Chapter 1

Physical World and Measurement

Topic 1

Physical World : Scope and excitement of Physics; Nature of Physical laws; Physics, Technology and Society

» Revision Notes

Scope and Excitement of Physics

- Scope of physics considers two domains: macroscopic and microscopic.
- Macroscopic domain relates to facts at labs, global and vast scales.
- Microscopic domain will have atomic, molecular and nuclear facts.
- Classical Physics contains macroscopic facts about Mechanics, Electrodynamics, Optics and Thermodynamics.
- Mechanics is based on Newton's laws of motion and gravitation law which deals with motion of particles, rigid and deformable bodies.
- Electrodynamics shows electric and magnetic facts linked with charged and magnetic bodies.
- Electrodynamics laws were given by Coulomb, Oersted, Ampere and Faraday which were finally summarized by Maxwell with its set of equations.
- Electrodynamics deals with motion of current-carrying conductor in magnetic field, response of a circuit, working of antenna, propagation of radio waves.
- Optics deals with facts related to light with involvement of telescopes, microscopes, colours of thin films.
- Thermodynamics concern with changes in internal energy, temperature, entropy of a system using external work and heat transfer.
- Scope of physics covers magnitude of physical quantities such as length, mass, time, energy, etc right from scale length of 10^{-14} m using electrons, protons till astronomical facts with scale length of galaxies of the order 10^{26} m.
- In scope of physics, time scale can be 10^{-22} s to 10^{18} s, mass scale can be 10^{-30} kg (electron) to 10^{55} kg (universe).

Nature of Physical laws

- Universe contains particles which are smaller than atoms as compared to stars which are quite far away.
- During any physical phenomenon, different forces and quantities, changes with time, but there are certain physical quantities (conserved quantities) which remain constant with time.
- In an external conservative force motion, total mechanical energy of a body is constant such as free fall of object under gravity.
- The kinetic energy and potential energy of an object changes continuously with time but their sum remains constant.
- Energy remains conserved while counting different forms of energy such as heat energy, mechanical energy, electrical energy etc.
- Law of conservation of energy is valid for all forces and for any transformation which occurs among various forms of energy.
- Law of conservation of energy is valid from microscopic to macroscopic domains which are applicable for analyzing atomic, nuclear and elementary particles.
- Earlier than Einstein's theory of relativity, law of conservation of mass acts as basic conservation law of nature because of durability of matter.
- Chemical reaction is a rearrangement of atoms among different molecules.
- Exothermic reaction is that in which the total binding energy of reacting molecules is less than the total binding energy of the product molecules.

- Endothermic reaction is that in which the total binding energy of reacting molecules is more than the total binding energy of the product molecules.
- Einstein's theory shows that mass m is equivalent to energy E given by $E = mc^2$ where c is speed of light in vacuum.
- In nuclear process, mass gets converted to energy and vice-versa which is released in nuclear power generation and nuclear explosions.
- Wolfgang Paul in 1931 predicted the existence of neutrino particle which gets emitted in β-decay with electron.
- Space is homogeneous with no preferred location in the universe.
- Laws of nature are same everywhere in the universe which gives rise to conservation of linear momentum.
- Isotropy of space underlies law of conservation of angular momentum.
- There are four fundamental forces in nature such as Gravitational Force, Weak Nuclear Force, Electromagnetic Force and Strong Nuclear Force.
- The strongest fundamental force is Strong Nuclear Force while the weakest fundamental force is Gravitational Force.
- Gravitational Force has maximum range length and acts between any two pieces of matter in Universe.
- Weak Nuclear Force has the short length range which is responsible for radioactive decay and neutrino interactions.
- Electromagnetic Force has long range which causes electric and magnetic effects due to repulsion among electrical charges or with bar magnet interaction.
- Strong Nuclear Force has short range having gluons particles that hold the nuclei of atoms together which acts as both attractive and repulsive.

Physical laws related to technology :

Principal of Physics	Technology
Bernoulli's theorem	Aeroplanes
Digital Electronics	Computers and Calculators
Electromagnetic Induction	Electricity Generation
Electromagnetic Waves Propagation	Radio, TV, Phones
Laws of Thermodynamics	Steam, petrol, or diesel Engine
Nuclear Chain Reaction	Nuclear reactor for power
Newton's Second & Third Law	Rocket propulsion
Population inversion	Lasers
X-rays	Medical Diagnosis
Ultra high magnetic fields	Superconductors

Contributions in Physics :

Contribution	Physicist
Unification of weak and e/m interactions	Abdus Salam
Law of Photo Electricity, Theory of Relativity	Albert Einstein
Principle of Buoyancy and Lever	Archimedes
Inelastic Scattering of light by molecules	C.V. Raman
Wave theory of Light	Christian Huygens
Law of Gravitation and Motion, Reflecting telescope	Isaac Newton
Law of Inertia	Galileo Galilei
Triple Helical structure of proteins	G.N. Ramachandran
Nuclear model of Atom	Earnest Rutherford
Cyclotron	E.O. Lawrence
Wave nature of matter	Louis de-Broglie
Electron	J.J. Thomson
Laws of Electromagnetic Induction	Michael Faraday
Expanding Universe	Edvin Hubble
Cascading of cosmic radiation	H.J. Bhabha
Theory of Nuclear Forces	Hideki Yukava
Thermal Ionization	M.N. Saha

Neutron	James Chadwick
Maxwell Electromagnetic theory of light	James Clerk
Quantum model of Hydrogen atom	Niels Bohr
Measurement of Electronic Charge	R.A. Millikan
Discovery of Radium, Polonium, Radioactivity	Marie S. Curie
Quantum Statistics	S.N Bose
Chandrashekhar limit, Structure of stars	S. Chandrashekhar
X-rays	W.K. Roentgen
Quantum Exclusion principle	Wolfgong Pauli
Quantum mechanics, Uncertainty principle	W. Heisenberg

Physics, Technology and Society

- Thermodynamics which is a branch of physics appears in order to understand and improve the working of heat engines.
- Steam engine played important role in Industrial Revolution in 18th century that left great impact on human civilization.
- Wireless Communication Technology shows discovery of laws of electricity and magnetism in 19th century.
- With invention of wireless technology, mobiles, tablets, computers, internet, T.V., radio etc are used for communication.
- In 1933, Ernest Rutherford dismissed the possibility of tapping energy from atoms.
- In 1938, nuclear fission was discovered by Otto Hahn and Lise Meitner on the Second World War.
- Invention of silicon chip technology triggered the computer revolution in last three decades of 20th century.
- The discovery of new sources of energy like conversion of solar energy, geothermal energy into electricity contributes a lot with fast ending of fossil fuels on earth.
- Electromagnetic waves transmit energy in the form of microwaves, infrared radiation, visible light, ultraviolet light, X-rays and gamma rays.
- The electromagnetic waves spread with the speed of light with the help of interrupted variations of electric and magnetic fields.
- Lightning is discharge of electricity that occurs in thunderstorm when ice and liquid particle above the freezing level collide and create electric field.
- In 1752, Ben Franklin discovered electricity by conducting a kite experiment using lightning rods and a key to conduct electricity and create charge by flying a kite in storm clouds.

Topic 2

Units and Measurements

- Need for measurement
- Units of measurement
- Systems of units; SI units, fundamental and derived units
- Length, mass and time measurements
- Accuracy and precision of measuring instruments
- Errors in measurement
- Significant figures
- Dimensions of physical quantities
- Dimensional analysis and its applications

» Revision Notes

Physical Quantities

- Quantities which can be measured like time, length, mass, force, work done, etc.
- Value of physical quantity is expressed as product of number and unit.
- In this, unit describes example or prototype of quantity for reference point while number shows ratio of value of quantity to unit.

- There are 7 base quantities such as Length (metre), mass (kilogram), time (second), electric current (ampere), thermodynamic temperature (kelvin), amount of substance (mole) and luminous intensity (candela)

Measurement

- Measurement is the comparison of quantity with standard of similar physical quantity.

Units

- Unit is a value, quantity or magnitude in terms of which other values, quantities or magnitudes are expressed. There are certain systems of units which have been in use like :
  (a) French System or C.G.S. system
  (b) British System or F.P.S. system
  (c) Meter-Kilogram-Second or M.K.S. system
  (d) International System of units or S.I. system
  Physical quantities are measured as per standard magnitude of similar physical quantity which is known as units like second, meter, kilogram, etc.
  Measurement of physical quantity = (Magnitude) × (Unit).
  There are three types of units
- Fundamental or base units
- Derived units
- Supplementary units

SI Multiple units and Non SI-Units

It is seen that commonly used measured quantities are :

- Distance Mass Time Temperature
- Volume Density Pressure Amount
- Concentration Energy Velocity Molarity
- Viscosity Electric charge
- All these quantities can be measured in variety of different ways.

System of Units

- **SI units :** It is an International System of Units which is latest metric system used all across the world.
- It is made of many basic units with which we can derive units for other quantities.

Gaussian units

- It is a system of unit which uses electrical and magnetic quantities like centimetre, gram and second.
- It includes certain dimensionless quantities like permittivity, magnetic permeability etc.
- It is known as symmetrical C.G.S. system where units of electrical quantities are equal to units of C.G.S absolute electrostatic system.
- It depends on absolute system of units having millimeter, milligram etc.
- It is used to measure magnetic quantities like length: centimeter (cm), mass: gram (g), time: second (s)
- It is sometimes called as cgs system for centimeter-gram-second.

- **British System :** In such system, the units of length: foot (ft), mass : pound and time : second
- **M.K.S. System :** In this, unit of mass is kilogram, unit of length is metre and unit of time is second.
- **C.G.S. System :** In CGS system, units of length, mass and time are centimeter, gram and second respectively.
  - Centimetre is $\left(\frac{1}{3}\right)^{rd}$ part of metre.
  - Meter is the distance between two lines on surface of Platinum Iridium bar kept at international Bureau of Weights and Measures at Serve's.
  - Kilogram is mass of a piece of cylinder having 39 mm height and 39 mm of diameter at 4°C Platinum Iridium alloy kept in vault at Serve's France by BIPM.
  - Second is $\left(\frac{1}{86400}\right)^{th}$ part of mean solar day.

Fundamental Units

- Fundamental units are set of units of particular physical quantities from where different other units can be obtained. These are the units which are neither derived from one another nor resolved in any other units.

➢ **SI Base Units :**

Qty.	Sym.	Unit	Unit Sym.	Unit Definition
length	l	meter	M	It is length of path covered by light in $\frac{1}{299792458}$ of a second.
mass	m	kilogram	kg	The mass of the International Prototype Kilogram.
time	t	second	s	It is duration of 9 192 631 770 periods of radiation corresponds to transition among two hyperfine levels of ground state of Caesium 133 atom at 0 K.
electric current	I	ampere	A	It is fixed current in two straight parallel conductors of infinite length with negligible cross-section which is 1 metre apart in vacuum appears among conductors with 2×10^{-7} newton/length.
thermodynamic temperature	T	kelvin	K	It is $\frac{1}{273.16}$ of thermodynamic temperature of triple point of water.
amount of substance	n	mole	Mol	It is amount of substance of system having many elementary entities with atoms in 0.012 kilogram of carbon 12.
luminous intensity	I_v	candela	Cd	It is the intensity of emitting monochromatic radiation of frequency 540×10^{12} hertz having radiant intensity in direction of $\frac{1}{683}$ watts/steradian.

Derived units

- ➢ These are units that are derived from fundamental units like Force which is Mass (m) × Acceleration (a).
- ➢ Such units are expressed algebraically using base units.
- ➢ These units are dimensionless which are expressed in two base units.
- ➢ Examples of derived units are area, volume, density, acceleration and frequency.

Dimensions :

- ➢ For physical quantity, dimensions are powers to which fundamental quantities shows given physical quantity.
- ➢ It is the power of base quantity that gets entered in an expression of physical quantity.
- ➢ It can be used to check the correctness of an equation.
- ➢ It also helps in verifying the equivalence of different mathematical expressions for a given quantity.

Dimensions of Physical Quantities

- ➢ Any quantity that is measured in units of length carries a dimension of length.
- ➢ Lengths, mass and time are taken as three base dimensions expressed as L, M and T.
- ➢ Dimensions of physical quantities give no idea about magnitude of quantity.
- ➢ If a quantity is not dependent on any of base units then such quantity is said to have zero dimensions.
- ➢ Quantity representing as a product of two identical dimensions will have two dimensions of that unit.
- ➢ If derived unit depends upon a^{th} power of mass unit, b^{th} power of length unit and c^{th} power of time unit, then dimension of such quantity be written as $[M^a] \times [L^b] \times [T^c]$ or $[M^aL^bT^c]$

Classification of Physical quantities :

- ➢ Dimensionless or Non dimensional variables
- ➢ These are quantities that are variable having no dimensions.
- ➢ Examples include angle, specific gravity, strain and efficiency of a machine.
- ➢ In this the quantities will have no dimensions since they are expressed as ratio of two identical units.

Dimensional variables

- ➢ These are variable quantities having some dimensions.
- ➢ Examples include force, momentum, velocity, power etc.

Dimensionless or Non - dimensional constants

- ➢ These include constant quantities having no dimensions.
- ➢ Examples include numbers 1, 2, 3…., π etc.

Dimensional constant

- ➢ It includes quantities having fixed value with certain dimensions.
- ➢ Examples include velocity of light in vacuum and gravitational constant.

Dimensional Formulae

- ➢ It is a formula which shows relation among derived unit and fundamental units.
- ➢ It is an expression that describes how and which fundamental unit enter in units of physical quantity.

- To have dimensional formula of a physical quantity, first express that quantity as physical quantities of mass, length and time.

Writing Dimensional Formula

- Write the formula for particular quantity with quantity on L.H.S. of General equation.
- Convert quantities on R.H.S. into fundamental quantities like mass, length and time.
- Use M for mass, L for length and T for time.
- Collect terms of M, L and T and find their resultant powers (a, b, c).
- Powers show the dimensions of quantity in mass, length and time.

Common Units with Dimensions

Physical quantities	Symbol	Dimensional formula	Units
Area	A	[L2]	m^2
Volume	V	$[L^3]$	m^3
Velocity	*v*	[L/T]	m/s
Angular velocity	ω	$[T^{-1}]$	1/s
Acceleration	a	$[LT^{-2}]$	m/s^2
Angular acceleration	α	$[T^{-2}]$	$1/s^2$
Force	F	$[MLT^{-2}]$	kg m/s^2
Energy	E	$[ML^2T^{-2}]$	kg m^2/s^2
Work	W	$[M\Lambda^2T^{-2}]$	kg m^2/s^2
Heat	Q	$[ML^2T^{-2}]$	kg m^2/s^2
Torque	τ	$[ML^2T^{-2}]$	kg m^2/s^2
Power	P	$[ML^2T^{-3}]$	kg m^2/s^3
Density	*d* or ρ	$[ML^{-3}]$	kg/m^3
Pressure	P	$[ML^{-1}T^{-2}]$	kg m^{-1}/s^2
Impulse	J	$[MLT^{-1}]$	kg m/s
Inertia	I	$[ML^2]$	kg m^2
Luminous flux	f	[C]	cd.sr
Illumination	E	$[CL^{-2}]$	cd sr/m^2
Entropy	S	$[ML^2T^{-2}K^{-1}]$	kg m^2/s^2K
Volume rate of flow	Q	$[L^3T^{-1}]$	m^3/s
Kinematic viscosity	η_k	$[L^2T^{-1}]$	m^2/s
Dynamic viscosity	η_α	$[ML^{-1}T^{-1}]$	kg/m-s
Specific weight	G	$[ML^{-2}T^{-2}]$	kg m^{-2}/s^2
Electric current	I	$[QT^{-1}]$	C/s
Emf, voltage, potential	E/V	$[ML^2T^{-2}Q^{-1}]$	kg m^2/s^2C
Resistance or impedance	R	$[ML^2T^{-1}Q^{-2}]$	$kgm^2/\sigma X^2$
Electric conductivity	S	$[M^{-2}L^{-2}TQ^2]$	$sC^2/kg\ m^3$
Capacitance	C	$[M^{-1}L^{-2}T^2Q^2]$	s^2C^2/kgm^2
Inductance	L	$[ML^2Q^{-2}]$	kg m^2/C^2
Current density	J	$[QT^{-1}L^{-2}]$	C/s m^2
Charge density	ρ	$[QL^{-3}]$	C/m^3
Magnetic flux/Magnetic induction	B	$[MT^{-1}Q^{-1}]$	kg/s C
Magnetic intensity	H	$[QL^{-1}T^{-1}]$	C/m s
Magnetic vector potential	A	$[MLT^{-1}Q^{-1}]$	kg m/s C
Electric field intensity	E	$[MLT^{-2}Q^{-1}]$	kg m/s^2 C
Electric displacement	D	$[QL^{-2}]$	C/m^2
Permeability	M	$[MLQ^{-2}]$	kg m/C^2

Permittivity	ε	$[T^2Q^2M^{-1}L^{-3}]$	s^2C^2/kgm^3
Dielectric constant	κ	$[M^0L^0T^0]$	——
Frequency	*f* or *n*	$[T^{-1}]$	s^{-1}
Angular frequency	ω	$[T^{-1}]$	s^{-1}

Dimensional Analysis

- It is a system of conversion involving factors from one unit of measurement to another.
- In this square brackets show dimension or units of physical quantity.
- It can be used to derive or check formulae by treating dimensions as algebraic quantities.
- In this the quantities can be added or subtracted when they have same dimensions with similarity on both sides of equation.
- It results in no numerical factors.

Applications

- It helps in checking the accuracy of a relation which connects many physical quantities by comparing dimensions of quantities on left side with dimensions of quantities on right side.
- The measurement of physical quantity is product of a number and unit, so on changing the unit of measurement, the associate number also gets changed.
- With use of dimensional formula of a quantity, we can change a unit from one system to another system of unit.
- With this, homogeneity of dimensions can be obtained using relationship among different physical quantities using idea of dependence of quantities on each other.

Changing of formula :

- Changing of quantity X_1 having M_1, L_1 and T_1 to X_2 value having M_2, L_2 and T_2.
- If $[M^aL^bT^c]$ is a dimensional formula of physical quantity, then magnitude in first system will be $X_1\ [M_1^aL_1^bT_1^c]$ and second be $X_2\ [M_2^aL_2^bT_2^c]$.
- Since quantity remains constant, its absolute value will be similar in both systems:
- $X_1\ [M_1^aL_1^bT_1^c] = X_2\ [M_2^aL_2^bT_2^c]$
- $X_2 = X_1\left[\frac{M_1}{M_2}\right]^a \cdot \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

Limitations :

- It is noted that idea of dimensional analysis is valid in simple cases and is not in general as :
- There exists no idea about magnitude of dimensionless variable and dimensionless constants.
- It is applicable only in case of quantities (M, L, T) and not above that. Using method of dimension we can not deduce or establish a relation which contains addition or subtraction like Y=A+B-C
- It is not applicable for trigonometric, exponential and logarithmic functions.

Principle of Homogeneity

- The principle of homogeneity says that dimensions of each term of dimensional equation on both sides be same.
- Formula having dimensions as mass, length, time, temperature, electricity have terms with similar dimensions.
- Terms with similar dimensions help to convert the units in one system to another system.
- Two or more physical quantities can be added/subtracted when they have same dimensions.

Error in Measurements

- The result of measurement by measuring instrument lead to uncertainty which is an error.
- All calculated quantities based on measured value contains an error.
- Error is the difference between measured value and true value.
- Final measurement can be taken in approximation if individual terms have error in the measurement.
- Errors in measurement be systematic errors or random errors.

Absolute error

Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times and the measured values be $a_1, a_2, a_3, \ldots.. a_n$ then arithmetic mean of these value is

$$a_m = \frac{a_1 + a_2 + \ldots a_n}{n}$$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots\ldots\ldots\ldots\ldots..$$
$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases.

Mean absolute error

It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus,

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ... + |\Delta a_n|}{n}$$

Relative error or Fractional error

The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured.

or, Relative error or Fractional error $= \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{\overline{\Delta a}}{a_m}$

Percentage error

When the relative/fractional error is expressed in percentage, we call it, percentage error.

or, Percentage error $= \frac{\overline{\Delta a}}{a_m} \times 100\%$

Random Error (Causes)

- Random error occurs irregularly with respect to sign and size.
- Such errors are due to random and unpredictable fluctuations in experimental conditions.
- The error is caused due to noise or smoke that gives positive or negative deviation from true value.
- The error of such type can be reduced by many readings for measuring value and applying statistical analysis for estimating true measurement readings.
- This type of error has no consistent effects as it moves observed scores up or down randomly.
- Such error adds variability to data and will not affect average performance of a group.

Calculation of Random Errors

- Arithmetic mean x_{Mean} of series of measurement as $n \to \infty$.

$x_{\text{Mean}} = \frac{x_1, x_2, x_3, ..., x_n}{n}$, $x_1, x_2, x_3, ..., x_n$ = series of measurement

$\Delta x_n = x_n - x_{\text{Mean}}$

When n is not infinite, mean of series of measurement is probable or presumable value :

$$x_p = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

If the frequency for measured value be $x_1, x_2, x_3, ..., x_n$ be $f_1, f_2, f_3, ..., f_n$, then mean value will be

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n}$$

Systematic Error

- Systematic errors tend to be in one direction either positive or negative.
- Such type of errors results from :
  - imperfect design or calibration of measuring instrument
  - imperfection in experimental technique or procedure
  - lack of proper setting of apparatus
- Carelessness of an individual in taking observation without observing proper precaution.
- Such errors can be lowered by improving experimental technique, choosing better instruments and removal of individual bias.

Errors in Mathematical operation

Error in sum of the quantities

If $x = a + b$, $\Delta a = \Delta b = \Delta x$ = absolute error in measurement and calculation.

The maximum absolute error in x is $\Delta x = \pm (\Delta a + \Delta b)$

Percentage error in the value of $x = \frac{(\Delta a + \Delta b)}{a + b} \times 100\%$

Error in difference of the quantities

Suppose $x = a - b$, $\Delta a = \Delta b = \Delta x$ = absolute error in measurement and calculation.

The maximum absolute error in x is $\Delta x = \pm (\Delta a + \Delta b)$

Percentage error in the value of $x = \frac{(\Delta a + \Delta b)}{a - b} \times 100\%$

Error in product of quantities

If $x = a \times b$, $\Delta a = \Delta b = \Delta x$ = absolute error in measurement and calculation.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of $x = \left(\frac{\Delta a}{a} \times 100 + \frac{\Delta b}{b} \times 100\right)\ \%$

Error in division of quantities

Suppose $x = \frac{a}{b}$,

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of $x = \left(\frac{\Delta a}{a} \times 100 + \frac{\Delta b}{b} \times 100\right)\ \%$

Error in quantity raised to some power

Suppose $x = \frac{a^n}{b^m}$, $\Delta a = \Delta b = \Delta x$ = absolute error in measurement and calculation.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$

Percentage error in the value of $x = \left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right) \times 100\ \%$

Error Reduction

Errors like measurement errors, random or systematic can be reduced by using :

- Pilot testing of instruments which improves performance of an equipment.
- Correct reading of instrument taken by skilled professional.
- Checking and punching of data using double entry and cross checking of it.
- Following certain statistical procedures which adjusts measurement error.
- Certain multiple measures.

Least count

- It is an error which is the smallest value that can be measured by measuring instrument.
- It is an error which is linked with resolution of instrument.
- It is a type of systematic and random error.
- The error can be reduced using high precision instruments and by enhancing experimental techniques.

Accuracy

- Accuracy of measurement is a measure of how close the measured value is with true value of certain quantity.
- It is the amount of uncertainty in measurement with respect to absolute standard.
- Its specifications carries effect of errors which results due to gain and offset parameters.

Precision of Measuring Instruments

- Precision tells about the resolution or limit of quantity measured.
- It is an instrument's degree of repeatability which shows how reliably the instrument can reproduce similar measurement over and over again.

Significant figures

- It shows number of important single digits (0 - 9) in a coefficient of expression.
- It arises due to rounding off an expression when a calculation is done.
- In a calculation, number of significant figures is equal to or less than number of significant figures in least precise expression or element.

Counting of Significant Figures

- In multiplication or division, final result should have many significant figures as in case of original number with least significant figures.
- In addition or subtraction, final result should have many decimal places as in case with least decimal places.

Rules for Arithmetic operations

- All non-zero integers are significant figures such as 123 (3 significant figures), 987654 (6 significant figures)
- Zeroes located between non-zero integers are significant figures like 701 (3 significant figures), 60204 (5 significant figures)
- All zeroes to the left of first non-zero digit and to the right of last are not significant such as 3.14 (3 significant figures), 15900000000000 (3 significant figures), 0.0078 (2 significant figures), 0.00000000017 (2 significant figures)

Rounding Off Uncertain Digits

Rounding-off measurements have certain rules :

(a) If digit to be dropped is < 5, then previous digit should be unchanged.
e.g.: if $x = 7.82$, then the round-off value will be 7.8.

(b) If digit to be dropped is > 5, then previous digit to be raised by 1.
e.g.: if $x = 6.77$, then the round-off value will be 6.8.

(c) If digit to be dropped is 5 followed by digits other than 0, then previous digit be raised by 1.
e.g.: if $x = 7.751$, then the round-off value will be 7.8.

(d) If digit to be dropped is 5 or 5 followed by 0, then previous digit remains same.
e.g.: if $x = 7.750$, then the round-off value will be 7.7.

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Chapter 2

Kinematics

Topic 1

Motion in a Straight Line

- Position-time graph
- Speed and velocity
- Elementary concepts of differentiation and integration for describing motion
- Uniform and non - uniform motion
- Average speed and instantaneous velocity
- Uniformly accelerated motion
- Velocity-time and position-time graphs
- Relations for uniformly accelerated motion (graphical treatment)

» Revision Notes

- **Displacement :**
  - Displacement of an object is equal to the length of the shortest path between the final and the initial points.
  - When initial and final points are same, then displacement is zero.
  - Its direction is from the initial point to the final point. It is a vector quantity. Its S.I. unit is m.
- **Distance :** Distance is actual path traversed by object from its initial position to final position. It is a scalar quantity.
  - Distance depends on path whereas displacement does not.
  - Distance is always greater than or equal to displacement.
  - Distance and displacement are equal in straight line motion without going back.
- **Speed :** Speed is the absolute value of velocity vector which is the ratio of distance covered by body to the time taken. SI unit of speed is meter per second (m/s).

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Total time taken}} = \frac{s}{t}$$

  where symbols have in usual meanings
- **Instantaneous speed :** When object covers a distance ds in infinite final time dt then speed is said to be Instantaneous speed *i.e.* $v = \frac{ds}{dt}$. It is scalar quantity.
- **Variable Speed :** It is a speed where particle covers equal distances in unequal intervals of time.
- **Uniform Speed :** It is the speed where particle covers equal distances in equal intervals of time.
- **Average Speed :** It is the ratio of total distance traveled by particle to total time taken to cover such distance. SI unit of average speed is meter per second (m/s).

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{s_1 + s_2 + s_3 + \ldots}{t_1 + t_2 + t_3 + \ldots}$$

- **Velocity :** Velocity is vector quantity having direction. It will be positive or negative.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

  - **Uniform Velocity :** It is velocity where particle covers equal displacements in equal interval of time.
  - **Variable Velocity :** It is the velocity in which a particle covers equal displacements in unequal intervals of time or unequal displacements in equal intervals of time.
  - **Average Velocity :** It is the velocity which describes as the ratio of total displacement to the total time interval where displacement occurs.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Total time taken}} = \frac{\text{Initial velocity} + \text{Final velocity}}{2}$$

$v_{av} = \frac{u+v}{2}$, where u is initial velocity and v is final velocity.

- **Acceleration :** Acceleration is the rate of change in velocity or direction or both having a direction, which is measured in m/s² (SI), cm/s² (CGS) with dimensions as $[M^{\circ}LT^{-2}]$.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken in change in velocity}} = \frac{v-u}{t}$$

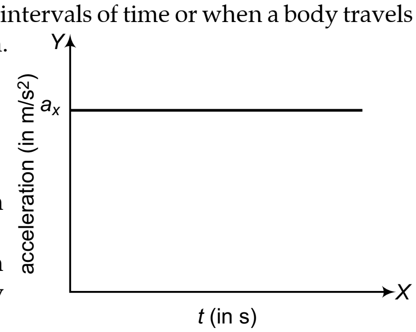
  - **Instantaneous Acceleration :** It is the rate of change of velocity at an instant of time. Further, it shows limit of average acceleration at a time interval Δt where it becomes infinitesimally small.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

  - **Average Acceleration :** It is the ratio of total change in velocity to total time taken where change in velocity results.

$$a_{av} = \frac{\Delta v}{\Delta t}$$

  - **Uniform/Constant Acceleration :** It describes movement of particle in uniform acceleration when velocity changes with constant rate.
  - **Variable Acceleration :** It describes that velocity of particle changes equally in different intervals of time, or shows unequal change in velocity in same time intervals.
  - **Gravitational Acceleration :** It describes about the acceleration on an object which results due to gravity. Every small body accelerates in gravitational field with the similar rate to the center of mass, irrespective of mass of body. On surface of the Earth, every object falls with acceleration 9.78 - 9.82 m/s² as per latitude.
- **Uniform Motion :** It is the motion in which a particle covers equal displacements in equal intervals of time.
  - Displacement is equal to distance covered.
  - No net force is needed for a body to undergo uniform motion.
  - The velocity is independent of choice of origin.
  - The velocity is independent of time interval.
  - It may be average and instantaneous velocity.
- **Non-Uniform Motion :** When an object covers unequal distances in equal intervals of time or when a body travels equal distances in unequal intervals of time is called non-uniform motion.
  The motion of a particle or a body is said to be non uniform if :
  - It covers unequal distances in equal intervals of time.
  - It covers equal distances in unequal intervals of time.



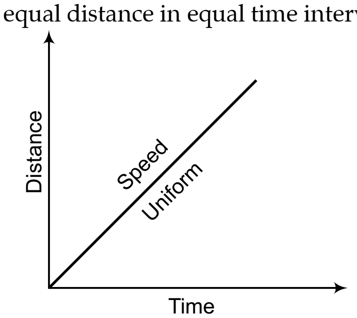
- **Uniformly Accelerated Motion :**
  - Uniformly accelerated motion is a motion in which the velocity of an object changes equally in every equal intervals of time.
  - In the acceleration-time graph for uniform accelerated body is shown with horizontal line where value of constant acceleration at any moment.
  - If $a_x(t)$ = constant, then $v_x(t) = u_x + a_x t$, where a_x shows constant value of $a_x(t)$.

Representing Motion

(a) Distance-Time Graphs :

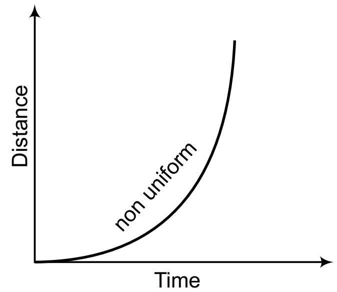
- Distance-time graph shows change in position of an object with respect to time.
- It can be used to calculate speed the moving body.

(b) Straight Line Graph : The distance-time graph for a body moving at uniform speed is always a straight line as body in uniform motion, body moves equal distance in equal time interval.

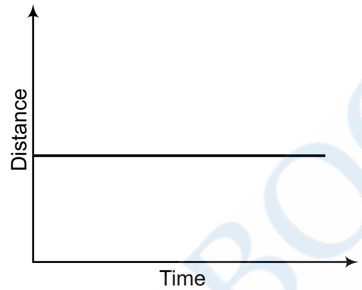


(c) Curved Graph :

- If the graph of distance - time is plotted for object having non-uniform speed, in such case the slope will not be a straight line, but shows increasing trend with velocity.

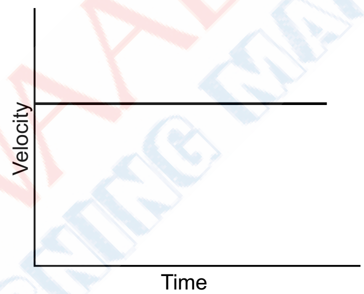


- If a body is at rest, then the distance time graph will show a parallel line with respect to time axis.

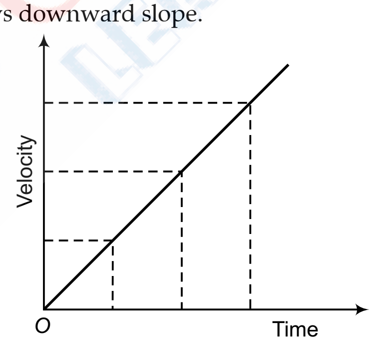


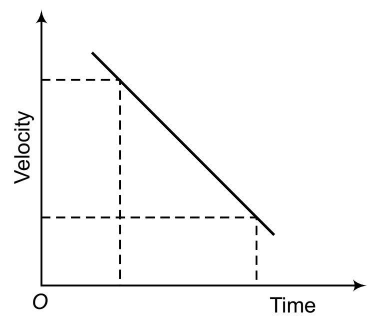
(d) Velocity-Time graph :

(a) When a body moves with uniform velocity in such case, the velocity-time graph for such body shows a straight line which is parallel to time axis.



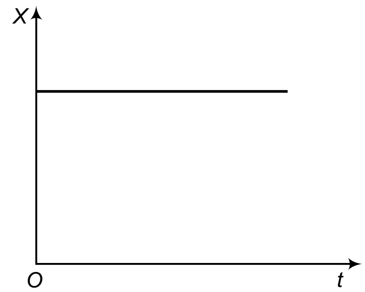
(b) When a body moves with non-uniform velocity, in such case, the velocity-time graph shows a straight line. When velocity increases with time, graph shows an upward slope while when velocity decreases with time, graph shows downward slope.



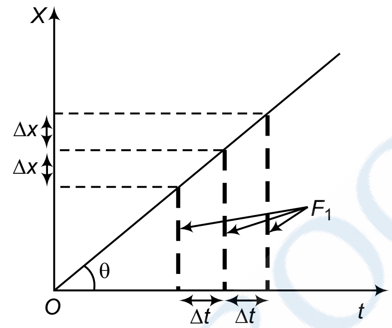


(e) Position-Time Graph :

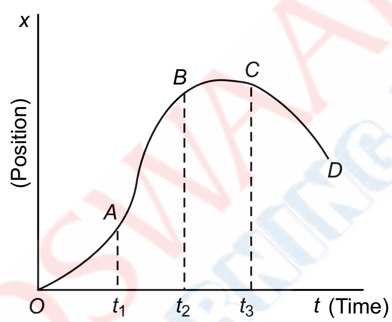
- Vertical axis shows position of an object while the slope describes velocity of an object.
- On drawing time (t) along x-axis and position (x) from origin O on y-axis, then graph result will be position-time graph.
- If position (x) remains constant and there will be variation in time, then particle is fixed in given reference frame, so straight line nature of position-time graph results parallel to time axis having state of rest where slope is zero.

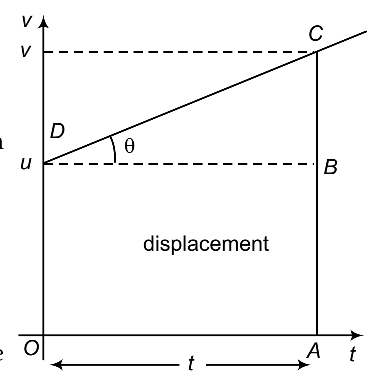


- If x-t graph is straight line inclined at certain angle ($\theta \neq 0$) with time axis, then the particle traverses equal displacement Δx in equal intervals of time Δt.



- Motion of particle will be uniform rectilinear motion where slope of the line is measured by $\frac{\Delta x}{\Delta t} = \tan\theta$ which shows uniform velocity of the particle.
- If x-t graph shows curve, motion is not uniform and is either increasing or decreasing as per the slope.





- If $t = t_1$ till $t = t_2$, then AB shows straight line showing uniform motion.
- Also, from $t = t_2$ to $t = t_3$, motion gets at rest in reference frame.

(f) Slope of tangent to curve at any point describing instantaneous acceleration

$$a = \frac{dv}{dt} = \tan\theta$$

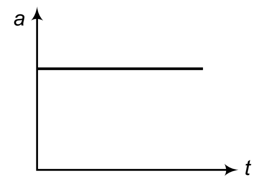
- Area under curve shows total displacement of particle

$$s = ut + \frac{1}{2}(v - u)t$$

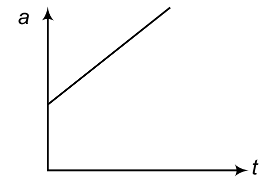
- Velocity-time graph shows straight line where acceleration of object is slope of line CD.
- For uniformly retarded motion, acceleration a < 0 which shows retardation or deceleration.
- In velocity-time graph DC of uniformly accelerated particle, points B and C correspond to velocities u and v where slope of straight line DC is described as $\tan\theta = \frac{BC}{DB} = \frac{v-u}{t-0}$

or, Acceleration $(a) = \frac{v-u}{t}$

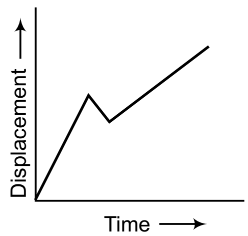
(g) Acceleration-Time Graph : Acceleration-time curves describe information about variation of acceleration with time. In these, area under the acceleration time curve shows change in velocity of particle in a given time interval.



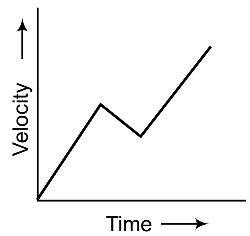
Motion under
constant acceleration



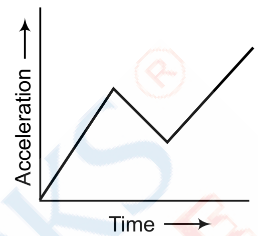
Motion under
uniform acceleration



d-t graph
Slope → Velocity
Directly measured → displacement
Area → X



v-t graph
Slope → acceleration
Directly measured → velocity
Area → Displacement



a-t graph
Slope → X
Directly measured → acceleration
Area → Velocity

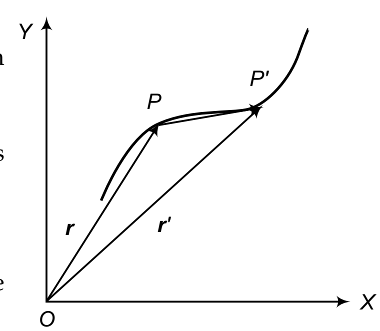
Topic 2

Concept of Vector and Motion in a Plane

- Scalar and vector quantities
- Position and displacement vectors
- General vectors and their notations
- Equality of vectors
- Multiplication of vectors by real number
- Addition and subtraction of vectors
- Relative velocity
- Unit vector
- Resolution of vector in a plane-rectangular components
- Scalar and Vector product of vectors
- Cases of uniform velocity and uniform acceleration
- Projectile motion
- Uniform circular motion

» Revision Notes

- **Scalar and Vector Quantities :** Physical quantities are classified as :
  - Scalar quantity
  - Vector quantity
- **Scalar Quantities :**
  - Scalar quantity is a quantity that is described by magnitude only, without direction.
  - Examples of scalar quantities are Mass, Charge, Pressure, etc.
- **Vector Quantities :**
  - Vector quantities are quantities that have magnitude and direction which follows vector laws of addition.
  - Examples of vector quantities are displacement, velocity, force, etc.
  - To describe vector quantities, direction is required while scalar quantities don't require direction.



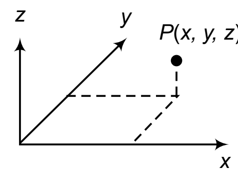
- **Position and Displacement Vectors :**

  Position Vector :
  - It is the vector of an object at time t which shows position of object relative to origin shown by straight line between origin and position at time t.

- In figure, *OP* and *OP′* are position vectors which are shown by ***r′*** and ***r*** where as PP′ is a displacement vector.

$$PP' = OP' - OP = \boldsymbol{r}' - \boldsymbol{r}$$

- In Cartesian and polar coordinates, use of unit vectors is common.
- A position vector ***r*** may be expressed in terms of the unit vectors.
- In 3 dimensions, Cartesian, spherical and polar coordinates are used with other coordinate systems for specific geometries.
  In cartesian coordinates

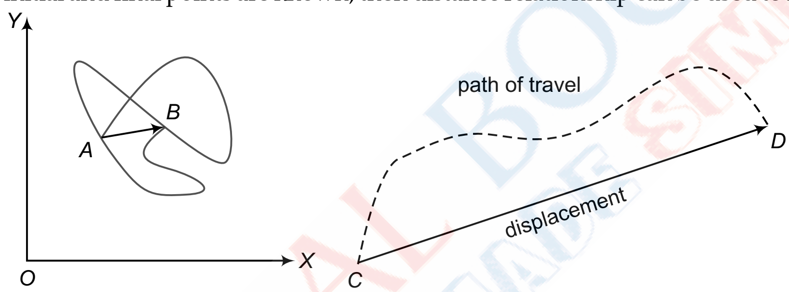


$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

- The position vector to general coordinate point (x, y, z) is written as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. This is a vector $\vec{r}$ which points from origin to point (x, y, z) having magnitude as $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

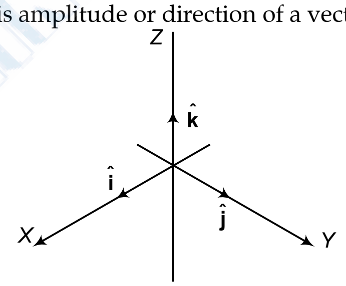
Displacement Vector :

- It is the vector of an object between two points which shows straight line between two points without any specific path.
- In this, path length is always equal or more than displacement. In figure shown, AB describes as displacement vector for any path which is followed.
- If position of initial and final points are known, then distance relationship can be used to find the displacement.



General Vectors Notations :

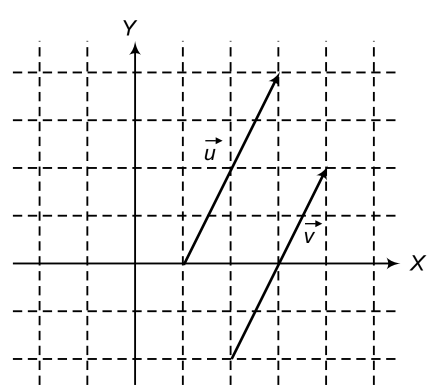
- When a vector is shown graphically, its magnitude is given by the length of an arrow and its direction is shown by head of that arrow.
- By giving magnitude of vector and angle of a particle from positive *x*-axis where the vector is pointing, the magnitude or length, of vector and angle is amplitude or direction of a vector.



- The basic vectors are all **unit vectors** as they have a length or magnitude of 1.
- If $\vec{u}$ is a unit vector, then it can be written as $\hat{u}$.
- In three dimensions, it shows three basic vectors, one along each coordinate axis that points in direction of $+x$-axis as $\hat{i}$.
- The vector pointing toward $+y$ axis is $\hat{j}$ and vector in z direction is $\hat{k}$.
- If a vector points three units along $+x$ axis, then multiply the vector $\hat{i}$ by scalar 3 to have $3\hat{i}$. Similarly it can be done to get vectors in y and z directions.

Equality of Vectors :

- If two vectors are equal, then they must have the same magnitude and direction.
- In the figure, two vectors look exactly alike in terms of length and direction irrespective of position are described as equal vectors.



- Vectors $\vec{u}$ and $\vec{v}$ have common length and direction, so they are equal and can be written as $\vec{u} = \vec{v}$.
- Two vectors, $\vec{a}$ and $\vec{b}$ are to be equal, if their components are equal.
- If vectors $\vec{a} = 3\hat{i} + 2u\hat{j}$ and $\vec{b} = (2+v)\hat{i} + 4\hat{j}$ are equal then the following statements should be correct as:

$$3 = 2 + v \quad \Rightarrow \quad v = 3 - 2 = 1 \qquad\qquad 2u = 4 \quad \Rightarrow \quad u = 4/2 = 2$$

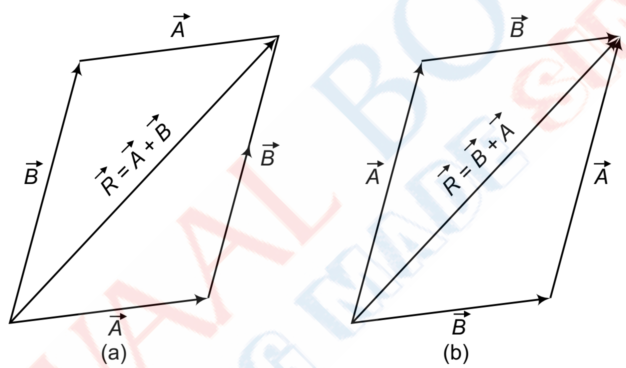
Multiplication of Vectors by Real Number :

- If a vector $\vec{A}$ is multiplied by real number (m) then the final vector will be $m\vec{A}$
- In general if real number λ is given with vector $\vec{a} = (a_1, a_2)$, then a new vector $\lambda\vec{a} = \lambda(a_1, a_2) = (\lambda a_1, \lambda a_2)$.

Addition and Subtraction of Vectors :

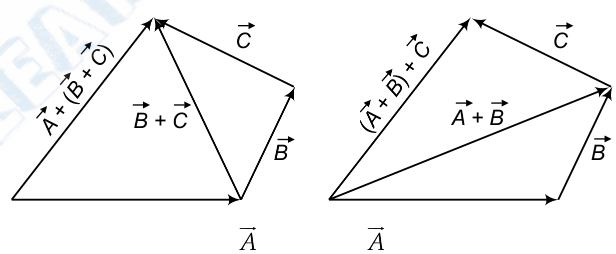
Addition of Vectors :

- To add two vectors $\vec{A}$ and $\vec{B}$, take second vector shown by an arrow and move it so that it's tail is at the head of first vector.
- $\vec{A}+\vec{B}$ is a vector which starts at tail of first arrow and goes to the head of second, so vectors $\vec{A}$, $\vec{B}$ and $\vec{A}+\vec{B}$, form a triangle.



- From the figure, resultant $\vec{R}$ is diagonal of parallelogram with sides $\vec{A}$ and $\vec{B}$ while other figure shows $\vec{A}+\vec{B} = \vec{B}+\vec{A}$, describes vector addition as commutative.

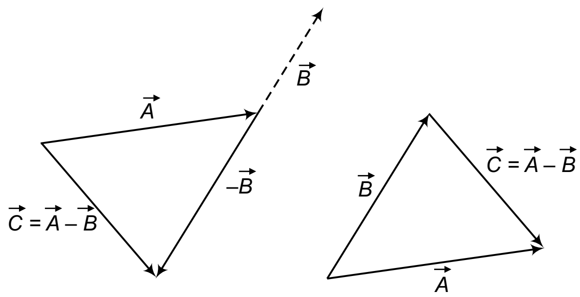
Associative Law



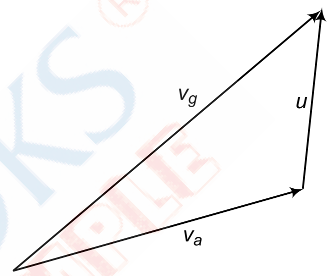
- When three or more vectors are added, their sum is independent of way in which individual vectors are grouped which is called as **associative law of addition** $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

Subtraction of Vectors :

- Subtracting vectors is way of adding two vectors where addition of second quantity is done with negative one and afterwards adding it to first quantity.
- Subtraction of two vectors is same as adding of negative of a vector, as $\vec{A} - \vec{B}$ is a vector where $-\vec{B}$ is added to vector $\vec{A}$, so $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.



- In vector subtraction, difference $\vec{A} - \vec{B}$ between two vectors $\vec{A}$ and $\vec{B}$ is that where adding of second vector is done to get the first. In this, vector $\vec{A} - \vec{B}$ points from tip of second vector to tip of the first.



Relative Velocity :

- Relative velocity of an object with respect to other object is velocity with which an object appears to have to an observer which is placed on other object that moves along with it.
- If an aeroplane moves with constant velocity v_a with respect to air, and air moves with constant velocity u with respect to ground, then vector velocity v_g of plane with respect to ground will be $v_g = v_a + u$

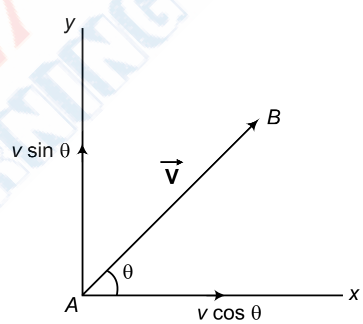
Resolution of a Vector in a Plane – Rectangular Components :

- Vector is a quantity having direction and magnitude where rectangular components of vector v are the vector along x and y axes which on adding results to vector $\vec{v}$.

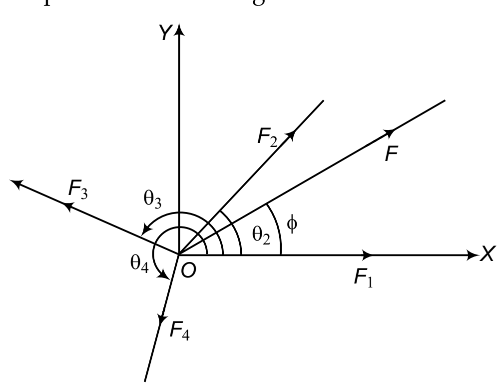
$$\overrightarrow{AX} = v \cos \theta$$

$$\overrightarrow{AY} = v \sin \theta$$

and $$\theta = \tan^{-1}\left(\frac{AY}{AX}\right)$$



- If multiple vectors are acting at a point as shown in figure



After resolving the given vectors along x-axis and y-axis

$$X = \sum_{i=1}^{4} F_i \cos\theta_i = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3 + F_4\cos\theta_4 \quad \text{Here, } \theta_1 = 0°$$

and
$$Y = \sum_{i=1}^{4} F_i \sin\theta_i = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3 + F_4\sin\theta_4$$

Let $\vec{F}$ be the resultant vector which makes angle ϕ with x-axis

$$F = \sqrt{(X)^2 + (Y)^2}$$

And
$$\phi = \tan^{-1}\left(\frac{Y}{X}\right)$$

- **Scalar and Vector Product of Vectors :**
  - Vectors can be multiplied by two ways.
  - A scalar product of two vectors results in a scalar quantity while vector product of two vectors results in vector quantity.
- **Scalar Product :**
  - Scalar product is a technique of multiplying two vectors where a scalar quantity is obtained.
  - It is known as dot product or inner product.
  - In this, two vector quantities are taken which return a single scalar quantity.
  - $\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$ $\{\vec{B}\cdot\vec{A} = \vec{A}\cdot\vec{B}\}$
  - Here, θ is the angle between vector $\vec{A}$ and $\vec{B}$ and can be calculated as $\phi = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right)$
  - The scalar product is an area bounded by the vectors.
- **Properties :**
  - If two vectors are orthogonal at 90°, then their scalar product is zero. *i.e.*, $\vec{A}\cdot\vec{B} = 0$
  - If two vectors $\vec{P}$ and $\vec{Q}$ are in same direction where angle between them is zero, then their dot product is the product of their magnitudes, $\vec{P}\,.\,\vec{Q} = |\vec{P}|\,|\vec{Q}|\cos 0° = |\vec{P}|\,|\vec{Q}|$ (cos 0° = 1)
  - The scalar product of a vector by itself is equal to square of its magnitude, $\vec{P}\,.\,\vec{P} = |P|^2$

$$|P| = \sqrt{\vec{P}\,.\,\vec{P}}$$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

- **Vector Product**
  - In vector product, result obtained will be vector quantity where vector product between two vectors is shown by cross product $\vec{A}\times\vec{B} = \vec{C}$ $\{\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}\}$
  - $\vec{C} = |\vec{A}|\,|\vec{B}|\sin\theta\,.\,\hat{n}$
  - Direction of vector $\vec{C}$ is shown by right hand rule where a thumb is kept along $\vec{A}$ with index finger along $\vec{B}$ and pointing middle finger perpendicular to the thumb with index finger in direction $\vec{C}$.
  - If $\vec{A}$ is parallel to $\vec{B}$, the vector product vanishes.

$$\hat{i}\times\hat{j} = \hat{k}$$

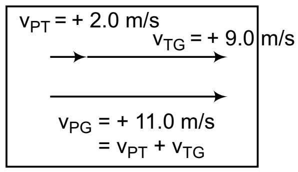
$$\hat{j}\times\hat{k} = \hat{i}$$

$$\hat{k}\times\hat{i} = \hat{j}$$

Cases of Uniform Velocity :

Case I :

- In relative velocity, passenger walking toward in front of moving train is observed by people sitting inside the train in right direction.
- If the train is moving with certain velocity relative to observer standing on ground, observer would happen to see passengers moving with velocity with result of walking motion and train's motion.



- In the figure, V_{PG} = velocity of passenger relative to ground, V_{PT} = velocity of passenger relative to train, V_{TG}= velocity of train relative to ground, then $V_{PG} = V_{PT} + V_{TG}$
- Consider v_{BS} in the velocity of boat relative to shore, v_{BW} in the velocity of boat relative to water and v_{WS} is the velocity of water (or current) relative to the shore, then
- Velocity v_{BS} of boat relative to shore results from combined motion of boat relative to water (v_{BW}) and motion of water relative to shore (v_{WS}) gives

$$v_{BS} = v_{BW} + v_{WS}$$

Cases of Uniform Acceleration :

Uniform Acceleration :

- A body is said to have a uniform acceleration if it travels in a straight line if its velocity increases by equal amounts in equal intervals of time.
- Motion of a freely falling body is uniformly accelerated.
- As acceleration is rate of change in velocity to corresponding time interval a, so it can be written as

$$a = \frac{v(t') - v(t)}{t' - t}$$

- In uniform velocity, there will be similar values of a in expression, immaterial of values of time t' and t.
- The variation of velocity vector with time is $v(t) = v(0) + a(t)$ or $v(t') = v(t) + a(t' - t)$
- The acceleration a in rectangular components is $a = a_x\hat{i} + a_y\hat{j}$ and change of $v(t)$ component is given as

$$v_x(t) = v_x(0) + a_x t$$
$$v_y(t) = v_y(0) + a_y t$$

- The magnitude of 'a' is given as $|\vec{a}| = a = \sqrt{(a_x^2 + a_y^2)}$

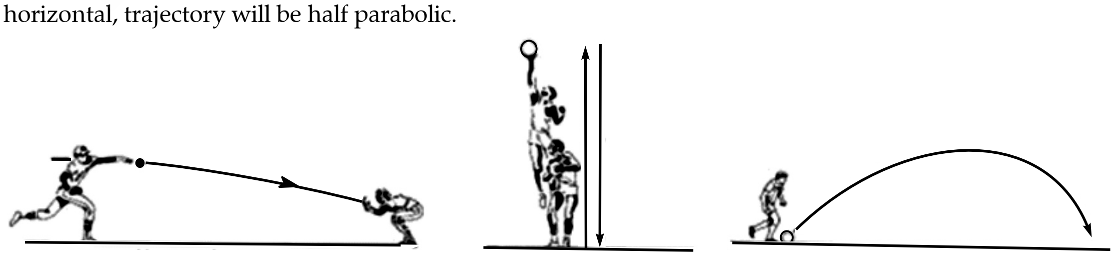
Projectile :

- A body in free fall which is subjected to force of gravity and air resistance only.
- Motion of bodies flung into air.
- It is a special case of linear kinematics.

Projectile Trajectory depends on :

Projection Angle : It shows direction of projection with respect to horizontal. If the angle is :

- perfectly vertical, trajectory will also be vertical
- oblique, trajectory will be parabolic
- horizontal, trajectory will be half parabolic.

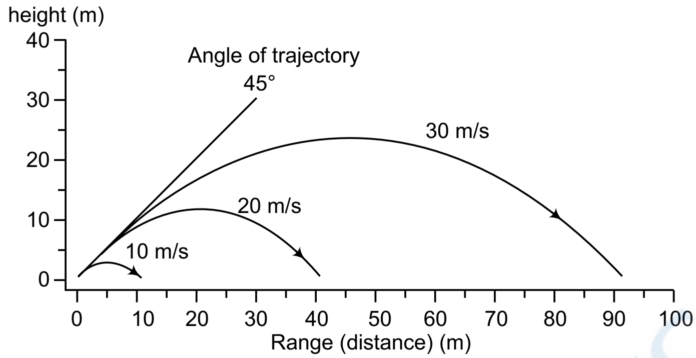


Horizontal trajectory Vertical trajectory Oblique trajectory

Projection Speed : It shows magnitude of projection velocity.

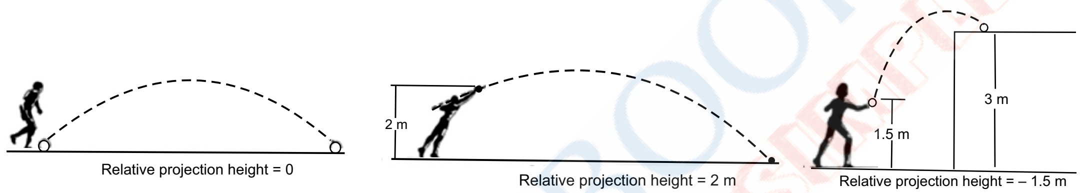
- With fixed projection angle, projection speed shows length of trajectory or range.
- In case of vertical projectile, speed shows apex.

- In case of oblique projectile, speed shows height of apex and horizontal range.



Relative projection height : It is the difference between projection height and landing height.

- If projection speed is fixed, more relative projection height gives longer flight time that increases horizontal displacement.



Factors Influencing Projectile Motion :

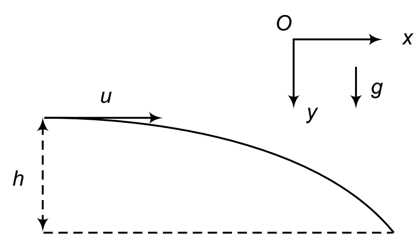
FACTORS INFLUENCING PROJECTILE MOTION (Neglecting Air Resistance)	
Variable	**Factors of Influence**
Flight time	Initial vertical velocity, Relative projection height
Horizontal displacement	Horizontal velocity, Initial vertical velocity
Vertical displacement	Relative projection height, Initial vertical velocity
Trajectory	Initial speed Projection angle, Relative projection height

Types of Projectile Motion :

(1) Oblique projectile motion

(2) Horizontal projectile motion

(3) Projectile motion on an inclined plane



Horizontal Projection : A body can be projected horizontally with initial velocity u along horizontal direction where the initial starting point will be origin.

When it is along x-axis :

1. Component of initial velocity $u_x = u$ along x-axis.
2. Acceleration $a_x = 0$ along x-axis as no force acts along horizontal direction
3. Component of velocity along x-axis at any instant t.

$$v_x = u_x + a_x t = u + 0$$
$$v_x = u$$
$$x = ut$$

Here, horizontal component of velocity will not change.

4. The displacement along x-axis,

$$x = ut$$

When it is along y-axis :

1. Component of initial velocity $u_y = 0$ along y-axis
2. Acceleration $a_y = g = 9.8 \text{ m/s}^2$ along y-axis is directed downwards.
3. Component of velocity along the y-axis at any instant t.

$$v_y = u_y + a_y t = 0 + gt$$
$$v_y = gt$$

4. The displacement along y-axis at any instant ***t***.

$$y = u_y t + (1/2)\, a_y t^2$$
$$y = 0 + (1/2)\, a_y t^2$$
$$y = 1/2 g t^2$$

- **Equation of a Trajectory :** Since, $x = ut$ so $t = x/u$ and $y = (1/2)gt^2$

  Putting value for t in $y = (1/2)gt^2$:

  $$y = kx^2 \text{ where } k = g/(2u^2)$$

  It shows the equation of parabola that is symmetrical about y-axis, hence the path of projectile, is projected horizontally from a height above ground, which is like a parabola.

- **Vertical Projectile Motion :**
  - Vertical projectile motion is similar to motion under constant acceleration.
  - In this, the graphs for the motion are similar with graphs for motion under constant acceleration.
  - Drawing of vertical projectile motion graphs involve: object moving upwards and object moving downwards.
  - In the graphs shown, an object thrown upwards with initial velocity where an object takes certain time to reach its maximum height after which it falls back to the ground.

  (a) Position vs. time graph (b) Velocity vs. time graph (c) Acceleration vs. time graph.

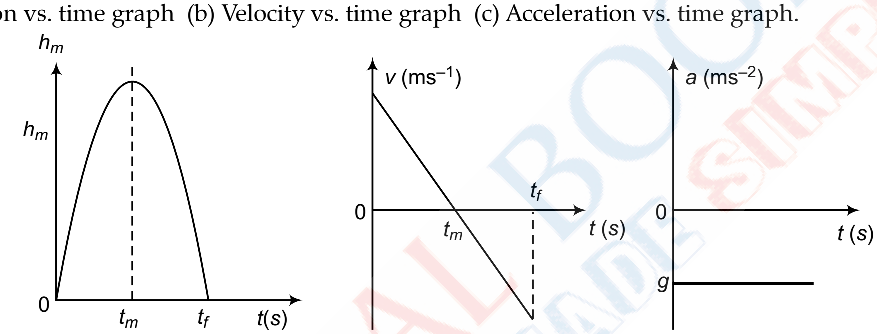


Fig. (a) Fig. (b) Fig. (c)

- **Features of projectile motion :**
  - In a projectile, the launching of an object at certain angle and returning of an object to certain height help in determining range or distance an object travels horizontally.
  - When the projectile is launched with a sharp angle, it spends more time in air, but when the projectile is fired at shallow angle, it will cover faster horizontal direction as compared to sharp firing angle of an object.
  - Maximum range or distance occurs when projectile is launched at an angle of 45° for same initial velocity.
  - If the projectile starts at point higher than its original position of landing, in such case, the ideal distance will not occur at 45° angle.

  Time of flight : $T = \dfrac{2u \sin\theta}{g}$; Maximum height : $H = \dfrac{u^2 \sin^2\theta}{2g}$; Horizontal range : $R = \dfrac{u^2 \sin 2\theta}{g}$

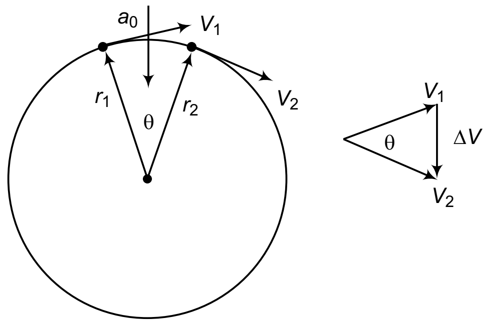
- **Uniform Circular Motion :**
  - An object moving along a circular path with constant or uniform speed, direction of its velocity will change.
  - If speed of moving object remains constant, the velocity will change due to change in its direction, hence $\Delta v = v_2 - v_1$ which will not be zero as of $|v_2| = v_2 = |v_1| = v_1 = v$, where v is speed that remains constant.
  - For small changes in time or for small angles θ, distance Δr is given as $\Delta r = v\,\Delta t$. it shows that :

    $$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

  - As $\dfrac{\Delta v}{\Delta t}$ is acceleration through which an object moves in circle describes centripetal acceleration a_c that is directed toward center of circle $a_c = v^2 / r$

- **Centripetal Acceleration :**
  - It is an acceleration which is acting on an object that undergoes uniform circular motion.
  - It always acts on the object along the radius towards the centre of the circular path.
  - The magnitude of centripetal acceleration

$$a = \frac{v^2}{r} = \omega^2 r = 4\pi^2 n^2 r = \frac{4\pi^2}{T^2} r$$



- Direction of centripetal acceleration is always same as that of velocity change Δv .
- As $\Delta t \rightarrow 0$, $\Delta \upsilon$ becomes perpendicular to velocity vector.

Centripetal Force :

- Centripetal force is that force which is required to move a body in a circular path with uniform speed.
- The force acts on the body along the radius and towards centre so, $F = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}$

Work Done :

- Work done by centripetal force will always because the force is perpendicular to velocity and instantaneous displacement.
- Work done = Increment in kinetic energy of body, W = F.S = F.S cosθ = F.S cos 90° = 0

❑❑❑

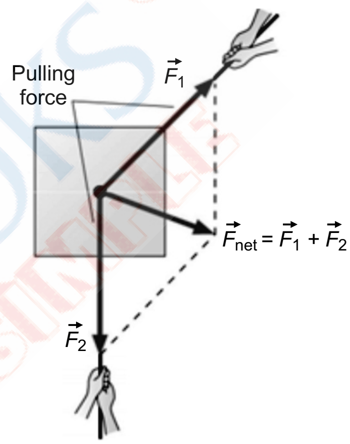
Chapter 3

Laws of Motion

» Revision Notes

Intuitive Concept of Force

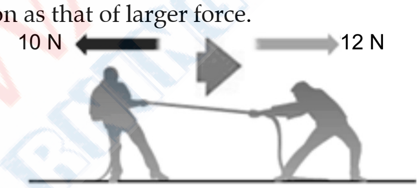
- Force is a push or pull which can be:
  - ***Contact Force***, which acts through direct contact between two objects such as Frictional Force, Tensional Force, Normal Force, Air Resistance Force, Applied Force, Spring Force.
  - ***Long Range Forces***, can act over distances like Gravitational Force, Electrical Force, Magnetic Force.



- Force is a vector quantity where net force is the vector sum of all forces.
- When forces are applied in the same direction, they are added to determine the size of the net force.

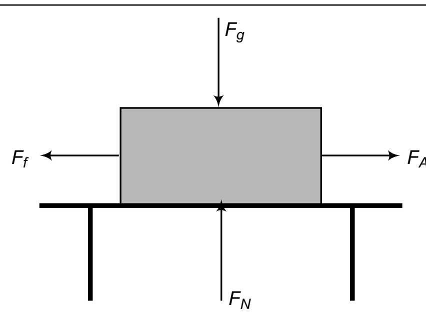
Net Force : 25N + 20N = 45N

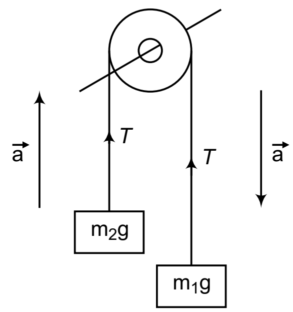
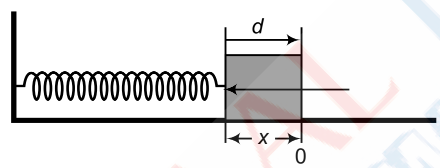
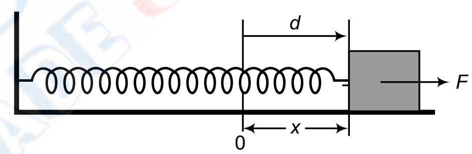
- When two forces are in opposite directions, then smaller force gets subtracted from larger to have net force which results in same direction as that of larger force.



Net Force :
12 N – 10 N = 2N

- There are many types of forces that are categorized as forces resulted from the contact or non-contact of two interacting objects.

Gravity Force F_g = Force of gravity on earth is always equal to the weight of the box	$F_{grav} = mg$, $g = 9.8$ m/s^2 and m = mass	
Friction Force F_f = It is the force which is exerted by surface on moving object or making an effort to move. $F_f = \mu \times F_n$ Here, μ is the coefficient of friction		**Applied Force** F_A = It is the force which is applied to a box by a person/object.
	Normal Force F_A = Normal force is a support force exerted normally in contact with the table.	

Tensional Force, T	Force transmitted by string, rope or wire when pulled tightly by forces acting from ends. The magnitude of tension is same everywhere in the rope.  1. $a = \dfrac{(m_1 - m_2)}{m_1 + m_2} g$ 2. $T = \dfrac{2m_1 m_2}{m_1 + m_2} \times g$
Spring Force $\mathbf{F}_{spring}$	Force exerted by compressed or stretched spring upon or by any object. pulling the block to the left  pulling the block to the right  Force F is exerted by a spring, $F_{spring} = -kx$ (Hooke's law)

- **Inertia**
  - Tendency of an object to resist changes in object's status.
  - More mass object has more inertia.
  - Law of inertia shows that a body will preserve its velocity and direction till no force in direction of its motion will act upon it.
  - Moment of inertia or rotational inertia (I) is measure of resistance of an object to change the rotation $I = mr^2$
  - Moment of inertia for cylinder about its axis is $I = \frac{1}{2}mr^2$
  - SI unit for inertia is kilogram.
- **Mass**
  - Mass is a quantity which depends on inertia of an object.
  - It is tendency of an object to resist changes in its state of motion that varies with mass.
- **Linear Momentum**
  - Linear momentum is quantity of motion contained in a body.
  - It is measured as force required to stop the body in unit time, momentum = Force ÷ time or $\frac{F}{\Delta t}$
  - It is a vector quantity and it's direction is same as that of velocity of body.
  - If object's velocity changes, its linear momentum also changes, at constant mass

$$\frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t} = ma = F$$

  - Its unit is kg–m/s or g-cm/s having its dimension as $[MLT^{-1}]$.
  - If two objects of different masses have same momentum, then lighter body will have greater velocity, $p = m_1 v_1 = m_2 v_2 =$ constant or, $m \propto \frac{1}{v}$
  - When there is net force $\vec{F}_{net} = \vec{0} = 0$, then $\Delta\vec{p} = \vec{0}$ and hence momentum gets conserved, $\vec{F}_{net}\Delta t = \Delta\vec{p}$

➢ **Law of Conservation of Linear Momentum**

- Law of conservation of linear momentum shows that total momentum of a system of particles remains constant as long as no external forces act upon it. or, $m_1\vec{v_1} + m_2\vec{v_2} + \ldots = \text{constant}$
- When no external forces act on colliding objects, then the vector sum of linear momentum of each body will remain constant which will not be affected by mutual interaction.

➢ **Cases of Conservation of Linear Momentum**

➢ **Case 1 : Recoil of gun:**

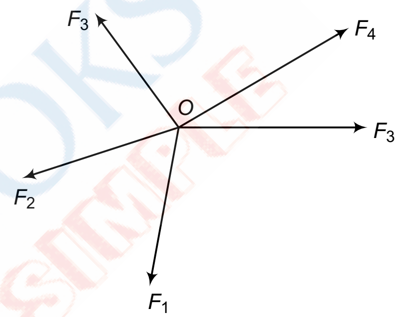
- The expression of recoil velocity of gun is given as $v_G = -\frac{B}{M} \times v_B$

  Here, M is the mass of gun and B is the mass of bullet.

  1. Gun recoils opposite to the direction of motion of bullet.
  2. Greater is the mass of gun, smaller will be recoil of gun.

➢ **Case 2 : Equilibrium of a Particle**

- When two forces F_1 and F_2, act on a particle, then as per Newton's 3rd law, the equilibrium needs $F_1 = -F_2$, where two forces on the particle to be equal and opposite.



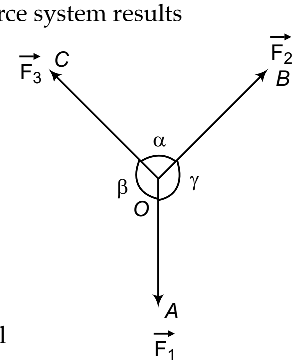
➢ **Case 3 : Equilibrium of concurrent forces**

- Concurrent forces acting at particular point in equilibrium, produces zero resultant.

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 = \vec{0}$$

- Equilibrium of a body needs translational as well as rotational equilibrium.
- According to Lami's theorem when forces F_1, F_2 and F_3 are in equilibrium, each force system results proportional to sine of angle between other two forces:

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



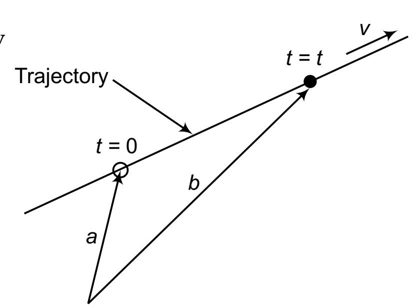
➢ **Conditions of Static Equilibrium of Concurrent Forces**

- Sum of all forces in x-direction or horizontal is zero: $\sum F_x = 0$ or $\sum F_H = 0$
- Sum of all forces in y-direction or vertical is zero: $\sum F_y = 0$ or $\sum F_V = 0$

➢ **Newton's first law of motion**

- Newton's first law of motion or law of inertia states that an object at rest will remain at rest and an object in motion will remain in motion with same velocity unless acted upon by an unbalanced force.
- In motion at constant velocity, speed and direction of the objects do not change with time.
- When an object is at rest or moving at a constant velocity, all the forces acting on it get balanced.
- A car traveling at uniform speed will continue to do so unless the brakes or friction are applied or it hits an object which slows its motion or stops it.
- Motion of an object in vector displacement form with constant velocity v and vector a at time t=0 shows object trajectory as a straight line which passes through point a which runs parallel to vector v.

$$b(t) = a + vt$$



➢ **Newton's second law of motion**

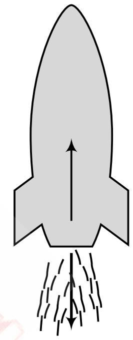
- Second law states that force causing acceleration is proportional to rate of change of momentum with time and acts in direction of change.

$$a \propto \frac{\Delta P}{\Delta t}$$

- In Newton's second law, object's acceleration (a) is directly proportional to net force (F) acting on it and inversely proportional to mass (m). (Hooke's law) *i.e.*

(i) $a \propto F$ **(ii)** $a \propto \frac{1}{m}$

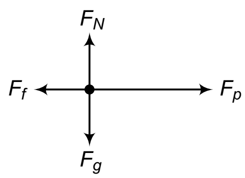
- If force F changes, velocity of a body with constant mass m changes from u to v in time t is given as $a = \frac{F}{m}$ as acceleration, $a = \frac{v-u}{t}$
- SI unit of force is Newton 'N' causing acceleration of 1 m/s^2 of mass of 1 kg.
- When many forces acts on an object, $\vec{F}_{net}$ is replaced with sum of force vectors; $\vec{F}_{net} = \Sigma\vec{F}_{sum}$



➢ **Newton's third law of motion**

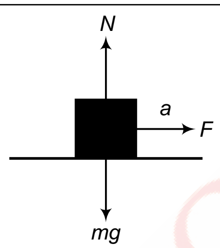
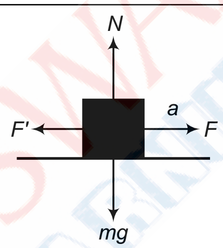
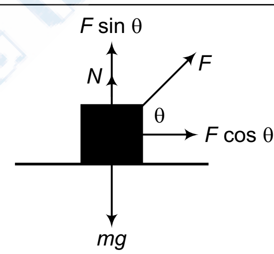
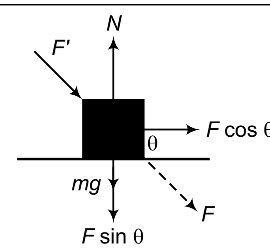
- Newton's third law states that for every action there is an equal and opposite reaction.
- In figure, rocket's action pushes with force of engines, and its reaction makes the rocket to push upwards with equal force.
- With Newton's 3rd law, in the rocket, gases which pushed out through the exhaust, will cause equal and opposite reaction.

➢ **Free Body Diagram**

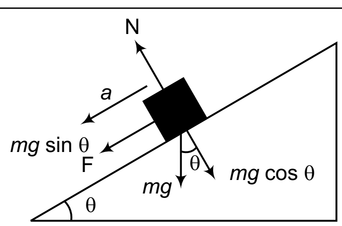
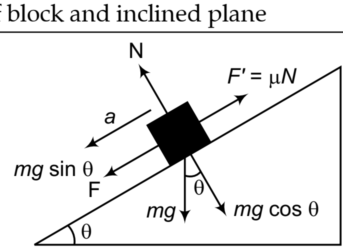
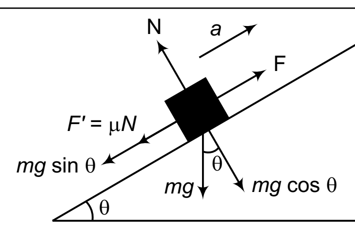


F_g = weight → downward
F_N = normal → perpendicular to surface
F_f = friction (air) → opposite to motion
F_p = push/pull
F_T = tension in string

➢ **Case 1: Horizontal plane**

Block resting on frictionless horizontal surface and is pulled horizontally with force F	Block resting on horizontal surface with μ as co-efficient of friction where it is pulled horizontally with force F	Block resting on horizontal frictionless surface is pulled at an angle θ to horizontal in upward direction	Block resting on horizontal frictionless plane is pushed with force F in downward direction at angle θ to horizontal
			
$N = mg$ $a = \frac{F}{m}$	$N = mg$ and $F - F' = ma$ $a = \frac{F - \mu mg}{m}$ as $F' = \mu mg$	$N = mg - F\sin\theta$ $a = \frac{F\cos\theta}{m}$	$N = mg + F\sin\theta$ $a = \frac{F\cos\theta}{m}$

➢ **Case 2: Inclined plane**

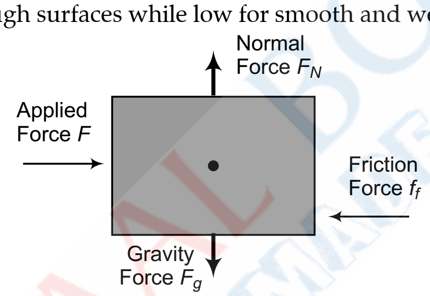
Block sliding down on frictionless inclined plane having angle of inclination θ	Block sliding down on inclined plane with angle of inclination θ and co-efficient of friction μ between surface of block and inclined plane	Block retardation in upward direction in rough inclined plane having coefficient of friction μ between surface of block and inclined plane.
		
$N = mg\cos\theta$ $F = mg sin\theta$ $a = g\sin\theta$	$N = mg\cos\theta, F' = \mu N$ $F = mg\ (\sin\theta - \mu\cos\theta)$ $a = g(\sin\theta - \mu\cos\theta)$	$N = mg\cos\theta, F' = \mu N$ $F = mg\ (\sin\theta + \mu\cos\theta)$ $a = g(\sin\theta + \mu\cos\theta)$

➢ **Apparent weight in lift**

Case 1	Case 2	Case 3	Case 4
When lift is at rest or moves with uniform velocity, then: *acceleration of the person = 0 *net force on person $F = 0$ or, $N - mg = 0$ or, $N = mg$ or, apparent weight = actual weight of person	When lift is moving upwards, then: *acceleration will be in upward for lift which shows positive acceleration. *there are two forces, weight of person downward ($-mg$) and normal force. $F = N - mg$ or $N = m(a + g)$ *apparent weight of person is more than actual weight, when elevator moves upward.	When lift is moving downward, then: *weight of person is downward and normal force works in upward when lift is moving downward, so $N = m(a - g)$ *apparent weight of person in elevator will be less than the actual weight.	In free fall of a body under gravity: * $a = -g$ *$N = m(g - g) = 0$ *apparent weight of person becomes zero or body becomes weightless.

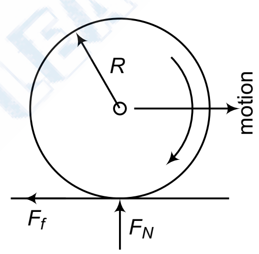
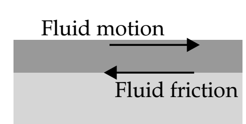
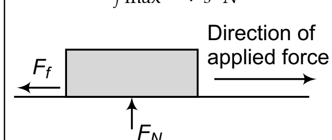
Friction

- It is a force which prevents or tries to prevent slipping or sliding of two surfaces in contact.
- It can be high for dry and rough surfaces while low for smooth and wet surfaces.



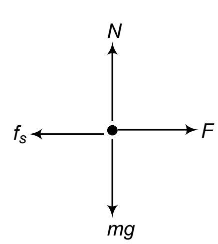
- Frictional resistance between two sliding surfaces is directly proportional to the force pressing the two surfaces $F_s \propto N$

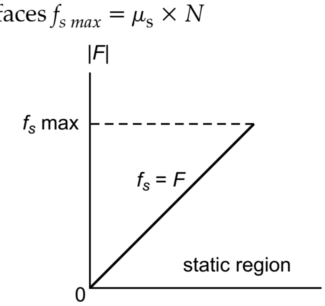
Types of friction

Sliding Friction	Rolling Friction	Fluid Friction	Static friction
Occurs due to pushing of object across a surface	Occurs when object rolls across a surface $F_f = \mu_r F_N$ 	Occurs when solid object moves through fluid which depends on speed, shape and fluid properties. 	Occurs when force applied to object will not cause object to move $F_{f\max} = \mu_s F_N$ 

Static Friction

- Static friction acts on an object when it is not in motion relative to the surface and will act in direction to cancel out applied force F.
- Maximum static friction $f_{s\,max}$ between two surfaces $f_{s\,max} = \mu_s \times N$





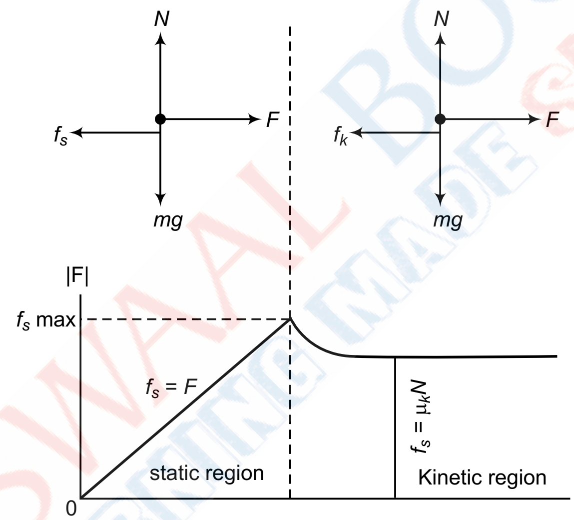
- Magnitude of static friction varies as per applied force F and maximum amount of static friction $f_{s,\,max}$
- If $F \le f_{s,\,max}$, then, object will not move, $f_s = F$.
- If $F > f_{s,\,max}$, then, object will move, $f_s = f_{s,\,max}$, but when object tends to moves, there will be no Static friction.
- If $F = f_{s,\,max}$, it shows tipping point, $f_{s,\,max} = \mu_s F_N$ where object tends to start moving.
- Coefficient of static friction $\mu_s = \dfrac{F_{f\,\max}}{F_N}$; Here, $0 \le \mu_s \le 1$
- If $\mu_s = 0$, no friction; $\mu_s = 0.5$, max friction force = half normal force; $\mu_s = 1.0$, max friction force = normal force.

Kinetic Friction

- Kinetic friction shows more force applied on two bodies in order to slide, due to which, objects will result in break free.
- It always acts opposite to direction of velocity and tends to act to slow down the speed of an object.
- Magnitude of kinetic frictional force is $f_k = \mu_k \times N$
- Coefficient of kinetic friction μ_k is a constant that depends on frictional force F_f and normal force F_N.

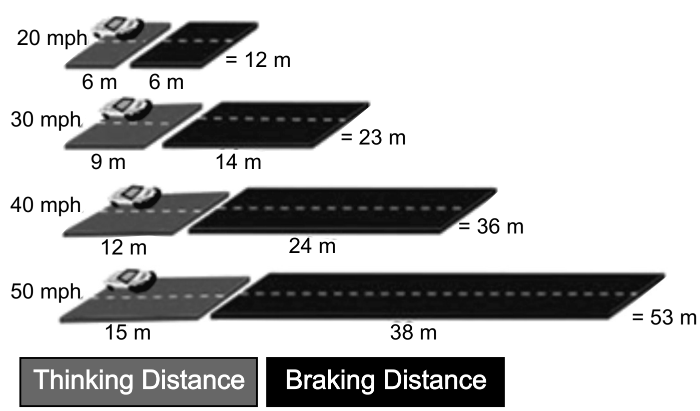
$$\mu_k = \frac{F_f}{F_N}$$

- Coefficient of kinetic friction μ_k is between 0 and 1
- If $\mu_k = 0$, no friction; $\mu_k = 0.5$, friction force = half normal force; $\mu_k = 1.0$, friction force = normal force.



Braking/Stopping distance

- It is distance covered by vehicle before coming to rest.
- Braking or stopping distance depends on:
  - coefficient of friction (μ) between the wheels and road
  - slope of the road
  - initial velocity (v)
- SI unit for stopping distance is meters.
- It can be calculated as $d = \dfrac{v^2}{2g\mu}$

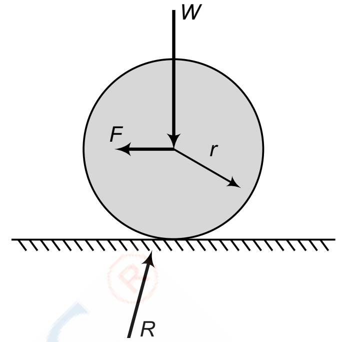


Laws of friction

- The first law states that when two bodies are in contact with each other, then direction of forces of friction on one at it's point of contact will be opposite to direction where point of contact tends to move relative to other.
- The Second Law states that when bodies are in equilibrium, force of friction prevents the motion which can be determined by using conditions of equilibrium of forces that acts on the body.
- The Third Law describes the ratio of limiting friction to normal reaction of surfaces that depends on such substances and not on magnitude of Normal reaction.
- The Fourth Law states that the amount of limiting friction which is independent of area of contact between surfaces and shape of surfaces shown where normal reaction remains unchanged.

- The Fifth Law states that during the motion, direction of friction is opposite to that of relative motion independent of velocity.

Rolling friction and lubrication



- It is a resistive force or rolling resistance which slows down the motion of rolling ball/wheel.
- It is the weakest friction compared to static/sliding friction.
- When a force F is applied to stationary wheel, there will be a small static rolling friction that resists the rolling motion.
- The general equation for rolling friction is $F_r = \mu_r N$, where F_r is resistive force of rolling friction, μ_r is coefficient of rolling friction, N is normal force pushing the wheel to the surface.

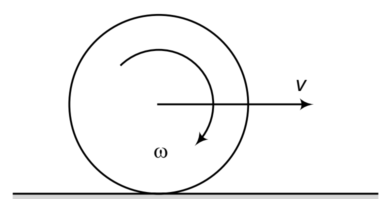
Rolling Without Slipping Friction

- Rolling without slipping occurs when spherical body rolls on a surface without skidding.
- If a body rolls without slipping, bottom of rolling body will be momentarily at rest, relative to constant observer.
- The distance, velocity and acceleration of center of mass of rolling body will be directly related to angle of rotation, angular velocity and angular acceleration about center of mass of body.
- A solid sphere of radius r, mass m, kept on rough inclined plane having coefficient μ, displacement L, moment of inertia I, acceleration a and velocity v with angular velocity ω, then on rolling:

Tangential velocity	Angular momentum	Rate of change of angular momentum	Total kinetic energy	Total torque
$\omega = \frac{v}{r}$	$I\omega = I\left(\frac{v}{r}\right) = \left(\frac{I}{r}\right)v$	$\frac{dI\omega}{dt} = \frac{d\left(\frac{I}{r}\right)v}{dt} = \left(\frac{I}{r}\right)a$	$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $= \frac{1}{2}mv^2 + \frac{1}{2}\mu_k mr^2\left(\frac{v}{r}\right)^2$ $= \frac{1}{2}(\mu_k + 1)mv^2$	$r\mu_0 mg\cos\theta \ge \left(\frac{I}{r}\right)a$

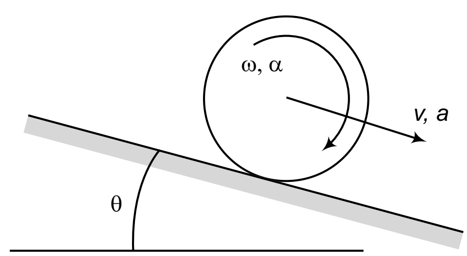
Cases of Rolling Friction

Free Rolling or Inertial Rolling:



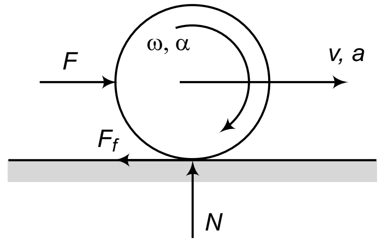
- A rigid cylinder rolls without slipping on rigid horizontal surface whose weight gets balanced by normal reaction from surface, as there is no acceleration perpendicular to the surface.
- The tangential force acting as frictional force at point of contact towards the left lowers the center-line velocity.
- In kinetic rolling, angular velocity and velocity are directly related by radius of wheel $v = r\omega$, so there will be no frictional force towards left and right.

