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Mutual fund performance: Using bespoke benchmarks to disentangle mandates, constraints and skill

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ABSTRACT

While no two mutual funds are alike in terms of their mandates and constraints, metrics used to evaluate fund performance relative to peers typically fail to account for these differences by relying on generic benchmark indices and rankings. We develop a methodology to construct a conditional multi-factor benchmark that explicitly incorporates the details of a given fund's mandates and constraints. The results suggest that (i) mandates and constraints are economically important and affect funds differently, (ii) in general, the average mutual fund has a much improved track record when comparing themselves to a bespoke benchmark, and (iii) the rank ordering of fund bespoke performance relative peers is significantly different than the original rank ordering suggesting advisors and board of directors would make better decisions regarding compensation and performance assessment respectively, if they incorporate the impact of mandates and constraints.

1. Introduction

Consider that as of 2019, actively managed U.S. mutual funds controlled \$13.9 trillion in total net assets.[2] Accurately measuring the performance of these funds is important to household investors, who hold the overwhelming majority of those assets as retirement savings, fund advisors, who must internally evaluate the performance of the fund's portfolio manager, and fund directors/trustees, who are required under the Investment Company Act of 1940 to annually review fund performance, among other items, as part of renewing the contract with the fund advisor. Traditionally, the basis for performance measurement is a comparison of the fund's return relative to a chosen benchmark within a defined asset category, e.g. large cap, growth, etc. and then ranked among a set of peer funds.

A crucial aspect of the appropriateness/accuracy of these performance assessments is the *comparability of a benchmark and a fund's performance*. A mutual fund's choice of a benchmark traditionally utilizes the same asset investment universe (i.e., domestic equities, corporate bonds, international equities, etc.), and matches the investment size (large, medium, small, micro, etc.) and style (i.e., growth, value, momentum, etc.) characteristics. The difference between the fund's return and the benchmark return is taken as a measure of performance, or lack thereof. While this measurement process may make sense for index and absolute return funds,

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2 Mutual fund statistics taken from the Investment Company Institute Fact Book: https://www.ici.org/pdf/2018_factbook.pdf.

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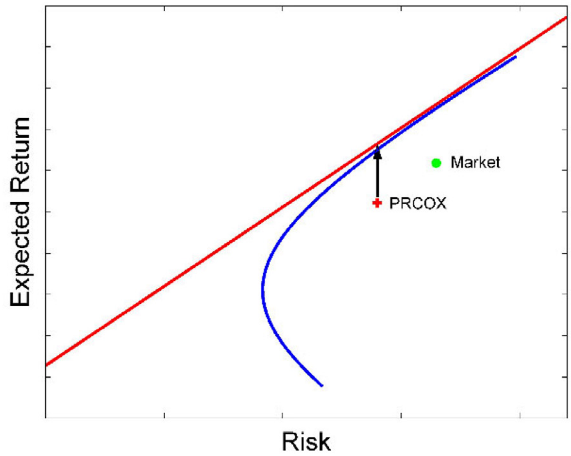
Panel A: Negative Alpha and Excessive Risk



Panel B: Adjustment for Mandates and Constraints

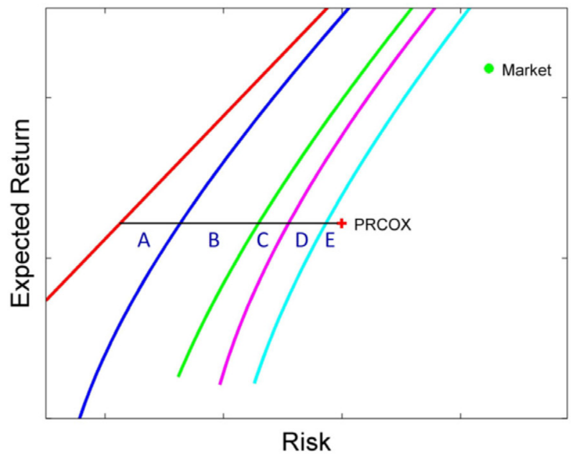


Chart 1. Large cap equity fund and the market portfolio in a mean–variance framework. The following charts illustrate an example of the standard comparison between a mutual fund's performance and its benchmark in a mean–variance framework (Panel A) and the impact of adjusting the minimum-variance frontier (bespoke benchmarks) for a series of mandates and constraints faced by the mutual fund (Panel B). In Panel B, the mandates and constraints listed are: A = Fully Invested, B = No Short Sales, C = Concentration Limit, D = Sector Weights match benchmark, E = residual or manager skill.

it ignores fundamental differences between an active mutual fund and benchmark, such as *fund mandates and constraints*, which are not imposed on the requisite benchmarks.

As an illustration of the comparability/measurement problem, consider a mutual fund that has a mandate to invest in small capitalization value stocks, with constraints to be fully invested at all times and have at least 60% of the portfolio companies be dividend-paying stocks. While a natural benchmark to use would be the Russell 2000 value index, the percentage of dividend paying stocks within the index is well below 60%. Moreover, because the Russell 2000 value index is static, and not subject to a fully-invested budget constraint, it does not need to sell a stock in order to buy another stock and vice versa like the active manager must. As such, differences between a mutual fund and a benchmark may be the results of mandates and constraints, manager skill or a combination of each. Interestingly, the mandates and constraints that are listed within mutual funds' prospectuses display substantial heterogeneity; thus, potentially having differential effects on the performance of funds.[3] Said differently, and the central problem our paper addresses, is that the conventional measure of fund performance is similar to comparing fruit of all varieties: apples, oranges, strawberries, etc. Thus, it is important academically, practically and regulatorily to find a method to make the comparison and measurement appropriate and meaningful.

[3] Note that if all active funds within the same size/style class were subject to the same mandates and constraints, the benchmark comparison would preserve the rank ordering of competing funds.

Our analysis develops precisely that methodology to account for the mandates and constraints of a fund manager by adjusting the relevant benchmark to mirror the fund manager's mandates and constraints — resulting in an apples-to-apples comparison. The intuition behind this methodology can be gleaned from taking a simple mean–variance perspective on performance. Consider a fund of all large capitalization equities over the period 1974–2013, which has a historical return of 13% and a standard deviation of 20%. Panel A of Chart 1 depicts the large capitalization equity fund within a standard minimum variance frontier calculation over the same period. Because the fund is below the Capital Market Line (CML), it delivers less return, or more risk, than its minimum variance benchmarks. It is standard to interpret the fund manager as generating negative alpha. It is this interpretation that is inappropriate, given the substantial differences in mandates and constraints between the fund and the benchmark. Consider now 'adjusting' the minimum-variance frontier to account for the fund specific mandates/constraints. Panel B of Chart 1 provides an illustration of how the frontier may be altered to accommodate the portfolio's unique set of mandates and constraints. This in turn alters the interpretation of alpha generation and skill. Thus, the models used as standard benchmarks, while full of intuition about the trade-off between risk and return, are built upon many strong assumptions. For example, in the Capital Asset Pricing Model (CAPM), two-fund separation exists in the presence of full information, simple and clear preferences over only risk and return, and *the absence of practical frictions facing the portfolio manager*.

Our results show that once benchmarks are properly adjusted, fund mandates and constraints display a wide range of effects on both the benchmark returns and manager's portfolio choice problem. Specifically, the investment universe (size and style) constraints cost funds an average excess risk of 190 basis points, with the small and value styles being the most costly; moreover, cash holdings and leverage add 177 basis points, limits on short-sales adds 80 basis points, and turnover restrictions contribute 141 basis points in average added costs.[4] In addition, the bespoke fund performance results in an average 30 to 40% reduction in fund manager underperformance with a corresponding increase in the variance of performance. Not surprisingly, given the heterogeneity in fund mandates and constraints, the bespoke ranking of peer performance is significantly different than the original ranking, especially for the highest ranked funds suggesting potentially different investment choices might be made by market participants if armed with the bespoke peer ranking.

Our analysis has important implications for academics and market participants alike. From an academic perspective, we provide a flexible methodology to properly compare a fund's performance to a benchmark. The methodology highlights that while basic asset pricing models are good at providing intuition regarding risk and return, they are poor at providing an accurate absolute and relative measurement of the risk/return tradeoff because of their failure to account for the reality of mandates and constraints. In addition, while all market participants are interested in accurate and meaningful performance measures, our results are particularly important for those constituencies who take fund mandates and constraints as given, or exogenous, with respect to their objective. Notable constituencies include (1) mutual fund advisors/management whose objective is to compensate and retain asset managers that are able to deliver the highest mandate/constraint adjusted performance, i.e. the best performance within the confines of the fund's given mandates and constraints, and (2) mutual fund directors/boards whose fiduciary responsibility is to ensure shareholders are receiving fund performance above and beyond that which can be attributed to the fund's mandates and constraints. To properly execute their objectives, both these constituencies require a method to partition a fund's performance into performance due to the given mandates and constraints, and performance due to the choices the portfolio manager makes.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 describes our methodology. Section 4 details our sample funds and the mandates and constraints they face. Section 5 provides model estimates and the costs of individual and joint mandates and constraints on candidate benchmarks. Section 6 concludes.

2. Relevant literature

Our work is related to three facets of the financial literature: parametric estimation of asset pricing models, incorporating frictions in asset pricing models, and mutual funds.

The need for parametric estimation arises because the traditional approaches (mean–variance optimization and factor-mimicking portfolios) are not feasible and flexible enough to capture fund mandates and constraints. Thus, we adopt the framework of Brandt et al. (2009) for developing a parametric portfolio in which the vector of portfolio weights is a function of a set of firm characteristic variables. We extend their original methodology to take account of mandates and constraints, essentially transforming a high-dimensional constrained portfolio choice problem of individual stocks into a low-dimensional problem expressed through characteristics.

Researchers have long acknowledged that frictions will deleteriously impact the performance of asset pricing models. However, the key difference among fund mandates and constraints and other studied frictions is that mandates and constraints are *heterogeneous across funds*, while most other frictions have a homogenous impact within the marketplace. One exception, which investigates a single constraint, is Briere and Szafarz (2017). Their investigation of the impact of short-selling constraints on factor-based portfolios concludes that accounting for this constraint substantially changes mean–variance performances. The goal of our paper is to account for the differential impacts of *multiple fund constraints and mandates* on fund managers and their respective benchmarks. Given few papers have addressed this issue, we believe our results make an important contribution to the literature.

The literature on mutual funds is vast, whereby a complete review is beyond the scope of this paper; however, there are few areas that are relevant to our work: rankings, benchmark choice and retail investor behavior. As mentioned earlier, there is considerable

[4] The added costs due to these mandates and constraints are average values across all applicable funds in our sample where the target excess return is 8%.

research which shows the importance and influence that fund rankings have on fund flows and AUM, representative work includes Blake and Morey (2000) and Del Guercio and Tkac (2008).

