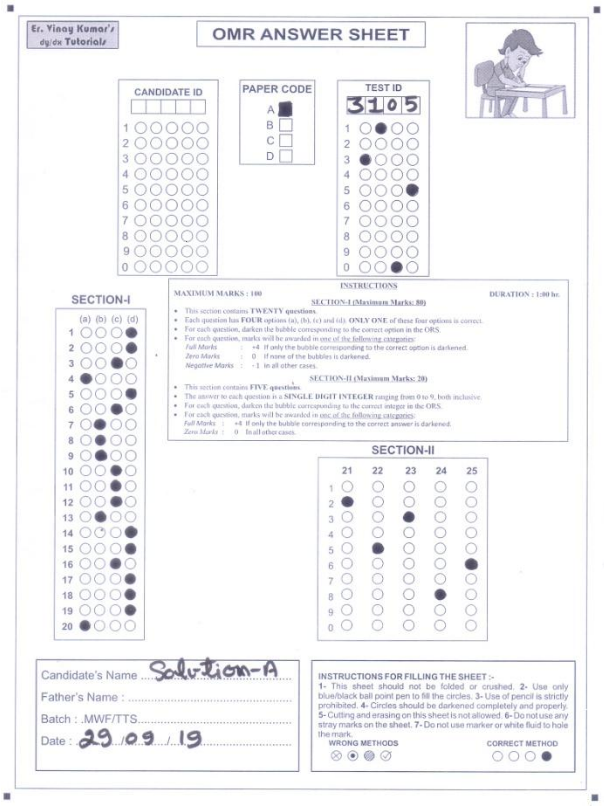
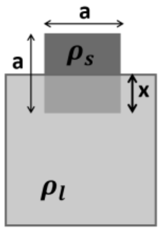
SET-A



Q1.

Sol:



$U = mg \Rightarrow a^2\, x\, \rho_l\, g = a^3\, \rho_s\, g$

$\Rightarrow x = \frac{\rho_s}{\rho_l} a \Rightarrow \frac{dx}{x} = \frac{d\rho_s}{\rho_s} - \frac{d\rho_l}{\rho_l} + \frac{da}{a}$

$\Rightarrow \frac{dx}{x} = -\gamma_s \Delta t - (-\gamma_l)\Delta t + \alpha_s \Delta t$

$\Rightarrow \frac{dx}{x} = (-\gamma_s + \gamma_l + \alpha_s)\Delta t$

$\Rightarrow \frac{dx}{x} = (-3\alpha_s + \gamma_l + \alpha_s)\,\Delta t \Rightarrow \boldsymbol{\frac{dx}{x} = (\gamma_l - 2\alpha_s)\,\Delta t}$

Q2. **Sol.**

$PV = \mu RT$ (PV: constant, RT: constant) $\Rightarrow P \propto \mu \Rightarrow \frac{P_{mix}}{P_{H_2}} = \frac{\mu_{mix}}{\mu_{H_2}}$

$\Rightarrow \frac{P_{mix}}{P_{H_2}} = \frac{\mu_{O_2} + \mu_{H_2}}{\mu_{H_2}}$

$\Rightarrow \frac{10}{P_{H_2}} = \frac{\frac{8}{32} + \frac{2}{2}}{\frac{2}{2}} \Rightarrow \frac{10}{P_{H_2}} = \frac{\frac{1}{4} + 1}{1} \Rightarrow \frac{10}{P_{H_2}} = \frac{5}{4}$

$\Rightarrow P_{H_2} = 8\ atm$

Q3. **Sol.**

$PV = \mu RT \Rightarrow PV = \frac{M}{M_o} RT \Rightarrow P \propto MT \Rightarrow M \propto \frac{P}{T}$

$\Rightarrow \frac{M_f}{M_i} = \frac{P_f}{P_i} \times \frac{T_i}{T_f} \Rightarrow \frac{M_f}{12} = \frac{P/3}{P} \times \frac{300}{400} \Rightarrow \frac{M_f}{12} = \frac{1}{3} \times \frac{3}{4} \Rightarrow M_f = 3$

$\Delta M = M_f - M_i = 12 - 3 = 9gm$

Q4. **Sol:**

$\mu_i = \mu_f$

$\frac{PV}{RT} + \frac{PV}{RT} = \frac{P'V}{RT} + \frac{P'V}{RT/4} \Rightarrow 2P = P' + 4P' \Rightarrow 5P' = 2P$

