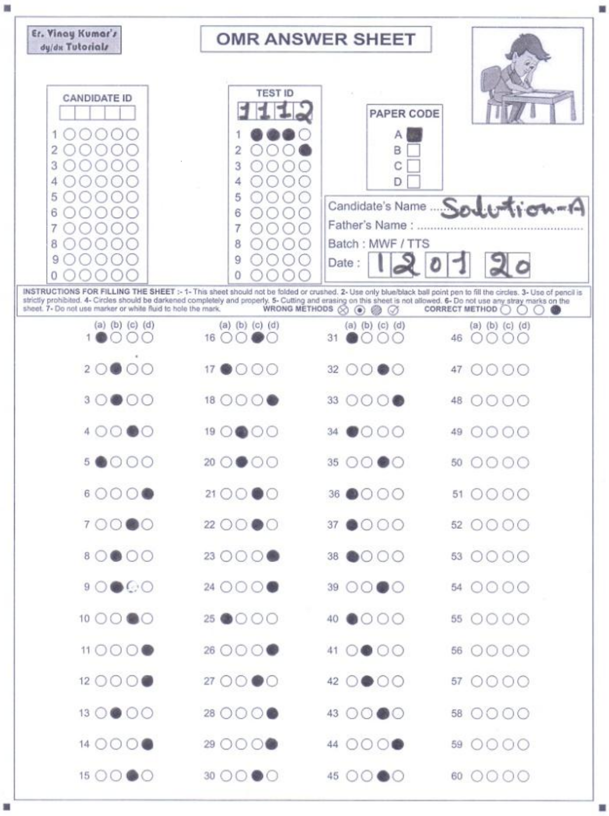
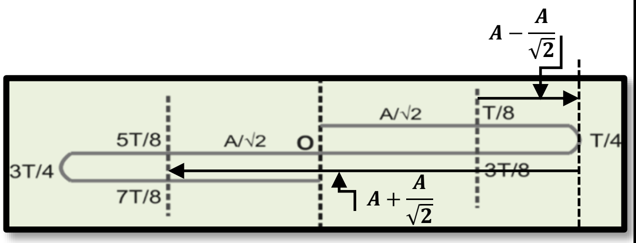
SET-A



Q1.



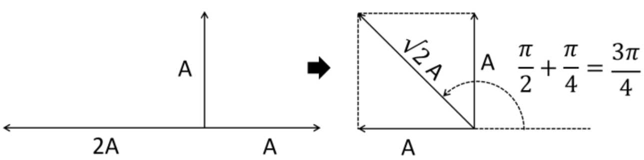
Distance is $= \left(A - \frac{A}{\sqrt{2}}\right) + A + \frac{A}{\sqrt{2}} = 2A$

Q2. $x = A[\cos\omega t + 1]$

$\Rightarrow x = A + A\cos\omega t \Rightarrow$ This is a +ve cos curve ($x = A \cos\omega t$) imposed on line $x = A$

Q4.

$x = A \sin \omega t + A \sin (\omega t + \pi/2) + 2A \sin (\omega t + \pi)$



Equation of resultant SHM is $x = \sqrt{2}A \sin (\omega t + 3\pi/4)$

Q5. Phase diff. $\pi/2$ and motion directions perpendicular to each other give orbital motion. Since amplitudes are different, therefore the orbit is elliptical.

Q6.

$$A = \frac{1}{\sqrt{[a\omega^2 - 2b\omega + c]}}$$

At resonance A should be maximum.

For A to be maximum $[a\omega^2 - 2b\omega + c]$ should be minimum⇨

$$D = a\omega^2 - 2b\omega + c \Rightarrow \frac{dD}{d\omega} = 2a\omega - 2b \quad (\to 0)$$

$$\Rightarrow 2a\omega - 2b = 0 \Rightarrow \boldsymbol{\omega = \frac{b}{a}} \Leftarrow \text{Resonance frequency}$$

