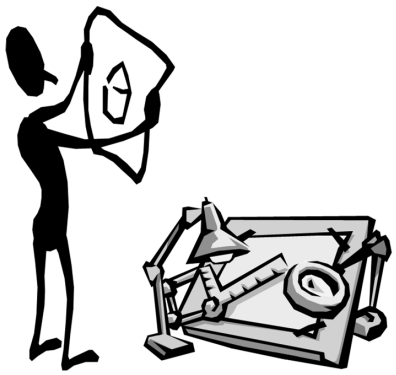
2^k Factorial Designs

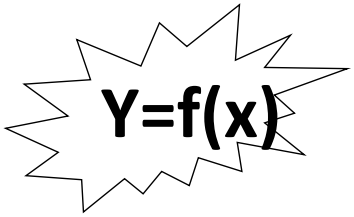


© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Experiments – Defined

*An experiment is a test or series of tests in which **purposeful changes** are made to **input variables** of a process or system so that changes in the **output responses** can be observed and identified*

Douglas C. Montgomery



© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

2^k Factorial Designs -- Notation

- 2^k designs are experiments where all factors have only two levels.
- Notation:
  - In General: How many factors and levels in a 2 x 2 x 2? How many treatment combinations in a full factorial?
  - How many factors and levels in a 2 x 2 x 2 x 2 x 2? How many treatment combinations in a full factorial? What does 2^5 equal?
  - How many factors and levels in a 2^7 experiment? How many treatment combinations?
- 2^k
  - How many factors and levels in a 2^k factorial?
  - How many treatment combinations?

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

DOE Method Demonstration

- A Green Belt wants to determine the impact of different levels of advertising on product recognition.
- He designs an experiment using:
  - Two different Television profiles (two vs. three in a week),
  - Two Radio profiles (daily vs. three times per week), and
  - Two different Newspaper profiles (weekend only vs. daily).
- He has identified enough “similar” market cities to run one replicate.

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

A Few Basics

- In a 2^k experiment:
  - One factor level is designated "low" and coded as a –1
  - The other level is designated as "high" and coded as a +1
- Standard order: Table of contrasts

TV	Radio	Press
-1	-1	-1
1	-1	-1
-1	1	-1
1	1	-1
-1	-1	1
1	-1	1
-1	1	1
1	1	1

PRACTICAL INTERPRETATION

RunOrder	tv	radio	newspaper
1	2x/week	daily	weekend
2	3x/week	daily	weekend
3	2x/week	3x/week	weekend
4	3x/week	3x/week	weekend
5	2x/week	daily	daily
6	3x/week	daily	daily
7	2x/week	3x/week	daily
8	3x/week	3x/week	daily

- Exercise
  - Create a 2^4 Factorial Design Matrix
  - How many runs are needed?

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Main Effects

- In a 2^k DOE:
  - The Main Effect for a factor is the average of all the data when the factor is at the high level minus the average of all the data when the factor is at the low level
  - Or:

$$MainEffect = \overline{\text{Response}}_{High} - \overline{\text{Response}}_{Low}$$

  - For our experiment the Main Effect for TV is:

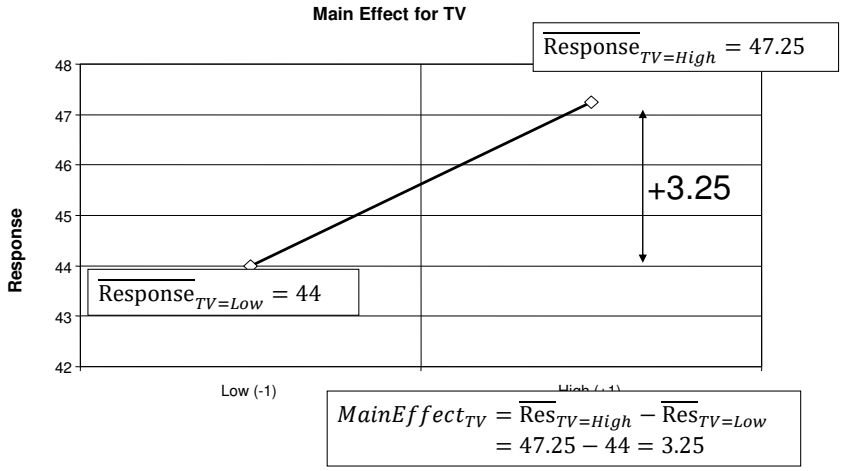
$$\text{TV} = \frac{+ \quad + \quad +}{} - \frac{+ \quad + \quad +}{}$$
$$=$$

TV	Radio	Press	Recognition
-1	-1	-1	43
1	-1	-1	45
-1	1	-1	45
1	1	-1	49
-1	-1	1	43
1	-1	1	46
-1	1	1	45
1	1	1	49

