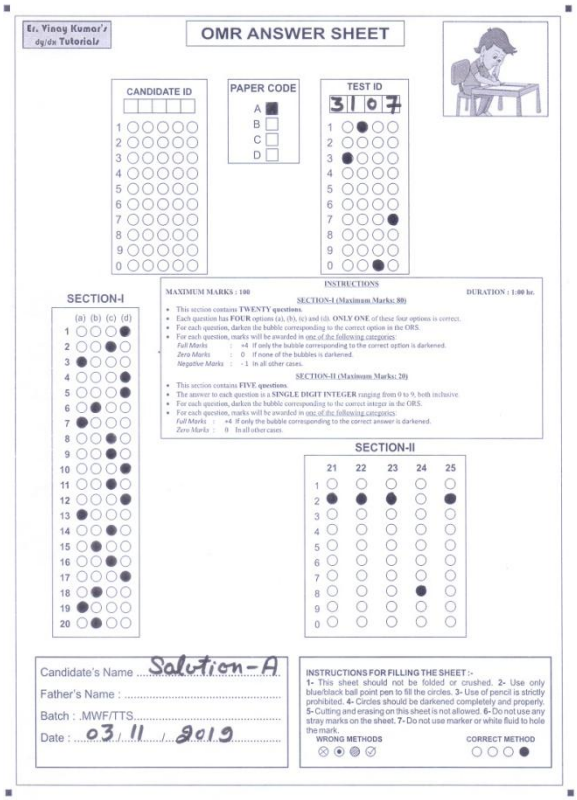
SET-A



Q1. $S = \frac{1}{2} g \cdot 2^2$ ..... (1)

$S_T = \frac{1}{2} g\,(2T-1)$ ..... (2)

$S = S_T \Rightarrow = \frac{1}{2} g \cdot 2^2 = \frac{1}{2} g\,(2T-1)$

$\Rightarrow 4 = 2T - 1 \Rightarrow T = 5/2\,\text{sec}.$

$H = \frac{1}{2} gT^2 = \frac{1}{2} \times 10 \times \left(\frac{5}{2}\right)^2$

$H = \frac{1}{2} \times 10 \times \frac{25}{4} \Rightarrow H = 31.25\text{m}$

Q2. A $U_A = 10$ m/s $a_A = 0$ ⇒ inframe of B : $a_r = 2\text{m/s}^2$ A $U_r = 10$ m/s

B $U_B = 0$ $a_B = 2\text{m/s}^2$ B

If the bodies meet again then Sr should be zero.

$S_r = U_r t + \frac{1}{2} a_r t^2 \Rightarrow 0 = 10 \times t + \frac{1}{2} \times (-2) \times t^2$

$\Rightarrow 0 = 10t - t^2 \Rightarrow t^2 = 10t \Rightarrow t = 10\,\text{sec}.$

Q4. $W = \Delta kE \Rightarrow -mgh = \frac{1}{2}\left(\frac{1}{2} m4^2\right) - \frac{1}{2} m4^2$

$\Rightarrow -mgh = -\frac{1}{4} m4^2 \Rightarrow gh = 4 \Rightarrow h = 0.4\text{m}$

Q6. $U = \frac{1}{2}\frac{\sigma^2}{E} \cdot V = \frac{1}{2}\frac{F^2}{A^2} \cdot Al = \frac{1}{2}\frac{F^2}{AE} \times l$

E & F are same in the given case $\Rightarrow U \propto \frac{l}{A}$

$\Rightarrow \frac{U_A}{U_B} = \frac{l_A}{l_B}\frac{A_B}{A_A} = \frac{l_A}{l_B} \times \left(\frac{d_B}{d_A}\right)^2 = \frac{3}{1} \times \left(\frac{1}{2}\right)^2$

$= \frac{3}{4}$

Q7. $P = -\frac{\Delta D/D}{\Delta l/l}$

$\Rightarrow 0.20 = -\frac{\Delta D/D}{2 \times 10^{-3}} \Rightarrow \frac{\Delta D}{D} = -4 \times 10^{-4}$ .....(1)

Volume of rod $V = \frac{\pi D^2}{4} \times l \Rightarrow V \propto D^2 \times l$