$$A = \frac{1}{\sqrt{[a\omega^2 - 2b\omega + c]}}$$

A becomes maximum at $\omega = \frac{b}{a}$

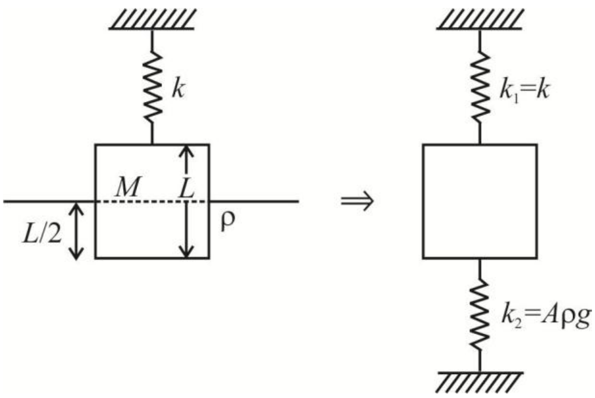
$$\Rightarrow A_{max} = \frac{1}{\sqrt{a\left(\frac{b}{a}\right)^2 - 2b\left(\frac{b}{a}\right) + c}} = \frac{1}{\sqrt{a \times \frac{b^2}{a^2} - \frac{2b^2}{a} + c}}$$

$$= \frac{1}{\sqrt{\frac{b^2}{a} - \frac{2b^2}{a} + c}} \Rightarrow A_{max} = \frac{1}{\sqrt{c - \frac{b^2}{a}}} \Rightarrow A_{max} = \frac{1}{\sqrt{\frac{ac - b^2}{a}}}$$

$$\Rightarrow \boldsymbol{A_{max} = \sqrt{\frac{a}{ac - b^2}}}$$

Q7. For the oscillation $x = A \sin \omega t$ the equation of kinetic energy is $K = \frac{E}{2}[1 + \cos 2\omega t]$ this shows that frequency of kinetic energy is double of the frequency of displacement.

Q8.

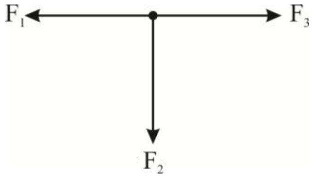


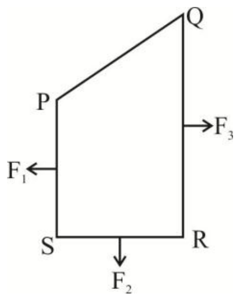
$$k_{eff} = k + A\rho g$$

$$\omega = \sqrt{\frac{k_{eff}}{M}} = \sqrt{\frac{k + A\rho g}{M}}$$

$$2\pi f = \left(\frac{k + A\rho g}{M}\right)^{1/2} \Rightarrow f = \frac{1}{2\pi}\left(\frac{k + A\rho g}{M}\right)^{1/2}$$

Q9. **(b)** As the net force on closed loop is equal to zero. So, force on *QP* will be equal and opposite to sum of forces on other 3 sides.





$$\Rightarrow \sqrt{(F_3 - F_1)^2 + F_2^2}$$

therefore $F_{QP} = \sqrt{(F_3 - F_1)^2 + F_2^2}$

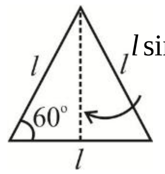
Q10. **(c)** Torque acting on equilateral triangle in a magnetic field B is

$$\tau = MB_o \sin\theta$$

$$\Rightarrow \tau = i\ AB\sin\theta\ [iA=\mathbf{M}]$$

Area of triangle

$$A = \frac{1}{2} \times l \times \frac{l\sqrt{3}}{2}$$



$$l \sin 60^o = \frac{l\sqrt{3}}{2}$$

$$\Rightarrow \quad A = \frac{\sqrt{3}}{4} l^2 \text{ and } \theta = 90^o$$

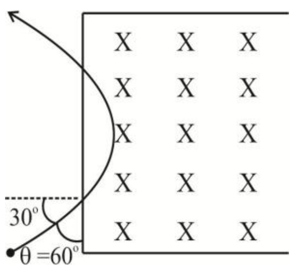
Substituting the given values in the expression for torque, we have

$$\tau = i \times \frac{\sqrt{3}}{4} l^2 B \sin 90^o = \frac{\sqrt{3}}{4} i l^2 B \qquad (\because \sin 90^o = 1)$$