Accelerated rolling on inclined surface:



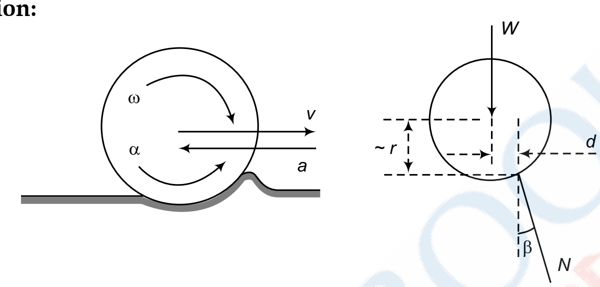
- In accelerated rolling on inclined plane, the surface is not horizontal, but it is at an angle θ with horizontal.
- In this, if required frictional force is more than maximum frictional force, then pure rolling is not possible.
- There results no rolling friction as there is no loss of energy since sum of kinetic energy and potential energy is constant.

- **Accelerated Motion on Horizontal Surface:**



- In accelerated motion on horizontal surface, there is an external force that acts on cylinder to make it to accelerate.
- Due to external tangential force on cylinder, it behaves similar to driven wheel.

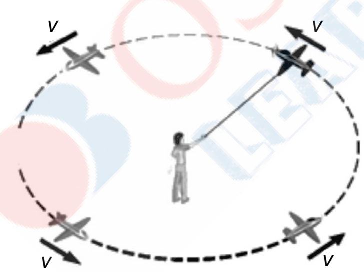
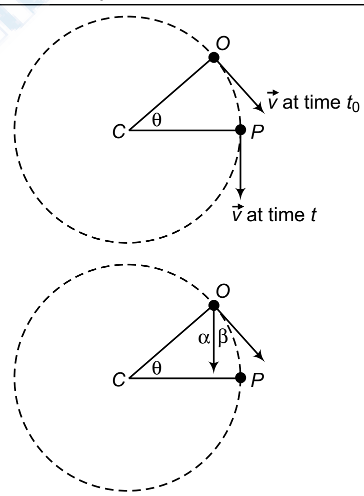
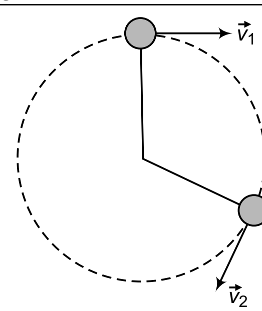
- **Rolling with Deformation:**



- For rolling friction, deformation occurs during rolling of rigid cylinder where normal reaction force gets shifted towards right at an angle β with vertical direction.
- It carries vertical component and horizontal component for normal force where net moment of normal reaction force about the centre, satisfies kinematic relation among angular and linear accelerations.

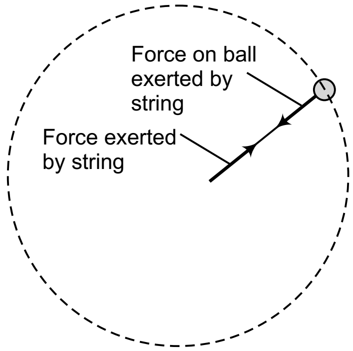
- **Dynamics of uniform circular motion**
  - As per Newton's First Law, there must be some external force which causes the uniform circular motion which could be in shape of:
    - string pulling on ball
    - friction of car tires against the road on a turn
    - gravity pulling on satellite
  - Forces which directs the acceleration towards the center of a circular path is known as Centripetal Force.

- **Uniform circular motion**

Object travelling at constant speed on circular path	Speed remains constant, direction of velocity vector varies	Instantaneous velocity to be tangential to the circle
 $\lvert\vec{r}\rvert = \lvert\vec{v}\rvert = \text{constant}$ $T = \frac{2\pi r}{v}$	 $\Delta\theta = \frac{\Delta s}{r} \approx \frac{v\Delta t}{r}$ $\Delta t \approx \frac{r\Delta\theta}{v}$ $a = \frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} \approx \frac{v^2}{r}$	 $a = \frac{v^2}{r} = \omega^2 r$

Centripetal force

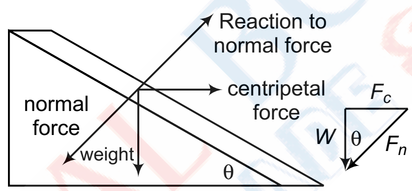
- Centripetal force is the net force required to produce centripetal acceleration whose direction always points toward the center of the circle and can repeatedly changes, as object moves.



- Centripetal force is proportional to square of velocity, so if speed is doubled, then centripetal force will be four times which keeps motion in a circle.

$$F_{centripetal} = \frac{mv^2}{r}$$

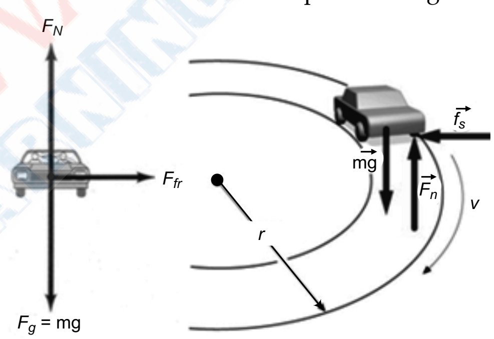
- Centripetal force on banked roads is given by free body diagram as $F_{centripetal} = W \times \tan\theta$



Examples of circular motion (vehicle on level circular road, vehicle on banked road)

Motion on level circular road

- Movement of vehicle on curved path with frictional force acting towards the centre of circular path undergoes centripetal force where wheels of vehicle leaves the curved path and regains straight line path.

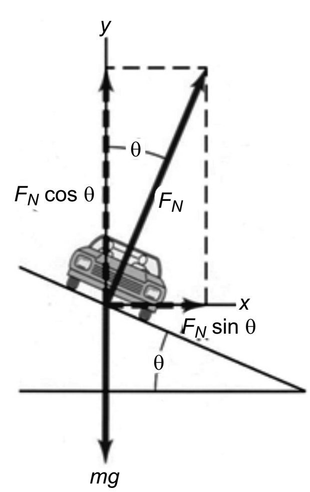


Acceleration : $a_y = 0 \text{ m/s}^2$

$$a_x = \frac{v^2}{r}$$

$$\Rightarrow \qquad F_C = F_{fr} \quad \Rightarrow \quad \frac{mv^2}{r} = \mu mg$$

Velocity $(v) = \sqrt{\mu rg}$



Motion on banked curve road

- Banked curve prevents skidding of vehicles.
- It tilts normal force F_n toward the center of curvature of the road so that the inward radial component $F_N \sin\theta$ can supply required centripetal force.

- There is no acceleration along y axis, so sum of forces in y plane will be zero,

$$F_N = \frac{F_W}{\cos\theta}$$

- The banking angle allowing the vehicle to travel in curve of radius r with constant speed v without frictional force is $\theta = \tan^{-1}\left(\frac{v^2}{r \cdot g}\right)$

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Chapter 4

Work, Energy and Power

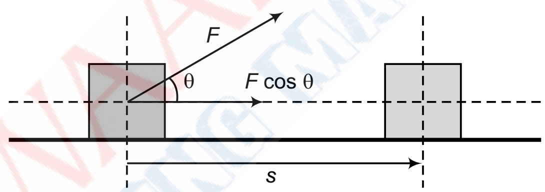
» Revision Notes

Work :

- Work is related to energy and is said to be done when a force acts on a body, displaces the body in the direction of force.
- Work is product of magnitude of displacement and component of force along displacement.

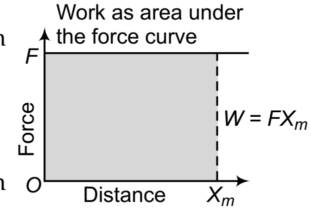
Work done by a Constant Force

- Work done W = $FS \cos\theta = \vec{F} \cdot \vec{S}$
- Work done is scalar quantity
- S.I. unit of work done is N-m or Joule (J)
- Dimensional formula of work done is $[M^1L^2T^{-2}]$



- **Conditions for Work Done**
  - work done by force is zero when point of application of force does not move (s = 0)
  - work done by force is zero when displacement is perpendicular to force ($\theta = 90°$)
  - when angle between force and displacement is acute ($\theta < 90°$), then work done by force is positive or in such case, work is done on the object.
  - when angle between force and displacement is obtuse ($\theta > 90°$), then work done by force is negative or in such case work is done by the object.
  - Force F acting on body with fixed magnitude acting at a constant angle θ from straight line path of the particle from s_1 to s_2 then,

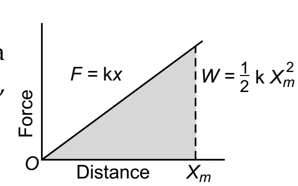
$$W = F\cos\theta \int_{s_1}^{s_2} ds = F\cos\theta\,(s_2 - s_1)$$



  - The work relationship $W = Fx$ applies to constant force along straight line which gives area of rectangle.

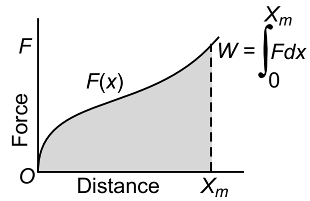
Work Done by Variable Force

- Force that changes with distance makes the work as area under curve.
- If work done in stretching a spring, where area under curve be determined as area of triangle, then integral of force over distance range is area under force curve, Work = $\int F(x)\,dx = \int kx\,dx = \frac{1}{2} k x^2$



- For any function x, work be calculated as area under curve using an integral

$$\text{Work} = \int_{x_1}^{x_2} F(x)\,dx$$



Energy :

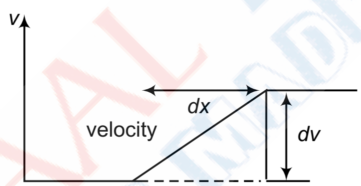
- A body which has the capacity to do work possesses energy.
- There are many forms of energy e.g. electrical, chemical, heat, nuclear, mechanical etc.
- SI unit of energy is same as for work (Joules J).
- In case of purely mechanical energy, energy is of two kinds, potential and kinetic.

Conservation of Energy :

- Law of conservation of energy states that total energy of system remains constant.
- Energy can neither be created nor be destroyed but may be converted from one form to another.
- On considering a body falling freely in air, mechanical energy is conserved, as potential energy is lost and equal amount of kinetic energy is gained due to decrease in height and increase in speed.
- If motion involves friction/collision, law of conservation of energy is true, but conservation of mechanical energy is not applicable as energy is converted to heat and sound.

Kinetic Energy :

- Kinetic energy is the energy due to motion of the object.
- For a body, kinetic energy is amount of work done before being brought to rest.
- The kinetic energy (KE) of an object of mass m that is moving with velocity v is $KE = \frac{1}{2}mv^2$



- If a constant force F applied to mass m in one dimension at distance x, so work done$W = \int Fdx$ which increases linearly over x.
- In one dimensions where force is applied over short distance dx shows mass m increases with velocity from v to $v + dv$.

Kinetic Energy and Work Done :

- If a body of mass m with increase in speed from u to v in distance s by applying a constant force F, produces an acceleration a, so $v^2 = u^2 + 2as$

  If an equation $\frac{1}{2}v^2 - \frac{1}{2}u^2 = as$ is multiplied by mass m, then an increase in kinetic energy shows $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs$ = work done

- On increasing the force $F = ma$, increase in kinetic energy where work done is Fs, gives relationship between kinetic energy, as Work done by forces acting on body = Change of kinetic energy in body (known as work-energy theorem).

  or, $$\Delta \text{KE} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W$$

Potential Energy :

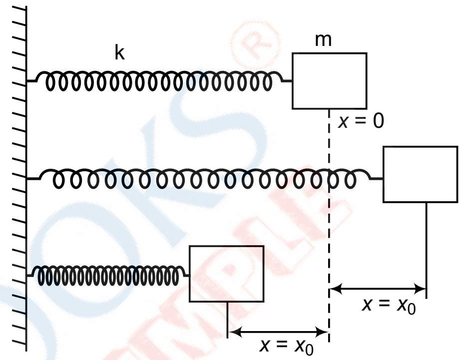
- Potential energy is energy that results from position or configuration of the object.
- An object has a capacity for doing a work as a result of its position in gravitational field, electric field or in magnetic field.
- Potential energy exists when an object having mass has a position inside a force field.
- The potential energy of an object is given by relation $PE = mgh$ where symbols have their usual meaning.
- If a force acting on an object is function of position, it is said to be conservative force shown by potential energy function for one-dimensional as $-dU/dx = F(x)$

- The potential energy is the integral representation as $U(x) = -\int F(x)\,dx + U(x_0)$ where arbitrary constant of integration shows any constant that can be added to potential energy.

Gravitational Potential Energy :

- Gravitational Potential Energy is $PE_g = mgh$, where symbols have usual meaning.
- If an object is dropped from rest at some height above earth's surface, it contains Gravitational Potential Energy and no Kinetic Energy.
- Before an object hits the ground, it has lost its entire Gravitational Potential Energy PE_g and gained an equal amount of Kinetic Energy.

Potential Energy of a Spring



- On slowly compressing/stretching a spring from its resting position, work is done without creating a kinetic energy
- Hooke's law shows that force exerted by a spring is $F_{spring} = -kx$, where symbols have usual meaning.
- In the absence of acceleration, applying force $F = -F_{spring}$, we see $\int dU_{spring} = \int dW = \int F\,dx = -\int F_{spring}\,dx = \int kx\,dx = \Delta(\frac{1}{2}kx^2)$
- If both U and stretching distance x are 0, $U_{spring} = \frac{1}{2}kx^2$
- It is seen that U_{spring} is always positive with respect to unstressed state where stretching (x > 0) and compressing (x < 0) require work, so potential energy is positive in all cases.

Conservative Forces :

- Conservative force is that force whose work done is independent of the path. It depends on initial and final position of the object.
- Conservative forces are Gravitational Force, Electrostatic Force, Magnetic Force , Elastic Force.
- Conservative forces hold the law of conservation of energy.
- Conservative force posseses specific quality where work = 0 when object on which they are exerted moves along closed path.
- Force is conservative, if line integral of any closed loop is zero, $\oint F\,dl = 0$

Conservation of mechanical energy (kinetic and potential energies) :

- Work-energy theorem shows that net work done by all forces acting on a system is the change in kinetic energy, $W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv^2{}_0 = \Delta KE$
- On considering conservative forces, $W_{net} = W_c$ and $Wc = \Delta KE$, when conservative force does some work, then system loses certain potential energy, $-\Delta PE = KE$ or $\Delta PE + \Delta KE = 0$
- Here, total kinetic and potential energy is constant for any process showing conservative forces as $KE_i + PE_i = KE_f + PE_f$, which is conservation of mechanical energy principle.

Non-conservative Forces :

- Non-conservative force is that for which work depends on path of the object.
- In non-conservative force, energy inside a system is not maintained, while it gets dissipated out of system, as a result of which, such forces are known as dissipative forces.
- A rubber band with an unstretched length gets stretched slowly until its extends a distance which is large compared to the unstretched length. The force is supplied by adding successive amount of weights to the end of the band. The band is then allowed to contract slowly until the unstretched length is reached by removing successive amount of weights.
- If an infinitesimal amount of work is done by force acting on infinitesimal displacement, then $dW = \vec{F}.d\vec{r} = |\vec{F}||d\vec{r}|\cos\theta$

Power :

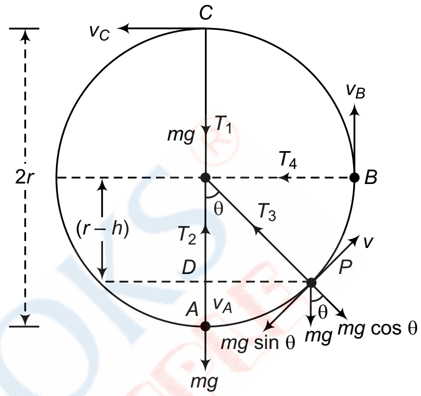
- Power is the rate at which work is done, or rate at which energy is transferred.
- Power = work done / time taken
- SI unit for power is watt (W).
- Power of 1W means that work is being done at rate of 1 J/s.
- Larger units for power are kilowatt kW, 1 kW = 1000 W = 10^3 W, megawatt MW, 1 MW = 1000000 W = 10^6 W.

- HP is another unit of power, 1 HP = 746 W.
- If work is done by machine in moving at speed v against constant force or resistance F, then

$$\text{Power} = Fv \qquad \left[\frac{W}{t} = F\frac{s}{t} \Rightarrow P = Fv\right]$$

Motion in a Vertical Circle

- If a body is released from certain height, it will travel vertically downward towards surface of earth due to force of gravitational attraction that is exerted on body by earth which produces an acceleration by force due to gravity (g).
- The value of g on surface of earth is 9.8 m/s^2 that is similar for all relevant bodies.
- A particle moving in a circle with the help of light and inextensible string fixed at the centre, makes an angle of θ with vertical shows total radial force at P



$$F = T - mg\cos\theta = \frac{mv^2}{r}$$

$$\text{Tension at point C (at the top)} = T_{min} = \frac{mv_c^2}{r} - mg$$

$$= \frac{m(rg)}{r} - mg \qquad \left(\because v_c = \sqrt{rg} = \text{critical velocity}\right)$$

$$T_{min} = 0 = T_1$$

$$\text{Tension at point B } (T_4) = \frac{mv_B^2}{r}$$

$$= \frac{m(3gr)}{r} \qquad (\text{Here, } v_B = \sqrt{3rg})$$

$$T_4 = 3mg$$

$$\text{Tension at point A } (T_2) = \frac{mv_A^2}{r} + mg$$

$$= \frac{m(5rg)}{r} + mg \qquad (\text{Here, } v_A = \sqrt{5rg})$$

$$T_{\max}(T_2) = 6mg$$

- With this, it is noted that the chances of the string to break will be maximum at bottom where it will not only support the object but also pulls it up out of it in straight-line. (When net force > 6 mg)

Collision :

- A collision is an event where momentum or kinetic energy is transferred from one object to another.
- In this, ratio of magnitudes of final and initial relative velocities is called coefficient of restitution shown as e as

$$e = \frac{v_f}{v_i}$$

Momentum and energy conservation in collision

- Law of conservation of linear momentum shows total momentum of system to remain same in magnitude and direction after collision till net external force on system remains zero.
- Collision for which total kinetic energy is conserved is called elastic collision.
- Inelastic collisions shows that total kinetic energy decreases after collision and lost kinetic energy gets converted to heat.
- Velocity of center of mass of system before and after collision will be equal in magnitude and direction in case of elastic collision.

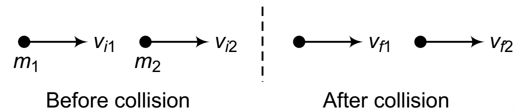
Types of Collision :

- Collisions in which there is no change in kinetic energy is elastic collision where $\Delta K = 0$.
- When magnitude of final relative velocity is less than magnitude of initial relative velocity, $e < 1$, then change in kinetic energy is negative, hence it shows inelastic collision as $\Delta K < 0$.

- When two objects stick together after collision, then relative final velocity is 0, $e = 0$, so it is called as totally inelastic collision where change in kinetic energy $\Delta K = -\frac{1}{2}\mu v_A^2 = -\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} v_A^2$, where μ is called reduced mass.
- When magnitude of final relative velocity is more than magnitude of initial relative velocity, $e > 1$, then change in kinetic energy is positive, so it is known as superelastic collisions where $\Delta K > 0$.

Elastic Collision

- In elastic collision, momentum and kinetic energy remain conserved.
- If two balls of masses m_1 and m_2 with velocities v_{i1} and v_{i2} colliding, then they will move with final velocities v_{f1} and v_{f2}.



- The total momentum and energy before and after collision results as

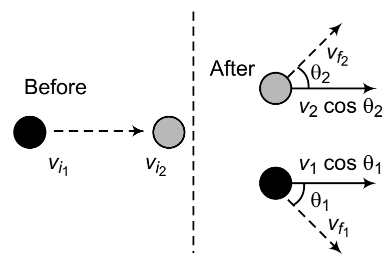
$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2} \text{ and } m_1 v_{i1}{}^2 + m_2 v_{i2}{}^2 = m_1 v_{f1}{}^2 + m_2 v_{f2}{}^2$$

Elastic Collisions in One Dimension

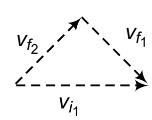
- Two objects with mass m_1 and m_2 moving in one dimension collide and subject to zero net force which are not in contact with one another, so both objects before and after collision result in constant velocity.
- Two velocities of objects before and after collision, exert large transitory force shows equal and opposite forces.
- Due to zero net external force, total momentum of system gets conserved, so momentum before and after collision in one-dimensional result as $m_1 v_1 + m_2 v_2 = m_1 u_{f_1} + m_2 u_{f_2}$.
- In elastic collision, total kinetic energy of two colliding objects remains same before and after collision, so

$$\frac{1}{2}m_1 v_{i1}{}^2 + \frac{1}{2}m_2 v_{i2}{}^2 = \frac{1}{2}m_1 u_{f1}{}^2 + \frac{1}{2}m_2 u_{f2}{}^2$$

Elastic Collisions in Two Dimension



- If an object collides elastically, then $m_1 v_{i1}{}^2 + m_2 v_{i2}{}^2 = m_1 v_{f1}{}^2 + m_2 v_{f2}{}^2$ appears as equation for energy and momentum conservation when initial velocities of two objects before impact, and final velocities after collision as per Newton's laws for complete trajectory of particles.
- Two balls on colliding with each other, tend to move in different direction.
- If masses of two balls are equal with one ball as stationary, then as per equation $m_1 v_{i_1}{}^2 + m_2 v_{i_2}{}^2 = m_1 v_{f_1}{}^2 + m_2 v_{f_2}{}^2$, $v_{i_1} = v_{f_1} + v_{f_2}$.
- As per velocity triangle, final velocities are perpendicular as per Pythagorean theorem.



$$v_{i_1}{}^2 = v_{f_1}{}^2 + v_{f_2}{}^2$$

Here, $v_{f_1} = v_1 \cos\theta_1$ and $v_{f_2} = v_2 \cos\theta_2$

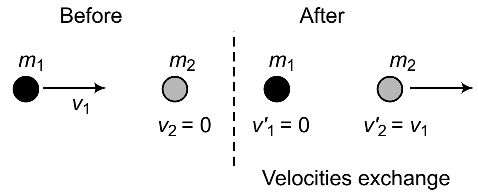
Perfectly Elastic Collision :

- Perfectly elastic collision shows law of conservation of momentum and kinetic energy during the time of collision.
- Before collision, momentum of small object is positive while momentum of big object is negative that shows total system momentum remains negative.
- Once the collision is done, momentum of small object becomes negative while momentum of big object results zero, so momentum of system lead to negative which gets conserved.
- Total system energy before collision gets added for objects while kinetic energy after collision is zero for one object.
- Kinetic energy before collision is equal to total kinetic energy after collision.

Head on Elastic Collisions :

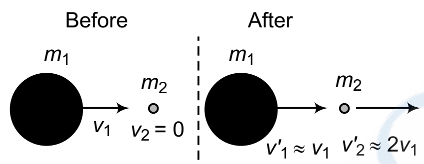
- There are three cases in Head on Elastic Collisions :
  - Equal Masses
  - Huge Projectile
  - Huge Target

Elastic Collision, Equal Masses $m_1 = m_2$



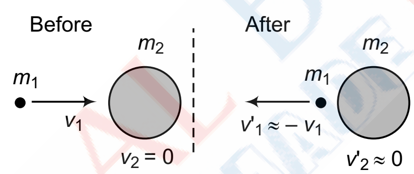
- In head-on collision, stationary object with equal mass, shows that projectile comes to rest and the target moves with equal velocity.
- In head-on elastic collision of equal masses, velocities get exchanged.

Elastic Collision, Huge Projectile $m_1 >> m_2$



- In head on elastic collision with huge projectile with very small target, velocity of target after collision becomes twice as that of projectile.
- In this, projectile velocity remains unchanged.

Elastic Collision, Huge Target $m_1 << m_2$



- In head on elastic collision between small projectile and huge target, projectile bounces back with similar speed.
- In this, target gains very small velocity.

Inelastic Collision

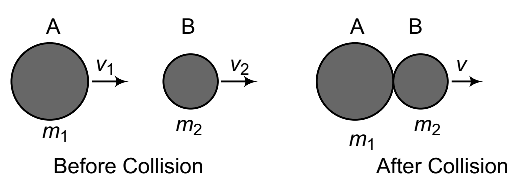
- Inelastic collision is a collision in which there is a loss of kinetic energy, but the momentum of the system gets conserved as some kinetic energy had been transferred to thermal energy, sound energy and material deformation.
- Total system kinetic energy before and after collision get added up and increase, so total kinetic energy before collision is not similar to total kinetic energy after collision.

Head on Inelastic Collision

- If two particles having masses m_1 and m_2 with velocities u_1 and u_2 before and after collision stick together and moves with similar velocity v, then from law of conservation of linear momentum, $m_1u_1 + m_2u_2 = (m_1 + m_2)\,v$ and $m = (m_1 + m_2)$.
- Kinetic energy before collision $KE_i = \frac{1}{2}m_1{u_1}^2 + \frac{1}{2}m_2u_2^2$ and KE after collision $KE_f = \frac{1}{2}(m_1 + m_2)v^2$
- As $KE_f < KE_i$, so energy loss there, after collision of particles.

$$\text{Loss of KE} = \frac{m_1m_2}{2\,(m_1 + m_2)}(u_1^2 - u_2^2)\,(1 - e^2)$$

Perfectly Inelastic Collision



- If two balls undergoes perfectly inelastic collision, they stick together after collision and results in final common velocity $v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$ and loss in $KE = \frac{m_1m_2}{2\,(m_1 + m_2)}(u_1 - u_2)^2$

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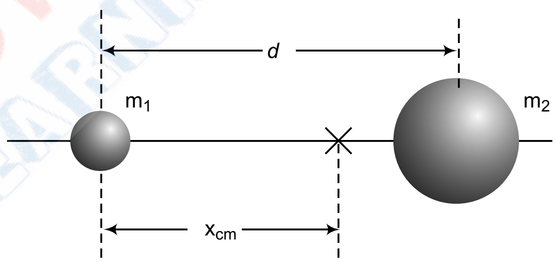
Chapter 5 Motion of System of Particles and Rigid Body

» Revision Notes

Centre of Mass of a Two-particle System :

- The center of mass of a system of particles is a particular point at which, the system's entire mass can be supposed to be concentrated.
- The centre of mass is a function of positions and masses of the particles in system.
- In rigid body, position of centre of mass is fixed in relation to the object.
- In loose distribution of masses in free space, the position of centre of mass is a point in space among them which corresponds to position of any individual mass.
- In uniform gravitational field, centre of mass is normally called as centre of gravity, a point where gravity acts.
- Center of mass in uniform gravitational field shows the unique point in an object or system that is applied to describe the system's response to external forces and torques.
- The idea of center of mass exists as average masses which are factored by certain distances from a reference point.
- The position of the center of mass of a system of two particles with mass m_1 and m_2, located at position x_1 and x_2 is $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$



- The center of mass lies between two masses which is nearer to heavy mass, so for system with more than two particles, position of center of mass is $x_{min} \leq x_{cm} \leq x_{max}$

Momentum Conservation

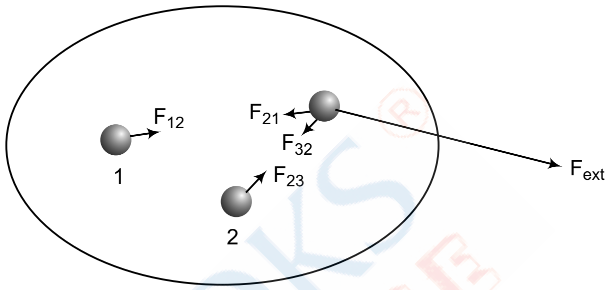
- Linear momentum p of object having mass 'm' and velocity 'v' is $p = mv$, which shows unit of momentum as (kg m/s) or (N s).
- As momentum is related to linear motion of an object, it is known as linear momentum.
- Under certain circumstances, linear momentum of system is conserved.
- Linear momentum of particle is related to net force acting on that object: $\sum F = ma$ or $\frac{dp}{dt}$

  This is also called Newton's 2nd law of motion.
- Rate of change of linear momentum of particle is net force acting on object that points in direction of force.
- If net force acting on object is zero, then its linear momentum is fixed, so total linear momentum p of system of particles is vector sum of individual linear momenta.
- As observed, linear momentum of system of particles as product of total mass M of system and velocity of center of mass is $p = \sum m_i v_i$ or Mv_{cm} where M is total mass of system.

- Net external force acting on system of particles is zero $F_{ext} = 0$ N, so linear momentum of system gets conserved.
- Law of conservation of linear momentum is fundamental and exact law of nature established as per Newton's law and holds true even where Newtonian mechanics fails.

Centre of Mass Motion

- In absence of external forces, centre of mass moves with constant velocity as $F_{ext} = Ma_{cm} = 0 \Rightarrow v_{cm}$ = constant.
- Centre of mass is applicable for all systems having internal forces covering magnetic fields, electric fields, chemical reactions etc.
- If total momentum for system of particles is $p = Mv_{cm}$ where M is total mass and v_{cm} is velocity of center of mass, then $Ma_{cm} = \sum m_i a_i$ where symbols have usual meaning.
- As per Newton's second law, $m_i a_i$ with force acting on mass m_i, shows $Ma_{cm} = \sum F_i = F_{ext}$ which shows that motion of center of mass is only determined by external forces.
- Forces exerted by one part of system on other parts of the system are internal forces because Newton's third law shows, sum of all internal forces cancel out.



Centre of Mass of Rigid Body

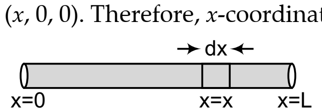
- Centre of mass of a system of particles predicts the possible motion of the system.
- Rigid Body has perfectly definite and unchanging shape when distance among all pairs of particles will remain unchanged.
- The center of mass of body does not always coincide with its intuitive geometric center, hence centre of mass of rigid body is point whose position is fixed with respect to body.
- The center of mass will tend to move due to all forces which will act on rigid body's particles $\sum m_i a_i = \sum F_i$
- The external force which act on a particle, it will not project that the body is rigid $\vec{F}_i = \sum \vec{F}_{ext}$
- If external force is applied to particle without internal forces, it allows particle to leave the body, so $F_i = \sum F_{ext} + \sum F_{int}$ which shows force that depends on particular model of internal forces.

Centre of Mass of Symmetrical Bodies

Object	Position of centre of mass
Uniform hollow sphere	Centre of the sphere
Uniform solid sphere	Centre of the sphere
Uniform circular ring	Centre of the ring
Uniform circular disc	Centre of the disc
Uniform rod	Centre of the rod
Plane lamina	Point of intersection of diagonals
Triangular plane lamina	Point of intersection of medians of the triangle
Rectangular or cubical block	Point of intersection of diagonals

Centre of Mass of Uniform Rod

- The centre of a mass of uniform rod is at its centre.
- Suppose a rod of mass M and length L is lying along the x-axis with its one end at $x = 0$ and the other at $x = L$.
  Mass per unit length of the rod $l = M/L$
  Let dm be the mass of the element dx situated at $x = x$.
  The coordinates of the element dx are $(x, 0, 0)$. Therefore, x-coordinate of COM of the rod is



$$x_{COM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L (x)(\lambda\,dx)}{\int_0^L \lambda\,dx} = \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

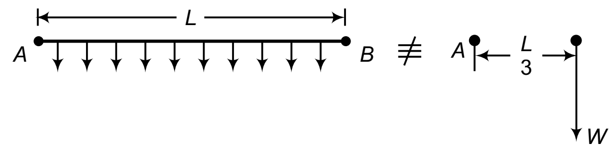
The y-coordinate of COM is

$$y_{COM} = \frac{\int y\,dm}{\int dm} = 0$$

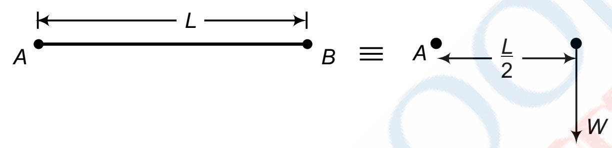
Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$, i.e., it lies at the centre of the rod.

- When a rod AB of weight w and other is a concentrated weight w, at a distance $\frac{L}{3}$ from point A act as two system, then these two systems are equivalent as far as the net gravitational pull in concerned, but the moment of gravitational forces of these two systems about point A is different.



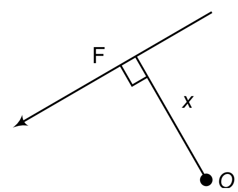
If weight w is kept at distance $\frac{L}{2}$ from A, then the moment of gravitational pull in the rod and the new system are same.



Moment of a Force, Torque and Angular Momentum

Moment of Force :

- The moment of a force (also known as torque) about a point or axis provides a measure of the tendency of the force to cause a body to rotate about the point or axis. A force causes a body to turn or rotate which results in turning effect.
- Moment of a force = force × perpendicular distance
- The moment M (turning effect) of a force about a point O is the product of the magnitude of the force (F) and the perp. distance (x) to the point of application.



- By convention, anti-clockwise moments are positive.
- Formula for torque vector is :

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- The magnitude of vector $\vec{r}$ is the moment arm length, hence the torque vector will be given as :

$$\vec{\tau} = \vec{r} \times F \sin\theta \times \hat{n}$$

In this, angle θ is the angle between the moment arm and the force and $\hat{n}$ is the unit vector.

The moment of force depends on :

- Size or magnitude of force
- Perpendicular distance between line of action of force and turning point

The moment of force can be calculated using :

Moment of force = Force × perpendicular distance from pivot to line of action of force

Moment = $F \times d$

Moment is measured in newton meters (Nm).

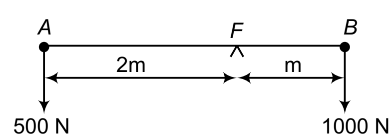
- The direction of the torque can be found using right-hand rule using curling of right hand from the moment arm vector to force vector and by following the direction of thumb.
- For an object which appears to be rotating clockwise, the direction of torque vector is down. For an object that appears to be rotating counter-clockwise, the torque vector points up.
- Tangential component of applied force F_{tan} has a magnitude of $F_{tan} = F \sin\theta$

The magnitude of torque $\tau = r \times F_{tan}$.

The Principle of Moments

It states that if the system is in equilibrium then total sum of the anticlockwise is equal to the total sum of the clockwise moment, i.e.,

Total anticlockwise moment = Total clockwise moment



From the given figure, 500 N × AF = 1000 N × BF

1000 Nm = 1000 Nm

Hence, the given case is in equilibrium.

For a rigid body acted upon by a system of coplanar forces, equilibrium is achieved when:

(i) the vector sum of the coplanar forces = 0

(ii) there is no net turning effect produced by the forces (the sum of clockwise and anti-clockwise moments = 0)

Angular Momentum

- If angular momentum of rotating object $\vec{L}$ is a result of linear momentum at a distance from axis of rotation, then angular momentum is $\vec{L} = \vec{r} \times \vec{p}$ or $\vec{L} = \vec{r} \times m\vec{v}$
- The SI units of angular momentum are kg m^2/s.
- In this, vector $\vec{p}$ is linear momentum, written as $\vec{p} = m\vec{v}$.
- The vector $\vec{r}$ is vector drawn from the axis of rotation to location of linear momentum vector. Angular momentum vector is written as $\vec{L} = \vec{r} \times m \times v \sin\theta \times \hat{n}$, where angle θ is angle between the moment arm and linear momentum and $\hat{n}$ in the unit vector.
- If moment arm and linear momentum are perpendicular, sine term is 1 and magnitude of the angular momentum results as $L = |L|$ (perpendicular linear momentum and moment arm)

  $$= r \times p \quad \text{or} \quad L = r \times m \times v$$

  The direction of angular momentum obtained with right-hand rule by curling right hand from moment arm vector to linear momentum or velocity vector that follows the direction of the thumb. For a point on an object rotating around the z axis, the linear velocity is $\vec{v} = \vec{r} \times \vec{\omega}$

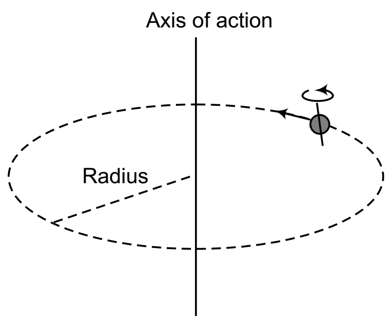
  A particle in rotating object having index number 'i' has angular momentum of magnitude,

  $$L_i = m_i \times r_i^2 \times \omega$$

  The total magnitude of angular momentum around z axis is sum of individual angular momenta,

  $$L = \sum L_i \text{ so, } L = (\sum m_i \times r_i^2)\,\omega \text{ or } \vec{L} = \vec{I} \times \vec{\omega}$$

Conservation of Angular Momentum and its Applications



- The conservation of motion for objects travelling in straight lines also has conservation of motion for objects that travels along the curved paths, and hence such conservation of rotational motion is conservation of angular momentum.
- An object that rotates has angular momentum while it is spinning, where angular momentum measures an object's ability to keep spinning.
- If L is angular momentum, I is moment of inertia and angular velocity as ω, so $\vec{L} = \vec{I} \times \vec{\omega}$
- Angular momentum is conserved when there is zero net torque applied to a system, where the system is the object or objects that are rotating.
- Torque and angular momentum are related through angular impulse equation.

  Angular impulse is the net torque (τ) times a change in time (t) which in turn equals a change in angular momentum as $\sum\tau \times \Delta t = \Delta L$
- If an angular momentum of system is conserved, there will be no change in total angular momentum, so equation with net torque equals zero as $0 \times \Delta t = \Delta L$ or $0 = \Delta L$
- The angular momentum will remain constant or conserved when net torque is zero as seen from Newton's 2nd law for rotational motion $\vec{\tau} = d\vec{L}/dt$.

  When net torque is zero, $d\vec{L}/dt = 0$, if change in angular momentum ΔL is zero, then angular momentum is constant, L = constant as $\tau = 0$, which shows law of conservation of angular momentum where torque acting on system about an axis results zero, which fixes total angular momentum of system about that axis.

Examples of Conservation of Angular Momentum

Example 1: Spinning Skater

In spin skating, a person will spin using the spreading of the arms and then further moving the arms closer to the body. Such motion will help in increasing the speed with which the skater rotates.

For case-1

$L_1 = I_1\,\omega_1$

Here, $I_1 = mr_1^2$

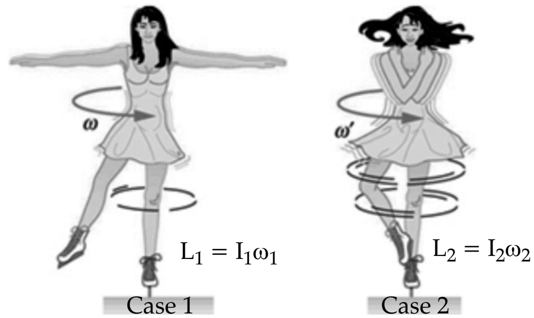
Since $r_1 > r_2 \Rightarrow I_1 > I_2$

But $L_1 = L_2$

or, $I_1\omega_1 = I_2\omega_2$

As, $I_1 > I_2 \Rightarrow \omega_1 < \omega_2$

Hence, in case 2, it rotates faster.



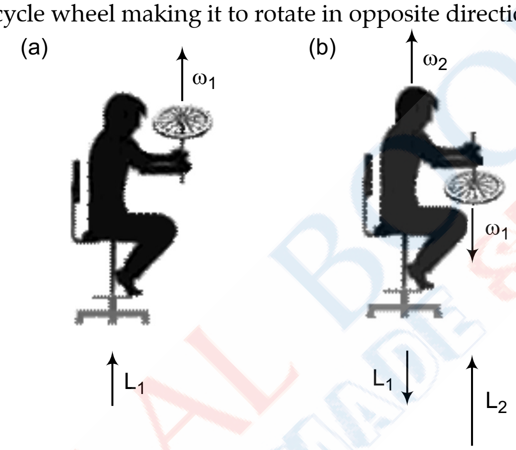
For case-2

$L_2 = I_2\omega_2$

Here, $I_2 = mr_2^2$

Example 2: Spinning bicycle wheel

Conservation of angular momentum is observed while the person holding a spinning bicycle wheel on a rotating chair. The person turns the bicycle wheel making it to rotate in opposite direction as described.



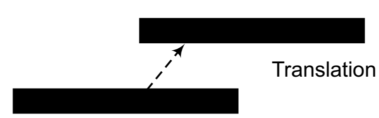
Initially, the wheel has angular momentum in an upward direction, but when the person turns the wheel, angular momentum of wheel gets reversed. As the person wheel chair system is an isolated system so, total angular momentum gets conserved and the person begins to rotate the wheel in opposite direction.

Now the vector sum of angular momentum in both the figures will be the same and momentum gets conserved.

Equilibrium of Rigid Bodies

- Rigid body is said to be in equilibrium, if forces acting on it does not change its state of rest or of uniform motion.
- If a body is at rest, it should remain at rest. While if the body is in motion, it should keep on moving with uniform velocity until a non-zero external force is applied on it.

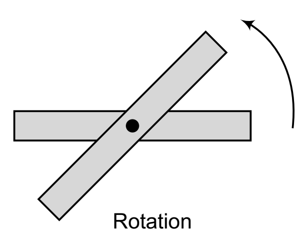
Translatory Equilibrium



- A body is said to be in translatory equilibrium, if its centre of mass, possesses through an inertial frame of reference having no linear acceleration.
- For a body to be in translatory equilibrium, the vector sum of all the external forces acting on the body should be zero.

  In such case, body at rest remains at rest. Such type of equilibrium is static equilibrium. While, if a body moving with uniform velocity, along a straight line keeps on moving, is known as dynamic translatory equilibrium.

Rotatory Equilibrium

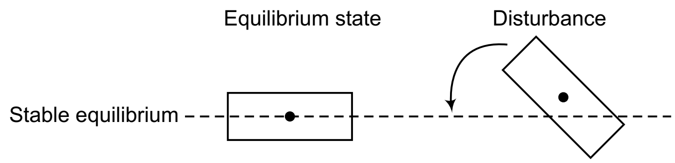


- A body is said to be in rotatory equilibrium if it possesses no angular acceleration about any axis in an inertial frame.
- For a body to be in rotatory equilibrium, the vector sum of all the external torques acting on the body should be zero.
- For a body to be in equilibrium, should satisfy following conditions:
  - vector sum of all external forces acting on the body should be removed.
  - vector sum of all external torque acting on the body should be removed.

State of Equilibrium

Equilibrium is classified as :

Stable equilibrium : The equilibrium of a body is said to stable, if on any movement, it tends to come back to its original position.

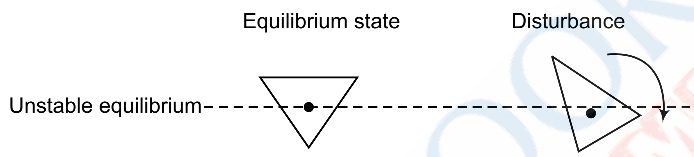


Condition for Stable Equilibrium

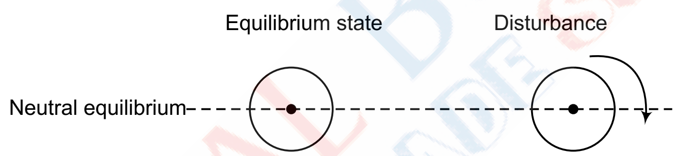
The degree of stability of a system depends upon :

- height of the centre of gravity of body from the surface of support
- vertical line drawn from center of gravity passing through the base
- low position of center of gravity
- broader base

Unstable equilibrium : The body is said to be in unstable equilibrium, if on any movement, shows no tendency to come back to its original position, but can move away from its position.

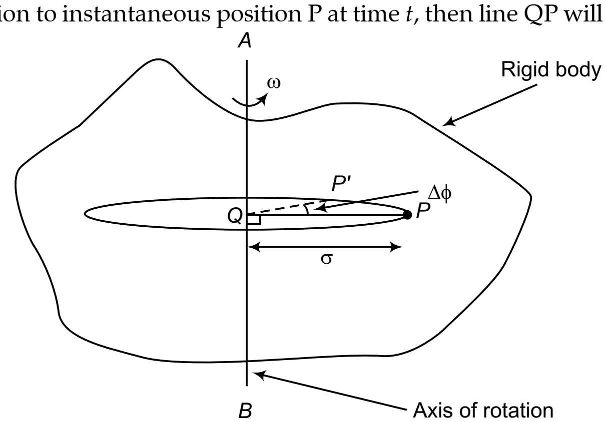


Neutral equilibrium : A body is said to be in neutral equilibrium if on any movement, it tends to remain in new/ current position.



Rigid Body Rotation

- A rigid body that undergoes rotational motion where axis of rotation corresponds to straight line that serves as locus of all points lying inside the body remains stationary during the rotation of the body.
- A rotating body with fixed axis of rotation along line AB has a point P lying inside the body that executes circular orbit is centred on line AB, in a plane which is perpendicular to it.
  If line QP is radius of orbit that links the axis of rotation to instantaneous position P at time t, then line QP will be normal to the line AB.
  At time $t + \Delta t$, the point P moves to P′ and radius QP rotates through angle $\Delta\phi$, so instantaneous angular *velocity* of the body is
  $\omega = \lim_{\Delta t \to 0} \Delta\phi / \Delta t = d\phi/dt$



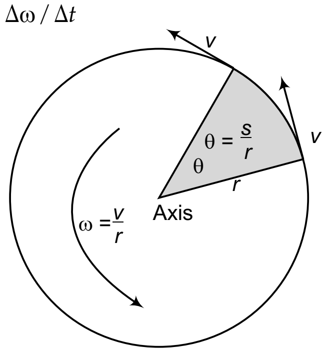
- If body is rotating rigidly, value of ω is same for all possible points P that lies in the body where linear speed v of point P is given as $v = \tau\omega$ where symbols have usual meanings.
- A rigidly rotating body will have high rotating speed that increases linearly with distance from the axis of rotation.
- The angular acceleration α (t) of rigid rotating body, is the time derivative of angular velocity which is $\alpha = d\omega / dt = d^2\phi/dt^2$, where symbols have usual meanings.
- For a body rotating with fixed angular velocity ω, angular acceleration is zero and rotation angle ϕ will tends to increase linearly with time is $\phi = \phi_0 + \omega t$ or $\phi_0 = \phi$ (at $t = 0$)
- For a body rotating with constant angular acceleration α, angular velocity increases linearly with time, so $\omega(t) = \omega_0 + \alpha t$ and rotation angle satisfies $\phi(t) = \phi_0 + \omega_0 t + 1/2\alpha t^2$ where $\omega_0 = \omega$ (at $t = 0$).

Equations of Rotational Motion

- The rotation in terms of angular displacement is described as time, angular velocity and angular acceleration.
- Angular velocity is the rate of change of angular displacement and angular acceleration is the rate of change of angular velocity.

$$\omega = \frac{d\theta}{dt} \text{ and } \alpha = \frac{d\omega}{dt}$$

- The averages of velocity and acceleration are defined by the relationships:
  - Average angular velocity : $\omega = \Delta\theta / \Delta t$
  - Average angular acceleration : $\alpha = \Delta\omega / \Delta t$



- If s is the arc length and the angular displacement is
  $$\theta = s/r$$
  So, in case of circular path, the angular velocity is $\omega = v/r$
  and the angular acceleration is $\alpha = a/r$
  The acceleration here is tangential acceleration. Apart from tangential acceleration, there always exists centripetal acceleration as $a_c = v^2/r$
- If an object is rotating about an axis, at each point on object, there is similar angular velocity.
- The tangential velocity of a point is proportional to distance from axis of rotation, so angular velocity with units rad/s is $v = \omega \times r$ or $\omega = v/r$
- if v is constant, then angle is $\theta = \theta_0 + \omega t$
- If α is constant, then rotation will be shown $\omega^2 = \omega_0 + 2\alpha\theta$

Comparison of Linear and Rotational Motions

Linear Motion		Rotational Motion	
Displacement	Δx	Angular Displacement	$\Delta\theta$
Velocity	$v = \frac{\Delta x}{\Delta t}$	Angular Velocity	$\omega = \frac{\Delta\theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	Angular Acceleration	$\alpha = \frac{\Delta\omega}{\Delta t}$
At constant acceleration, the equations of motion will be		At constant angular acceleration, the equations of motion will be	
(i)	$v = v_o + at$	(i)	$\omega = \omega_o + \alpha t$
(ii)	$v_{average} = \frac{1}{2}(v_o + v)$	(ii)	$\omega_{average} = \frac{1}{2}(\omega_o + \omega)$
(iii)	$x = x_o + v_o t + \frac{1}{2}at^2$	(iii)	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$
(iv)	$a = \frac{v^2 - v_o^2}{2(x - x_o)}$	(iv)	$\alpha = \frac{\omega^2 - \omega_o^2}{2(\theta - \theta_o)}$
Mass	m	Moment of Inertia	$I = \sum m_i r_i^2$
Linear Momentum	$p = mv$	Angular Momentum	$L = I\omega$
Force	$F = ma$	Torque	$\tau = I\alpha$
Power	$P = Fv$	Power	$P = \tau\omega$
Newton's 2nd Law	$\sum F = ma = \frac{dp}{dt}$	Newton's 2nd Law	$\sum \tau = I\alpha = \frac{dL}{dt}$
Equilibrium	$F_{net} = 0$	Equilibrium	$\tau_{net} = 0$
Kinetic Energy	$KE = \frac{1}{2}mv^2$	Rotational Kinetic Energy	$KE = \frac{1}{2}I\omega^2$

Moment of Inertia

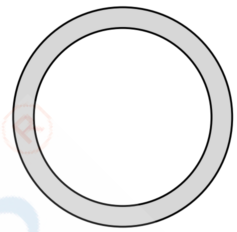
$$I = \Sigma \mu_i r_i^2$$
$$I = \lim_{\Delta m_i \to 0} \sum m_i r_i^2$$
$$I = \int r^2 dm$$

- Moment of inertia also known as rotational inertia is a rotational analog of mass for linear motion.
- It depends upon mass of an object and on quantity of mass distributed relative to axis of rotation.
- Moment of inertia of an object depends upon the position and location of axis of rotation.
- In point mass, moment of inertia (I) = mr^2.
- It is Continuous Mass Distribution.

  ***I* for a hoop**

  A uniform hoop : mass (M); radius (R)

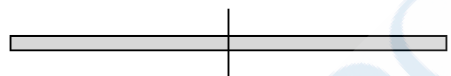
  $$I = MR^2$$



  ***I* for a thin rod**

  A uniform thin rod: mass (M); length (L); rotating about its center of mass.

  $$I = \frac{1}{12} ML^2$$



Radius of Gyration

- Radius of gyration is defined as the distance from the axis of rotation to a point where the total mass of the body gets concentrated, so that the moment of inertia about the axis may remain same.
- In terms of radius of gyration, moment of inertia of body of mass M is given as,

  $$\text{Inertia } (I) = MK^2$$

  $K = \sqrt{I/M}$, where symbols has usual meaning.
- Radius of gyration is the distance where the entire mass of object was packed together will result in similar moment of inertia.

Radius of Gyration of a Thin Rod

- The moment of inertia of uniform thin rod having mass M and length L about an axis through its center and perpendicular to its length is given as:

  $$I = ML^2 / 12$$
- If K is radius of gyration of the rod about the axis, then

  $$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}$$

Radius of Gyration of a Solid Sphere

- Moment of inertia for a solid sphere having radius R and mass M is

  $$I = \frac{2}{5} MR^2$$
- If K is radius of gyration of the solid sphere, then

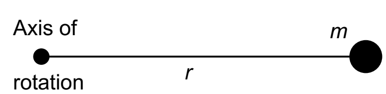
  $$I = MK^2$$

  Also
  $$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{2MR^2}{5M}} = \sqrt{\frac{2}{5}}\,R$$

Moment of Inertia with Respect to Specific Rotation Axis

- Moment of inertia of a point mass with respect to an axis is the product of mass times the distance from the axis squared.
- The moment of inertia of any extended object is created from the basic function.
- The moment of inertia of point mass is

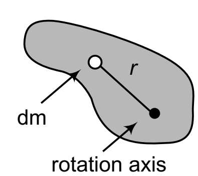
  $$I = m \times r^2$$



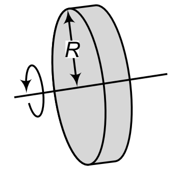
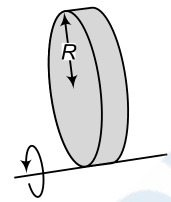
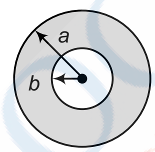
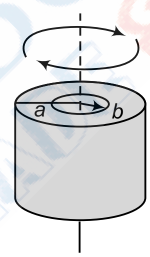
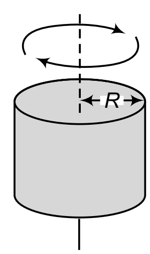
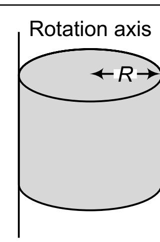
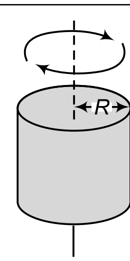
- The moment of inertia for infinitesimal mass element dm has same form which is known as differential element of mass with moment of inertia as:

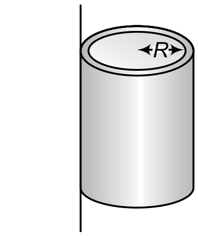
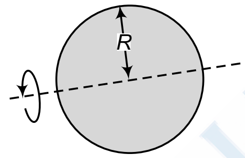
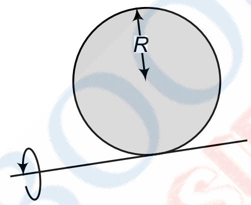
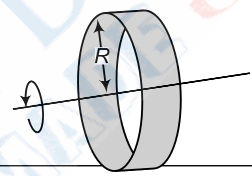
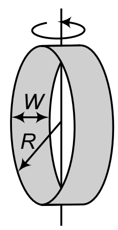
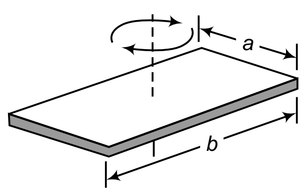
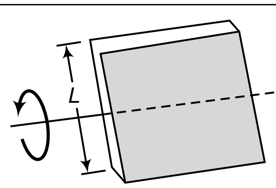
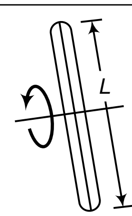
  $$dI = r^2\, dm$$
- The sum over all mass elements is an integral over the masses as :

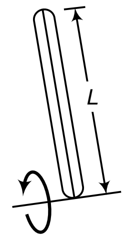
  $$I = \int dI$$
  $$= \int r^2\, dm$$



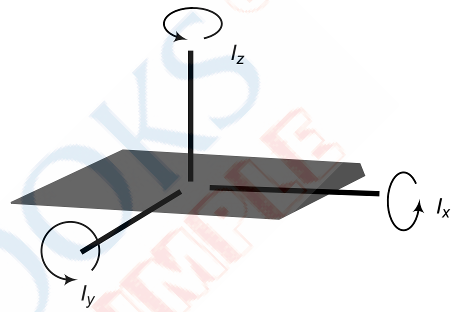
Values of Moments of Inertia for Geometrical Objects

Object	**Axis of Rotation**		Moment of Inertia
Solid Disk	Central axis of disk		MR^2
Solid Disk	Axis at Rim		$\frac{1}{2}MR^2$
Disk with a hole	Axis at center		$\frac{1}{2}M(a^2+b^2)$
Cylindrical Shell	Axis at center		$\frac{1}{2}M(a^2+b^2)$
Solid Cylinder	Central axis of cylinder		$\frac{1}{2}MR^2$
Solid Cylinder	Axis on surface		$\frac{3}{2}MR^2$
Hollow cylinder	Central axis of hollow cylinder		MR^2

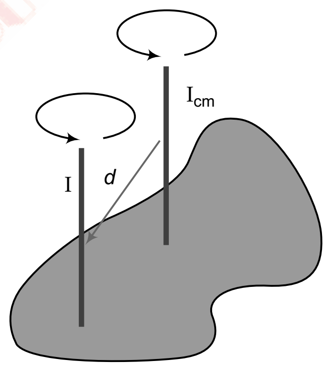
Object	Axis of Rotation		Moment of Inertia
Hollow cylinder	Axis on surface		$2\,MR^2$
Solid Sphere	Central axis of sphere		$\frac{2}{5}\,MR^2$
Solid sphere	Axis on surface		$\frac{7}{5}\,MR^2$
Hoop	Central axis of hoop		MR^2
Hoop	Axis through central diameter		$\frac{1}{2}\,MR^2 + \frac{1}{12}\,MW^2$
Rectangular plate	Axis through center		$\frac{1}{12}\,M\,(a^2 + b^2)$
Rectangular plate	Axis through center, in plane of plate		$\frac{1}{12}\,ML^2$
Thin Rod	Axis through mid point		$\frac{1}{12}\,ML^2$

Object	Axis of Rotation		Moment of Inertia
Thin Rod	Axis at one end		$\frac{1}{3}ML^2$

Parallel and Perpendicular Axes Theorems



- **Perpendicular Axes Theorem**
  - The theorem describes that the moment of inertia of a plane lamina, about an axis perpendicular to plane lamina is equal to sum of moments of inertia of lamina about two mutually perpendicular axes which are lying in plane of lamina and which intersects each other at a point, where perpendicular axis will pass through the body.
  - For a plane or flat object in x-y plane, and on rotating an object in z axis, moment of inertia will be I_z.
  - If the plane or object is rotated at x or y axis, then moment of inertia will be I_x and I_y so $I_z = I_x+I_y$
  - For plane object, moment of inertia about an axis perpendicular to plane is sum of the moments of inertia of two perpendicular axes at same point in the plane of object.



- **Parallel Axes Theorem**
  - Parallel axis theorem shows that moment of inertia of a body, about an axis, is the sum of moment of inertia of body about a parallel axis through its center of gravity with product of the mass of body and square of the distance between the axes.
  - The moment of inertia when the axis goes through the center of mass in a certain direction as I_{cm} and when this axis is displaced by a distance d but is parallel to original axis, then moment of inertia through an axis is $I = I_{cm}+ Md^2$

❑❑❑

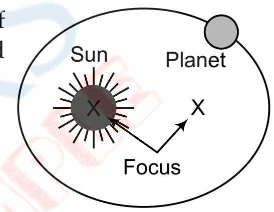
Chapter 6

Gravitation

» Revision Notes

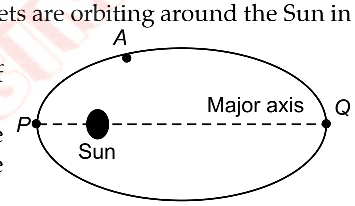
Kepler's laws of planetary motion

- Johannes Kepler after working on data painstakingly and without the help of telescope realized that the orbits of the planets are ellipses and based on it formulated the Laws of Planetary Motion.
  - **First Law :** The Law of Orbits/ Law of Ellipses
  - **Second Law :** The Law of Areas
  - **Third Law :** The Law of Periods/ Law of harmonies



Law of Ellipses

- Kepler's first law is referred to as the law of ellipses or orbits where the planets are orbiting around the Sun in a path which is in shape of an ellipse.
- In this, the orbits of the planets are ellipses, where the Sun is at one focus of an ellipse and every planets move in elliptical orbits.
- If A is a planet which revolves round the Sun, then the position P of the planet which is close to the Sun is known as **perigee** while position Q of the planet that is farthest from the Sun is known as **apogee**.



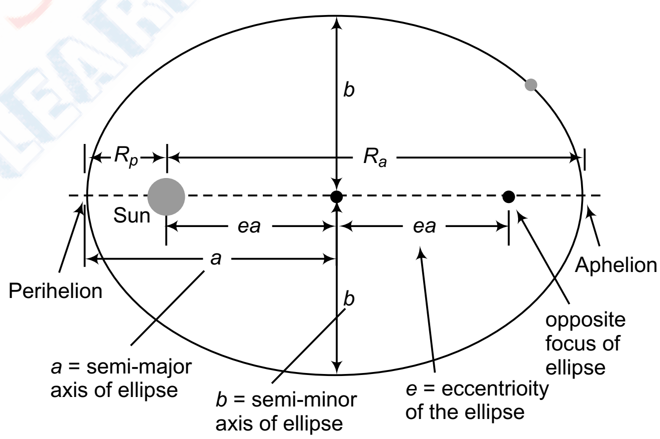
- The elliptical shape of an orbit is a result of the inverse square law of force of gravity where the eccentricity of an ellipse is described. Eccentricity of an ellipse is the ratio of the distance between the foci to the major axis of the ellipse. It is zero for a circle.
- Out of all planetary orbits, only Neptune has minimum eccentricity.

Since, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Here, $a > b$

So, $b^2 = a^2 (1 - e^2)$

and $e < 1$

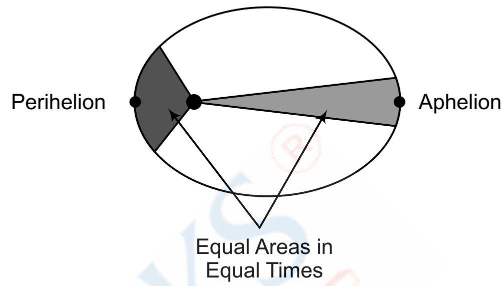


Ellipse Eccentricity of the planets

Planets	Eccentricity
Mercury	0.206
Venus	0.0068
Earth	0.0167
Mars	0.0934

Jupiter	0.0485
Saturn	0.0556
Uranus	0.0472
Neptune	0.0086

Law of Areas

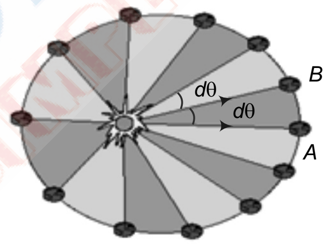


- Kepler's second law is known as law of equal areas.
- The law states that a line which connects a planet to the Sun sweeps out equal areas in equal intervals of time.
- A planet will move faster when it is nearer the Sun and will execute elliptical motion with constantly changing angular speed while moving about in an orbit.
- The second law shows that the planet moves fast when it is near perihelion while will move slow, when it is near the aphelion.
- If planet is moving from point A to B, then radius vector $\overrightarrow{OA}$ will sweep a small angle $d\theta$ at centre in small interval of time dt.

  $\overrightarrow{dA}/dt = r^2\omega / 2$

  Angular momentum is $L = mr^2\omega$ or $r^2\omega = L/2m$, which shows

  $\overrightarrow{dA}/dt = \vec{L}/m =$ constant



- As line of action of gravitational force passes through the axis, external torque will be zero, angular momentum is conserved, so dA/dt is constant.
- If a planet with smallest / greatest distance from center (r_1, r_2) is perihelion and aphelion and Sun/Earth as center, then distances are always measured from the center of the bodies.
- If planet is orbiting around the Sun, then velocity V_1 at perihelion will be maximum for orbit and V_2 is the velocity at aphelion will be minimum.
- The law of areas should be same, $r_1 V_1 = r_2 V_2$ or $V_1 : V_2 = r_2 : r_1$

Law of Periods

- Law of periods states that the square of the period of revolution of a planet around the Sun is directly proportional to cube of mean distance between the planet and Sun.

$$T^2 \propto R^3$$

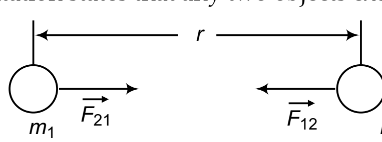
or, $$T^2 = \left(\frac{4\pi^2}{GM}\right) \times R^3$$

So, $$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

- The law is known as **law of harmonies** which compares the orbital time period and radius of an orbit of any planet with other planets and it gave rise to the law of gravitation.
- Mercury which is in the innermost planet will take 88 days to orbit the Sun while planet Neptune requires 165 years to orbit the Sun.

Universal Law of Gravitation

- Newton's Universal Law of Gravitation states that any two objects exert a force of attraction on each other.



- The magnitude of the force is proportional to the product of the central masses of the two objects, and is inversely proportional to square of the distance between them.
- If mass m_1 exerts a force F_{12} on mass m_2 and m_2 exerts a force F_{21} on m_1 then as per Newton's third law,

$$F_{12} = -F_{21}$$

- The magnitude of gravitational force is $F_{12} = Gm_1m_2/r^2$ where symbols have usual meaning
- Newton's gravitational constant is very small since the gravitational force between two 1 kg objects separated by 1 m, results in 6.67×10^{-11} Newtons.

- For an object of mass 'm' near the Earth's surface, $F_{grav} = -G \times \frac{Me\, m}{Re^2} = -mg$, where, M_e is 5.98×10^{24} kg, mass of Earth and R_e is 6.38×10^6 m, radius of Earth
- The acceleration due to gravity $g = G \times M_e / R^2_e = 9.8$ m/s^2

Importance :

- It is the force which is responsible for binding to the Earth
- It is responsible for motion of the moon around the Earth
- It is responsible for motion of planets around the Sun
- It is responsible for the formation of tides, happened due to gravitational force of both Sun and Moon on Earth.

Acceleration due to gravity and its variation with altitude and depth :

- It is an acceleration of a body due to the influence of gravitational force of Earth, shown by 'g'.
- The acceleration due to gravity on Earth comes from Earth's large mass.
- As acceleration of a body takes the direction of net force acting on it and only force will be the gravity, so acceleration will take the direction of gravity.

Variation of acceleration due to gravity :

- The acceleration due to gravity 'g' varies with altitude, depth, shape of earth and rotation of earth about its own axis.
- It decreases with height and depth.
- If the acceleration is g on surface of earth, then it is $g = \left(1 - \frac{2h}{R}\right)$ at height h and it is $g = \left(1 - \frac{d}{R}\right)$ at depth d.

Effect of altitude on acceleration due to gravity :

- If the Earth is considered a sphere of mass M and radius R with centre O and g is the acceleration due to gravity at point P on surface of Earth, then

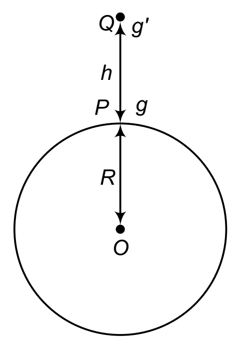
$$g = GM / R^2$$

- If g' is acceleration due to gravity at height h above the surface of Earth at point Q, then

$$g' = GM / (R + h)^2$$

or

$$g'/g = 1/(1 + h/R)^2 = [1 + h/R]^{-2}$$

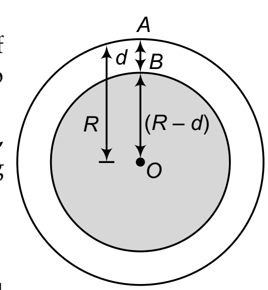


- If h is smaller than R, then h/R is small as compared to 1, so $g' = g\,(1 - 2h/R)$ [By Binomial Theorem]
- The acceleration due to gravity decreases with the height, as at height equal to radius of Earth $h = R$,

$$g' = gR^2 / (R + R)^2 = g/4$$

- The percentage decrease in acceleration due to gravity is $g' = (g - g')/g \times 100 = (2h/R) \times 100$

Effect of depth on acceleration due to gravity :



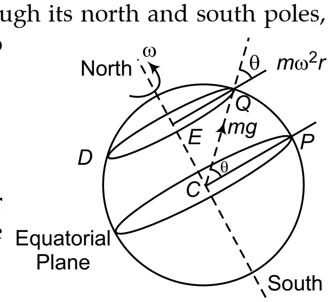
- If Earth is a sphere of mass M and radius R with centre O and g be the value of acceleration due to gravity at a point A on the surface of Earth, then $g = GM / R^2$ It ρ is density of material of Earth, then $g = (4/3) \times \pi GR \rho$
- If g' is acceleration due to gravity at point B with depth 'd' below the surface of Earth, then body at point B will show a gravity pull due to Earth with radius $(R - d)$ having mass M', so $M' = (4/3)\,\pi\,(R - d)^3\,\rho$ and $g' = (4/3)\,\pi\,G(R - d)\,\rho$
- The value of acceleration due to gravity decreases with depth, $g' = g(1 - d/R)$

Effect of latitude on acceleration due to gravity 'g' :

- The value of 'g' changes from place to place due to non-spherical shape of Earth and rotation of Earth.
- The Earth's shape with variation in g results as $g = GM/R^2$ as $g \propto 1/R^2$ since 'g' is inversely proportional to square of radius and is least at the equator and maximum at poles.

Effect of Earth's Rotation on acceleration due to gravity 'g' :

- If Earth is a sphere of mass M and radius R which rotates about an axis through its north and south poles, then effective value of acceleration due to gravity changes with latitude due to rotation of Earth.
- At point Q latitude is θ, then effective weight is $mg' = mg - m\omega^2 r \cos\theta$
- If r is radius of circle of rotation of Q, then $g' = g\,(1 - \omega^2 R/g \times \cos^2\theta)$



Gravitational Potential Energy :

- Gravitational potential energy is the energy possessed by masses as per their spatial arrangement and gravitational force which pulls them towards one another.

- If two or more bodies interact with each other, some work will be done in assembling such bodies together in respective places where total work done during assembling is related as gravitational potential energy of system.
- It is linked with the mass of an object and its position.
- The amount of gravitational potential energy of an object on Earth depends on mass of object and height of object, above the ground.
  It is equal to the work done in lifting an object to certain height against gravitational force,
  $$GPE = (\text{mass}) \times g \times (\text{height})$$
- If an object falls, it gives up its gravitational potential energy with gravitational force working on it that will speed up the object and gets converted into kinetic energy.
- Change in gravitational potential energy is $\Delta PE_{grav} = mgh$, where change in height is 'h' instead of 'Δh' where 'h' is positive when final height is more than initial height and vice versa.
- Due to gravitational force of attraction, two objects of masses m_1 and m_2 placed at a distance 'r' apart have ability to do work and will have potential energy, so $PE_{grav} = -Gm_1m_2/r$, where negative sign is a consequence of attractive nature of gravitational force.
- If objects are far apart, gravitational force moves the objects closer, that decrease the potential energy, so gravitational potential energy near Earth's surface of an object of 'm' with distance 'h' above the earth's surface is
  $$PE_{grav} = -G\,[mM_E/(R_E + h)]$$
- For distance $h << R_E$, $1/(R_E + h) \approx 1/R_E\,[1 - h/R_E]$ which is
  $$PE_{grav} \approx -G \times mM_E/[R_E\,(1 - h/R_E)] = \text{constant} + mgh$$

Gravitational Potential

- Two point masses, m and m' located at position vectors $\vec{r}$ and $\vec{r}'$ where acceleration g of mass m results in gravitational force that is exerted on it by mass m' will be
  $$g = Gm' \text{ x } (\vec{r}' - \vec{r})/|\vec{r}' - \vec{r}|^3$$
- The x component of acceleration,
  $$g_x = Gm'\,(x'-x)/[(x'-x)^2+(y'-y)^2+(z'-z)^2]^{3/2}$$
- The gravitational force exerted on some point mass by collection of other point masses is vector sum of forces exerted on former mass by each of latter masses taken in isolation.
- The gravitational potential generated by collection of point masses at a certain location in space is the sum of potentials generated at that location by each point mass taken in isolation.
- If there are N point masses, m_i (for i=1,N), located at position vectors $\vec{r}_i$, the gravitational potential generated at position vector $\vec{r}$ is $\Phi(\vec{r}) = -G\sum m_i/|\vec{r}_i - \vec{r}|$
- If there exists continuous mass distribution, then mass at position vector $\vec{r}'$ is $\rho(\vec{r}')\,d^3(\vec{r}')$
- By summing over space and taking the limit $d^3(\text{r}') \to 0$, we find that above equation will give the expression for gravitational potential $\Phi(\vec{r})$, by continuous mass distribution $\rho(\vec{r}')$
  $$\Phi(\vec{r}) = -G\int \rho(\vec{r}')/|\vec{r}_i - \vec{r}|\,x\,d^3(\vec{r}')$$

Gravitational Potential at a Point :

- Gravitational Potential at a point in a gravitational field is the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.
- The gravitational potential at a point is always negative and is maximum at infinity.
- In this, at a point, the energy is in object of mass 1kg falls from an infinite distance to distance "R" from the centre of mass of other object.
- Gravitational Potential at a point (V) = Work done (W) / mass (M)
- The dimensional formula of Gravitational Potential is $[M^0L^2T^{-2}]$ and SI unit of Gravitational Potential is J/ kg.
- From Newton's law of Gravity, gravitational force at a point is $((GMm)/r^2)$, where the force becomes smaller and smaller as moving further and further and at r = infinity, it tend to zero.
- Gravitational Potential at radius R is $-GM/R$ or Gravitational Potential Energy of a mass m at a Radius R is $-GMm/R$, where minus sign shows that energy is stored in the body, which makes the planet to revolve around the sun and not allowing them to fall.

Escape Velocity :

- It is the speed that an object needs to escape from the gravitational influence of a massive body without further propulsion.
- Escape velocity for Jupiter is many times that of Earth's as Jupiter is massive than that of earth.
- In this, the object must have greater energy than gravitational binding energy to escape the earth's gravitational field, hence $1/2\,mv^2 = GMm/R$ or $v = \sqrt{2gR}$
- For an initial velocity of object P at start of its escape from object Q, is that P goes infinitely far from the surface of Q of mass M with radius R resulting zero velocity of P.

Orbital Velocity of Satellite :

- Orbital velocity is the velocity given to the body to keep it in orbit.
- It is given to the artificial satellite so that it revolves round any particular planet.
- It depends only on the mass of the things being orbited and distance from the center of mass of the two objects, $V_{ORBIT} = \sqrt{GM/R}$
- Orbital Velocity depends on Universal Gravitational Constant $G = 6.67 \times 10^{-11}$, mass (M) of planet around which satellite orbits (5.98×10^{24} in case of Earth) and distance (R) between center of planet to the satellite.
- A satellite in orbit moves faster when it is close to the planet or other body that around with it orbits, and moves slower when it is farther away.
- When a satellite falls from high altitude to lower altitude, it gains speed and when it rises from low altitude to higher altitude, it loses speed.
- Speed (v) of a satellite in circular orbit is $v = \sqrt{GM/r}$
- The period T of a satellite in circular orbit is the orbit's circumference divided by satellite's speed as
$$T = 2\pi r/v$$
- Satellite in elliptical orbit moves faster as compared to circular speed while near perigee, and will move slower than circular speed while near apogee.
- The period of a satellite in any orbit, circular or elliptical is given by Kepler's third law,
$$T = 2 \times \pi \times \sqrt{r^3/G \times M}$$

Time Period of a Satellite

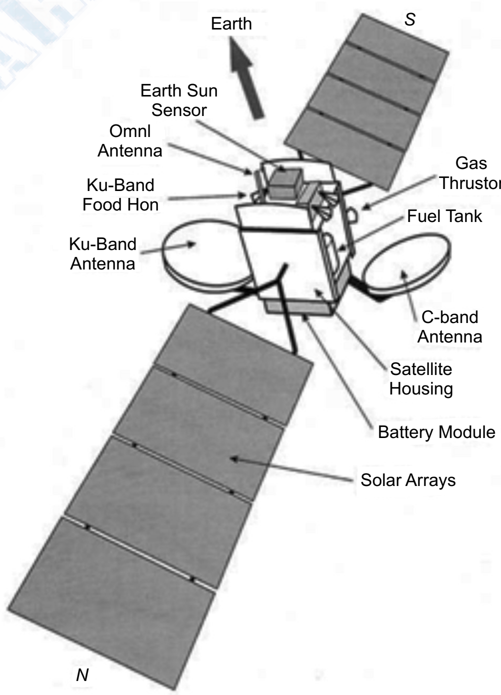
- Time taken by satellite to complete one revolution round the Earth is called time period.
- Time period is calculated by $T=2\pi r/v_0=2\pi(R+h)/v_0 = 2\pi(R+h)\sqrt{(R+h)/GM}$ which gives
$$T = 2\pi\sqrt{(R+h)^3/gR^2}$$
- If the satellite orbits very close to the Earth, then $h << R$, so $T = 2\pi\sqrt{R/g}$

Energy of an Orbiting Satellite

- A satellite revolving in a circular orbit round the Earth has potential energy and kinetic energy.
- If h is the height of the satellite above the Earth's surface and R is the radius of the Earth, then the radius of the orbit of satellite is $r = R+h$
- If m is mass of satellite, its potential energy is $E_P = -GMm/r$
- The satellite moves with an orbital velocity of $v_o = \sqrt{GM/(R+h)}$ has kinetic energy $E_K = 1/2\, mv_o^2$
- The total energy of the satellite is $E = E_P + E_K$ or $E = -GMm/2(R+h)$, where minus sign shows that satellite is bound to the Earth.

Geo-stationary Satellites :

- It is a particular type of satellite which is used in television and telephone communications.
- These types of satellites appear to remain in fixed positions at particular height above the equator.



- The satellites will help in transmitting certain programmes or events which are occurring in other countries as live presentations.
- The satellite is kept or positioned above 22,300 miles above the equator where force of gravity is cancelled by centrifugal force of rotating universe.
- In moving geostationary satellites, the probable speed should be about 7,000 m/h so as to maintain a stationary orbit above a fixed point on earth which is same speed as GPS satellite that used to orbit the earth twice a day.
- The satellite to appear fixed at a position above a certain place on the Earth, its orbital period around the Earth should be similar to rotational period of Earth about its axis.
- A satellite of mass '*m*' moving in circular orbit around the Earth at a distance '*r*' from the centre of the Earth, for synchronisation, its period of revolution around the Earth to be equal to the period of rotation of the Earth: 1 day = 24 hr = 86400 seconds.
- The speed of satellite in its orbit is $v = 2\pi r / T$
- The gravitational force on the satellite due to Earth is $F = GMm/r^2$
- For stable orbital motion, $4m\pi^2 r / T^2 = GMm/r^2$ or $r^3 = gR^2T^2/4\pi^2$
- Orbital radius of geo-stationary satellite is $r = [gR^2T^2 / 4\pi^2]^{1/3}$

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Chapter 7

Properties of Bulk Matter

Topic 1

Mechanical Properties of Solids

- Elastic Behaviours stress-strain, Hooke's law, Young's modulus, bulk modulus, shear modulus of rigidity, Poisson's ratio, elastic energy.

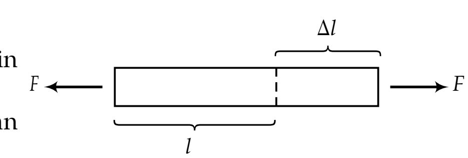
» Revision Notes

Elastic behaviour :

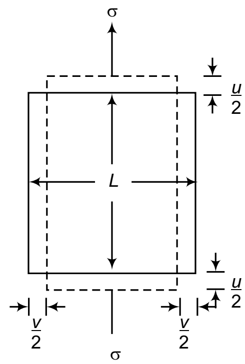
- Elasticity is the ability of an object/material to return to its normal shape after the removal of deforming force.
- Solids are made up of large number of atoms or molecules which are arranged in definite pattern known as lattice structure.
- Atoms or molecules are held in their positions by lattice or molecular forces which are attractive and tends to pull the particles together.
- There are also equal attractive forces which pull such molecules up and down and in and out positions.
- A repulsive force between the molecules will tend to repel the molecules if they get too close together.
- With the molecular forces, each molecule is in a position of equilibrium and on pulling one side of a solid material, effects pulling all such molecules from their equilibrium position.
- On removing the applied force, attractive molecular forces pull the molecules back to their original positions and the material returns to its original dimensions.
- On exerting a force on the material for compressing it, the molecules are again displaced from their equilibrium position.
- The repulsive molecular force saves them from getting too close together, where molecular displacement gets measured directly from reduction in size of original material.
- Once the compressive force is removed, repulsive molecular force causes atoms to return to their equilibrium position and making the solid to return to its original dimensions.
- Elastic properties of matter are signs of molecular forces which hold the solids together.

Stress-strain

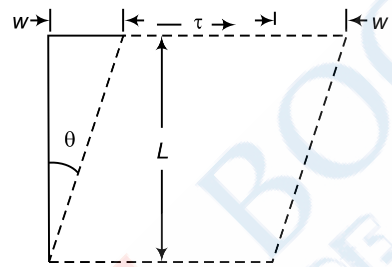
Strain :



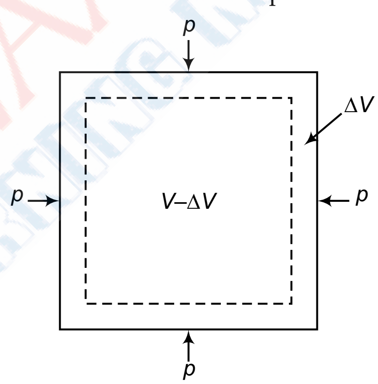
- Strain is the amount of deformation which an object faces in stretching in comparison to their original size and shape.
- Strain is the change in dimensions of a material as a result of an applied stress.
- The formula for strain is $\varepsilon = \Delta l/l$ where symbols have usual meanings
- Strain has no dimensions and units because of complete ratio.
- **Tensile Strain**
  - It is a function of stress which is applied.
  - A tensile stress (s) will produce a tensile strain (ε).
  - The nominal tensile strain is $\varepsilon_n = u / L$, while nominal lateral strain is $\varepsilon_n = -v / L$
  - The Poisson's ratio will be (σ) = $-$ lateral strain / tensile strain



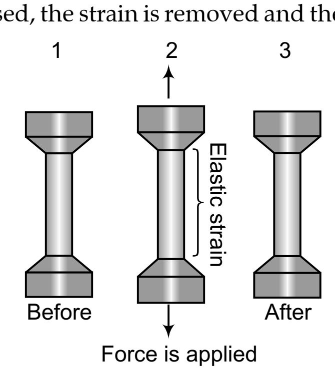
- **Shear Strain**
  - It is measured as a displacement of surface which is in direct contact with applied shear stress from its original position as $\tau = W/L = \tan\theta$



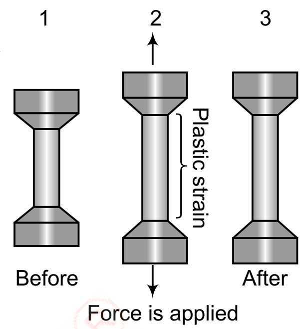
- **Volume Strain :**
  - It is the strain that is measured as a change in volume δV with respect to original volume V.
  - The strain is amount of elongation in three dimensions expressed as $\Delta V/V$.



- **Elastic Strain :**
  - When a material is stressed, there results a strain.
  - If stress is low, strain will be elastic strain which is caused by stretching of bonds in the material.
  - If the force on the material is released, the strain is removed and the material reverts to its initial dimensions.

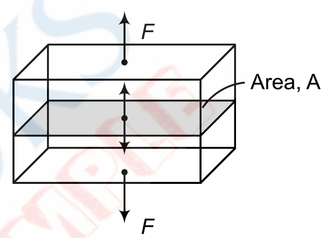


- **Plastic Strain**
  - If a higher stress is applied to the material, there results elastic strain along with small amount of inelastic or plastic strain.
  - The plastic strain is caused by rearrangement of atoms in the material which is not reversible.
  - When stress is removed, plastic deformation remains in the material.

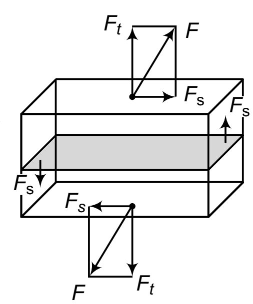


Stress

- Stress is the internal force per unit area having similar units as pressure.
- It is a complex quantity as compared to pressure as it varies with direction and surface of contacts.
- **Tensile Stress :**
  - A block which is kept on the floor and when a force tends to act on it, the floor will exert an equal and opposite force back on the block which makes the block to stop.
  - The force when acts on sections through the block parallel to top surface, allows the block to be in state of stress.
  - The intensity of stress s, is given by force which is divided by area of block surface as $s = F/A$
  - The stress is known as tensile stress as force is acting outwards at right angles to the block surface.

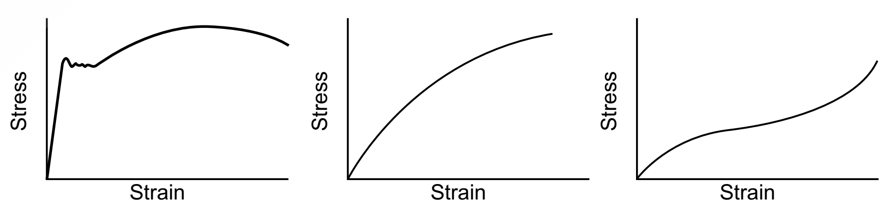


- **Shear Stress :**
  - When a force is applied on the block at right angle, it gets separated in two components; perpendicular to surface (F_t) and parallel to surface (F_s).
  - The component which is perpendicular to the surface creates a tensile stress and results in magnitude of F_t/A.
  - The component of stress which is acting parallel to surface is shear stress τ, and it's magnitude is given by $\tau = F_s/A$.



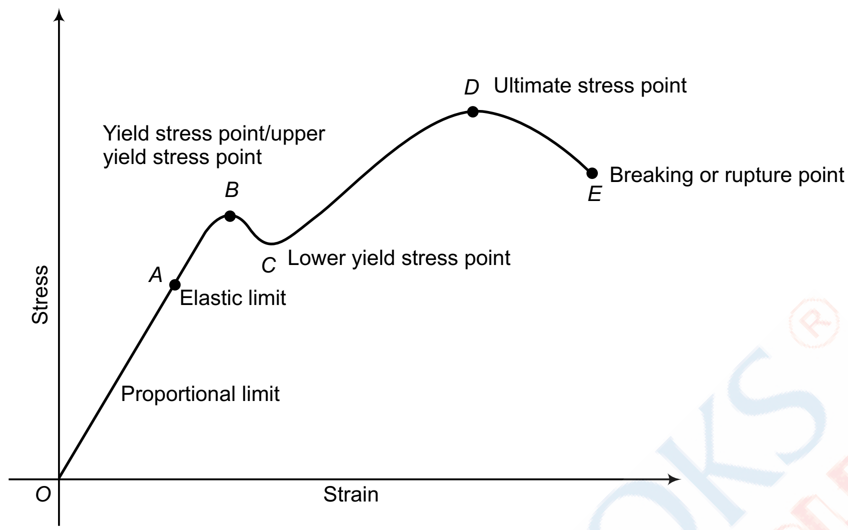
Stress-strain Curve

- In order to measure the mechanical properties of a material, we need to have the relationship between the stress and strain.
- As external forces is applied to object made of elastic material, they tend to produce changes in shape and size of object.
- The relationship among stress and strain for a particular material is described by stress-strain curve where the result is shown by the amount of deformation at certain intervals of tensile/compressive loading.
- Stress-strain curve is the graph where stress values of the material is plotted on y axis and corresponding strain values are plotted on x axis.
- Stress-strain curve also sometimes is known as stress-strain diagram.
- Stress-strain curve for different material is different which may vary as per temperature and loading conditions of the material.