$\Rightarrow \frac{\Delta V}{V} = 2\frac{\Delta D}{D} + \frac{\Delta l}{l}$

$\Rightarrow \frac{\Delta V}{V} = 2[-4 \times 10^{-4}] + 2 \times 10^{-3}$

$\Rightarrow \frac{\Delta V}{V} = -8 \times 10^{-4} + 20 \times 10^{-4}$

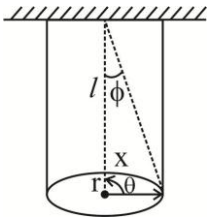
$\Rightarrow \frac{\Delta V}{V} = 12 \times 10^{-4}$

$\% \frac{\Delta V}{V} = 12 \times 10^{-4} \times 100 = 12 \times 10^{-2} = +0.12$

Q8. $x = \theta \cdot r = \phi \cdot l$

$30^\circ \times 4\text{mm} = \phi \times 1000\text{ mm}$

$\phi = 0.12^\circ$



Q9. $x_1 + x_2 + x_3 = l$

$v - v_P - v_P = 0$

$\Rightarrow v_P = v/2$ ..... (1)

$y_1 + y_2 = l'$

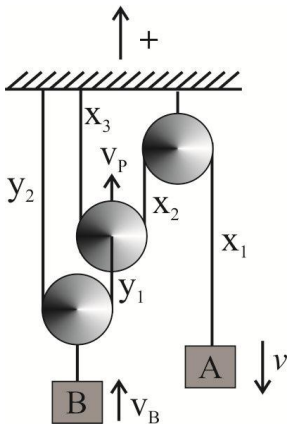
$\Rightarrow (v_P - v_B) - v_B = 0$

$\Rightarrow v_P = 2v_B$ ..... (2)

from (1) & (2) :

$\frac{v}{2} = 2v_B \Rightarrow v_B = \frac{v}{4}$

$Mv_0 = m_1 v_1 + m_2 v_2$



$$\Rightarrow \quad (m+m)v_0 = m(-v) + m\left(+\frac{v}{4}\right)$$

$$\Rightarrow \quad 2mv_0 = -mv + \frac{mv}{4}$$

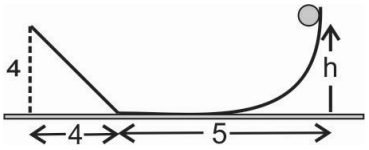
$$\Rightarrow \quad 2v_0 = -\frac{3v}{4} \quad \Rightarrow \quad v_0 = -\frac{3v}{8}$$

Q10. $Y_A = \tan 60^o = \sqrt{3}$

$$Y_B = \tan 30^o = \frac{1}{\sqrt{3}}$$

$$\frac{Y_A}{Y_B} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3 \quad \Rightarrow \quad Y_A = 3Y_B$$

Q11.



$$W = \Delta kE \quad \Rightarrow \quad W_{mg} + W_{fr} + W_R = kE_t - kE_i$$

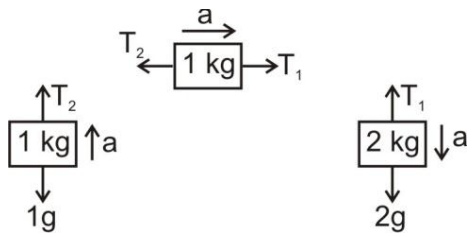
$$\Rightarrow \quad (mg \times 4 - mgh) - 0.4mg(4+5) + 0 = 0 - 0$$

$$\Rightarrow \quad 4 - h - 3.6 = 0 \quad \Rightarrow \quad h = 0.4m$$

Q12. 2a = 2g – T_1 …….… (1)

1a = T_1 – T_2 …… (2)

1a = T_2 – 1g …… (3)



(1) + (2) + (3) : $4a = g \quad \Rightarrow \quad a = g/4 = \frac{5}{2}$

$$m\vec{a}_0 = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3$$

$$4\vec{a}_0 = -2aj + 1 \cdot ai + 1 \cdot aj$$

$$\Rightarrow \quad 4\vec{a}_0 = ai - aj \quad \Rightarrow \quad \vec{a}_0 = \frac{a}{4}(i-j)$$