A mutual fund's *choice* of benchmark is also an important topic. The general consensus from that literature is that the choice of benchmark has a significant impact on performance, particularly if there is a mismatch between the investment universes. Sensoy (2009), Mateus et al. (2017), and Cremers et al. (2018) suggest that the benchmark choice may be strategic to bolster performance relative to peers. Finally, our analysis is related to the behavior of investors in mutual funds. Work by Palmiter (2016) and Friesen and Nguyen (2018) suggest that retail investors are less than savvy about their investment choices given they appear ignorant of fund characteristics and unresponsive to risks and fees. Given this characterization of retail investors, it is not difficult to argue that they would also be unaware of funds mandates and constraints.

3. Methodology

This section presents a constrained parametric approach to the traditional mean–variance portfolio choice problem and provides a description of the data, including information on fund mandates and constraints, which will be used to estimate the model.

3.1. Parametric benchmark portfolio policy

There are numerous ways in which benchmarks are calculated in finance. However, creating the proper benchmark to account for the differences across fund strategies (mandates, constraints) requires a parsimonious and feasible procedure for calculation. While mean–variance portfolio optimization may be the first method to spring to mind, it can be quickly dismissed as infeasible, and in addition, it is well known to be subject to unrealistic portfolio holdings. Similarly, factor-mimicking portfolios are not a suitable alternative, as it does not provide enough flexibility to capture the differences underlying various mandates and constraints. Consequently, we utilize a parametric portfolio approach as in Brandt et al. (2009), which provides both a feasible and flexible method for calculating bespoke benchmarks. The intuition behind this approach is similar in spirit to a change in mathematical basis, whereby the variation and impact of the various constraints and mandates are projected upon fund characteristics rather than onto a set of portfolio holdings.

Another interpretation of our approach, and one that we will carry through the remainder of the paper, is that any portfolio that a manager may choose can be decomposed into a set of beta strategies (long-only portfolios) and a set of alpha strategies (zero-cost hedged portfolios). The exposures to these strategies are the decision variables in the constrained parametric portfolio choice problem. In contrast to the traditional mean–variance portfolio choice approach, which models nonlinear constraints of the high-dimensional space of portfolio weights on individual assets, our constrained parametric portfolio choice framework specifies fund mandates and constraints as *linear restrictions* on a low-dimensional parameter space and the resulting constrained portfolio choice problem can be easily solved using quadratic programming.

3.2. Econometric model

We begin with a description of our baseline model, the parametric portfolio approach formulated in Brandt et al. (2009), and then describe the adaptations which allow our analysis of mandates and constraints.

Consider an investment universe of N_t stocks, whereby any portfolio is parameterized by active portfolio deviations from the market or benchmark portfolio at time t for stock i as a function of the firm's observable characteristics, $C_{t,i}$. The model starts with a single-period expected utility maximization over portfolio weights $w_{t,i}$:

$$\max_{\{w_{t,i}\}_{i=1}^{N_t}} \mathrm{E}_t\left[u\left(\sum_{i=1}^{N_t} w_{t,i} r_{t+1,i}\right)\right] \tag{1}$$

where $r_{t+1,i}$ is the gross (one plus) return on stock i from t to t+1 and the weights $w_{t,i}$ sum to one across stocks. In order to reduce the dimensionality of this maximization, Brandt et al. (2009) propose parameterizing the portfolio weights as a function of firm characteristics and a low-dimensional set of parameters θ, or $w_{t,i} = f\left(C_{t,i};\theta\right)$, in particular, they work with a simple linear parameterization:

$$w_{t,i} = \overline{w}_{t,i} + \frac{1}{N_t}\theta^\top \tilde{C}_{t,i} \tag{2}$$

where $\overline{w}_{t,i}$ are the weights of stock i in the market or benchmark portfolio and $\tilde{C}_{t,i}$ are firm characteristics that are now standardized (mean zero, standard deviation one) across stocks at each time t. The intuition of this parameterization is that the optimal portfolio weights are deviations from benchmark weights that depend only on the firms' standardized characteristics. The standardization ensures that the benchmark tilts sum to zero so that the sum of the portfolio weights equals the sum of the benchmark weights, which in turn, equals one. Finally, the authors argue that the $1/N_t$ normalization is required to keep the magnitude of tilts stable as the cross-section of stocks grows over time.

With this simple linear parameterization, the high dimensional optimization in Eq. (1) can be rewritten as much lower dimensional one with respect to the parameters θ. Moreover, since these parameters are time invariant by assumption, the conditional expectation can be conditioned down to an unconditional one using the law of iterated expectations, resulting in:

$$\max_{\theta} \mathrm{E}\left[u\left(\sum_{i=1}^{N_t}\left(\overline{w}_{t,i} + \frac{1}{N_t}\theta^\top \tilde{C}_{t,i}\right) r_{t+1,i}\right)\right] \tag{3}$$

Finally, Brandt et al. (2009) operationalize this parametric portfolio optimization problem by estimating the unconditional expectation with a sample average:

$$\max_{\theta} \frac{1}{T} \sum_{t=1}^{T} \left[u \left(\sum_{i=1}^{N_t} \left(\overline{w}_{t,i} + \frac{1}{N_t} \theta^{\top} \tilde{C}_{t,i} \right) r_{t+1,i} \right) \right] \tag{4}$$

Intuitively, the optimal parametric tilts from the benchmark portfolio maximize the realized utility of the portfolio in-sample.

Using the Brandt et al. (2009) approach, we adapt the model in three important ways to accommodate our investigation of mandates and constraints. For the first departure, we assign each stock to an industry, a size group, and a style group. Specifically, we consider in our application $I = 10$ industries based on top level SIC codes, small and large size groups based on the median firm capitalization, and value and growth style groups based on the median book-to-market ratio (i.e., $S = 4$ size and style groups). In the context of the parametric portfolio policy, we then associate each stock separately with (1) the average characteristics of the industry, size, and style group to which the firm belongs, and (2) the deviation of the firm's characteristics from these industry, size, and style group averages. This modeling choice allows the parametric portfolio to independently tilt into industry, size and style groups for broad group investments as well as into individual firms within each industry, size and style group for group-neutral stock investments. We refer to the average characteristics of the industry, size and style groups as *across group* characteristics and the firm specific deviations as *within group* characteristics.

As in Brandt et al. (2009), both sets of characteristics are cross-sectionally normalized to ensure that portfolio tilts from the market or benchmark portfolio add up to zero and are not affected by changes in the universe composition such as doubling the number of firms by simply splitting them up. Specifically, the *across group* characteristics are demeaned so that group tilts can be market or benchmark neutral and are scaled by the relative market capitalization of each firm versus the whole group to ensure that group tilts are market capitalization weighted. Intuitively, as the portfolio tilts from one group to another, it does so proportionally more for larger firms within the groups. The *within group* characteristics are, in addition to already being demeaned by construction, normalized by the relative market capitalization of the group versus the whole market. This normalization scales the active investment within each group by the market capitalization of the group instead of the number of firms within the group.

The second departure from Brandt et al. (2009) is that we split both the across and within group characteristics into positive and negative values with separate coefficients on each. This allows the portfolio to more aggressively overweight firms with positive characteristics than underweight firms with equally negative characteristics, or vice versa. However, by breaking the link between over and under-weights relative to the market or benchmark portfolio, the active tilts may no longer sum to zero or be self-funded. To compensate, we introduce another coefficient on the market or benchmark portfolio so that if the active tilts are net positive (negative) the allocation to the market or benchmark portfolio is adjusted appropriately less (more) than 100 percent so that the sum of portfolio weights still add up to one. This is the final modeling departure from Brandt et al. (2009).

To summarize, the optimal portfolio weight at time t for stock i, denoted $\omega_{t,i}$, is parameterized as follows:

$$\omega_{t,i} = \beta_0 \overline{\omega}_{t,i} + \overline{\beta}\overline{c}^{+}_{t,i} + \beta c^{+}_{t,i} + \overline{\alpha}\overline{c}_{t,i} + \alpha c_{t,i} \tag{5}$$

where $\overline{\omega}_{t,i}$ is the weight of the market or benchmark portfolio at time t for stock i, $\overline{c}_{t,i}$ is a vector of normalized and scaled average characteristics for the industry, size, and style group to which stock i belongs at time t (the *across group* characteristics), $c_{t,i}$ is a vector of differences between the firm's characteristics and the firm's industry, size, and style group average characteristics (the *within group* characteristics). $\overline{c}^{+}_{t,i}$ and $c^{+}_{t,i}$ are vectors that contain the positive values of across and within group characteristics, respectively, and zeros when the corresponding characteristics are negative.

$[\beta_0, \overline{\beta}, \beta, \overline{\alpha}, \alpha]$ are the parameters governing the optimal portfolio weights. $[\overline{\alpha}, \alpha]$ tilt the portfolio weight symmetrically away from the market or benchmark weight based on the across and within group characteristics of the firm. These are the zero-cost alpha strategies discussed in the previous section. $[\overline{\beta}, \beta]$ allow this tilt to be asymmetric, where positive (negative) values create a tilt that overweighs firms with positive (negative) characteristics more than it underweights firms with equally negative (positive) characteristics. Finally, β_0 scales the benchmark weight to allow for tilts that do not sum to zero in the cross-section, or are not self-funded. The three terms associated with the beta coefficients represent the long-only beta strategies. To reduce the number of free parameters, we assume that the loadings on the within across and within group characteristics are the same for all 10 industries. With K characteristics, this assumption reduces the number of coefficients to $1+16K$.[5]

An alternative interpretation of our parameterization is the common practice of funds attributing performance relative to the benchmark to "allocation" (*across groups*) and "selection" (*within groups*). "Selection" is that portion of the fund's return that is due to selecting outperforming stocks within a sector, while "allocation" is that portion of the return due to being in outperforming sectors, independent of which stocks were in the fund's portfolio. In our parameterization, selection skill comes from non-zero α and β loadings while allocation skill is due to non-zero $\overline{\alpha}$ and $\overline{\beta}$ loadings. In addition, it is common practice for a mutual fund to impose minimum and maximum allowable deviations from the benchmark weights in the requisite benchmark portfolio, sometimes called sector/industry bands. These limits would be corresponding upper constraints on the parametric portfolio parameters.

[5] Each of the four size and style groups has two coefficients on K across industry characteristics and two coefficients on K within industry, size and style group characteristics.