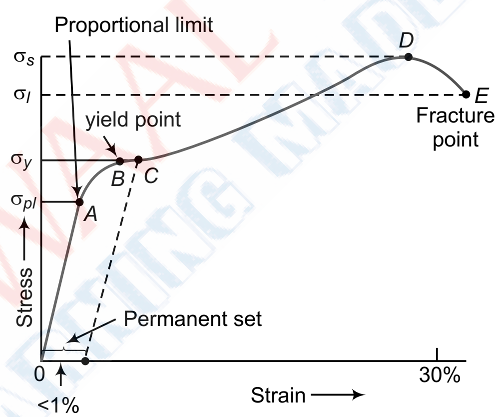


Mild Steel **Iron** **Rubber**

- The curve is used to read :
  - elastic deformation limit (yield point)
  - plastic deformation limit
  - maximum tensile strength
  - fracture or rupture point

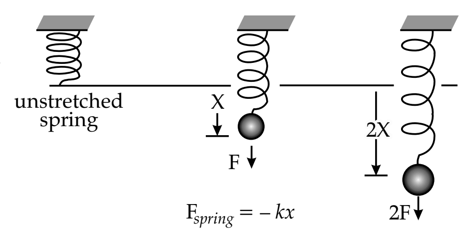


- Stress-strain curve has different regions and points. These regions and points are:
  - Proportional limit
  - Elastic limit
  - Yield point
  - Ultimate stress point
  - Fracture or breaking point.



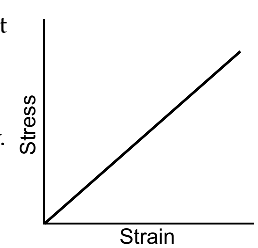
Hooke's Law

- According to this law, stress is directly proportional to the strain within the elastic limit, *i.e.*, stress ∝ strain.
- In this, force F needed to extend or compress a spring by distance x is proportional to that distance which is $F = -kx$
- Hooke's Law is applicable in case of elastic deformation of body, compatible with Newton law of static equilibrium.



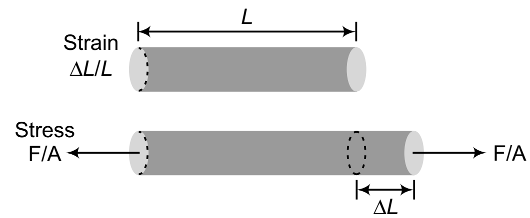
Young's Modulus

- It is a fundamental property of every material which can't be changed and is dependent on temperature and pressure.
- In this, a number shows how easy it is to deform a material.
- If a material is stretched, stress is directly proportional to strain upto limit of elasticity.
- The gradient of the graph gives the value of Young's Modulus for particular material.
- It is written as Young Modulus = tensile stress / tensile strain

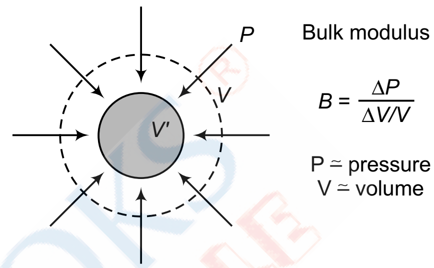


$$Y = [F/A] / [\Delta L/L] = FL / A\Delta L$$

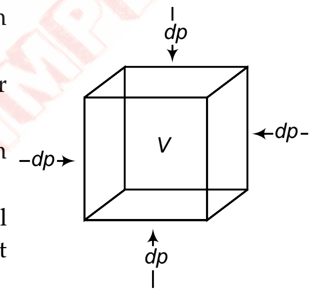
- Young's modulus can be used to calculate the elongation or compression of an object as long as the stress is less than the yield strength of the material.



Bulk Modulus



- It is a modulus associated with a volume strain, when a volume is compressed or expanded.
- The formula for bulk modulus is
  Bulk modulus = – (pressure applied / fractional change in volume)
- It is related to elastic modulus.
- The reciprocal of bulk modulus is compressibility of the substance.
- The bulk modulus of solid influences the speed of sound and other mechanical waves in the material.
- It is a factor which indicates amount of energy that is stored in solid material in Earth's crust.
- In this, elastic energy gets released violently in earthquake, so bulk modulus for Earth's crust materials is important in study of earthquakes.
- The bulk modulus is one of the factor in the speed of seismic waves from earthquakes.
- As Bulk Modulus of Elasticity or Volume Modulus is the property of the material which characterizes the compressibility of fluid by judging, how easily the unit volume of fluid gets changed due to change in pressure.



- Bulk Modulus can be calculated as

$$K = -dp / (dV / V_0) = -(p_1 - p_0) / \{(V_1 - V_0) / V_0\}$$

- Bulk Modulus for some fluids :

Fluid	Bulk Modulus	
	Imperial Units - BG (10^5 *psi*, lbf/in^2)	SI Units (10^9 *Pa*, *N/m*^{2})
Acetone	1.34	0.92
Benzene	1.5	1.05
Carbon Tetrachloride	1.91	1.32
Ethyl Alcohol	1.54	1.06
Gasoline	1.9	1.3
Glycerin	6.31	4.35
ISO 32 mineral oil	2.6	1.8
Kerosene	1.9	1.3
Mercury	41.4	28.5
Paraffin Oil	2.41	1.66
Petrol	1.55 - 2.16	1.07 - 1.49
Phosphate ester	4.4	3
SAE 30 Oil	2.2	1.5
Sea water	3.39	2.34
Sulfuric Acid	4.3	3.0
Water	3.12	2.15
Water - glycol	5	3.4
Water in oil emulsion	3.3	2.3

Shear Modulus of Rigidity

- It is also known as Modulus of rigidity that states the rate of change of unit shear stress with respect to unit shear strain for pure shear in the proportional limit.

- It is denoted by η and is expressed in pascals, [also in Gigapascals (GPa)].
- Modulus of rigidity value of a material can be obtained by torsion test.
- Shear Modulus $\eta = \dfrac{Y}{2(1+\sigma)}$ [For isotropic materials]
- The general formula of Shear Modulus is $\tau = \gamma \times G$, where τ = Shear stress in given member (N/m^2 or lbf/ft^2), γ = Shear strain, η = Shear Modulus or Modulus of Rigidity
- Shear modulus is coefficient of elasticity for shearing force which is the ratio of shear stress to the displacement per unit sample length.
- Shear modulus of rigidity is concerned with deformation of solid when it experiences a force parallel to its surfaces while its opposite face experiences an opposing force.
- The shear modulus of metals decreases with increasing temperature.

Poisson's Ratio

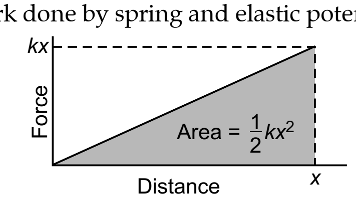
- It is the ratio of lateral strain to longitudinal strain in an elastic limit.
- In case of material with elastic limit, the ratio among lateral strain to longitudinal strain shows constant ratio known as poisson's ratio.
- Poisson's ratio = Lateral strain / Longitudinal strain
- It is denoted by symbol σ.
- The value of poisson's ratio varies from 0.25 to 0.33.

Elastic Energy

- It is also known as elastic potential energy which is the potential mechanical energy that is inside the configuration of a material or system as work is done to distort the material volume or shape.
- It is the potential energy which an object has in it due to deformed force.
- Any object that can be deformed and then return to its original shape will have elastic potential energy such as stretching of spring.
- It is equal to the work done to stretch a spring that depends on spring constant k and stretched distance.
- From Hooke's law, force required to stretch a spring is directly proportional to amount of stretch, as $F = -kx$, so work done is $\Delta PE = \dfrac{1}{2} kx^2$
- As the change in potential energy of an object between two positions is same for work done to move the object from one point to other, calculation of potential energy will be similar for calculating the work done.
- As force required to stretch a spring changes with distance, the calculation of work done is $W = \int kx\,dx = kx^2/2$
- The elastic potential energy is equal to work done by spring, the elastic potential energy is

  PE = Work = force × distance

  or PE = $(kx)\,x$ = PE = kx^2
- It is noted that, force and displacement are directly proportional to each other, so the straight line in graph passing through the origin shows that due to increase in force, displacement or distance also increases.
- The area under the curve is the work done by spring and elastic potential energy.



Topic 2

Mechanical Properties of Fluids

- Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure. Viscosity, Stokes's law, terminal velocity, streamline and turbulent flow, critical velocity. Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise.

» Revision Notes

Viscosity

- Viscosity is the quantity which shows fluid resistance to flow where the fluids resist the relative motion of immersed objects by motion of layers with different velocities.
- Viscosity of a fluid is a measure of its resistance to gradual deformation by shear, stress or tensile stress.
- Quantities that described viscosity are dynamic viscosity, kinematics/absolute viscosity
- Viscosity (η) or Dynamic viscosity is ratio of shearing stress (F/A) to velocity gradient ($\Delta v_x/\Delta z$ or dv_x/dz) in a fluid as $\eta = |(F/A)/(\Delta v_x/\Delta z)| = |(F/A)/(dv_x/dz)|$
- The Newton's second law of motion ($F = ma$), shows $F/A = \eta \Delta v_x / \Delta z$ or $F = m\Delta v / \Delta t$
- SI unit of viscosity is pascal second [Pa s].
- As pascal second is not commonly used unit today, so the common unit of viscosity is dyne second/ square centimeter [dyne s/cm^2] which is given the name poise P.

Kinematic Viscosity

Kinematic viscosity (η_k) is the ratio of the dynamic viscosity of a fluid to its density.

It is the measure of resistive flow of fluid under influence of gravity which can be measured using capillary viscometer.

The SI unit of kinematic viscosity is square meter per second [m^2/s] which is not commonly used.

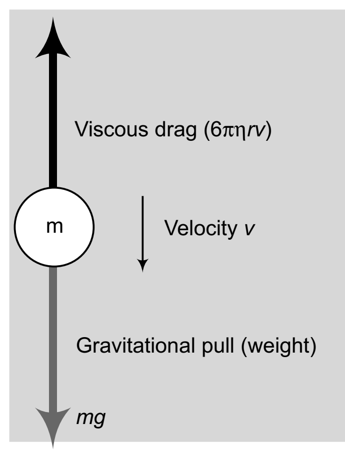
Kinematic viscosity has more common unit as square centimeter per second (cm^2/s).

It is noted that 1 square meter per second = 10 thousand stokes.

Stokes' Law

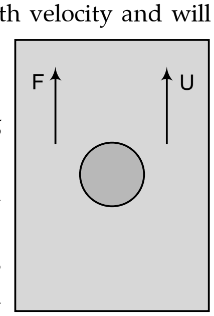
- In a viscous liquid and a sphere having a ball bearing of radius 1 millimeter, the liquid flows smoothly around the sphere as it falls with particular flow pattern.
- The velocity in such fluid flow pattern varies around the sphere, where the total viscous force on sphere with velocity gradient around the area of spherical surface. As per Stokes, drag force 'F' depends on size of the sphere having radius 'r' moving through fluid of viscosity 'η' at speed 'v' as $F = 6\pi\eta rv$
- The coefficient of viscosity 'η' has dimensional formula [$ML^{-1}T^{-1}$].
- When the density of material of sphere 'r' and liquid is 'σ', then effective gravitational force is $4/3\pi r^3 (r - \sigma)$, so viscosity $(\eta) = 2gr^2 (r - \sigma)/9v$

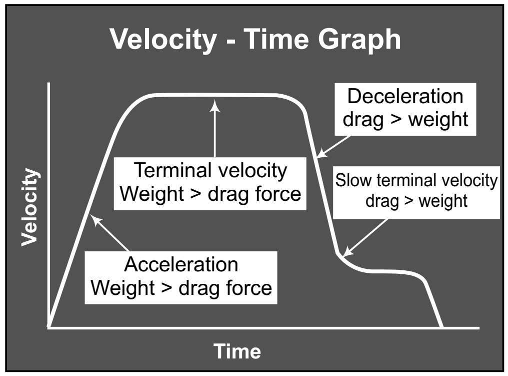
  The frictional drag is smaller for large spheres as compared to small spheres and terminal velocity of large sphere is more than that for small sphere of the same material
- Stokes' law shows that the frictional drag force (F) is directly proportional to weight of sphere where terminal velocity (v) is proportional to radius squared making velocity 'v' to be more for large sphere as compared to small sphere.



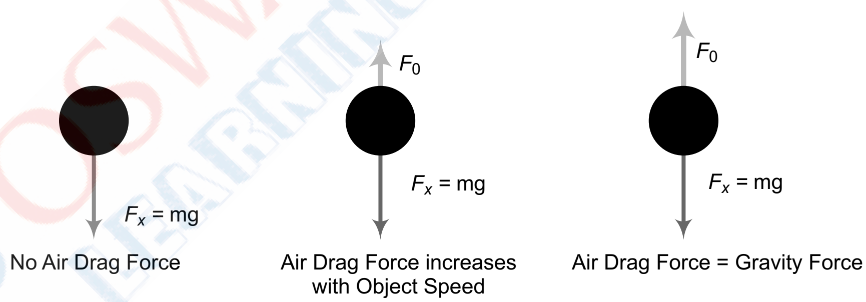
Terminal Velocity

- If an object falls under gravity, there will be three forces which act on it such as weight, drag force and up thrust.
- The weight acts downwards while drag force and upthrust act upwards, which result in net force which acts on the object towards the ground.
- An object falling under the influence of gravity or constant driving force, increases with velocity and will finally reach at maximum velocity where drag force equals driving force.
- The weight and up thrust of an object remains same throughout the motion.
- The drag force changes with speed of an object, where greater is the speed, more the drag force.
- As drag force increases with speed, at a certain point, total upward force and downward force results equal while the net force becomes zero.
- As there is no net force or resultant force, the acceleration of an object, at that time is zero and the object continues to move downwards with fixed velocity known as terminal velocity.

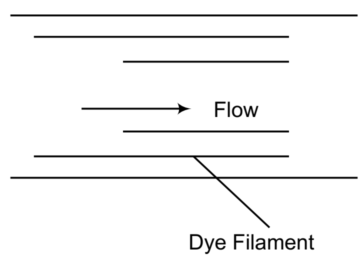




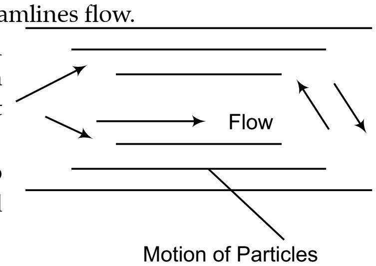
- Terminal velocity is maximum speed which an object achieves while falling through a fluid.
- If an object moves through fluid at low speeds with low turbulence, the terminal velocity is determined by viscous drag.
- The graph shows how the velocity of an object increases with time as it falls through viscous fluid.
- In the graph, the force of gravity is stronger than the resistance making the object to accelerate and when the gravity remains constant, viscous resistive force increases with increasing velocity and balances the object and the object stops accelerating.
- The relationship between velocity and viscous resistive force is known as drag force which depends on nature of fluid.
- In liquids, drag force is proportional to velocity, while in gases, drag force is proportional to square of velocity as $F_D = \frac{1}{2} C_D \rho A v^2$
- If a falling object reaches to terminal velocity (v), then drag force will be equal to the object's weight mg, where 'm' is mass of object and 'g' is acceleration due to gravity and $mg = \frac{1}{2} C_D \rho A v^2$ or $v = \sqrt{2mg / C_D \rho A}$
- If an air drag force that depends on size and shape of an object with object having larger surface area has low terminal velocity as compared to objects with small surface area.
- The weight of an object will affect the air drag force on an object and its terminal velocity.



Streamline and Turbulent Flow



- A liquid flowing in a pipe having disturbed pattern of fluid flow as a result of increase in velocity flow.
- When the flow is slow, the pattern is smooth, but when the flow is rapid, then there are disturbances in the fluid flow pattern in terms of velocity, directions and angle of flow.
- The injection of dye in the fluid, shows that at low velocities, dye filament remains unchanged and shows parallel lines in stream of flow.
- The type of flow pattern is known as Laminar flow or viscous flow or streamlines flow.



- If velocity of flow increases gradually, dye filament gets broken and spread across the cross section of pipe and the type of flow arrangement is known as turbulent flow where fluid particles will not move in parallel lines but will tend to move across the normal fluid flow direction.
- If a fluid particle in a stream gets disturbed, then due to inertia, it tends to move in new direction where viscous forces from surrounding fluid will tend to move in normal direction of flow.

- If shear force is large enough to overcome any deviation, there appears viscous or laminar flow. When shear forces are weaker and will not handle the inertia of particles, there result a turbulent flow.
- The ratio of inertia to viscous forces shows flow is laminar or turbulent is decided on basis of flow condition.
- The type of flow is described by Reynolds number R_e, which is an internationally recognized criteria for defining flow condition of fluid and is R_e = Inertia Forces / Viscous Forces = $\rho vD/\eta$
- Reynolds shows that values of R_e will have transition from laminar to turbulent flow where through experiments, if Reynolds number is less than 2000, flow is laminar while if Reynold's number is above 2000, flow is turbulent.

Critical Velocity

- Critical velocity is the maximum velocity of a fluid, above which, the streamline flow get change to turbulent flow.
- At maximum fluid velocity, the flow of fluid will remain as streamline flow while above the maximum fluid velocity; the flow of fluid becomes turbulent.
- Critical velocity (v_C) = ($R_e \times \eta$) / (ρr)
- Dimensional Formula of Reynolds number (R_e) = $[M^0L^0T^0]$
  Dimensional Formula of coefficient of viscosity (η) = $[M^1L^{-1}T^{-1}]$
  Dimensional Formula of Density of fluid (ρ) = $[M^1L^{-3}T^0]$
  Dimensional Formula of radius (r) = $[M^0L^1T^0]$
- Dimensional Formula of Critical velocity is $v_C = [M^0L^1T^{-1}]$
  SI unit of Critical velocity v_C is m/s.
- Critical velocity is further divided as lower critical velocity and upper critical velocity
- **Lower Critical Velocity**
  - It is the velocity at which the laminar flow stops.
  - In this, the flow enters from laminar to transition period which exists between laminar and turbulent flow.
  - If a laminar flow changes into turbulent, it does not change abruptly, but there results a transition period between such flows.
- **Upper Critical Velocity**
  - It is the velocity at which the turbulent flow starts.
  - In this, the flow enters from transition period to turbulent flow.

Bernoulli's Theorem and its Applications

- Bernoulli's theorem states that for an ideal liquid, that flows in continuous stream, the total energy of a particle remains the same, while the particle moves from one point to another.
- The theorem is law of conservation of energy where sum of all energy in a steady, streamlined, incompressible flow of fluid always remains constant,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

or,

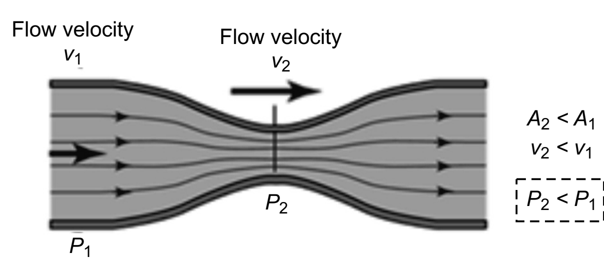
$$\frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g} + h = \text{constant}$$

Here, $\frac{P}{\rho g} \rightarrow$ Pressure head, $\frac{1}{2}\frac{v^2}{g} \rightarrow$ Velocity head and $h \rightarrow$ Gravitational head

- In a steady flow of constant density fluid in converging duct, without losses due to friction, the flow satisfies all restrictions governing the use of Bernoulli's equation.
- For an upstream and downstream of contraction, it is assumed that the velocity is constant over inlet and outlet areas and is parallel.
- When streamlines are parallel, pressure remains constant across the flow except for hydrostatic head differences.
- On ignoring the gravity, the pressure over inlet and outlet areas are constant.
- Along streamline on centerline, Bernoulli equation and one-dimensional continuity equation result as

$$P_1 - P_2 = \tfrac{1}{2} \rho (v_2^2 - v_1^2)$$

- Equation of continue, $A_1v_1 = A_2v_2$ = constant
- If the fluid passes over a solid body, the streamlines get closer and flow velocity increases while pressure decreases.

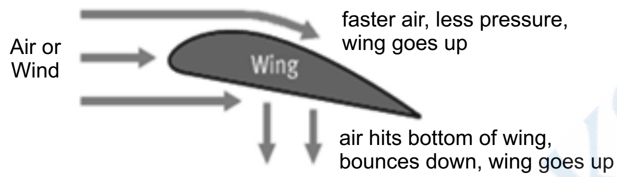


Bernoulli Equation as Conservation of Energy Principle

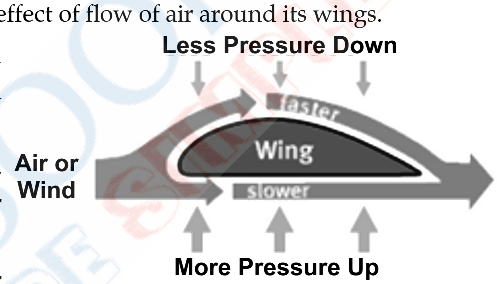
- Bernoulli Equation is a statement of conservation of energy principle for flowing fluids.
- It is the sum of kinetic and potential energies of masses at one place which is equal to sum at other place with work done on mass among the two points, as $KE_1 + PE_1 = W + KE_2 + PE_2$
- $1/2\, mv_1^2 + mgh_1 = \int F\, dx + 1/2\, mv_2^2 + mgh_2$

Applications of Bernoulli Principle

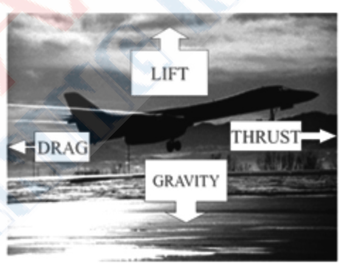
Aerofoil



- Flight of an aeroplane is based on Bernoulli's principle as the effect of flow of air around its wings.
- The shape of the aerofoil has rounded front edge and pointed trailing edge with top surface as curved and flat bottom surface.
- When wing in shape of aerofoil moves through the air or wind, the flow of wind/air over the top surface travels faster in order to cover long distance thereby creates low pressure region.
- The flow of air below the wing is slow which provides higher pressure region.

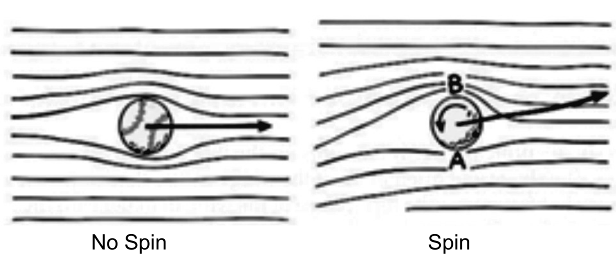


- The difference between top surface pressure and bottom surface pressure creates a net upward force, which finally lifts and helps the plane to take off.

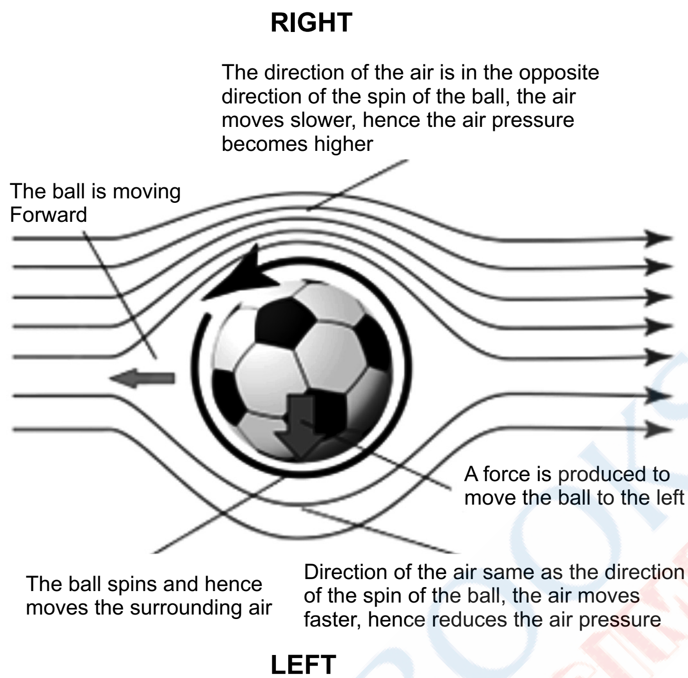


Spinning of Ball

- While playing baseball, football and volleyball a player is able to make the ball to move in a curved path by spinning it.
- The effect of spinning of ball is explained by Bernoulli's Principle where results a symmetric airflow in a ball which is not spinning.



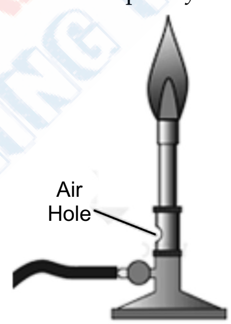
- Football, baseball or volleyball can be thrown or hit with a spin making the ball to go in a curve as one side of the ball experiences a reduced pressure.
- On sides of the ball, boundary layers move in similar direction as free stream air which carries around the ball before separating in turbulent flow.



- On side where boundary layer is opposed by free stream flow, there results a separation which shows net deflection of air stream in single direction behind the ball making Newton's 3rd law of reaction force on the ball in opposite direction giving an effective force in similar direction.

Bunsen Burner

- When a bunsen burner is connected to a gas supply, the gas flows at high velocity through a narrow passage in the burner which creates a region of low pressure.
- In this, outside air, which is at atmospheric pressure is drawn to mix with gas.
- The mixture of gas and air allows the gas to burn completely so as to produce a clean hot fire.



Bunsen Burner

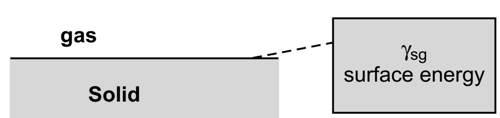
Carburetor

- Carburetor is a device whose function is to provide air-fuel mixture to automobile engine in required proportion.
- It mixes the correct amount of gasoline with air in order to allow the engine to run properly.
- It works on the Bernoulli Principle which shows that the velocity of an ideal gas increases thereby dropping the pressure.

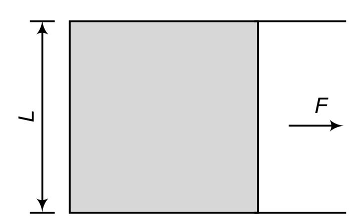
Surface Energy and Surface Tension

Surface Energy

- Surface energy is the energy difference which exists between bulk of material and surface of material.
- It is surface energy per unit surface area similar to surface tension measured.
- The unit of surface energy is J/m^2.



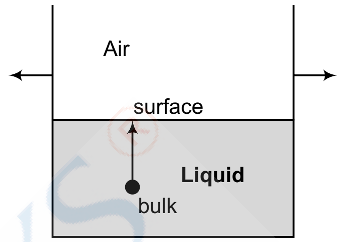
- If surface energy is provided by an external source, the liquid is said to be bubbling.
- It is an interfacial tension of solid-gas interface which is shown by γ_{sg}, where 's' is for solid and 'g' is for gas.
- If a solid has high surface energy, a strong interaction with the surface is possible as high surface energy is good for adhesion, while solids having low surface energy as plastics are difficult to coat and needs special treatment for more surface energy.



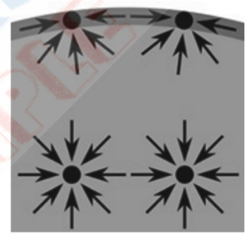
- In a thin film membrane where force F pulls on the membrane and stretches liquid film at per unit length L of surface, then in liquid film, membrane is pulled by a distance Δx with work done $F\Delta x$ and the increase in surface area $2L\Delta x$, then surface energy E is $E = F\Delta x / 2L\Delta x = F / 2L = \gamma$

Surface Tension

- Surface tension is a property of liquids which is managed by intermolecular interactions which exist from cohesive forces among the molecules in a liquid.
- It is the resistance of a fluid to deform or break which directly defines by intermolecular forces which are present on liquid surface.
- When substrate has high surface energy, it tends to attract with adhesive forces having low surface tension and resistance to deform or rupture, thereby producing good wetting of adhesive on substrate.
- In homogeneous liquid, all molecules in central parts of liquid have same amount of force which pull it to all sides and surrounding molecules pull the central molecule uniformly on all directions.

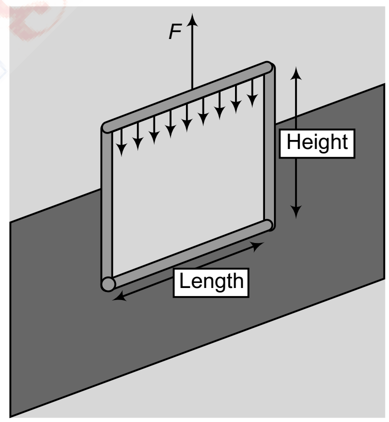


- The surface molecules have forces which act on it toward the liquid and air-liquid adhesive forces are not as strong as liquid-liquid cohesive forces, so surface molecules get attracted towards the center of the liquid forming a packed layer of molecules.
- The surface tension results as force which is parallel to surface that is perpendicular to unit length line drawn on the surface.
- The unit of surface tension is N/m or J/m^2.



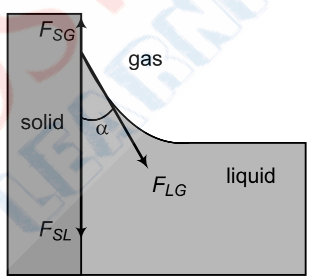
Measuring Surface Tension

- Surface tension is measured using wire frame method where a rectangular wire frame is suspended in a liquid and is pulled upwards with force F to balance the downward force of surface tension T.
- In this, the applied upward force F is acting up which balances the surface tension force $F_{down} = 2 \times T \times l$ from the two surfaces that clings to the top of frame, *i.e.*, $F_{down} = 2 \times T \times l$ or $T = F / 2 \times l$



Angle of Contact

- If a liquid surface coincides with solid surface, the angle between the two surfaces is known as angle of contact (α).
- If the angle α is less than 90^0, then liquid will spread which will wet the surface and when angle of contact is more than 90^0 , the liquid will form the droplets.



- The type of effect is seen with water on glass where for a clean glass-water boundary, angle of contact is zero which increases to pull of water if angle of contact is more than 90° for waxed surface.
- The forces, F_{SG} = upward force among solid and gas, F_{SL} = downward force among solid and liquid, F_{LG} = inclined force among liquid and gas are resolved vertically as force among solid and gas F_{SG} is much smaller than other two forces, then $F_{LG} \cos \alpha = F_{SL} - F_{SG}$ or $\cos \alpha = F_{SL} - F_{SG} / F_{LG} = F_{SL} / F_{LG}$
- If forces F_{SL} and F_{LG} are in similar direction, then cos α is positive, α is less than 90°, meniscus is positive and liquid will wet the surface.
- If forces F_{SL} and F_{LG} are in opposite directions, then cos α is negative, α is greater than 90° meniscus will be negative and liquid will not wet the surface.

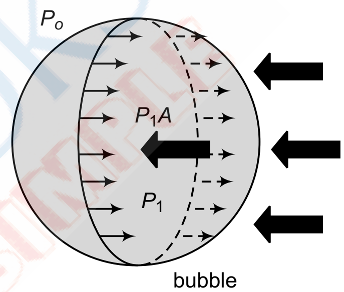
Contact angles for some interfaces

most organic liquids – glass	0° - 10°
mercury – copper	0°
pure water – glass	0°

water – glass	20°
kerosene – glass	26°
water – silver	90°
water – paraffin	106°
mercury – glass	148°

Excess of Pressure across curved surface

- In liquid, curved surface is formed due to difference in pressure which occurs between the atmosphere and liquid which is expressed in terms of area and surface tension.
- It is a fact that air needs to be blown into drop of soap solution so as to make a bubble which shows that pressure in the bubble is more than outside.
- The extra pressure will create a force which balances the inward pull of soap film of bubble as a result of surface tension.
- If a soap bubble of radius r with external pressure P_e and internal pressure P_i, then excess pressure P in bubble will be $P_i - P_e$
- The force acting from right to left as a result of internal excess pressure is PA, where A is area of section through the centre of bubble.
- If soap bubble is in equilibrium, forces will get balanced by force due to surface tension which acts from left to right which is $2 \times 2\pi r T = PA = P\pi r^2$ giving excess pressure in soap bubble $P = 4T/r$
- It is noted that air bubble in liquid has only one liquid-air surface where excess pressure in the bubble results as $P = 2T/r$



Application of surface tension; ideas to drops, bubbles and capillary rise

- Surface tension is an elastic tendency of fluid surface that uses least surface area.
- At liquid–air interfaces, surface tension occurs from more attraction of liquid molecules due to cohesion as compared to molecules in air due to adhesion.
- The results is that total effect is inward force at surface which causes liquid to behave like as its surface were covered with stretched elastic membrane.

Drops/Droplets

- It is seen that presence of molecules exists at the surface, which if small, have low potential energy where intermolecular attractive forces act to minimize surface area of a liquid.
- If the surface area is minimised, the wall tension will pull inwards from all sides that will lead to spherical shape.
- The shape which has smallest ratio of surface area to volume is sphere. So, very small quantities of liquids will form spherical drop. As drops get bigger with their weight get deform in tear shape.

- Surface tension is responsible for shape of liquid droplets.
- As easily deformed droplets of water tends to pull in spherical shape by cohesive forces of surface layer, so surface tension and adhesion determines the shape of drop on a twig.

Bubbles

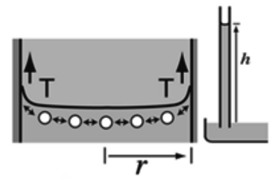
- The surface tension of water gives required wall tension for formation of bubbles with water.
- The tendency to minimize wall tension pulls the bubbles in spherical shapes.
- The interference colours shows thickness of soap film which ranges from few wavelengths of visible light.
- As soap film has less surface tension compared to pure water, it is nevertheless strong to maintain the bubble with such small thickness.
- The pressure difference among inside and outside of bubble depends on surface tension and radius of bubble, can be related by visualizing bubble as two hemispheres with no internal pressure for pushing the hemispheres by surface tension which acts around circumference of circle.

Capillary Action

- Capillary action takes place due to adhesion and surface tension.
- Adhesion of water to walls of a vessel results an upward force on liquid at edges which shows in meniscus that turns in the upward direction.
- The surface tension will act to hold the surface tightly, so instead of edge to move in upward direction, the whole liquid surface gets dragged upwards.

- Capillary action occurs when the adhesion to the walls is stronger as compared to cohesive forces that exist among the liquid molecules.
- In this, the height to which capillary action takes the water in a uniform circular tube is limited by surface tension.
- The force acting around the circumference will act in upward direction

$$F_{upward} = T \times 2\pi \times r$$



- If the height 'h' to which capillary action lifts the water, depend on the weight of water that surface tension lifts $T \times 2\pi \times r = \rho \times g\,(h \times \pi \times r^2)$
- The height to which the liquid can be lifted is given as $h = 2T / \rho \times g \times r$

Topic 3
Thermal Properties of Matter

- Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; C_p, C_v – calorimetry; change of state – latent heat capacity. Heat transfer : conduction, convection and radiation, thermal conductivity, Qualitative ideas of Blackbody radiation, Wein's displacement law, Stefan's law, Green house effect.

» Revision Notes

Heat

- Warming is possible by adding energy which can be either from light, electricity, friction, chemical reaction, nuclear reaction or any other kind of energy.
- All atom or molecule in the substance will move bit faster, if given some heat.
- When the energy which is added spreads throughout the substance, such energy is called as heat energy, thermal energy or heat.
- Heat is a form of energy having its units as Joule.
- Apart from unit Joule, other units of heat are Calories and BTU (British Thermal Unit).

Temperature

- It is the average kinetic energy of all molecules taken together which is basically the average energy of the particles in an object.
- It shows how hot or cold an object is, in degrees. (Degree of hotness or coldness)
- It is measured in Kelvin, Celsius and Fahrenheit scales.
- It is one of the principal quantities in study of thermodynamics.

Kinds of Temperature Scale

There are many kinds of temperature scale which can be classified based on empirically and theoretically. Basic relation is given as

$$\frac{C-0}{100} = \frac{F-32}{180} = \frac{K-273.15}{100} = \frac{R-0}{80}$$

Empirically based Scales

- Empirically based temperature scales depend directly on measurements of physical properties of materials which are valid only in convenient ranges of temperature.
- Empirically based thermometers, are direct measurements of physical properties of thermometric materials which can be re-calibrated using theoretical physical reasoning.

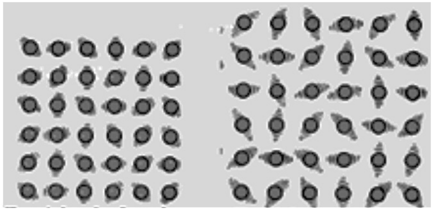
Theoretically based Scales

- Theoretically based temperature scales are based on theoretical arguments like those of thermodynamics, kinetic theory and quantum mechanics.
- These scales depend on theoretical properties of idealized devices and materials and are more or less comparable with practically feasible physical devices and materials.
- These scales are used to provide calibrating standards for practical empirically based thermometers.

Thermal Expansion

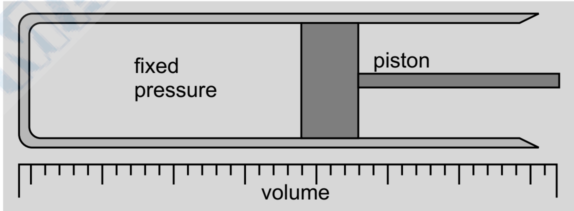
- It is the tendency of matter by virtue of which, it changes in shape, area and volume in response to change in temperature.

- It occurs when an object expands and becomes larger due to a change in object's temperature.
- Higher temperature results in faster movement of molecules on heating thereby increasing the size of objects.



Thermal Expansion of Solids, Liquids and Gases

- The three states of matter (solid, liquid and gas) expand when heated.
- The atoms themselves do not expand, but the volume they take up does.
- If a solid is heated, its molecules gain kinetic energy and atoms vibrate faster about their fixed points.
- As vibration become larger, molecules are pushed further apart and the solid expands slightly in all directions.
- The relative increase in the size of solids when heated is small.
- Change in colour with respect to the heat;
  (i) hottest part glows bright yellow (ii) Hot part glows red
  (iii) warm part shows slight colour change, (iv) cold part shows no change in colour.
- If a liquid is heated, its molecules gain kinetic energy and vibrate more vigorously.
- When vibration increases, the molecules are pushed further apart and liquid expands slightly in all directions.
- As the bonds between molecules in liquid are weak, so they expand more as compared to solids.
- If temperature increases it results in expansion of the liquid which means it rises up in the glass.
- The liquid level drops due to the expansion of its container which initially absorbed all the heat.
- After a while, the heat reaches the liquid it compensates for the expansion of the container and rises much more than the original level.
- The expansion of gases is much more larger than that of solids or liquids under the same rise in temperature.
- Molecules in the gases are more apart and are weakly bonded with each other.
- If the gases are heated, heat causes the molecules to move faster, thereby increasing the volume of gas more as compared to volume of solid or liquid.
- If temperature of a gas is increased, the molecules move faster and the collisions become more violent thus they spread away from each other causing the volume to increase.
- In gas, molecules keep on moving and travels fast thereby hitting each other and at the sides of the container.



- On increasing the temperature, the molecules will move faster which results in more violent collisions making the pressure to increase.

Thermal Expansions in Solids

(i) Linear thermal expansion

Coefficient of linear expansion $\alpha = \dfrac{L_f - L_i}{L_i \times (t_f - t_i)}$

(ii) Superficial thermal expansion

Coefficient of superficial expansion $\beta = \dfrac{A_f - A_i}{A_i \times (t_f - t_i)}$

(iii) Cubical thermal expansion

Coefficient of cubical expansion $\gamma = \dfrac{V_f - V_i}{V_i \times (t_f - t_i)}$

Here all alphabets are in their usual meanings.

Relation between α, β and γ

- $\alpha = \dfrac{\beta}{2} = \dfrac{\gamma}{3}$ or $\alpha : \beta : \gamma = 1 : 2 : 3$

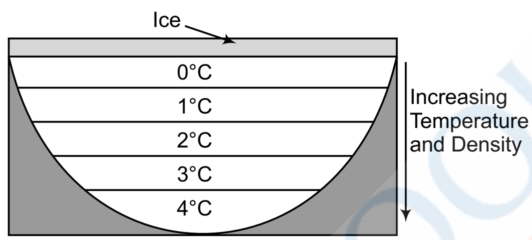
Remember:

- **Charles's law :** If pressure is constant, the volume of a fixed mass of a gas is directly proportional to the Kelvin temperature. Mathematically $V_1/T_1 = V_2/T_2$
- **Boyle's law :** If temperature is constant, the volume of a fixed mass of a gas is inversely proportional to the pressure. Mathematically $P_1V_1 = P_2V_2$

- **Gay-Lussac's law:** If volume is constant, the pressure of a fixed mass of a gas is directly proportional to the Kelvin temperature. Mathematically $T_1/P_1 = T_2/P_2$
- **Ideal gas law :** $PV = nRT$, where R = Universal gas constant.

Anomalous Expansion of Water

- Anomalous properties of water are those where behaviour of liquid water is different from what is found with other liquids.
- Frozen water (ice) shows anomalies when it is compared with other solids.
- Water being apparently a simple molecule (H_2O), has a complex and anomalous character due to inter-molecular hydrogen bonding.
- Anomalous expansion of water is an abnormal property of water where water will expand rather than contract when the temperature goes from 4°C to 0°C resulting in making it less dense.



- The density becomes less and less as it freezes since molecules of water normally form open crystal structures when in solid form.
- If the temperature falls below 0°C, water freezes as the surface freezes first.
- Similar to many materials where solid is denser than liquid form, ice is less dense than water, ice will not sink in water.
- As water gets heated from 0°C, it becomes denser up to a temperature of 4°C, before becoming less dense with increasing temperature.
- The anomalous behaviour of water is an important feature which played an important role in sustaining marine life.
- It is the unusual property of freezing of body of water, due to changes in weather, begins at the surface.
- As ice is less dense than water, it floats forming an insulating layer which considerably slows the thermal conduction from warmer water below to colder air above.

Specific Heat Capacity

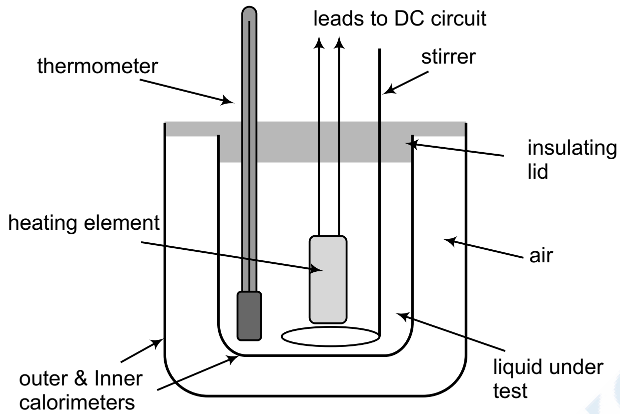
- Specific heat capacity is the amount of heat energy required to raise the temperature of unit degree of a substance of per unit of mass or specific heat capacity (C) is amount of heat in joules required to raise temperature of 1 kg of substance by 1 K.
- Specific heat capacity of a material is a physical property.
- It's SI unit is J/kg·K.
- It is reported in units of calories per gram degree Celsius.
- Molar heat capacity in J/mol·K and volumetric heat capacity in J/m^3·K.
- As water needs more time to boil as compared to alcohol, so water requires more heat as compared to alcohol in order to raise the same temperature.
- Water has high specific heat capacity.
- Equation for specific heat capacity : (C) = Heat Energy / (mass of substance × change in temperature)
- $(C) = \Delta Q / m \times \Delta\theta$

Specific Heat Capacity of a Liquid

- The heat energy supplied by element is given to liquid that generates temperature rise of $\Delta\theta$ where heater current (I) and voltage (V) gets monitored for time (t).

Specific Heat Capacity of Substances

Substance	**Cal g^{-1} K^{-1}**
H_2O (l)	4.184
Lithium	3.56
Ethyl alcohol	2.200
Ice @ 0°C	2.010
Steam @ 100°C	2.010
Vegetable oil	2.000
Sodium	1.23
Air	1.020
Magnesium	1.020
Aluminium	0.900
Concrete	0.880
Glass	0.840
Potassium	0.75
Sulphur	0.73
Calcium	0.650
Iron	0.444
Nickel	0.440
Zinc	0.39
Copper	0.385
Brass	0.380
Sand	0.290
Silver	0.240
Tin	0.21
Lead	0.160
Mercury	0.14
Gold	0.129



- The energy supplied by heater = VIt
- Energy absorbed by liquid and container = $m_L c_L \Delta\theta + m_C c_C \Delta\theta$
- Principle of calorimetry states

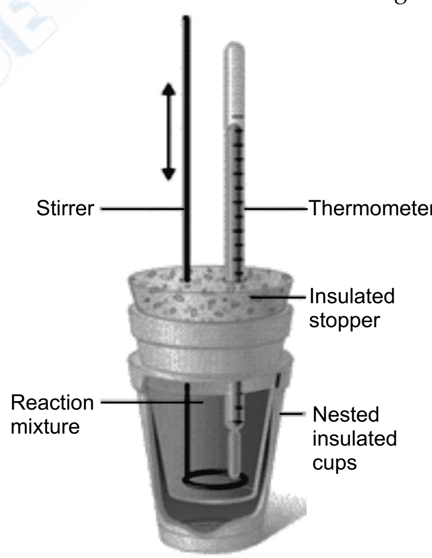
Heat give by hot object = Heat taken by cold object

$$VIt = m_L c_L \Delta\theta + m_C c_C \Delta\theta$$

Calorimetry

- Calorimetry is used to measure heat reaction or heat capacity.
- Calorimetry is of two types: measurements based on constant pressure and measurement based on constant volume.
- The heat capacity of calorimeter is obtained by transferring a required amount of heat in it and measuring its temperature increase.

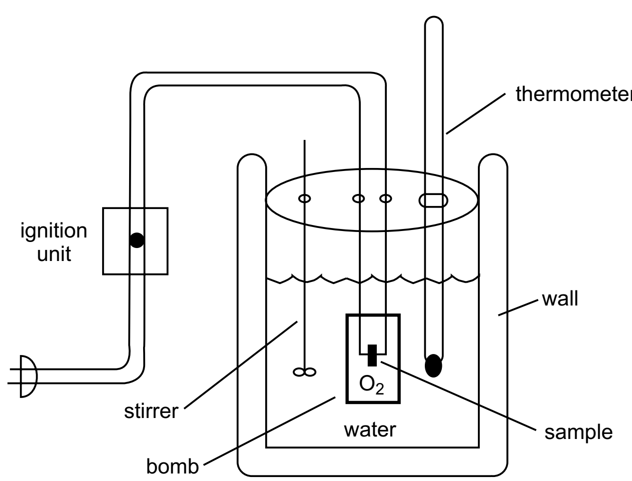
Constant Pressure Calorimetry



- It measure the changes in enthalpy of a reaction which takes place in liquid solution, during which pressure remains constant.
- The heat transferred to/from the solution for reaction to take place will be equal to the change in enthalpy ($\Delta H = q_p$).
- The example of constant pressure calorimeter is coffee-cup calorimeter which uses two nested Styrofoam cups and lid having holes for inserting a thermometer and stirring rod.
  In this, the inner cup holds the amount of liquid normally water to absorb heat from the reaction, while the outer cup being adiabatic do not absorb any heat.
  In coffee-cup, the data collected at constant-pressure calorimetry experiment calculate heating capacity of unknown substance.

Constant Volume Calorimetry

- Constant volume calorimetry (bomb calorimetry) is accurate than constant pressure coffee-cup calorimeter.
- The calorimeter is difficult to use as it needs a good reaction container for withstanding large of pressure changes that occurs in many chemical reactions.
- The constant pressure calorimeter are not suitable for studying reactions where one or more of reactants involving gas like combustion reaction.
- The enthalpy changes which is in combustion reactions can only be measured with the help of constant volume calorimeter so, bomb calorimeter is particularly used.
- In bomb calorimeter, reactants are kept in a steel cup in steel vessel having constant volume.
- The bomb is sealed and is filled with extra oxygen gas and is kept inside an insulated container which can keep a required volume of water.
- In bomb calorimeter, as combustion reactions are exothermic, so temperature of bath and calorimeter increases at the time of combustion.



- In bomb calorimeter, as volume of system is fixed, combustion reaction takes place under conditions where volume is constant, not the pressure.
- Bomb calorimetry is used to find the enthalpy of combustion, ΔH_{comb}, for hydrocarbons

$$C_xH_YO_{z\,(s)} + \frac{1}{2}\left(2X + \frac{Y}{2} - Z\right)O_{2\,(g)} \rightarrow X\,CO_{2\,(g)} + Y/2\,H_2O_{\,(l)}$$

- As combustion reactions are exothermic, ΔH_{comb} will be negative.
- The bomb calorimeter has sample, oxygen, stainless steel bomb and water where dewar prevents heat flow from calorimeter to outside, so $q_{calorimeter} = 0$
- As bomb is made from stainless steel, combustion reaction occurs at constant volume and there is no work, $w_{calorimeter} = -\int p\,dV = 0$
- The change in internal energy ΔU, for calorimeter is zero, so $\Delta U_{calorimeter} = q_{calorimeter} + w_{calorimeter} = 0$
- As calorimeter is isolated, so reactants will be the system while rest of calorimeter will be the surroundings.
- The change in internal energy of reactants on combustion is

$$dU_{tot} = dU_{sys} + dU_{surrounding} = 0 = -[(\delta U/\delta T)_v\,dT + (\delta U/\delta V)_T\,dV]$$

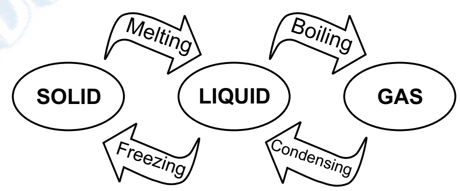
- As process with constant volume $dV = 0$, so as per heat capacity C_v :

$$dU_{sys} = -C_v\,dT$$

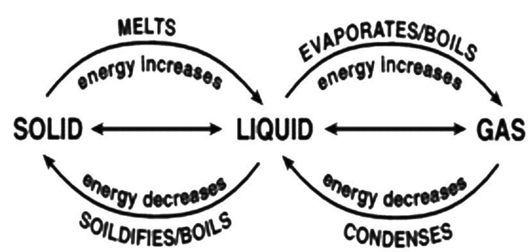
- If C_v is independent of T over small temperature ranges, then $\Delta U = -C_v\,\Delta T$, where C_v is heat capacity of surrounding.
- From the definition of enthalpy, $\Delta H = \Delta U + P\Delta V$
- As there is small expansion of work done by condensed phases, so $P\Delta V = 0$ for solids and liquids.
- If the gas is ideal, then $\Delta H = \Delta U + RT\Delta n_{gas}$

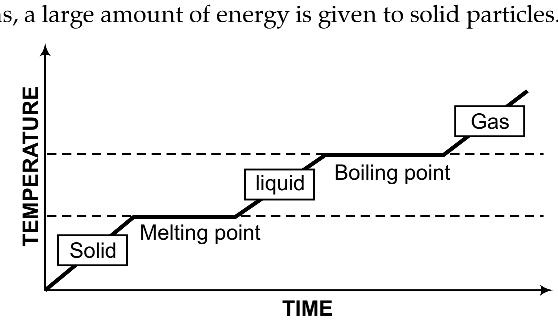
Change of State

- There are three states of matter: solids, liquids and gases.
- From the figure :
  - **Melting :** Change of state from solid to liquid.
  - **Vaporization :** Change of state from liquid to gas.
  - **Condensation :** Change of state from gas to liquid.
  - **Freezing :** Change of state from liquid to solid.
  - **Sublimation :** Change of state directly from solid to gas.
  - **Deposition :** Change of state directly from gas to solid.



- Solids, liquids and gases are made of particles and they differ on the basis of amount of energy present in these particles.
- Change of state will occur, when the particle energy is given to the molecules or is taken away by the molecules.
- On heating a solid, some of the energy is given to it which causes the particles to move apart resulting in change of state from solid to liquid or melting.
- If the energy is taken from a gas, it causes the gas to change its state to liquid or resulting in condense.
- The cooling of liquid results in freezing.
- For changing solid into gas, a large amount of energy is given to solid particles.





- The change from solid to liquid needs heat energy without any rise in temperature as all of the energy is used in changing the particles from solid to liquid.
- If the particles changes it state from liquid to gas, the heat is given to liquid for warming up.
- If more heat is supplied, it will not cause the temperature to rise as all the heat is applied to change the state from liquid to gas.

Latent Heat (L)

- Latent heat is the amount of heat which is taken in / or given out during change of state of material, *e.g.* solid to liquid or liquid to gas.
- It is written as $Q = mL$
  If a material changes it state from solid to liquid or liquid to solid, it is known as latent heat of fusion, while if a material changes it state from liquid to gas or gas to liquid, it is known as latent heat of vaporization.
- The phase changes involving latent heat energy are :

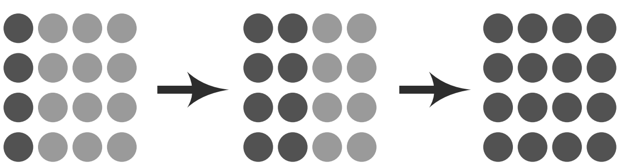
Phase change	Action	Symbol
solid to liquid	melting	L_F
liquid to solid	fusion	L_F
liquid to vapour	vaporization	L_V
vapour to liquid	condensation	L_V
solid to vapour	sublimation	L_S
vapour to solid	desublimation	L_S

Heat Transfer

- The transfer of heat can take place in three ways:
  - **Conduction :** In this process, heat is transferred between molecules that are in direct contact with each other, without the movement of particles.
  - **Convection :** In this process, transfer of heat that takes place due to movement of particles from one place to another within the medium.
  - **Radiation :** In this process, heat transfer takes place in the form of electromagnetic wave from on a place to another without supporting the medium.
- Heat transfer is an exchange of thermal energy which exists among physical objects.
- Heat normally flows from hotter to colder objects.
- Thermal equilibrium exists when all objects and environment caters similar temperature.
- It changes the internal energy of both the systems as per First Law of Thermodynamics.
- With Zeroth Law of Thermodynamics, thermodynamic equilibrium is established for two objects having similar temperature.
- If objects which are at different temperatures brought in contact, they will achieve thermal equilibrium.
- During thermal equilibrium, heat gets transferred among the objects, where amount of heat transferred ΔQ is proportional to temperature difference ΔT among the objects and mass of the object. Heat capacity c of an object which is written as $\Delta Q = cm\,\Delta T$

Conduction, Convection and Radiation

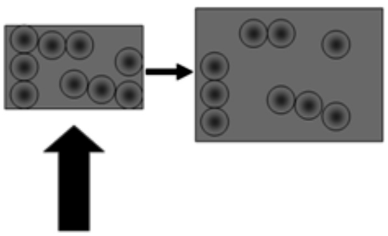
Conduction



- Conduction is the method of heat transfer through molecular disturbance in a material without any motion of particular material. In this process, molecules in hotter part of object start vibrating fast as compared to molecules, which are at cooler parts.
- The fast moving molecules are able to transfer the part of energy to slow moving molecules by transferring heat through object. In this, steady state is achieved when heat enters the object at one side which gets balanced by that, which is emitted from other side. During the process, the object's heat remains constant.
- Consider a metal rod where one end of it has high temperature. Here the energy gets transferred down the rod toward the colder end as high speed particles collide with the slower ones, results in the net transfer of energy to slower ones.
- In case of heat transfer among two plane surfaces, the rate of conduction of heat transfer is

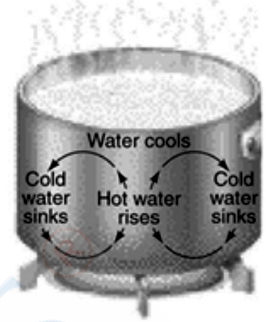
$$Q/t = kA\,(T_{hot} - T_{cold})\,/\,d$$

Convection

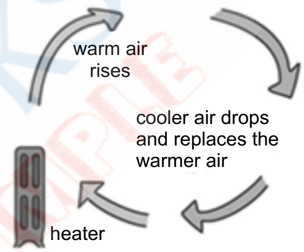


- Convection is the method of transferring heat using mass motion of fluid like air or water when heated fluid is made to move away from source of heat with energy in it.

- Convection above the hot surface takes place as hot air expands and becomes less dense and further rises. So hot water is less dense as compared to cold water and rises which results in convection currents that transport energy.
- Convection lead to circulation in a liquid, as in case of heating of bowl of water over a flame. The heated water expands and becomes more buoyant and cooler water will become dense near the surface, forming the circulation patterns.

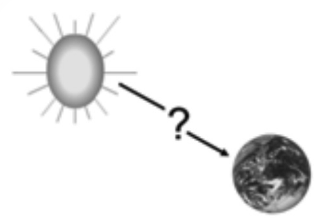


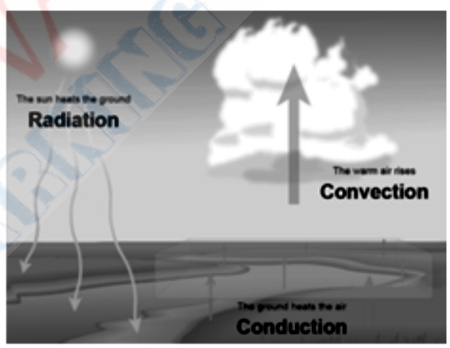
- Convection plays an important role in transporting energy from the center of the Sun to the surface, and in movements of hot magma beneath the surface of the earth.
- Convection is the movement of heat which can be done through the currents, since heated molecules tend to move from hot places to cold places.
  - Higher temperature fluid molecules expand the space in which they are.
  - The heated space is less dense as compared to the cooler space.
  - While heating, less dense fluid rises to replace the denser colder, fluid, that further sink in warmer areas.
  - The continuous of warming and cooling liquid or gas lead to convection current.
  - By warming one side of a container of fluid, the convection currents are able to transfer heat to other sides.



Radiation

- Radiation is the method in which the heat energy gets transferred through space by electromagnetic waves. Most of the electromagnetic radiations which appear on earth result from sun and are not visible, but only a small portion appears in shape of visible light.
- Thermal radiation is emitted as a result of random movements of atoms and molecules in matter. As atoms and molecules are made of charged particles like protons and electrons, their movements result in emission of electromagnetic radiation that carries energy away from the surface of body.





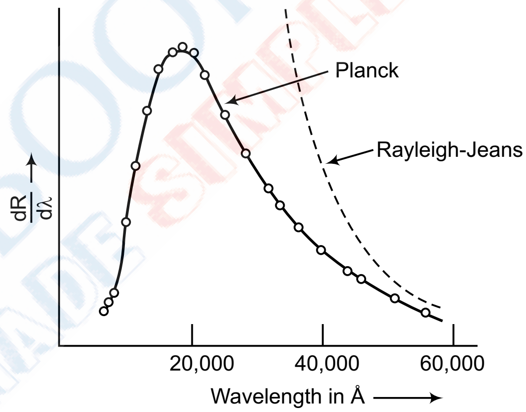
Thermal Conductivity

- Thermal conductivity is the property of the material which shows the ability to conduct heat. It is the quantity of heat which is transmitted by unit thickness of a material which is in direction normal to the surface of unit area due to unit temperature gradient under steady state conditions.
- Heat transfer through conduction involves transfer of energy in the material without movement of material. Conduction takes place when a temperature gradient appears in solid medium. The conductive heat flow takes place in direction of lowering temperature as high temperature equates higher molecular energy resulting in more molecular movement.
- The unit of thermal conductivity is W/(m K) in SI system and Btu/(hr ft °F) in Imperial system. As thermal conductivity is quantity of heat (Q) transmitted through unit time (t) in a normal direction to surface with unit area (A) due to unit temperature gradient $-\left(\frac{\Delta T}{\Delta x}\right)$, its equation can be shown as :

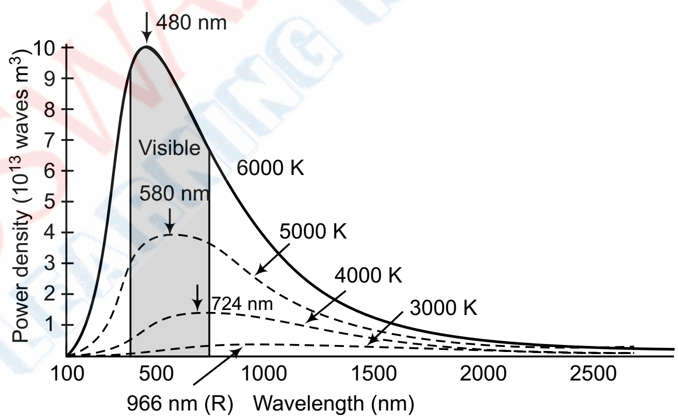
$$\text{Heat transfer } (Q) = KA\left(\frac{\Delta T}{\Delta x}\right)\times t$$

Here, K is the coefficient of thermal conductivity.

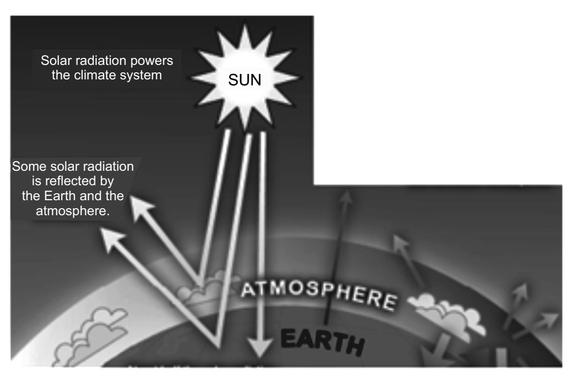
- **Qualitative ideas of Black body Radiation**
  - Black body are object or systems that absorb all radiations that are incident on them and re-radiate the energy that are part of radiating system and are not dependent on type of incident radiation. The radiated energy is obtained by standing wave of the cavity which is radiating.
  - In thermal equilibrium, the black body emits radiation at same rate as it absorbs. It is normally seen that objects around us in room temperature radiate in infrared as seen from the graph.
  - If we heat an object to 1500 degrees, a dull red glow visible with object as red hot, while if an object is heated to about 5000 degrees, it radiates throughout visible spectrum resulting as white hot.
  - If the plates of same material are in thermal equilibrium, then emissive power over absorption coefficient is similar as a function of wavelength, also when the plates are of different materials, then
  $$E_1(\lambda, T) / A_1(\lambda) = E_2(\lambda, T) / A_2(\lambda).$$
  - A black body is one that absorbs all radiation incident upon it, $A_{BB} = 1$. Hence, the black body's Emissive power, $E(\lambda, T)$ is universal.
  - If a black body having cavity with small hole where a light is incident on the hole tends to move inside the cavity that is not reflected back due to heavy reflections off walls of the cavity. The relation among energy density in a cavity, $u(\nu, T)$ and black body emissive power of black body is written as $E(\nu, T) = c/4\, u(\nu, T)$
  - The calculation of energy density in electromagnetic waves in a cavity was based on EM theory and equipartition, so as per Rayleigh and Jeans, $u(\nu, T) = (8\times\pi\times\nu^2/c^3)\, kT$.
  - Plank found a formula which fits for both long and short wavelength as $u(\nu, T) = (8\times\pi \times \nu^2/c^3) \times h\nu/e^{h\nu/kT} - 1$.
  - The graph shows comparison of Rayleigh and Jeans calculations and Plank calculation of data at 1600 degrees.



- **Wein's Displacement Law**
  - Wien's Displacement Law shows that black body with a wavelength (λ_{max}) carrying maximum energy is inversely proportional to absolute temperature (T),
  $$\lambda_{max} \times T = b$$
  Here, $b = 2.9 \times 10^{-3}$ mK in Wien's constant.



- **Stefan's Law**
  - Stefan's Law of Radiation states that the radiation energy of a body is proportional to the fourth power of the absolute temperature T of a body.
  - The rate of change of energy in a surrounding medium of absolute temperature T_s is given as
  $$H = e\sigma A\,(T^4 - T_s^4)$$
  Here, e = emissivity of the object, T = absolute temperature of the city,
  $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$ in Stefan-Boltzmann constant.
- **Green House Effect**
  - Green house effect is the exchange of incoming and outgoing radiation which warms the Earth due to incoming UV radiation which can easily pass through the glass walls of a green house that is absorbed by plants and hard surfaces.



- Weak IR radiation which is difficult to pass through the glass walls gets trapped inside which makes warming of the green house.
- The effect of green house results in increase of temperature of Earth by trapping heat in atmosphere making the Earths temperature higher.
- It is caused by interaction of sun's energy with green house gases like carbon dioxide, methane, nitrous oxide and fluorinated gases in Earth's atmosphere.
- The gases are able to trap the heat that results in causing green house effect.
- When sunlight reaches Earths surface, part of it gets absorbed which warms the ground while remaining is reflected back to space in form of heat.
- The green house gases which are in atmosphere will get absorb and redirect heat back towards the Earth.
- It is observed that greenhouse effect results as a major factor in keeping the Earth warm as it will help in keeping part of heat which could otherwise escape from atmosphere out in the space.

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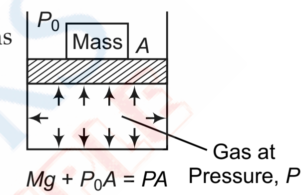
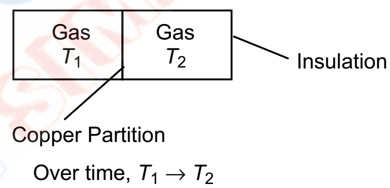
Chapter 8
Thermodynamics

» Revision Notes

Thermodynamic Equilibrium



- The state of a system in which properties of system do not change, as long as external conditions are unchanged is called equilibrium state.
- A system in thermodynamic equilibrium satisfies :
  - mechanical equilibrium (no unbalanced forces)
  - thermal equilibrium (no temperature differences)
  - chemical equilibrium (no reaction)



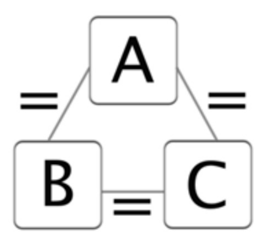
- The local state of a system at thermodynamic equilibrium is determined by the values of its intensive parameters, *e.g.*, temperature volume and pressure.
- Thermodynamic equilibrium is characterised by the minimum of a thermodynamic potential, such as the Helmholtz energy for system at constant temperature and volume, $F = U - T \cdot S$. Also, gibbs energy for system at constant pressure and volume is $G = H - T \cdot S$.
- A system that is in equilibrium experiences no changes when it is isolated from its surroundings.

Thermal Equilibrium :

- The condition under which two substances are in physical contact with each other resulting in no net flow of heat energy.
- Two substances in thermal equilibrium are said to be at the same temperature.
- If two cups of coffee were both left to evolve into thermal equilibrium with same surroundings, the two cups would be in thermal equilibrium with one another.
- The triple point of water is where solid, liquid, and gas of H_2O co-exist.
- The Kelvin temperature scale is thus defined as $T = 273.16 \text{ K} xp/p_{trip}$ where p_{trip} is pressure in fixed volume of gas when it is in thermal equilibrium with a mixture of water ice and water vapor at its triple point.
- The Celsius and Fahrenheit temperature scales are defined as $T_F = T_C \times 1.8 + 32$

Definition of Temperature (Zeroth Law of Thermodynamics) :

- The Zeroth Law of Thermodynamics states that if two bodies are each in thermal equilibrium with some third body, then they are also in equilibrium with each other.

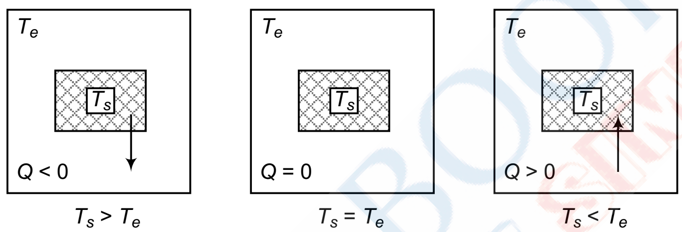


- For example, when three objects A,B and C from which A and C are in thermal equilibrium with B, then A is in thermal equilibrium with C which means that all three are at the same temperature, and it forms the basis for comparison of temperatures.
- In this, there results a measurable property that can be considered to be same for A and B, a property upon which heat transfer depends known as temperature.
- Thermal equilibrium means that when two bodies are brought into contact with each other and separated by a barrier that is permeable to heat, there will be no transfer of heat from one to the other.
- The Zeroth Law says that the temperature, defines the direction of heat flow, and it does not depend directly on the amount of energy that's involved.
- The Zeroth Law of Thermodynamics defines temperature and makes thermometers possible to be made.
- For thermometer to be useful, it must be first calibrated.
- Using logic of zeroth law, it is known that when two separate cups of boiling water are in thermal equilibrium with each other, the zeroth law allows to use thermometers in order to compare the temperatures of any object.

Heat, Work and Internal Energy

Heat (Q)

- Energy transferred across the boundary of a system in the form of heat always results from a difference in temperature between the system and its immediate surroundings.
- By convention, positive heat is that transferred from the surroundings to the system, resulting in an increase in internal energy of the system
- If an object is heated, energy of all of its particles increases, however, all the particles do not move in same direction as they move in all directions, and shows a disordered energy.
- For a system to do some work, it needs to move in particular direction.
- If a system of interest at temperature T_s is surrounded by an environment with temperature T_e and $T_s > T_e$, then heat will flow from the system to the environment.
- When $T_s < T_e$, then heat will flow from the environment into the system.
- Heat, presented by symbol Q and unit Joule, is positive when heat flows in the system and will be negative, if heat flows out of the system.
- Heat flow is a results of a temperature difference between two bodies, and the flow of heat is zero if $T_s = T_e$.

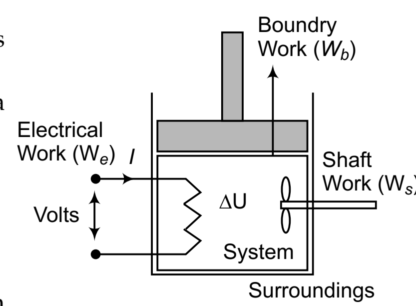


- Heat is not the only way in which energy can be transferred between a system and its environment.
- Energy can also be transferred between a system and its environment by means of work (W).
- Calorie is defined as the amount of heat that would raise the temperature of 1 g of water from 14.5°C to 15.5°C.
- The Joule and the calorie are related as follows: 1 cal = 4.1860 J

Work (W)

If a gas is allowed to expand, some of its random thermal energy is converted into work.

There appear three modes of work transfer across the boundary of a system.



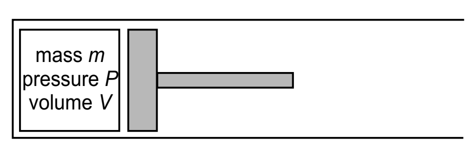
Three forms of work considered :

(a) Boundary work $W_b \Rightarrow$ Piston-Cylinder

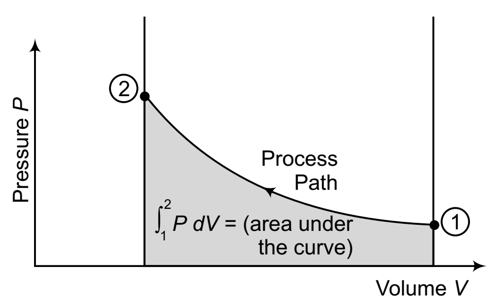
(b) Shaft work $W_s \Rightarrow$ Paddle Wheel

(c) Electrical work W_e = Volts. I. (Amps). time

- By convention, positive work is that which is done by the system on the surroundings, and negative work is that which is done by the surroundings on the system.
- As negative work results in increase in internal energy of the system, it shows negative sign in the energy equation.
- The boundary work is evaluated by integrating force F multiplied by the incremental distance moved dx between an initial state (1) to a final state (2).
- In a piston-cylinder device, force is replaced by the piston area A multiplied by pressure P, that replaces $A.dx$ by change in volume dV, as $W_{1\text{-}2} = \int F\,dx = \int PA\,dx = \int P\,dV = P\int dV$



- In a specific work done represented by P-V diagram, if the mass of the system is m [kg] then $W_{1\text{-}2} = \int P\,dV = m\int P\,dV = m \times W_{1\text{-}2}$



Internal Energy :

- Internal energy is related to temperature.
- It remains constant in a constant-temperature process.
- It is all the mechanical energy in all the components of a system.
- In monoatomic gas, it is the sum of the kinetic energies of all the gas atoms.
- In solid, internal energy is the sum of the kinetic and potential energies of all the particles that make up the solid.
- Internal energy in a system cannot be measured, but can be determined as change in internal energy E, that accompanies a change in the system.
- The change in internal energy that accompanies the transfer of heat q or work W, in or out of a system can be calculated as :

Reduces E	**Increases E**
$Q < 0$ (q negative), heat lost	$Q > 0$ (q positive), heat gained
$W < 0$ (W negative), work done by system	$W > 0$ (w positive), work done on system

First Law of Thermodynamics

The first law of thermodynamics is called as Law of Conservation of Energy.

It shows that energy can be transferred from one system to another in many forms.

Total amount of energy in the universe is constant.

Einstein's equation $E = \Delta mc^2$ describes the relationship between energy and matter.

Einstein suggested that energy and matter are interchangeable and showed that the quantity of energy and matter in the universe is fixed.

Engines work by converting heat energy into mechanical energy.

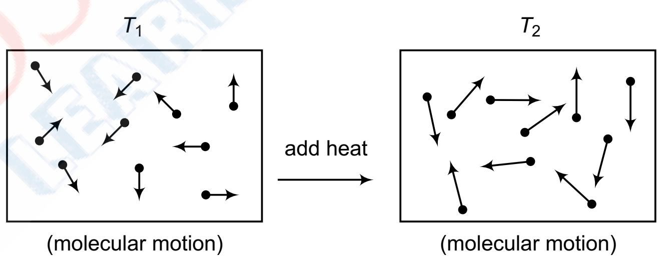
A system is an object whose behaviour can be monitored in relation to its surroundings.

The Laws of Thermodynamics were the results of work by nineteenth century physicists.

First Law of Thermodynamics defines the change in internal energy of a system which is equal to the sum of energy entering the system by heating and by work done.

There exists for every system a property called energy, E.

The system energy is considered as a sum of internal energy, kinetic energy, potential energy, and chemical energy.



First Law defines a useful property called energy.

If U is the internal energy which is function of the state of the system, then $U=U(p,T)$, or $U=U(p,v)$, or $U=U(v,T)$.

The change in energy of a system is equal to the difference between the heat added to the system and the work done by the system, $\Delta E = Q - W$.

In quasi-static processes $W = P_{sys}\, dV$, $dU = \delta Q - pdV$

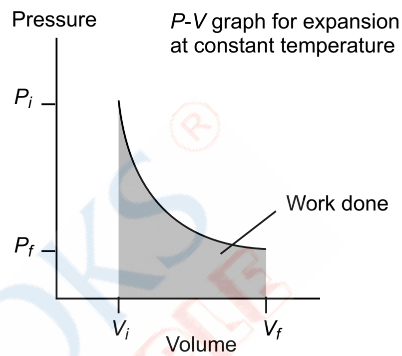
First Law in terms of Enthalpy

- In differential form, the first law can be defined by using pdV for dW by assuming a quasi-static or reversible process, $dU = \delta Q - \delta W$ (true for any process) or $dU = \delta Q - pdV$ (true for any quasi process)
- The enthalpy, $H = U + pV$ can be differentiated by applying chain rule to pV term
- $dH = dU + pdV + Vdp$
- $dH = \delta Q - \delta W + pdV + Vdp$
- $dH = pQ + Vdp$ (true for any quasi static process)

Isothermal and Adiabatic Processes

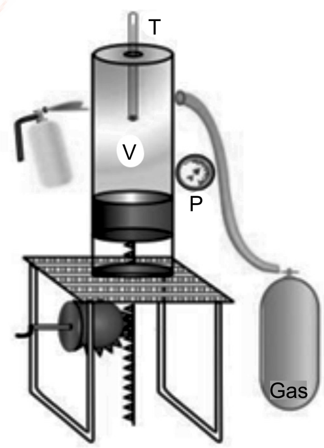
Isothermal Process :

- It is a thermodynamic process that takes place at constant temperature and in which the internal energy of a system remains unchanged.
- If a system immersed in a large constant-temperature bath and any work energy is done by the system, then temperature will remain constant.
- Transfer of energy as heat takes place in isothermal process if the process takes place as large number of gradual and small changes.
- If an air is inside the balloon which expands, its internal energy and temperature will decrease and the energy is transferred as heat from higher temp outside air to the air inside the balloon.
- An isothermal process is a thermodynamic process in which the temperature of the system stays constant $\Delta T = 0$ and work done by it, is the area under P-V curve $W = nRT \ln\left(\frac{v_f}{v_i}\right)$ which takes place when a system is in contact with an outside thermal reservoir where system changes slowly to allow to adjust to temperature.



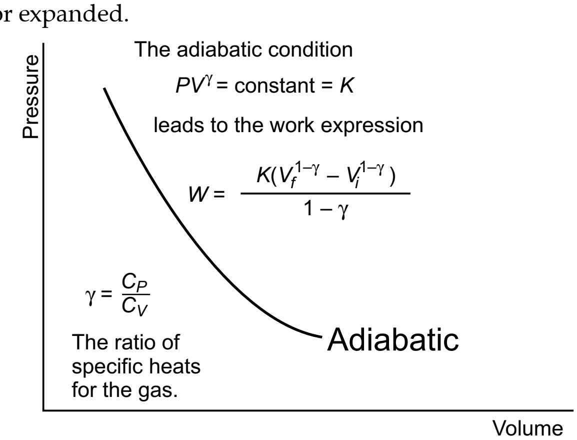
- If quantity n_0 of moles of gas of system remains constant, then internal energy E of the system will remain constant, $\Delta E = n_0 R\Delta T = 0$, so as per ideal gas law, $\Delta(PV) = 0$, so $P_iV_i = PV = P_fV_f$
- The graph shows isothermal process as hyperbolic line (T_0 = constant) which approaches both the V (abscissa) axis and P (ordinate) axis.
- For ideal gas, the line is called an *isotherm* and is $P = n_o RT_o/V$
- In an isothermal process, all heat accepted by the system from its surroundings must have its energy converted to work, which performs on the surroundings, so all energy that comes in the system comes out of the system which makes internal energy of the system constant.

$$Q = W$$

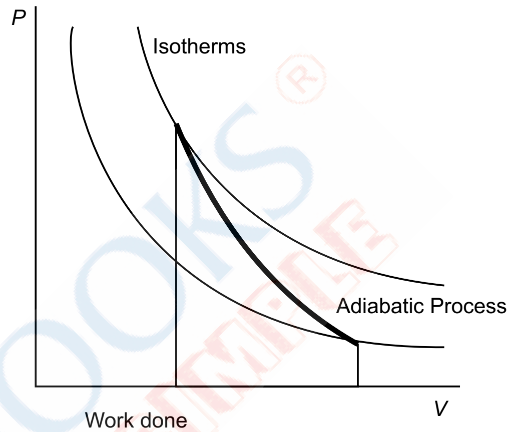


Adiabatic Process

- The process, during which the heat content of the system or certain quantity of the matter remains constant, is called as adiabatic process.
- In adiabatic process, no transfer of heat between the system and its surroundings takes place.
- In this, a decrease in internal energy is equal to the energy transferred from the gas as work.
- In this, no heat is added or removed from a system.
- An example of an adiabatic process is a gas expanding so quickly that no heat can be transferred.
- The expansion does work, and the temperature drops which happens with carbon dioxide fire extinguisher, with the gas coming out at high pressure and cooling as it expands at atmospheric pressure.
- Adiabatic expansion and compression of gases is found in many applications as refrigerators and internal combustion engines need the gases to be compressed or expanded.
- The wall of the system which does not allow the flow of heat through it, is called as adiabatic wall, while the wall which allows the flow of heat is called as diathermic wall.
- The first law of thermodynamics with Q=0 shows that all the change in internal energy is in the form of work done which shows constraint on heat engine process leading to adiabatic condition.
- The ratio of the specific heats $\gamma = C_P/C_V$ is a factor in determining the speed of sound in a gas and other adiabatic processes as well as this application to heat engines.
- The ratio $\gamma = 1.66$ for an ideal monoatomic gas and $\gamma = 1.4$ for air, which is predominantly a diatomic gas.

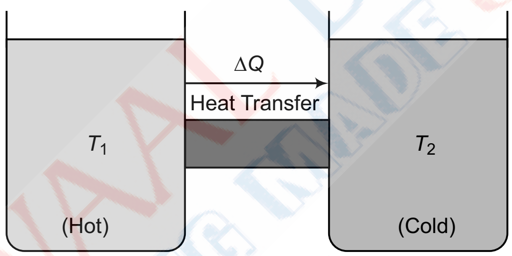


- Adiabatic describes things that are impermeable to heat transfer.
- An insulated wall approximates an adiabatic boundary. An adiabatic process is also reversible called an isentropic process.
- The opposite extreme, in which the maximum heat transfer with its surroundings occurs, causing the temperature to remain constant, is known as an isothermal process.
- Adiabatic heating and cooling are processes that commonly occur due to a change in the pressure of a gas which can be quantified using ideal gas law.
- There are three rates of adiabatic cooling for air.
- The ambient atmosphere lapse rate, which is the rate that air cools as one goes up in altitude.
- The dry adiabatic lapse rate, – 10°C per 1000 m rise.
- The wet adiabatic lapse rate, about – 6°C per 1000 m rise.
- Adiabatic cooling does not have to involve a fluid.
- To reach at very low temperatures, adiabatic demagnetization shows the change in magnetic field on a magnetic material that provides adiabatic cooling.
- During adiabatic process, internal energy of working substance will necessarily decrease.
- The mathematical equation for fluid undergoing an adiabatic process is PV^{γ} = constant



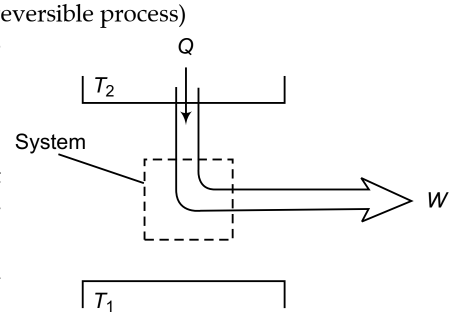
Second Law of Thermodynamics :

- Second Law of Thermodynamics is known as the Law of Increased Entropy.



ΔS = Entropy = $\frac{\Delta Q}{T}$

- The second law states that there exists a useful state variable called entropy S.
- If a system where change in entropy ΔS is equal to the heat transfer ΔQ divided by the temperature T, $\Delta S = \Delta Q / T$ where the combined entropy of the system and the environment remains constant if the process get reversed.
- If initial and final states of the system are "i" and "f", then $S_f = S_i$ (reversible process)
- The second law shows that if the physical process is irreversible, the combined entropy of the system and the environment increases.
- The final entropy needs to be greater than the initial entropy for an irreversible process, $S_f > S_i$.
- No process is possible whose sole result is the absorption of heat from a reservoir and the conversion of this heat into work. [Kelvin-Planck statement of the second law]
- No process is possible whose sole result is the transfer of heat from a cooler to a hotter body. [Clausius statement of the second law]



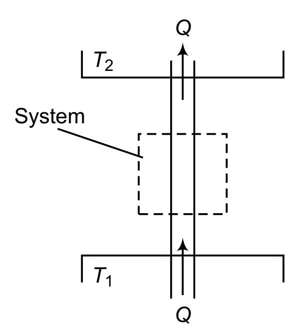
- For a reversible process, changes in property S is given by

$$dS = dQ_{\text{reversible}} / T$$

- The entropy change of any system and its surroundings, considered together, is positive and approaches zero for any process which approaches reversibility,

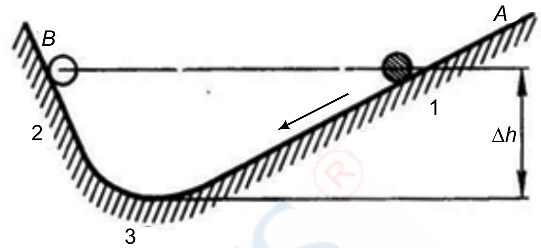
$$\Delta S_{total} \geq 0$$

- In an isolated system, a system that has no interaction with the surroundings, changes in the system have no effect on the surroundings, so first and second laws become, $\Delta E_{\text{system}} \geq 0$ and $\Delta S_{\text{total}} \geq 0$
- For isolated system, total energy ($E = U$ + Kinetic Energy + potential Energy) is constant. The entropy only increases or in the limit of reversible process remains constant.



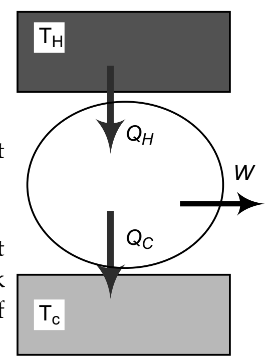
- The second law tells that the intensive variable is the temperature T, and the extensive state variable is the entropy S.
- The first law for a simple compressible substance in terms of state variables, so $dU = TdS = PdV$ which shows first and second law in terms of state variables for all processes.

Reversible and Irreversible Processes :



- A process is said to be reversible if after the process has been completed in forward and reverse directions, the system returns into its initial state.
- If two planes A and B with ball at point 1 on one of the planes and as the ball moves downward, it gets kinetic energy at the expense of potential energy.
- In this, at point 3, ball possesses a certain amount of kinetic energy, making it possible for the ball to rise to the other plane where its kinetic energy diminishes which is compensated by an increase in potential energy.
- If there is no friction between the ball and the surface over which it moves and the air offers no resistance to the motion of the ball, in accordance with laws of mechanics the ball shall rise to the same height from where its downward movement on plane A was initiated, *i.e.*, points 1 and 2 are arranged at the same height h above the horizon.
- The ball will cease to rise at point 2, its velocity will become equal to zero and the ball will begin to roll down, then rise to point 1, etc.
- Under the conditions specified above the considered process is reversible.
- For reversible processes, the reverse process is, so to say, a "mirror-image presentation" of the direct process.
- In a forward process, certain amount of heat is added to a system, then in reverse process an exactly equal amount of heat is rejected from the system, if a system does work in a forward process on surrounding medium.
- In reverse process, surrounding medium does work on the system, where absolute amount of this work being equals to the work done during the forward process.
- If a system undergoes expansion during a forward process, compression of the system will take place during the reverse process, etc.
- A process is said to be irreversible if after the process has been completed in the forward and reverse orders, the system fails to return into the initial state.
- To determine the maximum efficiency of a heat engine we need to define an ideal process and determine the efficiency of such a process.