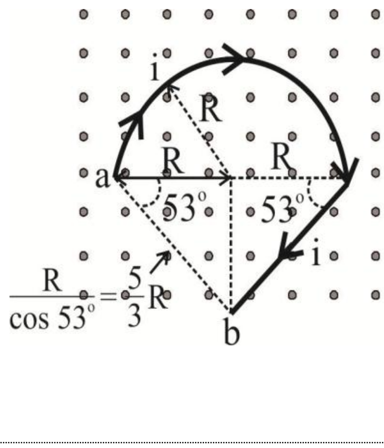
Hence, $l = 2\left(\frac{\tau}{\sqrt{3}Bi}\right)^{1/2}$

Q11. $\delta = 2\theta$

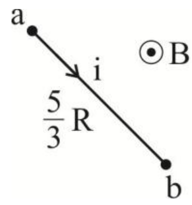
$$= 2 \times 60^o = 120^o$$



Q12.



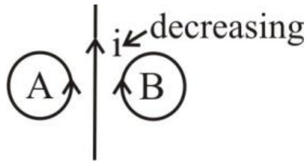
$\Rightarrow$



$\odot$B $\quad F = i\,(ab)\,B$

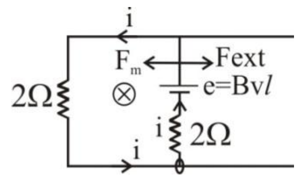
$$= i\frac{5}{3}RB$$

Q13. Since *i* is decreasing hence current in both the coils should in such direction so that they could support the current i



Q16. $i = \dfrac{Bvl}{2+2} = \dfrac{2 \times 2 \times 1}{4}$

= 1A

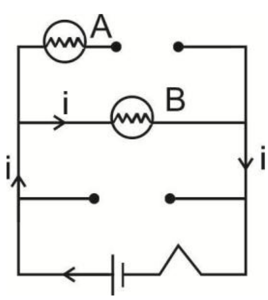


$$Fext = Fm = ilB = 1 \times 1 \times 2 = 2N$$

Q17. $Lxy = L + 4L - 2\sqrt{L \cdot 4L} = 5L - 4L = L$

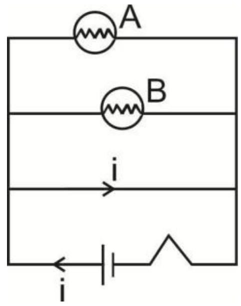
$$L_{eff} = \frac{5L \times L}{5L + L} = \frac{5}{6}L = 0.83L$$

Q18. Just at the moment the switch is connected inductance coils create insulation then the effective circuit at this moment is$\Rightarrow$



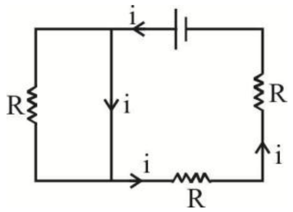
All the current passes through B and B immediately glows.

In steady state the coils act like zero resistance $\Rightarrow$



The whole circuit is short circuited and both the bulbs die out.

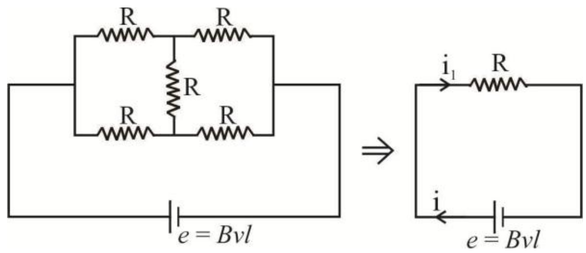
Q19. Effective circuit for calculating time constant is



$R_{eff} = R+R=2R$

$$\tau = \frac{L}{R_{eff}} = \frac{L}{2R}$$

Q20. Effective circuit is



$$i = \frac{Bvl}{R}$$

$$P = i^2 R = \left(\frac{Bvl}{R}\right)^2 R = \frac{B^2 v^2 l^2}{R}$$

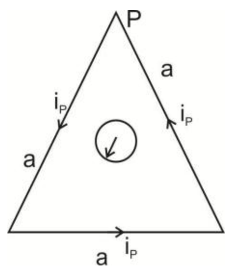
Q21. $\phi_S = B_P A_S$

$$= \frac{\mu_0}{4\pi}\frac{i_p}{a}\left[4\times 3\sin\frac{180^o}{3}\cdot\tan\frac{180^o}{3}\right]\pi r^2$$