3.3. Investment universe

We measure the impact of various mandates and constraints relative to the unconditional minimum variance frontier. Thus, we must define the unconditional investment universe as well as the size and the style groups. The investment universe includes all traded stocks, except those whose price is below \$5 per share, as well as stocks whose market capitalization is below the 20th percentile in the cross-section. These exclusions are meant to minimize the effect of extreme observations due to infrequent trading of illiquid securities on our results. We define the size and style groups based on market capitalization and the book-to-market ratio using NYSE breakpoints. A stock with a market equity below the 20th percentile is classified as "micro-cap" (thereby excluded as explained above), between the 20th and 50th percentile (median) as "small cap", and above the 50th percentile (median) as "large cap". Similarly, a firm with a book-to-market ratio above the 50th percentile (median) is classified as "value" and correspondingly below as "growth". Finally, we impose consistency between the investment universe and the size and style groups in two ways. First, we set the loadings associated with size and style groups outside the investment universe to zero. Second, we renormalize the market capitalization weights of all firms in the universe within each size and style group.

3.4. Mandates and constraints

There are numerous mandates and constraints that obfuscate the measurement of fund performance relative to a benchmark; thus, a comprehensive investigation of each one is infeasible. Moreover, fund mandates and constraints display incredible heterogeneity with respect to how widely they are communicated, the ease/difficulty with which they can be incorporated into our parametric portfolio problem, and whether they are self-imposed or part of market-wide financial regulation. The mandates and constraints we focus on in this paper are set out in the fund prospectus or Statement of Additional Information (SAI), are typically chosen by the fund, and are widely communicated to investors (see Section 4 below for a detailed description of our data). Specifically, we incorporate five mandates/constraints into our parametric benchmark models: investment universe, short-sales, borrowing/lending, portfolio turnover and transaction costs, as these are widely communicated to investors, parsimoniously modeled, and easily interpretable.

3.4.1. Investment universe

It is relatively common for a mutual fund to have a restricted or targeted investment universe such as growth versus value or large versus medium versus small capitalization firms. Universe restrictions are simple to impose in our framework by simply redefining and renormalizing the investment universe in Section 3.3.

3.4.2. Short sales

Some mutual funds are allowed to short-sell to a limited extent. Many funds are prohibited from short-selling all together. Suppose a fund is allowed to hold a total short position up to $q \geq 0$, then a sufficient condition for this constraint to be satisfied is $\left(\overline{\alpha}, \alpha\right) \iota_{2KS} \leq q$ since the long–short alpha strategies have a short position by design while the beta strategies are long-only by construction. Note that this parameter constraint is sufficient and likely overly restrictive, since some additional allocation to the alpha strategies can be offset by the long holdings of the beta strategies without producing a total net short position that exceeds the limit.

3.4.3. Borrowing/lending

Cash borrowing and lending constraints essentially place an upper ($\overline{\pi}$) and lower ($\underline{\pi}$) bound on the riskless asset, where $\overline{\pi} \in \{0, 1\}$ is the cash holding limit and $\underline{\pi} \leq 0$ is the borrowing limit. Translating these constraints to parameter restrictions obtains: $\left(\beta_0, \overline{\beta}, \beta\right) \iota_{2KS+1} \in [1 - \overline{\pi}, 1 - \underline{\pi}]$ since only the long-only strategies are impacted by borrowing and lending.

3.4.4. Portfolio turnover

A constraint on the extent of trading, or turnover, within the portfolio is modeled by requiring (monthly) turnover $TO(w) = \boldsymbol{E} \sum_{i=1}^{N} \left|\Delta w_{t,i}\right|$ to be bounded above by u. Given *all* strategies are impacted by a constraint on trade, the turnover constraint amounts to the following sufficient condition in terms of the parameters of the portfolio policy rule:

$$\beta_0 TO\left(\overline{w}\right) + \overline{\beta} TO\left(\overline{c}^+\right) + \beta TO\left(c^+\right) + \overline{\alpha} TO\left(\overline{c}\right) + \alpha TO\left(c\right) \leq u \tag{6}$$

3.4.5. Transaction costs

Finally, we model transaction costs by adding a function of trades to the objective function of portfolio return variance:

$$TC = \frac{1}{2} \eta \Delta w_t' \Sigma_t \Delta w_t \tag{7}$$

where η is a constant which governs the level of trading costs. We calibrate the median level of this parameter using the following mean–variance criterion:

$$E\left[\overline{w}_t R_{t+1} - R_{f,t}\right] - \frac{A}{2}[E\left(w_t R_{t+1}\right)^2 + \overline{\eta} E\left(\Delta \overline{w}_t R_{t+1}\right)^2] = 0 \tag{8}$$

This function sets equal the utility of holding a value-weighted market index, which is rebalanced each month, with the utility of holding cash, given the intensity of transaction cost η. Given standard assumptions, we obtain a medium transaction cost intensity of $\overline{\eta} = 50$.[6] We also consider two additional liquidity environments, one that is more liquid, where $\eta_{low} = \frac{1}{2}\overline{\eta} = 25$, and another that is more illiquid where $\eta_{high} = 2\overline{\eta} = 100$. The latter liquidity environment could be interpreted as either fragmented markets, markets with few liquidity providers, or perhaps periods of market downturns as in 1987, 1997 and 2008.

3.5. Constrained parametric portfolio choice problem

Combining the basic parametric portfolio problem with the parameterized mandates and constraints yields the following problem presented here in matrix notation:

$$\min_{\theta}\left(\frac{1}{2}\theta'\Omega_{\eta}\theta\right) \quad \text{where} \quad \Omega_{\eta} = E\left[X_t' R_{t+1}^{e} R_{t+1}^{e'} X_t\right] + \eta E[\Delta X_t' R_{t+1}^{e} R_{t+1}^{e'} \Delta X_t] \tag{9}$$

subject to the following constraints:

$$\begin{array}{ll} \theta'\mathbf{E}\left[X_t' R_{t+1}^{e}\right] = \mu^{e} & : \text{mean return target} \\ \theta' \boldsymbol{e}_{\alpha} \leq q & : \text{short sale} \\ \theta' \boldsymbol{e}_{\beta} \in \left[1-\overline{\pi}, 1-\underline{\pi}\right] & : \text{borrowing and lending} \\ \theta'\mathbf{E}\left|\Delta X_t'\right| \iota_N \leq u & : \text{turnover} \\ \theta \geq 0 & : \text{fund level long only} \end{array}$$

The vectors included in the above problem can be interpreted as follows. The vector $\boldsymbol{e}_{\beta}$ selects only those coefficients that correspond to the long-only strategy, i.e. $\beta_0, \overline{\beta},\ and\ \beta$; therefore, they are present in the mean return target and borrowing/lending constraint. Correspondingly, the vector $\boldsymbol{e}_{\alpha}$ selects those coefficients that pertain to the hedged portfolios, i.e. $\overline{\alpha}\ and\ \alpha$; thereby only included in the short-sale constraint. Note that any universe constraints are implicit in the initial dataset construction, and that we added a fund level constraint on the parameters which should be non-binding provided firm characteristics are signed appropriately.

4. Data

4.1. Market data

To empirically estimate the parametric model, we require equity returns and accounting data as well as information regarding mutual fund mandates and constraints. We describe each in turn.

We obtain monthly return, price per share, and shares outstanding of individual U.S. companies traded in the NYSE, AMEX, and NASDAQ from the Center for Research on Security Prices (CRSP). We then collect corresponding accounting variables as well as Standard Industry Classification (SIC) codes from the CRSP-Compustat merged data set. In the case of a missing SIC code, we complement it using data from CRSP. Stocks are then categorized into 10 industry sectors based on the definitions outlined on Kenneth French's Data Library. The sample period is January 1974 to December 2013.

For each firm, we construct three conditioning characteristic variables: size, or market equity; value, or book-to-market ratio; and return momentum. Specifically, market equity is defined as the log of the common share price times the number of common stock shares outstanding at the end of each June. Book-to-market is measured as the log of the ratio of book equity measured at the most recent fiscal year-end within the prior calendar year to the market equity measured at the end of the prior calendar year (December). Finally, momentum is defined as the one-year return lagged by one month to remove any short-term reversal effects.[7]

4.2. Fund mandates and constraints

The set of mutual funds we consider are all U.S. equity capital appreciation funds. Our mutual fund sample choice is based on a singular, clear objective (capital appreciation) as well as a parsimonious sample of 141 funds, making it feasible to collect the data. For our main analysis we further narrow the sample to the 71 mutual funds with at least a 10-year track record. In order to gather the mandate and constraint data for our funds, we reviewed the prospectus for each one of our sample (capital appreciation) funds. Specifically, we reviewed and hand collected data from the 485A and 485B Prospectus of Security (POS) as well as the Statement of Additional Information (SAI) that were submitted to the SEC within, or closest to, the fourth quarter of 2009.[8] In reviewing these prospectuses, we recorded data on 4 dimensions: security holdings, investment level, other securities, and benchmarks. See Table 1 for a specific list of variables coded.

The *security holding* describes the universe of securities that the fund is constrained to invest within. Common specifications include growth versus value, large, medium and small capitalization equities, and foreign holdings, which for most funds were relatively easy to identify. *Investment level* refers to the extent that the fund is able to buffer fund flows (in or out) with borrowing or the ability to hold cash (not be fully invested). The language surrounding the investment level tends to be broad in order to accommodate infrequent/rare occurrences. For example, many funds state they have the ability to borrow and hold cash on a

6 In solving for η, we assume that relative risk aversion is $A = 5$, the equity risk premium is $8\%/12 = 0.0067$, the variance of the market return is $\left(16\%^2\right)/12 = 0.0021$, the mean squared variance-adjusted turnover of the value-weighted market index is 1.2×10^{-5}.