Heat Engine :



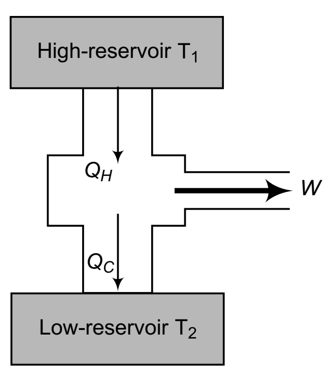
- A heat engine is a device that converts heat into another form of energy.
- Efficiency is normally stated as a percentage, η = (Energy out/Energy in) × 100%
- Heat engines move energy from a hot place to a cold place and convert some of that energy into mechanical energy.
- Heat engines require a difference in temperature to function.
- A heat engine is a device that uses heat to do work where in a full cycle of a heat engine, heat is added called Q_H, some of energy from input performs the work (W) while rest of heat is removed at surroundings cold (Q_C) which follows low of conservation of energy

$$Q_H = W + Q_C$$

- Efficiency η = work done/input heat = W/Q_H

$$\eta = Q_H - Q_C / Q_H = 1 - Q_C / Q_H$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{Q_C}{Q_H}$$



- It is the maximum possible efficiency for an engine having other losses that will reduce the efficiency.

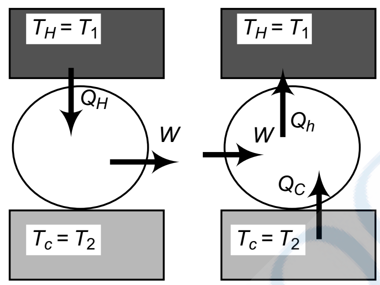
Examples of Heat Engines :

- Internal combustion
  - Piston engine
  - Gas turbine
  - Jet engine
- External combustion
  - nuclear reactors like CANDU reactor, pressurized water reactor

- coal-fired power plant
- natural gas power plant

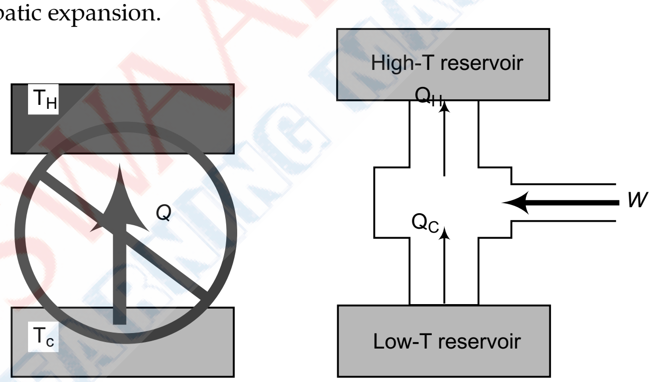
Heat Engine as Second Law

- It is observed that there is other form of Second Law of Thermodynamics which describes that it is not possible to make a heat engine which can only absorb heat from high-temperature region and convert all heat into work.
- It is impossible to design a heat engine which does not exhaust heat to environment nor having an efficiency of 1.00 or 100%.
- For engine to be 100% efficient, all input energy would need to be converted into useful output energy. The output of 100% efficient engine would be all work and no heat, no pollution, no emissions, no noise on vibrations.

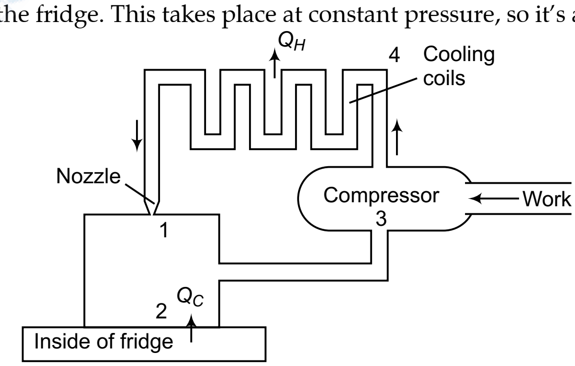


Refrigerator

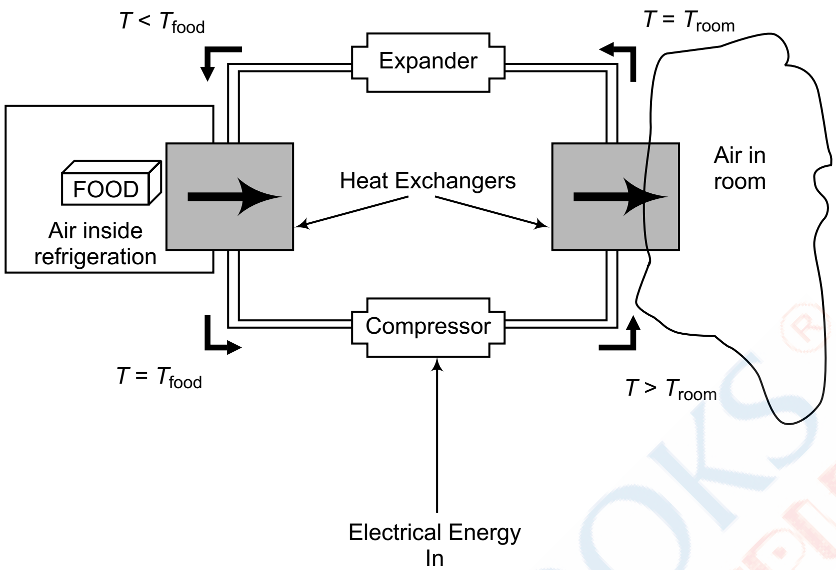
- A refrigerator is a heat engine which is designed to remove heat from a cold region and transfer it to a hot region, is essentially a heat engine operating in reverse.
- A refrigerator, consisting of a fluid pumped through a closed system, involves a four-step process.
  - **Step 1 :** The fluid passes through a nozzle and expands into a low-pressure area. Similar to the way carbon dioxide comes out of a fire extinguisher and cools down, the fluid turns into a gas and cools down. This is essentially an adiabatic expansion.



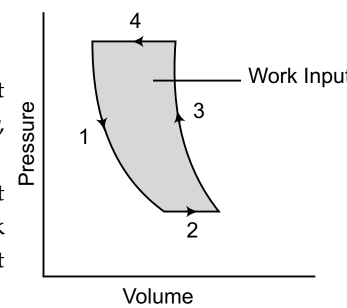
  - **Step 2 :** The cool gas is in thermal contact with the inner compartment of the fridge; it heats up as heat is transferred to it from the fridge. This takes place at constant pressure, so it's an isobaric expansion.



  - **Step 3 :** The gas is transferred to a compressor, which does most of the work in this process. The gas is compressed adiabatically, heating it and turning it back to a liquid.
  - **Step 4 :** The hot liquid passes through coils on the outside of the fridge, and heat is transferred to the room. This is an isobaric compression process.
- A refrigerator is rated by something known as the coefficient of performance, which is the ratio of the heat removed from the fridge to the work required to remove it.



- Coefficient of performance = Q_C/W
- Refrigeration cycles take in work from the surroundings and transfer heat from a low temperature reservoir to a high temperature reservoir.
- The objective of a refrigerator is to lower the internal energy of a body at low temperature and transfer that energy to the higher temperature surroundings and needs work to do this.
- The *P-V* (pressure-volume) graph is very useful for calculating the work done.
- For any kind of heat engine or refrigerator (reverse heat engine), the processes involved form a cycle on the *P-V* graph.



- The work is the area of the enclosed region on the graph.
- An idealized refrigerator is an engine which extracts heat from a cold heat reservoir (temperature T_1) and rejects it to a somewhat hotter heat reservoir, which is usually the environment (temperature T_2).
- If q_2 be the heat absorbed per cycle from cold reservoir, q_1 is the heat rejected per cycle into the hotter reservoir, and w be the external work done per cycle on the engine, then first law of thermodynamics tells us that $w + q_2 = q_1$.
- The second law says that $\frac{q_1}{T_1} + \frac{-q_2}{T_2} \geq 0$.
- From the first and second law $\frac{w}{T_1} \geq q_2\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$.
- The efficiency of a refrigerator (or coefficient of performance) is the ratio of the heat extracted per cycle from the cold reservoir to the work done per cycle on the engine, $\beta = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$.
- One joule of work done on the engine or pump, gives more than one joule of energy from the cooling.

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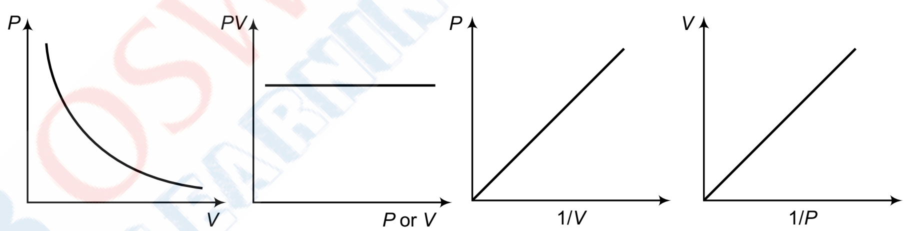
Chapter 9

Behaviour of Perfect Gas and Kinetic Theory

» Revision Notes

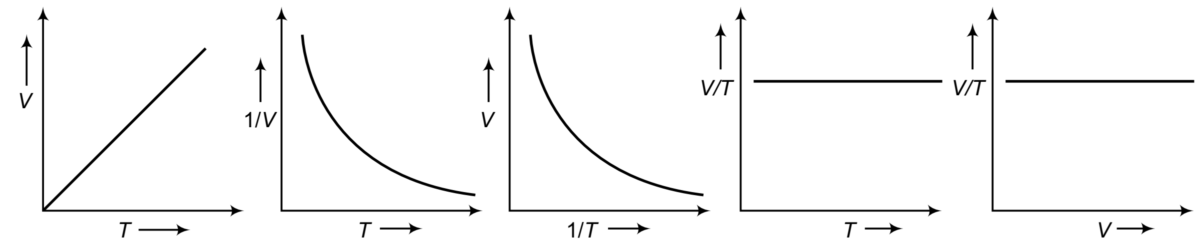
Equation of State of a Perfect Gas

- Equation of state explains the relationship between temperature, pressure and volume for a given substance or mixture of substances.
- The ideal gas law is a general explanation of both Boyle's law and Charles's law and states that for a particular quantity of gas, the product of volume 'V' and pressure 'P' is proportional to the absolute temperature 'T'.
- From the three important gas laws, relationships which exist among two physical properties of a gas, keeping other properties constant :
- **Boyle's Law :**
  - For a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure, *i.e.*, $V \propto 1/P$ or PV = constant
  - $P_1V_1 = P_2V_2$ or $P_1/P_2 = V_2/V_1$



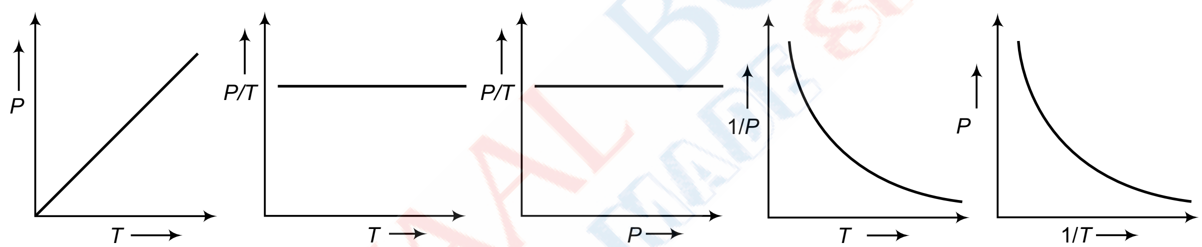
- **Charles' Law :**
  - If the pressure remains constant, the volume of given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its volume at 0°C for each 1°C rise or fall in temperature.
  - The Charles' law for centigrade scale is $V_t = V_0\left(1+\frac{1}{273.15}t\right)$
  - If pressure remains constant, volume of given mass of a gas is directly proportional to its absolute temperature, $V \propto T$ or $\frac{V}{T}$ = constant or $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ [If m and P are constant]
  - Also $\frac{V}{T} = \frac{m}{\rho T}$ = constant [As volume $V = \frac{m}{\rho}$]
  - Also ρT = constant or $\rho_1 T_1 = \rho_2 T_2$ [As m = constant]
  - As per kinetic theory of gases $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$ [As $v^2_{rms} \propto T$] or $P \propto \frac{\text{Mass of gas}}{V}T$

- Graphical representation of Charles law :



- **Gay-Lussac's law or pressure law :**
  - The volume remaining constant, the pressure of a given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its pressure at 0°C for each 1°C rise or fall in temperature.

    $P_t = P_0\left[1+\frac{1}{273.15}t\right]$ is known as pressure law for centigrade scale.
  - The volume remaining constant, the pressure of a given mass of a gas is directly proportional to its absolute temperature, $P \propto T$ or $\frac{P}{T} = \text{constant}$ or, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ [*m* and V are constant]
  - If mass and volume of gas remains constant then $P \propto T$, which is in accordance with Gay Lussac's law.



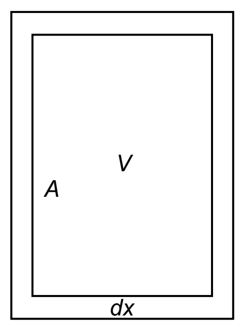
- **Avogadro's Law :**
  - $V \propto n$ where P and T are constants
  - The generalise gas law is $V \propto nT/P$, where if proportionality constant is "R", then, $V = R\,(nT/P)$
  - The simplified form of perfect gas equation is $PV = nRT$ or $PV = NkT$ where : R = Universal gas constant.
  - The ideal gas equation can be written as $PV = gRT/FW$ or $FW = gRT/PV$ which helps in determining the formula of weight of gas, if mass, temperature, volume and pressure of gas are known.
  - This is also called its equation of state that is sufficient to explain the behaviour of the substance. Here, FW is the formula weight and g is the mass of gas in gram.
- In equation $PV = nRT$
  - **Pressure** (P) **:** It is the pressure which is measured in pascals, Pa, sometimes written as newton/square metre or N/m^2
  - **Volume** (V) **:** It is the volume which is measured in cubic metre, m^3 where, $1\ m^3 = 1000\ dm^3$
  - **Number of moles (*n*) :** It is the number of moles which is measured by dividing mass in grams by mass of one mole in grams.
  - **Gas constant (*R*) :** It is the gas constant having an SI value as $8.31441\ J\ K^{-1}\ mol^{-1}$.
- **Graham's Law of Diffusion**
  - When two gases at the same pressure and temperature are allowed to diffuse into each other, the rate of diffusion of each gas is inversely proportional to the square root of the density of the gas.

    We know, $v_{rms} = \sqrt{\frac{3P}{\rho}}$ or $v_{rms} \propto \frac{1}{\sqrt{\rho}}$ and rate of diffusion of a gas is proportional to its rms velocity i.e.,

    $r \propto v_{rms}$ so $r \propto \frac{1}{\sqrt{\rho}}$ or $\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

Ideal Gas Equations	
For 1 mole or N_A molecule or M gram or 22.4 *litres* of gas	$PV = RT$
For μ mole of gas	$PV = \mu RT$
For 1 molecule of gas	$PV = \left(\frac{R}{N_A}\right)T = kT$
For N molecules of gas	$PV = NkT$
For 1 *gm* of gas	$PV = \left(\frac{R}{M}\right)T = rT$
for n *gm* of gas	$PV = nrT$

- **Universal Gas Constant (R) :**
  - Dimension $[ML^2T^{-2}\,mol^{-1}K^{-1}]$
  - $R = \dfrac{PV}{\mu T} = \dfrac{\text{Pressure} \times \text{Volume}}{\text{No. of moles} \times \text{Temperature}} = \dfrac{\text{Work done}}{\text{No. of moles} \times \text{Temperature}}$
  - Thus, universal gas constant signifies the work done by (or on) a gas per mole per kelvin.
  - S.T.P. value : $8.31 \dfrac{\text{Joule}}{\text{mole} \times \text{kelvin}} = 1.98 \dfrac{\text{cal}}{\text{mole} \times \text{kelvin}} = 0.8221 \dfrac{\text{litre} \times \text{atm}}{\text{mole} \times \text{kelvin}}$
- **Boltzman's Constant (k) :**
  - Dimension $[ML^2T^{-2}K^{-1}]$
  - $k = \dfrac{R}{N} = \dfrac{8.31}{6.023 \times 10^{23}} = 1.38 \times 10^{-23}$ Joule/kelvin
- **Specific Gas Constant (r) :**
  - Dimension $[M^0L^2T^{-2}K^{-1}]$
  - $r = \dfrac{R}{M}$;
  - Unit : $\dfrac{\text{Joule}}{\text{gm} \times \text{kelvin}}$
  - Since the value of M is different for different gases the value of r is different for different gases.
- **Work Done in compressing a gas**



  - If a gas in a container with volume V and surface area A and when a force normal to this area is applied for compressing the gas to certain distance dx, then as the work is force times the distance, $W=F\,dx$
  - If a pressure P is applied on the whole surface, the force will be pressure times the area, so $W=P\,A\,dx$
  - As area A times the distance moved dx is change in volume, then: $Adx = dV$ or $W=P\,dV$ which calculates the amount of work done in compressing the gas.
  - On compressing the gas, pressure will change which is a function of volume $P(V)$ and when gas is cooled, an isothermal compression will exists, where the pressure is related to volume as $PV = nRT$
  - The work to compress from volume V_1 to V_2 is $W = \int_{V_1}^{V_2} P(V)\,dV = \int_{V_1}^{V_2} (nRT/V)\,dV$ or $W = nRT \ln (V_2/V_1)$
  - In case of compression the gas $V_2 < V_1$, from First Law of Thermodynamics, U is internal energy, Q is heat, and W is work, $\Delta U = Q + W$, where,
    - $Q > 0$ for heat absorbed by system
    - $Q < 0$ for heat released by system
    - $W > 0$ for work done on system
    - $W < 0$ for work done by system
  - In a constant temperature system, work is added to system where equal amount of heat is being released, where there is no change in internal energy.
  - If n = number of moles of gas, C_v = molar heat capacity at constant volume and T = temperature, then internal energy U of ideal gas is $\Delta U = nC_v\Delta T$

- If there is no change in internal energy then, there is no change in temperature and process results as isothermal. In case of an isothermal process, change in internal energy will be zero, so putting value in First Law of Thermodynamics, $0 = Q + W$ or $Q = -W$
- The work term 'W' for compression is pressure volume work, so for P= pressure and V=volume, pressure-volume work will be $W = -PdV$
- If a gas is compressed, work is done on the system is $\Delta W_{system} = -P_{ext}\,\Delta V$, where P_{ext} = external pressure which is amount of pressure that surroundings are applying to the system.
  In the expression, (–) negative sign shows that compression that corresponds to work being done to the system.

Kinetic Theory of Gases

- Temperature and pressure are macroscopic properties of gases which are related to molecular motion known as microscopic phenomenon.
- The kinetic theory of gases correlates between macroscopic properties and microscopic phenomena.
- As pressure is proportional to average kinetic energy of all gas molecules, so Avogadro's principle relates to kinetic energies of different gases which are similar at same temperature.
- As molecular masses are different from gas to gas and if gases have same average kinetic energy, then average speed of a gas will be unique.
- Boltzmann and Maxwell extended their theory from the assumptions that show that the average kinetic energy of a gas will depend on its temperature.
- If 'u' be the root-mean-square speed of a gas with molar mass M and N is the Avogadro's number, then average kinetic energy will be $1/2\,(M/N)\,u^2$ or K.E $= (M/2N) \times u^2 = 3R\,T/2N = (3/2)\,kT$, where, M/N is mass of single molecule showing $u = \sqrt{(3k\,NT/M)} = \sqrt{(3\,RT/M)}$

Assumptions

The assumptions are :

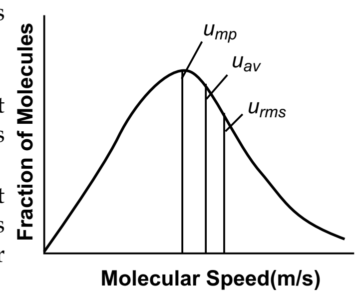
- Gases are made up of atoms and molecules which are in constant random motion in straight lines.
- The molecules behave as rigid spheres.
- Pressure is due to collisions between the molecules and the walls of the container.
- All collisions, both between the molecules themselves, and between the molecules and the walls of the container, are perfectly elastic. Hence, there will be no loss of kinetic energy during the collision.
- The temperature of the gas is proportional to the average kinetic energy of the molecules.
- There are no intermolecular forces between the gas molecules.
- The volume occupied by the molecules themselves is entirely negligible relative to the volume of the container.
- The particles are involved in random motion *i.e.*, there is no resultant force on them,
- Kinetic theory helps in deriving the equations that explain about pressure and temperature in terms of movement of individual molecules, $PV = (1/3)\,Nm\,(c^2)$ or $P = (1/3)\,\rho\,(c^2)$

Concept of Pressure

- It is seen from kinetic theory that the molecules which are inside a volume of a substance will be constantly moving around freely.
- The molecules collide with each other and with the walls of the container.
- The force of impact of a single collision is less but if taken together, the impact exerted will be too large with considerable force on surface of substance.
- If the molecules are hitting the surface of the substance at 90°angle, then they will exert maximum force while if the molecules hit the surface at an angle which is less than 90°, then the molecules will exert low force.
- Low forces cause the pressure which is exerted by the gas.
- The large number of collisions per unit area of substance, pressure will be large,

$$\text{Pressure} = \text{Force/Area or } P = F/A$$

- The direction of the force is perpendicular to the surface of container at every point.
- As gas molecules are in regular motion, they will have kinetic energy E_k.
  The kinetic energy for a molecule is calculated by $E_k = 1/2\,mu^2$
- The molecules in gas sample have average kinetic energy, so each molecule will result in distribution of kinetic energies as of distribution of speeds which appears from collisions that takes place between molecules in gas phase.



- As collision is elastic, each molecule will involve in collision.
- At the time of collision of two molecules, one molecule will be deflected at high speed while other molecule will be deflected at low speed which keeps the average kinetic energy unchanged.
- In the graph, most probable speed (u_{mp}) shows largest molecular speed that corresponds to distribution, average speed (u_{av}) shows mean speed of gas molecules, root-mean-square (rms) speed (u_{rms}) for molecules with similar same kinetic energy as average kinetic energy.

- All collisions are elastic where total kinetic energy and momentum are conserved.
- At ordinary temperature and pressure, molecular size is very small as compared to inter molecular distance between them.
- In case of gas, molecules are very far from each other and the size of molecules is small as compared to the distance between them due to which, interaction between them is negligible and as a result of no interaction among molecules, there will be no force among molecules and the molecules will move freely.

Kinetic Energy and Temperature

- The ideal gas law equation shows that the total kinetic energy of a collection of gas molecules is directly proportional to the absolute temperature of the gas.
- The ideal gas law can be rearranged to give an explicit expression for temperature.
- Temperature is a function only of the mean kinetic energy, the mean velocity and the mean molar mass.
- As absolute temperature decreases, the kinetic energy will also decrease and thus the mean velocity of the molecules also decreases.
- At the absolute zero of temperature (0 K), all motion of gas molecules would cease and the pressure would then also be zero
- The absolute zero of temperature has never been attained, although modern experiments have made it to temperatures as low as 0.01 K.
- The average velocity of the molecules of a gas is interpreted as the actual velocities are distributed over a very wide range as described by Maxwell's law of distribution of velocities.
- It is not necessary to use a Maxwell-Boltzmann distribution of velocities to explain either the nature of temperature or Charles' law, although it is the correct expression of the distribution.
- Charles' law can be obtained for any distribution in which the velocities of the gas molecules are a function of the nature of the gas and the absolute temperature only.
- As per kinetic molecular theory, the average kinetic energy of gas particles is proportional to the absolute temperature of the gas which is expressed with $\overline{E} = \frac{3}{2}kT$
- As average kinetic energy is related to absolute temperature and molecular speed, so the rms speed,

$$\overline{E_k} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT \quad \text{or} \quad v_{rms} = \sqrt{\frac{3kT}{m}}$$

Perfect Gas Equation

- Perfect gas equation is given by $PV = \mu RT$, where symbols have usual meanings.
- The behaviour of gas at particular situation if gas satisfies the equation as perfect or ideal gas is shown by $R = kNA$ where symbols have usual meanings.

Forms of Perfect Gas Equation

- $PV = \mu RT$
- $\mu = M/M_o$
- $PV = (N/N_A)RT$
- $P = (N/V)kT$
- $P = nkT$ as $n = N/V$ where N=number of molecules and V=volume
- $PV = nkT$
- $P = \rho RT/M_o$

RMS Speed of Gas Molecules

- RMS speed of the gas molecule is the square root of mean of squares of random velocities $v_1, v_2, v_3 \ldots v_n$ of the individual molecules of gas.

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \ldots v_n^2}{N}}$$

- The rms velocity is used instead of average velocity for a typical gas sample as the net velocity will result zero and as the particles are moving in all directions, the expression is used in determining the velocity of particles both in terms of diffusion and effusion rates.
- Root mean square speed is square root of mean of squares of the speed of different molecules *i.e.*
- From the expression for pressure of ideal gas $P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$
- (i) $v_{rms} = \sqrt{\frac{3PV}{mN}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3P}{\rho}}$ $\left[\text{As } \rho = \frac{\text{Mass of gas}}{V}\right]$

- (ii) $v_{rms} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3\mu RT}{\mu M}} = \sqrt{\frac{3RT}{M}}$
- (iii) $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3N_A kT}{N_A m}} = \sqrt{\frac{3kT}{m}}$ [As $M = N_A m$ and $R = N_A k$]
- Root mean square velocity $v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$

Average Speed

- It is the arithmetic mean of the speeds of molecules in a gas at given temperature,

$$v_{av} = \frac{v_1 + v_2 + v_3 + v_4 + ...}{N}$$

- As per kinetic theory of gases, average speed $v_{av} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$
- $v_{rms} > v_{av} > v_{mp}$ (order remembering trick : RAM)
- $v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} = \sqrt{3} : \sqrt{2.5} : \sqrt{2}$ [Here, $v_{mp} = \sqrt{\frac{2KT}{m}}$ is the most probable velocity]
- For oxygen gas molecules, v_{rms} = 461 m/s, v_{av} = 424.7 m/s and v_{rms} = 376.4 m/s

Degrees of Freedom

- Degree of freedom is the total number of independent variables whose values have to be specified for a system.
- Every kind of molecule has certain number of degrees of freedom, which are independent in the ways in which the molecule can store energy.
- The number of degrees of freedom of a system is the number of the parameters necessary to define the position of the system.
- If N is the number of particles moving freely in 's' dimensional space, then the degrees of freedom is $f = Ns$
- If there exists a constraints then $f = Ns - k$, where k is number of constraints.
- Degrees of freedom could be of translation, rotation and vibration where every molecule has $3N$ degrees of freedom, where N is number of atoms in molecule.
- The number N remains constant, even if the molecule is broken into fragments, with change in translational, rotational and vibrational degrees of freedom.
- The degree of freedom shows the way in which a molecule is free to move which could be x, yx, y and zx directions.

Constraints and Types

- Motion of a particle or system of particles is restricted by one or more conditions, so such limitations on motion of system are known as constraints.
- Degree of freedom of dynamic system is minimum number of independent co-ordinates needed to simplify the system completely along with the constraints.

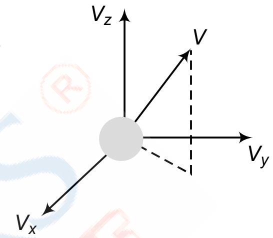
System	Types of Constraints
1. Rigid Body	Conservative in nature, scleronomic, holonomic, bilateral
2. Deformable bodies	Rheonomic, holonomic, bilateral, dissipative
3. Simple pendulum with rigid support	Scleronomic, holonomic, bilateral, conservative.
4. Pendulum with variable length	Rheonomic, holonomic, bilateral, dissipative
5. Rolling without sliding	Non-holonomic
6. Gas filled hollow sphere	Scleronomic, holonomic, unilateral, conservative.
7. Expanding or contracting gas filled container	holonomic, unilateral, dissipative, rheonomic

Types of Degrees of Freedom

- To specify the position in space of a molecule with 'n' nuclei, a '$3n$' coordinates are required (3 Cartesian coordinates for each nucleus).
- Degrees of freedom can be classified as translational, rotational or vibrational.
- The total number of $3N$ degrees of freedom results as:
- Non-linear molecules with :

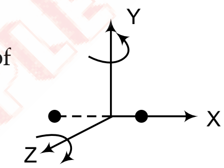
  - 3 degrees of freedom of translation
  - 3 degrees of freedom of rotation
  - 3N—6 degrees of freedom of vibration
- Linear molecules with :
  - 3 degrees of freedom of translation
  - 2 degrees of freedom of rotation
  - 3N—5 degrees of freedom of vibration

Monoatomic Molecules :

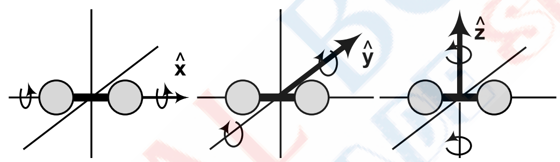


- The Molecules of mono atomic gas has one atom per molecule and can move in any direction in space so it can have three independent motions and with 3 degrees of freedom.
- In this, all the kinetic energy of monoatomic gas is in translational motion with a velocity v.
- Examples: He, Ne, Xe, and Kr.
- Total energy of each molecule is (3/2) kT.

Diatomic Molecules :

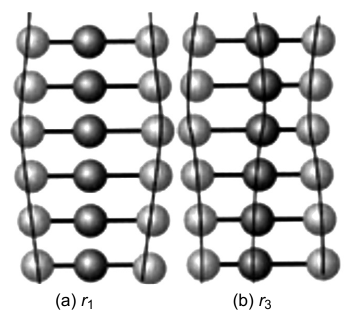


- The diatomic molecule will rotate about any axis at right angles to its own axis.
- It is always linear having 3 degrees of freedom of translation, 2 degrees of freedom of rotation and 1 degree of freedom of vibration.
- Examples: H_2, O_2, N_2, CO, Cl_2.
- Three translational degrees of freedom: n_x, n_y, n_z
- Three rotational degrees of freedom : about x, y and z axes.



- Total energy of each molecule is 5(1/2) kT = (5/2) kT

Triatomic Molecules

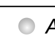
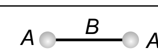
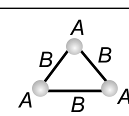
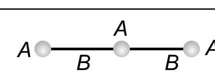


Symmetric & anti-symmetric vibration

- The triatomic gas molecule has three atoms per molecules which can be linear or non-linear, symmetric or asymmetric that rotates about any of three co-ordinate axes.
- Example : CO_2; N_2O; SO_2; H_2O
- A triatomic molecule has three translational and two accessible rotational degrees of freedom.
- The total energy of each molecule is 7(1/2) kT = (7/2) kT
- In this, non-linear molecules will have 3 degrees of freedom of translation, 3 degrees of freedom of rotation and 3N – 6 = 3 degrees of freedom of vibration.
- There are 9 possible degrees of freedom (3 translational, 3 rotational, and 3 vibrational), but only 7 are accessible at lower temperatures.

Linear Molecules

- It will have 3 degrees of freedom of translation, 2 degrees of freedom of rotation and 3N-5 = 4 degrees of freedom of vibration.

Atomicity of gas	Example	A	B	Degree of freedom $f = 3A - B$	Figure
Monoatomic	He, Ne, Ar	1	0	$f = 3$	
Diatomic	H_2, O_2	2	1	$f = 5$	
Triatomic non linear	H_2O	3	3	$f = 6$	
Triatomic linear	CO_2, $BeCl_2$	3	2	$f = 7$	

Law of Equi-partition of Energy (Statement Only)

- Equipartition of energy theorem states that energy is shared equally among all energetically accessible degrees of freedom of a system.

In this, a system will try to maximise its entropy by distributing the presently available energy equally among all accessible modes of motion.

- The equipartition law also simplifies by predicting that the available energy gets shared equally among the accessible modes of motion, and shows, about the amount of energy appears in each degree of freedom.

  Also, it shows that each quadratic degree of freedom possesses an energy $\frac{1}{2}kT$

- If kinetic and potential energies are associated with translational, rotational and vibrational energy, then:

  Translational degrees of freedom $K = \frac{1}{2}mv^2$

  Rotational degrees of freedom $K = \frac{1}{2}I\omega^2$

  Vibrational degrees of freedom $K = \frac{1}{2}mv^2$ and $V = \frac{1}{2}Kx^2$

- The three types of degrees of freedom all having quadratic dependence on velocity follow the equipartition theorem.
- As each degree of freedom has an energy of $\frac{1}{2}$ kT per molecule or $\frac{1}{2}$ RT per mole which is the equipartition of energy, the total energy of particle will depend on its movement as translational in x, y, z directions, rotational and vibrational.
- If the system possess degree of freedom f, then

Total energy associated with each molecule	$\frac{f}{2}kT$
Total energy associated with N molecules	$N\frac{f}{2}kT$
Total energy associated with each mole	$\frac{f}{2}RT$
Total energy associated with μ mole	$\frac{\mu f}{2}RT$
Total energy associated with each gram	$\frac{f}{2}rT$
Total energy associated with M_0 gram	$M_0\frac{f}{2}rT$

Specific Heat of Gases

- The specific heat of a gas is the amount of heat energy required to raise the temperature of 1 kg of gas through a unit degree.
- If the gas is compressed suddenly and no heat is supplied from outside *i.e.* $\Delta Q = 0$, while temperature of gas raises because of compression, then $C = \frac{\Delta Q}{m(\Delta T)} = 0$ *i.e.* $C = 0$
- If the gas is heated and allowed to expand at a rate such that rise in temperature results due to heat supplied will exactly be equal to fall in temperature as a result of expansion of gas, $\Delta T = 0$, $C = \frac{\Delta Q}{m(\Delta T)} = \frac{\Delta Q}{0} = \infty$ *i.e.* $C = \infty$
- If rate of expansion of gas is slow, then fall in temperature of gas as a result of expansion be smaller than rise in temperature of gas due to heat supplied.
  There exists some net rise in temperature of the gas *i.e.* ΔT will be positive,

$$C = \frac{\Delta Q}{m(\Delta T)} = \text{ positive } i.e. \ C = \text{ positive}$$

- If the gas were to expand fast, fall of temperature of gas due to expansion would be greater than rise in temperature due to heat supplied.

  There will be net fall in temperature of the gas *i.e.* ΔT will be negative, $C = \frac{\Delta Q}{m(-\Delta T)} =$ negative *i.e.* $C =$ negative

Specific Heat of Gas at Constant Volume (C_v)

- Specific heat of a gas at constant volume is quantity of heat required to raise the temperature of unit mass of gas through 1 K when its volume is kept constant, $C_v = \frac{(\Delta Q)_v}{m\Delta T}$

  Instead of unit mass, if 1 mole of gas is assumed, then specific heat will be molar specific heat at constant volume shown by C'_v.

- $C'_v = MC_v = \frac{M(\Delta Q)_v}{m\Delta T} = \frac{1}{\mu}\frac{(\Delta Q)_v}{\Delta T}$ $\left[\mu = \frac{m}{M}\right]$

Specific Heat of a Gas at Constant Pressure (C_p)

- Specific heat of gas at constant pressure is quantity of heat required to raise the temperature of unit mass of gas by 1 K when its pressure is kept constant, $C_p = \frac{(\Delta Q)_p}{m\Delta T}$

  Instead of unit mass, if 1 mole of gas is assumed, then specific heat will be molar specific heat at constant pressure shown by C'_p

  $C'_p = MC_p = \frac{M(\Delta Q)_p}{m\Delta T} = \frac{1}{\mu}\frac{(\Delta Q)_p}{\Delta T}$ $\left[\mu = \frac{m}{M}\right]$

Application to Specific Heat Capacities of Gases

- Specific heat or specific heat capacity s, is heat capacity/mass or $S = \frac{Q}{m(T_2 - T_1)}$ $[\because (T_2 > T_1)]$
- It is the amount of heat energy required for heating a substance of unit mass by 1°C.
- The higher the specific heat, the more energy required for heating the substance.

Car Radiator

- Water is pumped through the channels in the engine block to absorb heat.
- Water is used as the cooling agent due to its high specific heat capacity.
- The hot water flows to the radiator and is cooled by the air flows through the fins of the radiator.
- The cool water flows back to the engine again to capture more heat and this cycle is repeated continuously.

Cooking Utensils

- Cooking utensils are made of metal which has low specific heat capacity so that it need less heat to raise up the temperature.
- Handles of cooking utensils are made of substances with high specific heat capacities so that its temperature won't become too high even if it absorbs large amount of heat.

Thermal Radiator

- Thermal radiators are always used in cold countries to warm the house.
- Hot water is made to flow through a radiator. The heat given out from the radiator then warms the air of the house.
- The cold water is then flows back to the water tank. This process is repeated continuously.
- Water is used in the radiator because it has high specific heat capacity.

Sea Breeze

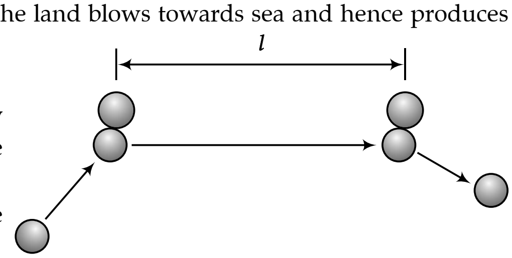
- Land has lower heat capacity than sea water. Therefore, in day time, the temperature of the land increases faster than the sea.
- Hot air (lower density) above the land rises. Cooler air from the sea flows towards land and hence produces sea breeze.

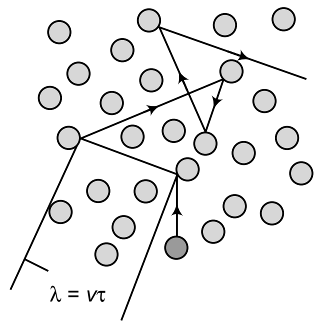
Land Breeze

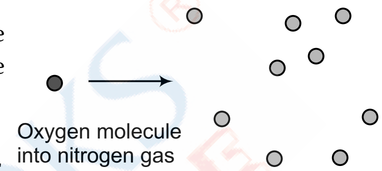
- Land has lower heat capacity than sea water. During night time, the temperature of the land drops faster than the sea.
- Hot air (lower density) above the sea rises. Cooler air from the land blows towards sea and hence produces land breeze.

Concept of Mean Free Path

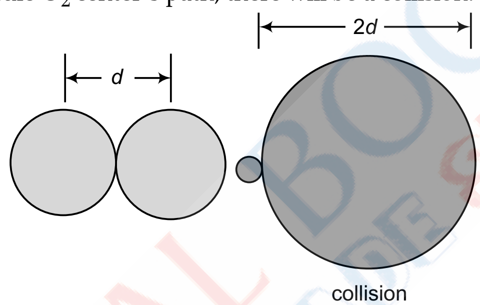
- Mean free path is the average distance which is travelled by a particle in a material before colliding with another particle in the material.
- It is inversely proportional to the density of particles in the material.



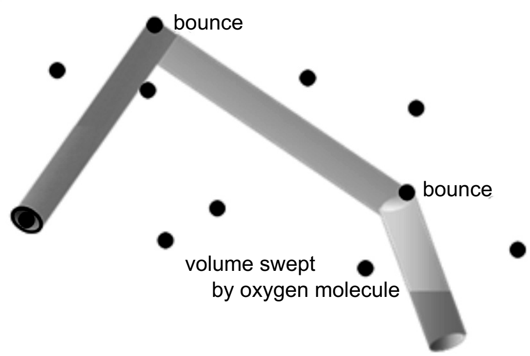




- If two spheres, Oxygen (O_2) and Nitrogen (N_2) of diameter 'd', the distance travelled by molecule O_2 before hitting molecule N_2 will be the average distance in gas before collision occurs, which shows the mean free path.
- Before collision, the molecule O_2 will move in straight line path.
- As molecules O_2 and N_2 being spheres of diameter d, if molecule N_2 center, lies inside diameter d of molecule O_2 center's path, there will be a collision.



- The molecule O_2 being sweeped out a volume of cylinder of radius 'd' which is centered on its path, which hits and deflects, when it encounters molecule N_2, which is centered in that cylinder.
- If the molecule is travelling at a distance 'x', the molecules will sweep out with volume of $\pi d^2 x$. Volume which is swept out will look like a pipe which is straight cylindrical sections having joints at collisions.
- The total volume of pipe will be $\pi d^2 L$, where L is total length or total distance which the molecule has travelled.
- If density of nitrogen molecule is n molecules/cubic meter, then number of N_2's molecules in the pipe volume will be, $\pi d^2 Ln$ which shows number of collisions, so average distance between collisions, is mean free path l, is mean free path l = total distance travelled/number of collisions, $= L/\pi d^2 Ln = 1/\pi d^2 n$
- The mean free path is inversely proportional to the probability that a molecule will collide with another molecule as it moves through the gas.



- As per the criteria, there is one molecule in collision tube which is swept out by molecular trajectory.

Factors affecting Mean Free Path

- **Density :** As gas density increases, molecules become closer to each other that decreases the mean free path.
- **Volume :** The mean free path decreases when there is increase in number of molecules or decrease in volume.
- **Radius of molecule :** Increasing the radii of molecules decreases the space which causes the molecule to run to each other that decreases the mean free path.
- Pressure, temperature and other factors which affect the density of molecules indirectly affects the mean free path.

Gaseous Mixture

- If two non-reactive gases are enclosed in a vessel having volume V, then in mixture, μ_1 moles of one gas are mixed with μ_2 moles of another gas. When N_A is Avogadro's number, then
- Number of molecules of first gas will be $N_1 = \mu_1 N_A$
- Number of molecules of second gas $N_2 = \mu_2 N_A$
- Total mole fraction is $\mu = (\mu_1 + \mu_2)$.

- If M_1 is molecular weight of first gas and M_2 is molecular weight of second gas, then molecular weight of mixture is $M = \dfrac{\mu_1 M_1 + \mu_2 M_2}{\mu_1 + \mu_2}$
- Specific heat of mixture at constant volume is :
- $$C_{V_{mix}} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2} = \frac{\mu_1\left(\frac{R}{\gamma_1 - 1}\right) + \mu_2\left(\frac{R}{\gamma_2 - 1}\right)}{\mu_1 + \mu_2} = \frac{R}{\mu_1 + \mu_2}\left[\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}\right]$$
- $$C_{V_{mix}} = \frac{R}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}\left[\frac{m_1}{M_1}\left(\frac{1}{\gamma_1 - 1}\right) + \frac{m_2}{M_2}\left(\frac{1}{\gamma_2 - 1}\right)\right]$$
- Specific heat of mixture at constant pressure is $C_{P_{mix}} = \dfrac{\mu_1 C_{P_1} + \mu_2 C_{P_2}}{\mu_1 + \mu_2}$

$$C_{P_{mix}} = \frac{\mu_1\left(\frac{\gamma_1}{\gamma_1 - 1}\right)R + \mu_2\left(\frac{\gamma_2}{\gamma_2 - 1}\right)R}{\mu_1 + \mu_2} = \frac{R}{\mu_1 + \mu_2}\left[\mu_1\left(\frac{\gamma_1}{\gamma_1 - 1}\right) + \mu_2\left(\frac{\gamma_2}{\gamma_2 - 1}\right)\right]$$

$$C_{P_{mix}} = \frac{R}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}\left[\frac{m_1}{M_1}\left(\frac{\gamma_1}{\gamma_1 - 1}\right) + \frac{m_2}{M_2}\left(\frac{\gamma_2}{\gamma_2 - 1}\right)\right]$$

- $$\gamma_{\text{mixture}} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{\frac{(\mu_1 C_{P_1} + \mu_2 C_{P_2})}{\mu_1 + \mu_2}}{\frac{(\mu_1 C_{V_1} + \mu_2 C_{V_2})}{\mu_1 + \mu_2}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2}}{\mu_1 C_{V_1} + \mu_2 C_{V_2}} = \frac{\left\{\mu_1\left(\frac{\gamma_1}{\gamma_1 - 1}\right)R + \mu_2\left(\frac{\gamma_2}{\gamma_2 - 1}\right)R\right\}}{\left\{\mu_1\left(\frac{R}{\gamma_1 - 1}\right) + \mu_2\left(\frac{R}{\gamma_2 - 1}\right)\right\}}$$
- $$\therefore \ \gamma_{\text{mixture}} = \frac{\frac{\mu_1\gamma_1}{\gamma_1 - 1} + \frac{\mu_2\gamma_2}{\gamma_2 - 1}}{\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}} = \frac{\mu_1\gamma_1(\gamma_2 - 1) + \mu_2\gamma_2(\gamma_1 - 1)}{\mu_1(\gamma_2 - 1) + \mu_2(\gamma_1 - 1)}$$

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Chapter 10

Oscillations and Waves

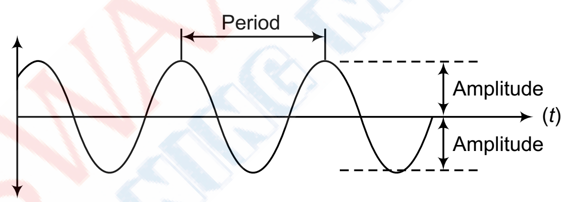
Topic 1

Oscillations

» Revision Notes

Periodic Motion

- Periodic Motion is a motion that repeats it path (or phase) in equal intervals of time.
- A function $f(t)$ is said to be periodic function of t, if there exists a positive real number T such that $f(t+T) = f(t)$ where smallest value of T is period of the function.
- The function is said to be periodic if its graph repeats itself after a fixed interval and the width of that interval is called its period.



- The function $f(t) = A \sin \omega t$ repeats itself after an interval of 2π, so $f(t) = A \sin \omega t$ is periodic with period 2π.
- The amplitude of sinusoidal functions is the maximum displacement from $y = 0$ and is clearly 1. A sinusoidal $y = A \sin nt$ has maximum value or amplitude A where n is usually a positive integer as $y = \sin 2t$ is sinusoidal of amplitude 1 and period $2\pi/2 = \pi$ as the period is π follows due to $\sin 2(t + \pi) = \sin(2t + 2\pi) = \sin 2t$ for any value of t.

Forces that cause periodic motion

- Periodic motion of a body about constant point on linear path under influence of force which acts towards the fixed point that is proportional to displacement of body from fixed point is known as simple harmonic motion.
- As observed, a body that performs simple harmonic motion is Simple Harmonic Oscillator where
  - particle oscillates on straight line
  - acceleration of particle is directed towards a fixed point on straight line
  - magnitude of acceleration is proportional to displacement of particle from fixed point
- There are many situations where external force makes an object to be in periodic motion which includes:
  - circular motion
  - back-and-forth motion

Circular or orbital motion

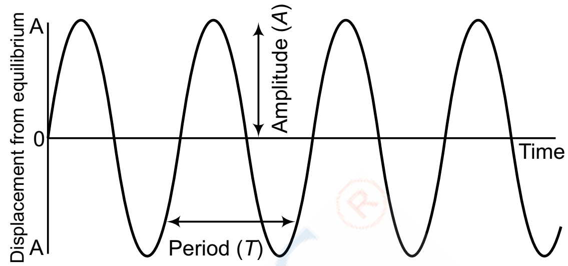
- If an object which is tied with a rope happens to swing, then that object is in periodic motion.
- The force which is preventing an object from flying out in straight line is the force which is being applied to the string.

Back-and-forth motion

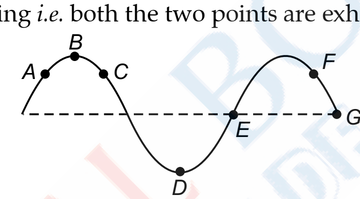
- The objects which cause back-and-forth motion will include :

- bouncing ball
- pendulum
- spring

Characteristics



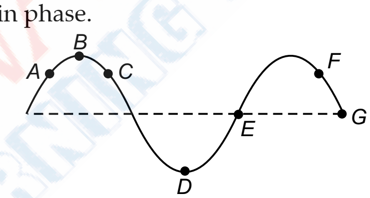
- Characteristics of periodic motion are the velocity of an object, period of motion and amplitude of motion.
- **Amplitude – A :** It is the maximum displacement either side from the mean position of the object.
- **Period – T :** It is the time to complete one cycle of motion, peak to peak or valley to valley.
- **Frequency – f :** It is the number of cycles completed in one time period, given as $f = 1/T$. It is measured in Hertz or per cycle.
- **Angular frequency – ω :** It is the rate of angular displacement in unit time. It is given as $\omega = \frac{d\theta}{dt} = 2\pi f$, where ω is angular velocity.
- **Phase :** Phase of a wave describes the state of motion as the wave sweeps through an element at a particular position.
- **In Phase** : Two points are said to be in-phase with each other when these two points are at the same position and they both are doing the same thing *i.e.* both the two points are exhibiting the same behaviour.



Points C and F are in phase with each other.

Out-of-phase

- Two points are said to be out of phase even though they are at the same points buy they are doing opposite thing i.e. both the points are exhibiting the different behaviour.
- Out of phase means which is not in phase.



Points B and D, E and G are out of phase by 180°

- **Completely out of phase :** In this, the waves are π radians or 180° apart where resulting amplitude is zero.

Period as a function of time

- The period of the rotation of the earth about its axis is one day, which is approx. 24 hours.
- The period T of the oscillating particle is time required to go through 1 cycle of its motion, which is the value of displacement at time when it equals the value of $x(t)$ at time $t + T$.
- Period T is the one cycle of its motion that shows change in phase from Φ to $2\pi + \Phi$,

$x(t) = a \times \cos(\omega \times t + \phi + 2\pi) - a \times \cos(\omega \times (t + T) + \phi)$

or $\omega t + \phi + 2\pi = \omega t + \phi + \omega T$

$\therefore \quad T = \frac{2\pi}{\omega}$

Frequency as a function of time

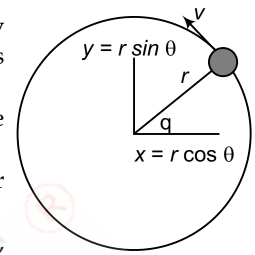
- A unit for frequency is one cycle per second which is defined as one Hertz (Hz) 1 Hz = 1 cycle/s.
- If the periodic motion occurs f times per second, then the time for one cycle is $1/f$, so $T = 1/f$ or $f = 1/T$.
- If an object executes periodic motion along x and y axes and if motion of an object occurs along x-axis, then the position of the object is given by its position function $x(t)$, or if motion is periodic, it shows $x(t + T) = x(t)$, where T is the period of the motion.
- The function $x(t)$ has any shape as long as $x(t) = x(t + T)$.
- In some cases, $x(t)$ has simple oscillating shape, that of a pure sinusoidal form.

Displacement as a function of time

- Displacement is a physical quantity that changes with time.

- Displacement is directly proportional to time and velocity.
- In general, displacement is directly proportional to time but proportional to the square of time when acceleration is constant.

Simple harmonic motion (S.H.M) and its equation



- An object undergoes a simple harmonic motion if acceleration of an object is directly proportional to its displacement from its mean position or acceleration is always directed towards the mean position.
- If an object moving in a uniform circular motion, position of an object in x and y plane are particular $x = r \times \cos\theta$ and $y = r \times \sin\theta$
- In uniform circular motion the angular velocity is constant and related to angular displacement as $\theta = \omega t$.
- An object experiences a simple harmonic motion while travelling in one dimension, given as $x = A \times \cos\omega t$, where A is amplitude of motion.
- Amplitude A is maximum at extreme position.
- The relation between angular velocity (ω), frequency (f) and time period (T) is given as $\omega = 2\times\pi\times f = 2\times\frac{\pi}{T}$.

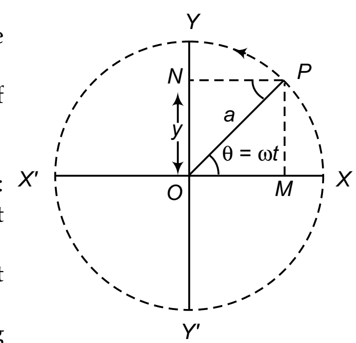
Velocity in SHM

- In simple harmonic motion, velocity keeps on changing.
- When displacement is maximum, the velocity zero and when displacement is zero, then velocity is maximum which is given as $v = -A \times \omega \sin\omega t$, where $A\omega$ is maximum speed.

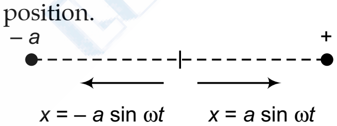
Acceleration in SHM

- In simple harmonic motion it is observed that acceleration also varies.
- If a mass is kept on a spring and when displacement is zero, acceleration will also be zero as spring exerts no force.
- If displacement is maximum, acceleration is also maximum as spring applies maximum force in the opposite direction of displacement.
- The acceleration is given as $y = -A \times \omega^2 \cos\omega t$, where $A\omega^2$ is maximum acceleration.
- The equation for acceleration is similar to equation for displacement as $y = -\omega^2 x$

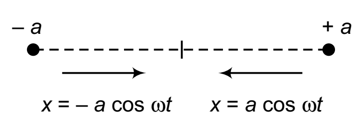
Displacement in S.H.M.



- In Simple Harmonic Motion, displacement of particle at an instant is the distance of particle from the mean position at that instant.
- It is the projection of uniform circular motion on any diameter of circle of reference.
- If projection is taken on y-axis, then
  $y = a \times \sin\omega t = a\sin(2\pi/T)\times t = a\sin 2\times\pi\times f\times t = a\sin(\omega t \pm \phi)$ where :
  - $y = a\sin\omega t$ then time is noted from instant when vibrating particle is at mean position.
  - $y = a\cos\omega t$ then time is noted from instant when vibrating particle is at extreme position.
  - $y = a\sin(\omega t \pm \phi)$ when vibrating particle is in phase leading or lagging from mean position.

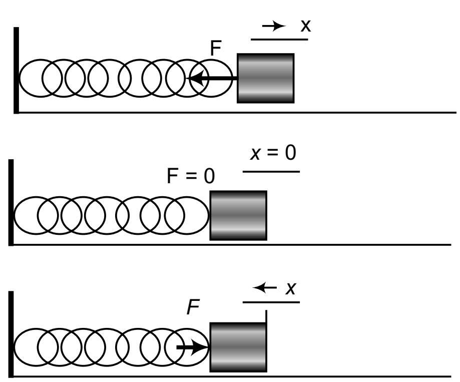


(A)



(B)

- If projection of P is on X-axis then equations of S.H.M. will be:
  - $a\cos(\omega t \pm \phi)$
  - $a\cos(2\pi/T \times t \pm \phi)$
  - $a\cos(2\pi f t \pm \phi)$
- Direction of displacement is always away from the mean position, particle either is moving away from or is coming towards the mean position.



Oscillations of a spring-restoring force and force constant

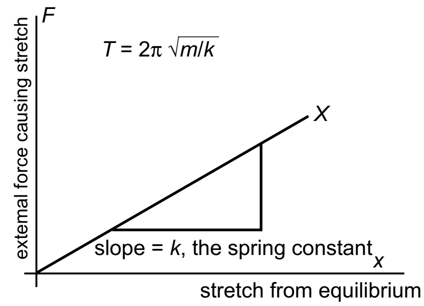
- In simple harmonic motion, oscillation frequency depends on the restoring force.
- The net force acting will be

$$F = -kx - kA\cos\omega t = ma = m(-a\omega^2\cos\omega t)$$

- The relationship between angular velocity, spring constant, and mass is $\omega^2 = k/m$

- The restoring force exerted by the spring is $F = -kx$,
- The acceleration on Simple Harmonic Oscillator (SHO) is

$$F_{net} = -k\,x = ma \quad \text{or} \quad a = -(k/m)x.$$



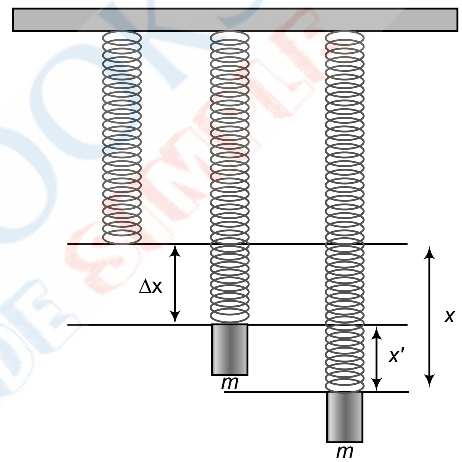
- In simple harmonic motion, the period remains the same even for motions with quite different amplitudes.
- It is often easy to set a vertical spring with a hanging mass, where force of gravity can change with respect to period of simple harmonic oscillator.
- If we have a vertical spring on which a mass m is hung; it will stretch a distance Δx due to the weight given as $\Delta x = mg/k$, where k is the spring constant of the spring.
- If the mass is pulling down, an extra distance x' is shown and the spring will exerting a force

$$F_{spring} = -k\,x = -k\,(x' + \Delta x) = -k\,x' - k\,\Delta x = -k\,x' - mg$$

- On adding the force of gravity,

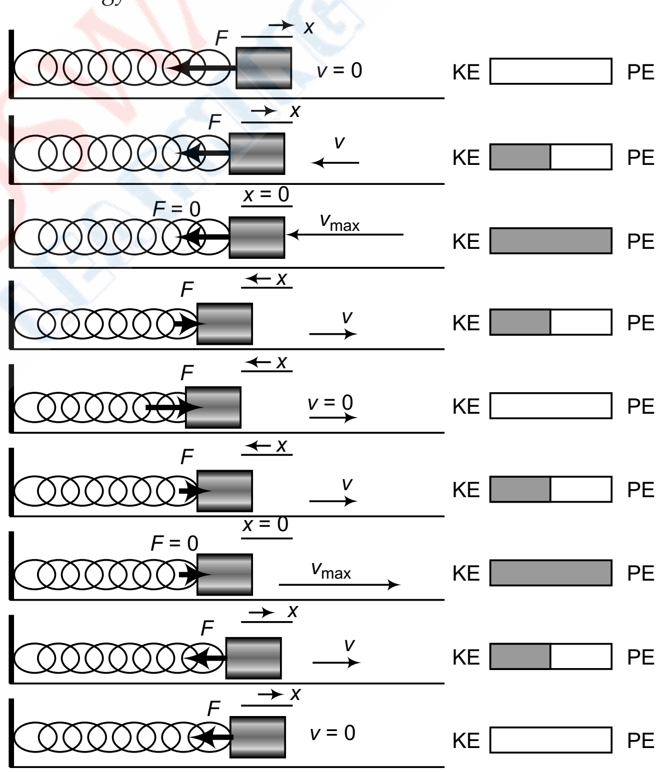
$$F_{net} = F_{spring} + mg = -k\,x' - mg + mg = -k\,x'$$

- It is similar to prototypical equation with displacement x' which can be measured from new equilibrium position.



Energy in S.H.M

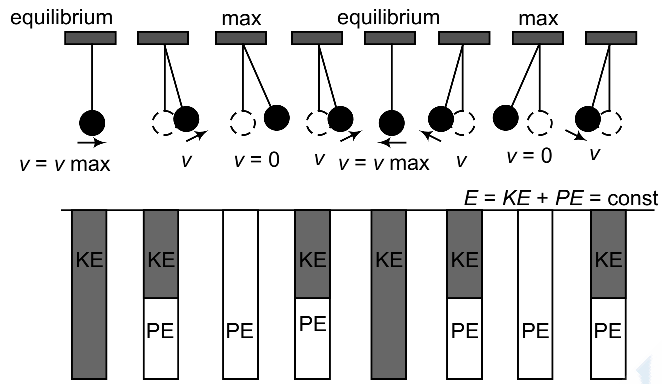
- The energy of a simple harmonic oscillator changes from one kind to another but the total energy remains the same.



$$E_{tot} = KE + PE = \text{constant}$$

$$E_{tot} = (1/2)\,mv^2 + (1/2)\,kx^2 = \text{constant}$$

- This is true of all simple harmonic oscillators. The example of a simple pendulum is shown here :



Kinetic and potential energies

- The equation of potential and kinetic energy of simple harmonic motion with function of time,

$$U = (½)\, kx^2 = (½)\, kA^2 \cos^2 (\omega t + \phi)$$

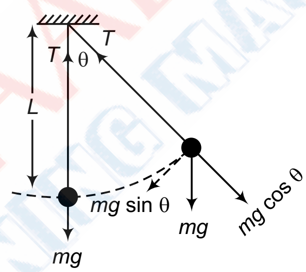
and $K = (½)\, mv^2 = (½)\, m\omega^2 A^2 \sin^2 (\omega t + \phi) = (½)\, kA^2 \sin^2 (\omega t + \phi)$ as $k = m\omega^2$

- $E = K + U = (½)\, kA^2 (\cos^2 (\omega t + \phi) + \sin^2 (\omega t + \phi)) = (½)\, kA^2$
- Thus total energy is constant and is equal to $(1/2)kA^2$.
- The equation expressing velocity as a function of position, then $v(x) = \sqrt{2/m\,(E - (1/2)KA^2)}$ where $A = \sqrt{2E/k}$,

  It shows that the amplitude of motion total energy is known if.

Simple pendulum: Derivation of expression for its time period

- A simple pendulum is a pendulum which consists of a heavy point mass suspended from a rigid support by means of an elastic inextensible string.



- Gravity provides the restoring force $\tau = -mgL \sin\theta$, or $\Sigma\tau = -mgL\sin\theta = I\alpha$, where L is length of pendulum.
- For small angular displacements, $\sin\theta \approx \theta$, so restoring equation will be $-mgL\,\theta = I\alpha$
- If an acceleration is proportional to and in opposite direction of displacement, then the motion acts as simple harmonic. $\alpha = -\omega^2\theta$
- In case of simple pendulum with all mass, the same distance from the suspension point, results in moment of inertia $I = mL^2$
- The equation relating the angular acceleration to the angular displacement for a simple pendulum is $\alpha = -(g/L)\,\theta$.
- The angular frequency of simple harmonic motion of simple pendulum is $\omega = \sqrt{g/L}$, where frequency is independent of the mass of the pendulum.
- Since, $T = \dfrac{2\pi}{\omega}$; so, $T = 2\pi\sqrt{\dfrac{L}{g}}$

Free, forced and damped oscillations (qualitative ideas only)

- There are three main types of simple harmonic motions :

  (a) Free oscillations : It is a simple harmonic motion with a constant amplitude and period having no external influences.

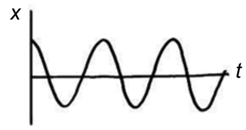
  (b) Damped oscillations : It is a simple harmonic motion but with a decreasing amplitude and varying period due to external or internal damping forces.

  (c) Forced oscillations : When object is forced to vibrate at a particular frequency by a periodic input of force.

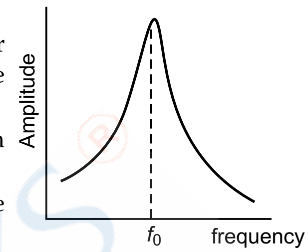
Free Vibrations

- F_1 oscillations where the total energy and amplitude of vibration stays the same over time.

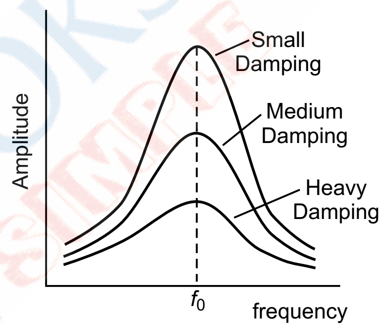
Free vibration



Forced vibrations:



- The vibrations that occur when an object is forced to vibrate at a particular frequency by a periodic input of force there are driven by external force are known as forced vibrations.
- Objects which are free to vibrate have one or more **natural frequency** at which they vibrate.
- If an object is being forced to vibrate at its natural frequency, **resonance** and large amplitude vibrations are observed.
- The resonant frequency is f_o (as shown in figure).
- The amplitude of the resonance peak decreases and the peak occurs at a lower frequency.

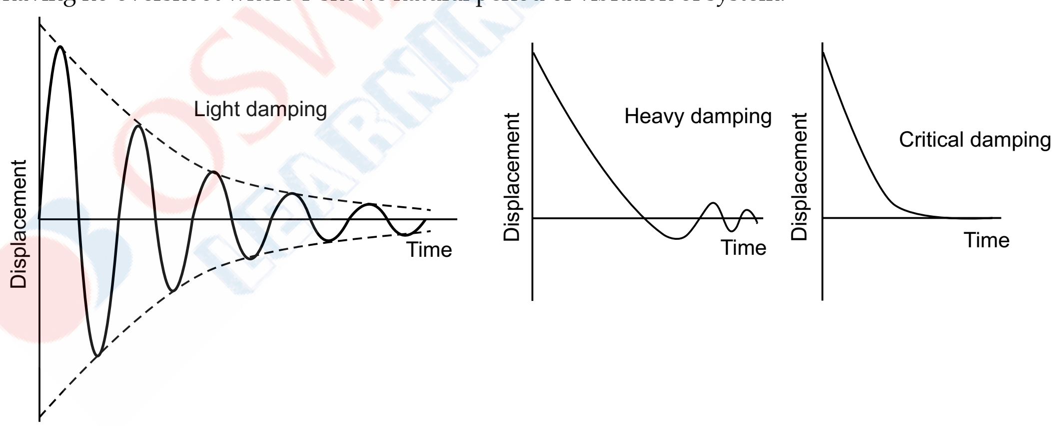


Damped oscillations:

- These are the oscillations that occur where energy is taken from the system where amplitude decays. It is of two types:

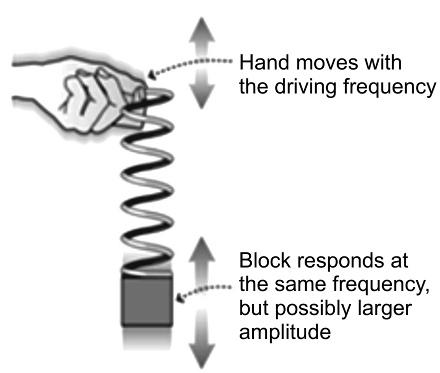
  (i) Natural damping:
  - internal forces in a spring,
  - fluids exerting a viscous drag.

  (ii) Artificial damping:
  - electromagnetic damping in galvanometers
  - coating of panels in cars to reduce vibrations
  - shock absorbers in cars
  - interference damping - gun mountings on ships
- It is observed that artificial damping are light, where the system oscillates about the midpoint as shown while heavy damping occurs where system takes a long time to reach equilibrium.
- Also, the critical damping is that where the system reaches equilibrium in shorter time as compared with T having no overshoot where T shows natural period of vibration of system.



Resonance

- If a vertical mass on a spring uses driving force (F) to move the support of spring up and down, then equation of motion of the mass is $m\,(d^2x/dt^2) = F_R - F_f + F$
- If a driving force itself oscillates, then $m\,(d^2x/dt^2) + b\,(dx/dt) + (kx) =$ $F_o \cos \omega_D t$, where ω_D is angular frequency of driving force, so the equation will be of the form $x = B \sin (\omega_D + \phi)$
  where amplitude of oscillations B, depends on parameters of the motion $\omega, \omega_D, b, m, F_o$, so $B = (F_o / m) / [(\omega^2 - \omega^2{}_D)^2 + b^2\omega^2{}_D / m^2]^{1/2}$
- The amplitude, B, has a maximum value when $\omega = \omega_D$ known as ***resonance*** which at a point B becomes extremely large if b is small.

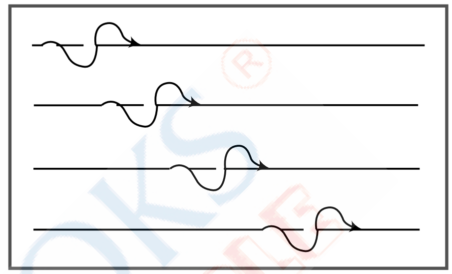


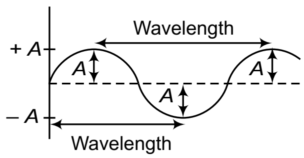
Topic 2
Waves

» Revision Notes

Wave motion

- A wave is a self-propagating disturbance in a medium.
- Waves carry energy, momentum, information, but not matter.
- Electromagnetic wave is the only kind of wave which does not require a medium.





- EM waves can travel in vacuum caused by electromagnetic field which generates itself as it propagates.
- Such wave rolls out it own area creating its own medium as it moves forward.
- In this, a signal is sent without sending any matter down the line by a disturbance in a medium without any movements.
- **Amplitude** is the maximum displacement either side from equilibrium.
- **Wavelength** is the distance moved by a wave in one cycle. where velocity = frequency × wavelength

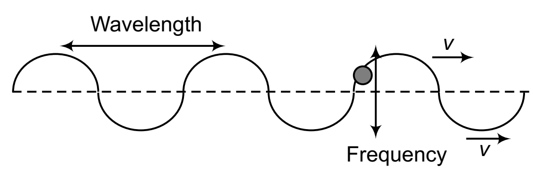
(Here, all aplhabets are in their usual meanings) $v = f \times \lambda$

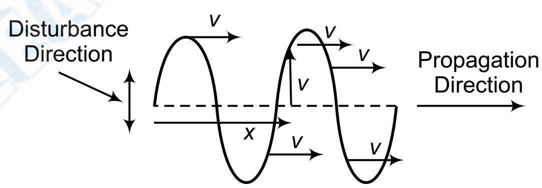
Examples:

- Sound waves (pressure waves) in air (or in any gas or solid or liquid).
- Waves on a stretched string.
- Waves on the surface of water.

Transverse and longitudinal waves

- In general, there are two kinds of waves :
  - Transverse wave
  - Longitudinal wave
- For **transverse waves**, the individual disturbance is perpendicular to the direction of the wave motion as a whole.

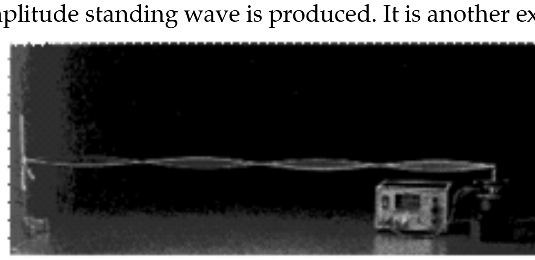




- In transverse wave, particle displacement is perpendicular to the direction of wave propagation where particles do not move along with the wave, but simply oscillate up and down about their individual equilibrium positions as the wave passes through them.

Transverse Standing Waves

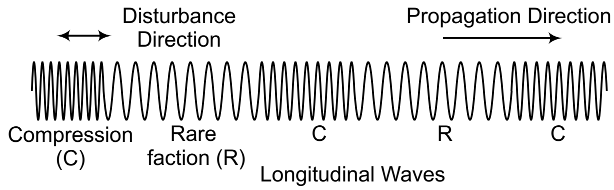
- If we twist the end of a string at the right frequency, the waves along the string reinforce the waves being reflected and a large-amplitude standing wave is produced. It is another example of resonance.



- In case of standing waves on a string, the ends are fixed so those are both nodes.
- To get standing waves, the whole number of loops between those ends needs to be fixed, so

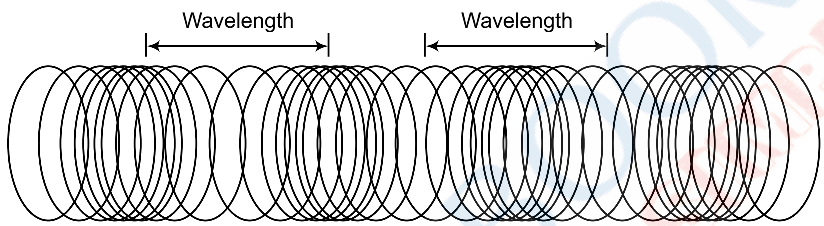
$$\text{Length} = (\text{whole number}) \times (\text{half wavelength}),$$
$$L = n(\lambda/2)$$

- For longitudinal waves, the individual disturbance is along the same direction as the wave motion as a whole.



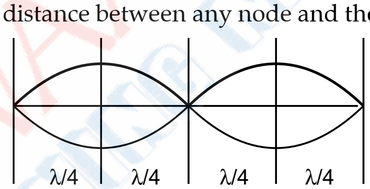
Longitudinal Waves

- In a longitudinal wave, the particle displacement is parallel to the direction of wave propagation.
- The particles do not move down the tube with the wave; they simply oscillate back and forth about their individual equilibrium positions.



Standing Waves (Longitudinal)

- Longitudinal waves support standing waves.
- It is a long, straight tube or pipe shaped wave.
- The standing waves are set up in every horn or woodwind instrument when a sound is played.
  - Wavelength may also be defined as the distance between a peak to the next, or the distance between a troughs to the next where the distance between any node and the anti-node next to it is $\lambda/4$.



The distance between a node and the antinode next to it is $\lambda/4$

- **Sound Waves**
  - Energy to which human ears are sensitive is called as sound.
  - All types of waves are produced in an elastic medium, irrespective of frequencies.
  - As per frequencies, waves are divided into:
- **Audible or sound waves :** These waves have range from 20 Hz to 20 kHz and are produced by vibrating bodies like vocal cords, stretched strings or membrane.
- **Infrasonic waves :** These waves have frequencies which lie below 20 Hz Example waves produced at the time of earth quake etc.
- **Ultrasonic waves :** These waves have frequencies more than 20 kHz. Human ear cannot detect such waves. As the velocity of sound in air will be 332 m/sec, hence wavelength for ultrasonics waves is less than 1.66 cm and infrasonics waves as more than 1.66 cm but less than 16.6 m.

Mach number

- It is the ratio of velocity of source to the velocity of sound.
- Mach Number = Velocity of source/Velocity of sound

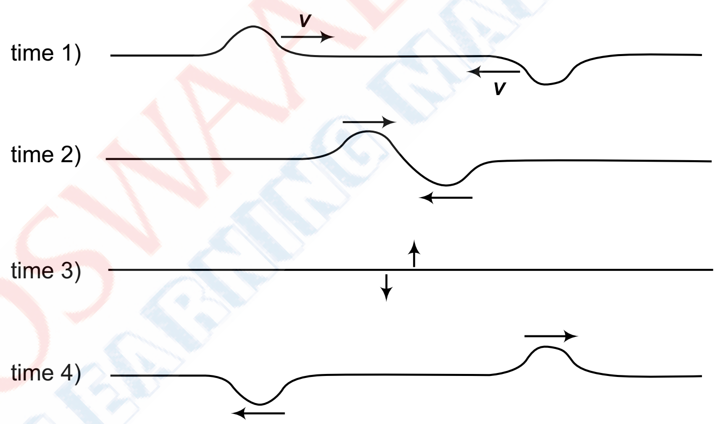
Speed of wave motion

- The wave speed is the distance a wave travels per unit time.
- A wave source with frequency 'f' moves with velocity v so $v = f\lambda$
- It is observed that wave speed v is a constant, independent of f and T.
- The wave speed v depends on the properties of the medium, not on the properties of the wave.

Examples:

- medium = string, properties = tension, mass per length
- medium = air, properties = temperature, mass per molecule, etc

- **Displacement Relation for Progressive Wave**
- **Progressive Wave**
  - Progressive waves are the waves that propagate in forward direction of medium with a finite velocity.
  - In this, energy and momentum are transmitted in the direction of propagation of waves without actual transmission of matter.
  - Here equal changes in pressure and density occur at all points of medium.
  - There are different forms of progressive wave function where displacement y is given as:
  - $y = A \sin(\omega t - kx)$
  - $y = A \sin(\omega t - 2\pi/\lambda x)$
  - $y = A \sin 2\pi [t/T - x/\lambda]$
  - $y = A \sin 2\pi (vt - x)$
  - $y = A \sin \omega (t - x/v)$
- **Remember**
  - Negative sign between t and x shows wave propagation along positive X-axis
  - Positive sign shows wave movement in negative X-axis direction
  - Coefficient of:
    - sin or cos functions shows $\omega t - kx$ in phase
    - t shows angular frequency $\omega = 2\pi n = 2\pi/T = vk$
    - x shows propagation constant or wave number $k = 2\pi/\lambda = \omega/v$
    - ratios of t to that of x shows wave or phase velocity $v = \omega/k$
- **Principle of superposition of waves**
  - If two or more waves are present in the same place, at the same time, the displacement of the total wave is the sum of the displacement of the individual wave $y_{tot}(x,t) = y_1(x,t) + y_2(x,t) \dots + y_n(x, t)$.
  - In such case, we have constructive or destructive interference depending on whether y_1 and y_2 add or subtract.



- **Superposition**
  - Two objects cannot occupy the same space at the same time, however, waves behave quite differently which passes through each other and continues on.
- **Interference of Waves**
  - When two waves of same frequency, wavelength and velocity move in same direction, on superposition, they results in interference.
  - Due to interference, resultant intensity of sound at that point is different from sum of intensities due to act of every wave.
  - The modification of intensity due to superposition of two or more waves is known as interference.
  - If at a point, two waves arrive with phase difference ϕ and equation of waves results as $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin(\omega t+\phi)$, then by superposition, $y = y_1 + y_2$ or $y = A \sin(\omega t+\theta)$ where $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$ or $\tan\theta = a_2 \sin\phi/(a_1+a_2 \cos\phi)$
  - As intensity $\propto A^2$ so $I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$ or $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$
  - **Constructive interference:**
    - Intensity is maximum when $\phi = 0, 2\pi, 4\pi, \dots 2\pi n$, where $n = 0,1,2 \dots$ or $x = 0, \lambda, 2\lambda, \dots n\lambda$, where $n = 0, 1 \dots$

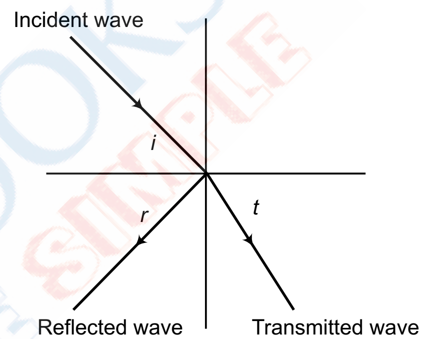
- $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2 \propto (A_1 + A_2)^2$
  - The intensity is maximum at points where path difference is integral multiple of wavelength λ.
  - The points of constructive interference called maxima.
- **Destructive interference:**
  - Intensity is minimum when $\phi = \pi, 3\pi, 5\pi, ...(2n-1)\pi$, where $n = 1,2,3$ ....or $x = \lambda/2, 3\lambda/2, ...(2n-1)\lambda/2$, where $n = 0, 1$ ....
- $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2 \propto (A_1 \sim A_2)^2$

Reflection and Refraction of waves

- It is observed that when waves are incident on boundary of two media, then a part of incident waves is returned back to initial medium which is known as reflection while remaining part of the waves gets absorbed and partly transmitted into other medium which is known as refraction.

Reflection

- Reflection is bouncing back of waves where a ray is incident on reflecting plane surface is reflected at the same angle from the surface.

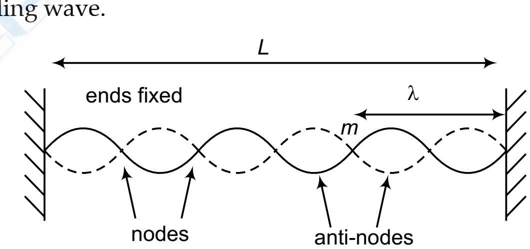


Refraction

- The speed of a wave changes when it changes the medium that it is travelling in.
- The wave is incident on the interface of two media at an angle and such speeding up or slowing down results the wave to change direction.
- **In case of reflection and refraction of sound :**
  - Frequency of wave remains unchanged where $\omega_i = \omega_r = \omega_t = \omega =$ constant
  - Incident ray, reflected ray, normal and refracted ray all lie in same plane
  - In case of reflection, angle of incidence (i) = angle of reflection (r)
  - In case of refraction, $\sin(i)/\sin(r) = v_i/v_t$
  - In reflection from denser medium, the phase changes by 180° while its direction gets reversed. If incident wave is $y = A_1 \sin(\omega t - kx)$, on reflection it becomes $y = A_r \sin(\omega t + kx + \pi) = -A_r \sin(\omega t + kx)$
  - In reflection from rarer medium, phase does not change and direction of wave gets reversed if incident wave is $y = A_I \sin(\omega t - kx)$ which on reflection becomes $y = A_r \sin(\omega t + kx)$
  - If there is a sound reflector at a distance 'd' from the source then time interval between original sound and its echo at the site of source will be $t = 2d/v$

Standing wave

- Two progressive waves having same amplitude and time period/frequency/wavelength travelling with similar speed along the same straight line in opposite directions superimpose forming another wave known as stationary wave or standing wave.



- If the ends of a string of length L are fixed, then standing waves are possible at certain resonant frequencies as $L = n \times \lambda/2$

Strings

- If a string under tension vibrates then, transverse harmonic waves tend to propagate along the length, while if a length of a string is fixed, there occur reflected waves.
- The incident and reflected waves superimpose so as to generate transverse stationary waves in a string.
- A single frequency sound wave irritates many people and a pleasant musical note is a mixture of frequencies which is equal to lowest or fundamental frequency + integer multiples of fundamentals where:
- f_1 = fundamental = 1^{st} harmonic
- $f_2 = 2 \cdot f_1$ = 1^{st} overtone = 2^{nd} harmonic
- $f_3 = 3 \cdot f_1$ = 2^{nd} overtone = 3^{rd} harmonic

- $f_n = n \cdot f_1 = (n-1)$ overtone $= n^{th}$ harmonic
- If a string of length L has its ends fixed, as in a stringed instrument, then only standing waves at certain resonant frequencies are possible :

$$L = \frac{\lambda}{2}, \quad \lambda = 2L$$

$$f = \frac{v}{\lambda} = \frac{v}{2L} = f_1 = \text{fundamental or } 1^{st} \text{ harmonic}$$

$$\lambda = L$$

$$f_2 = \frac{v}{\lambda} = \frac{v}{L} = 2f_1 = 2^{nd} \text{ harmonic}$$

$$L = 3\left(\frac{\lambda}{2}\right), \quad \lambda = \frac{2}{3}L$$

$$f = \frac{v}{\lambda} = 3\cdot\frac{v}{2L} = 3f_1 = 3^{rd} \text{ harmonic}$$

For the n^{th} harmonic (n = 1, 2, 3, ...),

$$L = n\cdot\frac{\lambda}{2}, \quad \lambda = \frac{2L}{n}, \quad f_n = \frac{v}{\lambda} = n\frac{v}{2L} = nf_1$$

- If the frequency of n^{th} harmonic is $n \times$ frequency of fundamental, then the wave speed v is constant by tension and mass of string which shows speed of wave $v = \sqrt{\frac{F_T}{m/L}}$
- If a guitar string is plucked, all of these resonant frequencies, or normal modes, are excited all at once, the listener hears all frequencies f_1, $f_2 = 2f_1$, $f_3 = 3f_1$, etc
- This mixture, gives the instrument a pleasing quality. The fundamental, or lowest frequency, establishes the "note".
- The fundamental $f_1 = v/(2L)$ can be adjusted (tuned) either by changing the tension in the string (changing v) or by holding string at a fret (changes L).

Organ pipes

- Organ pipes are musical instrument made for producing musical sound by blowing air into the pipe.
- In this, the longitudinal stationary waves are obtained by superimposition of incident and reflected longitudinal waves.
- If a standing wave $y = 2a \cos 2\pi vt/\lambda \times \sin 2\pi x/\lambda$ where $\lambda = 4L/(2n-1)$, the first normal mode of vibration is $n_1 = v/4L$ which is fundamental frequency or first harmonic while second normal mode of vibration $n_3 = v/\lambda_2 = 3v/4L = 3n_1$ known as third harmonic or first overtone.
- The third normal mode of vibration $n_3 = 5v/4L = 5n_1$ known as fifth harmonic or second overtone.

Nodes Position:

$$x = 0, 2L/(2n-1), 4L/(2n-1), 6L/(2n-1) \;..... \; 2nL/(2n-1)$$

- First mode of vibration, $x = 0$
- Second mode of vibration $x = 0$, $x = 2L/3$
- Third mode of vibration $x = 0$, $x = 2L/5, 4L/5$

Antinode Position: $x = L$

$$x = L/2n{-}1, 3L/2n{-}1, 5L/2n{-}1 .....L$$

- First mode of vibration, $x = L$
- Second mode of vibration $x = L/3$, $x = L$
- Third mode of vibration $x = L, L/5, 3L/5$

Parameters	Open organ pipe	Closed organ pipe
Fundamental frequency	$n_1 = v/2L$	$n_1 = v/4L$
Frequency of 1st overtone/ 2nd harmonic	$n_2 = 2n_1$	Not there
Frequency of 2nd overtone/ 3rd harmonic	$n_3 = 3n_1$	$n_3 = 3n_1$
Frequency ratio of overtones	2 : 3 : 4…	3 : 5 : 7…
Frequency ratio of harmonics	1 : 2 : 3 : 4…	1 : 3 : 5 : 7…
Wave Nature	Longitudinal stationary	Longitudinal stationary

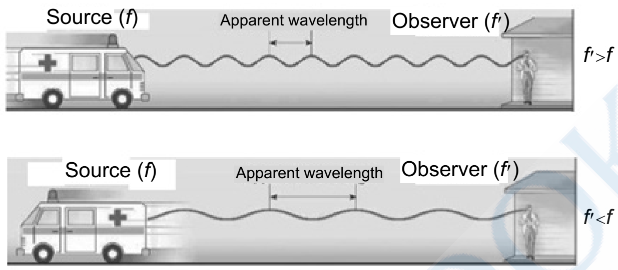
Beats

- It is observed that when two sound waves of different frequencies travelling in a medium along similar direction superimpose each other, then the intensity of resultant sound at a particular position rises and falls with respect to time.
- The mechanism with regular variation in intensity of sound with time at particular position is known as beats.
- A beat is produced when the intensity of sound is maximum at time $t = 0$.

- The beat period is the time interval among two successive beats.
- The frequency with which the number of beats are produced per second is known as beat frequency.
- In considering two sound moves of similar frequencies which on addition at same time, remain in phase.
- The sound waves tend to add constructively for a moment and next are out of phase and add destructively.
- The resulting sound wave increases in volume and further decrease in volume.
- The frequency of wavering soft and loud sound is equal to the difference in the two frequencies.

Doppler effect

- Doppler Effect is the apparent change in frequency or pitch when a sound source moves either toward or away from the observer/listener, or when the observer/listener moves either toward or away from the sound source.



- If the source is moving toward observer, the sound will be higher in pitch.
- If the source is moving away from observer, the sound will be lower in pitch.
- Let f be the frequency produced by source which is moving with velocity v_s. Further f' be the frequency observed by observer which is moving with velocity v_0, the Doppler Effect can be formulated as :

$$f' = f\left(\frac{v \pm v_0}{v \mp v_s}\right)$$; here, v in the velocity of sound

When both moves towards each other

$$f' = f\left(\frac{v + v_0}{v - v_s}\right)$$

$v_0 = 0$: If observer is stationary

$$f' = f\left(\frac{v}{v - v_s}\right)$$

$v_s = 0$: If source is stationary

$$f' = f\left(\frac{v + v_0}{v}\right)$$

When both moves away to each other

$$f' = f\left(\frac{v - v_0}{v + v_s}\right)$$

$v_0 = 0$: If observer is stationary

$$f' = f\left(\frac{v}{v + v_s}\right)$$

$v_s = 0$: If source is stationary

$$f' = f\left(\frac{v - v_0}{v}\right)$$

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Chapter 11

Electrostatics

Topic 1

Electric Charges and Fields

» Revision Notes

Electric Charges :

- The charge is a property of matter that causes it to experience a force when placed in an electromagnetic field.
- The charges of particles are basically the integer multiples of charge *e*; thereby making it electrically quantized.
- The SI unit of electric charge is Coulomb (C).
- Atoms are composed of electrons and protons and neutrons.
- The smallest amount of charge that exists is carried by an electron and a proton.
- Charge in an electron is $q_e = -1.602 \times 10^{-19}$ C and in proton is $q_p = 1.602 \times 10^{-19}$ C
- Charges are of two types : positive and negative.
- To charge an object means to transfer electrons from one object to another.

