sim W$ & N).

Suppose, for the purpose of reductio ad absurdum, that it is possible that someone truly occurrently believes that ($\sim W$ & N). Given this, it follows that there is a possible world μ', and a subject S, such that S truly occurrently believes at μ' some proposition that entails $\sim W$ but is not entailed by $\sim W$; for, $(\sim W \,\&\, N)$ entails $\sim W$ but is not entailed by $\sim W$. So:

(a′) someone truly occurrently believes at μ' a proposition that entails $\sim W$ but is not entailed by $\sim W$.

Furthermore, given our initial assumption, we know that S truly occurrently believes at μ' that $(\sim W \,\&\, N)$. Thus, both of these claims also hold (since S's belief is a *true* belief):

(b′) $\sim W$ is true at μ'; and

(c′) N is true at μ'.

However, given (a′), it is clearly false at μ' that no one at any time truly occurrently believes a proposition that entails $\sim W$ but is not entailed by $\sim W$, which is to say:

(d′) $\sim N$ is true at μ' (since N is the proposition, <No one at any time truly occurrently believes a proposition that entails $\sim W$ but is not entailed by $\sim W$>).

And given (c′) and (d′), it follows that $(N \,\&\, \sim N)$ is true at μ'. So, given our initial assumption that it is possible that someone truly occurrently believes that $(\sim W \,\&\, N)$, it follows that it is possible that $(N \,\&\, \sim N)$. But since it is not possible that $(N \,\&\, \sim N)$, we know that our initial assumption is false. That is:

Lemma 2 *It is not possible that someone truly occurrently believes that (*$\sim$W & N*).*

4 The paradox

With Lemmas 1 and 2 in mind, we can now provide an argument to derive a contradiction, as follows. Suppose that at some possible world μ^*, only one proposition is ever occurrently believed that entails W but is not entailed by W. Namely, Omar occurrently believes at μ^* that $(W \,\&\, R)$. Similarly, only one proposition is ever occurrently believed at μ^* that entails $\sim W$ but is not entailed by $\sim W$. Namely, Sarah occurrently believes at μ^* that $(\sim W \,\&\, N)$.

First, consider Omar's occurrent belief at μ^* that $(W \,\&\, R)$. We know that R is true at μ^*. For, R is the proposition, <No one at any time truly occurrently believes a proposition that entails W but is not entailed by W>. This is true at μ^*, since the only proposition ever occurrently believed at μ^* that entails W but is not entailed by W is the proposition $(W \,\&\, R)$; and given this, along with Lemma 1, it follows that there is no proposition ever *truly* occurrently believed at μ^* that entails W but is not entailed by W. So, in sum:

(i) Omar occurrently believes at μ^* that $(W \,\&\, R)$;

(ii) Omar does not *truly* occurrently believe at μ^* that $(W \,\&\, R)$; and

(iii) R is true at μ^*.

Given (i) – (iii), it follows that W is false at μ^*. So:

(iv) $\sim W$ is true at μ^*.

Next, consider Sarah's occurrent belief at μ^* that $(\sim W \,\&\, N)$. We know that N is true at μ^*. For, N is the proposition, <No one at any time truly occurrently believes a proposition that entails $\sim W$ but is not entailed by $\sim W$>. This is true at μ^*, since the only proposition ever occurrently believed at μ^* that entails $\sim W$ but is not entailed by $\sim W$ is the proposition ($\sim W$ & N); and given this, along with Lemma 2, it follows that there is no proposition ever *truly* occurrently believed at μ^* that entails $\sim W$ but is not entailed by $\sim W$. So, in sum:

(i′) Sarah occurrently believes at μ^* that ($\sim W$ & N);
(ii′) Sarah does not *truly* occurrently believe at μ^* that ($\sim W$ & N); and
(iii′) N is true at μ^*.
Given (i′) – (iii′), it follows that $\sim W$ is false at μ^*. So:
(iv′) W is true at μ^*.

Finally, given (iv) and (iv′), it follows that (W & $\sim W$) is true at μ^*. So, it is possible that (W & $\sim W$). Moreover, one can replace W with any contingent proposition and run the same type of argument. Therefore, for any contingent proposition p, one can employ the above line of reasoning to derive the conclusion that it is possible that (p & $\sim p$), thereby giving us the paradox.

5 Contrasts with the Liar Paradox

Consider the relationship between the above paradox and the Liar Paradox; that is, the paradox that results from the sentence, "This sentence is false." While there are several variations of the Liar Paradox, there is a common thread that runs through many attempts to solve the different variations of the paradox. Namely, such attempts often aim to show that at least one of the key claims used to generate the paradox has no truth value (Kripke 1975; Martin and Woodruff 1975; Quine 1976; Russell 1908; Tarski 1944).

Unlike the many variations of the Liar Paradox, the above paradox does not rely on the notion of sentences. Rather, it relies on the notions of occurrent belief and propositions. Given this, it seems implausible to maintain that the key feature responsible for generating the above paradox relies on assuming that certain sentences are truth-valued. For, such an assumption is never used in the above paradox. In light of this, if one hopes to avoid the type of paradox provided in this paper, it seems that one should look for other types of features that might be responsible for generating the derived contradiction.

6 Extending the paradox to other types of representational states and activities

Now, consider how the general ideas behind the above paradox regarding occurrent belief can be extended to a wide range of representational states and activities. A proof that is parallel to the one given earlier shows that it is impossible that someone truly *asserts* that (W & A). Moreover, if we let B be the proposition, <No one at any time truly asserts a proposition that entails $\sim W$ but is not entailed by $\sim W$>, then we can show, along similar lines, that it is impossible that someone truly asserts that ($\sim W$ & B). Next, we can consider a possible world ν^* at which only one proposition is ever asserted that entails W but is not entailed by W. Finally, we can argue, in much the way we did before, that both W and $\sim W$ must be true at ν^*, which of course is impossible.

In like manner, we can create further variations of the paradox by using other types of representational states and activities. For instance, we can run a similar argument to the one provided in Section 2–4 by replacing the notion of *occurrently believing that p* with any of the following alternative representational states and activities: *writing a sentence that expresses p*, *uttering a sentence that expresses p*, *hoping that p*, *imagining that p*. Given that the same general form of argumentation can be employed with all such versions of the paradox, it seems implausible to maintain that the key feature responsible for generating this type of paradox relies on assuming that certain sentences are truth-valued, especially since such an assumption in unneeded in many variations of the paradox.

7 Conclusion

To summarize, in Sections 2 and 3, I began by showing that there are certain contingently true propositions that no one can occurrently believe. In Section 4, I then used these results to further derive a contradiction, thereby giving us a paradox. Next, in Section 5, I looked at how the no-truth-value strategy that is often used in response to the Liar Paradox faces problems when responding to the type of paradox provided in this paper. Finally, in Section 6, I showed how the general ideas behind the paradox on occurrent belief can be extended to a wide range of representational states and activities: for example, writing a sentence, uttering a sentence, asserting, hoping, and imagining. Consequently, in order to avoid the type of paradox that is provided in this paper, it seems that one should look for features that reveal a single type of strategy for responding to all variations of the paradox, and one that differs from the no-truth-value strategy that is often used in response to the Liar Paradox. It is to such work that I hope to turn in future research.

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