7 For a further description of the industry sectors and market variables see Kenneth French's Data Library website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

8 See http://www.sec.gov/edgar/searchedgar/prospectus.htnm.

Table 1
Mutual Fund Mandates and Constraints. We select as our sample the set of Capital Appreciation mutual funds. In searching the CRSP mutual fund database, we identified 141 Capital Appreciation Mutual Funds as of June 2009, which we take as our sample. For each fund, we employ the following procedure to collect information on the securities, methods, and constraints they face. As each mutual fund is required to provide the Securities and Exchange Commission (SEC) with a prospectus (485POS) and an annual/semiannual report on holdings (N-30D), we review these two documents filed closest to the last quarter of 2009, and code the following information:

Security Holdings:			
Names:	Max # of securities, (U) Unconstrained		
Type:	Equity (E), Fixed Income (FI), or Both (B)		
Style:	Growth (G), Value (V)		
Size:	Large (L;%), Medium (M;%), Small (S;%)		
Industry:	Unconstrained (U), List		
Foreign:	Max %		
Investment Level:			
Ability to Borrow:	(Yes/No)	"Constrained buys"	
Ability to Hold Cash:	(Yes/No)	"Constrained sells"	***(N-30D file)***
Turnover:	(% Given)	"Constrained trade"	
Other Securities:			
Securities Lending:	(Yes;%, No)		
Shorting:	(Yes;%; No)		
Derivatives:	(Yes;%, No)		
Benchmark:			
None Referenced (NR), Specific Index (List)			
Volatility:			
Unconstrained (U), Managed (M), % or Index identified			

Table 2
Summary Statistics on Sample Mutual Fund Mandates and Constraints. We select as our sample the set of Capital Appreciation mutual funds. In searching the CRSP mutual fund database, we identified 141 Capital Appreciation Mutual Funds as of June 2009. For our main analysis, we further select 71 out of the 141 identified funds with more than 10 years of track record, which we take as our sample. For each fund, we employ the following procedure to collect information on the securities, methods and constraints they face. As each mutual fund is required to provide the Securities and Exchange Commission (SEC) with a prospectus (485POS) and an annual/semiannual report on holdings (N-30D), we review these two documents filed closest to, the last quarter of 2009. This table reports the average of key fund characteristics and constraints for both the whole sample (Row "All") and subsamples of funds with a focus on a certain size (large vs small) or style (growth vs value) groups. The last row shows the numerical values we choose to represent the unconstrained cases. For example, if the cash holdings of a fund without cash limit is capped by 1, or 100%.

Sample Funds	AUM million $	# of Names	Constraints			
			Leverage	Cash	Short-sale	Turnover
Large	1055.76	16	1.42	0.54	1.54	0.44
Small	541.17	12	1.70	0.70	1.58	0.56
Growth	1378.86	27	1.59	0.53	1.34	0.79
Value	1637.41	11	1.65	0.48	1.65	0.54
All	1147.49	71	1.48	0.59	1.46	0.73
Unconstrained			2.00	1.00	2.00	2.00

"temporary and defensive basis", and yet do so only on rare occasions. *Other securities* captures the extent to which the fund is able to use derivative contracts as well as taking a short position in securities and lending securities to other counterparties. *Benchmarks* are a key piece of information for our study as it defines the metric by which the fund has chosen to measure its own performance. Most funds specify a broad index that is representative of the security holding universe that the fund is constrained to invest within.

Recall the role of a prospectus is to provide investors accurate information about the investment strategy, risks, past performance, operations, restrictions, fees and management, so investors are able to make an informed decision about whether to invest. Not surprisingly, the ordering of topics, form of presentation and even the language used within these prospectuses often follows a common template.[9] Heuristically, we understand that the generalizations/legal language within the prospectuses does not always match practice. Moreover, we acknowledge that some of these mandates and constraints are explicit while others implied. For example, while a majority of funds have the ability to mitigate fund inflows/outflows, in practice they maintain a low cash position and borrow little, thereby investing when they receive inflows and selling when they are redeemed to remain fully invested.

After gathering and reviewing the prospectus mandate and constraint data, we take as the most relevant constraints to analyze: (1) mandates on the investment universe, (2) borrowing/lending constraints, (3) short-sale constraints, (4) turnover constraints, and (5) transaction costs.

[9] For example, see the following links for representative prospectuses within our sample. http://www.sec.gov/Archives/edgar/data/100334/000010033410000014/pea125-2010.htm http://www.sec.gov/Archives/edgar/data/275309/000072921809000035/main.htm.

Table 2 presents some summary statistics on the mandates/constraints facing our sample funds. The sample has a good mix of funds within each of the size/style groups, with large/growth having the largest number of funds and small/value having the smallest number. Note that some mandates/constraints are rather uniform, for example, cash and short-sale, while others, like leverage and turnover, have more variation.

5. Properties of constrained minimum variance frontiers

We perform two estimations of the constrained minimum variance frontiers, one in-sample and the other out-of-sample; each meant to address a different research question. The in-sample estimation suits our purpose of ex-post evaluation of fund performance relative to an appropriately constrained benchmark, while the out-of-sample estimation provides a realistic estimation of the constrained portfolio using rolling historical data. The out-of-sample estimation establishes a natural robustness check of the in-sample results with respect to overfitting the parameters and also provides a window into how real-time, rolling estimation may impact the mandate and constraint costs.

5.1. In-sample portfolio construction

We present the in-sample results in three complimentary ways. First, we present figures that show the impact of incorporating individual mandates and constraints on the minimum variance frontier. Second, we detail the changes to portfolio weights when incorporating mandates and constraints, and finally, we investigate the impact that mandates and constraints have on particular investment and trading strategies. In what follows we discuss each in turn.

We begin by addressing the following question: what would be the return variance of a portfolio constrained to have (i) the same mandates and constraints and, (ii) the same average return as the fund in consideration? To this end, we estimate the parameters of the constrained portfolio, theta, for any given value of mean excess return using the full-sample of data/information. We then construct the constrained portfolio based on the point estimates of parameters, $\hat{\theta}_{IN}$.

Charts 2–5 display the effects of the various mandates and constraints on the minimum variance frontier. Intuitively, the constrained minimum variance frontiers tend to be located inside (less return and more risk) the unconstrained or less constrained frontiers. The following analysis addresses each mandate/constraint individually.

Chart 2 displays the effects of constraining borrowing and lending. On one hand, we find that the borrowing constraint reduces the efficiency only marginally because beyond the tangent portfolio, the Sharpe Ratio of the benchmark without cash diminishes very slowly, which leaves little room for leverage to have a demonstrable difference (Panel A). On the other hand, limiting cash holdings can decrease the efficiency of the frontier substantially (Panel B). To provide some perspective on these effects, an unconstrained fund manager would be able to achieve an annualized average return of 8% in excess of the time-value of money at the cost of a relatively low level of risk, an annualized standard deviation of 6%. The counterpoint to that example is a fund manager who is allowed to hold no more than 20% of their portfolio in cash; they would have to induce a standard deviation of 10% to achieve the same level of return. Moreover, in the event that the fund was fully invested, the fund manager would generate a standard deviation of 13% for the same level of return.

Similarly, we examine the effects of the remaining mandates and constraints on the minimum variance frontier; however, we exclude the risk-free asset from the investment opportunity set to more clearly understand the links between the remaining mandates, constraints, and asset allocation decisions. Chart 3 displays the results for the constraint to refrain from short-selling. Interestingly, short-sale constraints appear to impact only the upper envelope (efficient portion) of the minimum variance frontier. As an example, for a relatively low return target, the imposition of short-sale constraints does not force a substantial increase in the standard deviation. Thus, our results suggest that the usefulness of being able to short-sell a stock only has an impact when it is being used as a financing vehicle for the purchase of a high return (high variance) asset.

Chart 4 displays the results of mandates related to the investment universe. Panel A displays the results for large versus small stocks and Panel B displays the results for growth versus value styles. The results in Panel A display an interesting contrast between low and high return targets. Notice that the two constrained frontiers intersect, whereby the frontier for large-cap stocks dominates that of small caps for returns below 13% and vice versa for the region above 13%. This suggests that during economic downturns, which are often accompanied by a “risk-off” environment, small-cap funds will face more headwinds than large-cap funds for the same level of target return. Perhaps not surprisingly, Panel B shows that limiting a manager’s investment universe imposes a cost whether the style is growth or value. However, value managers face a higher implied cost than do growth managers and the discrepancy increases with the level of the target return. The results suggest that value managers need to rely on high risk — high reward stocks, only a few of which generate the targeted return.

The results on asset trading constraints are presented in Chart 5. Panel A displays the bespoke minimum variance frontiers for various levels of transaction costs. In the context of our model, the reader should think of transaction costs broadly, including but not limited to: liquidity costs (bid–ask spreads, depth considerations, price impact), commissions or soft dollars, 12(b)-1 fees, back-office costs, e.g. TA fees. As intuition would serve, the results show that increasing levels of transaction costs impose less efficient frontiers in the way of parallel shifts in the frontier over the upper envelope (efficient) portion of the frontier. This suggests that funds that have poor trading capabilities, lack an in-house distribution system, or trade within inherently illiquid markets are at a disadvantage to funds not facing those same constraints. Panel B displays the results imposing various levels of turnover constraints. Intuitively, the tighter the restriction on turnover, the less trading is allowed to manage flows in and out of positions, and the lower the expected return where the turnover constraint is binding. Thus, funds with high expected return targets with constraints on turnover feel the impact of this constraint the most.

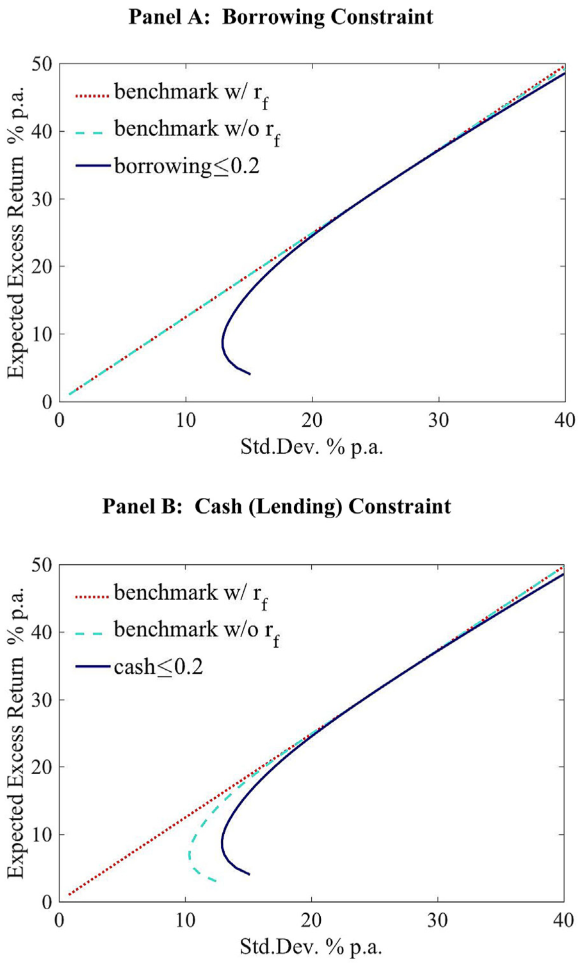


Chart 2. Impact of borrowing and lending constraints on the minimum variance frontier. The following charts illustrate the impact of imposing constraints on borrowing (Panel A) and cash holdings (Panel B) on the minimum-variance frontier with, and without, the presence of a riskless asset.

