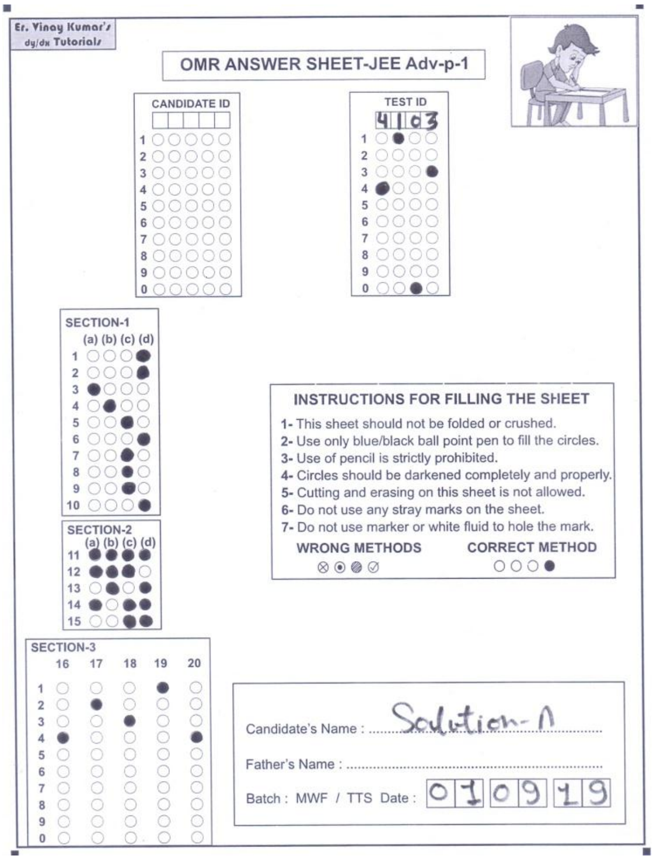
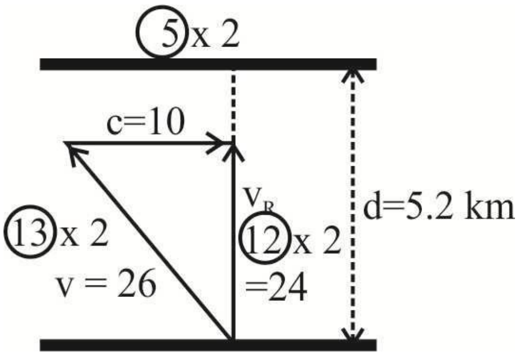
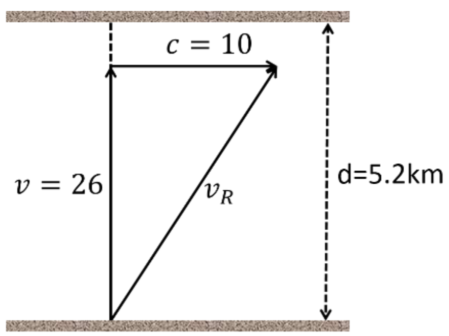


Q1. **For boat A**



$$t_A = \frac{d}{V_R} = \frac{5.2}{24} = 0.217\text{hr}$$

For boat B



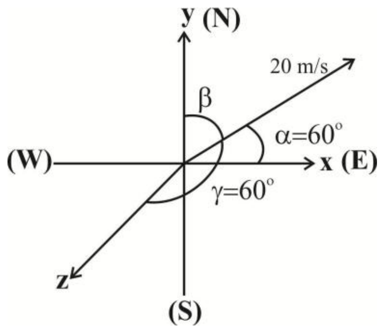
$$t_B = \frac{d}{v} = \frac{5.2}{26} = 0.2$$

$$\boldsymbol{t_A - t_B = 0.217 - 0.2 = 0.017hr}$$

Q2. $\alpha = 60^o$

$\beta = ?$

$\gamma = 60^o$



$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2 60^o + \cos^2\beta + \cos^2 60^o = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \cos^2\beta + \left(\frac{1}{2}\right)^2 = 1$$

$$\Rightarrow \cos^2\beta = \frac{1}{2} \Rightarrow \cos\beta = \frac{1}{\sqrt{2}} \Rightarrow \beta = 45^o$$

