

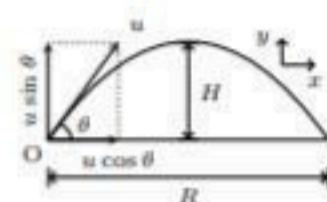
Motion in a straight line with constant a :

$$v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad v^2 - u^2 = 2as$$

0.1: Physical Constants

Speed of light	c	3×10^8 m/s
Planck constant	h	6.63×10^{-34} J s
	hc	1242 eV-nm
Gravitation constant	G	6.67×10^{-11} m 3 kg $^{-1}$ s $^{-2}$
Boltzmann constant	k	1.38×10^{-23} J/K
Molar gas constant	R	8.314 J/(mol K)
Avogadro's number	N_A	6.023×10^{23} mol $^{-1}$
Charge of electron	e	1.602×10^{-19} C
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$ N/A 2
Permitivity of vacuum	ϵ_0	8.85×10^{-12} F/m
Coulomb constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N m 2 /C 2
Faraday constant	F	96485 C/mol
Mass of electron	m_e	9.1×10^{-31} kg
Mass of proton	m_p	1.6726×10^{-27} kg
Mass of neutron	m_n	1.6749×10^{-27} kg
Atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit	u	931.49 MeV/c 2
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/(m 2 K 4)
Rydberg constant	R_∞	1.097×10^7 m $^{-1}$
Bohr magneton	μ_B	9.27×10^{-24} J/T
Bohr radius	a_0	0.529×10^{-10} m
Standard atmosphere	atm	1.01325×10^5 Pa
Wien displacement constant	b	2.9×10^{-3} m K

Relative Velocity: $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$



Projectile Motion:

$$\begin{aligned}x &= ut \cos \theta, \quad y = ut \sin \theta - \frac{1}{2}gt^2 \\y &= x \tan \theta - \frac{g}{2u^2 \cos^2 \theta} x^2 \\T &= \frac{2u \sin \theta}{g}, \quad R = \frac{u^2 \sin 2\theta}{g}, \quad H = \frac{u^2 \sin^2 \theta}{2g}\end{aligned}$$

1.3: Newton's Laws and Friction

Linear momentum: $\vec{p} = m\vec{v}$

Newton's first law: inertial frame.

Newton's second law: $\vec{F} = \frac{d\vec{p}}{dt}, \quad \vec{F} = m\vec{a}$

Newton's third law: $\vec{F}_{AB} = -\vec{F}_{BA}$

Frictional force: $f_{\text{static, max}} = \mu_s N, \quad f_{\text{kinetic}} = \mu_k N$

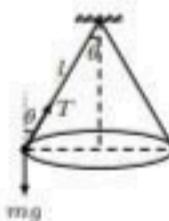
Banking angle: $\frac{v^2}{rg} = \tan \theta, \quad \frac{v^2}{rg} = \frac{\mu + \tan \theta}{1 - \mu \tan \theta}$

Centripetal force: $F_c = \frac{mv^2}{r}, \quad a_c = \frac{v^2}{r}$

Pseudo force: $\vec{F}_{\text{pseudo}} = -m\vec{a}_0, \quad F_{\text{centrifugal}} = -\frac{mv^2}{r}$

Minimum speed to complete vertical circle:

$$v_{\min, \text{bottom}} = \sqrt{5gl}, \quad v_{\min, \text{top}} = \sqrt{gl}$$



Conical pendulum: $T = 2\pi \sqrt{\frac{l \cos \theta}{g}}$

1 MECHANICS

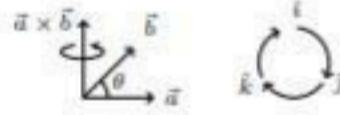
1.1: Vectors

Notation: $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

Magnitude: $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Dot product: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = ab \cos \theta$

Cross product:



$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y) \hat{i} + (a_z b_x - a_x b_z) \hat{j} + (a_x b_y - a_y b_x) \hat{k}$$