$$= \frac{\mu_0}{4\pi}\frac{i_p}{a}\left[12\times\sin 60^o \tan 60^o\right]\pi r^2$$

$$= \frac{\mu_0}{4}\frac{r^2}{a}\left[12\times\frac{\sqrt{3}}{2}\times\sqrt{3}\right]i_p$$

$$= \frac{\mu_0}{4}\frac{r^2}{a}[18]\, i_p = \left(\frac{9}{2}\mu_0\frac{r^2}{a}\right)i_p$$



$$\left.\begin{aligned}\phi_S &= \left(\frac{9}{2}\mu_0\frac{r^2}{a}\right)i_p \\ \text{also } \phi_S &= Mi_p\end{aligned}\right\} \Rightarrow M = \frac{9}{2}\mu_0\frac{r^2}{a}$$

Q24. connection if spring is parallel $\Rightarrow k_{eff}=k_1+k_2$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$$

And $$f' = \frac{1}{2\pi}\sqrt{\frac{4k_1+4k_2}{m}}$$

$$\Rightarrow \quad f' = \frac{1}{2\pi}\cdot 2\sqrt{\frac{k_1+k_2}{m}} = 2f$$

Q25. $KE = \dfrac{q^2 B_0^2 r^2}{2m}$

KE, B_0 are same,

$$\Rightarrow \text{constant} \propto \frac{q^2\, r^2}{m} \Rightarrow q^2 \propto \frac{m}{r^2}$$

$$\Rightarrow q \propto \frac{\sqrt{m}}{r} \Rightarrow \frac{q_1}{q_2} = \sqrt{\frac{m_1}{m_2}}\times\frac{r_2}{r_1}$$

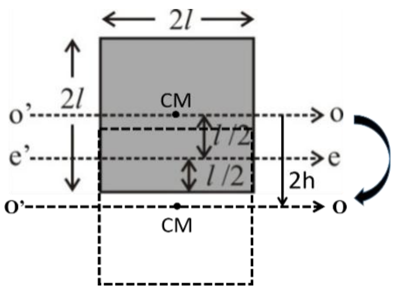
$$\Rightarrow \frac{q_1}{q_2} = \sqrt{\frac{m}{4m}}\times\frac{2}{1} = 1 \Rightarrow q_1 = q_2$$

Q26. $KE_t = \dfrac{KE}{\left[1+\dfrac{I}{mr^2}\right]} \quad\Rightarrow\quad \dfrac{1}{2}mv^2 = \dfrac{KE}{\left[1+\dfrac{I}{mr^2}\right]}$

Since m &KE are same for all the bodies,

$v^2 \propto \dfrac{1}{\left[1+\dfrac{I}{mr^2}\right]}$ and $\dfrac{I}{mr^2}$ is minimum for solid sphere therefore velocity is maximum for solid sphere

Q27.



$$mgh = \frac{1}{2}I\omega^2 \Rightarrow \quad mg\cdot 2\left(\frac{l}{2}\right) = \frac{1}{2}I_e\omega^2$$

$$\Rightarrow \quad mgl = \frac{1}{2}\left[\frac{m\,(2l)^2}{12} + m\left(\frac{l}{2}\right)^2\right]\omega$$

$$\Rightarrow \quad 2g = \left[\frac{l}{3}+\frac{l}{4}\right]\omega^2$$

$$\Rightarrow \quad 2g = \frac{7}{12}l\omega^2 \quad\Rightarrow\quad \omega = \sqrt{\frac{24}{7}\frac{g}{l}}$$

$$\Rightarrow \quad \omega = 2\sqrt{\frac{6}{7}\frac{g}{l}}$$

Q28. $ma = fr$ ………. (1)

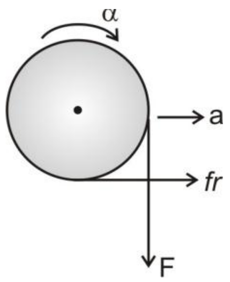
$I\alpha = F.r - fr.r$

$\Rightarrow \quad \frac{mr^2}{2}\cdot\frac{a}{r} = (F - fr)r$

$\Rightarrow \quad \frac{ma}{2} = F - fr$

$\Rightarrow \quad ma = 2F - 2fr$ ………. (2)