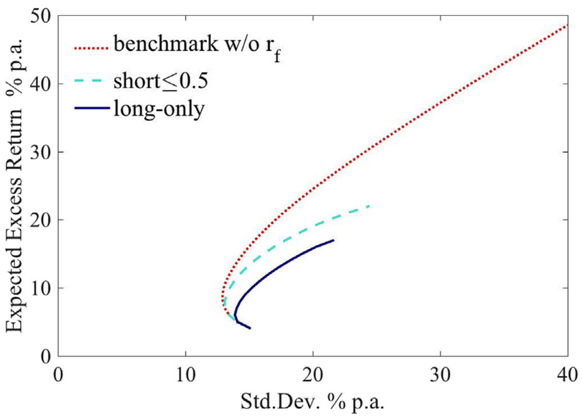


Chart 3. Impact of short-sale constraints on the minimum variance frontier. The following charts illustrate the impact of imposing constraints on short-sales on the minimum-variance frontier without the presence of a riskless asset.

As a complement to the impact of mandates and constraints on the minimum variance frontier, Table 3 presents the impact that these same mandates and constraints have on the portfolio weights. Panels A, B and C present results for a mean excess return of 4, 8, and 16%, respectively. A comparison of the various bespoke frontiers relative to the unconditional minimum-variance frontiers

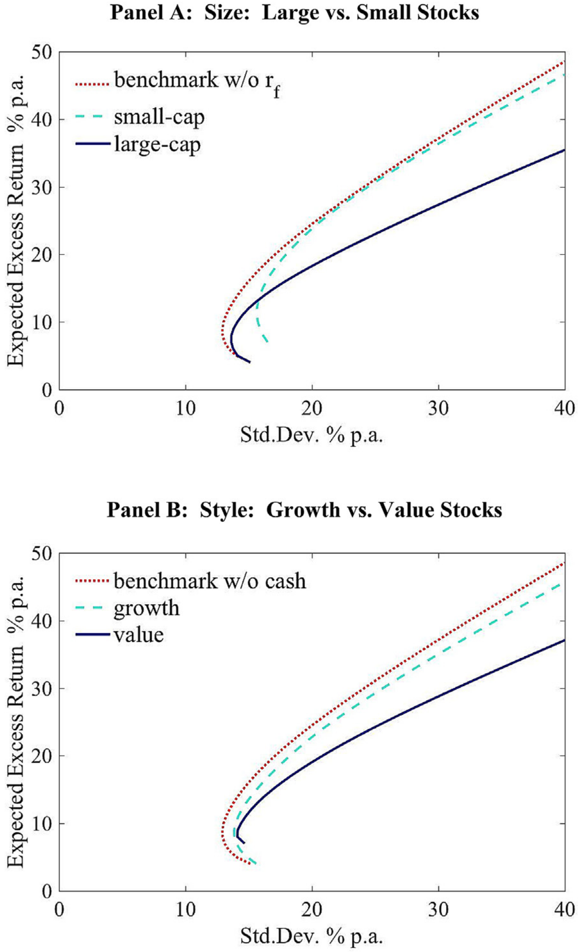


Chart 4. Impact of investment universe constraints on the minimum variance frontier. The following charts illustrate the impact of imposing constraints on the universe of stocks from which the fund can choose on the minimum-variance frontier without the presence of a riskless asset. Panel A addresses size — Large versus Small, while Panel B address style — Growth versus Value.

shows that mandates and constraints, other than the investment universe: (1) are less concentrated, as seen by smaller maximum and minimum weights, (2) have fewer short positions, and (3) have lower turnover. There are however, notable exceptions to the above generalization that are worth broaching. First, sector-neutral portfolios targeting higher excess returns (Panel C) require more concentrated weights, more short-selling and more intensive turnover than the unconstrained case, as all funds strain to reach a high expected excess return with their portfolio choices. Second, constraining the investment universe on size or style leads to more extreme tilts to active trading strategies in order to maintain a given expected excess return in their constrained investment universe. This is particularly true for small stock and value funds. Lastly, as expected, the imposition of transaction costs induces lower turnover as managers seek to avoid trading costs. However, transaction costs also have an odd effect in that for high targeted returns, portfolio managers resort to more extreme bets through concentrated portfolios and larger short positions. We suspect the pressure of a high return target induces managers to employ a buy-hold strategy where large positive bets are funded with large short positions.

Finally, we provide insights into how the imposition of mandates and constraints on the model impact traditional trading strategies. Intuitively, we investigate how a fund manager would reallocate among various trading strategies when faced with mandates and constraints. For brevity, we focus our attention on the impact of mandates and constraints within the environment of an 8% excess market premium. Table 4 displays our results. Panel A focuses on sector allocation (across group), while Panel B highlights stock selection (within group). Within each panel, changes to loadings on size, book-to-market and momentum are reported for size-style combinations. For the sector allocation strategies (Panel A), the value-small universes are unaffected by the constraints, while growth (large and small) and value-large are affected through loadings on style (book-to-market). The impact is particularly pronounced for constraints on short-sales and turnover and imposed transaction costs. Turning to Panel B, stock selection strategies, the growth style is unaffected by the imposed constraints; however, value-(large and small) are slightly altered through size (market capitalization), particularly in the presence of transaction costs.

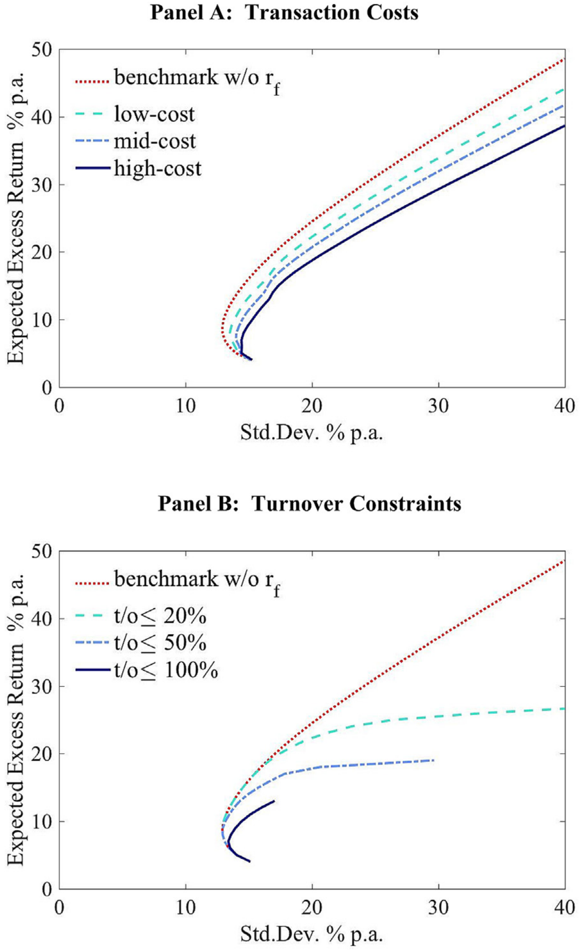


Chart 5. Impact of transaction cost constraints on the minimum variance frontier. The following charts illustrate the impact of various levels of transaction costs (Panel A) and turnover (Panel B) on the minimum-variance frontier without the presence of a riskless asset. For transaction costs, low, medium and high are defined as: $\frac{1}{2}$ normal costs, normal costs, and double costs, where normal is calibrated in a mean–variance framework.

5.2. Out-of-sample portfolio construction

Our out-of-sample analysis begins by using data from January 1964 through December 1973 to estimate the coefficients of the initial portfolio policy. Using those initial parameter estimates, we form the out-of-sample portfolio for the next month, January 1974. Then, in recursive fashion we expand the sample by one month, rebalancing the portfolio using the new parameter estimates month-by-month. Finally, using the entire sequence of out-of-sample portfolio estimates, we recalculate the relative performance of our sample capital appreciation funds against their respective bespoke benchmarks, over a 3, 5, and 10-year horizon as is feasible given the tenure of our sample funds.

As a robustness check, we analyze the correlation of returns between the in- and out-of-sample benchmark portfolios without cash for a spread of excess return targets. The out-of-sample portfolios are highly correlated with the in-sample portfolios, where the correlation for 4% excess return is 0.97 and drops monotonically to 0.93 at an excess return of 32%. Therefore, we are confident in both the model specifications and the ability to utilize a rolling portfolio rebalancing to investigate the impact of mandates and constraints. In addition, Chart 6 compares the performance of the in-sample and out-of-sample benchmarks on standard deviation of returns, average excess return, and the Sharpe ratio in Panels A, B and C, respectively. The standard deviation estimates are very similar, suggesting an even spread of return variation over time where the in-sample estimates provide lower standard deviations below 25%. In contrast, the average excess return between the two sets of estimates diverges sharply above the inflection point of 12%, with the in-sample estimates, perhaps not surprisingly, providing higher average excess returns. Finally, combining these two sets of results, we learn from the Sharpe Ratio results in Panel C that the two sets of estimates are similar for reasonable excess return targets, but diverge above a target of 20% where the out-of-sample portfolio deliver a maximum Sharpe ratio of 1.0 compared to 1.2 for the in-sample portfolio. Thus, in general, the relatively small difference between the performance of the two sets of portfolios over reasonable excess return targets lends strong support to our model and its specification.

Table 3

Portfolio Weights for the In-Sample Constrained Parametric Model. This table presents summary statistics regarding the constrained parametric portfolio weights given the target mean excess return of 4% p.a. (Panel A), 8% p.a. (Panel B), and 16% p.a. (Panel C). For each panel, we report the time-series averages of the mean absolute value of portfolio weights (weight, abs.val., %), the maximal weight (max weight, %), the minimal weight (min weight, %), the total weight of the short positions (short weights), the fraction of stocks being shorted (short fraction), and turnover, measured as absolute value of changes in portfolio weights summed across all stocks.