Conservation of Charge :

- Law of conservation of charge shows that electric charge can neither be created nor destroyed.
- In a closed system, the amount of charge remains the same.
- If two objects in an isolated system have net charge zero, and one object exchanges electrons with other, then the object with excess electrons will be negatively charged while the object with lower number of electrons will have positive charge of similar magnitude.
- If $Q(t)$ is quantity of electric charge in specific volume at time t and $Q(in)$ is the amount of charge flowing in volume between time t_1 and t_2 with $Q(out)$ as amount of charge flowing out of the volume at the same time period, then as per charge conservation: $Q(t_2) = Q(t_1) + Q(in) - Q(out)$

Coulomb's Law : Force between two Point Charges :

- Coulomb's law shows the relationship of force between two charged particles represented as

$$\vec{F} = (kq_a\, q_b / r^2_{ab}) \times \vec{r}_{ab} \text{ where } k = \text{constant} = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$$

- The charge on electrons and protons are equal and opposite, *i.e.*, $e = 1.602 \times 10^{-19}$ C.
- The values of k and G are $k = 8.99 \times 10^9$ Nm^2/C^2 and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

Forces between multiple Charges :

- In a system, with 'n' number of charges $q_1, q_2, q_3 \ldots q_n$ force on charge q_1 due to other charges $q_2, q_3 \ldots\ldots q_n$ can be calculated using Coulomb's Law.
- If force on charge q_1 due to charge q_2 is denoted as F_{12} then,

$$\vec{F}_{12} = (1/4\pi\varepsilon_0) \times (q_1q_2/r^2_{12}) \times \hat{r}_{12}$$

- The force on charge q_1 due to charge q_3, denoted by $\vec{F}_{13}$, is

$$\vec{F}_{13} = (1/4\pi\varepsilon_0) \times (q_1q_3/r^2_{13}) \times \hat{r}_{13}$$

- When there are three charges, a, b, and c, then the net force felt by charge a will be

$$\vec{F}_a = k\frac{q_a q_b}{r^3_{ab}} \times \vec{r}_{ab} + k\frac{q_a\, q_b}{r^3_{ac}} \times \vec{r}_{ac}$$

Superposition Principle :

- In principle of superposition, resultant force acting on any charge is equal to the vector sum of forces exerted by the other individual charges.

- If the resultant force acting on charge q_1 is vector sum of all the forces exerted on it by other charges, then $\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$
- At any point, total electric field due to a group of charges will be equal to the vector sum of electric fields of all charges.

$$\vec{E} = k \sum_{i=1}^{n} \frac{q_i}{r_i^2} \times \hat{r}_i$$

Continuous Charge Distribution

- Continuous charge distribution is a system in which the charge is uniformly distributed over the material.
- In this, all charges are closely held together with minor space among them.
- There are certain types of continuous charged distributions: Linear Charge Distribution, Surface Charge Distribution, Volume Distribution of charge

Linear Charge Density

- Linear Charge Density is denoted by λ which is measured in C/m.
- It can be mathematically written as $\lambda = dq / dl$
- If small element with length 'dl' is considered, then small amount of charge on this element will be $dq = \lambda \cdot dl$
- The small amount of force in this will be $dF = (q_o/4\pi\varepsilon_o)\, r^2 = dl \times q_o\, \lambda / 4\pi\, \varepsilon_o\, r^2$

Surface Charge Density

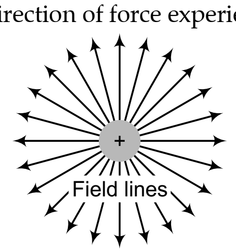
- Surface Charge Distribution Density is denoted by σ and is measured in C/m^2
- It can be mathematically written as $\sigma = dq / dS$
- Total force F on a small test charge q_0 is due to surface distribution of charge $F = \frac{1}{4\pi\varepsilon_0} \cdot q_0 \cdot \int \frac{\rho}{r^2}\, dS$

Volume Charge Density

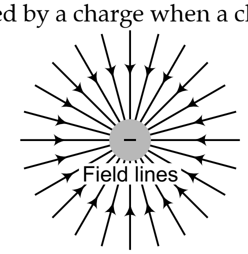
- Volume charge density is represent as ρ and is measured in C/m^3.
- It can be mathematically written as charge per unit volume, $\rho = dq / dV$ or $F = \frac{q_0}{4\pi\varepsilon_0} \cdot \int \frac{\sigma}{r^2}\, dV$

Electric Field

- Electric field is the region around a charged particle within which a force would be exerted on other charged particles.
- The electric field is a vector field with SI units as Newtons per coulomb N/C or volts per meter as (V/m).
- The strength or magnitude of field at given point is the force which is exerted on positive test charge of 1 coulomb placed at that point.
- Electric fields have electrical energy with energy density proportional to square of the field amplitude.
- The electric field which changes with time resulting due to motion of charged particles in the field, induced local magnetic field.
- As seen, an electric field can be visualized on paper by drawing lines of force, which shows the presence of size and strength of the field.
- Lines of force are field lines which start from positive charges and end at negative charges, and direction of field line at a point shows direction of force experienced by a charge when a charge is placed at a point.

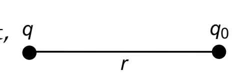


The electric field from an isolated positive charge



The electric field from an isolated negative charge

Electric Field due to a Point Charge



- A *point charge* is an electric charge regarded as concentrated in a mathematical point, without spatial extent.
- The electric field due the point charge (Q) is radial and decreases with increasing distance from the charge.
- The electric field strength of a point charge is $E = kQ/d^2$ where symbols have usual meanings.
- The resultant electric field arises due to many point charge is determined by using principle of superposition.

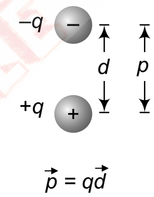
- If a point charge q in space sets an electric field, then a electrostatic force F acting on test charge q_0 at a point which is at distance r from source charge can be given as $F = kq\,q_0 / r^2$
- Electric field at a point is given by $E = \dfrac{F}{q_0}$ or $E = k\dfrac{q}{r^2}$

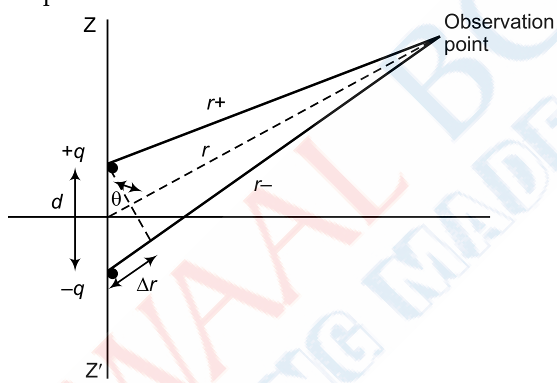
Electric Field Lines :

- Electric field is a set of lines showing direction of electric field.
- The field is strong at points where the field-lines are closely spaced and are weak at the points where field-lines are far apart.
  In this, magnitude of field is proportional to number of field-lines per unit area passing through small surface normal to the lines.
- The electric field-lines are linked with *positive* point charges radiates away from the charges.
- A tangent to the field-lines is always directed radially away from the charge.

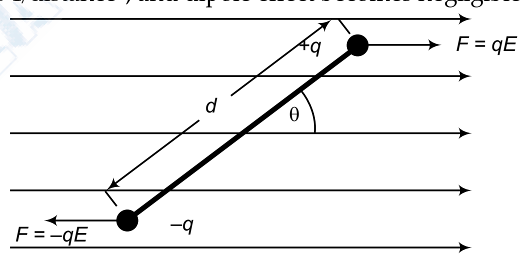
Electric Dipole :

- Electric dipole is a system which has a positive and negative charge of equal magnitude q, separated by a distance d.
- Electric dipole moment is a vector $\vec{p}$ with magnitude $p = qd$.
- The potential produced by electric dipole is calculated by summing the potential of the two point charges which produce it.





- If negative charge is placed on z-axis at point $z = -d/2$ and positive charge is placed on z-axis at point $z = +d/2$ then, for large r: $V(r) = kqd \cos\theta/r^2 = kp\cos\theta/r^2$
- In equipotential lines and field lines of an electric dipole, dipole potential does not have spherical symmetry and decreases as 1/distance^2, which is faster than Coulomb potential which decreases as 1/distance.
- The dipole field decreases as 1/distance^3, and dipole effect becomes negligible as distance increases.

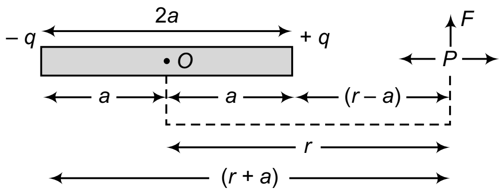


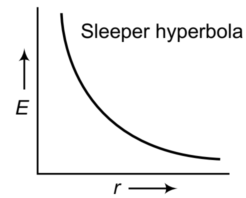
- In uniform electric field, net force on an electric dipole is zero.
- If dipole is not aligned with electric field, then torque acting on dipole is $\tau = pE\sin\theta$.
- To rotate dipole away from alignment, external torque is applied to do work, which is stored as potential energy that gets converted into other forms of energy.
- The potential energy of a dipole in an external field is $PE_{dipole} = -pE\cos\theta$
- If field is not uniform, then magnitude of electric force acting on positive charge will be different from the force which acts on negative charge, and net force will act on the dipole.

Electric Field due to a Dipole :

On Axial on End-on position

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}r}{(r^2 - a^2)^2}$$



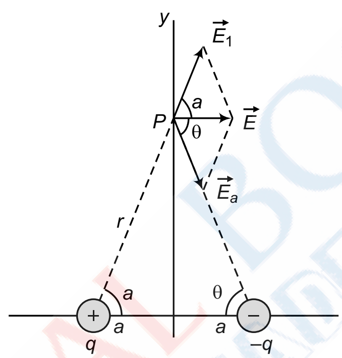


Since $r >> 2a$ $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$$

- **On Equatorial position**

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+a^2)^{3/2}}$$

since $r >> 2a$ $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$$



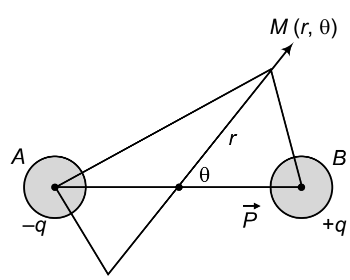
- **At any point M (r, θ)**

Radial component $E_R = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$

Transversal component $E_r = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$

Intensity of electric field $E = \sqrt{E_R^2 + E_r^2}$

or, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\sqrt{1+3\cos^2\theta}}{r^3}$

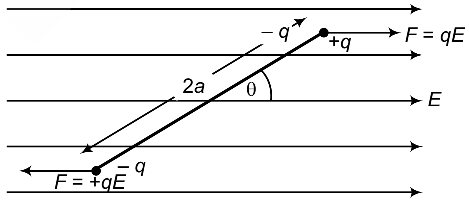


- **Torque on a dipole in uniform electric field**

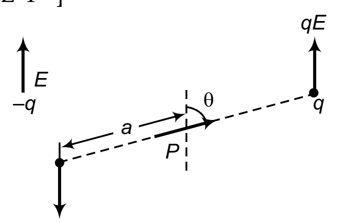
The moment of an electric dipole is defined as the cross product of electric dipole and intensity of electric field.

$\vec{\tau} = \vec{p}\times\vec{E}$ or, $\tau = pE\sin\theta$ here, $p = 2aq$

It is vector quantity. It SI unit is Nm and dimensional formula is $[ML^2T^{-2}]$



Dipole in uniform electric field



Dipole in non-uniform electric field

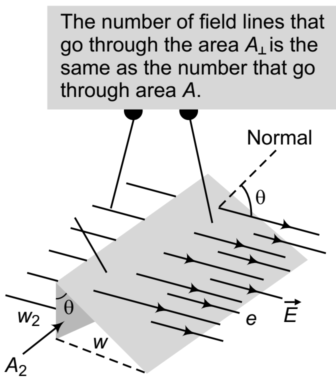
- **Electric flux**

It is defined as the dot product of intensity of electric field and surface of vector area.

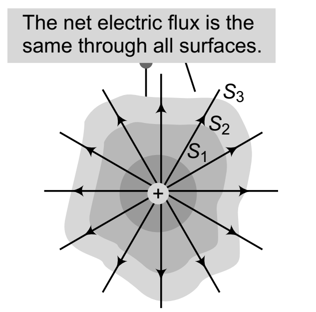
$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$$

- Here, θ is the angle between electric field lines and normal to surface of vector area.
- It is scalar quantity. It SI is $Nm^2\,C^{-1}$ and dimensional formula is $[ML^3T^{-3}A^{-1}]$

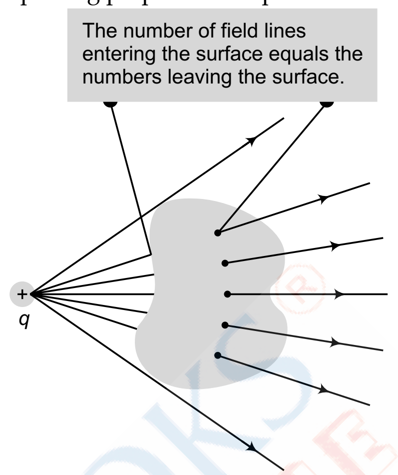
In physical significance, it is defined as the number of electric field lines passing perpendicular per unit area.



Field lines representing a uniform electric field penetrating an area A



Closed surfaces of various shapes surrounding a positive charge

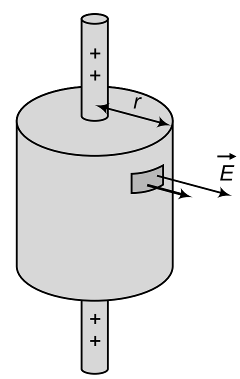


A point charge located outside a closed surface

Gauss's Theorem [*Karl Friedrich Gauss* (1777-1855)]

The net flux through and closed surface surrounding a point charge Q is given by $\frac{Q}{\varepsilon_0}$ and is independent of the shape of that surface.

Mathematically, $\Phi = \int_{surface} \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$



Applications of Gauss's Theorem

(a) Field due to infinitely long straight wire

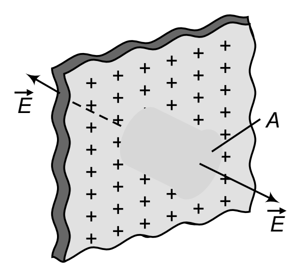
$$\Phi = \int_{surface} \vec{E} \cdot d\vec{A} = \frac{E}{\varepsilon_0}\int_{surface} dA = E\,(2\pi r l) = \frac{Q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

or, $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\lambda}{r}$$

Here, λ is uniform charge per unit length (line charge density)

- If $\lambda > 0$, the direction of field will be outwards
- If $\lambda < 0$, the direction of field will be inwards

(b) Field due to uniformly charged infinite plane sheet



$$\Phi = \int_{surface} \vec{E} \cdot d\vec{A} = E\int_{surface} dA = E(2A) = \frac{Q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

or, $$E = \frac{\sigma}{2\varepsilon_0}$$

Here, σ is the uniform charge per unit area (surface density)

- If $\sigma > 0$, the direction of field will be outwards
- If $\sigma < 0$, the direction of field will be inwards

(c) Field due to uniformly charged thin spherical shell (field inside and outside)

Consider a thin spherical shell of radius 'R' be charged by charge 'Q'. Let σ be uniform surface charge density.

Surface charge $\sigma = \frac{Q}{A} = \frac{Q}{4\pi R^2}$

Consider a Gaussian spherical surface of radius 'r' from the center of given spherical shell.

The electric field at each point of the Gaussian surface (being of spherical symmetry) has the same magnitude E and is along the radius vector at each point.

Apply Gauss Law

Electric Flux $= \int \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$

or, $$E \times (4\pi r^2) = \frac{Q}{\varepsilon_0}$$

or, $$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

or, $$E = \frac{\sigma R^2}{\varepsilon_0 r^2} \quad \text{as } Q = (4\pi R^2)\,\sigma \qquad \ldots\text{(i)}$$

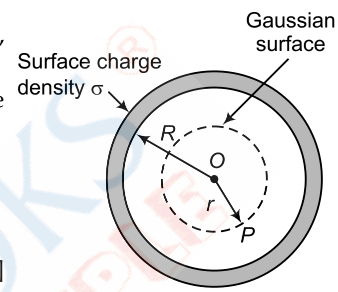
(1) Field Inside :

Let '*P*' be the point on Gaussian surface lies inside of the shell at a distance *r*, ($r < R$).

Since charges lie at the surface of shell so there would be no electric flux inside of the shell

$$\sigma = \frac{Q}{4\pi r^2} = 0 \quad \text{as } Q = 0$$

Hence, the electric field $E = \dfrac{\sigma R^2}{\varepsilon_0 r^2} = 0$ [here, $r < R$ and $\sigma = 0$]



(2) Field Outside

Let 'P' be the point on Gaussian surface lies outside of the shell at a distance *r*, ($r > R$).

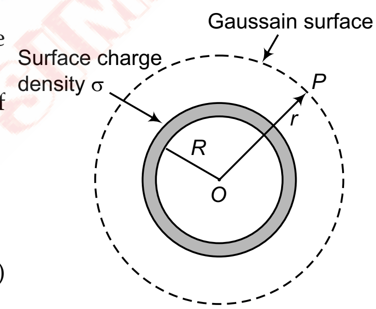
Since charges is uniformly distributed on Gaussian surface (being of spherical symmetry) then electric flux at point 'P' will be $\sigma = \dfrac{Q}{4\pi R^2}$

Hence, the electric field

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2} \qquad \ldots\text{(i)}$$

$$E = \frac{\left\{\dfrac{Q}{(4\pi R^2)}\right\} R^2}{\varepsilon_0 r^2} \quad \text{as } \sigma = \frac{Q}{(4\pi R^2)}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$



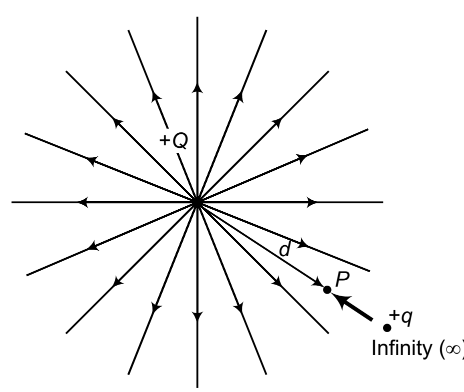
Topic 2

Electrostatic Potential and Capacitance

» Revision Notes

Electric Potential

- Electric potential at point is the work done in bringing unit positive test charge from infinity to that point in an electric field.
- The SI unit of electric potential is the volt.
- The potential at a point is a distance *r* from a charge Q given as $V = kQ/r$
- Electric potential is a measure of potential energy per unit charge.
- If potential at a point is known and a charge is placed at that point, then the potential energy linked with that charge is the charge multiplied by the potential.
- Electric potential similar to potential energy is a scalar and not a vector.
- When one positive charge moves away from another positive



charge its potential energy is decreased but when a negative charge is moved away from a positive charge (e.g. electron and proton) its potential energy increases.

Potential Difference

- The difference of electrical potentials between two points is known as potential difference or voltage.
- If the electrical force moves a charge a certain distance, it does work on that charge.

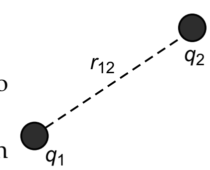
Work done = Force × Distance = (Charge Q) × (Potential)

The units of potential difference or potential is Joules / Coulomb called as Volts (V).

Electric Potential Due to a Point Charge

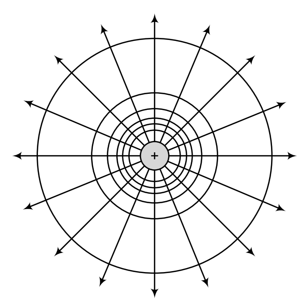
- $\Delta V_{BA} = V_B - V_A = -\int_A^B [kq/r^2]\, dr = -kq \int_A^B r^{-2}\, dr = kq\,[1/r_B - 1/r_A]$
- If change in electric potential energy on moving a charge q' from radius r_A to r_B due to a point charge q is $\Delta U_{BA} = kq'q[1/r_B - 1/r_A]$, then on analyzing the electric potential energy or electric potential relative to reference point, a reference point is considered as infinity where $r_A = \infty$ which shows that the electric potential at certain radius r is $V = kq1/r$
- If charge q' is brought at a distance r from charge q then it has electric potential energy $U = \frac{kq'q}{r}$
- The charges q_1 and q_2 at distance r_{12} apart having their electric potential energy is

$$U = kq_1\, q_2(1/r_{12})$$

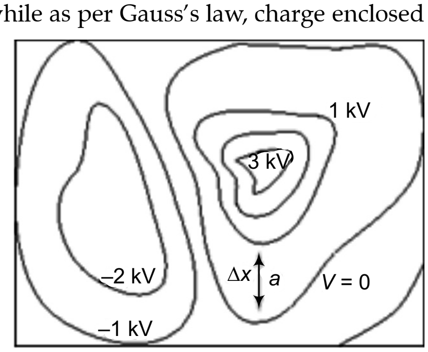


- The electric potential energy of two charges or electric potential at a point in space due to one electric charge is $V_{tot} = \Sigma V_i$
- It is the total or net electric potential which is the scalar sum of electric potential due to each of the individual point charges q_i.

Equipotential Surfaces



- An equipotential surface is a surface where every point on the surface has similar potential.
- Equipotential surfaces can be shown in two dimensions to provide quantitative view of electric potential.
- In an arrangement of equipotential lines, an electric field at point a can be obtained by calculating slope at a: $E = -\Delta V/\Delta x$ where all charges are at rest.
- If charges at rest reside on the surface of a conductor, the electric field outside the conductor perpendicular to surface at every point.
- At the surface, there can never be a component of E parallel to the surface.
- At Gaussian surface, flux through the Gaussian surface is not zero, while as per Gauss's law, charge enclosed by the Gaussian surface cannot be zero which contradicts the previous assumption of no charge in cavity.
- The Gauss's law shows that electric field at any point on the surface of a conductor is proportional to surface charge density at that point.



Rules for Equipotential Lines

- Electric field lines are perpendicular to equipotential lines and points from higher potential towards lower.
- A conductor forms an equipotential surface.
- The areas where equipotential surfaces are close to each other, the electric field is stronger.

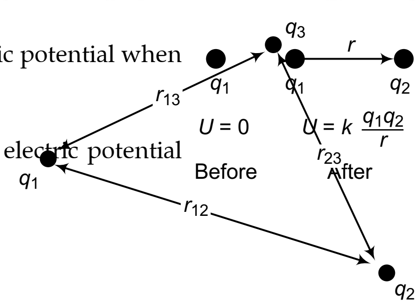
Electrical Potential Energy of a System of Two Point Charges

- If electric potential when q_1 is placed is $V(r_2) = V_2 = kq_1/r_{12}$ and electric potential when q_2 is placed into potential V_2 then,

$$U = q_2\, V_2 = k \times q_1q_2/r_{12}$$

- The electric potential when q_2 is placed is $V\,(r_1) = V_1 = kq_2/r_{12}$ and electric potential when q_1 is placed in potential V_1 then

$$U = q_1V_1 = k \times q_1q_2/r_{12}$$



- The electric potential energy of q_1 and q_2 is $U = (½)\, \Sigma\, q_iV_i$

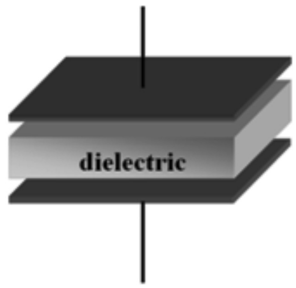
- **Conductors**
  - Conductor are the materials in which the electrical current flows easily.
  - In such materials, electrons can be transferred easily with in the lattice of material.
- **Insulators**
  - Insulators are the materials in which the electrical current does not flow easily.
  - The materials that do not easily transfer electrons are non-conductors like plastics and glass.
- **Free Charges**
  - Free charge is not subjected to certain constraints such as electrons in a conductor or in vacuum, which are free to move throughout the material.
- **Bound Charges**
  - Bound charge is that which cannot move macroscopically in response to external electromagnetic force.
  - Atoms of insulator undergo charge separation when subject to electric field, resulting in formation of dipoles which align themselves so as to counteract the applied field.
  - In dielectrics, there are no free electrons as the electrons are bound by molecular forces, so with an application of electric field, electrons get displaced which forms the dipole.
  - An external electric field which is applied to a dielectric material results in displacement of bound charges of elements which are bound to molecules and are not free to move around the material.
  - As per Gauss' Law, if surface S is enclose, then bound charge is $Q_{bound} = -\oint P.dS$ where P is polarization vector of dielectric material.
  - The electric field generated by charges on capacitor plates is given as

$$E_{free} = \sigma_{free}/\varepsilon_o$$

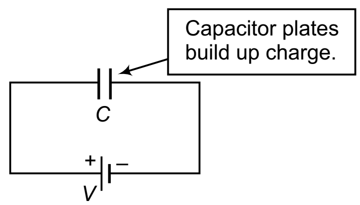
  - If charge density on surface of dielectric is (sigma)$_{bound}$, then field obtained by bound charges is

$$E_{bound} = \sigma_{bound}/\varepsilon_o$$

- **Dielectrics**
  - The electric field between the capacitor plates induces dipole moments in the material between the plates and reduces electric field in region between the plates.
  - A material in which the induced dipole moment is linearly proportional to the applied electric field is dielectric.
  - If a dielectric is placed in between the plates of a capacitor, then capacitance increases by a factor 'K' which is dielectric constant of material, that depends on material, so $C = K\varepsilon_o A / d$ where K is 1 for vacuum and air.

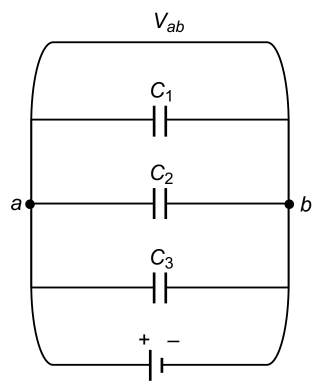


- **Capacitor**
  - A capacitor is a two parallel conducting plates separated by by air or insulating material.
  - The symbol representing a capacitor in an electric circuit looks like parallel plates ||.
  - When a capacitor is connected to an external potential, charges flow onto the plates and create a potential difference between the plates.
  - If the external potential is disconnected, charges remain on the plates, so capacitors are good for storing charge.
  - Capacitors release their stored charge all at once.
  - The magnitude of charge acquired by each plate of capacitor is $Q=CV$ where C is capacitance of the capacitor
  - The unit of capacitance is farad but most capacitors have values of C ranging from picofarads to microfarads (pF to μF) because the size of 1 farad in more than the size of Earth.
  - SI unit of capacitance is farad (F) where 1 farad capacitor when charged with 1 coulomb of electrical charge produces potential difference of 1 volt in between the plates.



- **Capacitance**
  - Capacitance is ability of a body to store an electrical charge.
  - Any object that can be electrically charged exhibits capacitance.
  - In parallel plate capacitor, capacitance is directly proportional to surface area of conductor plates and inversely proportional to distance between the plates.
  - Capacitance of a parallel-plate capacitor comprises of two parallel plates both having area A and is separated by distance d, $C = \varepsilon_r \varepsilon_o A/d$
  - If charges on plates are $+q$ and $-q$, and V be the voltage between the plates, then capacitance C is given as

$$C = q/V$$



- Capacitance is a function of physical dimensions of conductors and permittivity of dielectric which is independent of potential difference between the conductors and total charge on it.
- To calculate capacitance, we need to find geometry of conductors and dielectric properties of insulator.

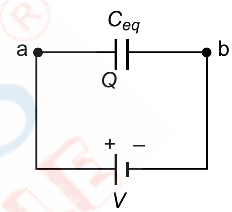
Capacitors Connected in Parallel

- In parallel capacitor circuit, potential difference or voltage drop from point a to point b must be equal to voltage $V = V_{ab}$.
- If a circuit components are connected in parallel, then charges across the components are

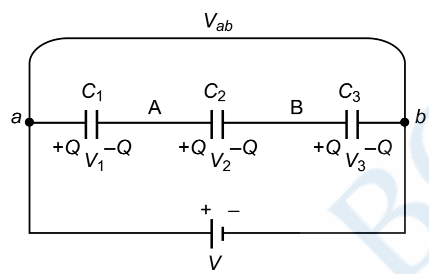
$$Q = CV \text{ or } Q_1 = C_1V, Q_2 = C_2V \text{ and } Q_3 = C_3V$$

- If capacitors arranged in parallel combination are replaced with single equivalent capacitor, then same charge will be stored in equivalent capacitor if voltage remains same, so

$$Q_1 + Q_2 + Q_3 = Q \text{ or } C_{eq}V = Q \text{ or } C_{eq} = \sum_{i=1}^{n} C_i$$

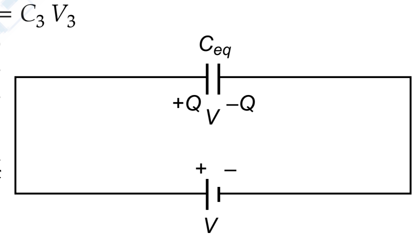


Capacitors Connected in Series



$$Q = C_1V_1 \qquad Q = C_2V_2 \qquad Q = C_3V_3$$

- If circuit components are connected in series, then the voltage drop across all the components is the sum of the voltage drops across the individual components, V_1, V_2 and V_3 across terminal a and b will be $V_{ab} = V$ or $V_{ab} = V_1 + V_2 + V_3$
- If all capacitors are replaced by single equivalent capacitor C_{eq}, then voltage V is same as total voltage drop across capacitors and amount of charge Q that flows out of battery remains same, so $Q = C_{eq}V$
- As $Q = C_1V_1$; $Q = C_2V_2$; $Q = C_3V_3$ and $V_{ab} = V_1 + V_2 + V_3$ then



$$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 \text{ or } 1/C_{eq} = \sum_{i=1}^{n} 1/C_i$$

Capacitance of a parallel plate Capacitor with and without dielectric medium between the plates

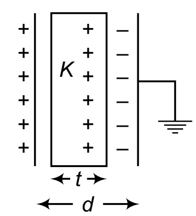
- A parallel plate capacitor consists of two metal plates parallel to each other and separated by a distance d.

Capacitance $C = \dfrac{KA\varepsilon_0}{d}$

For air capacitor $C_0 = \dfrac{A\varepsilon_0}{d}\ (K = 1)$

When a dielectric slab is inserted between the plates partially, then its capacitance.

$$C = \frac{A\varepsilon_0}{\left(d - t + \dfrac{t}{K}\right)}$$

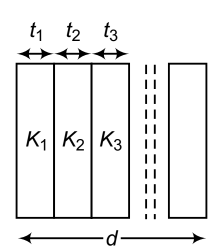


If a conducting (metal) slab is inserted between the plates, then

$$C = \frac{A\varepsilon_0}{(d - t)}$$

When more than one dielectric slabs are placed fully between the plates, then

$$C = \frac{A\varepsilon_0}{\left(\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3} + \ldots\right)}$$



The plates of a parallel plate capacitor attract each other with a force,

$$F = \frac{Q^2}{2A\varepsilon_0}$$

When a dielectric slab is placed between the plates of a capacitor then charge induced on its side due to polarization of dielectric is

$$q' = q\left(1 - \frac{1}{K}\right)$$

Energy Stored in a Capacitor

- Energy stored in a capacitor is equal to the work done to charge it.
- If a capacitor of capacitance C with charge $+q$ on a plate and $-q$ on other plate, then on moving a small element of charge 'dq' from one plate to other against potential difference, $V = q/C$ requires work dW,

$$dW = (q/C) \times dq$$

- The energy stored in capacitor is $dW = (q/C) \times dq$ or $W_{charging} = \int(q/C)\, dq = (½)\, Q^2/C = (½)\, CV^2$
- The energy stored in flat-plate capacitor is $W_{stored} = (½)\, CV^2 = [(½)\, \varepsilon_r\, \varepsilon_o\, A/d] \times V^2$
- When two capacitors C_1 and C_2 having potentials V_1 and V_2 respectively are connected with wire then

(i) Common potential $(V) = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

(ii) Loss of potential energy $(\Delta U) = \dfrac{1}{2}\dfrac{C_1C_2}{(C_1 + C_2)}(V_2 - V_1)^2$

Van de Graaff Generator

- It is an electrostatic generator.
- It uses a moving belt that accumulates charge on a hollow metal structure.

❐❐❐

Chapter 12

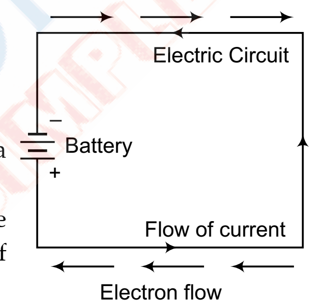
Current Electricity

» Revision Notes

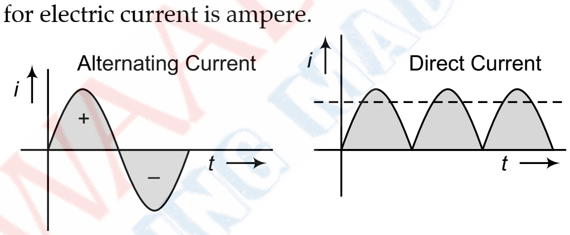
Electric Current

- Electric current can be defined as rate of flow of electric charge in a conductor.

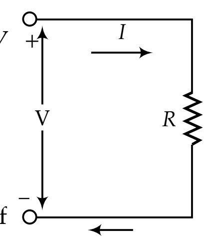
$$I = \frac{dQ}{dt}$$



- The direction of flow of current is opposite to that of the flow of electrons in a circuit.
- If a voltage source pushes in some direction, the electrons continue to move in that direction and this flow of electrons is known as Direct Current while if direction of current switches back and forth, it results in Alternating Current.
- The unit of measurement for electric current is ampere.

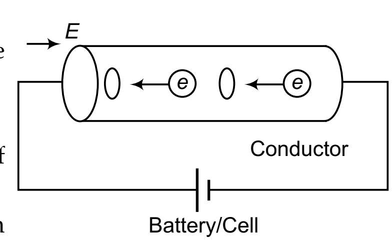


- Current in the circuit can be calculated using Ohm's Law when voltage of a circuit is V resistance is R then current $I = V/R$.
- The current I is used to calculate power as $P = I \times V$.
- Current in the circuit is shown with direction towards the ground
- Current in a circuit is measured using ammeter.
- When a voltage source is connected to a circuit, voltage will cause a uniform flow of electrons through the circuit
- Flow of electric current through a wire is just like the flowing of water through a pipe.



Flow of electric charges in a metallic conductor

- In a conductor, electric current flows freely as outer electrons of atoms are loosely bound and free to move through the lattice of material.
- The electric current flows when electrons move through a conductor, like metal wire as metals are good conductors of electricity.
- As current flows, electrons making the current collide with atoms of conductor thereby giving energy that appears in form of heat.
- The supply of voltage from battery, accelerates the free electron in the material.



Drift Velocity

- In the metal above absolute zero temperature there exists free electrons if substance is conductive.
- The free electrons in conductor moves randomly which collide with heavier atoms and change their direction.

- If a steady electric field is applied to conductor, the electrons start moving towards the positive terminal of applied electrical potential difference.
- During collision, electrons lose their kinetic energy and in presence of electric field, gets re-accelerated towards positive potential and regains kinetic energy.
- The applied electric field makes the electrons to drift in the opposite direction of the applied field. The average velocity of the electron in the presence of electric field in called the Drift velocity.
- In a metal conductor of length 'l' having cross-section area 'A' with 'n' as number of electrons per unit volume, shows charge $q = n \times A \times e \times l$, where symbols have usual meanings.
- If battery potential 'V' is connected across the circuit, then charge 'q' flows through the conductor in time t will have current

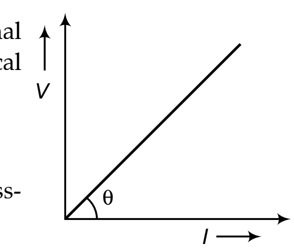
$$I = q/t \text{ or } I = n \times A \times e \times l/t$$

- The current density is $J = v_d \times e \times n$ is caused by steady flow of electrons due to drift velocity known as drift current density.

Mobility

- The electron mobility characterises how quickly an electron can move through a material, when pulled by an electric field.
- The mobility of electron (μ_e) is the drift velocity of electron per unit electric field applied, μ_e= drift velocity/electric field = v_d/E or $v_d = \mu_e E$
- The SI unit of mobility is $m^2s^{-1}V^{-1}$ or $ms^{-1}N^{-1}C$
- The drift velocity of holes is $v_p = \mu_p E$, where symbols have usual meanings.
- The drift current density due to free electrons is $J_n = en\mu_n E$, while drift current density due to holes is $J_p = ep\mu_p E$, where symbols have usual meanings
- The total drift current density is $J = J_n + J_p = en\mu_n E + ep\mu_p E$ or $J = e(n\mu_n + p\mu_p)E$
- 1 ampere of current is flow of 6.25×10^{18} electrons/second in cross-section of conductor.
- Electric field in a charged conductor is zero and is non zero in current carrying conductor.
- Electric field outside the current carrying conductor is zero.
- In a conductor, $JA = I$ = constant, as $J \propto 1/A$ as $J_1 A_1 = J_2 A_2$ known as continuity equation.
- If area of cross-section is fixed, $I \propto J$ where more current density, results in more current.

Ohm's Law



- Ohm's law shows that current (I) flowing through a conductor is directly proportional to the potential difference (V) across the ends of conductor, provided physical conditions of conductors remains constant, $V/I = R$ = a constant
- R is electrical resistance and it is expressed in ohms.
- If the potential difference is 1 V and flow of current is 1 A, then, resistance is 1 R.
- Electrical resistance between two points depends on conductor's length, cross-sectional area, temperature and material of conductor.
- Resistance of conductor proportional to length, $R \propto L$.
- The resistance of conductor is inversely proportional to its area, $R \propto 1/A$

$$R = \rho \frac{L}{A}$$

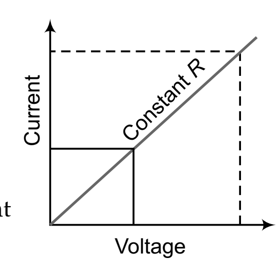
- The Resistivity $\rho = R\frac{A}{L}$ where symbols have usual meanings.
- If a resistor having resistance R_1, length L_1, area of cross section A_1 and radius r_1 is stretched to length L_2, area of cross-section A_2 and radius r_2 then new resistance will be

$$R_2 = R_1\left(\frac{l_2}{l_1}\right)\left(\frac{A_1}{A_2}\right) = R_1\left(\frac{l_2}{l_1}\right)\left(\frac{r_1}{r_2}\right)^2$$

But $A_1 l_1 = A_2 l_2$ so, $R_2 = R_1\left(\frac{l_2}{l_1}\right)^2 = R_1\left(\frac{A_1}{A_2}\right)^2 = R_1\left(\frac{r_1}{r_2}\right)^4$

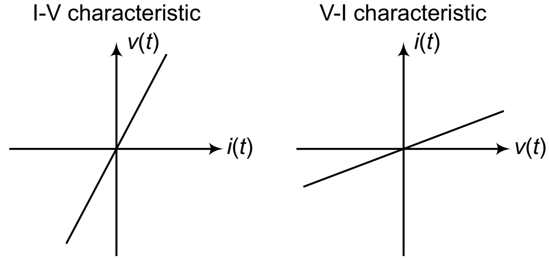
- The dimension of resistance is $[ML^2T^{-3}A^{-2}]$

Linear V-I Characteristics



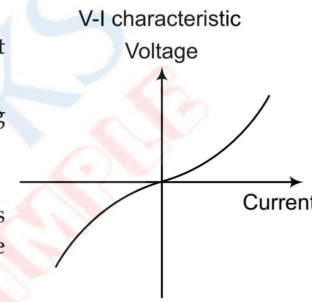
- A linear current–voltage characteristic curve or V- I has constant slope (or constant resistance).

- The *I-V* curve is a straight line through the origin with positive slope showing linear or ohmic resistance.
- Such circuits obeys Ohm's law where current is proportional to applied voltage over a wide range.



Linear Resistor

Nonlinear V-I Characteristics



- A circuit is said to have non-linear characteristic where resistance is not constant throughout and is a function of voltage or current.
- A curved line shows a nonlinear element like diodes and transistors having varying resistance for different values of voltage.

Electrical Energy

- Electrical energy is that which appears from electrical potential energy that is stored in an object due to objects position generated by movement of positive and negative charges.
- This energy is supplied by the combination of electric current and electric potential that is delivered by an electrical circuit in a given time or, $E = VIt$.

Electrical Power

- It is the energy per unit of time that depends on current and voltage, Power $= \dfrac{\text{electrical energy}}{\text{time}}$ $=$ current $\times$ voltage.
- $P = VI = I^2R = V^2/R$. The unit is Watt (W).

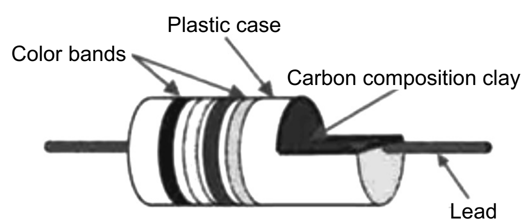
Electrical Resistivity

- Electrical resistivity is a measure of a material's property which opposes the flow of electric current.
- It is expressed in ohm-meters (Ωm) and is denoted as ρ (rho).
- A high resistivity shows that a material does not conduct electric charge.
- It is described as the relationship between the electrical field inside a material and the current density $\rho = E/J$, where symbols have usual meanings.

Electrical Conductivity

- The reciprocal of electrical resistivity or specific resistance ρ is electrical conductivity which is a measure of how material conducts electric current $\sigma = 1/\rho = J/E$
- It is measured in siemens/meter (S/m) and is shown by sigma (σ) where 1 siemen is reciprocal of 1 ohm-m.

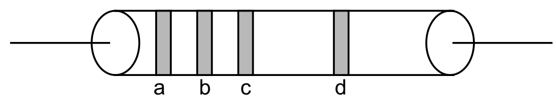
Carbon Resistors



- Resistors are of many types which are fixed and variable.
- The most common type of resistor applied for electronics use is the carbon resistor.
- Carbon resistor is made of different physical sizes as per power dissipation limits that ranges from 1 watt down to 1/8 watt.
- The resistor is made of carbon clay composition which is covered with plastic case.
- The lead of the resistor is made of tinned copper.

- Carbon resistor is available in very low cost and are durable and is sensitive to temperature.
- These resistor are available from 1 Ω value to 22 Mega ohm (MΩ) value with tolerance range of resistance from $\pm$ 5 to $\pm$ 20 %.
- It produces electric noise due to passage of electrical current from one carbon particle to other.

- The value of resistance is shown by four coloured bands marked on its surface with first three bands a,b,c showing value of resistance while fourth band d shows the tolerance of resistance.
- The colour of first and second band shows first and second significant figure of resistance and the third band c gives the power of ten by which, two significant digits are multiplied for obtaining value of resistance.



Colour Code for Carbon Resistors

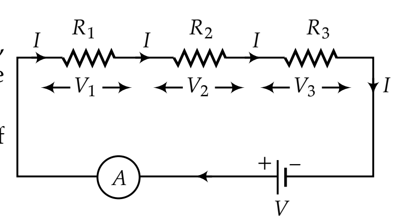
- Common carbon resistors has standard colour code that helps in determining the resistance and tolerance.
- There are mostly four colour bands in a carbon resistor that are on the body of resistor which shows unique digit.
- The table below shows the features and specifications of colour code for carbon resistors.

Colour	Figure(first and second band)	Multiplier(for third band)	Tolerance
Black	0	1	-
Brown	1	10	-
Red	2	10^2	-
Orange	3	10^3	-
Yellow	4	10^4	-
Green	5	10^5	-
Blue	6	10^6	-
Violet	7	10^7	-
Gray	8	10^8	-
White	9	10^9	-
Gold	-	10^{-1}	5%
Silver	-	10^{-2}	10%
no Colour	-	-	20%

- For example, carbon composition resistor with four color bands, with first band blue, second band yellow, third band red while fourth band golden, then we see that :
  - first digit of the number will be 6 (Blue ⇒ 6)
  - second digit of the number will be 4 (Yellow ⇒ 4)
  - multiplier of two digit number will be 10^2 (Red ⇒ 2)
  - electrical resistance value of a resistor will be $64 \times 10^2\ \Omega$ while tolerance will be ± 5 % as colour of fourth band is golden.

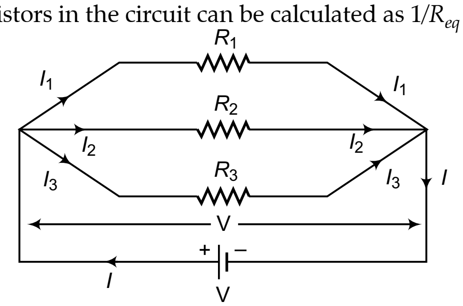
Resistors in Series

- A series circuit is a circuit in which resistors are arranged in a chain as R_1, R_2, and R_3, so the current has only one path to take. The current is the same through each resistor. $I = I_1 = I_2 = I_3$.
- The voltage drop across each resistor is not equal to V, but the sum of voltage drops, $V = V_1 + V_2 + V_3$
- The equivalent resistance of three resistors, $R_{eq} = R_1 + R_2 + R_3$



Resistors in Parallel

- In parallel circuit, resistors are arranged with their heads connected together and their tails connected together.
- In this, current can take more than one path as there are multiple paths for the current with potential difference remains same across the circuit. $V = V_1 = V_2 = V_3$
- In this, the current (I) from source gets divided among the resistors, $I = I_1 + I_2 + I_3$
- The equivalent resistance of resistors in the circuit can be calculated as $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$

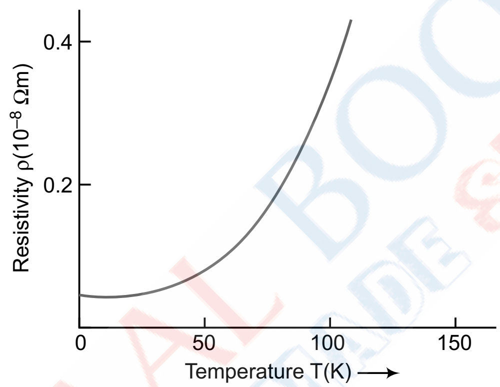


Remember

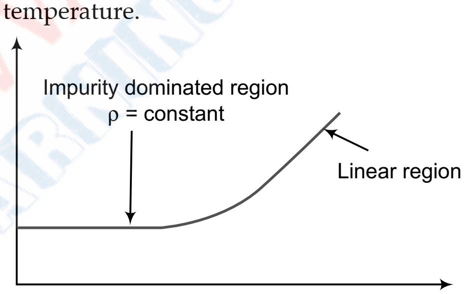
- If 'n' identical resistances are connected in series and then in parallel, then ratio of the equivalent resistance is $\frac{R_p}{R_s} = \frac{n^2}{1}$
- If equivalent resistance of R_1 and R_2 are in series and parallel are R_s and R_p then $R_1 = \frac{1}{2}\left[R_s + \sqrt{R_s^2 - 4R_sR_p}\right]$ $R_2 = \frac{1}{2}\left[R_s - \sqrt{R_s^2 - 4R_sR_p}\right]$.
- If a wire of resistance R is cut into n equal parts and then are collected to form a bundle, then equivalent resistance of the combination is $\frac{R}{n^2}$.

Temperature Dependence of Resistance

- Resistance of substances varies with temperature which depends on number of collisions which occurs between the charge carriers and atoms in a material. $R(T) = R_0\,[1 + \alpha\,(T - T_0)]$

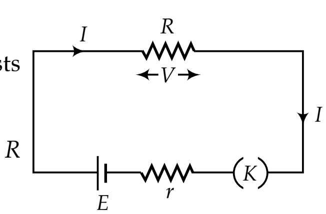


- As temperature increases, number of collisions also increases.
- The resistivity of a metallic conductor increases with increasing in temperature where ions of the conductor vibrates with more amplitude, making moving electrons to collide with ions which obstruct the drift of electrons and current at small temperature.



- Resistivity of metal is $\rho\,(T) = \rho_0\,[1 + \alpha\,(T - T_0)]$
  where α = temperature coefficient of resistivity.

Internal resistance of a cell



- Internal resistance is the resistance which are present within the cell that resists the current flow when connected to a circuit.
- It is provided by the electrolyte and electrodes which is present in the cell.
- The potential difference across the load or Terminal Voltage is $V = I \times R = [E/(R+r)] \times R$, where r is internal resistance of cell.

Electromotive Force

- Electromotive force (ε) or e.m.f. is the energy provided by a cell or battery per coulomb of charge passing through it. $\varepsilon = E/Q$, where symbols have usual meanings
- It is measured in volts (V).
- It is equal to the potential difference across the terminals of the cell in absence of current flowing it. $\varepsilon = V$

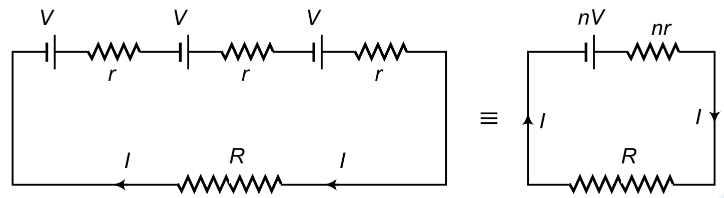
Cells in Series

- Cells connected in series has positive terminal of one cell connected with negative terminal of other cell that increases overall voltage.
- In series combination, positive terminal of first cell is connected to negative terminal of second cell while positive terminal of second cell is connected to negative terminal of third cell.

- In this, overall potential difference results as the sum of all individual potential differences of the cells in the circuit,

$$V = V_1 + V_2 + V_3$$

- If there are 'n' cells to be connected in a circuit with e.m.f and internal resistance of every cell as V and r, then total resistance $= nr + R$

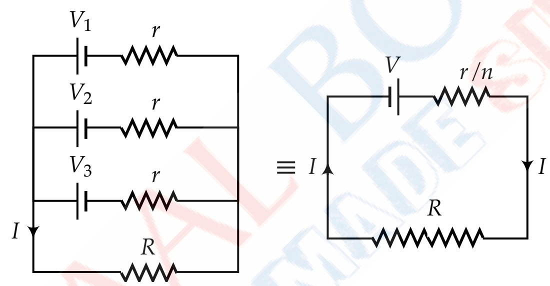


- If total current in the circuit is I, then I = Net EMF/Total Resistance or, $I = \dfrac{nV}{R + nr}$

Cells in Parallel

- In parallel combination, cells are joined such that the potential difference of the circuit is same for cells.
- In this, positive terminals of cells are connected together while negative terminals of cells. In parallel arrangements, potential difference between any two points in a circuit remains same.

$$V = V_1 = V_2 = V_3$$

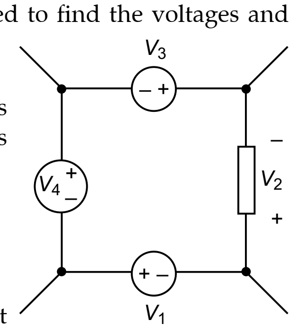


- In given figure, cells are joined in parallel across resistance R with e.m.f and internal resistance as V and r, then current I= Net E.M.F/Total resistance $= nV/(r + nR)$
- If combination of 'n' similar cells with emf E and internal resistance r, when 'a' cell is wrongly connected, then equivalent emf $E_{eq} = (n - 2a)E$ and equivalent internal resistance $r_{eq} = nr$.

Kirchhoff's laws

- There are two Kirchhoff laws, voltage law and current law which can be applied to find the voltages and currents in the circuits.

Kirchhoff's Voltage Law (KVL)



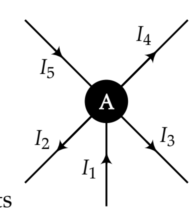
- This law states that for a closed loop series path, the algebraic sum of all voltages around any closed loop in a circuit is zero.It is also known as mesh law which shows that in a closed circuit network, algebraic sum of IR is equal to EMF in that path.

$$V_1 + V_2 + V_3 + V_4 = 0 \text{ or } \sum IR = 0$$

- It deals with the conservation of energy around a closed circuit path.

Kirchhoff's Current Law (KCL)

- It states that current flowing into a node (or a junction) must be equal to current flowing out of it, *i.e.*, algebraic sum of currents meeting at a junction is zero.



$$I_1 + I_5 = I_2 + I_3 + I_4 \text{ or } \sum I = 0$$

- This law is the consequence of charge conservation.

Applications of Kirchhoff's Laws

- The law is applied to find the values of unknown current, voltage in the circuit.
- The law is applied on any circuit and is useful in finding unknown values in complex circuits and networks.
- Kirchhoff's Laws are useful in understanding the transfer of energy through an electric circuit.

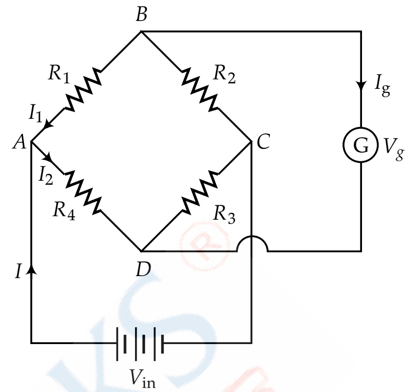
Wheatstone bridge

- Wheatstone bridge measures very low values of resistances upto milli-ohms range used for calibrating instruments like voltmeters, ammeters through long resistive slide wire.

- The bridge circuit is two simple series-parallel arrangements of resistances connected across the supply voltage terminal and ground which produces zero voltage difference among two parallel balanced branches.
- The has two input terminals and two output terminals with four resistors that are configured like a diamond arrangement.
- In the circuit, if voltage input V_{in} is given, then the currents flowing through arm ABC and ADC will depend on the resistances,

$$V_{in} = V_{ABC} = V_{ADC}$$

or $$I_{ABC} \times (R_1+R_2) = I_{ADC} \times (R_4+R_3)$$



- For balanced bridge, initial voltage reading V_g will result zero which shows the relationship among four resistances,

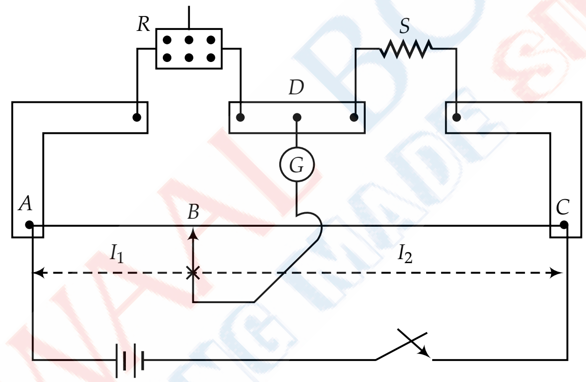
$$V_g = [(R_1R_3 - R_2 R_4)/ (R_1+R_2) (R_4+R_3)] \times V_{in} = 0$$

or $R_1 \times R_3 = R_2 R_4$

or $R_1/R_2 = R_4/R_3$

Metre bridge

- Meter bridge works on the principle of Wheatstone bridge which is used to find the resistance of an unknown conductor or unknown resistor.
- The bridge has straight uniform resistance wire which is stretched over a boxwood scale graduated in millimetres which is kept on the board.



- The ends of the wire are soldered or clamped to two stout copper or brass strips while the other end and longer strip is screwed to the board which is parallel to the wire.
- A resistance box with known resistance and unknown resistance are connected in two fixed gaps with battery connected between the two ends of the wire through a key.
- In this, terminals are provided on the strips for making required connections where a movable contact allows the contact to be made at any point along the wire.
- In this, one terminal of galvanometer is connected to point D while other terminal is joined to a jockey which slides on the bridge wire on adjusting required resistance value R in resistance box.
- By sliding jockey along the wire, a balance point is obtained at point B.
- As the circuit is similar to Wheatstone bridge , the condition of balanced will be $P/Q=R/S$, where resistance $P = \rho \times l_1/A$ and $Q = \rho \times l_2/A$, where symbols have usual meanings, so, $P/Q = (\rho \times l_1/A) \times (A/\rho l_2) = l_1/l_2$, here $l_1 = 1$ and $l_2 = 100 - l$

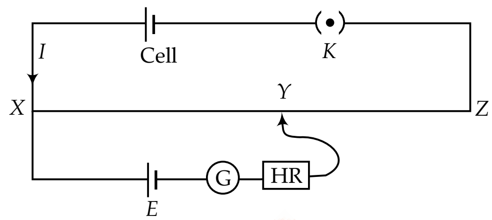
$$\frac{P}{Q} = \frac{R}{S} = \frac{l}{(100-l)}$$

or, $$S = R\left(\frac{100-l}{l}\right)$$

Potentiometer

- It is an accurate instrument used to compare emf's of a cells and the potential difference between two points of electric wire.
- It is preferred over voltmeter to measure emf of a cell as it measures true emf of the cell using null method where no current is drawn by galvanometer from the cell in balanced condition.
- It is a device which helps in comparing electromotive force of two cells, measuring internal resistance of a cell, measuring potential difference across a resistor

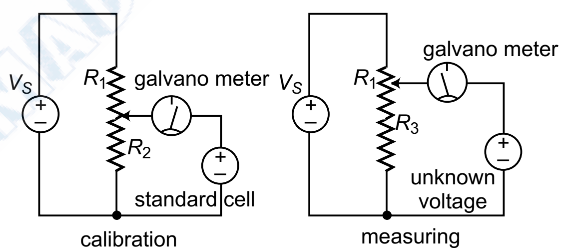
- It works on the principle that when a constant current flows through a wire of uniform cross sectional area, potential difference between its two points is directly proportional to the length of the wire between the two points.
- The circuit carries a cell connected across ends X and Z of potentiometer wire through a key K.
- A steady current 'I' flows through the potentiometer wire which forms the primary circuit.
- A primary cell is joined in series with positive terminal X of potentiometer, galvanometer, high resistance and jockey which forms the secondary circuit.



- When potential difference between the balancing lengths X and Y is equal to emf of cell, then no current flows through the galvanometer and galvanometer shows zero deflection.
- In this, if a steady current flows through the potentiometer, potential difference across a uniform wire is directly proportional to the length of the part across which the potential is measured, $E = I \times r \times L$ as I and r are constants, or $E \propto L$
- Smallest potential difference that can be measured by potentiometer is called **sensitivity of potentiometer.**
- Sensitivity of potentiometer is increased by increasing the length of the wire and decreasing the current in the wire using a rheostat.
- Sensitivity of potentiometer depends on the value of potential gradient K, which is the potential supplied by main battery which is divided by length of wire.
- Smaller the value of potential K, smaller will be the potential difference which a potentiometer will measure and greater will be the sensitivity of potentiometer.
- For a given potential difference, sensitivity of potentiometer tends to increase with increase in length of potentiometer wire.

Applications of Potentiometer in measuring potential difference

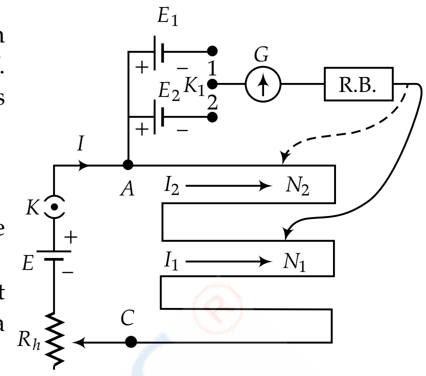
- In this, a fraction of known voltage from a resistive slide wire is compared with unknown voltage using a galvanometer.
- The sliding contact or wiper of potentiometer is adjusted with galvanometer connected among sliding contact and unknown voltage.
- With deflection in galvanometer, sliding tap gets adjusted till the galvanometer stops deflection from zero.



- At that point, galvanometer draws no current from unknown source and magnitude of voltage can therefore be calculated from position of sliding contact.
- In this, ends of uniform resistance wire R_1 is connected to regulated DC supply V_S for use as voltage divider.
- The potentiometer is calibrated by positioning the wiper (arrow) at the spot on R_1 wire which corresponds to voltage of standard cell, R_2/R_1 = Cell voltage/Vs
- The supply voltage V_S is adjusted till galvanometer shows zero showing voltage on R_2 equal to standard cell voltage. *Al*
- An unknown DC voltage, in series with the galvanometer is connected to wiper, across variable-length section R_3 of resistance wire.
- The wiper is moved till no current flows in or out of source of unknown voltage, as shown by the galvanometer in series with unknown voltage.
- The voltage across selected R_3 section of wire will then be equal to unknown voltage.
- The galvanometer reads zero or not zero, so on measuring unknown voltage and reading of galvanometer as zero, no current will be drawn from unknown voltage and reading will be independent of source's internal resistance.
- As resistance wire is made uniform in cross-section and resistivity where position of wiper being measured easily, it measures the unknown DC voltages which is more than or less than calibration voltage obtained by standard cell without drawing any current.
- If length of resistance R_1 wire is AB, where A is (–) negative end and B is (+) positive end and when the wiper is at point X at a distance AX on R_3 of resistance wire with galvanometer showing zero reading for unknown voltage, then distance AX is measured or read from a preprinted scale which is next to the resistance wire and the unknown voltage will be $V_u = V_s \times$ (AX/AB)

Applications of Potentiometer for comparing emf of two cells

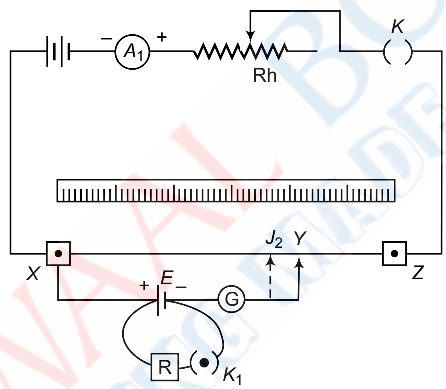
- In a potentiometer circuit where two cells of e.m.f. E_1 and E_2 with two way key K_1 and galvanometer G are connected, and if cell e.m.f. E_1 is connected, then at null point $E_1 = R_h l_1$ while if cell e.m.f. E_2 is connected, then at null point $E_2 = R_h l_2$, hence $E_1/E_2 = l_1/l_2$



E_1-Leclanche cell, E_2-Daniell cell

Measurement of Internal Resistance of a Cell

- Potentiometer measures the internal resistance of a cell.
- It consists of a long wire having high resistivity and low temperature coefficient with uniform cross sectional area and length.
- In this, the wires have high resistivity and low temperature coefficient that is stretched parallel to each other on wooden board carrying a metre scale joined in series with thick copper strips.
- For internal resistance of a cell E, a key 'K' is closed and a minimum current is maintained in potentiometer wire using a rheostat R_h.
- If the position of jockey Y is adjusted at different points of the wire and to find a point Y on wire where on pressing the jockey, galvanometer shows no deflection where length XY is l_1 of potentiometer wire.
- If emf of cell E = potential difference across the length l_1 of potentiometer wire, then $E = K \times l_1$ where, K = potential gradient across the wire.
- Closing of key K_1 such that resistance R comes in cell circuit and locate jockey position on potentiometer wire where galvanometer shows no deflection.



- If at point Y, galvanometer shows no deflection, then measure the length of wire XY (l_2) and then, potential difference between two poles of cell V which is equal to potential difference across length l_2 of potentiometer wire, so $V = k \times l_2$ or $E/V = l_1/l_2$
- The internal resistance r_1 of a cell with emf E if resistance R is connected in circuit is

$$r_1 = (E - V) \times R/V = [(E/V) - 1] \times R$$

- The internal resistance r_1 of the cell is $r_1 = [(l_1/l_2) - 1] \times R = [(l_1 - l_2)/l_2] \times R$

Chapter 13

Magnetic Effects of Current and Magnetism

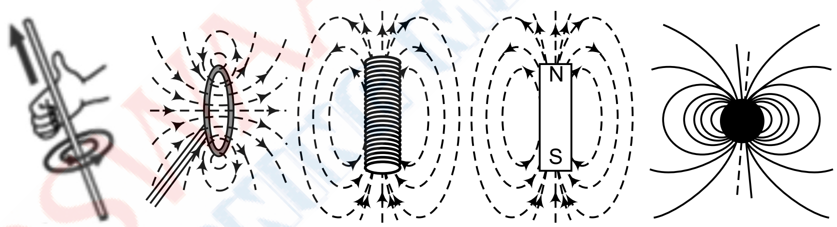
Topic 1

Magnetic Effect of Current

» Revision Notes

Concept of Magnetic Field

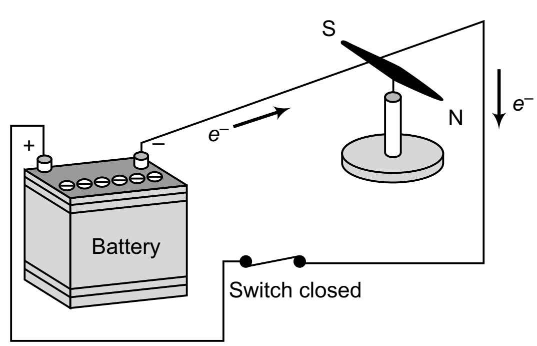
- Magnetic fields are invisible fields that exert magnetic force on substances which are sensitive to magnetism.
- The fields are generated inside the vicinity of a magnet by an electric current or changing electrical field.



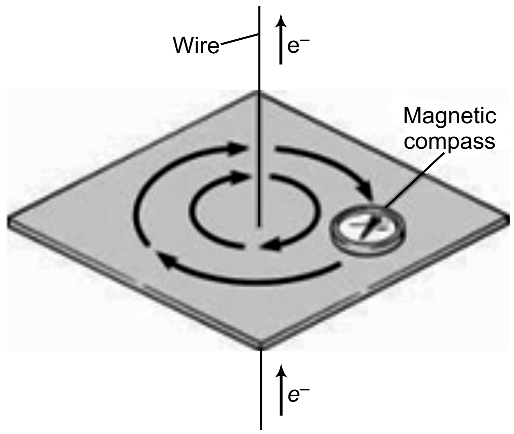
- The fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.
- SI unit to measure magnetic fields is Tesla, while smaller magnetic fields are measured in terms of Gauss (1 Tesla = 10,000 Gauss).
- The strength of field determines how hard the particle will be forced. More bunched the field lines be the stronger is the magnetic force.

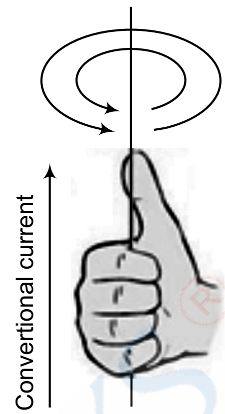
Oersted's Experiment

- Oersted in his experiment used a battery, switch, magnetic compass and wires.
- During the experiment Oersted observed that:
  - when there was no current, compass needle below a wire would show no deflection.
  - when there is a flow of current, compass needle gets deflected.
  - when the direction flow of current is reversed, compass needle deflects in opposite direction.



- From the observations, it is concluded that an electrical current produces a magnetic field which surrounds the wire.
- As per right hand rule, if thumb of right hand is in direction of current, magnetic field will wrap around the wire as other fingers.





- For flow of electrons, reverse everything and apply left hand thumb rule shown in figure Y.

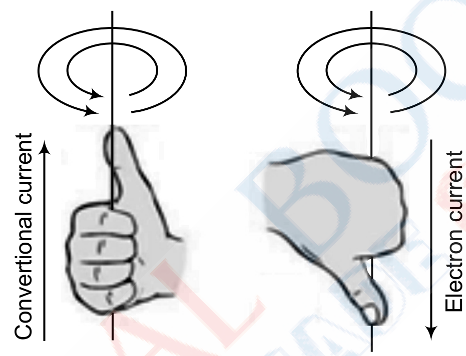


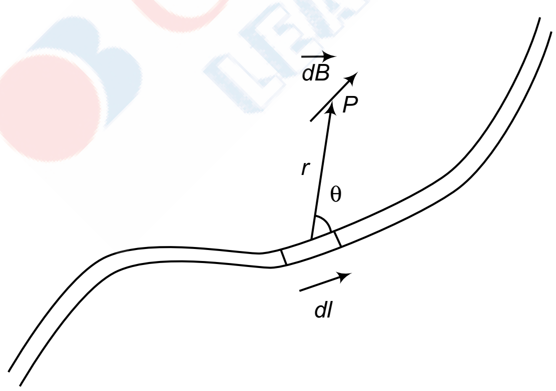
Figure X Figure Y

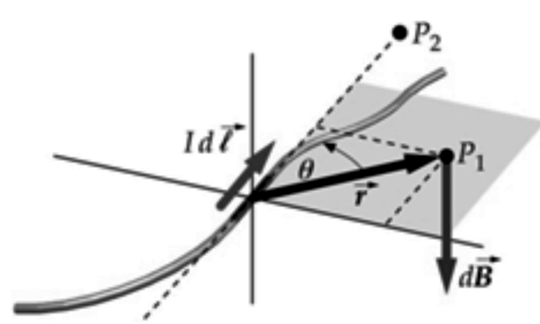
Biot – Savart law

- Biot-Savart law is used in order to determine the magnetic field intensity near a current carrying conductor.
- The magnetic of magnetic field dB at a distance r from a current carrying conductor of length dl is proportional to current I and to the length dl and is inversely proportional to square of distance (r)

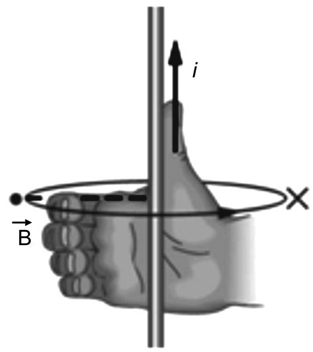
$$|dB| = \frac{\mu_0}{4\pi} \times \left(\frac{I\,dl\sin\theta}{r^2}\right)$$

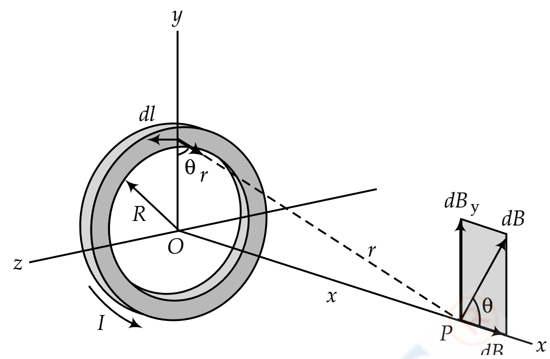
Application of Biot – Savart Law to Current carrying Circular Loop





- In a circular loop of radius R with centre O, plane of the coil is perpendicular to plane of paper and current I flowing as shown and P be any point on the axis passing to the centre.
- At point P, magnetic field from every element of length dl gets resolved in x component (dB_x) and y component (dB_y) where component dB_y for magnetic field from element of length dl on my side of ring which is equal in magnitude and opposite in direction to component dB_y for magnetic field produced by element of length dl on opposite side of ring 180° away Thus dB_y components get cancelled.





- The net magnetic field B at point P is sum of dB_x components for elements of length dl and the direction of net magnetic field along x-axis is directed away from circular loop.

$$\oint dB = \frac{\mu_o \cdot I}{4 \,.\, \pi} \cdot \oint \frac{dl}{R^2 + x^2} \cdot \cos\theta \qquad B = \frac{\mu_o \cdot I \cdot R}{4 \,.\, \pi \left(R^2 + x^2\right)^{3/2}} \cdot \oint dl$$

- If sum of elements of length dl around the closed current loop is circumference of current loop $\oint dl = 2\pi R$, then net magnetic field B at point P will be $B = \dfrac{\mu_o \cdot I \cdot R^2}{2 \cdot (R^2 + x^2)^{3/2}}$

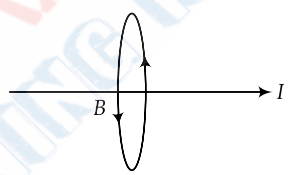
Ampere's Law

- This law states, the integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by the path and permeability of the medium.

$$\oint \vec{B} \,.\, \vec{dl} = \mu_0 I$$

Applications of Ampere's Law to Infinitely Long Straight Wire

- Ampere's law is applied to infinitely long wire for calculating a magnetic field strength $B = \mu_0 I/(2\pi r)$ with radial distance r from the wire.



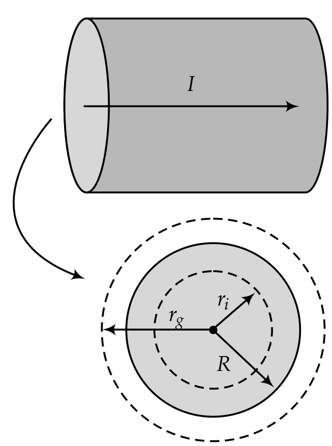
Magnetic field outside the wire, $r_o > R$

- $B \oint dl \propto I_{enc}$ so, $B \oint dl = \mu_0 I_{enc}$
- On integrating current carrying elements around path of circle.

$$\mathrm{B} \oint dl = \mu_0 I_{enc} \text{ or } B = \frac{\mu_0}{2\pi} \frac{I}{r_0}$$

- The equation for magnetic field around a long straight wire is

$$B = \frac{\mu_0 I}{2\pi r_0}$$



Magnetic field inside the wire, $r_i < R$

- If the current is distributed throughout the cross section, ratio of currents needs to be set up relating to cross-sectional area is

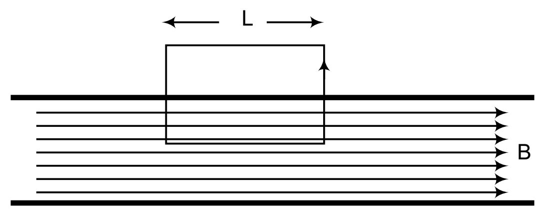
$$\frac{I_{enc}}{\pi r_i^2} = \frac{I}{\pi R^2} \Rightarrow I_{enc} = I \frac{r_i^2}{R^2}$$

So, $$\mathrm{B} \oint dl = \mu_0 \mathrm{I}_{enc} \text{ or } B(2\pi r_i) = \mu_o I \frac{r_i^2}{R^2}$$

- The magnetic field inside the wire is $B_{inside} = \dfrac{\mu_o I r_i}{2\pi R^2}$
- $B_{outside} = \dfrac{\mu_o I}{2\pi r_o} \quad \left(B \propto \dfrac{1}{r_o}\right)$; $\qquad B_{inside} = \dfrac{\mu_o I r_i}{2\pi R^2} \quad (B \propto r_i)$

Magnetic Field of Solenoid

- Solenoid is a long wire wound in a close-packed helix that carries current *I* where its length is much more than its diameter which generates a homogeneous magnetic field made of thin conducting wire wrapped in a tight helical coil with large number of turns.
- The magnetic field inside a solenoid can be obtained by adding the magnetic fields that is generated by *N* individual rings.



- The ideal solenoid has translational and rotational symmetry where magnetic field lines form closed loops, that cannot be directed along radial direction.
- The field lines in solenoid remain parallel with solenoid axis and magnitude of magnetic field can be obtained by applying Ampere's law.
- If all small current elements are integrated they add up to length of solenoid, *L*

$$B\oint dl = \mu_0 I_{enc} \text{ or } B(L) = \mu_0(NI) \text{ so } B = \mu_0 \frac{N}{L} I$$

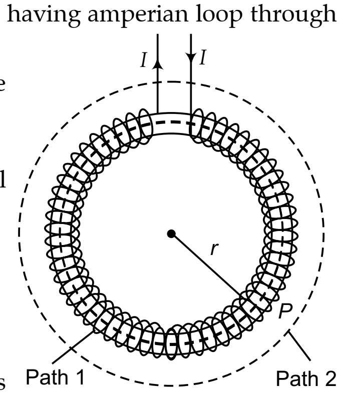
- For an infinite long solenoid where field is considered at external points of solenoid which equals to zero, we see $B_{solenoid} = \mu_o nI$; Here, $n = \frac{N}{L}$

Magnetic Field of a Toroid

- The magnetic field inside a ring is not uniformly distributed and is largest along the inner edge of the ring and smallest at outer edge of the ring where magnetic field *B* at point P in toroid having amperian loop through point P is concentric inside the toriod.
- The symmetry shows that field has equal magnitude at all points of the circle and is tangential to every point in a circle, $B \oint dl = 2 \times \pi \times r \times B$
- For *N* number of turns, net current crossing the area bounded by the circle will be *NI* where *I* is current in toroid, so Ampere law shows that

$$\oint B \times dl = \mu_o NI$$

or $$2 \times \pi \times r \times B = \mu_o NI \quad \text{or} \quad B = \frac{\mu_0 NI}{2\pi r}$$



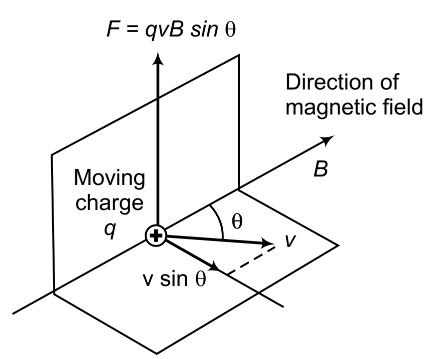
- The field *B* varies with *r* as field is not uniform over cross-section of core as path $l = 2\pi r$ is longer at outer side of section than at inner side.
- The net current passing through circular disc will be zero as current *NI* passes IN and OUT, so by Ampere's circuital law, field $B = 0$ outside the toroid.

Force on a Moving Charge in Uniform Magnetic and Electric Fields

- Ampere showed that wire carrying current experiences a force when placed in a magnetic field.
- The magnetic force acting on particle with charge *q* moving with velocity *v* is $\vec{F} = q(\vec{v} \times \vec{B})$
- The force *F* is always perpendicular to the direction of motion of the particle.
- If charged particle is moving in a uniform magnetic field, of strength *B* perpendicular to the velocity *v*, then magnitude of magnetic force is

$$F = qvB \text{ or } F = qvB \sin\theta$$

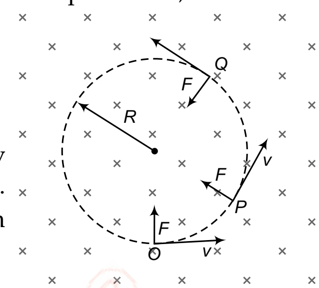
- The direction of *F* is shown (where moving charge is positive) as per Fleming's right hand rule that is on curling of right hand fingers in direction of *B*, right thumb will point in direction of *F*.



Path of Charged Particles in Uniform magnetic Field

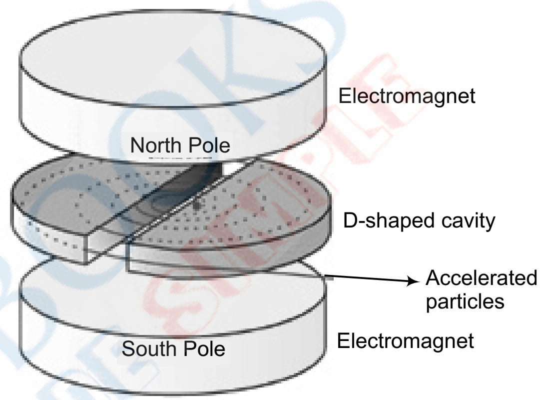
- The force, $F = qvB \sin\theta$ makes the particle to carry out uniform circular motion.

- The radius of circle is determined is shown by magnetic force, which is equal to centripetal force,
$$F = qvB \text{ or } F_c = mv^2/r$$
- The radius r of the orbit is $r = mv/qB = p/qB$; here, $p = mv$
- The distance travelled by particle in 1 revolution $d = 2\pi r = 2\pi mv/qB$
- The time T required to complete one revolution is $T = d/v = 2\pi m/qB$
- The frequency of motion is $\nu = 1/T = qB/2\pi m$ called as cyclotron frequency which is independent of energy of particle that depends on mass m and charge q.
- If an electron moving in uniform magnetic field describes circular motion with radius $r = mv/qB = p/qB$
- If electron is accelerated by potential $V_{0,}$ then final kinetic energy of electron is
$$K_f = eV_o$$

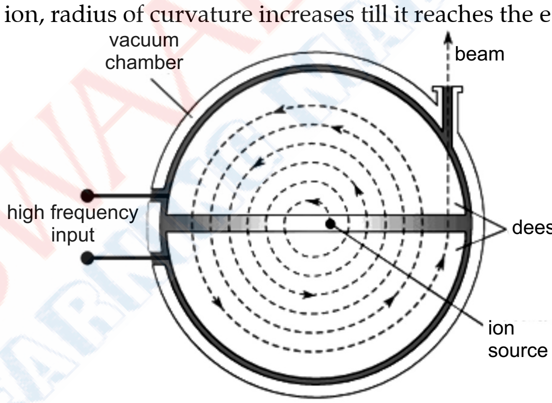


Cyclotron

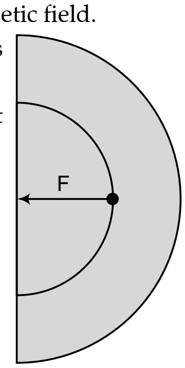
- In a evacuated cavity in between poles of a large electromagnet a metal piece cut into two D-shaped pieces having gap between them is placed.
- The oscillating high voltage is connected to the pieces of plates, which generates oscillating electric field in region between D shaped pieces.
- A charged particle is injected in center of cyclotron which carries uniform circular motion for first half of turn where frequency of motion of particle depends on mass, charge and magnetic field strength.
- The frequency of oscillator is selected such that every time the particle crosses the gap between the D shaped pieces that is accelerated by electric field.



- With increase of energy of ion, radius of curvature increases till it reaches the edge of cyclotron.



- In this, direction of motion of charged particle is perpendicular to direction of magnetic field having a uniform circular motion and shows spiral motion if direction of motion is not perpendicular to magnetic field.
- The magnetic force acting on particle is determined by component of velocity which is perpendicular to magnetic field.
- The projection of motion of particle on x-y plane is circular where magnetic field does not affect the component of motion parallel to field, and remains constant giving spiral motion.

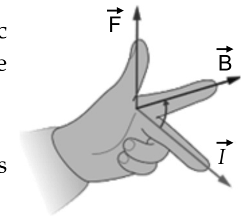


- In a D shape cyclotron length of path $= \dfrac{2\pi r}{2} = \pi r$
- The time an ion spends in each section of D shape is $\dfrac{T}{2}$.
- In D section, path of ion is circular and force towards the centre is $F = Bqv$ or $F = \dfrac{mv^2}{r}$

further $\dfrac{mv^2}{r} = Bqv$.

- The period of motion does not depend on radius of ion path or on speed of particles, $T = \dfrac{2\pi m}{Bq}$

Force on a Current-carrying Conductor in a Uniform Magnetic Field

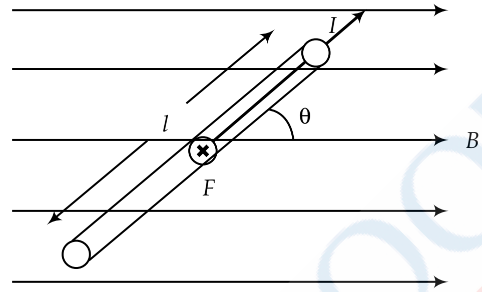


- In Fleming's Left-Hand Rule, current carrying conductor if placed in uniform magnetic field experiences a magnetic force whose direction is always perpendicular to plane with conductor and magnetic field,

$$F = B \times I \times l$$

- The factors affecting magnetic force on current-carrying conductor in a magnetic field is strength of magnetic field, current flowing through wire, length of wire.
- In a conductor with length l, area of cross section A in uniform magnetic field with 'n' electrons per unit volume and 'v_d' drift velocity of electron with charge 'q', the current is

$$I = nqv_dA$$

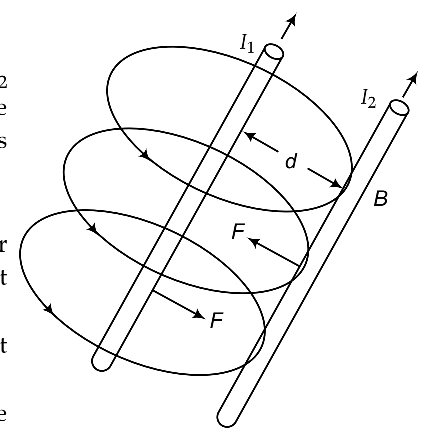


- The electrons experiences a Lorentz's magnetic force due to motion of electron in magnetic field,

$$\vec{F} = q(\vec{v}_d \times \vec{B}) = qv_dB \sin\theta \times n$$

- If conductor is perpendicular to magnetic field, force is maximum and if it is parallel to the magnetic field, force is zero.

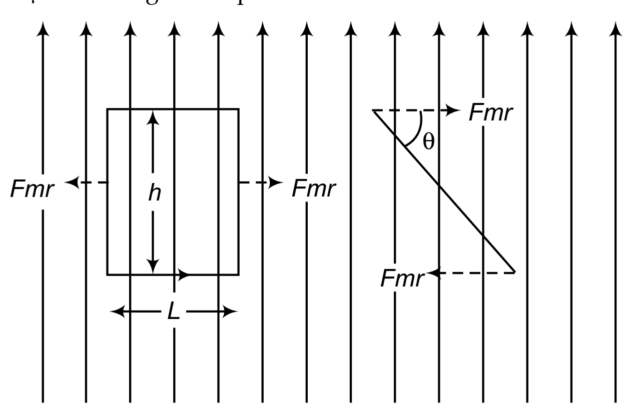
Force between Two Parallel Current-carrying Conductors



- Two current-carrying conductors exert magnetic forces on each other.
- In a two long, straight wires of length l carrying currents I_1 and I_2 separated by distance d, magnetic force exerted by one wire onto the other is directed toward the other wire, hence wires carrying currents in same direction attract each other, $F = \mu_0 I_1 I_2 / 2\pi d$

Torque Experienced by a Current Loop in Uniform Magnetic Field

- The force is exerted on current-carrying conductor when the conductor is placed in external magnetic field which produces torque on current loop in a magnetic field.
- A uniform magnetic field exerts no net force on a current loop but it does exert a net torque.
- If a rectangular current loop is placed in a uniform magnetic field, the angle between the normal of current loop and magnetic field is θ and magnetic forces acting on top and bottom sections of the current loop is $F_{mb} = F_{mt} = I \times L \times B$
- The torque exerted on the current loop with respect to axis is $\vec{\tau} = \Sigma \vec{r} \times \vec{F} = I \times L \times h \times B \sin\theta$
- Here, $\tau = \mu B \sin\theta$; where μ is the magnetic dipole moment.



- The work done against magnetic field to rotate the current loop by angle $d\theta$ is equal to $\tau d\theta$ and the change in potential energy of current loop when it rotates between θ_0 and θ_1 is

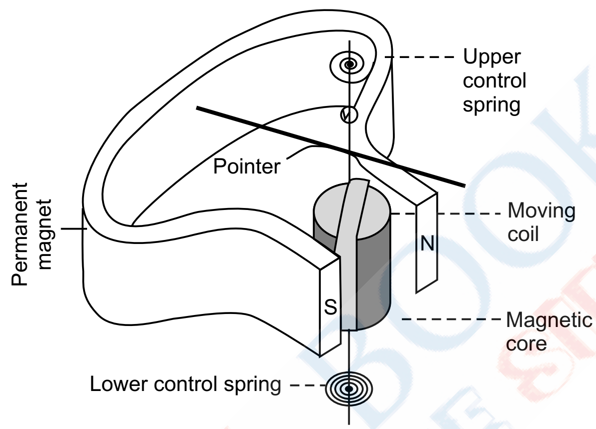
$$U(\theta_1) = U(\theta_0) - \int dW = U(\theta_0) + \int_{\theta_0}^{\theta} \mu \times B \sin(\theta)\, d\theta$$

or $$U(\theta_1) = \int_0^{\theta_1} \mu \times B \sin\theta\, d\theta = -\mu \times B \cos\theta_1$$

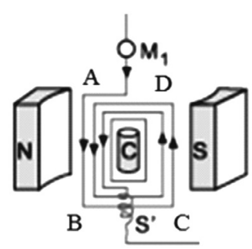
- The potential energy of current loop is minimum when μ and B are parallel and is maximum when μ and B are anti-parallel.