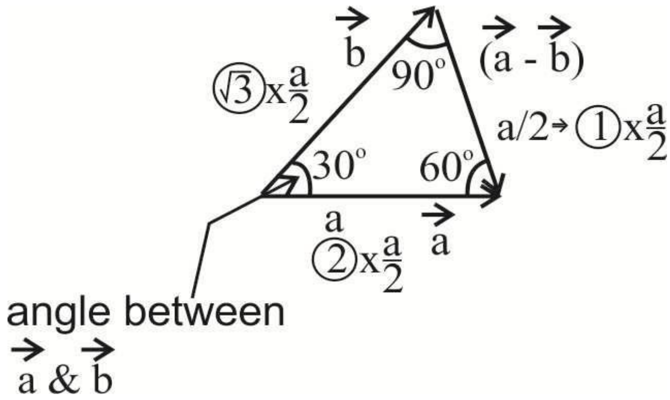
$$v_x = v\cos\alpha = 20\cos 60^o = 20\times\frac{1}{2} = 10$$

$$v_y = v\cos\beta = 20\cos 45^o = 20\times\frac{1}{\sqrt{2}} = 10\sqrt{2}$$

$$v_z = v\cos\gamma = 20\cos 60^o = 20\times\frac{1}{2} = 10$$

$$\vec{v} = v_x i + v_y i + v_z k \Rightarrow \vec{v} = 10i + 10\sqrt{2}j + 10k$$

Q3.



or

$$(\vec{a}-\vec{b}) \perp \vec{b} \Rightarrow (\vec{a}-\vec{b})\cdot\vec{b} = 0$$

$$\Rightarrow \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{b} = 0 \Rightarrow ab\cos\theta - b^2 = 0$$

$$\Rightarrow ab\cos\theta = b^2 \Rightarrow \cos\theta = \frac{b}{a} \;......(1)$$

$$|\vec{a}-\vec{b}| = \frac{1}{2}|\vec{a}| \Rightarrow \sqrt{a^2 + b^2 - 2ab\cos\theta} = \frac{1}{2}a$$

$$\Rightarrow \; a^2+b^2-2ab\cos\theta=\frac{a^2}{4} \;\ldots\ldots(2)$$

From (1) & (2) $\Rightarrow \; a^2+b^2-2ab\times\frac{b}{a}=\frac{a^2}{4}$

$$\Rightarrow \; a^2+b^2-2b^2=\frac{a^2}{4} \;\Rightarrow\; a^2-\frac{a^2}{4}=b^2$$

$$\Rightarrow \; \frac{3a^2}{4}=b^2 \;\Rightarrow\; \frac{b^2}{a^2}=\frac{3}{4}$$

$$\Rightarrow \; \frac{b}{a}=\frac{\sqrt{3}}{2} \;\ldots.\;(3)$$

From (1) & (3) $\Rightarrow \; \cos\theta\frac{b}{a}=\frac{\sqrt{3}}{2}$

$$\Rightarrow \; \theta=30^o=\pi/6$$

Q4. $(\vec{a}+\vec{b})\perp\vec{a} \;\Rightarrow\; (\vec{a}+\vec{b})\cdot\vec{a}=0$

$$\Rightarrow \; \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}=0 \;\Rightarrow\; a^2+ba\cos\theta=0$$

$$\Rightarrow \; a\,(a+b\cos\theta)=0 \;\Rightarrow\; a+b\cos\theta=0$$

$$\Rightarrow \; \cos\theta=-\frac{a}{b} \;\ldots\ldots\ldots(1)$$

$$(2\vec{a}+\vec{b})\perp\vec{b} \;\Rightarrow\; 2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}=0$$