$(2)-(1): \quad 0 = 2F - 3fr \quad \Rightarrow \quad fr = 2F/3$



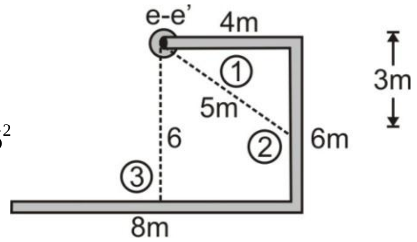
Q29. $I_1 = \frac{1\times 4^2}{3} = \frac{16}{3}$

$I_2 = \frac{1\times 6^2}{12} + 1\times 5^2$

$= 3 + 25 = 28$

$I_3 = \frac{1\times 8^2}{12} + 1\times 6^2 = \frac{16}{3} + 36$

$I = I_1 + I_2 + I_3 = \frac{16}{3} + 28 + \frac{16}{3} + 37 = 74.67$



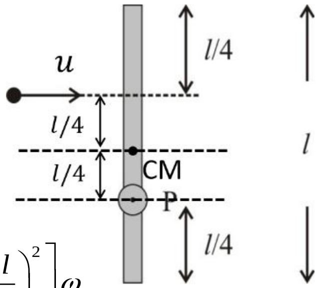
Q30. $l_p \quad \Rightarrow \quad conserved$

$\Rightarrow mu\frac{l}{2} + 0 = I_p\omega$

$\Rightarrow \quad mu\frac{l}{2} = \left[\frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2\right]\omega$

$\Rightarrow \quad mu\frac{l}{2} = ml^2\left[\frac{l}{12} + \frac{1}{16}\right]\omega$

$\Rightarrow \quad u = l\left[\frac{l}{6} + \frac{1}{8}\right]\omega \quad \Rightarrow \quad \omega = \frac{24}{7}\frac{u}{l}$



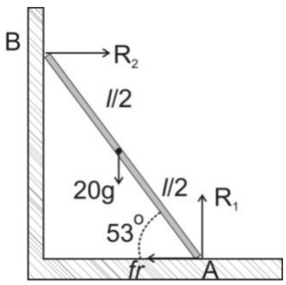
Q31. $\Sigma\tau_A = 0$

$R_2 l\sin 53^o - 20g\times\frac{l}{2}\cos 53^o = 0$

$\Rightarrow \quad R_2 = \frac{300}{4} = 75N$

$\Sigma F_x = 0$

$\Rightarrow \quad R_2 = fr = 75N$



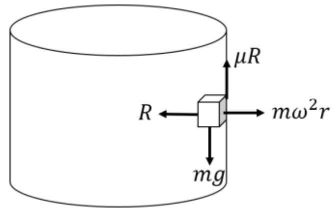
Q32. $R = m\omega^2 r$ …. (1)

$\mu R = mg$ …. (2)

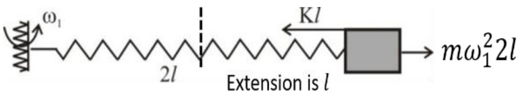
$\frac{(1)}{(2)} \Rightarrow \frac{1}{\mu} = \frac{\omega^2 r}{g}$

$\Rightarrow \quad \omega = \sqrt{\frac{g}{\mu r}} = \sqrt{\frac{10}{0.4\times 1}}$

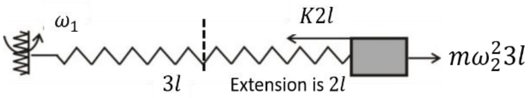
$\Rightarrow \quad \omega = \sqrt{25} = 5\, rad/s$



Q33.