Moving Coil Galvanometer

- Galvanometer measures the strength of a weak electric current.



- The D'Arsonval movement galvanometers are moving coil type and pivoted coil type.
- The moving coil type comprises of coil of 10 to 20 turns of insulated copper wire wounded around a rectangular frame suspended from a fine conducting wire or thin flat strip between the poles of a horseshoe magnet having soft iron core that concentrate the magnetic flux in surrounding area of coil.
- If a beam of light strikes the mirror M, it is reflected to a scale, when electrical current passes through the coil producing torque.
- If α is angle which a plane of coil makes with direction of magnetic field, then torque experienced by coil ABCD, is $\tau = nIBA\sin\alpha$
- The wire inside the coil will get twisted due to torque and a restoring torque appears in the wire, that restores the coil back to its original position.
- If θ is angular deflection produced due to rotation of coil and k is restoring torque (unit twist of strip), then total restoring torque = $k \times \theta$
- The voltage sensitivity of galvanometer is deflection produced in galvanometer when a unit voltage is applied across two terminals of galvanometer $V_s = \dfrac{\theta}{V}$



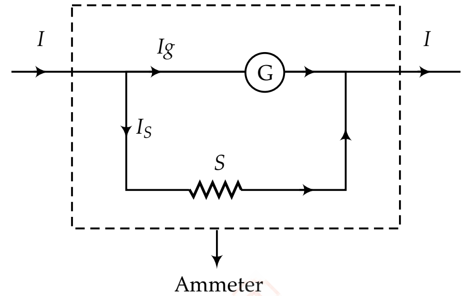
Current Sensitivity

- Deflection produced in coil is directly proportional to current flowing through the galvanometer.
- If θ is deflection in galvanometer during flow of current I, then current sensitivity $I_s = \theta/I = nBA/k$
- The unit of current sensitivity is rad/A or div/A.
- A galvanometer is said to be sensitive if it produces large deflection for a small current.
- The current sensitivities range from 0.01 microampere to 0.0001 microampere per mm division or a scale at a distance one meter.
- Current sensitivity of a galvanometer increases by increasing number of turns, increasing magnetic induction, increasing area of coil, decreasing couple per unit twist of suspension wire
- If current I_1 in coil gives a scale reading of s_1, then current sensitivity is $I_s = I_1/s_1$.

Ammeter

- Ammeter measures currents that serves as D'Arsonval movement wired placed in parallel with low resistance shunt.
  In this, bulk current goes through shunt with small amount going through the movement.
- Ammeter if connected in series, it is important for its resistance to be low so as not to reduce the amount of current flow, while taking current readings.

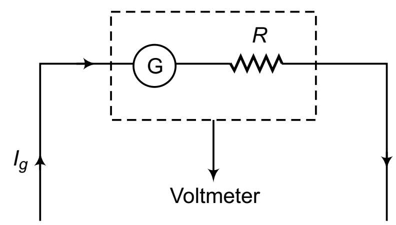
- **Galvanometer into Ammeter**
  - In this, large amount of current cannot be passed, as it may damage the coil, so a galvanometer is converted into ammeter by connecting a low resistance in parallel with it, is called shunt.
    With this, when large current flows in a circuit, only a small fraction of current passes through the galvanometer and the remaining larger portion of the current passes through the low resistance.



  - Value of shunt resistance depends on fraction of total current which is needed to be passed through the galvanometer.
  - If I_g is maximum current that can be passed through the galvanometer, then current I_g gives full scale deflection in galvanometer.
  - The current through shunt resistance is $I_s = (I - I_g)$ as galvanometer and shunt resistance are in parallel with common potential.
  - The shunt resistance quiet small due to I_g being fraction of I, so effective resistance of ammeter R_a will be

$$1/R_a = 1/G + 1/S = \frac{GS}{G+S}.$$

  - If ammeter is connected in series, it does not appreciably change the resistance and current in the circuit, so an ideal ammeter is one that has zero resistance.
- **Voltmeter**
  - It is an instrument for measuring potential differences that serves as basic D'Arsonval movement wire connected in series with high resistance.
  - It requires to measure the potential difference among two points in a circuit without appreciable change in potential difference.
  - If connected in parallel at time of readings, it correctly and accurately measure the readings, where very little amount of current is allowed to pass through it.
  - It measures the voltage drop as, ohm's law shows voltage drop V proportional to current I that helps in scaling of current to voltage drop.
- **Galvanometer into Voltmeter**
  - Galvanometer can be converted to voltmeter when it is connected with high resistance in series.
    In this, scale gets calibrated in volt where value of resistance which is connected in series selects the range of the voltmeter.



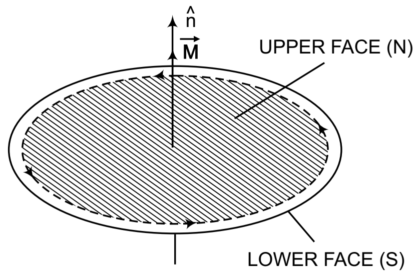
  - If G = Galvanometer resistance, I_g = Current which produces full scale deflection in galvanometer, V = Range of voltmeter, R = Resistance in series with galvanometer, then as R is connected in series with galvanometer, the current through galvanometer will $I_g = \frac{V}{G+R}$ or, $R = \frac{V}{I_g} - G$
  - The potential difference measured by voltmeter is very less as compared to actual potential difference which eliminates error when voltmeter has high resistance serving as ideal voltmeter with infinite resistance.
  - Field in moving coil galvanometer is radial in nature in order to have linear relation among current and deflection.
  - Rectangular current loop is in arbitrary orientation in external magnetic field, hence no work is required to rotate the loop about an axis perpendicular to it's plane.
  - Moving coil galvanometer can be made ballistic by using non-conducting frame instead of a metallic frame.

Topic 2

Magnetism and Magnetic Matter

» Revision Notes

- **Current Loop as Magnetic Dipole**
  - In a closed wire loop where electric current flows is kept in a uniform magnetic field, generation of torque rotates the loop perpendicular to direction of magnetic field.



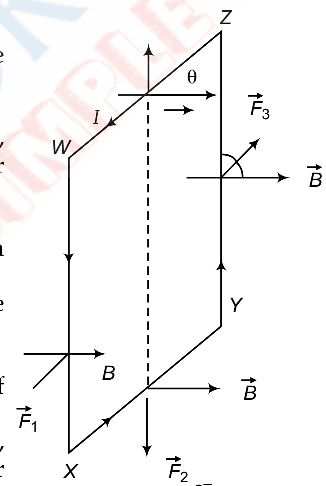
- If current carrying loop is observed from top, current direction is anticlockwise direction with north polarity while in lower face of loop, current direction is clockwise with south polarity.
- If a rectangular coil is kept in an external magnetic field where current 'I' is flowing through the coil, each part of the coil experiences Lorentz force F_1, F_2, F_3 and F_4 where forces F_2 and F_4 are equal in magnitude acting in opposite directions along same straight line which cancels each other,

$$F_1 = I \times (WX \times B) = I \times lB \ \text{ as } \theta = 90°$$

- The force F_1 will act in direction perpendicular to plane,

$$F_3 = I \times (YZ \times B) = I \times lB \text{ as } \theta = 90°$$

- Forces F_1 and F_3 constitute a couple which rotates the coil in anticlockwise direction, so torque τ = force × couple arm = $IlBb \cos\theta = NIAB \cos\theta$

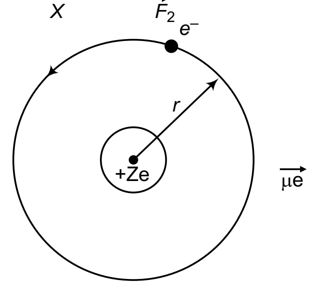


Magnetic Dipole Moment

- It is the measure of magnetic strength of magnet or current-carrying coil, expressed as torque per unit magnetic flux density produced when magnet or coil is set with its axis perpendicular to magnetic field.
- It is a vector quantity.
- It is the amount of current flowing through the loop which is multiplied by area that encompass by loop, with direction established by right hand rule.
- It can be thought of as a vector pointing from south to north of magnetic dipole which is equal to the length of dipole times the strength of poles.

Magnetic Dipole Moment of Revolving Electron

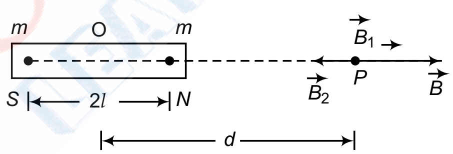
- Bohr's atom shows revolution of electrons around stationary heavy nucleus of charge $+Ze$.
- If an electron moves in circular path, there is a generation of electric current, $i = e/T$ where $T = 2\pi r/v$ and frequency of revolution $\nu = v/2\pi r$ having angular velocity, $\omega = v/r$, so current is : $i = ev/2\pi r = e\omega/2\pi$



Magnetic Field Intensity due to Magnetic Dipole along its Axis

- A bar magnet is of length $2l$ and pole strength m. Point P is on the axis of magnet at distance d from its center. $(d - l)$ is distance of P form N-pole of magnet. Magnetic field intensity at P due to north-pole of magnet is

$$B_1 = \frac{\mu_0}{4\pi}\frac{m}{r^2} = \frac{\mu_0}{4\pi}\frac{m}{(d-l)^2}$$



- The south pole of magnet is at a distance $r = d + l$ from P, the magnetic field intensity at P due to S-pole is $B_2 = \frac{\mu_0}{4\pi}\frac{m}{r^2} = \frac{\mu_0}{4\pi}\frac{m}{(d+l)^2}$ is directed towards the S-pole of the magnet.
- The magnetic field intensity B at P is resultant of

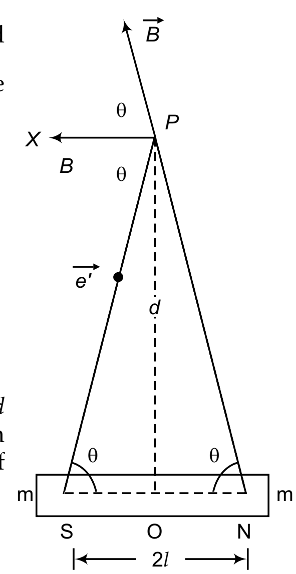
$$B = B_1 + (-B_2) = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{2md}{(d^2 - l^2)}$$

- If length of magnet is very small, $d >> l$ and magnetic field intensity is

$$B = \frac{\mu_0}{4\pi}\frac{2m}{d^3}$$

Magnetic field intensity due to magnetic dipole perpendicular to its axis

- If point P is on perpendicular line of bar magnet which bisects the magnet and d be the distance of the point P from the centre O of magnet. $2l$ is the magnet length with M as dipole moment then magnetic field at point P due to North pole of magnet is $B = \frac{\mu_0}{4\pi}\frac{2m}{(d^2 + l^2)}$



- The magnetic field at point P due to North pole of magnet is

$$B' = \mu_o m/4\pi(d^2 + l^2)$$

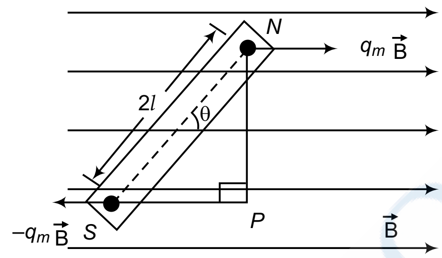
- The magnetic field intensity due to magnetic dipole perpendicular to its axis

$$B_2 = B\cos\theta - B'\cos\theta = 2B'\cos\theta$$
$$= \mu_o(m \times 2l)/4\pi(d^2 + l^2)^{3/2} = \mu_o M/4\pi(d^2 + l^2)^{3/2}$$

- If $d >> l$, then $B = \frac{\mu_0}{4\pi}\frac{m}{d^3}$

Torque on magnetic dipole (bar magnet) in uniform magnetic field

- Uniform Magnetic Field is a region where magnitude and direction of magnetic field results same at every point.

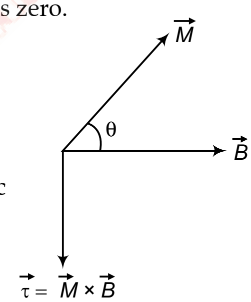


- If a bar magnet is placed in uniform magnetic field B, of pole strength q_m and $-q_m$ experience force $q_m B$ and $-q_m B$ along and opposite to direction of magnetic field B, so net force on bar magnet is zero.
- If a bar magnet placed in rectangular loop carrying current I in presence of uniform magnetic field, then

  Torque = Force × Perpendicular Distance or $\tau = q_m B \times (PN) = q_m B \times 2l \sin\theta$

  $\tau = MB\sin\theta$

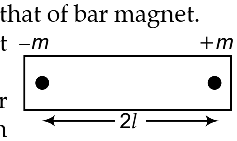
  or, $\vec{\tau} = \vec{M} \times \vec{B}$



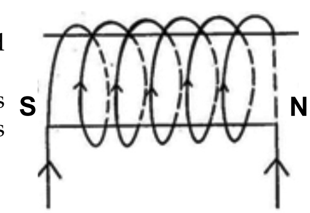
- Torque acting on bar magnet is maximum when it is kept perpendicular to the magnetic field.
  - If $\theta = 0°$ then $\tau = 0$
  - If $\theta = 90°$ then $\tau_{max} = MB$
  - If $\theta = 180°$ then $\tau = 0$

Bar magnet as a solenoid

- Solenoid is long straight coil of wire used to generate uniform magnetic field similar to that of bar magnet.
- If bar magnet of two equal and opposite magnetic poles, separated by distance, a magnet acts as called magnetic dipole.



- If 'm' is pole strength and $2l$ is separation between poles, then magnetic moment of bar magnet, $M = m \times 2l$, where magnetic dipole moment is vector and its direction is from $-m$ to $+m$.
- The current carrying solenoid is similar to bar magnet if bar magnet is suspended freely from its mid-point. It stays in north-south direction.
- In bar magnet, lines of magnetic field leaving and entering forms a closed lines where the field direction goes out from North Pole of a magnet and enters towards the South Pole.



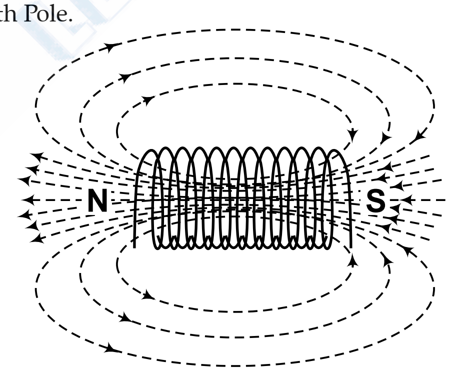


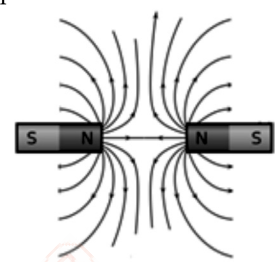
Figure (a)

Magnetic Field Lines

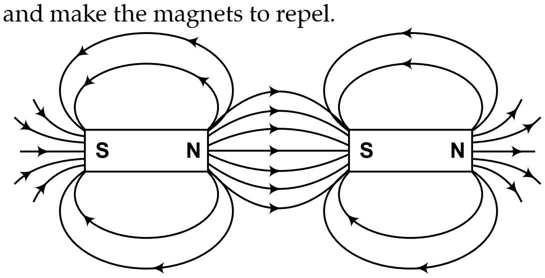
- Magnetic field is a region around the magnet where other magnet or magnetic material experiences a force.
- The fields around a magnet can be shown using field lines which acts at certain distance.
- In a magnet, magnetic field lines extend from north pole to south pole and never cross each other.
- The magnetic field of a magnet is strongest at its poles.
- Strength of magnetic field lines are shown by their distances, so closer is the magnetic the field lines, stronger is the magnetic field. Refer figure (a).

- The density of magnetic field lines decreases when they move from an area of higher permeability to area of lower permeability while their density decreases with increasing distance from the poles.

Field Lines between Two Magnets

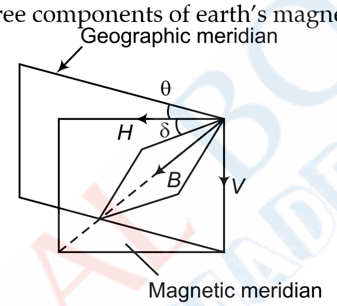


- If two magnets are kept closer to each other, then field lines around the magnets form distinctive patterns as per attraction or repulsion of magnets.
- If two like poles are placed next to each other, field lines drawn from North Pole to South Pole which shows attraction of two magnets.
- If two unlike poles are placed next to each other, field lines are pushed away from other magnet and make the magnets to repel.



Earth's Magnetic Field and Magnetic Elements

- Earth's magnetism results due to magnetic effect of current that flows in a liquid-core at the center of the Earth.
- Earth is a huge magnet. There are three components of earth's magnetism.



(i) Magnetic Declination (θ) : The smaller angle subtended between the magnetic meridian and geographic meridian is called magnetic declination.

(ii) Magnetic Inclination or Magnetic Dip (δ) : The smaller angle subtended between the magnetic axis and horizontal is called magnetic inclination on magnetic dip.

(iii) Horizontal and Vertical Component of Earth's Magnetic Field (*H*) : If B is the intensity of earth's magnetic field then horizontal component of Earth's magnetic field $H = B \cos \delta$.

It acts from south to north direction.

Vertical component of earth's magnetic field $V = B \sin \delta$

$$\therefore \quad B = \sqrt{H^2 + V^2} \quad \text{and} \quad \tan \delta = \frac{V}{H}$$

Angle of dip is zero at magnetic equator and 90° at poles.

Magnetic Meridian : A vertical plane passing through the magnetic axis is called magnetic meridian.

Geographic Meridian : A vertical plane passing through the geographic axis is called geographic meridian.

Magnetic Map : Magnetic map is obtained by drawing lines on the surface of earth, which passes through different places having same magnetic elements.

The main lines drawn on earth's surface are given below :

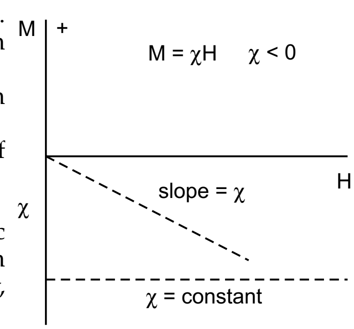
(i) Isogonic Line : A line joining places of equal declination is called an isogonic line.

(ii) Agonic Line : A line joining places of zero declination is called an agonic line.

(iii) Isoclinic Line : A line joining places of equal inclination or dip is called an isoclinic line.

(iv) Aclinic Line : A line joining places of zero inclination or dip is called an aclinic line.

(v) Isodynamic Line : A line joining places of equal horizontal component of earth's magnetic field (*H*) is called an isodynamic line.



Diamagnetic Materials

- The diamagnetic substances are made of atoms with no net magnetic moments where orbital shells are filled without unpaired electrons which when exposed to magnetic field produces magnetic moment, susceptibility, intensity of magnetization are negative and small.
- If graph is drawn with *M vs H*, it is observed that when the field is zero with magnetization resulting as zero.
- The well known diamagnetic substances having in units of 10^{-8} m^3/kg, Quartz (SiO_2) –0.62, Calcite ($CaCO_3$) – 0.48, Water – 0.90

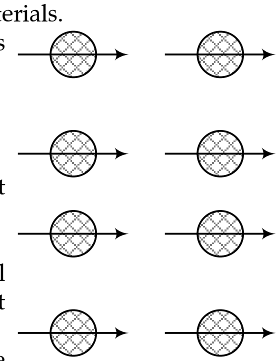
- These are repelled by strong magnet.
- Relative permeability is less than unity.
- In non-uniform magnetic field, these experience attraction towards weaker parts of the magnetic field.
- The diamagnetic substance rod when suspended between poles pieces of magnet remains with its axis perpendicular to magnetic field by poles.
- With strong magnetic field across U-tube, the liquid inside the limb gets depressed.
- Magnetic susceptibility of diamagnetic substances is independent of magnetizing field and temperature.

Paramagnetic Materials

- In this, the atoms or ions have net magnetic moment due to unpaired electrons present in partially filled orbital.
- If the atom has unpaired electrons (as iron) where each magnetic moment does not interact magnetically where magnetization is zero on removing of field.
- The graph of M *vs* H and χ *vs* T shows that in presence of magnetic field, there will be partial alignment of atomic magnetic moments in direction of field which shows net positive magnetization and positive susceptibility.
- The efficiency of magnetic field in aligning the moments is opposed by random temperature effect.
- At normal temperatures and moderate fields, paramagnetic susceptibility is small and if the temperature is very low or field is very high, the paramagnetic susceptibility is independent of applied field.
- Examples include, Montmorillonite (clay)–13, Nontronite (Fe-rich clay)–65, Biotite (silicate)–79, Siderite (carbonate)–100, Pyrite (sulfide)–30.
- It gets attracted by strong magnet.
- It has magnetic properties such as magnetic moment, magnetic susceptibility, intensity of magnetism which are positive while relative permeability is more than unity.
- In non-uniform magnetic field, paramagnetic substance gets attracted towards stronger part of field.
- On free suspension of rod between the poles of magnet shows parallel axis to magnetic field.
- Paramagnetic liquid in U-tube on strong magnetic field shows rise in liquid level in limb.
- Susceptibility and permeability remain intact with varying magnetizing field.
- Susceptibility of substance is inversely proportional to absolute temperature.

Ferromagnetic Materials

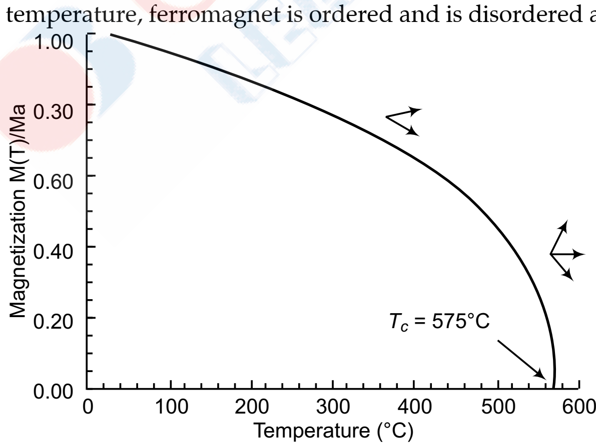
- Ferromagnetic materials show parallel alignment of moments that results in large total magnetization in the absence of magnetic field.
- The elements Fe, Ni, and Co and many of their alloys are typical ferromagnetic materials.
- There exists two distinct characteristics of ferromagnetic materials like spontaneous magnetization and existence of magnetic ordering temperature.
- They are attracted by weak magnet.
- They have very large and positive susceptibility.
- Very high relative permeability.
- Magnetization intensity is proportional to magnetizing field for smaller values that varies rapidly for moderate values and has constant value for larger values of H.



Parallel alignment

Curie Temperature

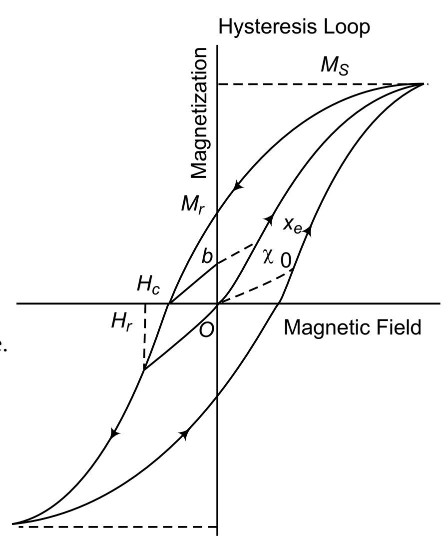
- In ferromagnet, electronic exchange forces exist which are very large where thermal energy eventually overcomes the exchange and lead to randomizing effect at particular temperature called as Curie temperature, T_C.
- In graph of magnetization vs temperature for magnetite, below the Curie temperature, ferromagnet is ordered and is disordered above it.



- The saturation magnetization goes to zero at the Curie temperature.
- It is an intrinsic property and is a diagnostic parameter which is used for mineral identification.

Hysteresis

- In ferromagnets, Curie temperature and saturation magnetization retain a memory of applied field on removal, which shows behaviour of hysteresis.



- The graph of variation of magnetization with magnetic field is called as hysteresis loop.
- It has a hysteresis property as coercivity, H_r, which shows reverse field which is applied and then removed, thereby reducing saturation remanence to zero.
- The hysteresis parameters are not exclusively intrinsic property but depends on grain size, domain state, stresses, temperature.

Electromagnets

- Electromagnets make fields through large currents in wires, hence electromagnets are coils of wire which behave like bar magnets with a distinct north and south pole when an electrical current passes through the coil.
- The static magnetic field produced by each coil loop is assembled with its neighbour having combined magnetic field that concentrates like single wire loop.

Factors affecting Strength of Electromagnets

- The main factors that affect the strength of an electromagnet are number of turns on the coil of wire around the core, strength of the current applied, material of the coil.
- The magnetic field strength of electromagnet depends on type of core material used for purpose of core to concentrate the magnetic flux in a defined path.
- The air cored coils are considered but introduction of other materials into core has large controlling effect on strength of magnetic field.

Permanent Magnets

- Magnet exerts a magnetic force on other materials without physical contact which made them to attract or repel.
- These are the magnets that exert force on objects without any outside influence.
- **Ceramic**
  - Ceramic or Ferrite are magnets made of iron oxide and barium/strontium carbonate.
  - They are made by pressing and sintering and are brittle.
  - These magnets are made in different grades.
  - They have good balance of magnetic strength, resistance to demagnetizing and economy.
- **Alnico**
  - These magnets are prepared from aluminium, nickel and cobalt having good temperature stability, good resistance to demagnetization.
  - They can be easily demagnetized.
  - The magnets are made from casting or sintering.
  - The common grades of magnets are 5 and 8 which are anisotropic grades for preferred magnetic orientation.
- **Samarium Cobalt**
  - It is a rare Earth magnet which is highly resistant to oxidation having high magnetic strength and temperature resistance as compared to Alnico or Ceramic material.
  - These magnets are Sm_1Co_5 and Sm_2Co_{17} with 1– 5 and 2 – 17 grades.
  - They have good temperature characteristics and can withstand temperatures up to 300° C.
  - Due to high cost of material, samarium, samarium cobalt magnets are used for applications where high temperature and corrosion resistance is critical.
- **Neodymium Iron Boron**
  - It is a type of rare Earth magnetic material having similar features as that of Samarium Cobalt, but is easily oxidized.
  - They have high energy products and are costly normally used in selective applications.
  - They have high energy products and low manufacturing costs and are highly corrosive.
- **Injection Molded**
  - These magnets are made of resin and magnetic powders.
  - They are made from injection molding.
  - They have low magnetic strength.

Flexible Magnets

- They are similar to injection molded magnets made of flat strips and sheets having low magnetic strength.
- These magnets are flexible as per their materials which can be applied in compound with magnetic powders.
- Vinyl is used in such magnet as binder.

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Chapter 14

Electromagnetic Induction and Alternating Currents

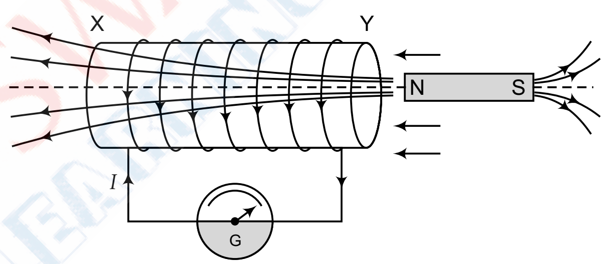
Topic 1

Electromagnetic Induction

» Revision Notes

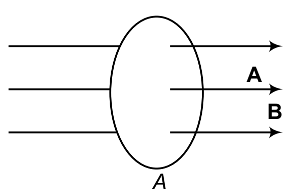
Electromagnetic Induction

- Electromagnetic induction is the production of electromotive force across an electrical conductor in a changing of magnetic flux or magnetic field.
- Michael Faraday is credited the discovery of induction in 1831.
- Electromagnetic induction has found many applications, including electrical components such as inductors and transformers, and devices such as electric motors and generators.

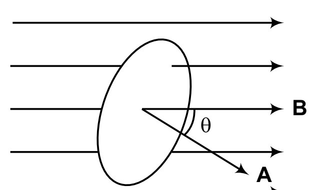


Magnetic Flux

- The magnetic flux linked with any surface is equal to total number of magnetic lines of force passing normally through it.
- Suppose, we consider small area dA in field B, then $\phi = \int \vec{B} \cdot \vec{dA} = \vec{B} \cdot \vec{A}$



Magnetic Flux, $\phi = BA$ = Maximum value



Magnetic Flux, $\phi = \vec{B} \cdot \vec{A} = BA \cos \theta$

- SI unit of magnetic flux is Weber (Wb).
- CGS unit of magnetic flux is Maxwell (Mx).

$$1 \text{ Wb} = 10^8 \text{ Mx} = 1 \text{ Tm}^2$$

- Magnetic flux is a scalar quantity and its dimensional formula is $[ML^2T^{-2}A^{-1}]$

Faraday's Law of Electromagnetic Induction

- **First Law** Whenever magnetic flux linked with the closed loop or circuit changes, an emf induces in the loop or circuit which lasts so long as change in flux continuous.
- **Second Law** The induced emf in a closed loop or circuit is directly proportional to the rate of change of magnetic flux linked with the closed loop or circuit

  i.e., $$e \propto \frac{(-)\,\Delta\phi}{\Delta t} \Rightarrow e = -N\frac{\Delta\phi}{\Delta t}$$

  where, N = number of turns in loop.

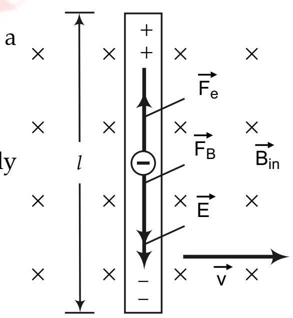
  Negative sign indicates the Lenz's law.
- If N is the number of turns and R is the resistance of a coil, the magnetic flux linked with its each turn changes by $d\phi$ in short time interval dt, then induced current flowing through the coil is

  $$I = \frac{|e|}{R} = -\frac{1}{R}\left(N\frac{\Delta\phi}{\Delta t}\right)$$
- If induced current is produced in a coil rotated in a uniform magnetic field, then

  $$I = \frac{NBA\omega\sin\omega t}{R} = I_0 \sin\omega t$$

  where, $I_0 = \dfrac{NBA\omega}{R} =$ Peak value of induced current

Motional EMF



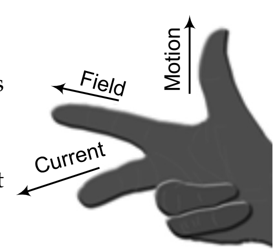
- An emf induced by the motion of the conductor across the magnetic field is a motional electromotive force.
- The equation is given by $E_m = -\int(\vec{v}\times\vec{B})\cdot\vec{dl} = -vBl$
- This equation is true as long as the velocity, field and length are mutually perpendicular.
- The minus sign associated with the Lenz's law.

  $$E_m = -\frac{d\,\Phi_B}{dt} = -\frac{d\,(Blx)}{dt} = -Bl\frac{dx}{dt}$$
  $$E_m = -vlB$$
- The induced emf developed between two ends of the conductor of length 'l' rotating about one end with angular velocity 'ω' in a direction perpendicular to the magnetic field is given by

  $$E_m = \frac{B\omega l^2}{2}$$
- The power required to move a conductor rod in a magnetic field is,

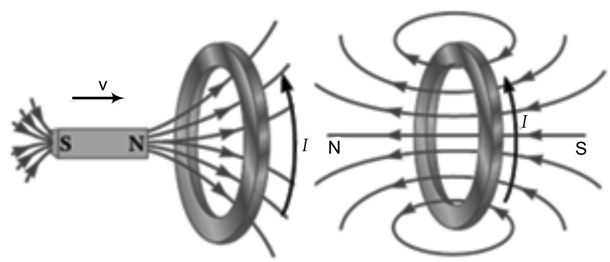
  $$P = \frac{B^2 l^2 v^2}{R}$$

  Here, all alphabets are in their usual meanings.
- The direction of induced e.m.f and current is determined by Lenz's law or Fleming's **Right Hand Rule**.

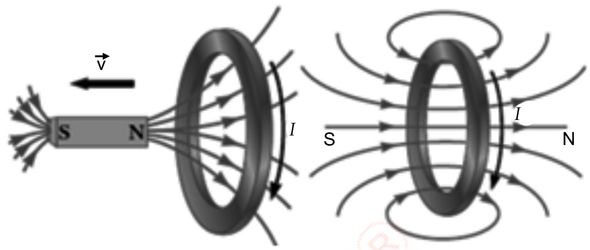


Lenz's Law

- According to Lenz's law, the direction of induced E.M.F. and hence induced current is such that it opposes the cause (say motion of the magnet) which produces it.
- Lenz's law is in accordance with the law of conservation of energy.
- **Case-I : When a magnet is moving towards the coil.**
  - If north pole of magnet approaches towards the coil, magnetic flux linked with the coil will increase.
  - With Faraday's law, if there is change in flux, an emf and current is induced in the coil where the current creates its own magnetic field.
  - With Lenz law, magnetic field obtained tends to oppose the increasing flux by the coil if approaching coil side gets north polarity.



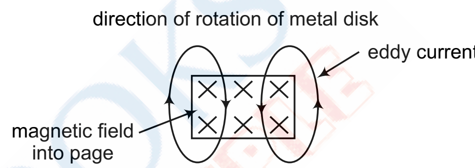
- **Case-II : When a magnet is moving away from the coil.**
  - If north pole of magnet is moving away from the coil, magnetic flux linked with the coil decrease.
  - With Faraday's law, emf and current is induced in the coil, where current creates its own magnetic field.
  - With Lenz's law, magnetic field obtained opposes the decreasing flux through coil, if approaching coil side gets south polarity.
  - The magnetic polarity of coil helps in finding the direction of induced current with the help of right hand rule, where current flows in clockwise direction.



Eddy Currents

- The induced circulating currents or swirling currents produced in a conductor itself due to change in magnetic flux linked with the conductor are called eddy currents.
- The magnitude of eddy currents is given by

$$i = -\frac{E_m}{R} = +\frac{d\phi}{dt} \times \frac{1}{R}$$



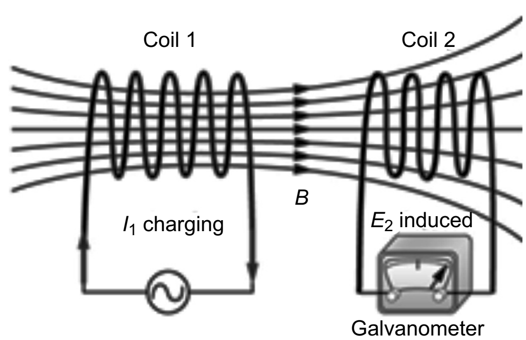
- These currents were discovered by Foucault, so they are also known as Foucault's currents.
- The direction of eddy currents is given by Lenz's law.
- The production of eddy currents in a metallic block leads to loss of electrical energy in form of heat. This heat breaks the insulation used in electrical appliances.
- Eddy currents may cause unwanted dampening effect.

Self Inductance

- The turn induces an EMF in the same coil which opposes the change in flux.
- Self-inductance takes place if changing current in a circuit results in induced emf that opposes the change in circuit which happens as some magnetic flux if produced in circuit tends to pass through same circuit.
- If magnetic flux Φ_B is passing through N turns of coil which is proportional to current I in the coil, then proportionality constant is self inductance, $L = N\frac{\Phi_B}{I}$
- The induced EMF E, due to self-inductance of coil is $E = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
- Self Inductance is measured in Henry (H), $\Phi = B \cdot A = \mu_0 NI\frac{A}{l}$ or $L = \mu_0 N^2 \frac{A}{l}$

Mutual Inductance

- If two coils are placed near to each other in same plane, then changing current in one coil induces an EMF in other coil.
- Faraday's law shows that EMF E_2 induced in coil 2, is proportional to rate of change of magnetic flux passing through it.
- If $\Phi_{B_{2,1}}$ be magnetic flux in each loop of coil 2 created by the current in coil 1 and if coil 2 has N_2 turns, then $N_2\Phi_{B_{2,1}}$ is total flux in coil 2 and Mutual Inductance as per Faraday's Law shows change in current in coil 1 to EMF induced in coil 2, $M_{2,1} = \frac{N_2\Phi_{B_{2,1}}}{I_1}$



- Faraday's law shows EMF $E_2 = -N_2\frac{d\Phi_{B_{2,1}}}{dt}$ and magnetic flux $\Phi_{B_{2,1}} = M_{2,1}\frac{I_1}{N_2}$ or $\frac{d\Phi_{B_{2,1}}}{dt} = \frac{M_{2,1}}{N_2}\frac{dI_1}{dt}$

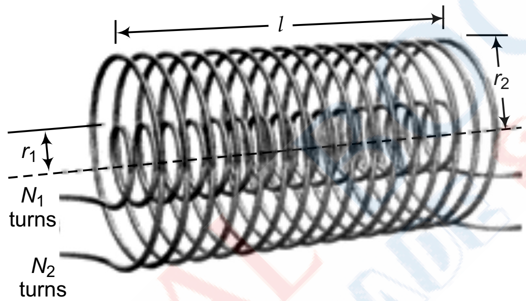
- The change in current in coil 1 to EMF induced in coil 2 shows $E_2 = -N_2 \frac{M_{2,1}}{N_2} \frac{dI_1}{dt} = -M_{2,1} \frac{dI_1}{dt}$
- The mutual inductance of coil 2 with respect to coil 1 is constant that depends on geometry of two coils.
- If change in current in coil 2 induces an EMF in coil 1, then $E_1 = -M_{1,2} \frac{dI_2}{dt}$ or mutual inductances $M_{1,2} = M_{2,1}$

  so that $E_1 = -M \frac{dI_2}{dt}$, $E_2 = -M \frac{dI_1}{dt}$
- The SI Units for mutual inductance is Henry (H), 1H = 1V·s/A= 1Ω·s

Coaxial Solenoid

- If a long thin solenoid of length l, radius r_1, with N_1 turns has coil 2 that is wrapped around coil 1, of radius r_2, with N_2 turns, then mutual inductance of coils is $M = \mu_0 n_1 n_2 l \pi r_1^2$
- Coefficient of coupling

  $K = \sqrt{\frac{M}{L_1 L_2}}$



Remember

- With Faraday's law of induction, mutual inductance of two coils is

  $$M_{12} = N_{12}\Phi_{12}/I_1 \text{ and } M_{21} = N_1\Phi_{21}/I_2$$
- Induced emf in coil 2 due to change in current in coil 1 is $\varepsilon_2 = -MdI/dt$
- Self-inductance of coil with N turns is $L = N\Phi B/I$
- Self-induced emf responding to change in current in coil current is $\varepsilon_L = -LdI/dt$
- Inductance of solenoid with N turns, cross sectional area A and length l is $L = \mu_0 N^2 A/l$
- If battery supplies emf ε connected to inductor and resistor in series at time $t = 0$, then current in RL circuit as a function of time is $I(t) = \varepsilon/R(1 - e^{-i/t})$
- Magnetic energy stored in inductor with current I passing through is $E_B = (1/2)LI^2$
- Magnetic energy density at a point with magnetic field B is $U_B = \frac{B^2}{2\mu_0}$.

For two inductors having inductance L_1 and L_2

- Coefficient of coupling $K = \sqrt{\frac{M}{L_1 L_2}}$
- For series

  When L_1 and L_2 are connected in series and current is flowing in the same direction :

  Equivalent Inductance $L = L_1 + L_2 + 2M$ … for $k = 1$

  $L = L_1 + L_2$ … for $k = 0$

  When current is flowing in opposite direction :

  Equivalent Inductance $L = L_1 + L_2 - 2M$ … for $k = 1$

  $= L_1 + L_2$ … for $k = 0$

- For parallel

When L_1 and L_2 are connected in parallel and current is flowing in the same direction :

Equivalent Inductance $L = \dfrac{L_1L_2 - M^2}{L_1 + L_2 + 2M} \cdots$ for $k = 1$

$= \dfrac{L_1L_2}{L_1 + L_2} \cdots$ for $k = 0$

When current is flowing in opposite direction :

Equivalent Inductance $L = \dfrac{L_1L_2 - M^2}{L_1 + L_2 - 2M} \cdots$ for $k = 1$

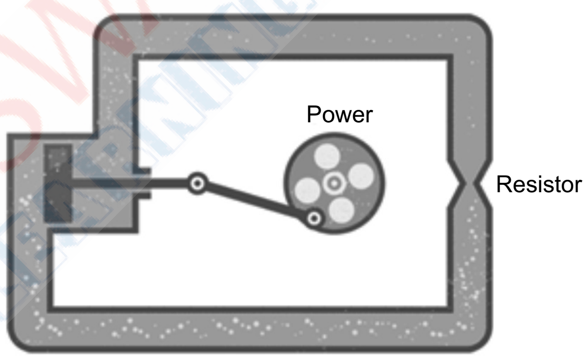
$L = \dfrac{L_1L_2}{L_1 + L_2} \cdots$ for $k = 0$

Topic 2
Alternating Current

» Revision Notes

Alternating currents

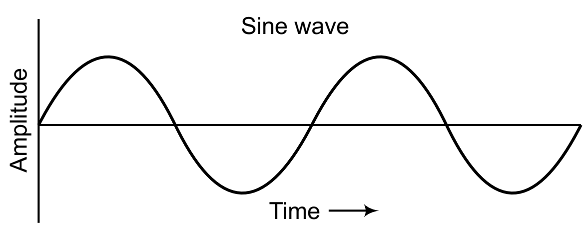
- It is the rate of flow of charge which changes the magnitude and direction periodically as a result of which, fashion voltage also varies in the same fashion along with current. or, $i = i_0 \sin \omega t$
- It is produced by using an alternator which is an electrical generator that produces current when charge carriers in a conductor or semiconductor periodically reverse the direction of movement.



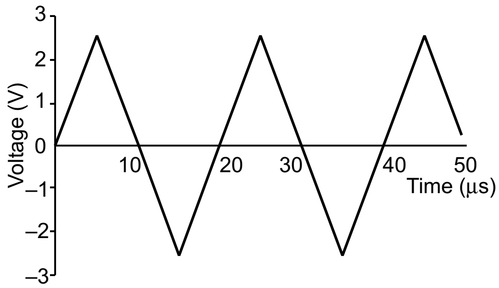
- The loop of wire is rotated in magnetic field that induces a current along the wire.
- As wire turns and enters different magnetic polarity, voltage and current alternate on the wire.

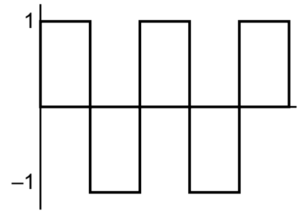
Waveform

- Sine wave is the most common type of alternating waveform.



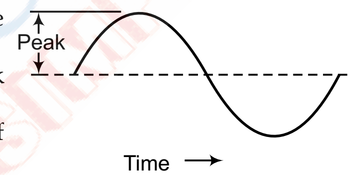
- The other types of alternating waveforms include square wave and triangle wave which are used in digital and switching electronics and for testing linear electronics devices, like amplifiers.





Peak and rms Value of Alternating Current/Voltage

- The alternating voltage or current is expressed in terms of peak (maximum) value, average (mean) value or effective (rms) value.
- In specifying alternating voltage or current, the peak or maximum value is rarely used as it has value which is twice of each cycle.
- The average or mean value cannot be used as it is positive as much as it is negative, so average value results zero.
- The average value over half cycle is used which is not a logical choice as that of effective (virtual/rms) value, which relates to power developed in resistance using alternating voltage or current.
- To express intensity or magnitude/amplitude of AC quantity, measure the peak height on waveform, known as peak or crest value of AC waveform.
- To measure total height between opposite peaks, measure peak-to-peak value of AC waveform.
- To express amplitude of different waveforms, average the values of points of waveform and find average value.



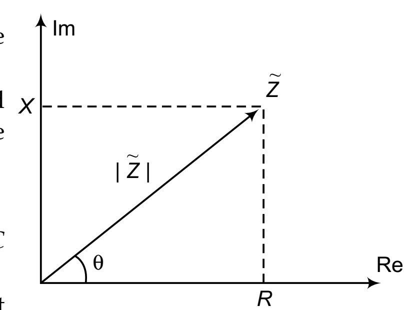
- Averaging of all points on waveform shows the average value. For many waveforms result may be zero as all positive points get cancel with negative points over complete cycle.
- The average value of AC current shows equal amount of charges in DC current, $V_{avg} = (2/\pi) \times V_{peak}$
- The RMS value of AC current shows equal amount of power in DC current, $V_{rms} = V_{peak}/\sqrt{2}$
- The AC current takes less amount of charges to supply same amount of DC power, Power = $I^2 \times R$

Reactance (Ω) (X)

- Reactance are capacitive reactance and inductive reactance which are created by capacitor and inductor respectively.
- Reactance is expressed in (Ω) ohms and behaves in same way as resistance for resisting the flow of current through a circuit.
- The formula for inductive reactance is $X_L = 2\pi fL$ and for capacitive reactance is, $X_C = 1/2\pi fC$

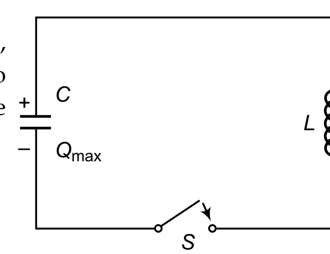
Impedance (Ω) (Z)

- It includes resistance and reactance in opposition to electric currents.
- It is measured in Ω.
- The opposite of impedance is admittance, which is measure of allowance of current.
- In a complex impedance plane, impedance is Z, resistance is R and reactance is X, the total impedance will be the square root of sum of the squares of resistance and reactance, $Z = \sqrt{(R^2) + (X^2)}$



***LC* Oscillations (qualitative treatment only)**

- LC circuit is a resonant electric circuit with inductor L and capacitor C joined together.
- The circuit acts as electrical resonator for storing energy, oscillating at circuit's resonant frequency.
- In an ideal capacitor and inductor circuit with no common internal resistance, for capacitor C to charge with initial charge q_o, a capacitor is connected to inductance L where flow of current i is analysed with voltage V across the components,

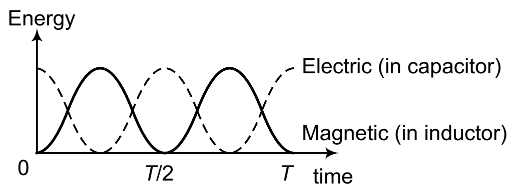


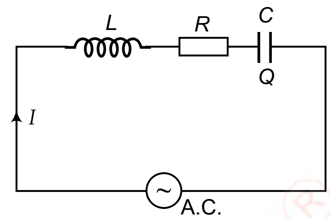
$$V = -L\frac{dI}{dt} = \frac{Q}{C} \text{ or } I = dQ/dt \text{ or } \frac{dI}{dt} = \frac{d^2Q}{dt^2} \text{ so } \frac{d^2Q}{dt^2} = -\frac{Q}{LC}$$

- The equation $\frac{d^2Q}{dt^2} = -\frac{Q}{LC}$ has a solution $Q = Q_0 \cos \omega_0 t$, where $\omega_0 = \frac{1}{\sqrt{LC}}$

i.e., charge oscillates with frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$.

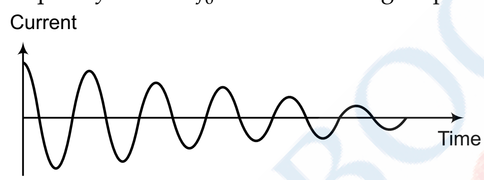
- Stored energies alternate between the capacitor and inductor.





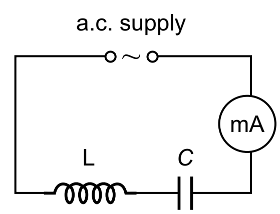
Damping in LC circuit

- In LCR circuit, total potential difference across circuit is $\dfrac{Q}{C} + L\dfrac{dI}{dt} + IR = 0$ or, $L\dfrac{d^2Q}{dt^2} + R\dfrac{dQ}{dt} + \dfrac{Q}{C} = 0$
- Due to inevitable resistance, there is energy loss through emission of EM waves due to oscillation of electrons.
- The current oscillates with frequency close to f_0 with decreasing amplitude.



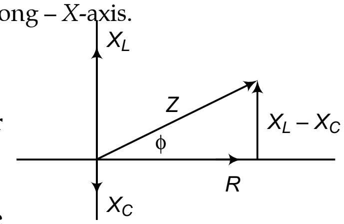
LCR Series Circuit

- In LCR circuit, total resistance to flow current is known as impedance, Z which is combination of resistance, capacitive reactance, and inductive reactance.
- In RLC circuit, resistance and reactance are added as vectors due to phase relationships.
- With a resistor, voltage and current are in phase; with capacitor, current is 90° ahead of voltage; and with inductor, voltage leads current by 90°.

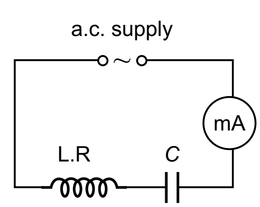


- The resistance R drawn along $+x$-axis of x-y coordinate system where inductive reactance is 90° to it which is drawn along $+y$-axis where capacitive reactance is 90° to resistance drawn along $-X$-axis.
- The impedance, Z, is sum of vectors given as $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- The current and voltage in RLC circuit is related by $V = IZ$.
- The phase relationship between current and voltage is obtained from vector diagram, where its angle between impedance Z and resistance R is

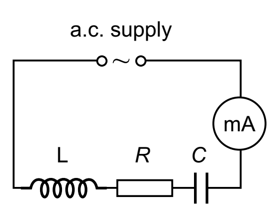
$$\tan \Phi = (X_L - X_C)/R$$



- If angle is positive, voltage leads the current and if angle is negative, voltage lags the current.
- Power dissipated in RLC circuit is $P = VI \cos \phi$
- Impedance $Z = V/I = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$
- $V_0^2 = I_0^2\left[\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2\right]$
- $(V_C)_{max} = \dfrac{I_0}{\omega C} = \dfrac{V_0}{\left[\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2\right]^{1/2}} \times \dfrac{1}{\omega C}$

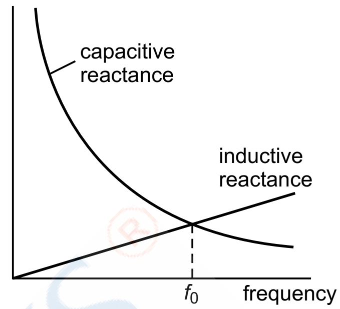


- $I_{max} = I_0 = \dfrac{V_0}{\left[\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2\right]^{1/2}}$ or $(E_C)_{max} = \dfrac{1}{2}C(V_c)^2_{max} = \dfrac{V_0^2}{2\left[\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2\right]} \times \dfrac{1}{\omega^2 C}$
- $(E_L)_{max} = \dfrac{1}{2}LI_0^2 = \dfrac{LV_0^2}{2\left[\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2\right]}$



- If $(E_C)_{max} = (E_L)_{max}$ then, we have $\frac{1}{\omega^2 C} = L$ or $\omega = \frac{1}{\sqrt{LC}}$

Resonance



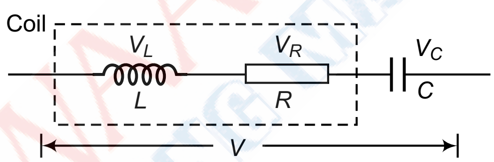
- In AC circuits, there exists special condition when capacitive impedance is equal to inductive impedance where impedance equation is $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- If inductive and capacitive reactance are equal then, impedance is minimized and is equal to reactance $Z = R$, where phase angle is 0 and power factor is 1 with maximum current and more power transfer that results into resonance.
- For resonance to be occurred :

$$X_L = X_C$$

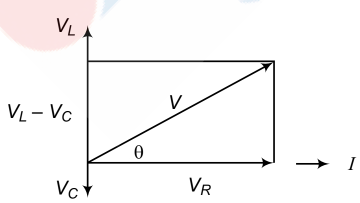
$$\Rightarrow \quad 2\pi f L = \frac{1}{2\pi f_0 C} \quad \Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

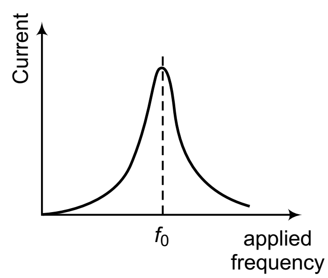
where f_0 = resonant frequency

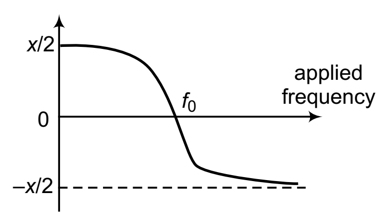
- The AM/FM radio uses resonant circuit where current rises sharply at resonance.
- If radio is tuned to required radio station that broadcasts at particular frequency, it shows that circuit has resonance frequency similar to station's broadcast frequency.
- The oscillator frequency varies till maximum current is observed in *LCR* circuit.
- The damping of induced current reduces sharpness in response that is produced by increasing resistance *R*.
- If $V = V_0 \sin \omega t$ is applied across capacitor *C* then, corresponding current, $I = \frac{dQ}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$
- $I = \omega C V_0 \cos \omega t$, which shows that voltage across capacitor lags behind the current by $\pi/2$.
- If current $I = I_0 \sin \omega t$, passes through an inductor *L*, then e.m.f = $L\frac{dI}{dt} = L\frac{d}{dt}(I_0 \sin \omega t)$



- The $\omega L I_0 \cos \omega t$ shows that voltage across inductor is ahead of current by $\pi/2$.
- In pure resistance *R*, current and voltage are in phase.
- The instantaneous voltages across components by phasor is : $V_L = \omega L I$, $\pi/2$ ahead of I; $V_R = IR$ in phase with I; $V_C = \frac{1}{\omega C} I$, $\pi/2$ lag behind I.



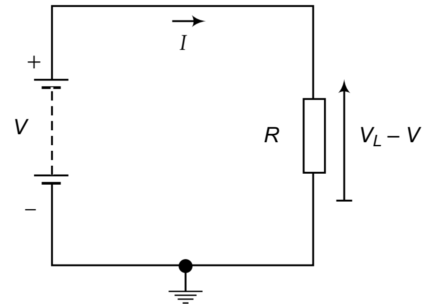




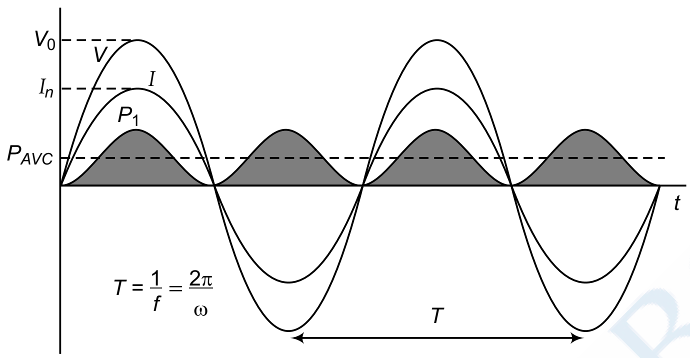
- In the phasor diagram, at low frequencies, $V_C >> V_L$ and phase angle θ is – ve that tends to $-\pi/2$, *I* ahead by $\pi/2$; at high frequencies $V_L >> V_C$ and θ as + ve, which tend $+\pi/2$, that *I* lags applied voltage by $\pi/2$; at resonance, $V_L = V_C$ at natural oscillation frequency, $I = V/R$ and is maximum, at resonance $\theta = 0$, current is in phase with applied voltage.

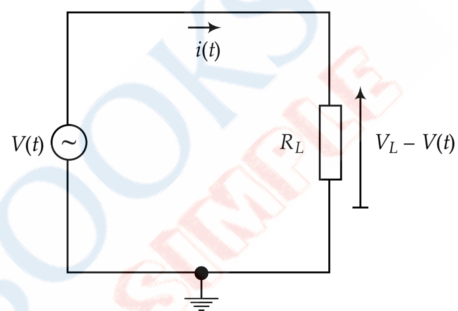
Resistive Power Dissipation

- Power dissipation in resistor supplied by DC battery where voltage across resistor is fixed with current flowing through it, is constant and invariant with time, $P = VI = \frac{V^2}{R} = I^2 R$



- In AC voltage source circuit, voltage across resistor varies with time and current flows through it.





- In resistor, voltage and current are in phase with each other so that waveforms of each are similar with similar function of time as in case of sinusoidal source.
- Power is the product of voltage and current is a function of time that varies in sinusoidal manner, so

$$V(t) = V_m \sin \omega t \quad \text{or} \quad i(t) = I_m \sin \omega t \quad \text{and} \quad I_m = V_m / R \quad \text{or} \quad P_i(t) = V(t)i(t)$$

Instantaneous Power

- $P_i(t) = V(t)i(t)$ where $V(t) = V_m \cos \omega t$ and $i(t) = I_m \cos \omega t$ or $P_i = V_m I_m \cos^2 \omega t$ showing instantaneous power as $P_i = \dfrac{V_m I_m}{2}(1 + \cos 2\omega t)$

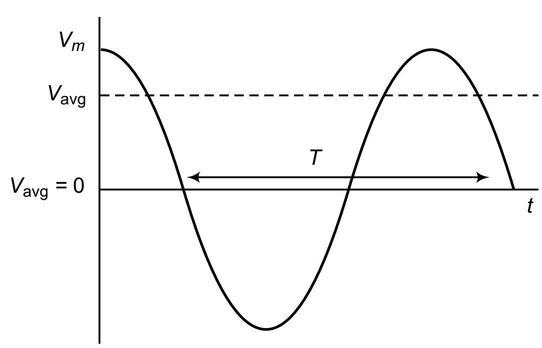
Average Power

- It is the average value of instantaneous power.
- In periodic source, average power is obtained over one full cycle of source which is delivered to the load, so equivalent value of constant power $P_{AVE} = \dfrac{1}{T}\int_0^T \dfrac{1}{2} V_m I_m (1+\cos 2\omega t)$

$$P_{AVE} = \frac{V_m I_m}{2T}\int_0^T 1.dt + \frac{V_m I_m}{2T}\int_0^T \cos 2\omega t.dt$$

$$P_{AVE} = \frac{V_m I_m}{2T} \times T \quad = \frac{V_m I_m}{2}$$

- For resistive load $I_m = \dfrac{V_m}{R}$ then $P_{AVE} = \dfrac{V_m I_m}{2} \quad = \dfrac{I_m{}^2 R}{2}$

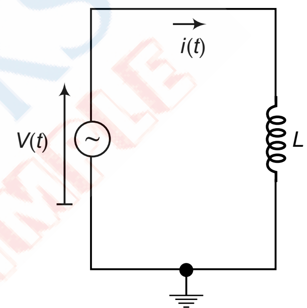


- From RMS value of a sinusoidal waveform, root of mean of square, $V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt}$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2 \omega t.dt} = \sqrt{\frac{V_m^2}{2T}\int_0^T (1+\cos 2\omega t).dt} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

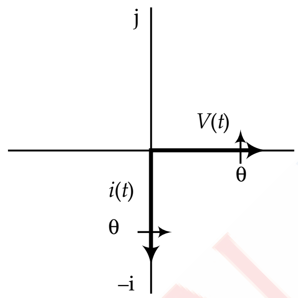
- The rms value of AC sinusoidal source voltage is such value of voltage that delivers similar average power to load as DC supply having similar value, $P_{AVE} = V_{RMS} I_{RMS} = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R$
- For sinusoidal source, instantaneous power varies from 0, when $V(t) = 0$, $i(t) = 0$ to maximum, when $V(t) = V_m$ and $i(t) = I_m$.
- For sinusoidal source, average power is half of peak power.

Power in Inductive Load



- The average power is determined in same way to that for resistive load,

$$V(t) = V_m \sin \omega t \text{ or } i(t) = -I_m \cos \omega t$$

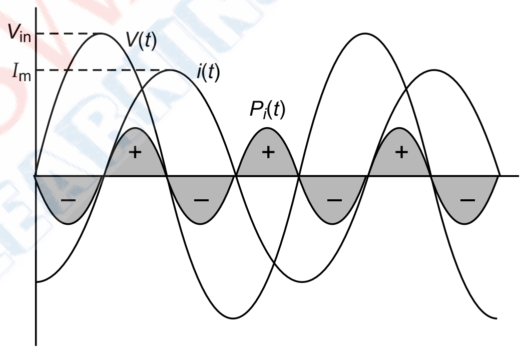


- In the waveform, instantaneous power is alternately positive and negative at twice frequency of source.

$$P_i = V(t)\, i(t) = -V_m \sin \omega t\, I_m \cos \omega t$$

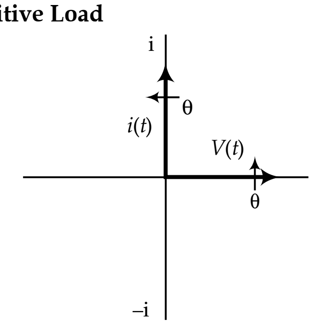
or,

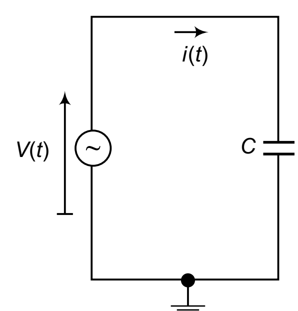
$$P_i = \frac{-V_m I_m}{2} \sin 2\omega t$$



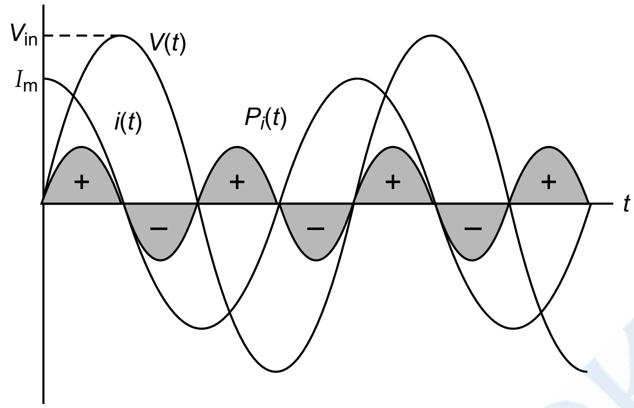
- Average power $P_{AVE} = -\frac{1}{2T}\int_0^T V_m I_m \sin 2\omega t$
- Average power dissipated in pure inductance is zero $P_{AVE} = \frac{V_m I_m}{4\omega T}|1-1| = 0$

Power in Purely Capacitive Load





- The average power dissipated by capacitor is $V(t) = V_m \sin\omega t$ and $i(t) = I_m \cos\omega t$ then $X_C = \frac{1}{\omega C}$



- The instantaneous power in waveform alternates between positive and negative phases at twice frequency of source., $P_i = V(t)\, i(t) = V_m \sin\omega t \cdot I_m \cos\omega t$

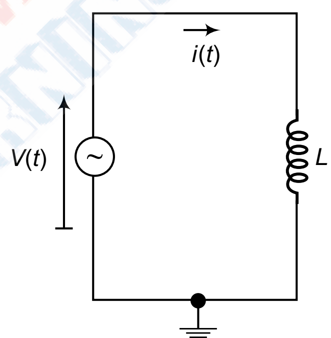
and $$P_i = \frac{V_m I_m}{2} \sin 2\omega t$$

- For capacitor, average power dissipated in capacitor is zero as power is drawn from supply and stored as energy in capacitor for quarter of cycle and returned to source during quarter cycle, $P_{AVE} = -\frac{V_m I_m}{2T}\int_0^T \sin 2\omega t = 0$

Imaginary Power

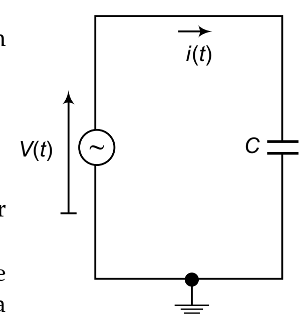
- The power transferred to inductor and capacitor in reactive circuits is temporarily stored and returned to source, hence power that is not dissipated is considered as imaginary power,

$$i(t) = \frac{V(t)}{Z_L} = \frac{V(t)}{\hat{j}\,\omega L} = -\hat{j}\frac{V(t)}{\omega L} \text{ So } P_i = -\frac{1}{2} V_m I_m \sin 2\omega t$$



- The imaginary power will be $P_i = j\frac{V_m^{\ 2}}{2\omega L}\sin 2\omega t$ [For inductive circuit]
- In a capacitive circuit, inductance and capacitor consume imaginary power, which is not dissipated energy or power which they draw from the supply,

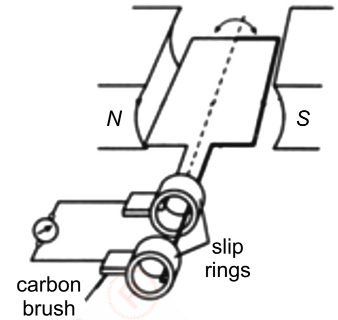
$$P_i = j\frac{1}{2}V_m^2 \omega C \sin 2\omega t$$



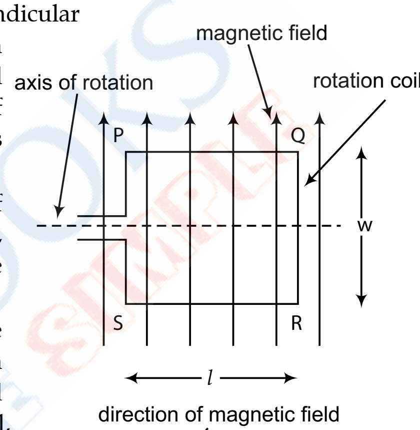
Wattless Current

- It is also known as ideal current which appears in AC circuit where average power consumed in the circuit corresponds to zero.
- In an inductive or capacitive AC circuits with no ohmic resistance, phase angle $\phi = \pi/2$ and $\cos\phi = 0$, the circuit does not consumes any power, and offers a resistance to flow.

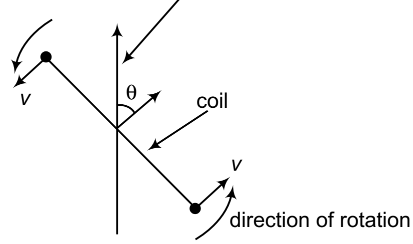
AC Generator



- A.C. Generator or an alternator is a device which converts mechanical energy into alternating electric energy having field winding and armature winding.
- Field winding on poles needs DC current supplied from slip rings and carbon brushes where poles are similar with symmetry having alternate polarities.
- Armature winding has stator made of alloy with copper windings placed in slots that act as three phase winding where three terminals of connection along with fourth neutral terminal in a terminal box.
- A.C generator works on the principle of electromagnetic induction where an induced emf is produced by rotating coil in magnetic field where flux linking coil changes continuously showing continuous fluctuating emf.
- If l is length of coil along axis of rotation, and w is width of coil perpendicular to its axis, so if coil rotates with fixed angular velocity ω in uniform magnetic field B, then velocity v with which two long sides of coil (PQ and RS) moves through magnetic field results as product of angular velocity of rotation ω with distance $l/2$ of each side from axis of rotation, $v = w\omega/2$



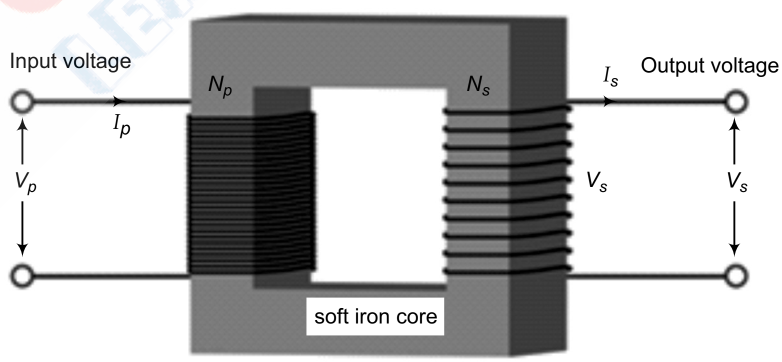
- The motional emf induced in each side is $\varepsilon = B\perp(l \times v)$, shows that if direction of magnetic field subtends an angle θ with normal to coil, then $B\perp = B\sin\theta$, so magnitude of motional emf generated will be $\varepsilon_{ab} = Bwl\omega \sin\theta/2$ and $\varepsilon_{ab} = BA\omega\sin\theta/2$
- If direction of rotation of coil is such that side PQ is moving in page while RS is moving out of page, then motional emf induced in side PQ will act from P to Q and motional emf induce in side RS will act from R to S, so both *emfs* acts in clockwise direction around the coil, having net emf ε acting around the coil as $2\varepsilon_{ab}$.



- If coil having N turns, then net emf is $2N\varepsilon_{ab}$ then emf generated around a steadily rotating, multi-turn coil in uniform magnetic field is $\varepsilon = NBA\omega\sin(\omega t) = \varepsilon_{max}\sin(2\pi ft)$ where $\varepsilon_{max} = 2\pi NBAf$ is the peak emf by generator.
- If load of resistance R is connected across terminal of generator by connecting two ends of the coil with rotating rings using metal brushes, current I flowing in load will be $I = \varepsilon/R = \varepsilon_{max}/R \sin(2\pi ft)$ where current keeps on changing its direction.

Transformers

- Consider N_p, N_s be the number of turns in primary and secondary coils and V_p and V_s are their respective voltages.



$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

- A transformer is a static electrical device that transfers electrical energy between two or more circuits. A varying current in one coil of the transformer produces a varying magnetic flux, which in turn, induces a varying EMF across a second coil wound around round the same core.
- In transformer, relationship between number of turns in each coil and voltage across each winding is $V_s/V_p = N_s/N_p$, where symbols have usual meanings.

- $\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = k$

  where k = **transformation ratio** or **turns ratio**
- $k < 1$ for step down transformer (here, $N_p < N_s$)
- $k > 1$ for step up transformer (here, $N_p > N_s$)
- For an ideal transformer,
- Output power = Input power

$$V_s I_s = V_p I_p$$

- Efficiency, $\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_s I_s}{V_p I_p}$
- For an ideal transformer $\eta = 100\%$ but in practice, it is not possible because of certain loss of energy.

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Chapter 15

Electromagnetic Waves

» Revision Notes

Need for displacement current

- Displacement current is a quantity that appears in Maxwell's equations in terms of rate of change of electric displacement field.
- It has similar units as of electric current.
- As per Ampere's law,

$$\oint_c \vec{B} \cdot \vec{dl} = \mu_0 i$$

- This equation is valid only for steady current or when the electric field at the surface doesn't change with time.
- When a changing electric current flows through the wire, charge on the plates of the capacitor changes with time. Thus a changing electric field is set up between the plates of the capacitor.
- Maxwell discovered the concept of displacement current. Displacement current is that which results due to the displacement of electrons. The displacement of electrons is caused by the time varying electric field.
- The generalized form of Ampere's law $\oint B.dl = \mu_0 I$, which includes both conduction current I_C (current flowing through the conducting wire) and displacement current I_C as given by Maxwell *i.e.*, $\oint B \, . \, dl = \mu_0 \, (I_C + I_D)$

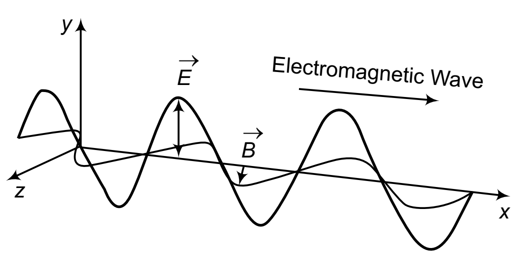
  where $I_C = \frac{dq}{dt}$, where $\frac{dq}{dt}$ is rate of flow of charge.

  and $\qquad I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$

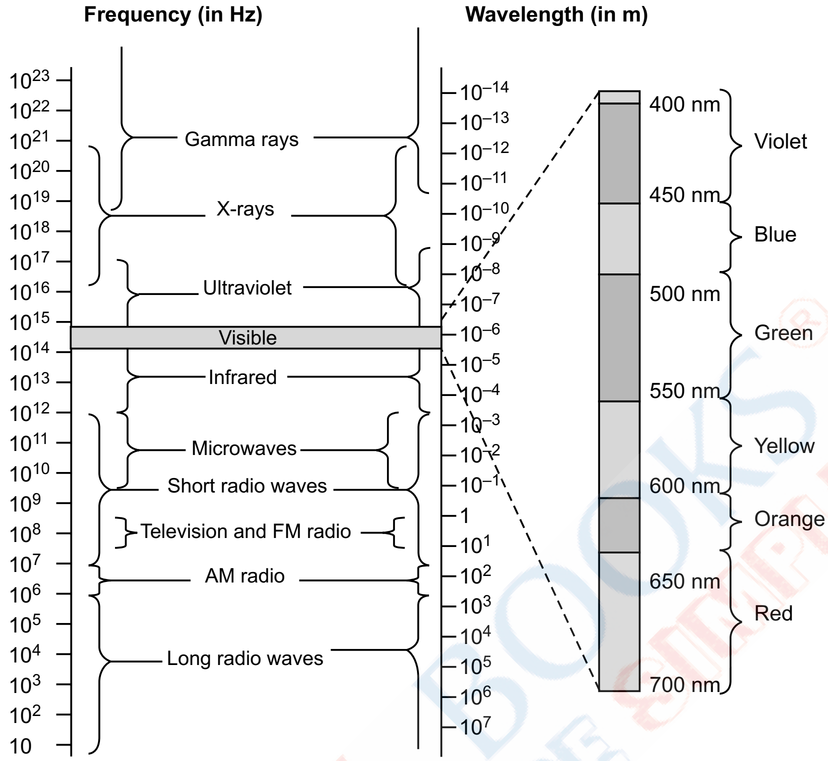
  Here Φ_E is electric flux.
  Therefore, modified Ampere's law may be expressed as

$$\int \vec{B} \, . \, \vec{dl} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Electromagnetic waves



- These waves are propagated by oscillations of electric and magnetic fields where change in electric field produces a changing magnetic field and change in magnetic field produces a changing electric field, so electromagnetic wave is self propagating which needs no medium to travel.
- A charge oscillating harmonically is a source of electro-magnetic wave of same frequency.
- Electromagnetic waves travel at a speed of light and are described by their wavelength and frequency.
- In Maxwell's Equations, pair of electric and magnetic waves propagate together with a speed of c = 299,792,458 m/s known as speed of light.
- EM radiations are classified as per the frequency and wavelength of wave such as radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.



- **Characteristics of Electromagnetic waves (qualitative idea only)**
  - Magnetic field oscillates in phase with electric field.
  - Electric field is perpendicular to magnetic field and fields are directed at right-angles to direction of propagation of wave $\vec{E} \times \vec{B}$.
  - For z-directed wave, electric field is free to oscillate in any direction which lies in x-y plane.
  - The relation between maximum amplitudes of electric and magnetic fields is $E_0 = c \times B_0$
  - There is no constraint on possible frequency or wavelength of electromagnetic waves, but propagation velocity of electromagnetic waves is fixed and have value $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

- **Transverse nature of Electromagnetic waves**
  - In electromagnetic wave, electric and magnetic field vectors oscillate perpendicular to the direction of propagation of electromagnetic wave showing transverse nature.
  - At any instant, the electric and magnetic fields varying sinusoidally with x can be expressed by equation

$$E_y = E_0 \sin(x - ct) \qquad \text{...(1)}$$

and

$$B_z = B_0 \sin(x - ct) \qquad \text{...(2)}$$

Here E_0 and B_0 are the amplitudes of electric and magnetic fields along y-axis and z-axis respectively.
Faraday's law of electromagnetic induction states that

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt} \qquad \text{...(3)}$$

From equation (3), it can be proved that for a plane electromagnetic wave propagating along x-axis,

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \text{...(4)}$$

i.e., a magnetic field varying with time gives rise to an electric field varying in space.
From equation (1), we have

$$\frac{\partial E_y}{\partial x} = E_0 \cos(x - ct)$$

and from equation (2), we have

$$\frac{\partial B_z}{\partial t} = -cB_0 \cos(x - ct)$$

Substituting for $\frac{\partial E_y}{\partial z}$ and $\frac{\partial B_z}{\partial t}$ in equation (4)

$$E_0 = cB_0 \text{ or } c = \frac{E_0}{B_0}$$

It shows that the velocity of electromagnetic waves is equal to the ratio of amplitudes of electric and magnetic fields.
Velocity of electromagnetic waves is also given by

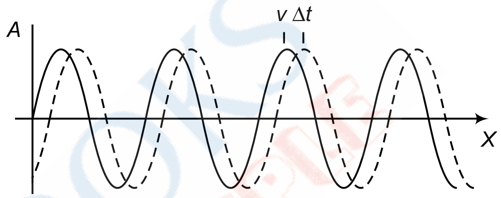
$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Phase, Group, and Signal Velocity

- If a plane EM wave is travelling along x-direction, the sinusoidal nature of the wave is shown is figure. The magnitudes of the instantaneous values are given :

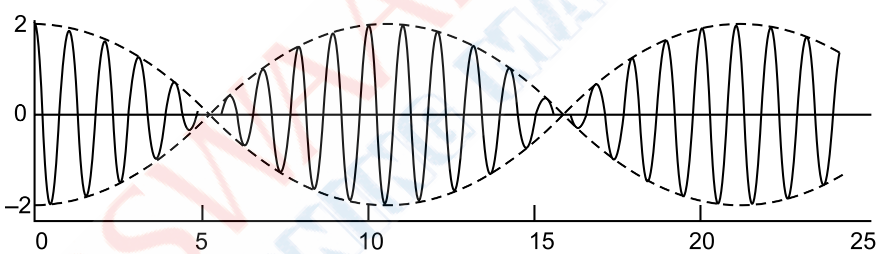
$$E_y = E_0 \sin(kx - wt) \quad \text{...(i)}$$

$$B_z = B_0 \sin(kx - wt) \quad \text{...(ii)}$$



where $k = \frac{2\pi}{\lambda}$ is the propagation constant.

- Phase velocity (v_p) $\frac{\lambda}{T} = \frac{\omega}{k}$
- Group velocity (v_g) $= \frac{\partial \omega}{\partial k}$
- Signal velocity (v_s) $= \frac{c}{\sqrt{\varepsilon_n \mu_n}}$



Remember

(i) Electromagnetic waves are produced by accelerated or oscillating charge.

(ii) Electromagnetic waves propagate in form of varying electric and magnetic fields that are perpendicular to each other and to direction of propagation of wave (showing transverse nature).

(iii) Electromagnetic waves obey superposition principle.

(iv) In free space or vacuum, electromagnetic waves travel with speed of light $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

(v) The velocity of electromagnetic waves is independent of amplitude of field vectors and depends only on electric and magnetic properties of medium in which they propagate.

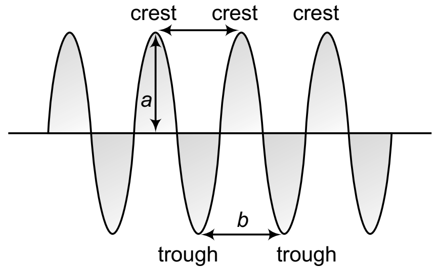
(vi) The energy of electromagnetic waves is divided between electric field and magnetic field vectors.

(vii) In free space, relation between amplitudes of electric fields and magnetic fields is $c = E_0/B_0$

(viii) Since electromagnetic waves are chargeless, they are not deflected by electric and magnetic fields.

Types of Electromagnetic waves (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays)

- Electromagnetic waves require no medium to travel or propagate.
- Varying Electric and magnetic fields create electromagnetic waves.
- Electromagnetic waves are transverse waves which measure their amplitude, wavelength, or distance between highest/lowest points.
- In electromagnetic waves a crest is the highest point of the wave and trough he lowest point of wave in a cycle.



a = Amplitude
b = Wavelength

Electromagnetic spectrum is divided into following regions :

- The electromagnetic spectrum is the distribution of electromagnetic radiation in terms of energy, frequency or wavelength. The electromagnetic radiation can be described as a stream of photons travelling in a wave like pattern, at the speed of light.

Type of Radiation	Frequency Range	Wavelength Range
Gamma rays	$< 3 \times 10^{20}$	< 1 fm
X-rays	$3 \times 10^{17} - 3 \times 10^{20}$	1 fm – 1 nm
Ultraviolet	$7.5 \times 10^{14} - 3 \times 10^{17}$	1 nm – 400 nm
Visible	$4 \times 10^{14} - 7.5 \times 10^{14}$	0.4 μm – 0.75 μm
Near-infrared	$10^{14} - 7.5 \times 10^{14}$	0.75 μm – 3.0 μm
Midwave infrared	$5 \times 10^{13} - 10^{14}$	3.0 μm – 6 μm
Long wave infrared	$2 \times 10^{13} - 5 \times 10^{13}$	6.0 μm – 15 μm
Extreme infrared	$3 \times 10^{13} - 2 \times 10^{13}$	15 μm – 10 μm
Micro and radio waves	$< 3 \times 10^{11}$	> 1 mm

Uses of Electromagnetic Waves

Band Designation	Applications
Audible	Acoustics
Extremely Low Frequency (ELF) Radio	Electronics, Submarine Communications
Infra Low Frequency (ILF)	Not applicable
Very Low Frequency (VLF) Radio	Navigation, Weather
Low Frequency (LF) Radio	Navigation, Maritime Communications, Information and Weather Systems, Time Systems
Medium Frequency (MF) Radio	Navigation, AM Radio, Mobile Radio
High Frequency (HF) Radio	Citizens Band Radio, Mobile Radio, Maritime Radio
Very High Frequency (VHF) Radio	Amateur (Ham) Radio, VHF TV, FM Radio, Mobile Satellite, Mobile Radio, Fixed Radio
Ultra High Frequency (UHF) Radio	Microwave, Satellite, UHF TV, Paging, Cordless Telephone, Cellular and PCS Telephony, Wireless LAN (WiFi)
Super High Frequency (SHF) Radio	Microwave, Satellite, Wireless LAN (WiFi)
Extremely High Frequency (EHF) Radio	Microwave, Satellite, Radio-location
Infrared Light (IR)	Wireless LAN Bridges, Wireless LANs, Fiber Optics Remote control
Visible Light	Photographic plate, photocells
Ultraviolet (UV)	Photocells, kill bacteria and germs.
X-rays	In medical, Geiger tubes, ionisation chamber.
Gamma and Cosmic Rays	In medical (cancer cell killing)

Types of Electromagnetic Waves, Wavelength Range, Production and Detection

Type of Radiation	Wavelength Range	Production	Detection
Radio	$> 1.0 \times 10^{-1}$ m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1 m – 1.0×10^{-3} m	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1.0×10^{-3} m – 700×10^{-9} m	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700×10^{-9} m – 400×10^{-9} m	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eyes Photocells photographic film
Ultraviolet	400×10^{-9} m – 1.0×10^{-9} m	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1.0×10^{-9} m – 1.0×10^{-12} m	X-ray tubes or inner shell electrons	Photographic film Geiger tubes ionisation chamber
Gamma rays	$< 1.0 \times 10^{-12}$ m	Radioactive decay of the nucleus	Photographic film Geiger tubes Ionisation chamber

Facts and Uses

- Electromagnetic waves are of particular importance as they are only source of information regarding Universe.
- Radio waves and microwaves have provided much knowledge about centre of Galaxy.
- Infrared radiation is useful for detecting proto-stars which are not yet hot enough to emit visible radiation.
- The visible radiation is the mainstay of astronomy.
- Satellite based ultraviolet observations have yielded invaluable insights into the structure and distribution of distant galaxies.
- X-ray and Y-ray astronomy usually concentrate on exotic objects in the Galaxy such as pulsars and supernova remnants.

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Chapter 16

Optics

Topic 1

Ray Optics and Optical Instruments

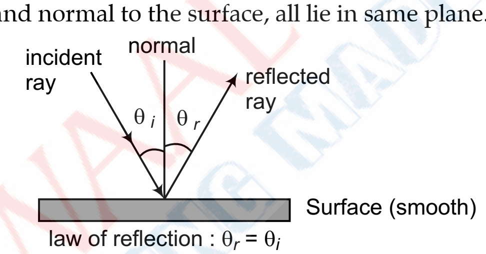
» Revision Notes

Reflection of light

- When a ray of light falls on any polished, smooth and shiny surface then bouncing back of in the same medium is known as reflection of light.

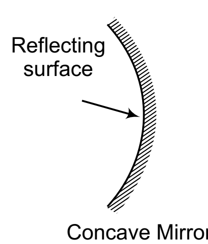
Laws of Reflection

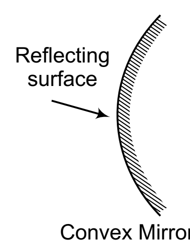
- The angle of reflection equals to the angle of incidence.
- Incident ray, reflected ray and normal to the surface, all lie in same plane.

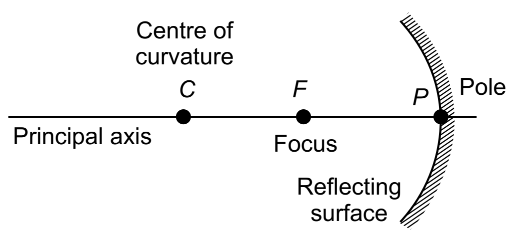


Reflection of light through spherical mirrors

- Spherical mirror is a piece cut out of spherical surface which can be concave and convex.
- For spherical mirrors, surface of a spoon is a perfect example which is not a mirror but acts similar to it.
- The concave mirror is shown as inner surface of a spoon while convex mirror is shown by outer surface of a spoon.
- The centre of the mirror is the pole P.



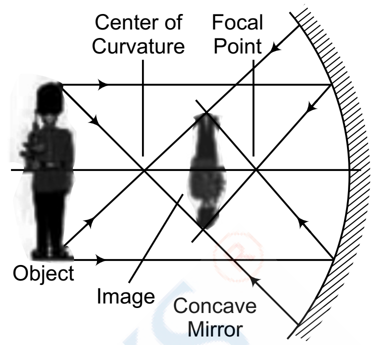




- The centre of the sphere of which mirror is a part, is center of curvature C and the distance of C from pole is called radius of curvature.
- The line joining the pole of mirror to center of curvature is principal axis.
- The midway point between the pole and center of curvature is focal point F and its distance from pole is called focal length 'f'.
- Ray diagrams determine the position of the image formed at a place where rays intersect.
- For finding the image, two rays are required where:

1. **First ray is drawn from tip of the object parallel to principal axis which reflects from the mirror surface and passes through focal point or can be extended back to pass through focal point.**
2. **Second ray is ray which is drawn from the tip of the object to the mirror through center of curvature. This ray hits the mirror at 90° angle and retraces its way. OR**

- A ray that is drawn from the tip of the object through the focal point which reflects off the mirror, parallel to principal axis.



Ray diagram for Concave Mirror

- Ray diagram for concave mirror shows an object placed beyond the center of curvature from the mirror where image lies between the focal point F and center of curvature C.
  In this, image formed is inverted and real as light rays actually passes through the point where the image is located.