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Graphing Main Effects

Advertise.mtw



© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

New Product Promotion Example

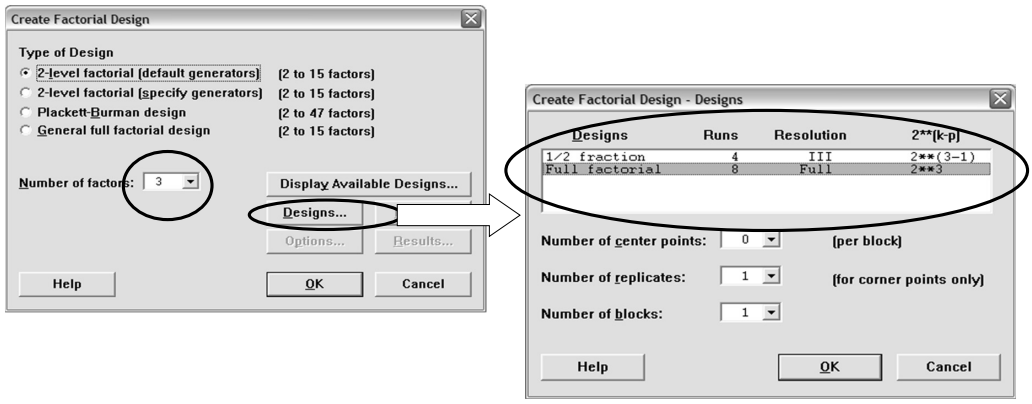
- Step 1: State the practical problem
  - The Black/Green Belt would like to improve recognition for a new product.
- Step 2: State the factors and levels of interest
  - TV Profile: 2x/week, 3x/week
  - Radio Profile: Daily, 3x/week
  - Newspaper Profile: Weekend only, Daily
- Step 3: Select the sample size
  - Given: 1 replicate (How many runs?)

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Step 4: Design the Experiment

Advertise.mtw

- In Minitab: Stat>DOE>Factorial>Create Factorial Design

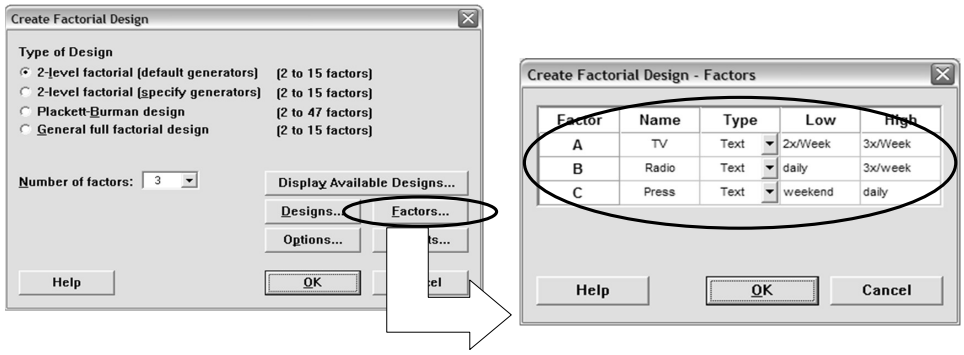


© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Step 4: Design the Experiment (cont.)

Advertise.mtw

- Select the Factors button:



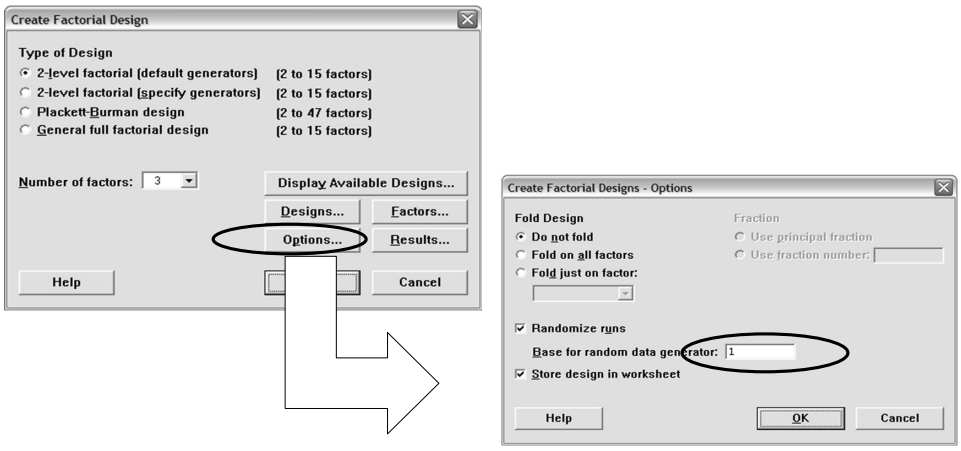
? ***Why doesn't Minitab need the level values?***

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Step 4: Design the Experiment (cont.)