$\Rightarrow \quad kl = m\omega_1^2 2l$ ………. (1)



$\Rightarrow \quad k\cdot 2l = m\omega_2^2\cdot 3$ ………. (2)

$\frac{(1)}{(2)} \quad \Rightarrow \quad \frac{1}{2} = \frac{\omega_1^2}{\omega_2^2}\times\frac{2}{3} \quad \Rightarrow \quad \frac{\omega_1}{\omega_2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Q34. $v = t^2 - t + 1 \quad \Rightarrow \quad \frac{dv}{dt} = 2t - 1 \quad \Rightarrow \quad a_\tau = 2t - 1$

$P = F_\tau v = ma_\tau v = 1\times(2t-1)(t^2 - t + 1)$

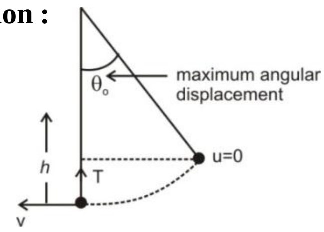
$\Rightarrow P_{(at=2)} = 1\times(2\times 2 - 1)(2^2 - 2 + 1) = 3\times 3 = 9W$

Q35. Tension at mean position :

$T = \frac{mv^2}{r} + mg\cos\theta$

$\Rightarrow \quad 2mg = \frac{mv^2}{r} + mg$

$\Rightarrow \quad \frac{v^2}{r} = g \Rightarrow \quad v^2 = gr$ …….. (1)



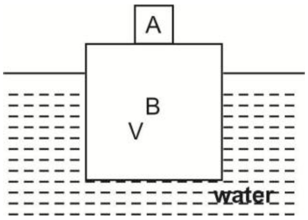
$W = \Delta KE \; : \; mgh = \frac{1}{2}mv^2$

$\Rightarrow \quad 2g\cdot r(1-\cos\theta_o) = v^2$ ……… (2)

(1) = (2) : $2gr\,(1-\cos\theta_o) = gr \;\Rightarrow\; 1-\cos\theta_o = \frac{1}{2}$

$\Rightarrow \quad \cos\theta_o = \frac{1}{2} \quad \Rightarrow \quad \theta_o = 60^o$

Q36. Let volume of block B be V.



U = mg

$0.9V\rho_\omega g = (m_A + m_B)\, g$

$\Rightarrow\ 0.9V\rho_\omega = [m_A + V\times(0.6\rho_\omega)]$

$\Rightarrow\ m_A = 0.3V\rho_\omega$ …….(1)

$m_B = V(0.6\rho_\omega)\ \Rightarrow\ 12 = 0.6V\rho_\omega$

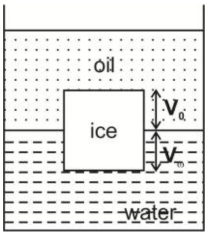
$\Rightarrow\ V\rho_\omega = 20$ ……. (2)

From (1) & (2) $\Rightarrow\ m_A = 0.3\times 20 = 6kg$

Q37. $S.G.\ of\ oil = \dfrac{loss\ of\ weight\ in\ oil}{loss\ of\ weight\ in\ water}$

$= \dfrac{160-136}{160-130} = \dfrac{24}{30} = 0.8$

Q38. U = mg



$\Rightarrow\ V_0\rho_0 g + V_\omega\rho_\omega g = (V_0 + V_\omega)\rho_i\, g$

$\Rightarrow\ \dfrac{V_0}{V_\omega}\cdot\rho_0 + 1\times\rho_\omega = \left(\dfrac{V_0}{V_\omega}+1\right)\rho_i$

$\Rightarrow\ \dfrac{V_0}{V_\omega}\times 0.8\rho_\omega + \rho_\omega = \left(\dfrac{V_0}{V_\omega}+1\right)\times 0.9\rho_\omega$

$\Rightarrow\ 0.8\dfrac{V_0}{V_\omega} + 1 = 0.9\dfrac{V_0}{V_\omega} + 0.9$

$\Rightarrow\ 0.1\dfrac{V_0}{V_\omega} = 0.1\ \Rightarrow\ \dfrac{V_0}{V_\omega} = 1\ \Rightarrow\ 1:1$

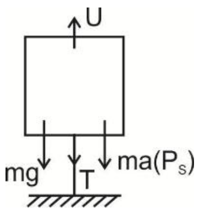
Q39. $R = \dfrac{\rho\, v\, r}{\eta}\ \Rightarrow\ 2000 = \dfrac{1000\times v_c\times\left(\dfrac{2\times 10^{-2}}{2}\right)}{10^{-3}}$