The Cartesian Sign Convension

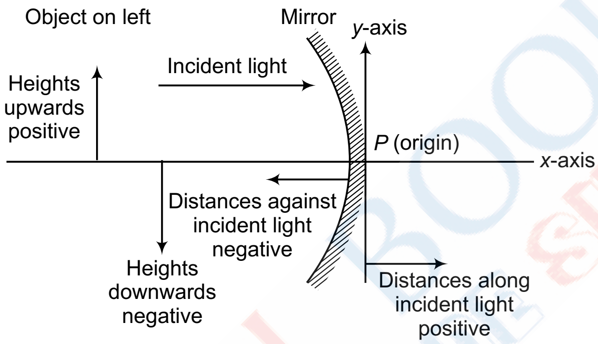


Image formation in concave mirror for different positions of object

Position of the object	Position of the image	Size of the image	Nature of the Image
At infinity	At the focus *F*	Highly diminished, point sized	Real and inverted
Beyond *C*	Between *F* and *C*	Diminished	Real and inverted
At *C*	At *C*	Same size	Real and inverted
Between *C* and *F*	Beyond *C*	Enlarged	Real and inverted
At *F*	At infinity	Highly Enlarged	Real and inverted
Between *P* and *F*	Behind the mirror	Enlarged	Virtual and erect

Image formation in convex mirror for different positions of object

Position of the object	Position of the image	Size of the image	Nature of the Image
At infinity	At the focus *F*, behind the mirror	Highly diminished, point sized	Virtual and Erect
Between infinity and the pole P of the mirror	Between *P* and *F*, behind the mirror	Diminished	Virtual and Erect

Mirror formula

- A mirror formula gives the relationship between the distance of image v, distance of object u, and the focal length of a mirror, $1/f = 1/u + 1/v$ where symbols have usual meanings.
- The mirror formula is applicable both in spherical mirrors and in plane mirrors.

Magnification by Mirror

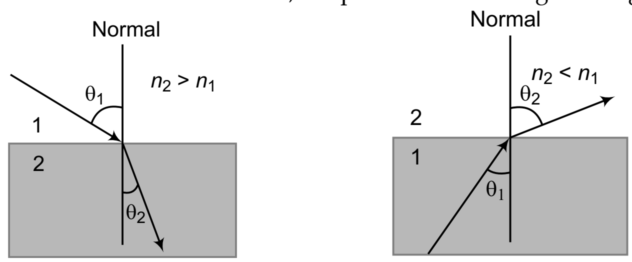
$$m = -\frac{v}{u} = \frac{I}{O}$$

Here, all alphabets are in their usual meanings.

Refraction of light

- Refraction of light is change in direction if it passes from one optically transparent medium to another.
- If the speed of light in a vacuum as 3.00×10^8 m/s and if the light travels through glass, diamond or plastic, it travels at different speed.
- The speed of light in given material is related to index of refraction n, which is ratio of speed of light in vacuum to speed of light in medium, $n = c / v$. It has no unit.

- If light travels from one medium to another, its speed and wavelength changes.



Snell's law : $n_1 \sin \theta_1 = n_2 \sin \theta_2$ or, equivalently, $\sin \theta_1 / \sin \theta_2 = v_1/v_2 = n_2/n_1$

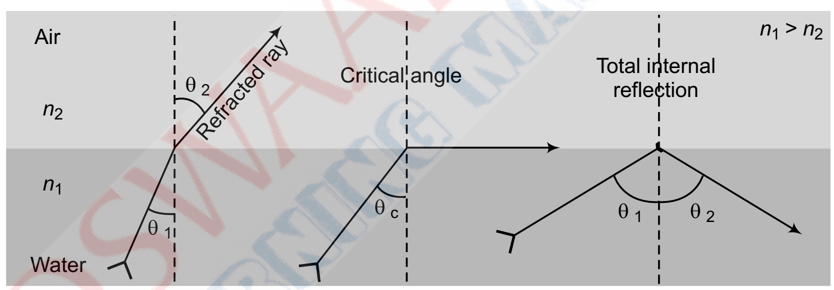
- If speed and wavelength changes, frequency of light is constant then relation between frequency, wavelength and speed is $v = f \times \lambda$
- Refractive index (n) of a medium is $n = \sin i / \sin r$, where greater the value of refractive index, greater is bending effect of light, if it passes from air to that medium.

Laws of Refraction

- First law of refraction shows that incident ray, refracted ray and normal to the interface lies on the same plane.
- Second law of refraction shows that for two given media, ratio $\sin i/\sin r$ = constant, where i is angle of incidence and r is angle of refraction, (Snell's law).

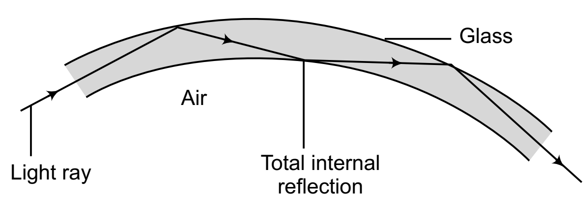
Total internal reflection and its applications

- Total internal reflection is complete reflection of ray of light in optically denser medium from the surrounding surfaces of optically less dense media back into denser medium.
- Light ray travel from an optically denser medium to less dense medium.
- The angle of incidence must be greater than critical angle.



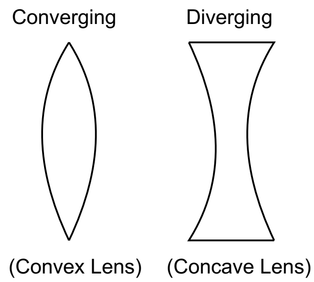
- If ray of light passes from optically denser to less dense medium, then critical angle θ_c is angle of incidence at which the angle of refraction is 90°.
- $$\theta_c = \sin^{-1}\left(\frac{1}{n}\right)$$

Optical fibres

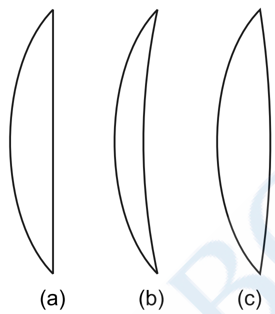


- Optical fibre is thin rod of high quality glass that absorbs very little light.
- In this, light undergoes total internal reflection to transmit data from one end to other.
- They have polished surfaces coated with material of good refractive index so that there is no loss of light through sides of fibre.
- The fibers are used in bundles to transmit and receive light from inaccessible places, by repeated totally internal reflections inside the fibre.
- These fibres replaces metal telecommunication cables, where messages are encoded as digital pulses of light instead of fluctuating electric current having more data and are reliable.
- Bundled optical fibres are used in endoscopes for inspecting inaccessible parts of machines/living body.

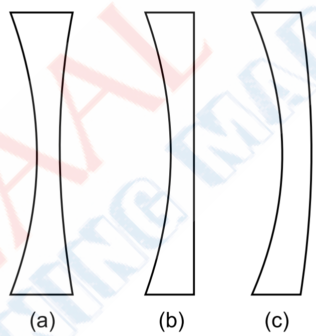
Lenses



- A lens is a transparent medium bounded by two curved surfaces.
- The converging lenses can be plano-convex, convex meniscus, biconvex.



- Concave lens can be biconcave, plano–concave and of concave meniscus



- Nature position and relative size of image formed by a convex lens for various positions of the object.

Position of the object	Position of the image	Relative size of the image	Nature of the Image
At infinity	At focus F_2	Highly diminished, point sized	Real and inverted
Beyond $2F_1$	Between F_2 and $2F_2$	Diminished	Real and inverted
At $2F_1$	At $2F_2$	Same size	Real and inverted
Between F_1 and $2F_1$	Beyond $2F_2$	Enlarged	Real and inverted
At focus F_1	At infinity	Infinitely large or highly enlarged	Real and inverted
Between focus F_1 and optical centre O	On the same side of the lens as the object	Enlarged	Virtual and erect

Nature, position and relative size of the image formed by a concave lens for various positions of the object

Position of the object	Position of the image	Relative size of the image	Nature of the Image
At infinity	At the focus F_1	Highly diminished, point-sized	Virtual and Erect
Between infinity and optical centre O of the lens	Between focus F_1 and optical centre O	Diminished	Virtual and Erect

Lens formula

- $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

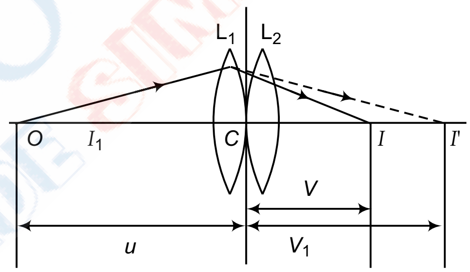
where v = image distance, u = object distance, f = focal length

- **Lens maker's formula** $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

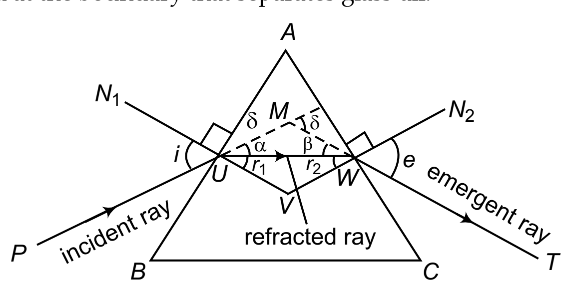
  where f = focal length, n = Refractive Index

- **Lens Magnification**
  - Magnification $(m) = \frac{h_i}{h_0} = \frac{v}{u}$

    where h_i = height of image, h_0 = height of object
  - Magnification depends on focal length and focusing distance of lens
  - Magnification does not depends on aperture or size of image
- **Power of lens**
  - Power of a lens is the ability of the lens to converge or diverge a beam of light that falls on it.
  - $P = \frac{1}{f}$ (measured in Diopter D)
- **Combination of thin lenses in contact**
  - If two lenses $L_1$and L_2 with focal length f_1 and f_2 are kept in contact with each other and an object is placed at point O beyond the focus of lens L_1 on same principal axis, then lens L_1 produces an image at point I_1 which act as an object for lens L_2 and final image at point I where common optical centre C is selected.
  - For image I_1 produced by lens L_1, $1/v_1 - 1/u = 1/f_1$ while for final image I, produced by lens L_2, $1/v - 1/v_1 = 1/f_2$ then combination of lens are change by single lens with focal length F forming image of O at same point I, $1/v - 1/u = 1/F$ where F is focal length of combination of lens.



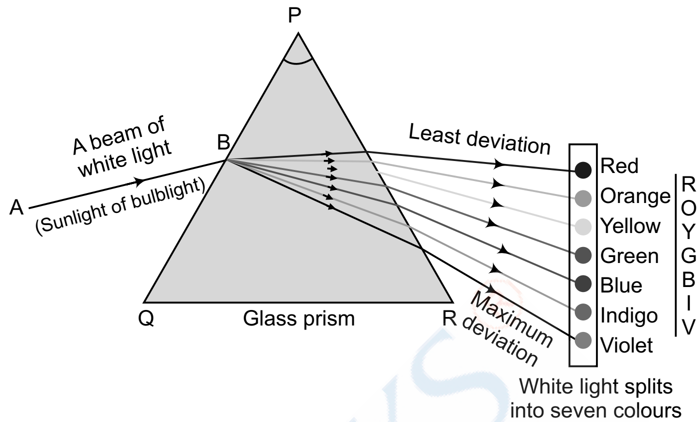
  - Thin lenses of focal lengths f_1, f_2, f_3...will have effective focal length as $1/f = 1/f_1 + 1/f_2$....and power, $P = P_1 + P_2 + P_3 + ...$, which is total powers of individual lenses.
- **Refraction of light through a prism**
  - A prism is a transparent, geometric, optical object with two polished plane faces inclined in relation to each other from where light gets reflected or refracted.
  - The rectangular sides of glass prism are not parallel to each other while its triangular ends are parallel.
  - Refraction of light by rectangular sides of prism is different in behaviour as compared to refraction of ray by rectangular slab.
  - If a glass prism with A is refracting angle of prism, 'i' is angle of incidence, 'e' is angle of emergence, r_1 and r_2 are refracting angles, δ is angle of deviation and μ is refractive index of glass.
  - If light incident from air to glass in a prism gets deviated due to refraction that occurs at the boundary separating air-glass and at the boundary that separates glass-air.



  - In a quad AUVW in figure, where $A = r_1 + r_2$. In triangle MUW δ exterior angle as sum of interior, angles α and β, $\delta = \alpha + \beta$ where $\alpha = i - r_1$ and $\beta = e - r_2$, $2i = A + \delta$ which shows that $\sin i / \sin r_1 = \mu$ and $\sin e / \sin r_2 = \mu$,

$$\mu = \sin (A + \delta/2) / \sin (A/2)$$

Dispersion of light through a prism

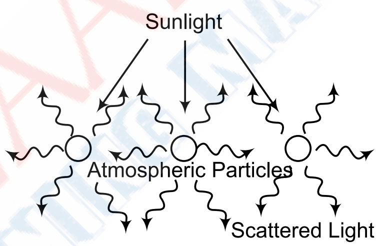


- Dispersion is separation of light into its constituent colours.
- If light ray is incident on the interface between two transparent media, it gets refracted.
- The change in velocity and direction of beam of light from air-glass shows decomposition of white light ray in its component colours which passing through a prism known as dispersion.
- The refraction angle depends on wavelength of colour where shorter wavelength leads to greater angle of refraction.
- The white light is a mixture of many colours with different wavelengths. If white light ray passes through refraction which gets separated into constituent colours resulting in rainbow.
- The change in velocity of light as observed if light passes from air to glass depends on wavelength of light.

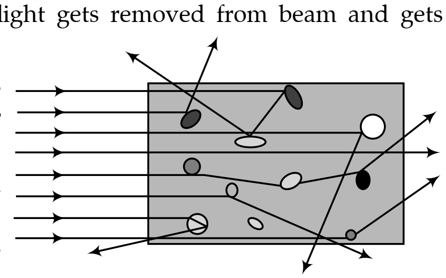
As, $\delta = A(\mu - 1), \mu \propto \frac{1}{v}$ and $v = \nu\lambda$

- If in air-glass, lower is wavelength, lower is the velocity of light, so red light rays move faster as compared to violet light rays.
- Dispersive power $\omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\mu_v - \mu_r}{\mu_y}$

Scattering of light



- The light beam, which lights the rough surface where part of light gets removed from beam and gets redistributed in several directions is called scattering.



- Light scattering is deflection of ray from straight path that takes place due to irregularities in propagation medium, particles or interfacing between two media.
- Scattering of light takes place if a particle of light is fully absorbed and is further emitted.
- The light molecules those are scattered in different directions are much smaller as compared to wavelength of visible light, so amount of scattering depends on wavelength known as **Rayleigh scattering**. As, $I \propto \frac{1}{\lambda^4}$

Blue colour of sky

- Sunlight after reaching Earth's atmosphere gets scattered or deflected by small molecules of gas in air as molecules are much smaller than wavelength of visible light, so amount of scattering depends on wavelength.
- As light moves as waves of different wavelengths where some are short giving blue light and some are long giving red light.
- The appearance of red, blue and green types of colour receptors or cones in eyes of our retina responds such colours strongly to light at certain wavelengths.
- Due to stimulation of colours in different proportions, our visual system frames such colours what we see.
- The shorter the wavelengths of colours violet and blue, more strongly they are scattered, hence more blue light is scattered towards our eyes as compared to other colours.

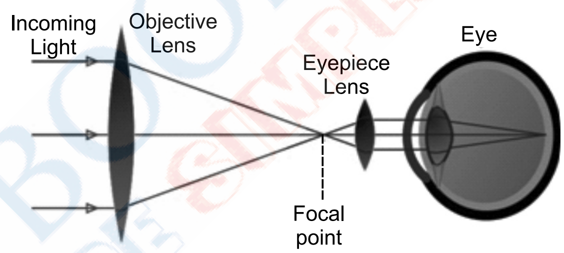
- The sky doesn't actually look purple as violet light is scattered more strongly in comparison to blue since there is not much violet in sunlight, as eyes are more sensitive towards the blue and green.
- While seeing at sky, red cones result in less scattering with less strong to orange and yellow wavelengths while green colour cone respond to yellow and more strongly scattered green and green-blue wavelengths.
- The blue cones are stimulated by colours near blue wavelengths, those are strongly scattered, so if there were no indigo and violet in spectrum, sky would appear blue with green tinge.
- The total effect shows that red and green cones are stimulated equally by light from the sky, while blue stimulates more strongly which makes sky to appear with blue colour.

Reddish appearance of Sun at sunrise and sunset

- If Sun is high in sky, it appears its true colour, white and at sunrise and sunset, we see redder Sun as sunlight is passing through a thick layer of atmosphere which scatters the blue and green light along the way, allowing red light to pass through and illuminate the clouds in beautiful array of red, orange and pink.
- If the air is clear, sunset appears yellow as light from the sun passes long distance through air and some of blue light gets scattered and if air is polluted with small particles, natural or otherwise, sunset appears more red.
- Sunsets over sea is orange due to salt particles in air, showing effective Tyndall scattering.

Telescope :

- Telescope is an optical instrument with two optical elements; objective and eyepiece.
- Objective is a large lens that collects light from distant object and forms an image in focal plane showing faithful representation of object while eyepiece is smooth magnifying glass by which an image can be seen.

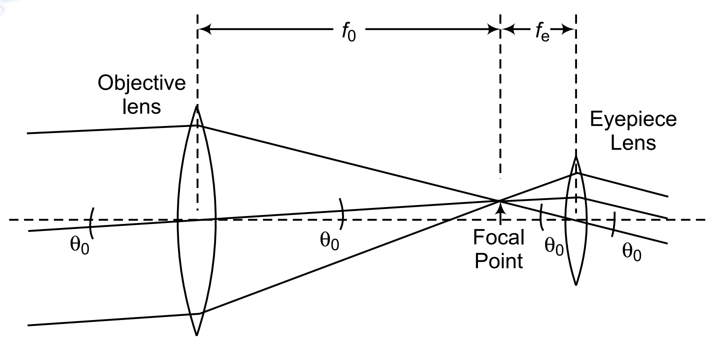


Resolving Power of Telescope

- Resolving power of a telescope is ability of an instrument to produce images of two closely spaced objects/ sources where plane waves from each source after passing through an aperture from diffraction pattern results as characteristics of an aperture.
- It is the inverse of the distance or angular separation among two objects that can be resolved when seen through optical instrument.
- In telescopes, very close objects like binary stars or stars of galaxies subtend very small angles on the telescope.
- To resolve, very large apertures are required and using Rayleigh's criterion for resolving power, angular separation between two objects as $\sin\theta = 1.22\frac{\lambda}{d}$ (where θ is very small)

Magnification of Telescope

- Magnification is the ratio of apparent size to actual size of an object.
- It depends on telescope focal length and eyepiece focal length.
- A useful range of magnification for many telescopes is 50x to 250x.
- If the image gets magnified for observer, the position of each feature in image moves towards larger and larger angle off the centerline where magnification shows ratio of angle at eyepiece to angle by objective lens.

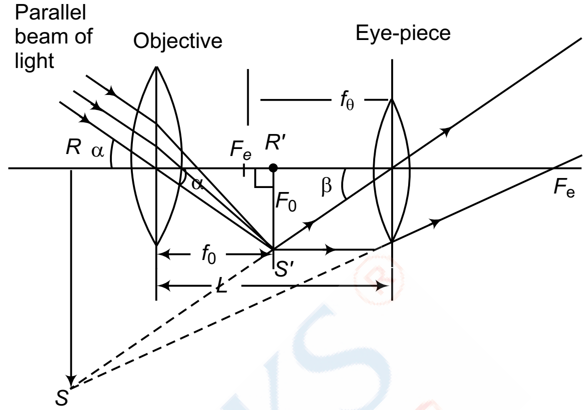


- If θ_o is angle seen at objective, θ_e is angle at eyepiece, f_o is focal length of objective, f_e is focal length of eyepiece, then light rays from a distant point arrives at objective in parallel and passes through center of lens, as ray is not bent and forms a straight line by the lens.
- If angle of incoming ray with centerline is θ_o and same at front and back of the lens, then line passes from center of objective to focal point at a distance f_0 from objective, so θ_0 is h/f_0 and other line from focal plane to center of eyepiece, the angle at eyepiece θ_e is h/f_e then telescope's magnification $M = \theta_e / \theta_o = h/f_e / h/f_o$ or $M = f_o / f_e$

$$m = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right) \quad \text{(for distinct vision)}$$

$$= -\frac{f_o}{f_e} \quad \text{(for relaxed eye)}$$

- In telescope, there are lowest and highest magnification where lowest magnification is ratio of the telescope's aperture to eye's pupil which for 6″or 15cm aperture is 6″ × 25.4mm/inch / 7mm eye-pupil = 22x.
- The highest magnification is magnification where eye's resolving power is fully utilized.



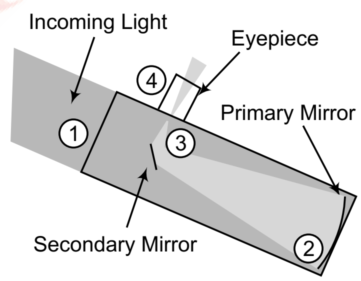
Types of Telescope

- Telescope are refractors and reflectors

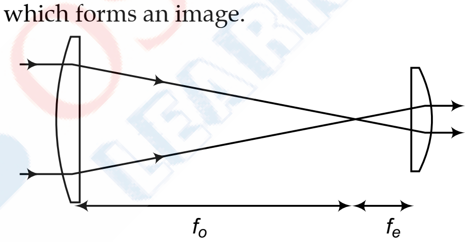
Refracting Telescope

- Refractor telescope is tube with two lenses – one at the front end with little magnification, 'objective lens' while other at bottom end closest to user's eye, 'eyepiece'.
- In Refractor telescopes, large objective lens collects lots of light from object far away and then refracts or 'bends' that light, bringing it to a point near the bottom end of the telescope.
- The smaller eyepiece lens further magnifies that point and brings it in focus at user's eye.
  In this, the tube hold the lenses at right distance from each other by keeping out light, dust and moisture that interferes with image.
- The two convex lenses where objective is bigger in size with more focal length compared to eye-piece. The object being far away. Incident parallel beam of light form intermediate image R'S' in focal plane.
- The intermediate image is allowed to be in focal length distance of eye-piece with final image RS being bigger in size. Magnification power of optical instrument is $M = \alpha/\beta = (R'S'/f_e)/(R'S'/f_o) = f_o/f_e$

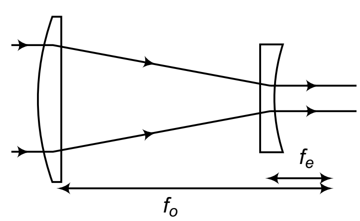
Reflecting Telescopes



- A reflecting telescope is similar to refracting telescope which uses mirrors to reflect the light internally.
- In the telescope light enters at one end (1) and is reflected by a concave mirror at other end (2) towards a smaller mirror known as secondary mirror.
- The mirror reflects (3) the light into the eyepiece, which is usually mounted on the side of the telescope (4).
- Telescope can be Keplerian telescope, Galilean telescope, Terrestrial telescope.
- Keplerian telescope has a converging lens eyepiece while Galilean telescope has a diverging lens eyepiece.
- The telescope by itself is not an image forming system where an eye of observer or camera is attached to telescope which forms an image.

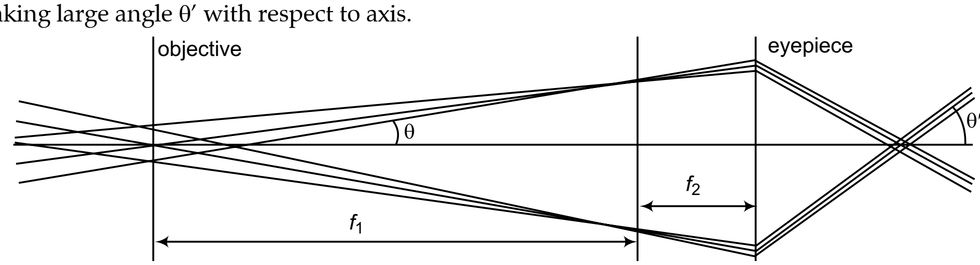


Keplerian telescope



Galilean telescope

- Telescope is used for collecting light so as to increase the angle which a distant object subtends at the eye.
- If an eye is relaxed for distant viewing, telescope produces an angular magnification and an incident, parallel beam from distant source point makes an angle θ with respect to optical axis that emerges as parallel beam making large angle θ′ with respect to axis.

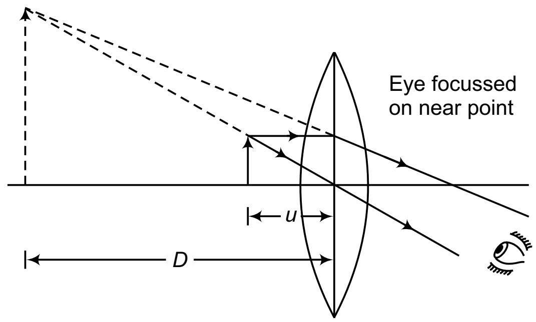


Simple Microscope : Convex lens behaves as simple microscope.

The magnifying power of the simple microscope :

(i) For least distance of distinct vision $m = 1 + \frac{D}{f}$

where, D is the least distance of distinct vision of the eye and f is focal length of the lens.



(ii) For relaxed eye :

$$m = \frac{D}{f}$$

From above formulae, it is clear that for larger magnifying power the focal length of the convex lens should be small.

Also note that angular magnification by optical instruments is the linear magnification by lenses only. It means magnification of an instrument means how many times it enlarges the image of object. So this is just as

$$m = \frac{h'}{h}$$

where, h is size of object (in one dimension) and h' is the size of image.

Compound Microscope : For much large magnification, compound microscope is used. It is a combination of two convex lenses hence the magnification of each lens is compounded.

- These two lenses are placed co-axially and the distance between them is adjustable.
- The lens towards the object is called objective and that towards the eye is called eyepiece.
- The final image formed by the compound microscope is magnified and inverted.
- Total magnification by compound lens

$$m = m_o \times m_e$$

where, m_o is magnification by objective lens and m_e is magnification by eyepiece.

- For least distance of distinct vision magnification by object lens is

$$m_o = \frac{v_o}{u_o} = \frac{L}{f_o}$$

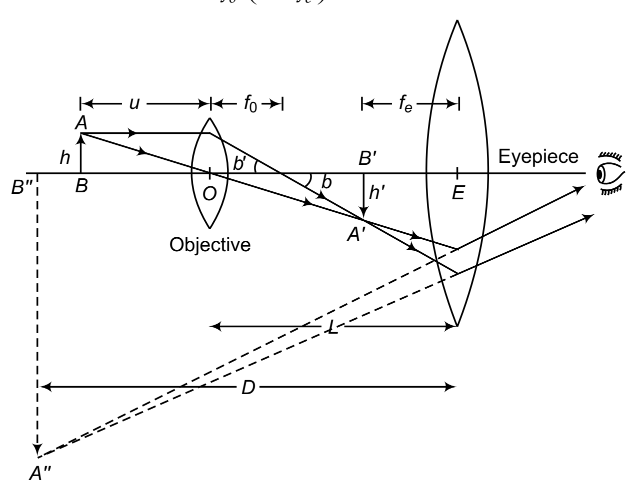
where, L is the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_0). It is called the tube length of the compound microscope.

Eyepiece lens acts as simple microscope.

Magnification by eyepiece lens is

$$m_e = 1 + \frac{D}{f_o}$$

Hence, Magnification by compound lens $= \frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$



For relaxed eye (normal adjustment)

For relaxed eye the magnification by objective lens remains same, the magnification by eyepiece is $+\frac{D}{f_e}$

Hence, the total magnification of compound microscope in relaxed eye condition is

$$m = \frac{L}{f_o} \times \frac{D}{f_e}$$

Properties of Compound Microscope :

- For large magnification of a compound microscope, both f_o and f_e should be small.
- If the length of the microscope tube increases, then its magnifying power increases.
- Generally f_o is much smaller. So that objective is placed very near to principle focus.
- The aperture of the eyepiece is generally small so that whole of the light may enter the eye.
- The aperture of the objective is also small so that the field of view may be restricted.

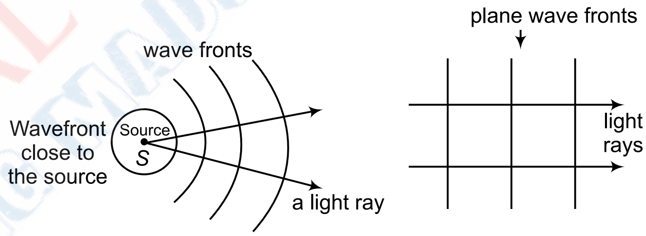
Topic 2

Wave Optics

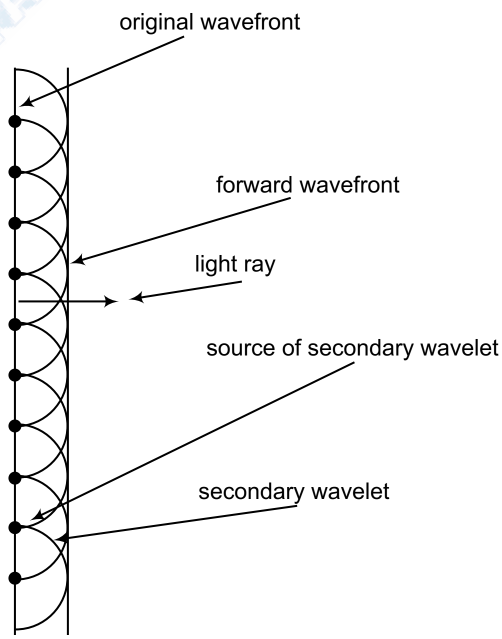
» Revision Notes

Wavefront

- Wavefront is an imaginary surface where an optical wave has constant phase with its surface having maximum or minimum value.
- If similar waves with common origin travel through a homogeneous medium, the corresponding crests and troughs at that instant are in phase with similar fractions of cyclic motion.



- The direction of propagation of wave is always be perpendicular to surface of wavefront at every point, so wavefront of a point source is a sphere and wave propagates radially outward where radius of a sphere is perpendicular to its circumference at every point.

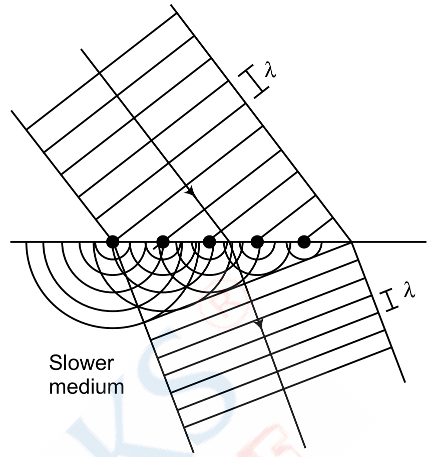


Huygen's principle

- The principle shows that, every point on wavefront acts as a point source which emits spherical wavelets that travels with velocity of light in a medium.

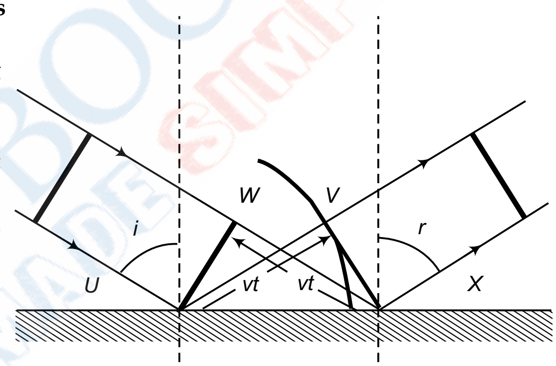
  In this, each point on wavefront be considered as a source of secondary spherical wavelets that spread out in forward direction at speed of light.
- The new wavefront is tangential to all the surface of secondary wavelets and a plane light wave propagates though free space at the speed of light, *c*.
- The light rays get associated with wavefront which propagates in straight lines, so it is easy to visualize refraction where a point on wavefront at boundary of different media serves as source of light that propagates with change in direction.

  In this, speed of light is smaller in second medium and waves do not travel as far in given time as new wavefront changes direction showing why a ray changes direction to become closer to perpendicular when it slows down.



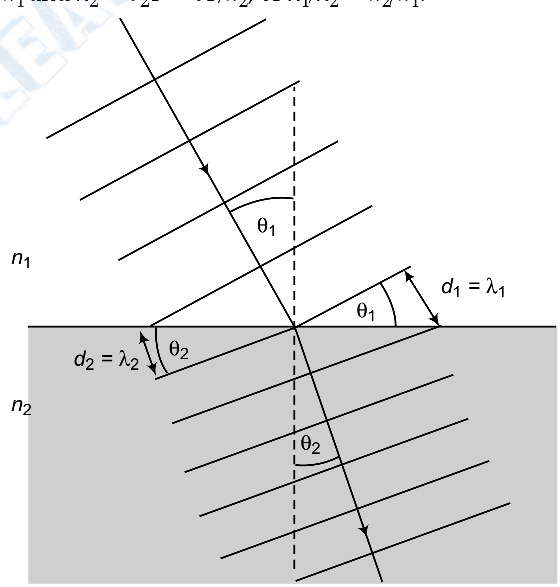
Reflection of plane wave at plane surface using wavefronts

- The reflection of plane wave from a plane mirror shows how the position of reflected wavefront is formed using wave theory.
- The secondary wavelet reflected from U where distance from U to V is same as that from W to X and so wavefront reforms here in same shape as incident wavefront.
- The reflected wavefronts move in the same medium speed of waves before and after reflection.



Refraction of plane wave at plane surface using wavefronts

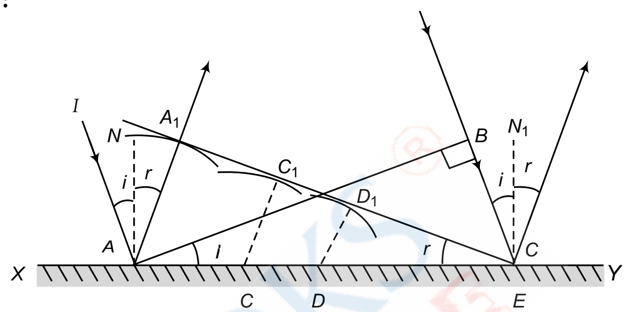
- Refraction is change in direction of propagation of wave when the wave passes from one medium to another, and changes its speed.
- Light waves are refracted when crossing the boundary from one transparent medium into another as speed of light is different in different media.
- If the light waves encounter plane surface of a piece of glass after travelling initially by air, then speed of light in glass or water is less than the speed of light in vacuum or air.
- The distance between wavefronts is shorter in glass than in air, as waves travel a smaller distance per period T.
- If f is frequency of wave and $T = 1/f$ is period, i.e. time interval between successive crests passing a fixed point in space, $\lambda_1 = v_1T = cT/n_1$ and $\lambda_2 = v_2T = cT/n_2$, or $\lambda_1/\lambda_2 = n_2/n_1$.



- In wavefront, light rays which approach surface at an angle, the rays will bend as wave passes from air to glass which results as wavefronts do not travel as far in one cycle in a glass in comparison with air.

- If a wavefront is half in glass which travels with less distance in glass as compared to air that helps in bending at the centre, then ray, which is perpendicular to wavefront bends at an angle to the edge and amount of bending depends on angle of incidence and on indices of refraction of glass and air showing change in speed, $\lambda_1/\lambda_2 = \sin\theta_1/\sin\theta_2$
  or, $\lambda_1/\lambda_2 = n_2/n_1$ or $n_2/n_1 = \sin\theta_1/\sin\theta_2$ known as Snell's law or law of refraction.

Proof of laws of reflection using Huygen's principle :

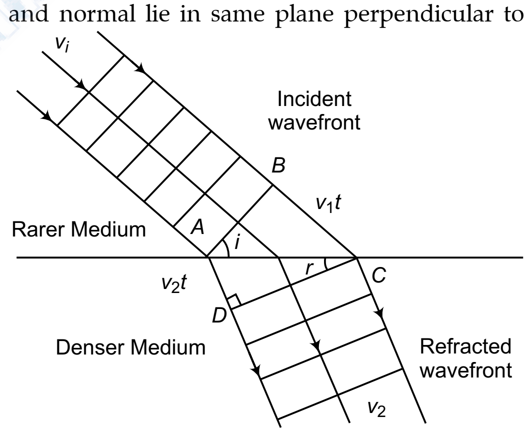


- Reflection is sudden change in direction of propagation of wave that strikes the boundary between two different media.
- If light is incident on surface, it gets re-emitted without any change in frequency known as Reflection of Light.
- If incoming rays are incident on a surface where wavefront are plane waves with infinite parallel planes to each other with fixed amplitude, the wave AB falls on reflecting surface and is incident on wavefront perpendicular to incident ray at an angle i on the surface.
- The Huygens's principle shows that each point on wave AB acts as source of secondary wavelets where A and B are new sources that emit secondary waves where velocity of propagation of waves is 'v' and time taken is 't'.
- If 'vt' is distance travelled by secondary wavelets, then AA_1 and BE are secondary waves.
- As new wavefront needs to be a tangents line that joins two secondary waves, so reflected waves act perpendicular to new wavefront.
- A_1E is new tangential line which connects the secondary wavelets, so in ΔABE and ΔAA_1E, AE is common, $\angle B = \angle A_1 = 90°$.
- $AA_1 = BE$, so triangles are congruent triangles, hence $\angle i = \angle r$

Laws :

- **First Law**: Angle of Incidence = Angle of Reflection
- **Second law**: The incident wavefront, reflected wavefront and normal lie in same plane perpendicular to reflecting surface.

Proof of laws of refraction using Huygen's principle :

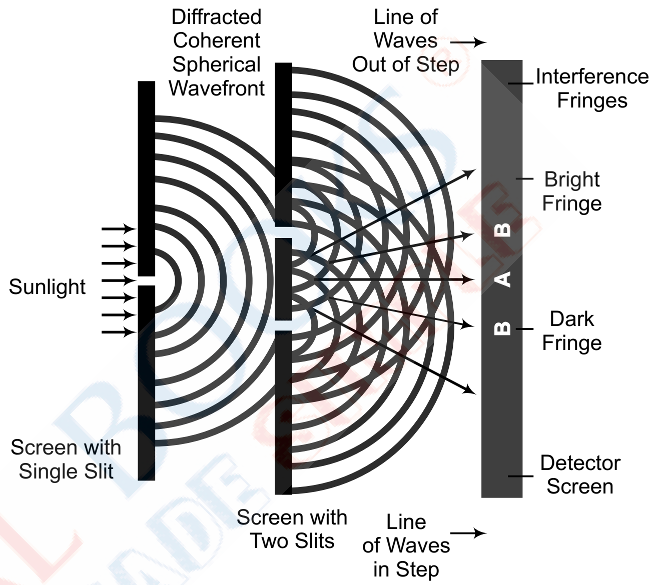


- Refraction is change in velocity of light if it passes from one medium to another.
- If a plane wavefront AB is incident on surface where v_1 and v_2 being velocities of incident ray and refracted ray of medium 1 and medium 2 ($v_1 > v_2$), velocity of waves depends on the medium.
- The Huygens's principle shows that, A and C forms the source of secondary spherical wavelets, where t is time taken from B to reach C, so BC = v_1t in medium 1 and AD = v_2t in medium 2.
- If CD is tangent drawn from point C to sphere, then AD and CD will act as refracted wavefronts, so ΔABC and ΔADC gives $\sin i / \sin r = (BC/AC) / (AD/AC) = BC/AD = v_1t/v_2t = v_1/v_2 = \mu$, refractive index of the medium.
- Refractive Index is ratio of velocity of light in vacuum to velocity of light in other medium.
- Snell's Law of refraction, $\sin i / \sin r = v_1/v_2 = n$ (constant) is proved using Huygens's principle where incident wavefront, the refracted wavefront and the normal lie in the same plane.

Interference : (Young's double slit experiment)

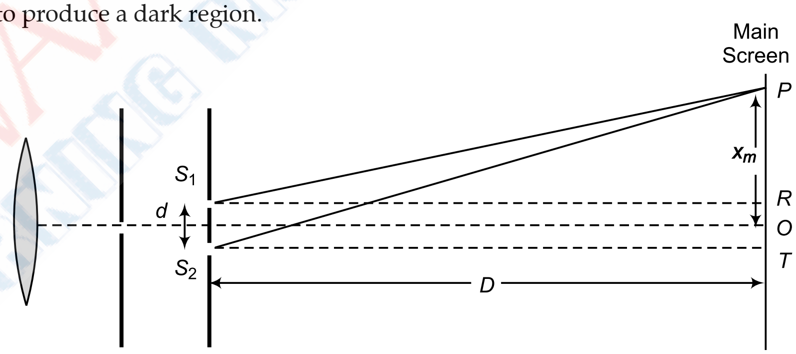
- Thomas Young's experiment strongly inferred wave-like nature of light where light was made of waves and where certain type of interaction takes place if two light waves met together.

  In this, light source is kept at certain distance behind the slit screen A that possesses pair of closely spaced narrow slits and light emerges from pinholes and spreads as spherical waves which overlaps resulting in interference pattern, projected on measurement screen B.
- The simulation of pattern of light was observed on screen B where bright lines of light from two slits form constructive interferences along with dark lines giving destructive interferences.
- With sunlight diffracted through small slit as source of coherent illumination, Young's projected light rays from slit to other screen with two slits placed side by side.



- If slits were large, then two overlapping patches of light appears on the screen and if size of slits are reduced and kept closer to slits, then light passes through the slits and screen shows distinct bands of light separated by dark regions in serial order.
- The coherent wavefront of light which impacts the twin slits gets divided into two new wavefronts those are different in their steps from each other. The light waves from each of slits travel equal distance so as to reach to point A on the screen and show basic requirements for constructive interference by adding together to give bright interference fringe on screen.
- The wave coming from slit close to point B not far to travel to reach its destination, as compared to wave travelling from other slit and the wave from closer slit arrives at point B before wave from far slit.
- The waves do not arrive at point B in phase and undergo destructive interference to produce a dark region.

Expression for fringe width :



- If two slits S_1 and S_2 separated by distance 'd' illuminated by monochromatic light of wavelength λ, where main screen is at a distance of D from the slits S_1, S_2 then waves from two slits superimpose on each other forming interference pattern on the screen and point O is equidistant from S_1 and S_2, forming path difference between waves as 0. So point O is of maximum intensity, central maximum while other point P at distance 'x' from O, path difference at P = $S_2P - S_1P$, or $S_1S_2 = RT = d$ or $S_1R = S_2T = D = xd/D$
- In bright fringes, path difference is integral multiple of wavelength i.e., $n\lambda = xd/D$ or $x = n\lambda D/d$ where x is distance of n^{th} bright fringe from point O where $x_1 = \lambda D/d$, distance of 1^{st} bright fringe, $x_n = n\lambda D/d$ as distance of n^{th} bright fringe from point. So separation between centers of two consecutive bright fringe is width of dark fringe, $\lambda_1 = x_n - x_{n-1} = \lambda D/d$ as dark fringes, $x_n = (2n-1)\lambda D/2d$
- The separation between centers of two consecutive dark interference fringes is the width of bright fringe, $\beta_2 = x_n - x_{n-1} = \lambda D/d$ while separation between the centers of two consecutive dark interference fringes shows width of a bright fringe where all bright and dark fringes result equal in width as $b_1 = b_2$.
- Fringe width is the distance between two successive bright fringes or two successive dark fringes.
- In interference pattern, the fringe width is constant for all fringes where all bright and dark fringes are equally spaced.
- Fringe width is independent of order of fringe and is directly proportional to wavelength of the light used.

Coherent sources

- Coherent sources are those sources of light that give continuous light waves of similar wavelength and frequency and are in same phase or have a constant phase difference.
- For light waves emitted by two sources of light to be coherent, the initial phase difference between the waves should be constant in time.

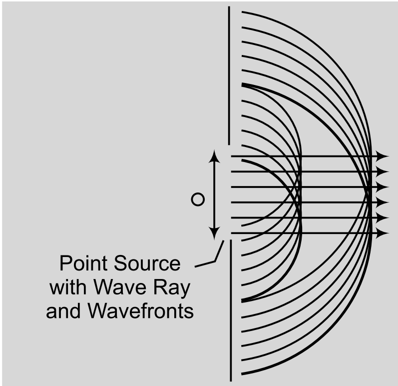
- If phase difference changes continuously with time, then the sources are incoherent
- Two independent sources of light are not coherent and hence cannot produce interference as light beam is emitted by millions of atoms those radiate independently so that phase difference between waves fluctuates randomly many times per second.
- Two coherent sources can be obtained either by a source and obtain virtual image or obtain two virtual images of similar source as any change in phase in real source results in simultaneous and equal change in the images.

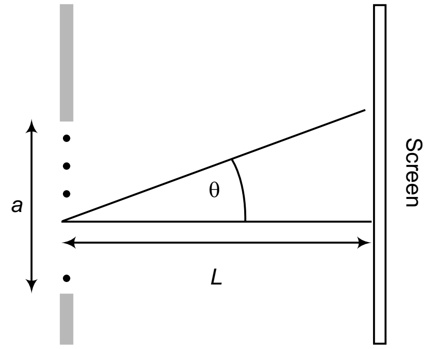
Sustained interference of light

- Interference is phenomenon of redistribution of light energy in medium on account of superposition of light waves from two coherent sources.
- At the points, where resultant intensity of light is maximum, interference is constructive while at point where resultant intensity of light is minimum, interference is destructive.
- Interference pattern where positions of maximum and minimum intensity of light remain fixed with time is sustained or permanent interference pattern.
- Sustained and observable interference pattern should have following conditions:
  - The two sources should be coherent *i.e.* the sources should emit light of similar frequency with constant phase difference.
  - Two sources should be very narrow.
  - The sources should lie very close to each other to form distinct and broad fringes.
  - The interfering waves should have equal amplitude for making minimum intensity to be zero for general illumination.
  - Two interfering waves must propagate along same line for making vibrations to be on common line.
  - The separation between two sources must be very small for making satisfying fringes of maximum and minimum intensity to lie close together for clear visibility.

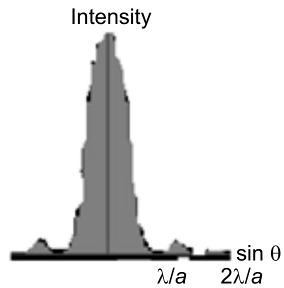
Diffraction due to a single slit

- In a single slit diffraction pattern, there are certain assumptions:
  - Slit size is small as compared to wavelength of light.
  - Screen is far away.
  - Cylindrical waves shown in 2D diagrams as circular waves.
  - Intensity at any point on screen is independent of angle made between ray and normal line between the slit and screen.
  - If a slit of width a, light of wavelength λ is smaller than λ', if light meet the slit, is pattern of resulting wave be obtained by treating every point in aperture as a point source from where new waves spread out.





- If L is distance between the slit and the screen and θ is angle between the wave ray to a point on screen and normal line between slit and screen, then single slit diffraction patterns observed on the screen where there are more blending between bright and dark bands.
- The pattern in intensity bar graph where intensity of light in diffraction pattern serves as a function of $\sin \theta$, much light is concentrated in broad central diffraction maximum and minor secondary bands on either side of central maximum are present.



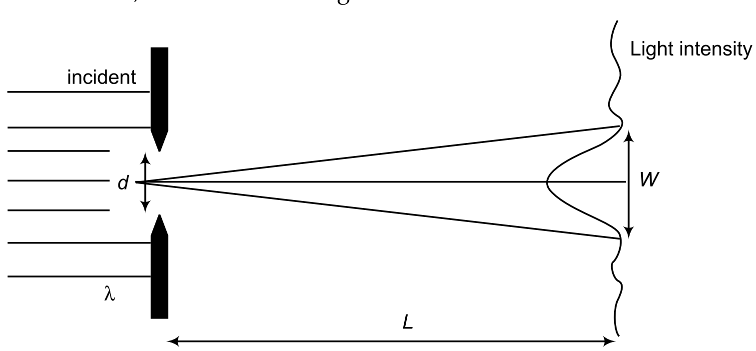
Central Diffraction Maximum

- The first diffraction minimum takes place at an angle $\sin\theta = \lambda / a$. Width of central diffraction maximum is inversely proportional to width of slit and if width size a increases, the angle θ where intensity initially is zero decreases, resulting in narrower central band, where $a \sin\theta$ is path difference between two light rays.

  In this, parallel wave rays show bottom wave with $2/3^{rd}$ of its cycle when top wave begins its cycle showing two light rays having path difference of $(2/3) \times 2\pi$ or $4\pi / 3$.
- As slit width a, is few hundred nanometers in size, with light waves from very top and bottom of slit are essentially right on top of each other with waves in between showing interference, and resultant wave's amplitude equal to sum of individual wave amplitudes showing superposition of waves. So for top and bottom light waves, the phase difference is equal to path difference as $4\pi / 3$.
- If $a \sin\theta = \lambda$, the path difference between top and bottom light rays result equal to one wavelength (λ) which are in phase and path difference between top ray and ray just below midpoint of slit ($a/2$) is half a wavelength ($\lambda/2$).
- The path difference of half wavelength corresponds to phase difference of π as top wave and wave below the midpoint of slit are out of phase and cancels each other.
- The general equation for points of zero intensity in diffraction pattern of a single slit is $a \sin\theta = m\lambda$, where $m = 1,2,3, \ldots$
- If y is distance from center of central diffraction maximum to first diffraction minimum then angle θ is related to distance y and distance to screen L by $\tan\theta = y / L$
- As angle θ is very small, $\cos\theta \sim \lambda$, so $\tan\theta \sim \sin\theta$ and shows $\sin\theta = \lambda / a \sim y / L$, or $y = L\lambda / a$
- The distance to screen, width of slit and wavelength of light, $y = L\lambda/a$ are applied to find where first diffraction minimum takes place in single slit diffraction pattern.

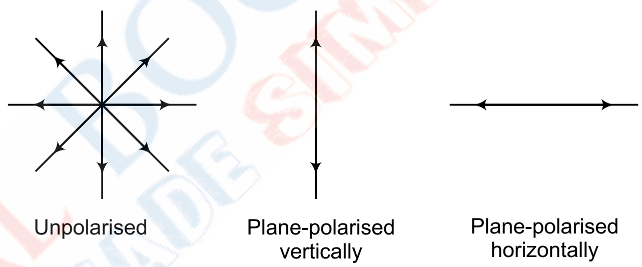
Diffraction due to width of central maximum

- By virtue of diffraction light tends to bend while passing through a slit or opening.
- The diffraction pattern in terms of intensity and angle of deviation from the central position is shown, where central maximum is twice the width of other maxima which have similar width.
- Width of central maximum is distance between two minima closest to center of diffraction pattern.
- As minima are symmetrical about the center of pattern, so only the distance to one of the minima needs to be obtained where width of central maximum results as twice of the distance.
- In a single slit diffraction showing relationship between the width of a slit and wavelength as λ/W using the laser using lens to produce intense parallel beam as shown then the equation relating width of slit d, W the width of central maximum, λ as laser wavelength and L the distance from the screen to slit, $W = 2L\lambda/d$

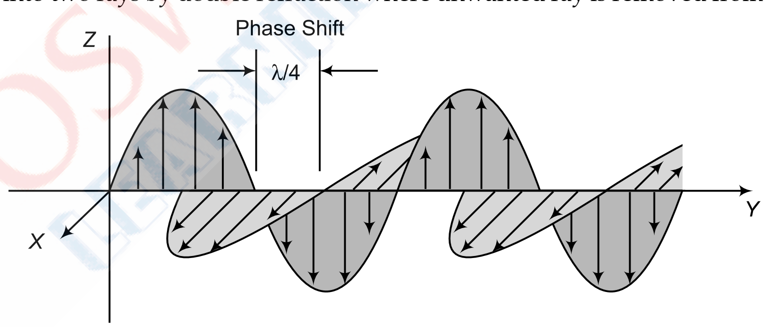


Resolving power of microscopes and astronomical telescopes

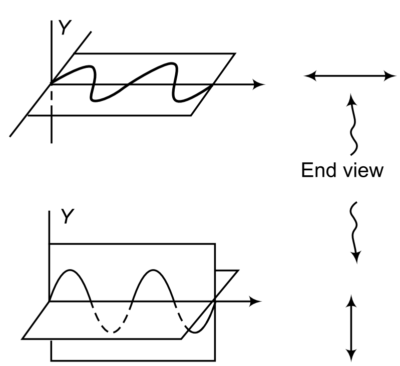
- **Microscope Resolving Power**
  - Resolving power is smallest distance between two separate points of object, if viewed with an optical instrument that can still be seen as distinguishable, $d = 0.61\ \lambda/NA$ where symbols have usual meanings.
  - Resolving power of objective lens is measured by its ability to differentiate two lines or points in an object.
  - If resolving power is more, smaller is the minimum distance between two lines or points which can be distinguished and if numerical aperture is large, resolving power is more.
  - The resolving power increases if minimum distance d seen between two points in image, $d = 0.61\ \lambda / NA$, which increases by either decreasing the wavelength, λ or increasing numerical aperture NA.
- **Resolving Power of Telescope**
  - Resolving power is important power of telescope that serves as an ability showing details about the images.
  - It is measured in absolute smallest angle which can be resolved.
  - The absolute minimum resolvable angle in arc seconds λ_R = 252,000 × (observation wavelength)/(objective diameter).
  - The wavelength and diameter are measured in same length units.
  - The telescope with one arc second resolution is able to see a dime from about 3.7 kilometers away.
  - The resolving power of telescope is reciprocal of smallest angle subtended at objective lens of telescope by two point objects that distinguishes and is $= D/d = a / 1.22\ \lambda$ where symbols have usual meanings.
- **Polarisation**
  - Polarisation is a phenomenon in which the oscillations are made to occur in one plane only.
  - It is measured with polarimeter.
  - Transverse waves oscillating in orientation perpendicular to direction of travel are polarized, if there is a plane having line of direction of propagation of wave and line along which the particles vibrate.



  - As longitudinal waves have particles vibrating back and forth in line of propagation, so is no plane specified by two parallel lines which cannot be polarised.
  - Polarisation can be plane polarized light, circularly polarized light, elliptically polarized light
  - Unpolarized light can be converted into single polarized beam using Nicol prism, a device that separates incident light into two rays by double refraction where unwanted ray is removed from the beam by reflection.

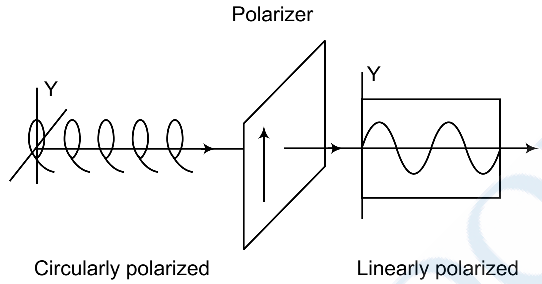


  - Vertically polarized light is that where there is no amplitude in x ($E_0x = 0$), but there is one component in y (vertical) direction.



Linearly polarized light

- Circular polarization has phase difference $\phi = 90°$ and $E_{0x} = E_{0y}$, where $\frac{E_x}{E_{0x}} = \cos\theta, \frac{E_y}{E_{0y}} = \sin\theta$ having equation of circle, $\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 = \cos^2\theta + \sin^2\theta = 1$
- Elliptical polarization is combination of linear polarisation and circular polarization.
- Light is linearly polarized where light wave lie on horizontal or vertical plane.
- Polaroid shows class of material that absorbs light oscillations in one direction but not component oriented at right angles.

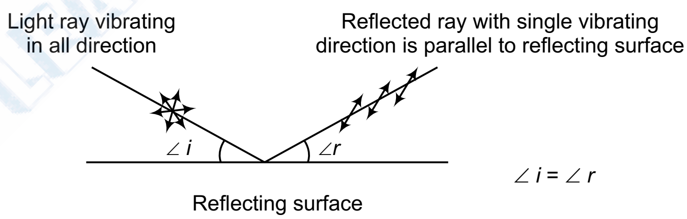


- The polarizing material has long particles, rods or plates, aligned parallel to each other which transmits one plane of polarized light and absorbs the perpendicular light.
- The polarizer can transform circularly polarized light into linearly polarized light.
- The direction of polarisation of X-ray photon can change as a result of scattering/diffraction where change is maximum or no change, as per initial polarisation in plane with pre and post scattered X-rays.
- The component of polarisation resolved along new direction is reduced by cosine of scattering angle, so reduction in intensity results by $\cos^2 2\theta$ as intensity is proportional to square of amplitude.
- The component of polarisation which is unaffected and remains unchanged, by diffraction process, so reduction factor is 1.
- The mixture polarisation occurs with diffractometer as X-ray sources produce unpolarised X-rays, where X-rays polarised is equal in all directions and reduction factor shows mean of cases,

Case (1) : Polarisation in plane of scattering,	$P = \cos^2 2\theta$
Case (2) : Polarisation perpendicular to plane of scattering,	$P = 1$
Case (3) : Unpolarised X-rays,	$P = (1 + \cos^2 2\theta)/2$

Plane Polarised Light

- A polarized light which vibrates in single plane, perpendicular to direction of propagation is known as plane polarised light.



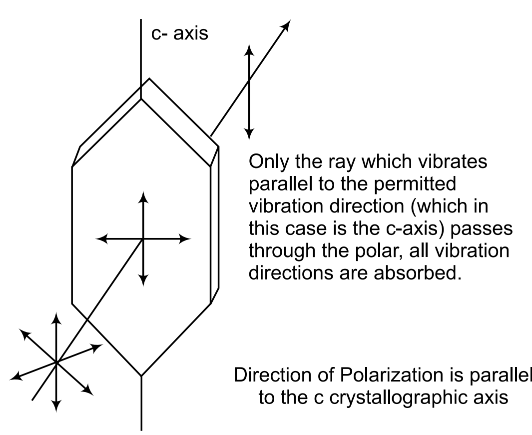
- In plane polarised light, electric vectors of majority are oriented parallel to each other.
- Plane polarized light is produced by reflection, selective absorption, double refraction and scattering.

Reflection

- Unpolarized light on striking a smooth surface reflects and is polarized such that its vibration direction is parallel to reflecting surface.
- The reflected light is completely polarized when angle between reflected and refracted ray is 90°.

Selective Absorption

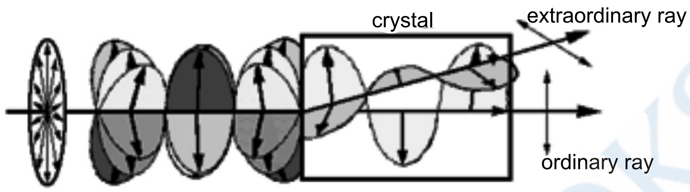
- The plane polarized light is produced in microscopes using polarized filters.



- The anisotropic materials strongly absorbs the light that vibrates in one direction and easily transmit light that vibrates at right angles.
- On entering in anisotropic material, unpolarized light gets split in two plane polarized rays whose vibration directions are perpendicular to each other with each ray having about 1/2 the total light energy.
- If anisotropic material is thick and strongly pleochroic, then one ray gets completely absorbed while other ray passes through the material in order to emerge and retain its polarization.

Double Refraction

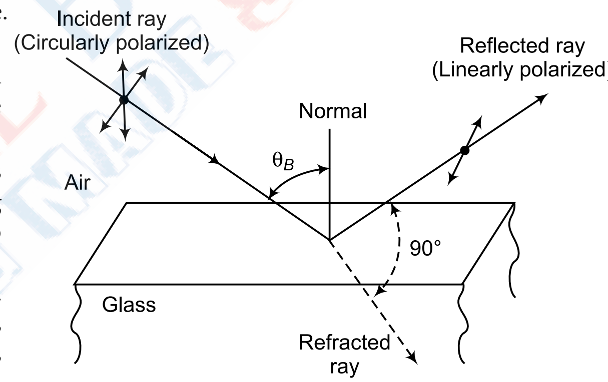
- If ray of light reaches at anisotropic crystal, it splits into two rays of polarised light which vibrate in perpendicular planes where one ray obeys laws of refraction while other ray other does not obey laws of refraction.



- If rays have different refractive indices with different directions of vibration, then these rays leave the crystal, following parallel paths as their planes of vibration are perpendicular.
- The ordinary and extraordinary component in rays follow different paths in crystal, on leaving the crystal and both rays follow parallel paths which result in splitting of crystal in two waves that follows common path but vibrate in two perpendicular planes.
- The velocity of propagation is shown by direction of vibration where each wave vibrate with different velocity, so on leaving the crystal, both rays are out of phase.

Brewster's law

- Polarized light is produced by transmission through polarizers or by reflection from the surface of transparent material.
- The incident light, wave-oscillation vectors shows polarization of reflected light in plane of reflecting surface, where refracted beam have mixture of two orientations.
- If incident light is incident at Brewster angle θ_B, reflected light gets fully polarized and at angles other than Brewster angle, reflected light gets partially polarized.



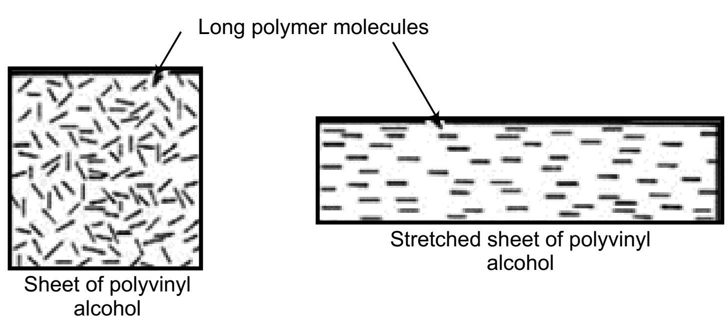
- The Brewster angle is related to index of refraction n of the material by $\tan\theta_B = n(\text{material})/n(\text{air}) = n(\text{material})$
- For glass, $n = 1.5$ and $\theta_B = 56.3°$ and for water, $n = 1.33$ and $\theta_B = 53°$.

Uses of plane polarised light and Polaroids

- Polarized light is produced by using tourmaline crystal which removes one of the polarized rays by absorption.
- Applications of Polarized Light:
  - Polarized light is used in photo elastic stress analysis.
  - For 3-D view, special type of glasses, which have polaroids with perpendicular axes, are used.
  - Polarized light is useful to find size and shape of viruses.
  - Polarized light helps in many practical applications in industry and engineering.

Polaroids

- Is made by heating and stretching a sheet of PVA laminated to support sheet of cellulose acetate treated with iodine solution.



Crystal polarizers

- It is an anisotropic crystal, where electrons are bound with different springs as per orientation.
- In this, different spring constants show different propagation speeds and different indices of refraction with two output beams.
- Crystal polarizers used as beam displacers, beam splitters, polarizers, analyzers.

- Examples: Nicol prism, Glan-Thompson polarizer, Glan or Glan-Foucault prism, Wollaston prism, Thin-film polarizer,

- **Circular polarizers**
  - In this, input light is unpolarized while output light is circularly polarized which is made of linear polarizer glued to quarter-wave plate which is oriented at 45° with respect to other.
- **Applications of Polaroids:**
  - Polaroids are used in sun glasses. They reduce the intensity and the glare by cutting down the horizontally polarized light.
  - Polaroids are used in the laboratory to produce and analyse plane polarised light.
  - Polaroids are widely used as polarising sun glasses.
  - They are used to eliminate the head light glare in motor cars.
  - They are used to improve colour contrasts in old oil paintings.
  - Polaroid films are used to produce three-dimensional moving pictures.
  - They are used as glass windows in trains and aeroplanes to control the intensity of light. In aeroplane one polaroid is fixed outside the window while the other is fitted inside which can be rotated. The intensity of light can be adjusted by rotating the inner polaroid.
  - Aerial pictures may be taken from slightly different angles and when viewed through polaroids give a better perception of depth.
  - In calculators and watches, letters and numbers are formed by liquid crystal display (LCD) through polarisation of light.
  - Polarisation is also used to study size and shape of molecules.

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Chapter 17
Dual Nature of Matter and Radiation

» Revision Notes

Dual Nature of Radiation

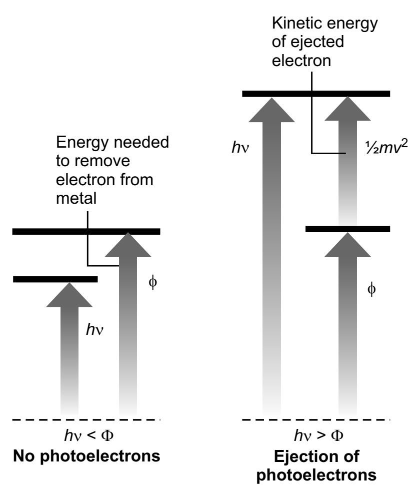
- In a conductor at room temperature, free electrons move randomly without leaving conductor surface where certain external energy known as work function W is required to emit electrons from metal surface.
- The energy required to liberate an electron from metal surface appears from various source like heat, light, electric field, etc. These emissions are :
  - **Thermionic emission :** Emission of electron from its metallic surface when heal is imparted on it.
  - **Field emission :** If a conductor is kept under strong electric field, free electrons experience an electric force in opposite direction of field and after certain limit, electrons will be coming out of metal surface.
  - **Secondary emission :** The emission of electrons from metal surface by bombarding high speed electrons or particles.
  - **Photoelectric emission :** The emission of free electrons from metal surface when light is imparted on it, where electrons so emitted are photoelectrons.
- The phenomena like interference, diffraction and polarisation are based on wave nature of light while photoelectric and compton effect are based on quantum theory of light which shows that light radiation has dual natural that behaves as wave and particle.

Light has properties of radiation and material. There are certain radiation properties which light possess are :

- It spreads in form of waves
- It can be experienced, but cannot be touched.

There are certain material properties which light possess are :

- It moves in straight line in form of ray.
- It has momentum.
- It has certain amount of energy.

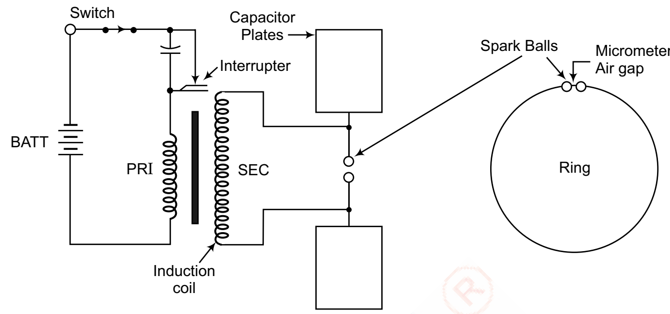


Photoelectric Effect

- It involves emission or ejection of electrons from a metal surface in response to incident light.
- It is interpreted as photons and conservation of energy with the equation $h\nu = \Phi + \frac{1}{2}mv^2$
- The energy inside incident light gets absorbed by electrons in metal which gives required energy to electrons to emit from the surface.
- Maxwell's wave theory of light shows that more intense the incident light, more will be the energy with which the electrons being ejected from metal.

- The average energy carried by ejected electron increases with intensity of incident light.

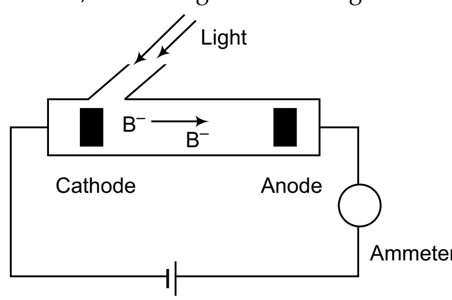
Hertz Experiment



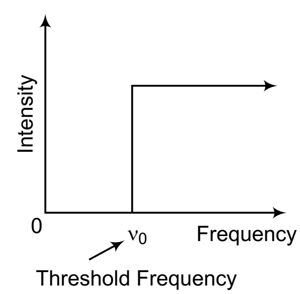
- In an experiment, Hertz tested Maxwell's hypothesis using an oscillator of brass knobs there are joined with induction coil, separated by small gap allowing sparks to jump.
- He observed that if Maxwell's predictions were correct, electromagnetic waves would be able to transmit during every series of sparks.
- Hertz made a receiver of looped wire at the ends of which, the knobs are separated by small gap. The receiver was kept away from oscillator. As per theory, if electromagnetic waves were spreading from oscillator sparks, they would induce a current in the loop which send the sparks across the gap. It happened when Hertz turned on oscillator and produce first transmission and reception of electromagnetic waves.
- In an observation, electrical conductors reflect the waves which can be focused by concave reflector.
- Non-conductors allow most of waves to pass through and photoelectric effect and production and reception of electromagnetic waves using discharging of spark occur.
- With high voltage spark across detector loop were enhanced when emitter plate was illuminated by ultraviolet light from arc length.
- Maximum spark length was produced when the apparatus was kept in dark box.
- The small receiver spark became more vigorous when exposed to ultraviolet light from transmitter spark.

Lenard's Experiment

- Philipp von Lenard made observations on change in electron energy with the change in light frequency.
- In an experiment, Lenard measured electron potential energy and found that the maximum electronic kinetic energy is proportional to the frequency of light.
- The experiment tested theories proposed by classical mechanics as shown, where a light on reaching cathode, emits electrons which travels in vacuum tube till they reached the anode.



- Lenard determined the amount of electrons which reached the anode by measuring the current through the wire with a battery and run series of experiments by varying frequency and intensity of light.
- Lenard found that below certain threshold frequency, there was no emission of electrons while above the threshold frequency, the current was directly proportional to light intensity.
- The current which appeared instantaneously after light was turned on was observed by varying potential and change in current, to determine the kinetic energy of ejected electrons.



- Lenard found that higher frequency of light increases kinetic energy of electrons, while on changing the light intensity there is no effect on the kinetic energy.
- A minimum frequency of light is required for photoelectric effect to be observed, and once that minimum frequency exceeds, an immediate emission of photoelectrons takes place.
- By changing intensity of light and measuring the stopping potential of photoelectrons, the maximum kinetic energy $(\max E_k = qV_{stop})$ becomes independent of intensity.
- By changing frequency of light and measuring stopping potential, maximum kinetic energy $(\max E_k = qV_{stop})$ results more for higher frequencies of light.

Einstein's photoelectric equation - particle nature of light

- Einstein using the theory explained photo electric emission, where photon of energy $E = h\nu$ when falls on metal surface, transfers the energy to electrons of metal.
- If energy of photon is more than minimum energy required by electrons for leaving the metal surface, then emission of electrons takes place instantaneously.
- After absorbing photon, electron either leaves the surface or dissipates its energy in metal in a short interval where there is no chance for absorption of other photon.

- An increase in intensity of light source increases the number of photon and photo electrons without increase in energy of photo electron while increase in frequency increases the energy of photons and photo electrons.
- As minimum amount of energy is required to remove electron from a surface which equals the work function, so the equation is Energy Supplied = Energy Consumed in ejecting an electron + maximum kinetic energy of electron or $eV = eV_0 + \frac{1}{2}mv^2$
- The Einstein's photoelectric equation, $K_{max} = h\nu - \Phi$ shows that :
  - Photoelectric current (i) is directly proportional to intensity (I) of radiation where as intensity increases, number of photons received by metal surface increases, making number of electrons to emit more thereby increasing photoelectric current.
  - As photoelectric current is maximum, saturation current is more with increasing intensity of incident light
  - In metal, certain minimum frequency is required below which no photoelectric effect occurs known as **threshold frequency.**
  - Stopping potential (V_o) and Maximum kinetic energy (K_{max}) do not depend on intensity as intensity is number of photons in unit area and unit time where photoelectric effect takes place when one electron takes one photon.
  - Stopping potential (V_o) and Maximum kinetic energy (K_{max}) is directly proportional to frequency (ν).

Matter Waves

- Photons are light particles while matter comprises of atoms which results from mixture of protons, neutrons and electrons having dimensions of order 10^{-10} m to 10^{-15} m.
- Louis de Broglie proposed that wave function is linked with particles having nonzero amplitude, so magnitude of wave function of a particle at any point is proportional to probability of finding the particle at that point.
- Spreading of wavelengths shows uncertainty in momentum where uncertainty principle holds for material particles, so for minimum value for product $\Delta x\ \Delta p$, order of λ is $\Delta x \Delta p \sim \lambda$
- For particle, we cannot predict its position and momentum with absolute certainty, hence product of uncertainties is of order $h/2\pi$ or greater.

de Broglie's explanation

- Light is a form of energy that behaves like waves and particles.
- Matter behaves like particles and waves.
- de Broglie shows that, if particle of mass m, moves with velocity v, then it behaves like a wave with wavelength as $\lambda = h/(mv)$.
- The matter wave is sometimes referred to as de Broglie wave and λ as de Broglie wavelength.
- The wave associated with moving particle is matter wave or de Broglie wave that controls the particle in all respect.
- The intensity of a matter wave at a point shows probability of associated particle being present, so if intensity of matter wave is large in certain region, then a greater probability of particle is observed.
- Planck's quantum theory shows that if energy of photon of radiation of frequency ν and wavelength λ is $E = h\nu$, where h = Planck's constant, and if photon is considered as particle of mass m, then Einstein's energy mass relation shows energy **E** of photon as $E = mc^2$
- The quantity mc, is momentum p of photon with mass m travelling at velocity c, **so**

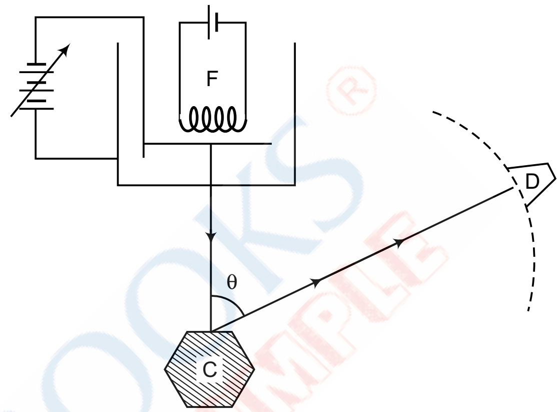
$$p = mc = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{mc} = \frac{h}{p}$$

  which shows de Broglie wavelength for photon.
- If a material particle with mass m moves with velocity v, then momentum is $p = mv$ and as per de Broglie, wavelength λ of wave associated with moving particle, $\lambda = \frac{h}{p} = \frac{h}{mv}$ which is de Broglie wave equation for moving material particle where waves associated with moving particle are matter waves or de Broglie waves.
- The kinetic energy, $K = p^2/2m \Rightarrow \lambda = h/\sqrt{2mK}$
- The energy of a photon in terms of its frequency ν, is $E = h\nu$, so as per special theory of relativity, a new expression appears in terms of velocity of light where m is relativistic mass of light that is non-zero as it travels with velocity c, so energy $E = mc^2$
- The idea of dual nature of matter is important for microscopic bodies while for larger bodies, wavelengths of associated waves are very small that cannot be measured by any methods, so larger bodies have no wavelengths.

- **de Broglie wavelength associated with charged particles**
  - For electrons ($m_e = 9.1 \times 10^{-31}$ kg) : $\lambda = h/\sqrt{2mqV} = 12.27/\sqrt{V}$ Å
  - For protons ($m_p = 1.67 \times 10^{-27}$ kg) : $\lambda = 0.286/\sqrt{V}$ Å
  - For deuterons ($m_d = 2 \times 1.67 \times 10^{-27}$ kg) : $\lambda = 0.202/\sqrt{V}$ Å
  - For α-particles ($m_a = 4 \times 1.67 \times 10^{-27}$ kg) : $\lambda = 0.101/\sqrt{V}$ Å
- **de Broglie wavelength associated with uncharged Particles**
  - For neutrons ($m_n = 1.67 \times 10^{-27}$ kg) :

    $\lambda = h/\sqrt{2mK} = 6.62 \times 10^{-34}/\sqrt{2 \times 1.67 \times 10^{-27} K}$
  - For thermal neutrons at ordinary temperatures:

    $K = kT$ or $\lambda = h/\sqrt{2mkT} = 30.835/\sqrt{T}$
  - For gas molecules : $\lambda = h/mv_{rms}$
  - For gas molecules at T K, $E = 3/2\, kT$ or $\lambda = h/\sqrt{3mkT}$
- **Davisson - Germer Experiment**



Davisson - Germer experiment shows wave nature of electron, which confirms earlier hypothesis of de Broglie.

Using wave-particle duality, the experiment shows major step towards development of quantum mechanics in which vacuum apparatus measure energies of electrons that gets scattered from metal surface.

Davisson and Germer allowed electrons from heated filament to accelerate by voltage that strikes the surface of nickel metal. The electron beam that was directed at nickel surface, was rotated to observe angular dependence of scattered electrons.

The electron detector was kept on an arc so that it could rotate to find electrons at different angles.

Davisson and Germer found that at certain angles there was a peak in intensity of scattered electron beam whose peak shows wave behavior for electrons that can be interpreted by Bragg law for having lattice space values in nickel crystal.

In experiment, data of repeated peaks of scattered electron intensity is obtained with increasing accelerating voltage that was collected at particular scattering angle.

- **Observations**

The experiment shows :

- wave nature of electron for first time
- diffraction effects of electron in crystal diffraction experiment
- strong peak at 55 V
- angle of scattering at 50°
- pattern of deflected electrons which is similar to diffraction pattern of waves
- wavelength that corresponds to electron at $\lambda = 0.165$ nm
- strong agreement with De Broglie's hypothesis

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Chapter 18

Atoms and Nuclei

Topic 1

Atoms

» Revision Notes

Alpha-Particle Scattering Experiment and Rutherford's Nuclear Model of Atom

- In 1897-98, the first model of an atom was proposed by J.J. Thomson, famously known as the **Plum-pudding model** or the **watermelon model.** He proposed that an atom is made up of a positively charged ball with electrons embedded in it. Further, the negative and positive charges were equal in number, making the atom electrically neutral.
- Rutherford, along with his assistants – H. Geiger and E. Marsden – started performing experiments to study the structure of an atom. In 1911, they performed **the Alpha particle scattering experiment,** which led to the birth of the **'nuclear model of an atom'** – a major step towards how we see the atom today.
- They took a thin gold foil having a thickness of 2.1×10^{-7} m and placed it in the centre of a rotatable detector made of zinc sulfide and a microscope. Then, they directed a beam of 5.5 MeV alpha particles emitted from a radioactive source at the foil. Lead bricks collimated these alpha particles as they passed through them.
- After hitting the foil, the scattering of these alpha particles could be studied by the brief flashes on the screen. Rutherford and his team expected to learn more about the structure of the atom from the results of this experiment.
- Here is what they found :

1. Most of the alpha particles passed through the foil without suffering any collisions.
2. Around 0.14% of the incident alpha particles scattered by more than 1°.
3. Around 1 in 8000 alpha particles deflected by more than 90°.

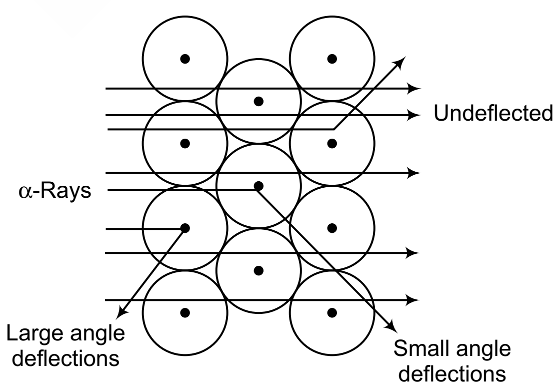


Fig. (1)

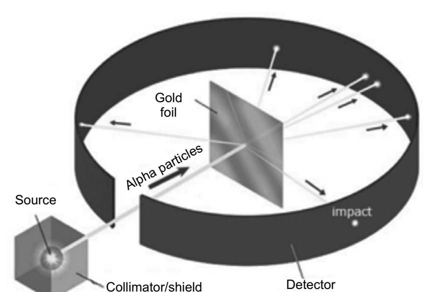
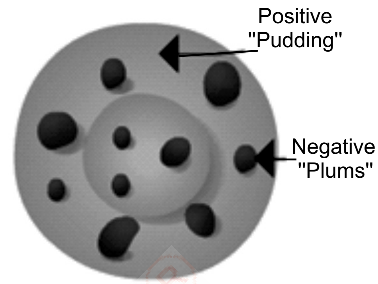


Fig. (2)

- The results of this experiment were not in sync with the plum-pudding model of the atom as suggested by Thomson. Rutherford concluded that since alpha particles are positively charged, for them to be deflected back, they needed a large repelling force. He further argued that for this to happen, the positive charge of the atom needs to be concentrated in the centre, unlike scattered in the earlier accepted model.

  Hence, when the incident alpha particle came very close to the positive mass in the centre of the atom, it would repel leading to a deflection. On the other hand, if it passes through at a fair distance from this mass, then there would be no deflection and it would simply pass through.



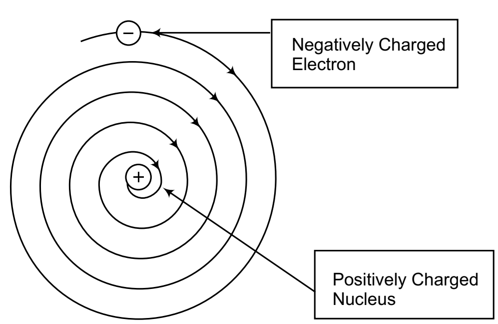
- He then suggested the 'nuclear model of an atom' wherein the entire positive charge and most of the mass of the atom is concentrated in the nucleus. Also, the electrons are moving in orbits around the nucleus akin to the planets and the Sun. Further, Rutherford also concluded from his experiments that the size of the nucleus is between 10^{-15} and 10^{-14} m.
- According to Kinetic theory, the size of an atom is around 10^{-10} m or around 10,000 to 1,00,000 times the size of the nucleus proposed by Rutherford. Hence, the distance of the electrons from the nucleus should be around 10,000 to 1,00,000 times the size of the nucleus.
- This eventually implies that most of the atom is empty space and explains why most alpha particles went right through the foil. And, these particles are deflected or scattered through a large angle on coming close to the nucleus. Also, the electrons having negligible mass do not affect the trajectory of these incident alpha particles.
- The trajectory traced by an alpha particle depends on the impact parameter of the collision. The **impact parameter** is simply the perpendicular distance of each alpha particle from the centre of the nucleus. Since in a beam all alpha particles have the same kinetic energy, the scattering of these particles depends solely on the impact parameter.

  Hence, the particles with a small impact parameter or the particles closer to the nucleus, experience large angle of scattering. On the other hand, those with a large impact parameter suffer no deflection or scattering at all. Finally, those particles having zero impact parameter or a head-on collision with the nucleus rebound back.
- Coming to the experiment, Rutherford and his team observed that a really small fraction of the incident alpha particles was rebounding back. Hence, only a small number of particles were colliding head-on with the nucleus. This, subsequently, led them to believe that the mass of the atom is concentrated in a very small volume.
- In a nutshell, Rutherford's nuclear model of the atom describes it as an electrically neutral sphere with (1) A small and positively charged nucleus at the centre and (2) Surrounded by revolving electrons in their dynamically stable orbits.
- The centripetal force that keeps the electrons in their orbits is an outcome of the electrostatic force of attraction between :

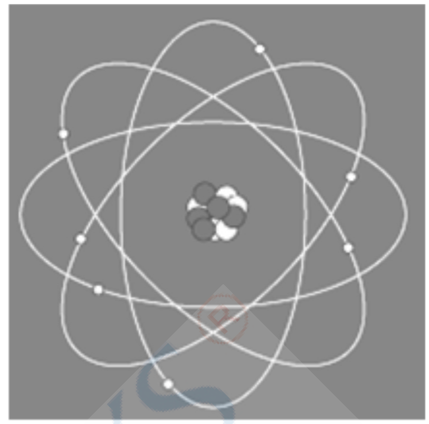
  1. The positively charged nucleus and
  2. The negatively charged revolving electrons.

Defects in Rutherford's Theory

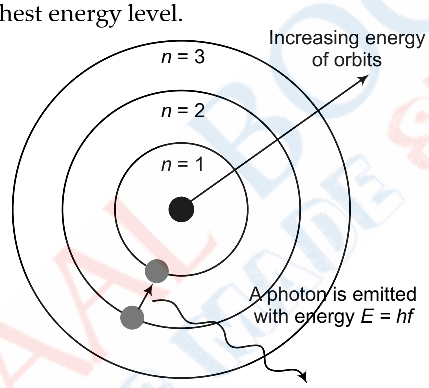
- Rutherford's model suffered certain drawbacks as it could not explain the stability of an atom in-spite of revolving electrons around the nucleus.
- The electrons revolving around emit radiation and subsequently lose energy.
- The loss of energy slows down the electrons which gradually move towards the nucleus in a spiral path and fall in nucleus resulting into collapse of atom and unstability; that is untrue. There were two fundamental defects in Rutherford's atomic model :
  - As per electromagnetic theory, electron when accelerated must emit energy as motion of electrons around nucleus that accelerates the motion, so it should radiate energy, as this does not happen. If such thing exists, then with continuous loss of energy orbit of electron, there will be a regular decline making electron to fall in nucleus which is against actual situation thereby resulting unstable atom.
  - If electrons emit continuous energy, they should form continuous spectrum, but actually line spectrum is obtained.



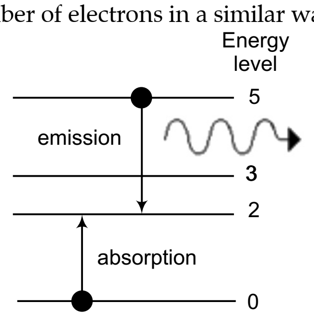
Bohr's Model of Atom



- Bohr atomic model and the models after that explain the properties of atomic electrons on the basis of certain allowed possible values. The model explained how an atom absorbs or emits radiation when electrons on subatomic level jump between the allowed and stationary states.
- He modified the problems and limitations associated with Rutherford's model of an atom.
- Bohr modified Rutherford's model where he explained that electrons move around in fixed orbital shells. Furthermore, he explained that each orbital shell has fixed energy levels.
- Postulates of Bohr Atomic Model :
  1. Every circular orbit has a certain amount of fixed energy and these circular orbits are termed orbit shells. The electrons do not radiate energy as long as they continue to revolve round the nucleus in the fixed orbital shells.
  2. The different energy levels are denoted by integers such as $n = 1$ or $n = 2$ or $n = 3$ and so on. These are called as quantum numbers. The range of quantum number may very and begin from the lowest energy level (nucleus side $n = 1$) to highest energy level.



     The different energy levels or orbits are represented in two ways such as 1, 2, 3, 4, ... , K, L, M, N, ... shells. The lowest energy level of the electron is called the ground state.
  3. The change in energy occurs when the electrons jump from one energy level to other. In an atom, the electrons move from lower to higher energy level by acquiring the required energy. However, when an electron loses energy it moves from higher to lower energy level.
- Electronic distribution of various orbits or energy levels can be calculated by the formula $2n^2$. Here, 'n' denotes the number of orbits.
- The number of electrons in K shell (1st orbit) can be calculated by $2n^2 = 2 \times 1^2 = 2$. Thus, maximum number of electrons in 1st orbit = 2.
- Similarly, the number of electrons in L shell (2nd orbit) = $2 \times 2^2 = 8$. Thus, maximum number of electrons in 2nd orbit = 8.
- We can determine the maximum number of electrons in a similar way.



Bohr atom

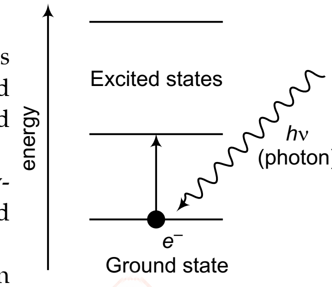
Limitations of Bohr's Model

Bohr's atomic model had few limitations. There are :

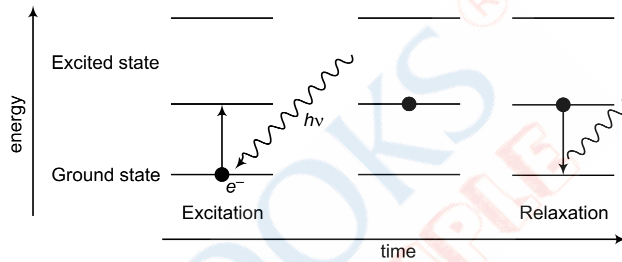
- Failure to explain Zeeman Effect (how atomic spectra are affected by high magnetic fields).
- It contradicts, Heisenberg Uncertainty Principle.