Panel A: Mean Excess Return of 4%													
Uncon	Sector neutral	Short sale		Universe				Transaction cost			Turnover		
	w/o rf	<0.5	0	Growth	Value	Small	Large	Low	Med	High	<0.2	<0.5	<1
Weight, abs.val.(%)	0.16	0.13	0.07	0.20	0.22	0.18	0.21	0.11	0.10	0.09	0.10	0.16	0.16
Max weight (%)	9.54	9.12	8.40	8.45	13.82	14.67	9.55	5.70	4.63	3.93	7.71	9.54	9.54
Min weight (%)	−1.11	−1.02	0.00	−1.44	−0.24	−0.52	−1.03	−0.53	−0.37	−0.21	−0.89	−1.11	−1.11
Short weights	−0.64	−0.43	0.00	−0.37	−0.12	−0.12	−0.28	−0.29	−0.21	−0.11	−0.23	−0.64	−0.64
Short fraction	0.26	0.30	0.00	0.20	0.28	0.35	0.31	0.25	0.24	0.21	0.10	0.26	0.26
Turnover	0.37	0.30	0.18	0.23	0.18	0.21	0.28	0.20	0.16	0.12	0.18	0.37	0.37
Panel B: Mean Excess Return of 8% (Current market premium)													
	Uncon	Short sale		Universe				Transaction cost			Turnover		
	w/o rf	<0.5	0	Growth	Value	Small	Large	Low	Med	High	<0.2	<0.5	<1
Weight, abs.val.(%)	0.22	0.16	0.08	0.28	0.37	0.26	0.27	0.16	0.14	0.11	0.13	0.22	0.22
Max weight (%)	8.25	7.91	7.51	7.09	10.34	12.57	8.40	4.86	4.14	3.60	6.63	8.25	8.25
Min weight (%)	−1.65	−1.48	0.00	−1.99	−1.08	−1.01	−1.70	−1.00	−0.68	−0.37	−1.11	−1.65	−1.65
Short weights	−0.75	−0.41	0.00	−0.48	−0.34	−0.18	−0.36	−0.44	−0.28	−0.12	−0.25	−0.74	−0.75
Short fraction	0.32	0.31	0.00	0.25	0.32	0.35	0.29	0.27	0.24	0.20	0.13	0.32	0.32
Turnover	0.41	0.32	0.19	0.28	0.33	0.22	0.33	0.25	0.19	0.13	0.18	0.41	0.41
Panel C: Mean Excess Return of 16%													
	Uncon	Short sale		Universe				Transaction cost			Turnover		
	w/o rf	<0.5	0	Growth	Value	Small	Large	Low	Med	High	<0.2	<0.5	<1
Weight, abs.val.(%)	0.28	0.15	0.08	0.48	0.40	0.48	0.52	0.21	0.20	0.20	–	0.21	0.28
Max weight (%)	6.10	4.16	5.24	4.35	4.70	5.67	5.44	1.68	1.62	1.55	–	2.84	6.08
Min weight (%)	−3.02	−1.25	0.00	−3.80	−3.18	−2.59	−4.48	−2.18	−2.02	−1.83	–	−2.77	−3.02
Short weights	−1.12	−0.37	0.00	−1.14	−0.40	−0.74	−1.10	−0.69	−0.62	−0.60	–	−0.69	−1.11
Short fraction	0.30	0.22	0.00	0.31	0.18	0.34	0.39	0.30	0.29	0.31	–	0.27	0.30
Turnover	0.76	0.53	0.42	0.70	0.72	0.64	0.79	0.51	0.48	0.47	–	0.40	0.75

5.3. Measuring mutual fund performance and rank

In this section, we analyze an essential question. If fund benchmarks were properly adjusted to account for mandates and constraints facing the funds, thus, allowing an apples-to-apples comparison — how would this impact the distribution and rank of relative performance of mutual funds? Said differently, how is the cross-section of relative fund performance altered, if at all, when properly accounting for mandates and constraints?

We illustrate our procedure of building the requisite bespoke skill distribution by first providing two single fund examples to highlight how we account for individual mandates and constraints. Specifically, we use the *Value Line Large Companies Fund* and the *Janus Investment Fund* as our examples because they span the entire sample period (January 1974 to December 2013) and they both face a broad set of mandates and constraints. The *Value Line Fund* is a large-cap fund with December 2013 AUM of $0.21B and the S&P 500 as its chosen benchmark; the *Janus Fund* is a value fund with December 2013 AUM of $1.75B and the FTSE EPRA/NAREIT Developed and Global Indices as its chosen benchmark.

We begin by considering the unconstrained benchmark coupled with the risk-free asset, then we sequentially account for mandates and constraints, adding a single additional constraint in each successive step. Because the contribution of each constraint to the shift in the minimum variance frontier is not independent, we fix the order that constraints will be addressed. Specifically, the constraint ordering we employ is: investment universe, borrowing/lending, short-sale, turnover, and transaction costs.[10]

Chart 7 illustrates how mandates and constraints are incorporated to sequentially create the bespoke benchmarks, Panel A displays the *Value Line Fund* and Panel B displays the *Janus Investment fund*. A number of interesting results emerge from this simple comparison. First, the impact of mandates and constraints can be very different across individual funds. For example, consider the mandate to be fully-invested with a zero-cash balance (the second adjustment from the benchmark). The adjustment for the *Janus Investment Fund* is much larger than the adjustment for the *Value Line Large Companies Fund*, likely due to the more illiquid nature of value stocks. Second, if the difference between the standard deviations of a benchmark and the two sample funds is smaller, then we observe that this difference relative to the bespoke benchmark adjusted for mandates and constraints is higher in both cases. Lastly, the performances of the two funds diverge from each other (namely, have a wider spread) after adjusting for mandates and constraints. Relative to the unconstrained benchmark, the fund performances are -13% and -11%, respectively (200

[10] We perform robustness checks on the ordering of our mandates and constraints, and while the estimates vary slightly, the results are both quantitatively and qualitatively similar and are available upon request.

Table 4

Impact on Trading Strategies for the In-Sample Constrained Parametric Model. This table presents the coefficients regarding the constrained parametric portfolio weights given the target mean excess return of 8% p.a. Panel A shows the coefficients for the market and sector-level beta strategies, Panel B for the firm-level beta strategies, Panel C for the sector-level alpha strategies, and Panel D for the firm-level alpha strategies.

Panel A: Coefficients for market and sector-level beta strategies

	Benchmark	Short-sale		Universe				Transaction cost			Turnover		
	w/o rf	<0.5	0	growth	value	small	large	low	med	high	<0.2	<0.5	<1
Small-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0.155	0.196	0.189	0.226	0	0.801	0	0.120	0.089	0.059	0.142	0.155	0.155
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0
Small-Value													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0.002	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0
Large-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0.254	0.186	0.014	0.297	0	0	0.207	0.115	0.083	0.059	0.052	0.254	0.254
MOM	0	0	0.129	0	0	0	0	0	0	0	0	0	0
Large-Value													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0.427	0.431	0.417	0	0.527	0	0.442	0.192	0.131	0.086	0.370	0.427	0.427
MOM	0	0	0.072	0	0.001	0	0	0.006	0.009	0.010	0	0	0

Panel B: Coefficients for firm-level beta strategies

	Benchmark	Short-sale		Universe				Transaction cost			Turnover		
	w/o rf	<0.5	0	growth	value	small	large	low	med	high	<0.2	<0.5	<1
Small-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0
Small-Value													
ME	0	0	0.180	0	0	0.088	0	0	0.020	0.059	0.075	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0
Large-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0
Large-Value													
ME	0	0	0	0	0	0	0	0	0.038	0.080	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0

(*continued on next page*)

basis point difference), while relative to the bespoke constrained benchmark the performances are −6.1% and −2.7% (340 basis points difference), respectively.

With our two examples as a backdrop, we turn our attention to constructing the full sample distribution of bespoke performance levels and fund rankings. Note that to be included in this analysis we require our sample funds to have a minimum of 10 years of data. We adopt this investment horizon screen to focus attention on fund performance over the long-run. This screen results in 71 funds with which we repeat the process applied to the example funds above, namely, we estimate the parameters of the unconstrained and constrained portfolio policies by matching the time-series of the fund return and the market data.

To facilitate our assessment of fund performance, we define two comparison metrics. The first is *excess risk* defined as $\sigma_i - \sigma_b$, where σ_i is the standard deviation of fund i's excess returns and σ_b is the standard deviation of the benchmark portfolio's excess returns. The second is the *return-adjusted excess risk* defined as $\frac{\sigma_i - \sigma_b}{\mu_i}$, where the numerator is the *excess risk* metric defined above and μ_i is the mean excess return of fund i. The return-adjusted excess risk effectively measures the excessive risk per unit of expected

Table 4 (*continued*).

Panel C: Coefficients for sector-level alpha strategies

	Benchmark	Short-sale		Universe				Transaction cost			Turnover		
	w/o rf	<0.5	0	growth	value	small	large	low	med	high	<0.2	<0.5	<1
Small-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0.009	0.009	0.005	0	0	0
Small-Value													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0.001	0.004	0.004	0	0	0
Large-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0.004	0.003	0.003	0	0	0
Large-Value													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0

Panel D: Coefficients for firm-level alpha strategies

	Benchmark	Short-sale		Universe				Transaction cost			Turnover		
	w/o rf	<0.5	0	growth	value	small	large	low	med	high	<0.2	<0.5	<1
Small-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0.413	0.371	0	0.500	0	0.268	0	0.252	0.174	0.095	0.278	0.413	0.413
MOM	0	0	0	0	0	0	0	0	0	0	0	0	0
Small-Value													
ME	0.318	0.003	0	0	0.306	0.004	0	0.178	0.107	0.030	0	0.318	0.318
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0.044	0.045	0	0	0.020	0	0	0.041	0.027	0.021	0	0.044	0.044
Large-Growth													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0.046	0	0	0.397	0.098	0.085	0.055	0	0	0
MOM	0.036	0.066	0	0.074	0	0	0.140	0	0	0	0	0.036	0.036
Large-Value													
ME	0	0	0	0	0	0	0	0	0	0	0	0	0
BTM	0	0	0	0	0	0	0	0	0	0	0	0	0
MOM	0.065	0.015	0	0	0.167	0	0.085	0.021	0.014	0.009	0	0.065	0.065

excess return and can be interpreted as the additional risk the fund manager needs to take for each percentage point of expected excess return earned by the fund.[11]

Our fund performance results are presented in Chart 8 and Table 5. Chart 8 displays the distribution of return-adjusted excess risk measured against the original and bespoke benchmarks and Table 5 displays the associated statistical comparisons between the two distributions. Chart 8 shows a dramatic shift in the distribution of return-adjusted excess risk toward zero, suggesting far less under-performance relative to their bespoke benchmarks than implied by the original benchmarks. Table 5 compares the moments of the two distributions. The results show a monotonic and statistically significant reduction in fund underperformance (mean) for both metrics with the inclusion of each additional mandate and constraint. In addition, the standard deviation of both metrics tends to increase with each added mandate and constraint, which suggests more of a performance discrepancy between funds than was previous appreciated. Skewness and kurtosis are more ambiguous, although there appears to be less skewness and kurtosis for the bespoke distribution of return-adjusted excess risk.