$\Rightarrow \boldsymbol{P' = 2P/5}$

Q5. **Sol:**

$v_{rms} = \sqrt{\frac{3RT}{M_o}} \quad \Rightarrow v_{H_2} = \sqrt{\frac{3RT_{H_2}}{M_{o_{H_2}}}}$ ....(1)

$\Rightarrow v_{N_2} = \sqrt{\frac{3RT_{N_2}}{M_{o_{N_2}}}}$ ....(2)

Equating (1) and (2): $\Rightarrow v_{H_2} = v_{N_2}$

$\Rightarrow \sqrt{\frac{3RT_{H_2}}{M_{o_{H_2}}}} = \sqrt{\frac{3RT_{N_2}}{M_{o_{N_2}}}} \Rightarrow \sqrt{\frac{T_{H_2}}{M_{o_{H_2}}}} = \sqrt{\frac{T_{N_2}}{M_{o_{N_2}}}}$

$\Rightarrow \frac{T_{H_2}}{M_{o_{H_2}}} = \frac{T_{N_2}}{M_{o_{N_2}}}$

$\Rightarrow \frac{T_{H_2}}{2} = \frac{273+7}{28} \qquad \Rightarrow \frac{T_{H_2}}{2} = \frac{280}{28}$

$\Rightarrow \frac{T_{H_2}}{2} = 10 \Rightarrow \boldsymbol{T_{H_2} = 20\ K}$

Q6. **Sol:**

Both the gases have same temperature in the mixture

$K.E._{trans.} = \frac{3}{2} \frac{M}{M_o} RT \Rightarrow K.E. \propto \frac{M}{M_o}$

$\frac{N_2}{O_2} \Rightarrow \frac{KE_1}{KE_2} = \frac{M_1}{M_2} \times \frac{M_{o_2}}{M_{o_1}} = \frac{7}{4} \times \frac{32}{28} = 2$

Q7. **Sol:**

$U = U_{H_2} + U_{He}$

$\Rightarrow U = f_{O_2}\left[\frac{1}{2}\mu_{H_2} RT\right] + f_{He}\left[\frac{1}{2}\mu_{He} RT\right]$

$\Rightarrow U = 5\left[\frac{1}{2} \times 3 \times RT\right] + 3\left[\frac{1}{2} \times 5 \times RT\right]$

$\Rightarrow U = 7.5RT + 7.5RT \Rightarrow U = 15RT$

Q8. $\rho_{t2} = \rho_{t1}(1 - \alpha\,\Delta t)$

$\Rightarrow 0.992 = 0.998\ [1 - \alpha(40 - 20)]$

$\Rightarrow 1 - 20\alpha = \frac{0.992}{0.998} \Rightarrow 20\alpha = 1 - \frac{0.992}{0.998}$

$\Rightarrow 20\alpha = \frac{0.998 - 0.992}{0.998} \Rightarrow 20\alpha = \frac{0.006}{0.998}$

$\Rightarrow \alpha = 3 \times 10^{-4} = 0.0003\ per^{o}C$

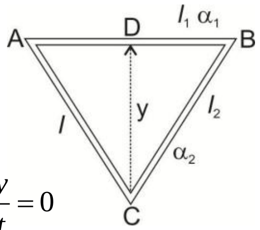
Q9. $L_{measured} = L_{actual}\ (1 - \alpha\,\Delta t)$

$$25 = L_{actual}\ [1 - \alpha\,(0 - 20)]$$

$$\Rightarrow\ 25 = L_{actual}\ [1 + 20\,\alpha]$$

$$\Rightarrow\ L_{actual} = \frac{25}{1 + 20\,\alpha} < 25$$

Q10. $y^2 = l_2^2 - l_1^2$



$$\Rightarrow\ 2y\frac{dy}{dt} = 2l_2\frac{dl_2}{dt} - 2l_1\frac{dl}{dt}$$

$y \Rightarrow$ does not change $\Rightarrow\ \frac{dy}{dt} = 0$

$$0 = 2l_2\frac{dl_2}{dt} = 2l_1\frac{dl_1}{dt}\ \Rightarrow\ l_2\frac{dl_2}{dt} = l_1\frac{dl}{dt}$$

$$\Rightarrow\ l_2(\alpha_2 l_2) = l_1(\alpha_1 l_1)$$

$$\Rightarrow\ \alpha_2\ l_1^2 = \alpha_2\ l_2^2\ \ldots\ldots(1)$$

$$l_2 = 2l_1\ \ \ldots\ldots(2)$$

$\Rightarrow$ from eq. (1) & (2) $\Rightarrow\ \alpha_1\ (l_1)^2 = \alpha_2\ (2l_1)^2$

$$\Rightarrow\ \alpha_1\ l_1^2 = 4\alpha_2\ l_1^2\ \ \Rightarrow\ \alpha_1 = 4\alpha_2$$