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

1.2: Kinematics

Average and Instantaneous Vel. and Accel.:

$$\vec{v}_{\text{av}} = \Delta \vec{r} / \Delta t,$$

$$\vec{a}_{\text{av}} = \Delta \vec{v} / \Delta t$$

$$\vec{v}_{\text{inst}} = d\vec{r}/dt$$

$$\vec{a}_{\text{inst}} = d\vec{v}/dt$$

1.4: Work, Power and Energy

Work: $W = \vec{F} \cdot \vec{S} = FS \cos \theta, \quad W = \int \vec{F} \cdot d\vec{S}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $F = -\partial U/\partial x$ for conservative forces.

$$U_{\text{gravitational}} = mgh, \quad U_{\text{spring}} = \frac{1}{2}kx^2$$

Work done by conservative forces is path independent and depends only on initial and final points:
 $\oint \vec{F}_{\text{conservative}} \cdot d\vec{r} = 0$.

Work-energy theorem: $W = \Delta K$

Mechanical energy: $E = U + K$. Conserved if forces are conservative in nature.

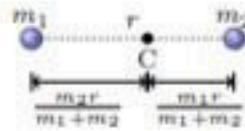
Power $P_{av} = \frac{\Delta W}{\Delta t}$, $P_{inst} = \vec{F} \cdot \vec{v}$

1.5: Centre of Mass and Collision

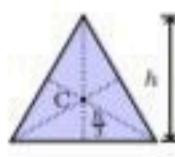
Centre of mass: $x_{cm} = \frac{\sum x_i m_i}{\sum m_i}$, $x_{cm} = \frac{\int x dm}{\int dm}$

CM of few useful configurations:

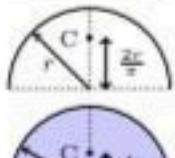
1. m_1, m_2 separated by r :



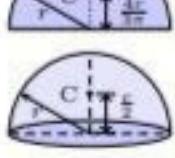
2. Triangle (CM ≡ Centroid) $y_c = \frac{h}{3}$



3. Semicircular ring: $y_c = \frac{2r}{\pi}$



4. Semicircular disc: $y_c = \frac{4r}{3\pi}$



5. Hemispherical shell: $y_c = \frac{r}{2}$



6. Solid Hemisphere: $y_c = \frac{3r}{8}$



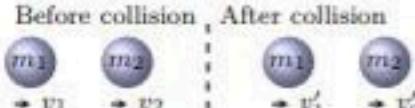
7. Cone: the height of CM from the base is $h/4$ for the solid cone and $h/3$ for the hollow cone.

Motion of the CM: $M = \sum m_i$

$$\vec{v}_{cm} = \frac{\sum m_i \vec{v}_i}{M}, \quad \vec{p}_{cm} = M \vec{v}_{cm}, \quad \vec{a}_{cm} = \frac{\vec{F}_{ext}}{M}$$

Impulse: $\vec{J} = \int \vec{F} dt = \Delta \vec{p}$

Collision:



Momentum conservation: $m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$
Elastic Collision: $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v'_1^2 + \frac{1}{2} m_2 v'_2^2$
Coefficient of restitution:

$$e = \frac{-(v'_1 - v'_2)}{v_1 - v_2} = \begin{cases} 1, & \text{completely elastic} \\ 0, & \text{completely inelastic} \end{cases}$$

If $v_2 = 0$ and $m_1 \ll m_2$ then $v'_1 = -v_1$.

If $v_2 = 0$ and $m_1 \gg m_2$ then $v'_2 = 2v_1$.

Elastic collision with $m_1 = m_2$: $v'_1 = v_2$ and $v'_2 = v_1$.

1.6: Rigid Body Dynamics

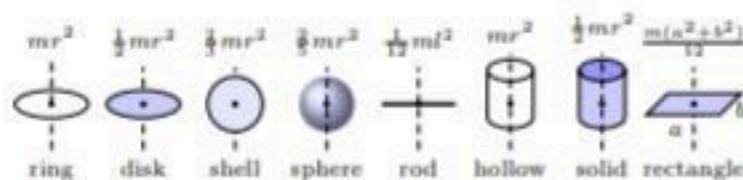
Angular velocity: $\omega_{av} = \frac{\Delta \theta}{\Delta t}$, $\omega = \frac{d\theta}{dt}$, $\vec{v} = \vec{\omega} \times \vec{r}$

Angular Accel.: $\alpha_{av} = \frac{\Delta \omega}{\Delta t}$, $\alpha = \frac{d\omega}{dt}$, $\vec{a} = \vec{\alpha} \times \vec{r}$