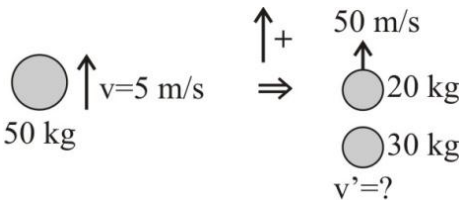
$$a_o = \frac{a}{4}\sqrt{2} = \frac{5}{2\times 4} \times \sqrt{2} = \frac{5\sqrt{2}}{8} m/s^2$$

Q13. After 5 sec, velocity of body :

$$v = u + at$$

$$\Rightarrow \quad v = 100 - g \times 5 = 100 - 9.8 \times 5 = 100 - 49$$

= 51 m/s



$P_i = P_f$ $50 \times 51 = 150 \times 20 + 30v'$

$$\Rightarrow \quad 2550 = 3000 + 30v'$$

$$\Rightarrow \quad 30v' = -450 \quad \Rightarrow \quad v' = -15 \ m/s$$

Q14. $\frac{KE_f}{KE_i} = \frac{3}{4}$

$$\Rightarrow \quad \frac{\Delta KE}{KE_i} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{\Delta KE}{KE_i} = \frac{1}{4} \quad \Rightarrow \quad \frac{\frac{1}{2}\mu u_r^2(1-e^2)}{\frac{1}{2}mv^2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{\frac{1}{2} \times \left(\frac{m \times m}{m+m}\right)(v-0)^2(1-e^2)}{\frac{1}{2}mv^2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{\frac{1}{2} \times \frac{m}{2} \times v^2(1-e^2)}{\frac{1}{2}mv^2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{(1-e^2)}{2} = \frac{1}{4} \quad \Rightarrow \quad 1 - e^2 = \frac{1}{2} \quad \Rightarrow \quad e^2 = \frac{1}{2}$$

$$e = \frac{1}{\sqrt{2}}$$

Q15. $\Delta KE = \frac{1}{2}\mu U_r^2 (1-e^2)$

$$= \frac{1}{2} \times \left(\frac{M \times M}{M+M}\right)(U+2U)^2(1-0)$$

$$= \frac{1}{2} \times \frac{M^2}{2M} \times 9U^2 = \frac{9}{4}MU^2$$

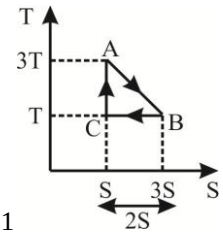
Q16. $Q_{AB} = \frac{1}{2}(3T+T)2S = 4TS$

$Q_{BC} = -T \times 2S = -2TS$

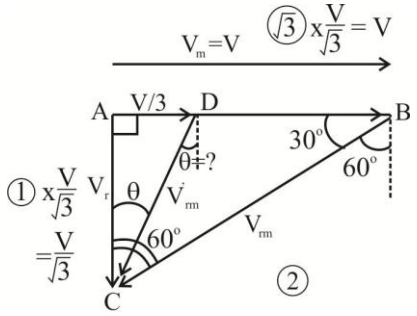
$Q_{CA} = 0$

$Q_h = 6TS, \ Q_l = 2TS$

$$\Rightarrow \quad \eta = 1 - \frac{Q_l}{Q_h} = 1 - \frac{2TS}{4TS} = 1 - \frac{1}{2} = \frac{1}{2}$$



Q19.



In Δ ACD

$$\tan\theta=\frac{V/3}{V/\sqrt{3}}=\frac{\sqrt{3}}{3}=\frac{1}{\sqrt{3}}$$

$\theta=30^\circ$

Q20. For vectors to be coplanar $\left(\vec{A}\times\vec{B}\right)\cdot\vec{B}=0$

$\vec{A}=2i-3j+k$

$\vec{C}=i+j+4k$

$\vec{A}\times\vec{C}=-13i-7j+5k$

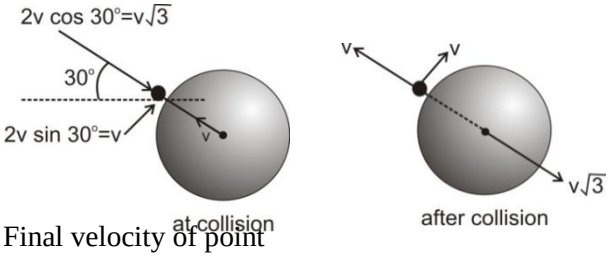
$\left(\vec{A}\times\vec{C}\right)\cdot\vec{B}=0$

$(-13i-7j+5k)\cdot(i+j+2\,mk)$

$-13-7+10m=0 \;\Rightarrow\; 10\,m=20$

$\Rightarrow\; m=2$

Q21.