Q11. $\Delta l_1 = \alpha_a\ l_1\ \Delta t\ \ \ldots\ldots\ (1)$

$$\Delta l_2 = \alpha_s\ l_2\ \Delta t\ \ldots\ldots\ (2)$$

$$\Delta l_1 = \Delta l_2\ \ \Rightarrow\ \alpha_a\ l_1\ \Delta t = \alpha_s\ l_2\ \Delta t$$

$$\Rightarrow\ \alpha_a\ l_1 = \alpha_s\ l_2$$

$$\Rightarrow\ \frac{l_1}{l_2} = \frac{\alpha_s}{\alpha_a}$$

$$\Rightarrow\ \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_s + \alpha_a}$$

Q14. $v_B = \tan 37^o = \frac{3}{4} = 0.75\text{m/s}\ \ (+)$

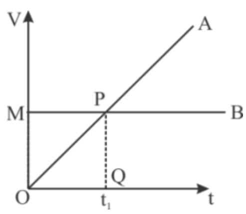
$$v_A = \tan 53^o = \frac{4}{3}\ \text{m/s} = 1.33\ \ (-)$$

$$v_{BA} = v_B - v_A$$

$$= 0.75 - (-1.33) = 2.08\ \text{m/s}$$

Q16. **(a)** slope of graph A is constant $\Rightarrow$ acc. is constant

(b) slope of graph B is zero $\Rightarrow$ acc. is zero therefore velocity is constant



(c) In t_1 time the distance travelled by A = Area MPQO

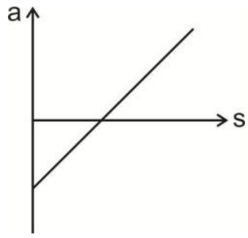
In t_1 the distance travelled by B = area OPQ.

Area OPQ < Area MPQO i.e. the distance traveled are different for A & B hence they can not cross each other at t_1.

Q17. The equation of the given v/s graph

$$\Rightarrow\ v = -ms + c\ \ \ldots\ldots\ldots(1)$$

differentiating the equation with time $\Rightarrow$



$$\frac{dv}{dt} = -m\frac{ds}{dt} + 0$$

$$\Rightarrow\ a = -mv\ \ \ldots\ldots\ldots(2)$$

from eq. (1) & (2)

$\Rightarrow$ a = - m (- ms + c)

$\Rightarrow$ a = m^2s – mc

$\Rightarrow$ straight line with positive slope and negative intercept.

Q18. $s^2 = 2t + 1\ \ \Rightarrow\ \ 2s\frac{ds}{dt} = 2\ \ \ \Rightarrow\ \ sv = 1$

$$\Rightarrow\ v = \frac{1}{s}\ \ \Rightarrow\ \ v = s^{-1}\ \ \ldots\ldots..(1)$$

$$\Rightarrow\ \frac{dv}{dt} = -1s^{-2}\frac{ds}{dt}\ \ \Rightarrow\ \ a = -\frac{v}{s^2}\ \ \ \ldots\ldots..(2)$$

From eq. (1) & (2)

$$\Rightarrow\ a = -\frac{\frac{1}{s}}{s^2}\ \ \Rightarrow\ \ a = -\frac{1}{s^3}\ \ \Rightarrow\ \ a \propto \frac{1}{s^3}$$

Q19. $t_1 + t_2 = \frac{2u_y}{g}\ \ \Rightarrow\ \ 2 + 4 = \frac{2u_y}{10}$

$$\Rightarrow\ u_y = 30\ m/s$$

$$\tan 37^o = \frac{u_y}{u_x}\ \Rightarrow\ \frac{3}{4} = \frac{30}{u_x}\ \Rightarrow\ u_x = 40\ m/s$$

$$R = \frac{2u_x u_y}{g} = \frac{2 \times 40 \times 30}{10} = 240\ m$$

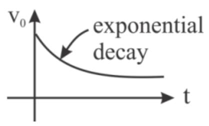
Q20. $a \propto v\ \ \Rightarrow\ \ a = -kv\ (retardation)$

$$\Rightarrow\ \frac{dv}{dt} = -kv\ \Rightarrow\ \frac{dv}{v} = -kdt$$

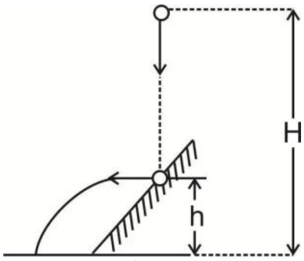
$$\Rightarrow\ \int_{v_0}^{v}\frac{dv}{v} = -\int_0^t kdt\ \Rightarrow\ \ln\frac{v}{v_0} = -kt$$

$$\Rightarrow\ \frac{v}{v_0} = e^{-kt}\ \Rightarrow\ v = v_0 e^{-kt}$$

$\Rightarrow$



Q21. Let time taken by ball to fall down is t1 by H-h and to reach the ground is t2 after impact.