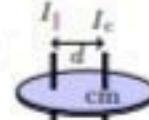
Rotation about an axis with constant α :

$$\omega = \omega_0 + \alpha t, \quad \theta = \omega_0 t + \frac{1}{2} \alpha t^2, \quad \omega^2 - \omega_0^2 = 2\alpha\theta$$

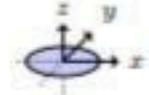
Moment of Inertia: $I = \sum_i m_i r_i^2$, $I = \int r^2 dm$



Theorem of Parallel Axes: $I_{\parallel} = I_{cm} + md^2$



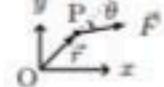
Theorem of Perp. Axes: $I_z = I_x + I_y$



Radius of Gyration: $k = \sqrt{I/m}$

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$, $\vec{L} = I\vec{\omega}$

Torque: $\vec{\tau} = \vec{r} \times \vec{F}$, $\vec{\tau} = \frac{d\vec{L}}{dt}$, $\tau = I\alpha$



Conservation of \vec{L} : $\vec{\tau}_{ext} = 0 \implies \vec{L} = \text{const.}$

Equilibrium condition: $\sum \vec{F} = \vec{0}$, $\sum \vec{\tau} = \vec{0}$

Kinetic Energy: $K_{rot} = \frac{1}{2} I \omega^2$

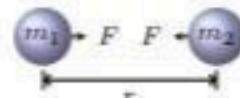
Dynamics:

$$\vec{r}_{cm} = I_{cm} \vec{\alpha}, \quad \vec{F}_{ext} = m \vec{a}_{cm}, \quad \vec{p}_{cm} = m \vec{v}_{cm}$$

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2, \quad \vec{L} = I_{cm} \vec{\omega} + \vec{r}_{cm} \times m \vec{v}_{cm}$$

1.7: Gravitation

Gravitational force: $F = G \frac{m_1 m_2}{r^2}$



Potential energy: $U = -\frac{GMm}{r}$

Gravitational acceleration: $g = \frac{GM}{R^2}$

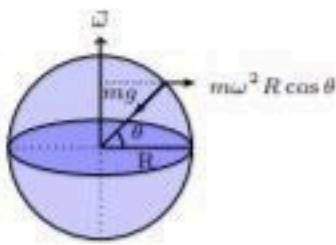
Variation of g with depth: $g_{\text{inside}} \approx g(1 - \frac{h}{R})$

Variation of g with height: $g_{\text{outside}} \approx g(1 - \frac{2h}{R})$

Effect of non-spherical earth shape on g :
 $g_{\text{at pole}} > g_{\text{at equator}}$ ($\because R_e - R_p \approx 21 \text{ km}$)

Effect of earth rotation on apparent weight:

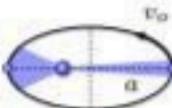
$$mg'_\theta = mg - m\omega^2 R \cos^2 \theta$$



$$\text{Orbital velocity of satellite: } v_o = \sqrt{\frac{GM}{R}}$$

$$\text{Escape velocity: } v_e = \sqrt{\frac{2GM}{R}}$$

Kepler's laws:



First: Elliptical orbit with sun at one of the focus.

Second: Areal velocity is constant. ($\therefore d\vec{L}/dt = 0$).

Third: $T^2 \propto a^3$. In circular orbit $T^2 = \frac{4\pi^2}{GM} a^3$.

1.8: Simple Harmonic Motion

Hooke's law: $F = -kx$ (for small elongation x .)

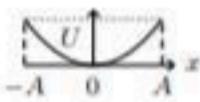
$$\text{Acceleration: } a = \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

$$\text{Time period: } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

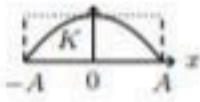
$$\text{Displacement: } x = A \sin(\omega t + \phi)$$

$$\text{Velocity: } v = A\omega \cos(\omega t + \phi) = \pm \omega \sqrt{A^2 - x^2}$$

$$\text{Potential energy: } U = \frac{1}{2}kx^2$$



$$\text{Kinetic energy } K = \frac{1}{2}mv^2$$



$$\text{Total energy: } E = U + K = \frac{1}{2}m\omega^2 A^2$$

$$\text{Simple pendulum: } T = 2\pi\sqrt{\frac{l}{g}}$$



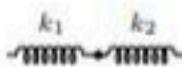
$$\text{Physical Pendulum: } T = 2\pi\sqrt{\frac{l}{mgI}}$$