As the distribution of fund performance is altered with bespoke benchmarks, is it natural to hypothesize that the relative rank of funds would also change. Chart 9 displays the bespoke performance rankings and those rankings based on traditional performance for

[11] We present the differences in terms of risk, instead of the more common return (alpha) approach, for two reasons. First, changing the minimum-variance optimization to maximize return given risk poses an estimation challenge in that it penalizes highly constrained portfolios (e.g., no leverage, no short-sale, low turnover) to achieve a high risk target. Second, comparisons of Sharpe Ratios are also problematic, as managers with higher target return would be labeled as less skilled.

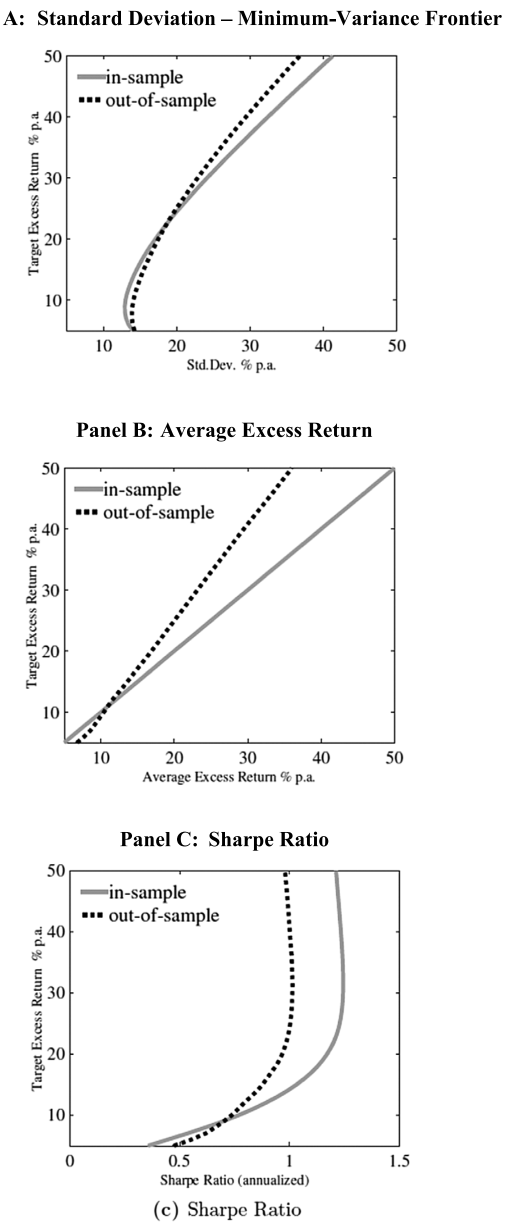


Chart 6. Comparison of in-sample and out-of-sample model performance. This series of charts compares the standard deviation, average excess return, and Sharpe ratio results within the in-sample and out-of-sample bespoke model estimations. The in-sample estimation is conducted over the period January 1974 through December 2013 while the initial out-of-sample estimation is conducted over the period January 1964 through December 1973 and then re-estimated recursively through December 2013, adding one month of data each iteration.

our sample of 71 funds partitioned into the four fund subgroups, large, small, growth and value, in panels A through D, respectively. As above, performance is measured as return-adjusted excess risk.[12] The chart is constructed so the diagonal represents the original rankings of funds and the vertical bars represent the bespoke rankings; thus, bars above (below) the diagonal represent a rank improvement (deterioration) with the bespoke benchmark. The results include a number of noteworthy observations. First, the bespoke rankings are statistically different for all subgroups. In particular, tests of the equality of matched pairs of observations using the Wilcoxon matched-pairs signed-ranks test can be rejected at the 1% level for each group. Second, of the four groups, the large-capitalization funds display the least change in the rankings, likely because the stock universe is small and liquidity ample

[12] While rankings based on risk-adjusted returns are theoretically appropriate, we acknowledge that market participants traditionally measure performance rank as a simple return difference between the fund and respective benchmark. As a robustness check we have compared the simple return differential rank with the bespoke rank and the results are both quantitatively and qualitatively similar to those shown.

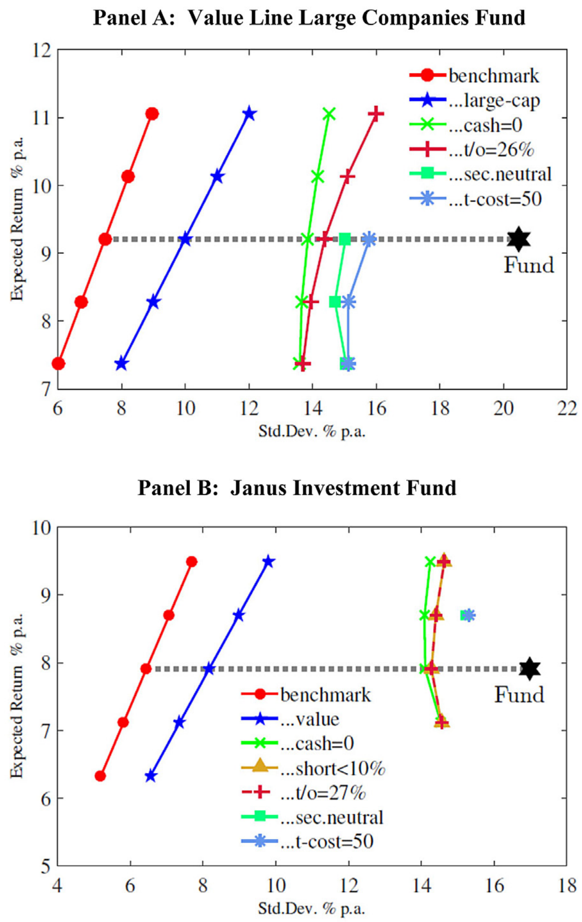


Chart 7. Examples of imposing fund-specific mandates and constraints. This chart provides two examples of imposing fund-specific mandates and constraints on the minimum-variance frontier and the resulting implied skill level of the manager. Each panel illustrates a series of bespoke benchmarks that are applicable to our example funds. Panel A details the *Value Line Large Companies Fund,* which is a large-capitalization fund with December 2013 AUM of \$0.21B and the S&P 500 as its chosen benchmark. Panel B shows the *Janus Investment Fund,* which is a value fund with December 2013 AUM of \$1.75B and the FTSE EPRA/NAREIT Developed and Global indices as its chosen benchmark.

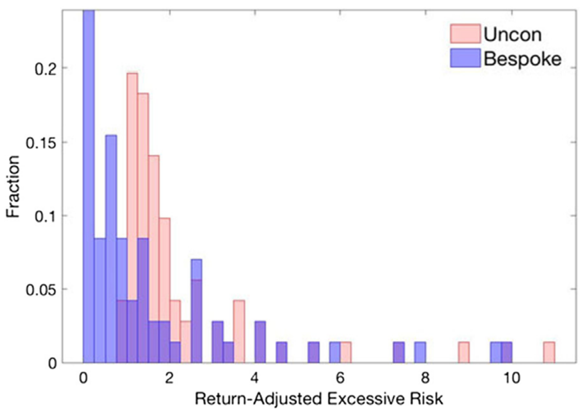


Chart 8. Comparison of standard and bespoke fund performance. The following chart compares fund performance measured against the standard (unconditional) benchmark to the bespoke benchmark for 71 of the sample funds that had at least a 10-year track record. The bespoke benchmark for each fund was estimated imposing the fund-specific constraints applicable to that fund. We define two comparison metrics; the first is *return-adjusted excess risk* defined as $\frac{\sigma_i-\sigma_b}{\mu_i}$, where μ_i is the mean excess return of fund i. The *return-adjusted excess risk* effectively measures the excessive risk per unit of expected excess return and can be interpreted as the amount of additional risks the fund manager needs to take for each percentage point of the expected excess return earned by the manager.

Table 5

Comparison of Standard and Bespoke Skill Distributions. We define two comparison metrics the first is *excess risk* defined $\sigma_i - \sigma_b$, where σ_i is the standard deviation of fund i's excess returns and σ_b is the standard deviation of excess returns to the benchmark portfolio, the second is *return-adjusted excess risk* defined as $\frac{\sigma_i - \sigma_b}{\mu_i}$, where μ_i is the mean excess return of fund i. The *return-adjusted excess risk* effectively measures the excessive risk per unit of expected excess return and can be interpreted as the amount of additional risks the fund manager needs to take for each percentage point of the expected excess return earned by the manager. Mandate and constraints, 1 through 4, are: Investment universe (size and style), cash holding and leverage, short-sale, and turnover, respectively.

Panel A: Summary Statistics of Performance Metrics

Metrics	Unconstrained	Bespoke with mandates and constraints:			
		1	1 + 2	1 + 2 + 3	1 + 2 + 3 + 4
Excess Risk					
Mean	14.39	12.49	10.71	9.92	8.51
Std. Dev.	4.16	4.91	6.02	6.07	6.53
Skew	0.40	0.42	0.25	0.32	0.52
Kurtosis	3.23	2.79	2.42	2.46	2.32
Return-adjusted excess risk					
Mean	2.37	2.13	1.92	1.82	1.67
Std. Dev.	2.02	2.05	2.16	2.12	2.18
Skew	2.56	2.49	2.32	2.21	2.18
Kurtosis	9.57	9.22	8.42	7.76	7.55

Panel B: Significance of Bespoke Benchmarking on Fund Manager Skills

Metrics	Unconstrained	Bespoke with mandates and constraints:			
		1	1 + 2	1 + 2 + 3	1 + 2 + 3 + 4
Excess Risk					
Level	14.39*	12.49*	10.71*	9.92*	8.51*
Change		−1.90*	−1.77*	−0.80*	−1.41*
Return-adjusted excess risk					
Level	2.37*	2.13*	1.92*	1.82*	1.67*
Change		−0.25*	−0.21*	−0.09*	−0.15*

*Indicates statistical significance at the level of 1%.

relative to the other groups. Third, ranking changes appear to be concentrated at the top of the rankings (best) rather than at the bottom, perhaps because the performance differential among funds is decreasing with rank. The most substantial change occurs within the rank of the top three funds for small, growth and value, with the highest ranked fund changing for small and value. As an example, consider the value group in Panel D. The top two funds in the original ranking were the Third Avenue Value Fund ($2.55B AUM, MSCI World Index as benchmark) and the Franklin Balance Sheet Investment Fund ($1.72B AUM, Russell 3000 Value Index as benchmark), respectively. After taking account of individual fund mandates and constraints, the Third Avenue Value Fund falls to third, the Franklin Balance Sheet Investment Fund remains second and the Gabelli Value Fund ($0.64B AUM with S&P500 as benchmark) takes over the top-ranked spot moving up from fourth place.