- Unable to explain how to determine the spectra of larger atoms.

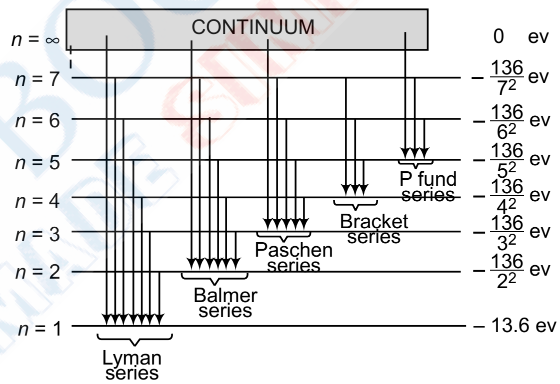
Energy levels



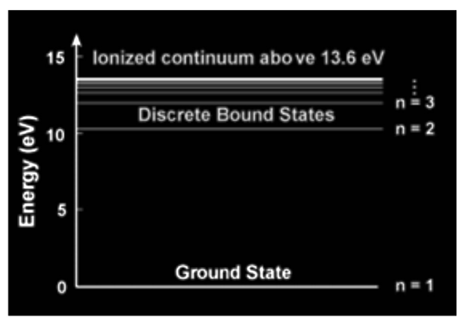
- The energy level of electron shows how easily it can be removed from its orbital, electrons in orbital's close to an atomic nucleus tend to be stable and needs lot of energy to remove, as electrons farther away are less stable and needs less energy.
- Completely removing an electron from atom is called ionisation, as negatively-charged electron and positively-charged ion, so energy required is called ionisation energy.
- If energy supplied is equal to or greater than ionisation energy, ionisation occurs.
- If energy supplied is less than ionisation energy, ionisation cannot occur, but electrons are excited into higher energy levels called energy states.
- In normal conditions, electrons surrounding an atom are in ground state as they occupy the orbitals closest to nucleus that are most stable.



- Higher energy states exist that is occupied by electrons if there is some energy inside.
- Excited states are unstable as these are without constant energy input that maintains higher energy level where electrons quickly relax back to ground state.
- As energy has to go somewhere and the atom must emit a photon with energy equal to difference between the two levels which is known as luminescence.
- In energy level diagram, horizontal lines show values of energy of electron and vertical lines containing arrow show the transition of electron from one energy level to another energy level.



- Each energy level is labelled with quantum number n (n = 1, 2, 3,...) and energy of a particular level is $E_n = -Rhc\,(1/n^2)$
- The quantized energy levels for hydrogen atom where levels are labelled by integer n called as quantum number where lowest energy state is ground state.
- The states with successively more energy than the ground state are first excited state, second excited state and so on.
- Beyond ionization potential where single electron of hydrogen atom is no longer bound to the atom and energy levels forms continuum.



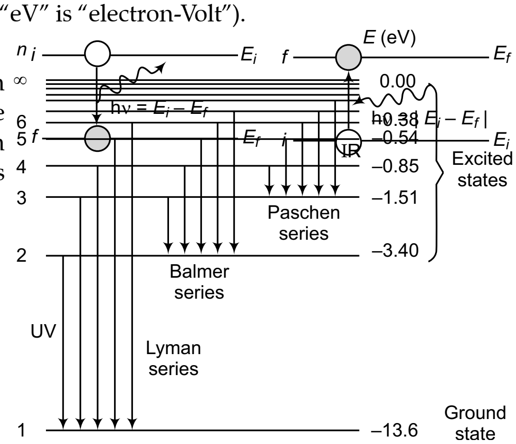
- In hydrogen, continuum starts at 13.6 eV above the ground ("eV" is "electron-Volt").

Hydrogen spectrum

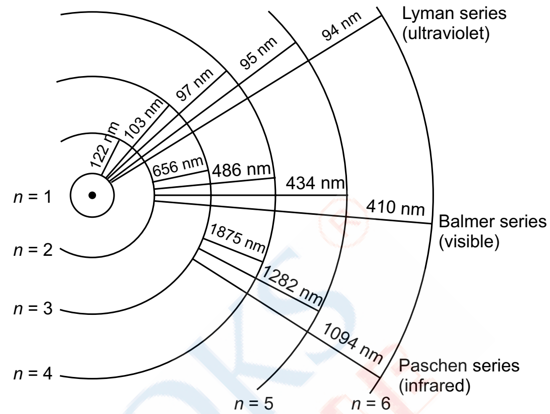
- Bohr atomic model shows that spectrum series arises when electron jumps from initial stationary orbit with principle quantum number n_i to final orbit with principle quantum number n_f, so energy change with such orbits are emitted as photon of frequency ν,

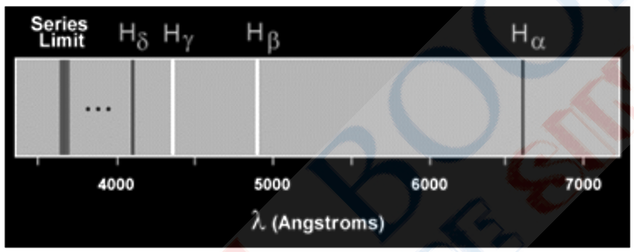
$$\Delta E = h\nu = E_i - E_f$$

- Spectrum of hydrogen is important as most of universe is made of hydrogen.
- Emission or absorption processes in hydrogen results in rise to series which are sequences of lines that corresponds to atomic transitions which ends or begins with similar atomic state in hydrogen.



- Balmer Series involves transitions starting (absorption) or ending (emission) with first excited state of hydrogen.
- The various series of lines are named as per the lowest energy level involved in transitions that give rise to the lines.
- Lyman series involve jumps to or from ground state (n = 1); Balmer series where all lines are in the visible region corresponds to n = 2, Paschen series to n = 3, Brackett series to n = 4 and Pfund series to n = 5.
- Lyman series is in ultraviolet while Balmer series is in visible and Paschen, Brackett, Pfund and Humphreys series are in infrared regions.
- Niels Bohr proposed a theory of hydrogen atom that shows origin of its spectrum that led to entire new concept of atomic structure where wavelengths of hydrogen spectrum be calculated by Rydberg formula, $1/\lambda_{vac} = R_H (1/n_1^2 - 1/n_2^2)$





- By setting n_1 to 1 and letting n_2 run from 2 to infinity, spectral lines known as Lyman series converges to 91 nm are obtained and other series of spectral lines is obtained using values of n_1 and n_2:

n_1	n_2	Name	Converge toward
1	$2 \rightarrow \infty$	Lyman series	91 nm – 122 nm
2	$3 \rightarrow \infty$	Balmer series	365 nm – 657 nm
3	$4 \rightarrow \infty$	Paschen series	821 nm – 1876 nm
4	$5 \rightarrow \infty$	Brackett series	1459 nm – 4053 nm
5	$6 \rightarrow \infty$	Pfund series	2280 nm – 7462 nm
6	$7 \rightarrow \infty$	Humphreys series	3283 nm –

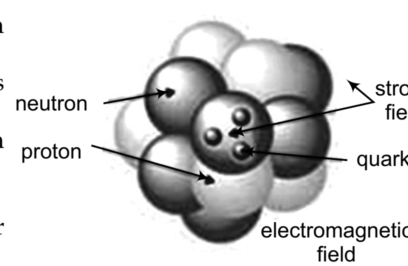
Series spectra for hydrogen are known after their discoverers, as Lyman, Paschen, Brackett, and Pfund series and their wavelengths are represented by formulae similar to the Balmer formula.

Topic 2
Nuclei

» Revision Notes

Composition of Nucleus

- Composition of atomic nuclei includes protons and neutrons in which nucleus of hydrogen atom has only single proton.
- The charge on a proton inside nucleus is $+1.6 \times 10^{-19}$ C with mass as 1836 times of electron.
- Neutrons are uncharged particles with its mass slightly more than that of a proton.
- Neutrons and protons together are known as nucleons.
- The number of protons in nuclei of an element is similar to number of electrons in neutral atom of that element.
- All nuclei of a given element may not have similar number of neutrons.



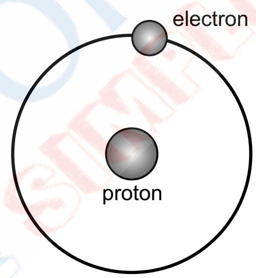
- Elements having similar number of protons with different number of neutrons in their nucleus are known as isotopes.

Size

- The size of nucleus was estimated initially through Rutherford scattering experiment where incident alpha particles on deflection by target nucleus having distance not exceeding 10^{-14}m were shown.
- Apart from Rutherford's scattering experiment, experiments like fast electrons and neutron scattering were done to obtain nuclear dimensions.
- As electrons interact with nucleus by electric forces, so electron scattering experiments describe distribution of charge in the nucleus.
- As neutron interacts with nucleus by nuclear forces, neutron scattering provides details on distribution of nuclear matter and volume of nucleus is directly proportional to number of nucleons which is mass number A.
- If R is nuclear radius, relationship between R and A is $R = R_0A^{1/3}$ where $R_0 \cong 1.2 \times 10^{-15} \cong 1.2$ fm and is known as nuclear radius parameter.

Atomic Mass

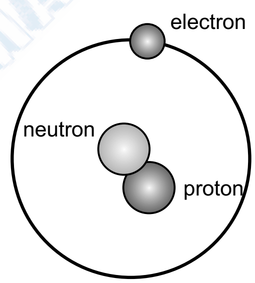
- Atoms are basic units of matter and tiny components of element that cannot be broken into simpler material.
- Atomic mass is equal to sum of individual particle masses of an atom.
- Atomic mass = Number of protons + number of neutrons
- There are three fundamental particles of an atom : protons, neutrons, electrons.
- Protons and neutrons are big sized, particles these are present in nucleus of an atom.
- Atomic mass is measured in amu.
- Atomic mass can be calculated by looking in periodic table, adding number of neutrons and protons in single atom and in calculating weighted average of all atoms of element.



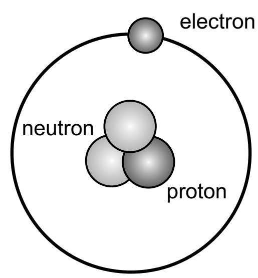
H hydrogen

Isotopes

- Isotopes are atoms with similar number of protons and electrons but different number of neutrons.
- If value of number of neutrons in an atom changes, it does not affect the element.
- Isotope of an element is shown by the nucleon number that is sum of number of protons and number of neutrons in an atomic nucleus.
- In ^{16}O, oxygen-16 with 8 protons and 8 neutrons and in ^{12}C, carbon-12 with 6 protons and 6 neutrons.
- Isotopes are Radioactive isotopes and Stable isotopes.
- Stable isotopes has stable mixture of protons and neutrons with stable nuclei these do not undergo decay, so such are isotopes of carbon, potassium, calcium and vanadium.
- Radioactive isotopes have unstable mixture of protons and neutrons having unstable nuclei. Due to their instability, they undergo decay thereby emitting alpha, beta and gamma rays.



^{3}H or D deuterium



^{3}H or T tritium

Isotopes of Hydrogen

- As hydrogen is simplest nucleus, its isotopes shows different effects in a nuclei.
- There are three isotopes of hydrogen : protium, deuterium, tritium ($_1H^1$, $_1H^2$, $_1H^3$)
- The isotopes have one single proton ($Z = 1$)with different number of neutrons.
- In protium, there is no neutron, deuterium has one while tritium has two neutrons.
- The atoms of isotopes having one electron to balance the charge of one proton.
- If a proton and neutron combines to form deuterium, reaction can be written $_0n^1 + {_1H^1} \rightarrow {_1H^2} + \gamma$ where energy balances equation in which mass is written in atomic mass units (u) or (MeV)/c^2.
- If tritium is formed by adding a neutron to deuterium, $_0n^1 + {_1H^2} \rightarrow {_1H^3} + \gamma$, a larger amount of energy is released (6.2504 MeV).

Isotopes of Carbon

- Carbon has three isotopes; carbon-12, carbon-13, carbon-14
- ^{14}C is a radioactive isotope of carbon while isotopes, ^{12}C and ^{13}C are stable isotopes.

- C-12 is stable and abundant isotope of carbon having 6 protons and 6 neutrons. As of balance neutron to proton ratio in C-12, it is stable and found in living organisms.
- C-14 is a radioactive isotope having half life of 5570 years having 6 protons and 8 neutrons.

Properties of Isotopes

- They have same number of protons as their atomic number is same.
- Their chemical properties are same.
- Their electronic configuration is same as they have same number of protons
- They have different physical properties like mass, density, melting point, etc.

Isobars

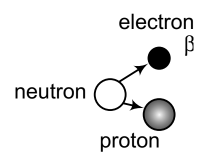
- Isobars are atoms having similar mass number but different atomic numbers.
- Isobars have different atomic numbers as they have different numbers of protons.
- They have similar atomic mass as they have enough neutrons to make same total of nucleons.
- In $_{32}Ge^{76}$ and $_{34}Se^{76}$ isobars where $_{32}Ge^{76}$ nucleus has 32 protons and 44 neutrons while $_{34}Se^{76}$ has 34 protons and 42 neutrons with nucleus having 76 nucleons.
- Isobars occur at different places in the Periodic Table.
- Isobars are not atoms of similar element as their atomic numbers are different, but are different elements with different atomic numbers having similar mass number e.g. $_{18}Ar^{40}$ and $_{20}Ca^{40}$.

Isotones

- Isotones are any of two or more species of atoms or nuclei having same number of neutrons.
- Chlorine-37 and potassium-39 are isotones as the nucleus of these species of chlorine has 17 protons and 20 neutrons, while nucleus of the species of potassium has 19 protons and 20 neutrons.
- Isotones are atoms that belong to different elements with same number of neutrons.
- Example: Carbon and Nitrogen having same number of neutrons $_6C^{13}$ and $_7N^{14}$

Radioactivity alpha, beta and gamma particles/rays and their properties

- Alpha, beta and gamma rays/particles are three most common forms of radiation emitted by unstable or radioactive isotopes.
- Protons in nucleus are positively charged particles and repel each other.
- The force that overcomes the repulsion and holds them together is strong force or strong nuclear force and it acts between the neutrons and protons in nucleus only at very short range.
- If nucleus has too high or too low ratio of neutrons to protons, it becomes unstable and radioactive.
- Alpha, beta and gamma decay are result of three fundamental forces working in nucleus which are 'strong' force, 'weak' force and 'electromagnetic' force.
- In all three cases, emission of radiation increases nucleus's stability by adjusting its proton: neutron ratio.
- In **alpha radiation**, nucleus attempts to find stability by emitting 'alpha particle' which is similar to helium nucleus having two protons and two neutrons.
- Alpha particle has large amount of energy due to its mass, so kinetic energy of particle gets transferred to objects on which it strikes.
- It is an ionizing radiation since it has two positive charge, which can stripe electrons away from neutral atoms.
- In addition to release of alpha particle, gamma rays are also released, $_{92}U^{235} \rightarrow {}_2He^4\,(\alpha) + {}_{90}Th^{231} + \gamma$
- **Beta radiation** involves transformation of proton into neutron through emission of electron, or reverse process by emission of positron, $_0n^1 \rightarrow {}_{-1}\beta + {}_1p^1 + \bar{\nu}$ where –1 on beta particle is not negative number of protons, but is used to balance the equation on charge.



- The symbol ($\bar{\nu}$) shown represents anti-neutrino where neutrino is chargeless, massless particle which moves with speed of light.
- In this, there are positive beta particles known as positrons and as beta particle is less massive compared to alpha particle, it can penetrate through greater thickness in materials and can be stopped easily.
- As number of protons changes during beta decay, element which releases beta particle change during the reaction.
- **Gamma radiation** is loss of energy by nucleus which is similar to emission of light or X-rays by energetic atoms.
- The radiation is instantaneous, and brings nucleus down to stable energetic state.
- A gamma ray is way for nucleus to lose energy and move in more stable state, just as electrons to be stable orbitals, nucleus to be in lowest energy state and there result no change in substance by emission of gamma ray.

Units of Radiations

(1) Roentgen (R)

- The roentgen is a unit used to measure a quantity called exposure.
- The roentgen measures the energy produced by gamma radiation in a cubic centimeter of air.
- It is used to describe an amount of gamma and X-rays, and only in air.
- One roentgen is equal to depositing in dry air enough energy to cause 2.58×10^{-4} coulombs per kg.
- It is measure of ionizations of molecules in mass of air.
- The main advantage of this unit is that it is easy to measure directly, but it is limited because it is only for deposition in air, and only for gamma and X-rays.

(2) Rad (Radiation Absorbed Dose)

- Different materials that receive same exposure may not absorb same amount of energy.
- The rad is a unit used to measure a quantity called radiation absorbed dose.
- It translates to amount of energy actually absorbed in some material, and is used for any type of radiation and any material.
- One rad is the absorption of 100 ergs per gram of material and its exposure results in about one rad of absorbed dose.
- The unit rad can be used for any type of radiation, but it does not describe the biological effects of the different radiations.

(3) Rem (Roentgen Equivalent Man)

- The rem is unit used to derive a quantity called Roentgen equivalent dose.
- It relates to absorbed dose in human tissue to effective biological damage by radiation.
- For gamma rays and beta particles, 1 rad of exposure results in 1 rem of dose.

(4) Curie (Ci)

- The curie is a unit used to measure a radioactivity.
- One curie is the number of particles per second from 1 gram of Radium = 3.7×10^{10} counts/second = 37 billion cps. = 37 billion Becquerel.
- Often radioactivity is expressed in smaller units like: thousandths (mCi), one millionths (uCi) or even billionths (nCi) of a curie.
- The relationship between becquerels and curies is: 3.7×10^{10} Bq in one curie.
  - 1 microcurie = 1 μCi = 37,000 Bq = 37,000 cps.
  - 1 microcurie = 2.22×10^6 disintegrations / minute = 2,220,000 cpm.
  - 1 nanocurie = 1 billionth of a curie = 2,220 disintegrations / minute.
  - 1 picocurie = 2.2 disintegrations / min.

(5) Becquerel (Bq)

- 1 Bq = 1 count per second = 1 event per second.
- The Becquerel is a unit used to measure a radioactivity.
- One Becquerel is that quantity of a radioactive material that has 1 transformations in one second.
- Often radioactivity is expressed in larger units like: thousands (kBq), one millions (MBq)or even billions (GBq) of a becquerels.
- With one Becquerel being equal to one transformation per second, there are 3.7×10^{10} Bq in one curie.

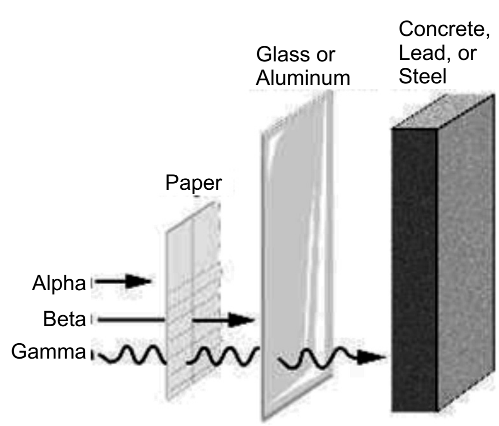
Properties

Alpha

- The alpha particle is heaviest and positively charged particle which is produced when the heaviest elements decay.
- These are fast moving helium nucleus which contains two protons and two neutrons having high energy in MeV and are stopped by its large mass.
- They have less penetration power but more ionization power and cannot penetrate the skin.

Beta

- They are highly energetic and fast moving electrons which are released from inside of a nucleus.
- These particle are energetic electrons which give off by nuclei of unstable isotopes to restore an energy balance.
- They leave the nuclei at a speed of 270,000 kilometres/second.



- They are negatively charged and have negligible mass having energies in the range of few hundred keV to several MeV.
- These particles are electrons that ejects from nucleus at high speed and have more penetration power as compared to alpha particles and can easily travel through the skin.

Gamma

- These are photons that are similar to light having very high energy X-ray typically from several keV to several MeV.
- These are waves from high frequency end of electromagnetic spectrum which do not have any mass.
- They are least ionizing but due to major penetrating, it is difficult to stop them from entering the body.
- As per their energy, they can be stopped by thin piece of aluminium foil or can penetrate several inches of lead.
- It is capable of damaging living cells as it slows down by transferring its energy to surrounding cell components.
- The table shows the characteristics of alpha, beta and gamma radiations.

Property	**α ray**	**β ray**	**γ ray**
Nature	Positive charged particles, ${}_2He^4$ nucleus	Negatively charged particles (electrons)	Uncharged electromagnetic radiation
Charge	+ 2e	– e	0
Mass	6.6466×10^{-27} kg	9.109×10^{-31} kg	0
Range	Approx 10 cm in air, can be stopped by 1mm of Al	Upto few m in, can be stopped by 200 cm of Al	Several m in air, can be stopped by 2500 cm of Pb
Natural Sources	Al natural radioisotopes e.g. ${}_{92}U^{236}$	Al radioisotopes e.g. ${}_{29}Co^{68}$	Excited nuclei formed due to α, β decay

Radioactive decay law

- Radioactive decay is spontaneous radioactive disintegration of an atomic nucleus, resulting in the release of energy.
- Some atoms are stable while others are unstable and decay, emitting radiation to achieve a stable state.
- The emissions from an unstable atom's nucleus, as it decays, can be in the form of alpha, beta or gamma radiation.
- Radioactive decay is a random process and prediction about decay of individual nucleus with larger numbers of nuclei cannot be done.

Activity of Sample

- The activity of sample is average number of disintegrations per second. Its unit is becquerel (Bq), activity $= \dfrac{\Delta N}{\Delta t}$
- One becquerel is one decay per second.

Decay constant λ

- The decay constant λ is probability that a nucleus will decay per second so its unit is s^{-1}.
- Activity = (decay constant) × (number of undecayed nuclei), $\dfrac{\Delta N}{\Delta t} = -\lambda N$

Decay Law

- Law of radioactive decay is important law of radioactivity that shows when nucleus undergo decay by emission of alpha particle or beta electron, it undergoes transformation and allows conversion of radium into radon, or tritium into helium.
  In this, number of atom in radioactive substance will inevitably decrease which also decreases number of emissions per second.
- The decay rate is activity of particular sample which is directly related to number of nuclei present, $N = N_0 e^{-\lambda t}$
- If nucleus regains stability after emission of particle, then form of decay law is simple and decay of such type is exponential decay.

$$T_{1/2} = \frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda} \approx 0.693\,\tau$$

Radioactive half-life ($T_{1/2}$); Radioactive decay constant (λ); Mean lifetime (τ)

- It is observed that easy measure of radioactive decay is period of time known as half-life which is amount of time taken for given sample of substance to be half.
- The half-life of any substance is a characteristic property of its nucleus which does not change.
- Half lives varies from seconds (*e.g.*, radon-224 half life = 55 seconds) to millions of years (*e.g.*, potassium-40 half life = 1.3×10^9 years).

- It has an implication for radioactive waste from nuclear power stations which is required to be stored safely for longer time.
- If end-result of radiation process is itself radioactive, then form of decay becomes more complex.
- A few grams of any substance has millions of billions of billions of atoms even in smallest possible sample number of radioactive nuclei which is unthinkable.

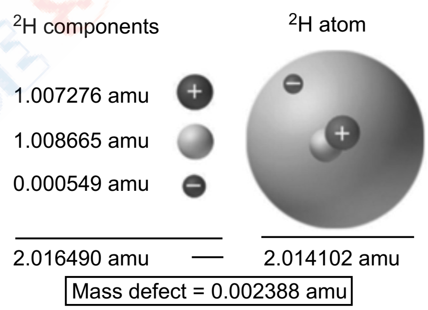
Mass-energy relation

- In an equation showing how much mass appears to increase due to speed, $m = \frac{m_0}{\sqrt{1-(v^2/c^2)}}$, the energy required in high speed movement where object on getting faster makes mass to increase and if more mass an object exists, more energy is required to move an object, so $E = \frac{1}{2}mv^2$
- If mass (m) and speed of light (c) are related in some way, then Einstein shows, if speed (v) becomes very low, then mass increases as mc^2 so both kinetic energy and mass increase due to motion for low speeds $E \approx mc^2 + \frac{1}{2}mv^2$
- Relativistic kinetic energy = $E - mc^2$ or E = Relativistic kinetic energy + mc^2, where first part is kinetic which depends on speed of moving body while other part results from increase in mass and does not depend on speed of body.
- The relation derived as mass-energy relationship discovered by Einstein, $E = mc^2$ which shows that amount of energy possessed by object is equal to its mass which is multiplied by square of speed of light and energy and mass are interchangeable with each other where energy gets converted to mass and mass to energy.

Mass defect

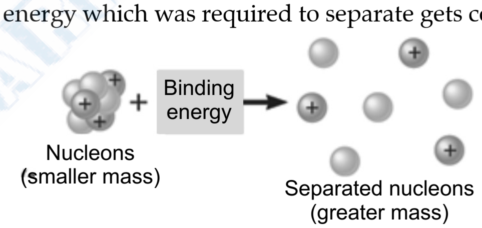
- If mass of nucleus of an element is measured and the sum of mass of protons and neutrons which makes the nucleus is measured, then mass of the nucleus is found to be slightly lighter and the difference in mass is mass defect.
- Mass defect ΔM for a nucleus with A protons and B neutrons be calculated as $\Delta M = Am_p + Bm_n - M_n$,
- Mass defect of neutral (number of proton = number of electron) whole atom ΔM_a :

$$\Delta M_a = Am_p + Bm_n + Am_e - M_a,$$



Binding energy per nucleon

- Binding Energy per Nucleon or Nuclear Binding Energy is energy which is required to separate a stable nucleus into its constituent protons and neutrons.
- If nucleons are separated, the energy which was required to separate gets converted into mass.



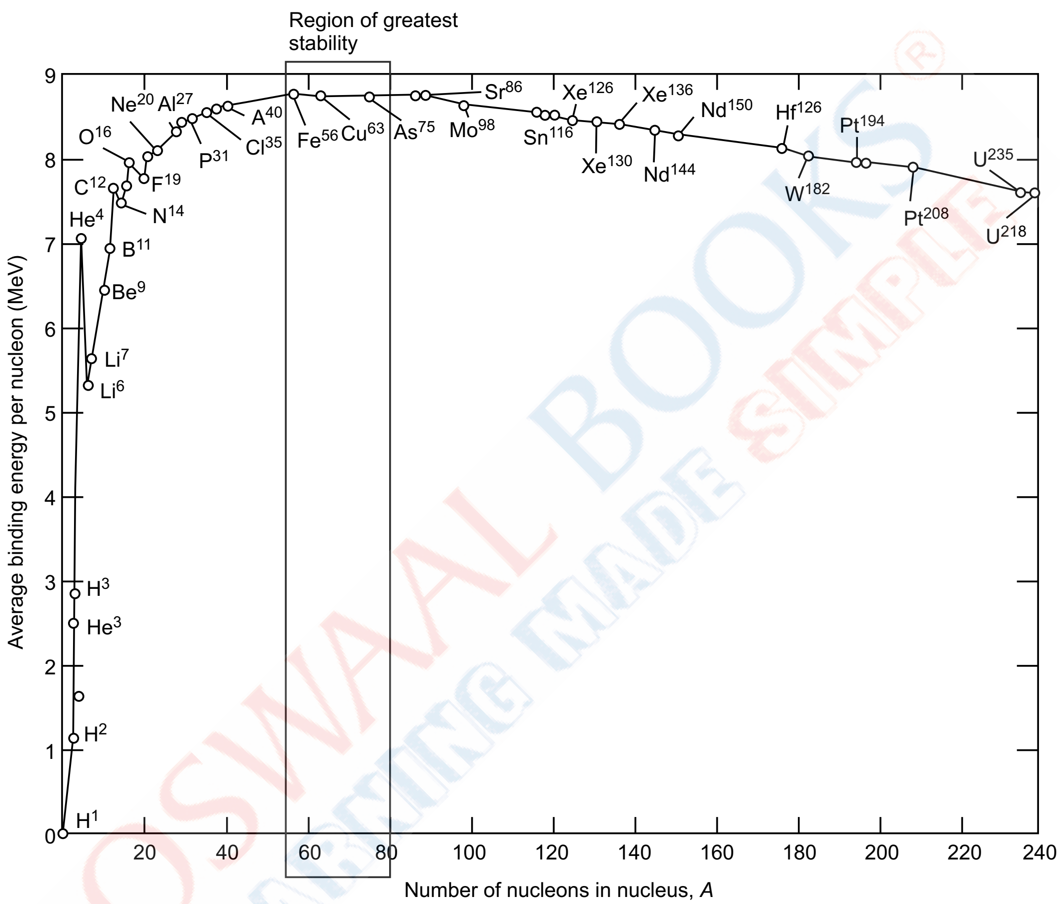
- More stable the nucleus is, greater is the amount of energy required to break it apart and the required energy is called as binding energy of the nucleus.
- Binding Energy per nucleon = Binding Energy/ Nucleon Number
- Binding energy of a nucleus can be determined from mass defect: Binding energy = (Mass defect) $c^2 = (\Delta m)c^2$
- To find binding energy, add masses of each protons, neutrons and electrons and subtract the mass of atom, and further convert's mass difference into energy.

Variation of Binding Energy per Nucleon with Mass Number

- Knowledge of binding energy per nucleon of a nucleus gives an idea about stability of that nucleus.
- Greater is binding energy per nucleon, more stable is the nucleus.
- In a graph between binding energy per nucleon (B_n) and mass-number (A) of all known nuclei, curve is called binding energy curve which gives following information's about nuclei.

1. The binding energy of each nucleus is positive, so to decompose a nucleus into its constituent nucleons external energy must be given.
2. The binding energy per nucleon is relatively small for lighter nuclei and it increases rapidly with increase of mass number (A) and becomes maximum and then decreases steadily.

3. The nuclei having mass number 56 and near 56 have maximum binding energy per nucleon (nearly 8.8 MeV); so these nuclei are relatively more stable.
4. For nuclei with mass number above 56, binding energy per nucleon decreases steadily and becomes 8 MeV at mass number 180.
5. For nuclei with mass number below 56, binding energy per nucleon decreases.
6. The binding energy curve shows that binding energy per nucleon of very light and very heavy nuclei is generally less than that of middle nuclei which indicates that very light and very heavy nuclei are less stable or unstable.



Nuclear Reactions

- A nuclear reaction is a process in which atoms collide with other atoms and lose some of their original mass.
- Due to principle of energy conservation, lost mass must reappear as generated energy, as per Einstein's equation $E = mc^2$.
- The two types of nuclear reactions used to produce energy are fission and fusion.

Nuclear Fission

- In fission reaction, heavy atomic nucleus is split into smaller nuclei, other particles and radiation.
- In typical reaction, atom of uranium 235 absorbs a neutron and splits into two lighter atoms, barium and krypton, emitting radiation and neutrons.
- Under special circumstances, emitted neutrons can split further atoms, which bring about more splitting, producing a very fast chain reaction.
- Nuclear power plants exploit the process of fission to create energy.
- If an incoming neutron strikes uranium nucleus, it will break into fragments where some fragments are chemical elements like barium or krypton while some are free neutrons.
- The free neutrons strike other uranium nuclei, creating a chain reaction.

$$_{92}U^{235} + 1\ _0n^1 \longrightarrow \left[_{92}U^{236}\right] \longrightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3\ _0n^1 + 200 \text{ MeV}$$

Nuclear fusion

- In fusion reaction, two or more light atomic nuclei fuse to form a single heavier nucleus.
- The mass change in the process is source of nuclear energy.
- Fusion within the cores of sun and other stars generates their radiating energy by fusing two hydrogen atoms to produce a helium atom.
- Fusion of deuterium in tritium creates helium-4, frees a remaining neutron, and releases energy as $E = Mc^2$, where two atoms are fused and very small amount of mass gets converted to large amount of energy.

$$_1H^2 + _1H^2 \longrightarrow _2He^3 + _0n^1 + 3.27 \text{ MeV}$$

$$_1H^2 + _1H^2 \longrightarrow _1He^3 + _1H^1 + 4.03 \text{ MeV}$$

$$_1H^2 + _1H^3 \longrightarrow _2He^4 + _0n^1 + 17.59 \text{ MeV}$$

$$_1H^2 + _2He^3 \longrightarrow _2He^4 + _1H^1 + 18.3 \text{ MeV}$$

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Chapter 19

Electronic Devices

» Revision Notes

Semiconductor Materials

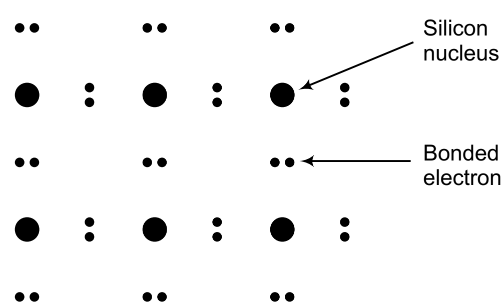
- Solid-state materials are grouped as insulators, semiconductors and conductors.
- Elemental semiconductors are made of single species of atoms like silicon (Si), germanium (Ge), tin (Sn), selenium (Se), tellurium (Te) along with many compound semiconductors that has two or more elements such as Gallium arsenide (GaAs) which is a compound of gallium (Ga) and arsenic (As).
- Intrinsic semiconductor material are the semiconductor materials in which the semiconducting properties of material occur naturally.
- Extrinsic semiconductor materials are semiconducting properties of the materials that are manufactured by us to make material behave in the manner as required.

Energy bands in conductors, insulators and semiconductors (qualitative ideas only)

- Electrons in isolated atom occupy discrete energy levels.
- Atoms are close to each other, they use energy levels of its neighbours and if atoms are regularly arranged in crystal lattice of solid, energy levels are grouped together in band that is continuous range of energies instead of single level.
- There are groups of energies called as band gap.
- Similar to energy levels of an atom, electrons fill lower bands first.
- The Fermi level describes about levels of electrons that fill up, but there are electrons with energies above this.

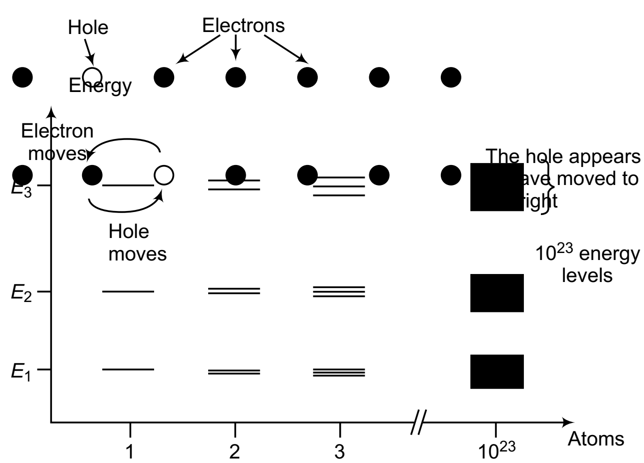
Bonding in Semiconductors

- The common semiconductors silicon and germanium has valence with 4 outer electrons for bonding.
- In pure crystal, each atom is bonded covalently to another four atoms with its outer electrons bonded having free electrons available to conduct which makes the resistance very large.
- Few electrons those are available come from imperfections in crystal lattice and thermal ionisation due to heating.
- A higher temperature results in more free electrons which increases conductivity and decreases resistance, as in case of thermistor.

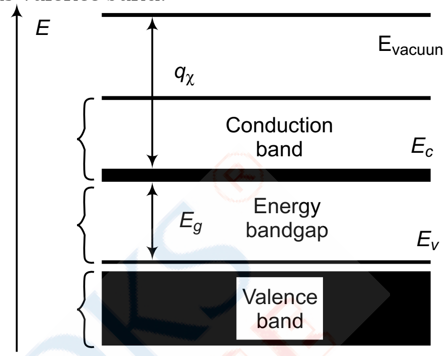


Holes

- If electron leaves its position in crystal lattice, it is known as positive hole, *i.e.*, a space left behind which is positively charged, which gets filled by electron from neighbouring atoms that leaves a hole there.
- In intrinsic semiconductor, number of holes is equal to number of electrons where small currents has drifting electrons in one direction and drifting holes in other direction.
- In Bohr model of atom, there exist sharp distinct energy levels that occupy electrons. If there are many atoms, they are interdependent and in such case, the discrete energy levels are fanned out.



- In silicon crystal, there are 10^{23} atoms per cubic centimeter, so the individual energy levels are no longer distinguishable from each other and thus form broad energy ranges.
- The width of energy bands depends on bounding of electrons.
- The valence electrons in highest energy level interact strongly with neighbouring atoms.
- The energy bands of each atom merge to continuous band known as valence band.
- In energy band diagram of semiconductors, energy is plotted as function of wave number, $\bar{v}$ along the crystallographic directions in crystal as band diagram depends on direction in crystal.
- The energy band diagrams contain many completely-filled and empty bands along with multiple partially-filled bands.



Doping

- The electrical properties of semiconductors are important in electronic devices like transistors, diodes and light dependent resistors which can be altered by adding small amounts of impurities.
- Doping is adding of such impurities to semiconductors and after this, the semiconductors devices are known as extrinsic semiconductors.

n-type Semiconductors

- If an impurity arsenic (As) with 5 outer electrons is present in crystal lattice, then 4 of its electrons is used in bonding with silicon and fifth electron is free to move and conducts.
- Due to extra electrons, fermi level gets closer to conduction band in intrinsic semiconductor and semiconductor is known as **N-type semiconductor**, where conduction is done by movement of free electrons those are negatively charged.

p-type Semiconductors

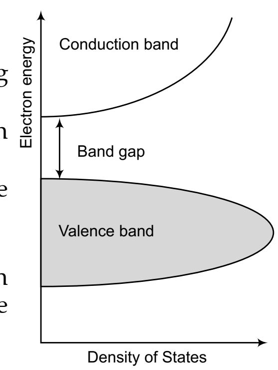
- P-type semiconductor is doped with indium (In) with 3 outer electrons and the hole in semiconductor occurs in crystal lattice where there is no electron.
- Due to lack of electrons, fermi level gets closer to valence band in intrinsic semiconductor where electron from next atom move into hole as conduction takes place by movement of positive holes and is known as **p-type** semiconductor.

Valence Band

- It is the highest range of electron energies where electrons are present at absolute zero temperature.
- The valence electrons are bound to individual atoms, as opposed to conduction electrons, which can move freely within the atomic lattice of the material.

Conduction Band

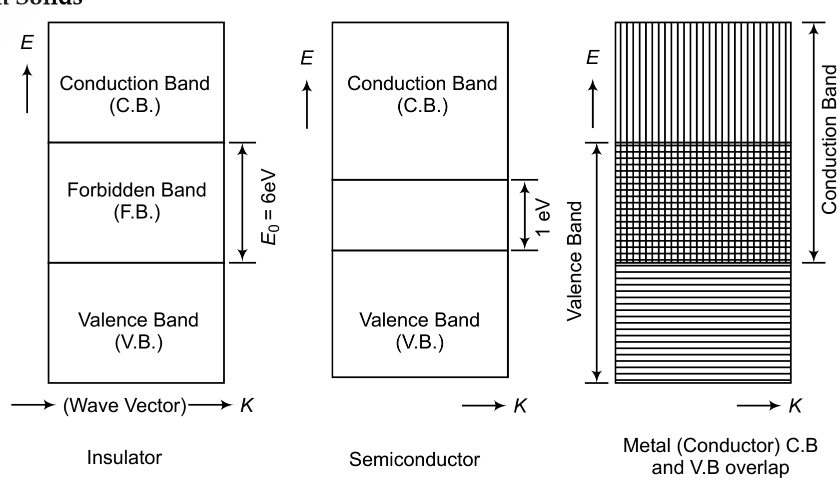
- The band having range of electron energies that makes the electron free from binding with atoms to move in atomic lattice of material is known as conduction band.
- In conductors having many free electrons, the conduction band overlaps with valence band.
- Electrons in conduction band are mobile charge carriers in solids which are responsible for conduction of electric currents in metals and other electrical conductors.



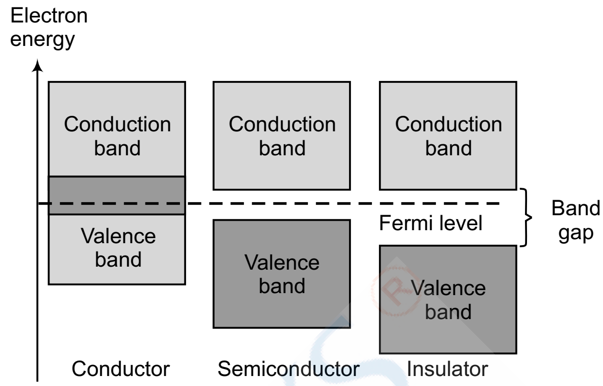
Band Gap

- It is the distance between the valence band of electrons and conduction band which describes the minimum energy that is needed to excite an electron up to a state in the conduction band where it can participate in conduction.

Energy Bands in Solids



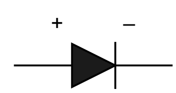
- The first unfilled band above valence band is conduction band. In case of an insulator, gap between valence band and conduction band which is larger with not enough energy to move electrons from valence band to conduction band for conduction and there result no electrical conduction in an insulator.
- If applied voltage is high then sufficient electrons get lifted to conduction band which makes current to flow.
- In a gas, voltage is referred to as striking voltage in context of fluorescent lamp, as this is voltage at which the gas starts to conduct and lamp lights up.



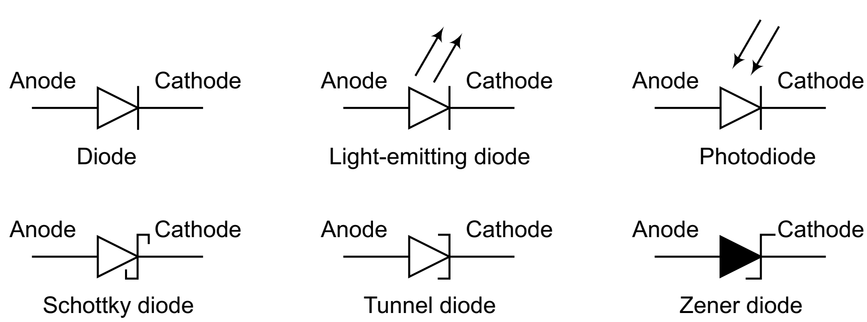
- In semiconductor, gap between valence band and conduction band is small while at room temperature there is sufficient energy present to move electrons from valence band to conduction band making conduction to take place.
- With increase in temperature, conductivity of semiconductor increases as electrons have enough energy to jump to conduction band.
- The electron bands also control optical properties of materials.
- The exact energies available in such bands control at which, frequencies of material gets absorbed.
- The available energy states are known as bands.
- In insulators, electrons in valence band are separated by large gap known as forbidden energy gap.
- Insulators have empty conduction band, but completely filled valence band.
- In conductors, valence band overlaps the conduction band.
- Semiconductor are the material whose electrical properties lies in between insulators and conductors.
- Semiconductors are materials having almost empty conduction band and almost filled valence band.
- In semiconductor, there appears small gap between the valence and conduction bands where thermal excitations are able to bridge the gap.
- The small gap in semiconductor is due to presence of small percentage of doping material which increases conductivity drastically.
- The outermost electrons of an atom are known as valence electrons having highest energy or least binding energy.
- The band of energy occupied by valence electrons is known as valence band which is the highest occupied band that can be completely or partially filled with electrons and is never empty.

Semiconductor Diode

- A semiconductor diode is made of crystalline piece of semiconductor material with a p–n junction across the electrical terminals.



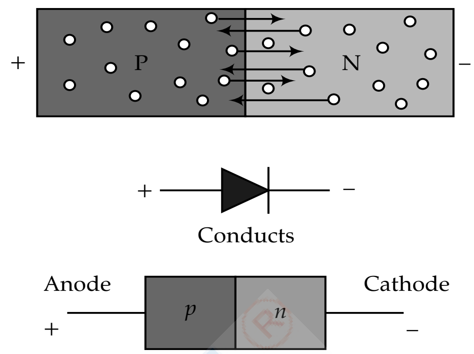
- There are many types of semiconductor diode as shown with their circuit symbol.



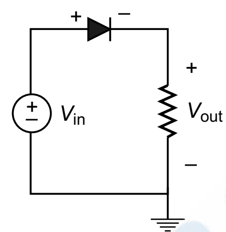
- Semiconductor diode can be made either from Silicon or Germanium and each differs in size and properties.

Diode or *p–n* Junction Diode

- Diode is a semiconductor device that is doped in different proportions as *p* and *n* types which forms *p-n* junction where "*n*" layer has donor dopants with excess electrons while "*p*" layer has acceptor dopants having less electrons.
- If diode is ON, it has no voltage across it and acts as short circuit while if diode is OFF, current is zero and it acts as open circuit.
- Diode is a two terminal device made by P type and N type materials. If junction is between metal and semiconductor, it is called Schottky diode.



- In PN junction, there is a transfer of charge through junction due to concentration gradient of charge carriers with barrier potential.

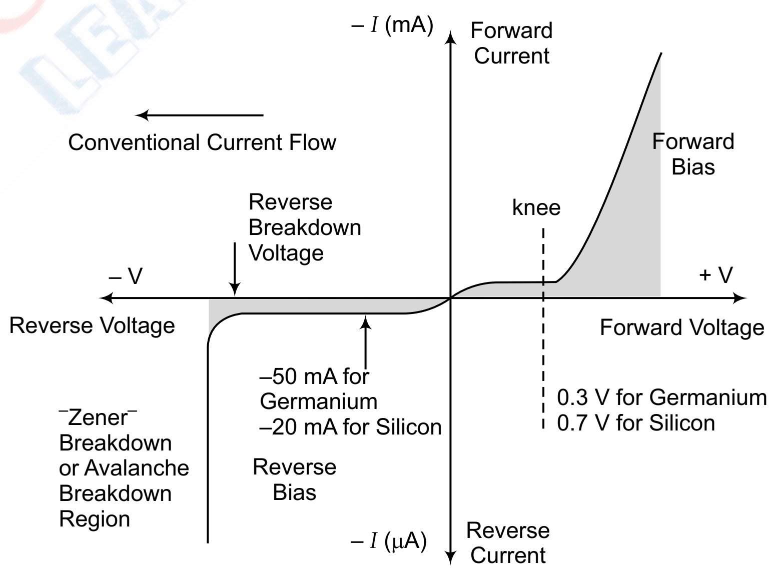


Characteristics of Diode

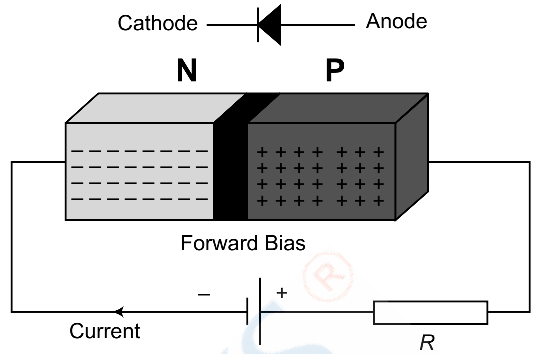
- Diodes are two terminal devices like resistors and capacitors.
- In diodes, current is directly related to voltage, like in a resistor. They're not like capacitors where current is related to the time derivative of voltage or inductors where the derivative of current is related to voltage.
- In diodes, the current is not linearly related to voltage, like in a resistor.
- Diodes only consume power and do not produce power like battery and are called passive devices.

I-V characteristics of diode in forward and reverse bias

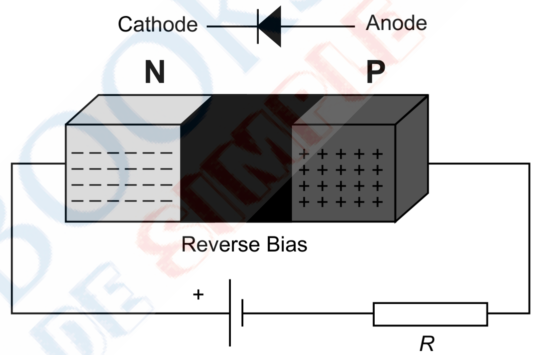
- Before using *p-n* junction as practical device or rectifying device, first the junction needs to be biased by connecting a voltage potential across it.
- In the I-V characteristics, on voltage axis, "reverse bias" is external voltage potential that increases the potential barrier while an external voltage which decreases the potential barrier is "forward bias".
- In forward bias, if external voltage is applied where negative of battery is connected to *n* side while positive of battery connected to *p* side of a diode of a diode then voltage potential gets reduced and more current can flow across the junction which decreases the width of *p-n* junction.



- If positive terminal of battery repels majority carriers holes in P-region and negative terminal repels majoirty carries electrons in N-region, those are pushed towards the junction and there is an increase in concentration of charge carriers near the junction, where recombination takes place and width of depletion region tends to decrease.

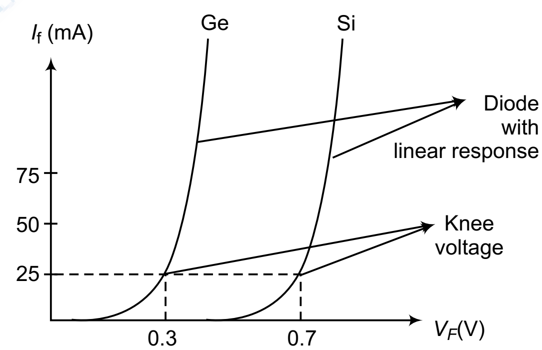


- As forward bias voltage rises, depletion region continue to reduce its width which results in more and more carriers recombination.
- In reverse bias, if external voltage is applied in reverse direction where positive terminal of battery is connected to N side while negative terminal of battery is connected to P side then barrier potential increases and minority charge carriers flow across junction, so current flows very small and is independent of external voltage.
- Beyond certain voltage, diode breaks down with Avalanche breakdown mechanism or Zener breakdown mechanism.



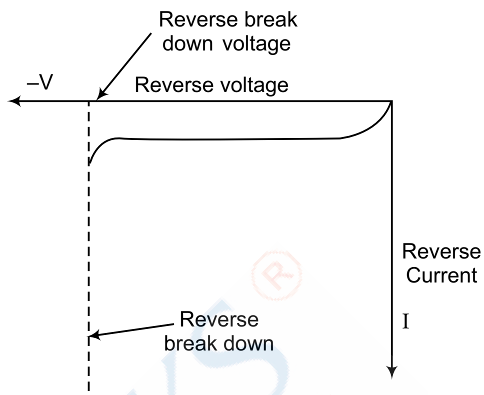
- The negative terminal of battery attracts majority carriers, holes in P-region and positive terminal attracts majority carries electrons in N-region which pulls them away from the junction which decreases concentration of charge carriers near junction thereby increasing the width of depletion region.
- The minority carriers which are diffusing across the junction get sufficient energy from the increased voltage to break the covalent bonds and generate more electron hole pairs.
- The generated pairs will break more bonds which is multiplicative, so at breakdown condition, the current will suddenly shoot up due to extra pairs.

Forward Bias Features



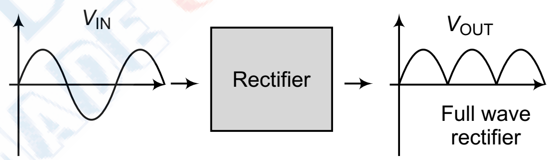
- If applied external forward bias increases, width of depletion layer becomes thin and forward current in *p-n* junction diode starts increasing strongly after KNEE point.
- A small amount of current known as reverse saturation current exists due to presence of contact potential and related electric field, while electrons and holes are freely cross the junction causing diffusion current that flows in opposite direction to reverse saturation current.
- Using forward biasing, height of potential barrier gets reduced while majority carrier current in *p-n* junction diode increases by exponential factor making the total amount of current as $I = I_s \times (e^{V/kT})$, where I_s = constant.
- Excess of free majority charge carrier holes and electrons that enters N and P regions will act as minority carriers that recombine with local majority carriers in N and P regions that decreases with distance from PN junction.
- Forward characteristic of *p-n* junction diode is non linear showing that resistance is not constant during the operation of *p-n* junction while slope of forward characteristic of *p-n* junction diode becomes quiet steep
- The steep characteristics shows that resistance is very low in forward bias of junction diode where forward current becomes directly proportional to external power supply and inversely proportional to internal resistance of junction diode.
- Using forward bias to *p-n* junction diode results in low impedance path for junction diode which allows conduction of large amount of infinite current that flows above KNEE point with application of small amount of external potential.
- Potential difference across the junction or at the two N and P regions is maintained constant by action of depletion layer.
- Maximum amount of current conducted is kept limited by load resistor as when diode conducts more current than normal, then excess current results in dissipation of heat which leads to severe damage of the device.

Reverse Bias Features



- In reverse bias condition, majority charge carriers are attracted away from the depletion layer by their respective battery terminals connected to *p-n* junction.
- Positive terminal attracts the electrons away from the junction in N side and negative terminal attracts the holes away from the junction in P side.
- By attraction of electrons and holes, width of potential barrier increases that obstruct the flow of majority carriers in N side and P side.
- Width of free space charge layer increases, thereby electric field at *p-n* junction increases and the *p-n* junction diode acts as a resistor.
- The current that flows in *p-n* junction diode is small leakage current due to minority carriers generated at depletion layer or minority carriers which drift across *p-n* junction making depletion layer a high impedance path that acts as an insulator.
- As reverse bias potential to *p-n* junction diode increases, there is a reverse voltage breakdown where diode current is controlled by external circuit.
- Reverse breakdown depends on doping levels of P and N regions.
- With increase in reverse bias, *p-n* junction diode becomes short circuited due to overheat and maximum circuit current flows in *p-n* junction diode.

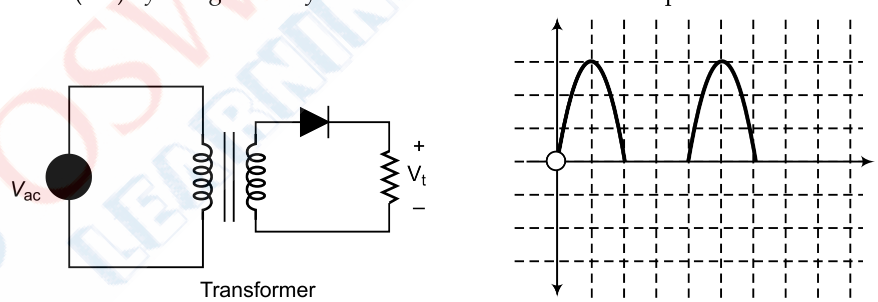
Diode as a Rectifier



- Rectifier is a circuit which converts AC supply into unidirectional DC supply.
- The bridge rectifier circuits uses semiconductor diode for converting AC as it allows the current to flow in one direction only.

Half wave Rectification

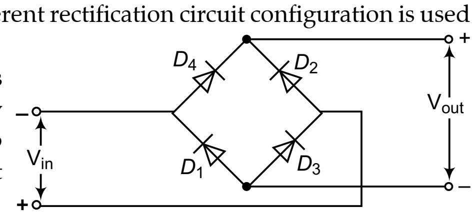
- Rectification is one of the most famous applications of diode where alternating current (AC) gets converted into direct current (DC) by using one way flow of electrons which reduces power to resistive load.



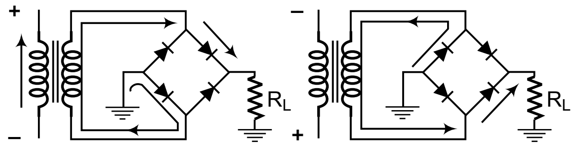
- A circuit used for converting AC into DC is a half-wave rectifier with single diode allowing current to flow in one direction where AC power source V_{ac} is connected to diode and resistor in series.
- Positive part of a cycle of V_{ac} a positive voltage produces on secondary side of transformer which forward bias the diode and diode starts passing the current, making the voltage drop across the load.
- Negative part of a cycle of V_{ac} produced negative voltage of secondary side and diode is reverse biased that does not pass any current.
- The voltage drop over the load is now zero. So, voltage waveform over the load resistor shows positive side of sinusoidal cycle while clamping off negative side of sinusoidal cycle.

Full wave Rectification

- To rectify AC power for using both half cycles of sine wave, a different rectification circuit configuration is used which is known as full wave rectification.
- A simple kind of full wave rectifier uses a center tap transformers with two diodes where in first half-cycle, if source voltage polarity is positive (+) on top and negative (–) on bottom, then only top diode conducts while bottom diode blocks the current where first half of sine wave is positive on top and negative on bottom.

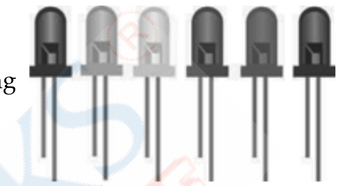


- Another type of fullwave rectifier uses 4 diodes and a normal transformer. Positive half of input AC, diodes D_1 and D_2 are forward biased while diodes D_3 and D_4 are reverse biased which makes load current flows through D_1 and D_2 diodes.
- Negative half cycle of input AC, diodes D_3 and D_4 are forward biased while diodes D_1 and D_2 are reverse biased which makes load current to flow through D_3 and D_4 diodes.
- The Full-wave rectifiers are used in power supplies to convert AC voltages to DC voltages.
- A large capacitor in parallel with the output load resistor reduces the ripple from the rectification process.

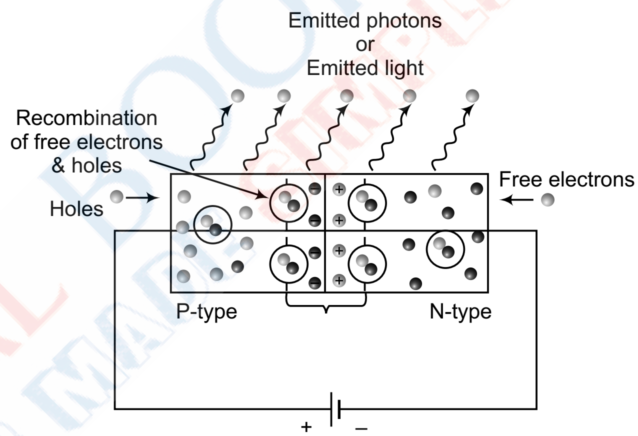


Current flow positive half-cycle   Current flow negative half-cycle

LED

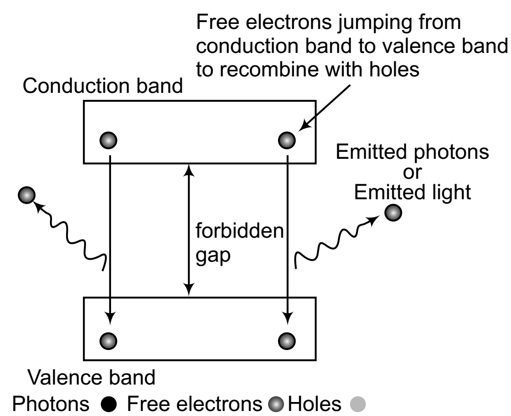


- Light Emitting Diode or LED is most widely used semiconductor diodes among all the different types of semiconductor diodes available today.
- It emits visible light or invisible infrared light when forward biased.
- The LEDs which emit invisible infrared light are used for remote controls.
- LED is a light source that uses semiconductors and electroluminescence to create light.
- Light emitting diode is a *p-n* junction diode which is specially doped.
- If diode is forward biased, electrons & holes move fast across the junction and combine constantly by removing one another.
- The electrons which move from *n*-type to *p*-type silicon combine with holes and disappear which results in atom with more stability which gives little burst of energy in form of tiny packet or photon of light.
- The free electrons in conduction band do not remain for longer period, so after short period, free electrons tend to lose energy in form of light which further recombine with holes in valence band, making the charge carrier to emit light energy.



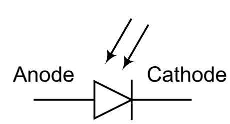
There are different types of light emitting diodes :

- Gallium Arsenide (GaAs) – infra-red
- Gallium Arsenide Phosphide (GaAsP) – red to infra-red, orange
- Aluminium Gallium Arsenide Phosphide (AlGaAsP) – high-brightness red, orange-red, orange, and yellow
- Gallium Phosphide (GaP) – red, yellow and green
- Aluminium Gallium Phosphide (AlGaP) – green
- Gallium Nitride (GaN) – green, emerald green
- Gallium Indium Nitride (GaInN) – near ultraviolet, bluish-green and blue
- Silicon Carbide (SiC) – blue as a substrate
- Zinc Selenide (ZnSe) – blue
- Aluminium Gallium Nitride (AlGaN) – ultraviolet

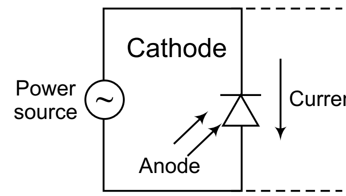


Photodiode

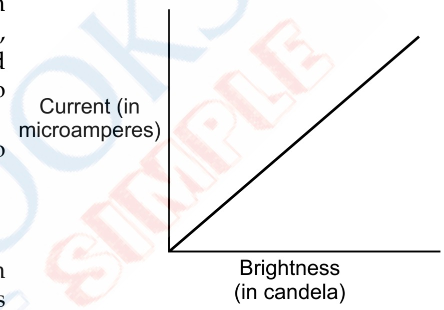
- Photodiode is a transducer which takes light energy and converts it to electrical energy.
- It creates electricity when exposed to light.
- It conducts electric current in similar proportion as the amount of light falls on it.
- Photo diode has two terminals anode and cathode with the arrows indicating that the light rays falling on photo diode.
- Photodiode is *p-n* junction which consumes light energy to generate electric current.
- It is referred to as photo-detector, photo-sensor or light detector and is specially designed to operate in reverse bias condition.



- It is sensitive to light as when light falls on it, it easily converts light into electric current.
- There are mainly three types of photo diodes: *p-n* junction photo diode, Avalanche photo diode and PIN photo diode.
- Photodiode operates in reverse bias in a circuit where anode gets connected to the ground of a circuit and cathode gets connected to the positive voltage supply of the circuit where current flows from cathode to anode if exposed to light.



- The photodiodes connected to power source in circuit produces microamperes of current which is not enough power so as to drive many electronic devices.
- The voltage source is used in conjunction which increases output current so that there should be enough current to drive a load.
- The photodiode provides extra current to circuit from the light it receives.
- As photodiodes have linear response to light, they can function easily in light meters where if a circuit is exposed to more light, photodiode produces more current and if the circuit is connected with galvanometer, then galvanometer shifts more right due to increased light intensity on photodiode.



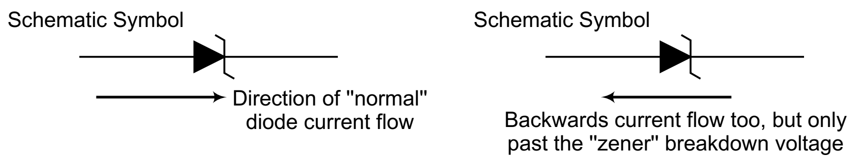
- A photo sensitive diode can be operated mainly in two modes; photo conductive mode and photo voltaic mode.
- The photo detectors are operated in photo conductive mode.
- Solar cells are operated in Photo voltaic mode.
- If photo generation takes place at a distance of diffusion length order, then photo generated electron hole pairs are removed across by applied reverse bias field and such mode of operation of photo diode is known as photo conductive mode.
- The photo current varies almost linearly with incident light flux or optical power, where current is $I = I_S + I_O\,(1 - e^{V/(\eta \times V_t)})$
- The minimum energy of light required for photo generation due to intrinsic excitation is forbidden gap energy E_G where wavelength associated with critical energy is known as critical wavelength $\lambda = 1.24/\,E_G$ (in eV).
- Photodiode is capable of converting light energy to electrical energy and can be expressed as a percentage known as Quantum Efficiency (Q.E.).
- The sensitivity of photodiode is expressed in practical units of amps of photodiode current per watt of incident illumination where QE is related to photodiode response as $1.24 \times 10^5\, R/\lambda$ (nm).
- In certain design, for minimum detectable light of photodiode, minimum incident power is required to generate photocurrent. It is similar to total photodiode noise current which is noise equivalent power or NEP – noise current/responsivity.

Solar Cell

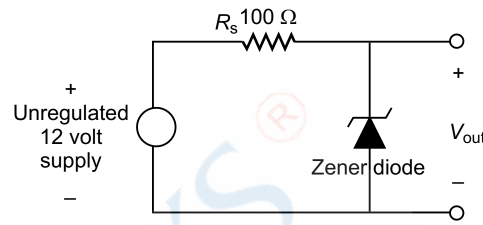
- Solar cell is an electronic device which absorbs sunlight and converts it into electricity.
- It is compact in size and is bundled with larger units for making solar panels.
- It is a sandwich of two different layers of silicon those are doped for allowing electricity to flow through them in a particular direction.
- In solar cell, lower layer is doped so that it has less electrons known as *p*-type or positive-type silicon while upper layer is doped to give more electrons known as *n*-type or negative-type silicon.
  In this, there are *n*-type silicon and *p*-type silicon layers those generate electricity with the help of sunlight to make electrons to jump across the junction between different types of silicon material.
- If sunlight falls on the cell, photons bombard the upper surface and carry their energy down through the cell to *p*-type layer.
- The electron uses energy to jump across the barrier into the upper *n*-type layer and escapes out into the circuit.

Zener Diode and their Characteristics

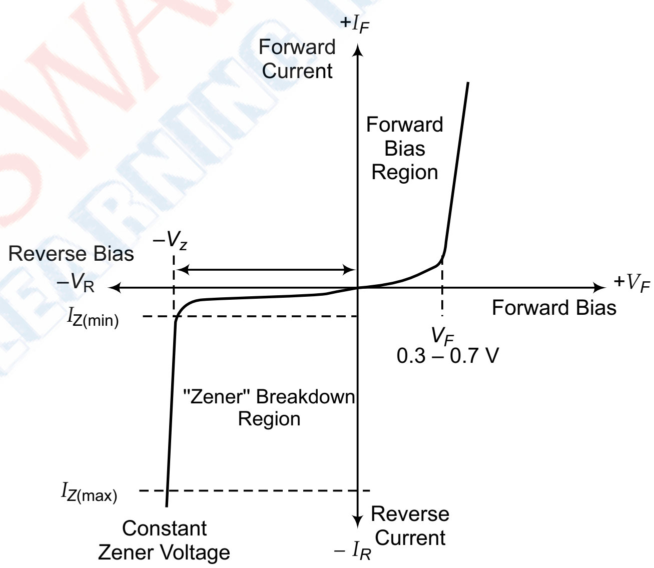
- Zener Diode is commonly used for making reference voltages for Voltage Regulators and to protect other electronic devices from voltage surges.
- It is a special type of semiconductor diode which allows current to flow in one direction and in opposite direction if exposed to high voltage.
- It is a *p-n* junction semiconductor device which is designed to operate in reverse breakdown region.
- The breakdown voltage of Zener diode is carefully set by controlling doping level during manufacturing.



- It allows electric current in forward direction similar to normal diode. It also allows electric current in reverse direction when applied reverse voltage is more than zener voltage.



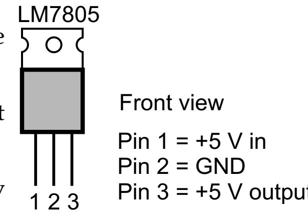
- It is always connected in reverse direction as it is specifically designed to work in reverse direction and allows fixed stable voltage to be taken from unstable voltage source like battery of renewable energy system.
  In this, breakdown voltages have been designed to sit at particular voltage level.
- There are two breakdown mechanisms: Zener Breakdown and Avalanche Breakdown
- **Zener Breakdown** is electrical breakdown mechanism that occurs in reverse biased *p-n* Junctions due to high electric field density across depletion region that breaks certain covalent bonds which leads to many minority carriers.
- **Avalanche breakdown** is different from zener breakdown which results due to collision of minority carriers in depletion region which are accelerated by reverse biased electric field where energy of minority carriers is sufficient to release electron hole pairs from covalent bonds by collision.
- Avalanche effect is mainly above 5.6 V for silicon diodes and has positive temperature coefficient where zener and avalanche effects are equally important which cancel positive and negative temperature dependency and result as component of choice in temperature critical applications.
- The I-V Characteristics Curve of zener diode shows current-voltage relationship of zener diode where right half side is part where zener diode receives forward voltage – positive voltage across its anode to cathode terminals.



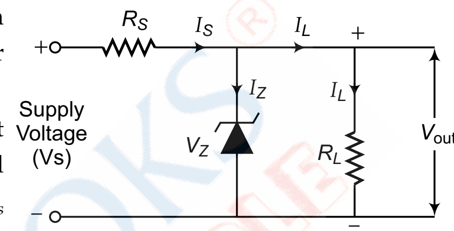
- In this region, diode is in forward biased while the current is small till it spikes exponentially up, once the voltage reaches a certain point known as threshold voltage.
- In left half side, zener diode receives positive voltage across its cathode to anode terminals where diode is reverse biased.
- While receiving reverse voltage, current is very small known as leakage current that flows through the diode and on hitting breakdown voltage, current sharply increases, known as avalanche current.
- The breakdown voltage point is important, not because of avalanche current, but when the voltage of zener diode reaches at such point, it remains constant at this voltage inspite of increase in current, so zener diode is useful for voltage regulation.

Voltage Regulator

- To step down 12 voltage supply, a special three-terminal device known as voltage regulator is used.
- Voltage regulator is special semiconductor device that has been designed to act as an ideal battery.
- Voltage regulator has 3 pins where Pin 1 (V_{IN}) is connected to positive battery terminal, Pin 2 (GND) is connected to ground while Pin 3 is 5 volt regulated output.



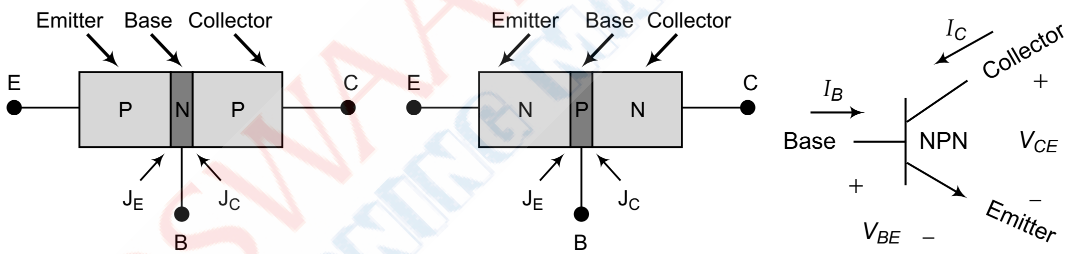
- Voltage regulation is measure of ability of circuit to maintain constant voltage output under variation either in input voltage or load current.
- In zener diode voltage regulator circuit, resistor R_s is used to limit reverse current through diode to safer value, V_s and R_s are selected such that diode operates in breakdown region and series resistor R_s limits current through it, $I_s = (V_s - V_z)/R_s$ or $I_s = I_z + I_L$



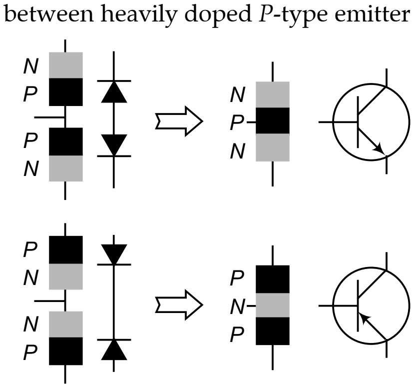
- Zener diode maintains constant voltage across load as long as supply voltage is more than zener voltage.
- In this, if input voltage increases, current through Zener diode increases while voltage drop remains constant.

Junction Transistor

- A junction transistor is a bipolar device having two *p-n* junctions that combine to form either an *n-p-n* or *p-n-p* transistor with three electrodes: emitter, base, collector.
- In junction transistor there is small current flowing in the base which causes larger current to flow in the collector.

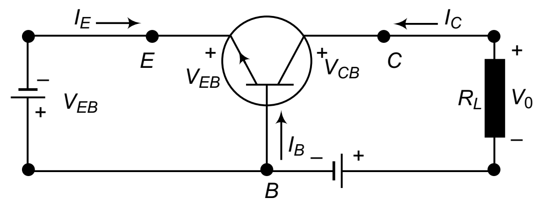


- In this, if small current in base increases, there will be more current in collector up to maximum of saturation value and if current in base decreases to zero, the current in collector will be zero.
- The idea that the current in base causes large current in collector, makes $I_C = \beta I_B$
- As collector current I_C is not saturated at its maximum, β is constant (ranging 30 – 100). So such property shows that BJT can be used as an amplifier or switch.
- In NPN transistor, a thin and lightly doped *P*-type base is sandwiched between heavily doped *N*-type emitter and other *N*-type collector.
- In PNP transistor, a thin and lightly doped *N*-type base is sandwiched between heavily doped *P*-type emitter and other *P*-type collector.
- Bipolar transistors work as current-controlled current regulators which restrict the amount of current passed as per smaller controlling current.
- In this, main current moves from collector to emitter or from emitter to collector as per type of transistor PNP or NPN.
- The small current that controls the main current goes from base to emitter, or from emitter to base which depends on kind of transistor PNP or NPN.



Principle of BJT

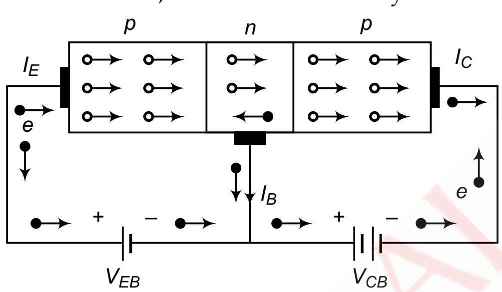
- In PNP transistor circuit, BE junction is forward biased while CB junction is reverse biased.
- In this, width of depletion region of CB junction is higher than BE junction.
- The forward bias of BE junction decreases the barrier potential and produces electrons to flow from the emitter to the base.
- The base being thin and lightly doped has very few holes and less electrons from the emitter for recombination in base region with holes that flow out from base terminal.



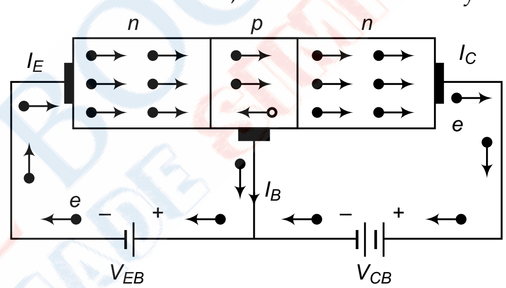
- The recombination initiates the base current flow which leaves large number of electrons that passes reverse bias collector junction for collector current. Kirchhoff Current Law shows $I_E = I_B + I_C$ where base current is very less as compared to emitter and collector current $I_E \sim I_C$.
- It is observed that the operation of PNP transistor is similar to NPN transistor where there are holes instead of electrons.

Transistor Action

- *N-P-N* and *P-N-P* transistors will behave in similar way except change in biasing and major charge carriers.
- In *P-N-P* transistors, conduction is done by holes while in *N-P-N* transistors, conduction is done by electrons.

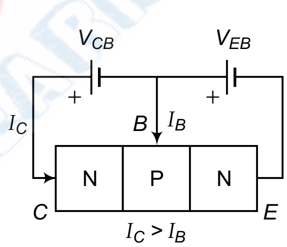


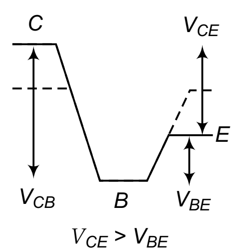
Basic Connections of *P-N-P* Transistor



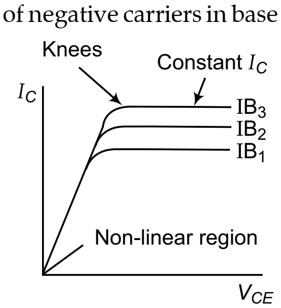
Basic Connections of *N-P-N* Transistor

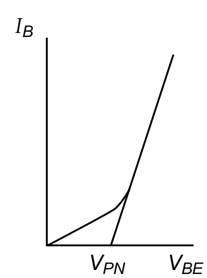
- In *N-P-N* transistor, forward bias causes electrons in *N*-type emitter to flow towards the base which constitutes emitter current I_E and electrons while flowing through *P*-type base will tend to combine with holes.
- As base is lightly doped and thin, so only few electrons combine with holes to form base current I_B while the remaining electrons cross in collector region to form collector current I_C so emitter current I_E is equal to the sum of collector current I_C and base current I_B, *i.e.*, or $I_E = I_B + I_C$





- In *N-P-N* transistor with collector, emitter and base are shorted together, form two depletion regions surround the base where diffusion of negative carriers in base and positive carriers out of base results in electric potential.



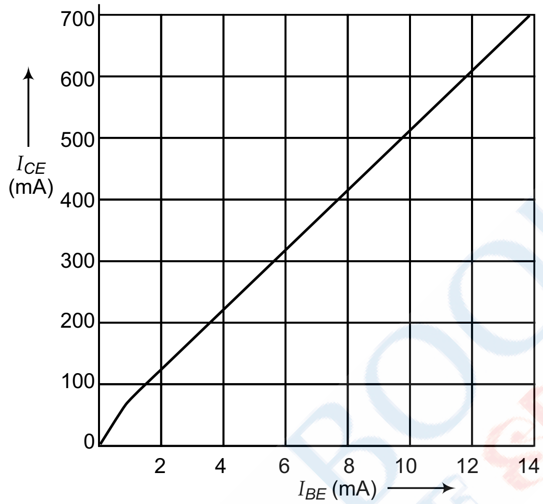


- If *N-P-N* transistor is biased for normal operation, base terminal is positive with respect to emitter and negative with respect collector. If properly biased, transistor acts as $I_C > I_B$, where depletion region at reverse-biased base-collector junction grows and supports the increased electric potential changes.

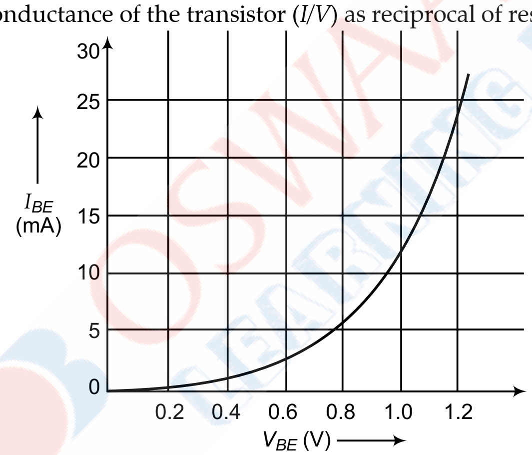
- In typical transistor 99% charge carriers from emitter to collector form collector current I_C which is less than emitter current I_E, so $\alpha = I_C/I_E$, where α is 0.99.
- In transistor characteristics curve, a small current flows into base that controls larger current flow in collector, so $I_C = \alpha\, I_B = h_{fe} \times I_B$ or $I_E = I_C + I_B$ so $\beta = \alpha / 1 - \alpha$.

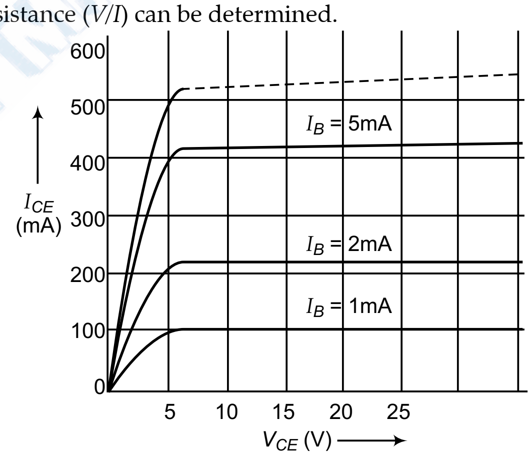
Characteristics of Transistor

- A bipolar junction transistor operates in three modes: Common Base (CB) mode, Common Emitter (CE) mode, Common Collector (CC) mode.



- From the transfer characteristic graph of I_{CE} vs I_{BE}, current gain is described as h_{fe} is calculate. Low gain transistors have current gain of 20-50 and high gain transistors have current gain 300-800.
- The input characteristic of BJT with base emitter current I_{BE} against base emitter voltage V_{BE}, where input conductance of the transistor (I/V) as reciprocal of resistance (V/I) can be determined.



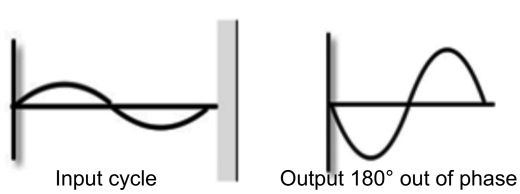


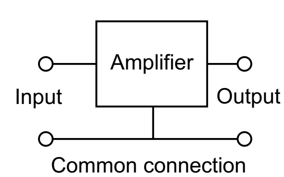
- In the output characteristic of BJT, vertical line describes change in collector emitter current I_{CE} and horizontal line describes change in collector emitter voltage V_{CE} has not effect on I_{CE} in the horizontal portion of graph. The slope of the linear portion of the character gives output conduction β for a given V_{CE}

$$\beta = \frac{\Delta I_{CE}}{\Delta I_B}$$

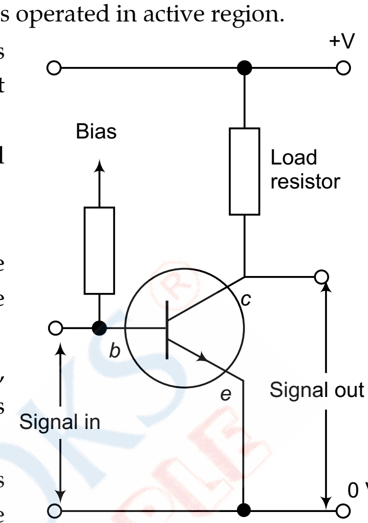
Transistor as an amplifier (common emitter configuration)

- Amplifier is an electronic device that increases the strength of weak signal by 180° phase.
- If an input of low amplitude is provided, in such case an amplifier converts low amplitude to high amplitude signals.



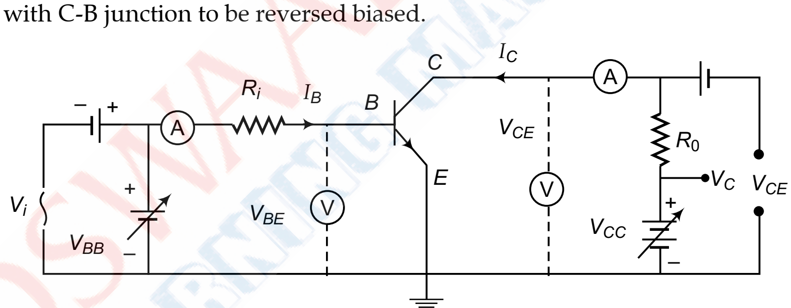


- Amplifier is *N-P-N* transistor with common emitter configuration is shown in the below figure.
- Transistor is current sensitive device with larger current gain (β) when it is operated in active region.
- The amplifier has two input and two output terminals. A transistor used as an amplifier should have one of its three terminals common to both input and output.
- The choice of the terminal used as the common connection has marked effect on the performance of an amplifier.



Common Emitter Mode

- The common function of transistor is to be used in Common Emitter mode where small changes in base/emitter current result in large changes in the collector/emitter current.
- For voltage amplification, load resistor is connected in collector circuit, so that a change in collector current results in change in voltage that is developed across the load resistor.
- The value of load resistor affects the voltage gain of an amplifier. Larger is the load resistor, larger is the change in voltage which results from change in collector current.
- In this, output waveform is in anti-phase to input waveform which occurs due to increase in base/emitter voltage that causes an increase in base current and causes increase in collector current.
- If collector current increases, voltage drop across the load resistor also increases and as voltage on top end of load resistor does not change, voltage on bottom end should tend to decrease which increase in base/emitter voltage resulting into decrease in collector emitter voltage.
- In transistor CE amplifier with *N-P-N* transistor, transistor to work as an amplifier *EB* junction should be forward biased with C-B junction to be reversed biased.



- Here voltage sources V_{BB} and V_{CC} provide required bias and signal voltage V_i is amplified between the base and emitter with development of amplified voltage V_0 across load resistor R_0.
- If input signal varies, base current varies which produces large change in collector current, so input voltage is $V_{BB} = V_{BE} + I_B R_i$ or $\Delta V_i = \Delta I_B \cdot R_i$
- As output voltage is $V_0 = V_{CC} - I_C R_0$ so changes in output voltage $\Delta V_0 = 0 - \Delta I_C . R_0$
- The amplification of voltage is $A_v = -\beta_{ac} R_o/R_i$ where negative sign shows if input voltage increases in positive direction, output voltage increases in negative direction showing phase reversal, so amplification of power or power gain is $A_p = \beta_{ac}^2 R_0/R_i$

Logic Gates

In electronics, digital systems are constructed using basic logic gates.

- AND gate
- OR gate
- NOT gate

} Basic gates

- NAND gate
- NOR gate

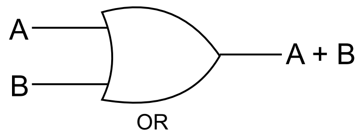
} Universal gates

- EXOR gate
- EXNOR gate

} Other gates

The basic working of these gates along with the truth tables are described.

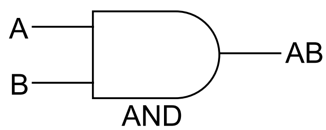
OR Gates



2 Input OR gate		
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

- The OR gate circuit representation has two inputs and single output.
- The output of the gate is high (1) if **one or more** than one of its inputs are high.
- The output of OR gate is shown with plus (+) sign *i.e.*, $A + B$.

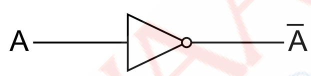
AND Gates



2 Input AND gate		
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

In an AND gate representation, the output is **high** (1) if **all** the inputs are high.
In this the dot (.) shows the AND operation i.e. A.B.
With two input and one output, the truth table of AND gate can be written as shown.

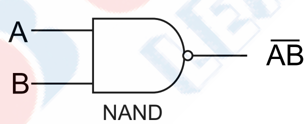
NOT Gates



NOT gate	
A	$\overline{A}$
0	1
1	0

- In NOT gate circuit with single input and single output, the output is inverted representation of the input.
- Due to inverted output, the circuit is also called as *inverter circuit*.
- In this, if input variable is A, the output is an inverted variable of A known as NOT A; output shown as A′ or $\overline{A}$.
- NOT gate can be obtained by NAND gate or NOR gate.

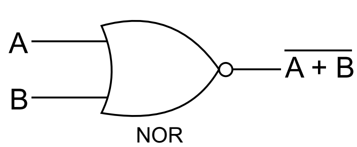
NAND gates



2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

- NAND gate is also called as NOT-AND gate which is same as AND gate followed by NOT gate.
- The output of NAND gates is high if **any** of its inputs are low.
- The output of NAND gate is AND gate with bubble on output as due to bubble at output, output is the inverted value of input.

NOR gates



2 Input NOR gate		
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

- NOR gate is called as NOT-OR gate which is similar as OR gate along with NOT gate.
- The output of NOR gate is low if **any** of its inputs are high.
- The symbol of NOR gate is OR gate with bubble on output, so due to small bubble at output, output is inverted representation of input.