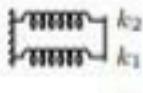
$$\text{Torsional Pendulum: } T = 2\pi\sqrt{\frac{l}{k}}$$



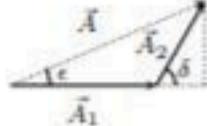
$$\text{Springs in series: } \frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2}$$



$$\text{Springs in parallel: } k_{\text{eq}} = k_1 + k_2$$



Superposition of two SHM's:



$$x_1 = A_1 \sin \omega t, \quad x_2 = A_2 \sin(\omega t + \delta)$$

$$x = x_1 + x_2 = A \sin(\omega t + \epsilon)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta}$$

$$\tan \epsilon = \frac{A_2 \sin \delta}{A_1 + A_2 \cos \delta}$$

1.9: Properties of Matter

$$\text{Modulus of rigidity: } Y = \frac{F/A}{\Delta l/l}, \quad B = -V \frac{\Delta P}{\Delta V}, \quad \eta = \frac{F}{Ad}$$

$$\text{Compressibility: } K = \frac{1}{B} = -\frac{1}{V} \frac{dV}{dP}$$

$$\text{Poisson's ratio: } \sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\Delta D/D}{\Delta l/l}$$

$$\text{Elastic energy: } U = \frac{1}{2} \text{ stress} \times \text{strain} \times \text{volume}$$

$$\text{Surface tension: } S = F/l$$

$$\text{Surface energy: } U = SA$$

$$\text{Excess pressure in bubble:}$$

$$\Delta p_{\text{air}} = 2S/R, \quad \Delta p_{\text{soap}} = 4S/R$$

$$\text{Capillary rise: } h = \frac{2S \cos \theta}{r \rho g}$$

$$\text{Hydrostatic pressure: } p = \rho gh$$

$$\text{Buoyant force: } F_B = \rho V g = \text{Weight of displaced liquid}$$

$$\text{Equation of continuity: } A_1 v_1 = A_2 v_2 \quad v_1 \rightarrow v_2$$

$$\text{Bernoulli's equation: } p + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

$$\text{Torricelli's theorem: } v_{\text{efflux}} = \sqrt{2gh}$$

$$\text{Viscous force: } F = -\eta A \frac{dv}{dx}$$

$$\text{Stoke's law: } F = 6\pi\eta rv$$



$$\text{Poiseuilli's equation: } \frac{\text{Volume flow}}{\text{time}} = \frac{\pi p r^4}{8\eta l}$$



$$\text{Terminal velocity: } v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

2 Waves

2.1: Waves Motion

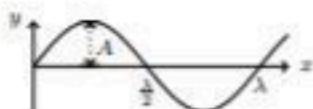
General equation of wave: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$.

Notation: Amplitude A , Frequency ν , Wavelength λ , Period T , Angular Frequency ω , Wave Number k ,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega}, \quad v = \nu\lambda, \quad k = \frac{2\pi}{\lambda}$$

Progressive wave travelling with speed v :

$$y = f(t - x/v), \text{ towards } +x; \quad y = f(t + x/v), \text{ towards } -x$$



Progressive sine wave:

$$y = A \sin(kx - \omega t) = A \sin(2\pi(x/\lambda - t/T))$$

2.2: Waves on a String

Speed of waves on a string with mass per unit length μ and tension T : $v = \sqrt{T/\mu}$

Transmitted power: $P_{av} = 2\pi^2 \mu v A^2 \nu^2$

Interference:

$$y_1 = A_1 \sin(kx - \omega t), \quad y_2 = A_2 \sin(kx - \omega t + \delta)$$

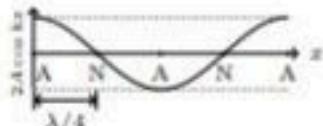
$$y = y_1 + y_2 = A \sin(kx - \omega t + \epsilon)$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta}$$

$$\tan \epsilon = \frac{A_2 \sin \delta}{A_1 + A_2 \cos \delta}$$

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive.} \end{cases}$$

Standing Waves:

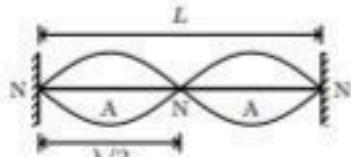


$$y_1 = A_1 \sin(kx - \omega t), \quad y_2 = A_2 \sin(kx + \omega t)$$

$$y = y_1 + y_2 = (2A \cos kx) \sin \omega t$$

$$x = \begin{cases} \left(n + \frac{1}{2}\right) \frac{\lambda}{2}, & \text{nodes; } n = 0, 1, 2, \dots \\ n \frac{\lambda}{2}, & \text{antinodes. } n = 0, 1, 2, \dots \end{cases}$$

String fixed at both ends:



- Boundary conditions: $y = 0$ at $x = 0$ and at $x = L$

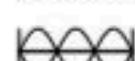
- Allowed Freq.: $L = n \frac{\lambda}{2}$, $\nu = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$, $n = 1, 2, 3, \dots$

- Fundamental/1st harmonics: $\nu_0 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$

- 1st overtone/2nd harmonics: $\nu_1 = \frac{2}{2L} \sqrt{\frac{T}{\mu}}$

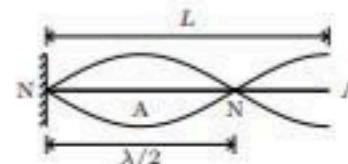


- 2nd overtone/3rd harmonics: $\nu_2 = \frac{3}{2L} \sqrt{\frac{T}{\mu}}$



- All harmonics are present.

String fixed at one end:



- Boundary conditions: $y = 0$ at $x = 0$

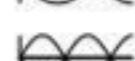
- Allowed Freq.: $L = (2n+1) \frac{\lambda}{4}$, $\nu = \frac{2n+1}{4L} \sqrt{\frac{T}{\mu}}$, $n = 0, 1, 2, \dots$



- Fundamental/1st harmonics: $\nu_0 = \frac{1}{4L} \sqrt{\frac{T}{\mu}}$



- 1st overtone/2nd harmonics: $\nu_1 = \frac{3}{4L} \sqrt{\frac{T}{\mu}}$



- 2nd overtone/5th harmonics: $\nu_2 = \frac{5}{4L} \sqrt{\frac{T}{\mu}}$

- Only odd harmonics are present.

Sonometer: $\nu \propto \frac{1}{L}$, $\nu \propto \sqrt{T}$, $\nu \propto \frac{1}{\sqrt{\mu}}$, $\nu = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$

2.3: Sound Waves

Displacement wave: $s = s_0 \sin \omega(t - x/v)$

Pressure wave: $p = p_0 \cos \omega(t - x/v)$, $p_0 = (B\omega/v)s_0$

Speed of sound waves:

$$v_{\text{liquid}} = \sqrt{\frac{B}{\rho}}, \quad v_{\text{solid}} = \sqrt{\frac{Y}{\rho}}, \quad v_{\text{gas}} = \sqrt{\frac{\gamma P}{\rho}}$$

$$\text{Intensity: } I = \frac{2\pi^2 B}{v} s_0^2 v^2 = \frac{p_0^2 v}{2B} = \frac{p_0^2}{2\rho v}$$

Standing longitudinal waves:

$$p_1 = p_0 \sin \omega(t - x/v), \quad p_2 = p_0 \sin \omega(t + x/v)$$

$$p = p_1 + p_2 = 2p_0 \cos kx \sin \omega t$$



Closed organ pipe:

- Boundary condition: $y = 0$ at $x = 0$

- Allowed freq.: $L = (2n+1) \frac{\lambda}{4}$, $\nu = (2n+1) \frac{v}{4L}$, $n = 0, 1, 2, \dots$



- Fundamental/1st harmonics: $\nu_0 = \frac{v}{4L}$



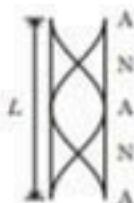
- 1st overtone/3rd harmonics: $\nu_1 = 3\nu_0 = \frac{3v}{4L}$

5. 2nd overtone/5th harmonics: $\nu_2 = 5\nu_0 = \frac{5v}{4L}$

6. Only odd harmonics are present.



Open organ pipe:



1. Boundary condition: $y = 0$ at $x = 0$

Allowed freq.: $L = n\frac{\lambda}{2}$, $\nu = n\frac{v}{4L}$, $n = 1, 2, \dots$

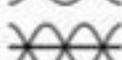
2. Fundamental/1st harmonics: $\nu_0 = \frac{v}{2L}$



3. 1st overtone/2nd harmonics: $\nu_1 = 2\nu_0 = \frac{2v}{2L}$

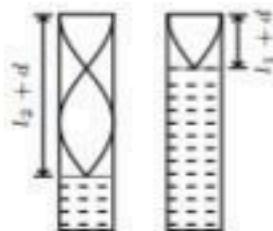


4. 2nd overtone/3rd harmonics: $\nu_2 = 3\nu_0 = \frac{3v}{2L}$



5. All harmonics are present.

Resonance column:



$$l_1 + d = \frac{\lambda}{2}, \quad l_2 + d = \frac{3\lambda}{4}, \quad v = 2(l_2 - l_1)\nu$$

Beats: two waves of almost equal frequencies $\omega_1 \approx \omega_2$

$$p_1 = p_0 \sin \omega_1(t - x/v), \quad p_2 = p_0 \sin \omega_2(t - x/v)$$

$$p = p_1 + p_2 = 2p_0 \cos \Delta\omega(t - x/v) \sin \omega(t - x/v)$$

$$\omega = (\omega_1 + \omega_2)/2, \quad \Delta\omega = \omega_1 - \omega_2 \quad (\text{beats freq.})$$

Doppler Effect:

$$\nu = \frac{v + u_o}{v - u_s} \nu_0$$

where, v is the speed of sound in the medium, u_0 is the speed of the observer w.r.t. the medium, considered positive when it moves towards the source and negative when it moves away from the source, and u_s is the speed of the source w.r.t. the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

2.4: Light Waves

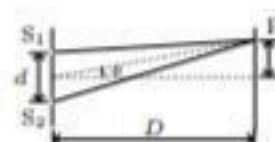
Plane Wave: $E = E_0 \sin \omega(t - \frac{x}{v})$, $I = I_0$



Spherical Wave: $E = \frac{nE_0}{r} \sin \omega(t - \frac{r}{v})$, $I = \frac{I_0}{r^2}$



Path difference: $\Delta x = \frac{dy}{D}$



Phase difference: $\delta = \frac{2\pi}{\lambda} \Delta x$

Interference Conditions: for integer n ,

$$\delta = \begin{cases} 2n\pi, & \text{constructive;} \\ (2n+1)\pi, & \text{destructive,} \end{cases}$$

$$\Delta x = \begin{cases} n\lambda, & \text{constructive;} \\ (n + \frac{1}{2})\lambda, & \text{destructive} \end{cases}$$

Intensity:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta,$$

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2, \quad I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$I_1 = I_2 : I = 4I_0 \cos^2 \frac{\delta}{2}, \quad I_{\max} = 4I_0, \quad I_{\min} = 0$$

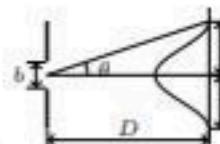
Fringe width: $w = \frac{\lambda D}{d}$

Optical path: $\Delta x' = \mu \Delta x$

Interference of waves transmitted through thin film

$$\Delta x = 2\mu d = \begin{cases} n\lambda, & \text{constructive;} \\ (n + \frac{1}{2})\lambda, & \text{destructive.} \end{cases}$$

Diffraction from a single slit:



$$\text{For Minima: } n\lambda = b \sin \theta \approx b(y/D)$$

Resolution: $\sin \theta = \frac{1.22\lambda}{b}$

Law of Malus: $I = I_0 \cos^2 \theta$



4 Heat and Thermodynamics

4.1: Heat and Temperature

Temp. scales: $F = 32 + \frac{9}{5}C$, $K = C + 273.16$

Ideal gas equation: $pV = nRT$, n : number of moles

van der Waals equation: $(p + \frac{a}{V^2})(V - b) = nRT$

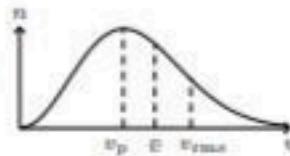
Thermal expansion: $L = L_0(1 + \alpha\Delta T)$,
 $A = A_0(1 + \beta\Delta T)$, $V = V_0(1 + \gamma\Delta T)$, $\gamma = 2\beta = 3\alpha$