Advertise.mtw

- Select the Options… button:



© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Step 4: Minitab Output

Advertise.mtw

- Click "OK" on the main DOE Menu

↓	C1	C2	C3	C4	C5-T	C6-T	C7-T
	StdOrder	RunOrder	CenterPt	Blocks	TV	Radio	Press
1	2	1	1	1	3x/Week	daily	weekend
2	8	2	1	1	3x/Week	3x/week	daily
3	6	3	1	1	3x/Week	daily	daily
4	1	4	1	1	2x/Week	daily	weekend
5	4	5	1	1	3x/Week	3x/week	weekend
6	7	6	1	1	2x/Week	3x/week	daily
7	3	7	1	1	2x/Week	3x/week	weekend
8	5	8	1	1	2x/Week	daily	daily

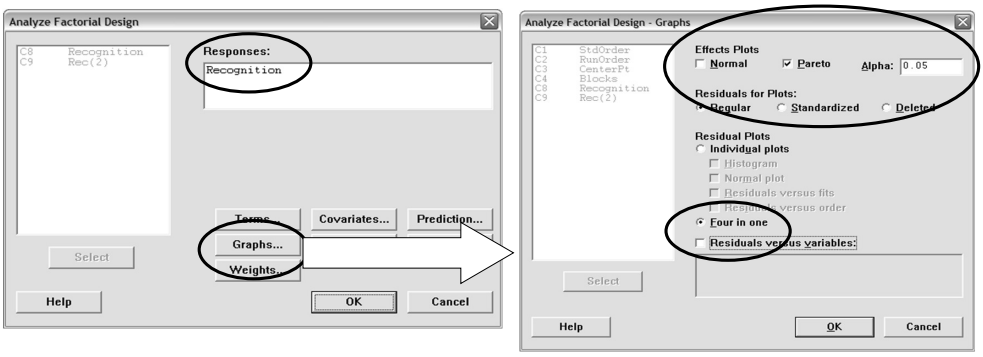
- Step 5: Run the experiment/Collect the data

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Step 6: Analyze the Full Model

Advertise.mtw

- Open Minitab worksheet
  - Select Stat>DOE>Factorial>Analyze Factorial Designs…



© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Minitab Output – Full Model

Advertise.mtw

Estimated Effects and Coefficients
for Recognition (coded units)

Term	Effect	Coef
Constant		45.6250
TV	3.2500	1.6250
Radio	2.7500	1.3750
Press	0.2500	0.1250
TV*Radio	0.7500	0.3750
TV*Press	0.2500	0.1250
Radio*Press	-0.2500	-0.1250
TV*Radio*Press	-0.2500	-0.1250

? ***Do these look familiar?***

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Step 11: Mathematical Model

- From the Minitab analysis:

Term	Effect	Coef
Constant		45.6250
TV	3.2500	1.6250
Radio	2.7500	1.3750
TV*Radio	0.7500	0.3750

$$(Recognition) = 45.6 + 1.6 \times TV + 1.4 \times Radio + 0.4 \times (TV * Radio)$$

The focus of Six Sigma: Y=f(x)!

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

The Mathematical (Empirical) Model

- What is the value for Recognition if all of the levels were set high?

$$Rec = 45.6 + 1.6 \times 1 + 1.4 \times 1 + 0.4 \times (1 * 1) = 49$$

  - Low?

$$Rec = 45.6 + 1.6 \times -1 + 1.4 \times -1 + 0.4 \times (-1 * -1) = 43$$

- Where should they be set to maximize Recognition?
  - They should all be set high
- What is the value when all effects are set to zero?

$$Rec = 45.6 + 1.6 \times 0 + 1.4 \times 0 + 0.4 \times (0 * 0) = 45.6$$

- What does this value represent?
  - The grand average of the experiment, or, if all effects are linear, the expected value when the levels are at the midpoints

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.

Step 12: Translate into Practical Terms

- Conclusions:
  - On average, if the process parameters are set neither high nor low, the mean Recognition be 45.6
  - TV & Radio advertising are both important to recognition will, the effects are not simply additive.
  - The interaction suggests a synergistic effect between TV and Radio advertising
- Recommendations:
  - To maximize Recognition set TV & Radio advertising to the high level. The estimate for Recognition at this operating point is 49.

$$Recognition = 45.6 + 1.6 \times TV + 1.4 \times Radio + 0.4 \times (TV * Radio)$$

Step 13: Replicate the optimum conditions. Plan the next experiment and institutionalize the change.

Practical results are the goal of experimentation

© August 21, Institute for Industrial Performance & Engagement. Unpublished proprietary work available only under license. All rights reserved.