$$H-h=\frac{1}{2}gt_t^2 \Rightarrow t_1=\sqrt{\frac{2(H-h)}{g}}$$

$$h=\frac{gt_2^2}{2} \Rightarrow t_2=\sqrt{\frac{2h}{g}}$$

So, total time taken by the ball to reach ground is

$$T=t_1+t_2=\sqrt{\frac{2}{g}}[\sqrt{H-h}+\sqrt{h}]$$

For T to be minimum $\frac{dT}{dh}=0$

$$\frac{dT}{dh}=\sqrt{\frac{2}{g}}\left[\frac{1\times(-1)}{2\sqrt{H-h}}+\frac{1}{2\sqrt{h}}\right]=0$$

$$\Rightarrow \sqrt{H-h}=\sqrt{h} \Rightarrow H-h=h$$

$$\Rightarrow 2h=H \Rightarrow \frac{H}{h}=2$$

Ans. 2

Q22. $\vec{u}_1=25\,i$

$\vec{u}_2=-100\,i$

At time t,

$\vec{v}_1=\vec{u}_1+\vec{a}\,t \Rightarrow \vec{v}_1=25i-10\,j\cdot t$ ….. (1)

$\vec{v}_2=\vec{u}_2+\vec{a}\,t \Rightarrow \vec{v}_2=-100\,i-10\,j\,t$ ….. (2)

$\vec{v}_1\perp\vec{v}_2 \Rightarrow \vec{v}_1\cdot\vec{v}_2=0$

$\Rightarrow (25\,i-10\,jt)\cdot(-100\,i-10\,j\,t)=0$

$\Rightarrow -2500+100\,t^2=0 \Rightarrow 100\,t^2=2500$

$\Rightarrow t^2=25 \Rightarrow t=5\text{sec.}$

Q23. $F=-kv \Rightarrow ma=-kv \Rightarrow a=-\frac{k}{m}v$

$$\Rightarrow \frac{dv}{dt}=-\frac{k}{m}v \Rightarrow ds\frac{dv}{dt}=-\frac{k}{m}v\,ds$$

$$\Rightarrow v\cdot dv=-\frac{k}{m}v\,ds \Rightarrow dv=-\frac{k}{m}ds$$

$$\Rightarrow \int_{v_o}^{v_o/4}dv=-\frac{k}{m}\int_o^s ds \Rightarrow \frac{v_o}{4}-v_o=-\frac{k}{m}\cdot s$$

$$\Rightarrow -\frac{3v_o}{4}=-\frac{k}{m}\cdot s \Rightarrow s=\frac{3}{4}\frac{mv_o}{k}$$

Q24. $x=3t^2-t^3$

at $t=0$ $x_1=3\times0-0=0$

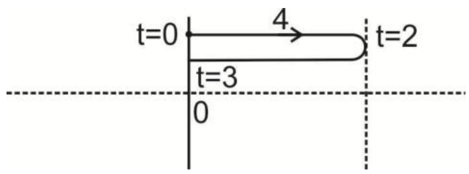
for turning points

$$\Rightarrow \frac{dx}{dt}=0 \Rightarrow \frac{d}{dt}(3t^2-t^3)=0$$

$$\Rightarrow 6t-3t^2=0 \Rightarrow 3t(t-2)=0 \Rightarrow t=2$$

at $t=2$ $x_2=3\times2^2-2^3=12-8=4m$

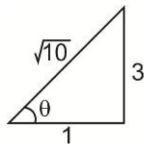
at $t=3$ $x_3=3\times3^2-3^3=27-27=0$



Distance = 4+4 **8 m**

Q25. $\frac{PE_P}{KE_P}=\tan^2\theta \Rightarrow \frac{9}{1}=\tan^2\theta \Rightarrow \tan\theta=\frac{3}{1}$

$$R=\frac{2u^2\sin\theta\cos\theta}{g}$$



$$\Rightarrow R=\frac{2\times10^2\times\frac{3}{\sqrt{10}}\times\frac{1}{\sqrt{10}}}{10}=6m$$