Final velocity of point mass

$\Rightarrow\quad \sqrt{v^2+v^2}=v\sqrt{2}$

Q22. $\begin{cases} mass\ of\ \ disc=\sigma\cdot\pi R^2 \\ mass\ of\ \ cut\ \ disc=\sigma\cdot\pi\left(\frac{R}{2}\right)^2=\frac{1}{4}\sigma\cdot\pi R^2 \end{cases}$

$$x_o=\frac{m_1x_1-m_2x_2+m_3x_3}{m_1-m_2+m_3}$$

$$=\frac{\sigma\pi R^2\times 0-\frac{1}{4}\sigma\pi R^2\times\left(-\frac{R}{2}\right)+\frac{1}{4}\sigma\pi R^2\times\left(\frac{R}{2}\right)}{\sigma\pi R^2-\frac{1}{4}\sigma\pi R^2+\frac{1}{4}\sigma\pi R^2}$$

$$=\frac{\frac{R}{8}+\frac{R}{8}}{1-\frac{1}{4}+\frac{1}{4}}=\frac{2R}{8}=\frac{R}{4}=\frac{8}{4}=2cm$$

Q23. at maximum speed the kinetic energy becomes maximum or potential energy becomes minimum.

$KE_{maximum} = E_{total} - U_{minimum}$ ..... (1)

for $U_{minimum}$.

$$\frac{dU}{dx}=u \;\Rightarrow\; \frac{d}{dx}[20+(x-2)^2]=0$$

$$\Rightarrow\quad 2(x-2)=0 \;\Rightarrow\; x=2$$

$u_{\min}=20+(2-2)^2=20J$ ..... (2)

at x=2

$E=U+KE=U_{x=5}+KE_{x=5}=20+(5-2)^2+20$

$= 20 + 9 + 20 = 49$ J .... (3)

from eq. (1), (2) & (3) $KE_{max} = 49 - 20 = 29$ J

$$\frac{1}{2}mv_{\max}^2=29 \;\Rightarrow\; \frac{1}{2}\times 1\times v^2=29$$

$$\Rightarrow\; v_{\max}^2=2\times 29$$

$$\Rightarrow\; v_{\max}^2=58 \;\Rightarrow\; v_{\max}=\sqrt{58}\,m/s$$

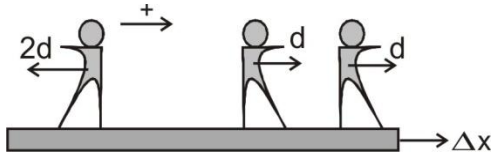
$$\Rightarrow\; v_{\max}=\sqrt{(60-2)}\,m/s$$

Q24. $m\Delta x_o=m_1\Delta x_1+m_2\Delta x_2+m_3\Delta x_3$

$8m\times 0=m[\Delta x-2d]+2m[\Delta x+d]+m[\Delta x+d]+4m\Delta x$

$0=\Delta x-2d+2\Delta x+2d+\Delta x+d+4\Delta x$

$\Rightarrow\quad 8\Delta x=-d \quad\Rightarrow\quad \Delta x=-d/8$



Q25. Velocity of centre of mass :

$$2mv_0=mv+m2v \quad\Rightarrow\quad v_0=\frac{3}{2}v$$

in the frame of centre of mass : $\frac{3v}{2}$

at maximum extension the bodies stop moving i.e. all the kinetic energy is converted into potential energy of extended spring

$$\frac{1}{2}kx^2=\frac{1}{2}m\left(\frac{v}{2}\right)^2 \;\Rightarrow\; kx^2=\frac{mv^2}{2} \;\Rightarrow\; x=v\sqrt{\frac{m}{2k}}$$