$\Rightarrow\ 2000\times 10^{-3} = 1000\times v_c\times 10^{-2}$

$\Rightarrow\ v_c = \dfrac{2000\times 10^{-3}}{1000\times 10^{-2}}$

$\Rightarrow\ v_c = \dfrac{2}{10} = 0.2\ m/s = 20\ cm/s$

Q40. $U = mg + T + ma$



$\Rightarrow\ T = U - mg - ma$

$= Vdg - V\rho g - V\rho a$

$= V\,[dg - \rho\,(g+a)]$

Q41. $v_0\ \alpha\ r^2\rho_s\ \Rightarrow\ v_0\ \alpha\dfrac{m^{2/3}}{\rho_s^{\ 2/3}}\cdot\rho_s$

$\Rightarrow\ v_0\ \alpha\ m^{2/3}\ \rho_s^{1/3}$

$$\left[\begin{array}{l} M = \dfrac{4}{3}\pi r^3\rho_s \\ M\alpha r^3\rho_s \\ r^3\alpha\dfrac{m}{\rho_s}\ \Rightarrow\ r\,\alpha\dfrac{m^{1/3}}{\rho_s^{\ 1/3}} \end{array}\right]$$

Since material of spheres is same

$v_0\ \alpha\ m^{2/3}\ \Rightarrow\ \dfrac{v_{0_2}}{v_{0_1}} = \left(\dfrac{m_2}{m_1}\right)^{2/3} = \left(\dfrac{8m}{m}\right)^{2/3} = \dfrac{4}{1}$

$\Rightarrow\ \dfrac{v_{0_2}}{v_0} = 4\ \Rightarrow\ v_{0_2} = 4v_0$

Q42. Velocity of efflux when the hole is at depth h, $v = \sqrt{2gh}$

Rate of flow of water from square hole

$Q_1 = a_1 v_1 = L^2\sqrt{2gh}$ …..(1)

Rate of flow of water from circular hole

$Q_2 = a_2 v_2 = \pi R^2\sqrt{2g(4y)}$

According to problem $Q_1 = Q_2$

$\Rightarrow\ L^2\sqrt{2gy} = \pi R^2\sqrt{2g(4y)}\ \Rightarrow\ R = \dfrac{L}{\sqrt{2\pi}}$

Q43. $a_{max} = \omega^2 A\ \Rightarrow \alpha = \omega^2 A$ …..(1)

$v_{max} = \omega A\ \Rightarrow \beta = \omega A \Rightarrow \omega = \dfrac{\beta}{A}$ … … (2)

From eq (1) and (2): $\alpha = \omega^2 A\ \Rightarrow \alpha = \left(\dfrac{\beta}{A}\right)^2 A$

$\Rightarrow \alpha = \dfrac{\beta^2}{A^2}A \Rightarrow \alpha = \dfrac{\beta^2}{A}\ \Rightarrow \boldsymbol{A = \dfrac{\beta^2}{\alpha}}$

Q44. $v = \omega\sqrt{A^2 - x^2}\ \Rightarrow \dfrac{\omega A}{4} = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \dfrac{A}{4} = \sqrt{A^2 - x^2}\ \Rightarrow \left(\dfrac{A}{4}\right)^2 = A^2 - x^2$

$\Rightarrow \dfrac{A^2}{16} = A^2 - x^2\ \Rightarrow x^2 = \dfrac{15A^2}{16} \Rightarrow \boldsymbol{x = \dfrac{\sqrt{15}A}{4}}$

Q45. $A = A_0 e^{-bt}\ \Rightarrow \dfrac{A}{A_0} = e^{-bt}\ \Rightarrow \dfrac{80}{100} = e^{-5b}$

$\Rightarrow 0.8 = e^{-5b}$ ….(1)

$\dfrac{A}{A_0} = e^{-bt}\ \Rightarrow \dfrac{\alpha}{100} = e^{-b(5+5)}\ \Rightarrow \alpha = 100e^{-10b}$

$\Rightarrow \alpha = 100\left(e^{-5b}\right)^2$ ….(2)

From eq (1) and (2): $\Rightarrow \alpha = 100(0.8)^2\ \Rightarrow \boldsymbol{\alpha = 64}$