The *Janus* and *Value Line Fund* examples and altered fund rankings put in clear view the statistically and economically significant impact of mandates and constraints on mutual fund performance. We acknowledge that while mutual fund constituencies universally want accurate performance assessments, their interest in accurately measuring the *components*, mandates/constraints and manager skill, is predicated on whether they take mandates and constraints as *exogenous* or *endogenous* with respect to their goal or objective.

Mutual fund advisors and fund board of directors necessarily take fund mandates and constraints as fixed rules within which the fund manager must operate and are therefore exogenous to the objectives of performance-based compensation and the relative value-add of the advisor's management, respectively. Consider a mutual fund advisor's objective of compensating and retaining skillful portfolio managers. Since mandates and constraints are specified in the fund prospectus and very rarely, if ever change, the fund advisor would want to retain and reward/compensate portfolio managers who exhibit skill *within* the mutual fund's given mandates and constraints. Indeed, the fund advisor would prefer to employ the portfolio manager with the highest skill among the set of portfolio managers that operate within that investing environment. Hence, from a compensation and retention standpoint fund performance should be measured abstracting from the inherent characteristics of the fund that are captured in its mandates and constraints. Indeed, the findings of Khorana (1996) and Daniel et al. (1997) are consistent with measuring performance based on fund manager skill, rather than exogenous factors impacting performance. In addition, anecdotally, a Chief Investment Officer (CIO) of a prominent fund family acknowledged that despite the externally visible constraints in the prospectus, portfolio managers are compensated, not by their performance relative to the public benchmarks, but by their internal benchmark model which is subject to the constraints they impose on each manager to measure their alpha generation through stock selection on top of constraints factor exposure.

Mutual fund board members have a similar perspective whereby mandates and constraints are rarely considered a choice variable instead being treated as exogenous with respect to their fiduciary responsibility to ensure the value/performance delivered is appropriate for the fees charged. The Investment Company Act of 1940 requires that mutual fund board of directors annually

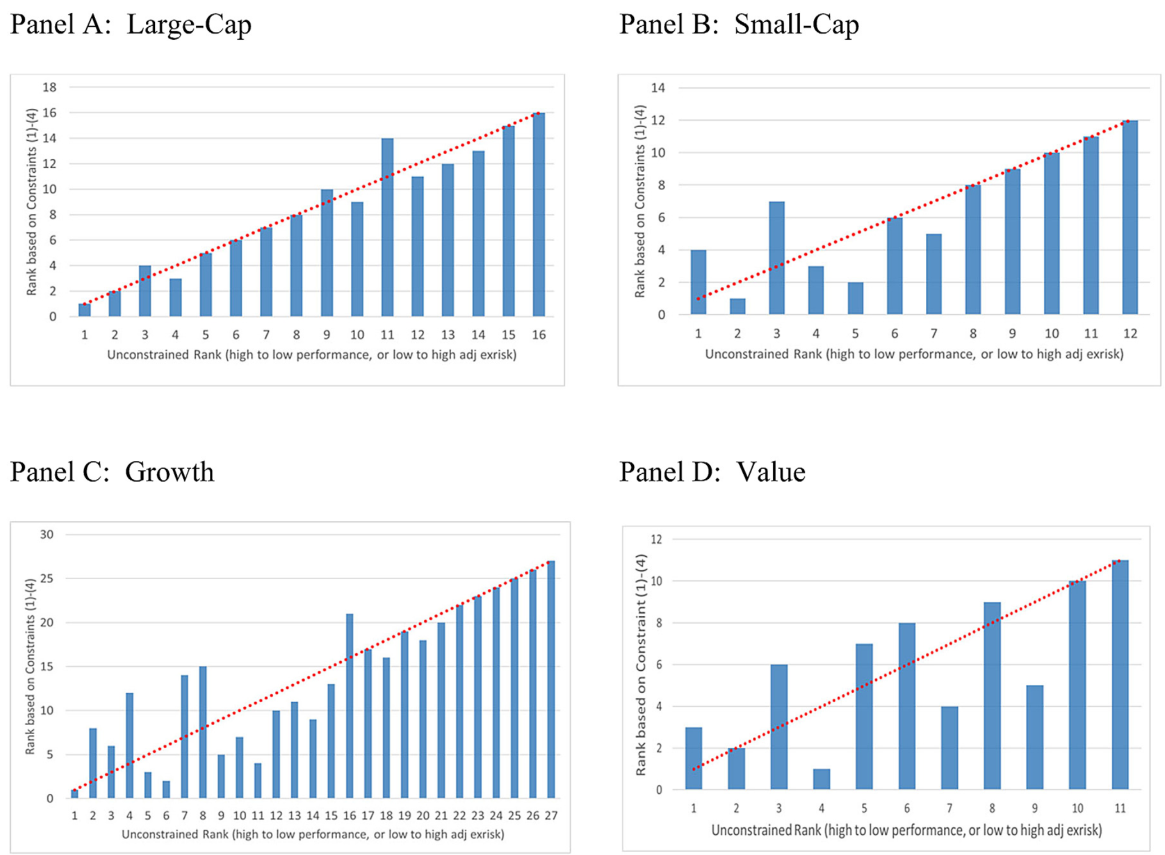


Chart 9. Change in group rankings based on bespoke benchmark performance. The following chart compares fund rankings based on bespoke performance and rankings based on traditionally-measured performance for our sample of 71 funds partitioned by large-cap, small-cap, growth and value groups respectively. The diagonal line in each panel represents the group ranking based on traditionally-measured performance and the vertical bars represent the rankings based the bespoke performance. Tests of the equality of matched pairs of observations for each group using the Wilcoxon matched-pairs signed-ranks test can be rejected at the 1% level.

review and approve the fund advisory contract. The fund directors are specifically asked to consider a number of factors, known as the *Gartenberg Factors* after the current precedent setting legal case, when making their determination. Specifically, directors must consider and evaluate: (1) the nature, extent, and quality of the services provided by the advisor, including the investment process used by the advisor; (2) *the performance of the funds in comparison to their benchmark indices and a peer group of mutual funds;* (3) the management fees and total operating expenses of the funds, including comparative information with respect to a peer group of mutual funds; (4) the profitability of the advisor with respect to the funds; and (5) the extent to which economies of scale may be realized as the funds grow. The proper evaluation of each of these Gartenberg Factors necessitates assessing the value provided the shareholder by the advisor abstracting from the impact of the mandates and constraints imposed on the fund. Consistent with the need for directors to focus on manager skill in conducting their review, research by Ding and Wermers (2012) provide evidence that funds with superior ‘‘internal governance’’ are better able to monitor performance and terminate underperforming, inexperienced managers. Thus, from a practical perspective for the fund advisor and a regulatory perspective for the fund board of directors, the importance of being able to properly assess the performance of a mutual fund abstracting from the exogenously given mandates and constraints can hardly be overstated.

6. Conclusion

We develop a methodology to incorporate frictions into financial settings where heretofore their impact could not be quantified. We apply our methodology to the lingering academic quandary regarding the appropriateness of comparing mutual fund performance to a benchmark that does not share the same mandates and constraints. The approach utilizes a parametric re-mapping of portfolio weights having imposed individual fund mandates and constraints. Our results demonstrate that fund mandates and constraints are pervasive and impose costs on funds that are economically important. Consistent with their importance, they impact both fund portfolio weights and manager trading strategies.

By constructing bespoke benchmarks for a sample of mutual funds, we are able to improve on the performance evaluation of mutual funds and their relative ranking. A comparison of fund performance relative to its appropriate bespoke benchmark within our sample shows a monotonic increase in relative performance with the inclusion of each additional mandate and constraint, where the aggregate increase is between 30 and 40%. More important than the increase in relative performance is the change in the ranking of peer funds. Performance rankings that account for fund mandates and constraints are significantly and economically different than traditional rankings which fail to do so. Funds investing in small-cap, growth and value display the largest ranking changes, particularly for the top ranked funds. Armed with these bespoke rankings, mutual fund advisors and board of directors would likely make different decisions suggesting an improved allocation of capital and oversight among mutual funds.

From the practitioner's perspective, our results are important for mutual fund advisors/management and fund directors/boards. For mutual fund advisors/management should be evaluating portfolio managers based on mandate/constraint adjusted performance for compensation and retention purposes. In addition, in order for mutual fund directors/boards to execute their fiduciary responsibility to their shareholders, they should be comparing the fund advisor's performance against an appropriate bespoke benchmark during their annual 15(c)-3 review of the fund's advisory contract.

References

Blake, C., Morey, M., 2000. Morningstar ratings and mutual fund performance. J. Financ. Quant. Anal. 35, 451–483.

Brandt, M., Santa-Clara, P., Valkanov, R., 2009. Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns. Rev. Financ. Stud. 22, 3411–3447.

Briere, M., Szafarz, A., 2017. In: Jurczenko, E. (Ed.), Factor Investing: The Rocky Road from Long Only to Long Short in Factor Investing. ISTE Press Ltd.

Cremers, M., Fulkerson, J., Riley, T., 2018. Benchmark Discrepancies and Mutual Fund Performance Evaluation. University of Notre Dame, Working Paper.

Daniel, K., Grinblatt, M., Titman, S., Wermers, R., 1997. Measuring mutual fund performance with characteristic-based benchmarks. J. Finance 52, 1035–1058.

Del Guercio, D., Tkac, P., 2008. Star power: The effect of morningstar ratings on mutual fund flow. J. Financ. Quant. Anal. 43, 907–936.

Ding, B., Wermers, R., 2012. Mutual Fund Performance and Governance Structure: The Role of Portfolio Managers and Board of Directors. University of Maryland, Working Paper.

Friesen, G., Nguyen, V., 2018. The Economic Impact of Mutual Fund Investor Behaviors. University of Nebraska, Working Paper.

Khorana, A., 1996. Top management turnover an empirical investigation of mutual fund managers. J. Financ. Econ. 40, 403–427.

Mateus, I., Mateus, C., Todorovic, N., 2017. The Impact of Benchmark Choice on US Mutual Fund Benchmark-Adjusted Performance and Ranking. University of Greenwich, Working Paper.

Palmiter, A., 2016. The mutual fund investor. In: Elgar HandBook of Mutual Fund Regulation. http://dx.doi.org/10.2139/ssrn.2853506, (forthcoming).

Sensoy, B., 2009. Performance evaluation and self-designated benchmark indexes in the mutual fund industry. J. Financ. Econ. 92, 25–39.