Thermal stress of a material: $\frac{F}{A} = Y \frac{\Delta L}{L}$

4.2: Kinetic Theory of Gases

General: $M = mN_A$, $k = R/N_A$

Maxwell distribution of speed:



RMS speed: $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}}$

Average speed: $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$

Most probable speed: $v_p = \sqrt{\frac{2kT}{m}}$

Pressure: $p = \frac{1}{3}\rho v_{rms}^2$

Equipartition of energy: $K = \frac{1}{2}kT$ for each degree of freedom. Thus, $K = \frac{f}{2}kT$ for molecule having f degrees of freedoms.

Internal energy of n moles of an ideal gas is $U = \frac{1}{2}nRT$.

4.3: Specific Heat

Specific heat: $s = \frac{Q}{m\Delta T}$

Latent heat: $L = Q/m$

Specific heat at constant volume: $C_v = \left. \frac{\Delta Q}{n\Delta T} \right|_V$

Specific heat at constant pressure: $C_p = \left. \frac{\Delta Q}{n\Delta T} \right|_p$

Relation between C_p and C_v : $C_p - C_v = R$

Ratio of specific heats: $\gamma = C_p/C_v$

Relation between U and C_v : $\Delta U = nC_v\Delta T$

Specific heat of gas mixture:

$$C_v = \frac{n_1 C_{v1} + n_2 C_{v2}}{n_1 + n_2}, \quad \gamma = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}}$$

Molar internal energy of an ideal gas: $U = \frac{1}{2}fRT$,
 $f = 3$ for monatomic and $f = 5$ for diatomic gas.

4.4: Thermodynamic Processes

First law of thermodynamics: $\Delta Q = \Delta U + \Delta W$

Work done by the gas:

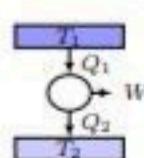
$$\Delta W = p\Delta V, \quad W = \int_{V_1}^{V_2} pdV$$

$$W_{\text{isothermal}} = nRT \ln \left(\frac{V_2}{V_1} \right)$$

$$W_{\text{isobaric}} = p(V_2 - V_1)$$

$$W_{\text{adiabatic}} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$$

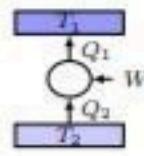
$$W_{\text{isochoric}} = 0$$



Efficiency of the heat engine:

$$\eta = \frac{\text{work done by the engine}}{\text{heat supplied to it}} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta_{\text{Carnot}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$



Coeff. of performance of refrigerator:

$$\text{COP} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\text{Entropy: } \Delta S = \frac{\Delta Q}{T}, \quad S_f - S_i = \int_i^f \frac{\Delta Q}{T}$$

$$\text{Const. } T : \Delta S = \frac{Q}{T}, \quad \text{Varying } T : \Delta S = ms \ln \frac{T_f}{T_i}$$

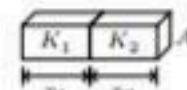
Adiabatic process: $\Delta Q = 0$, $pV^\gamma = \text{constant}$

4.5: Heat Transfer

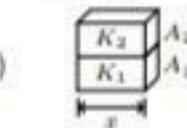
Conduction: $\frac{\Delta Q}{\Delta t} = -KA \frac{\Delta T}{x}$

Thermal resistance: $R = \frac{x}{KA}$

$$R_{\text{series}} = R_1 + R_2 = \frac{1}{A} \left(\frac{x_1}{K_1} + \frac{x_2}{K_2} \right)$$



$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{A} (K_1 A_1 + K_2 A_2)$$



Kirchhoff's Law: $\frac{\text{emissive power}}{\text{absorptive power}} = \frac{E_{\text{body}}}{a_{\text{body}}} = E_{\text{blackbody}}$

Wien's displacement law: $\lambda_m T = b$



Stefan-Boltzmann law: $\frac{\Delta Q}{\Delta t} = \sigma e A T^4$

Newton's law of cooling: $\frac{dT}{dt} = -bA(T - T_0)$