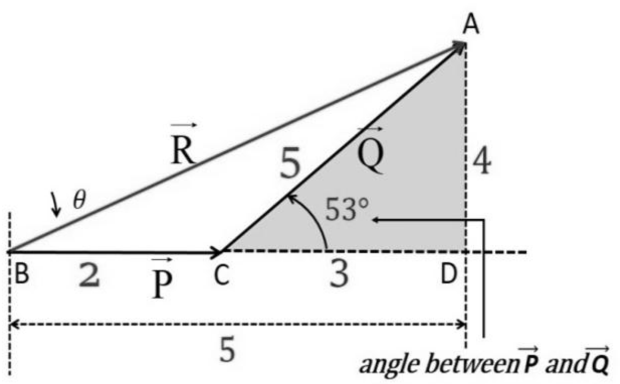
$$\Rightarrow \; 2ab\cos\theta+b^2=0$$

$$2a\cos+b=0 \;\Rightarrow\; \cos\theta=-\frac{b}{2a}$$

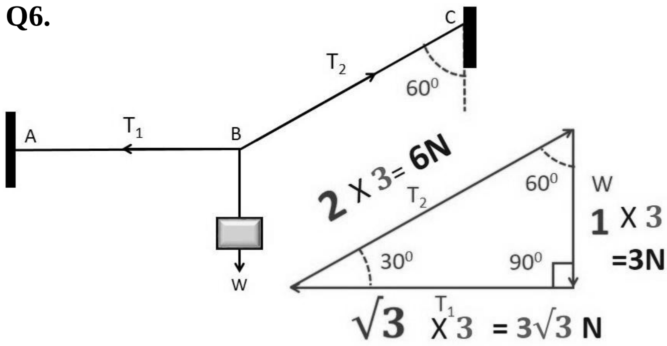
Equating (1) & (2) $-\frac{a}{b}=-\frac{b}{2a}$

$$\Rightarrow \; \frac{b^2}{a^2}=2 \;\Rightarrow\; \frac{b}{a}=\sqrt{2}$$

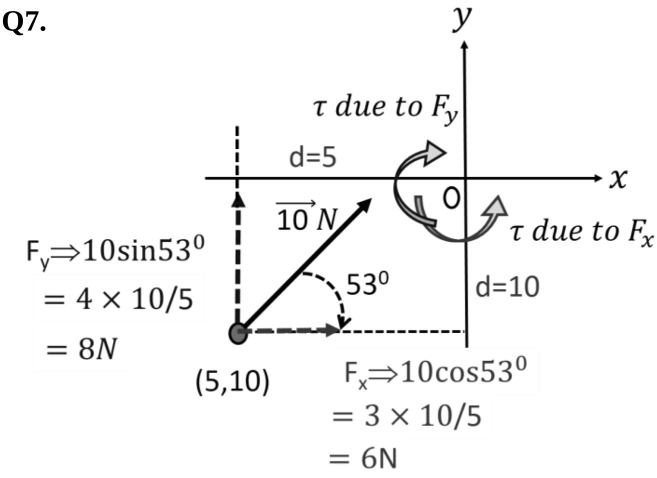
Q5. $\theta=\tan^{-1}\left(\frac{4}{5}\right) \Rightarrow tan\theta=\left(\frac{4}{5}\right)=\left(\frac{AD}{BD}\right)$ In Δ ABD



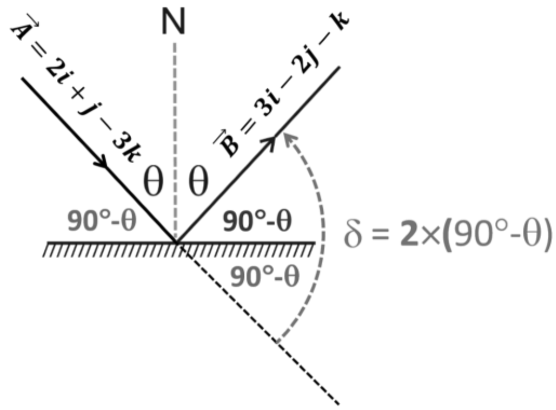
Q6.



Q7.



Q8.



$$cos\delta=\frac{(2i+j-3k)(3i-2j-k)}{\sqrt{2^2+1^2+3^2}.\sqrt{2^2+1^2+3^2}}$$

$$\Rightarrow cos\delta=\frac{6-2+3}{\sqrt{14}.\sqrt{14}} \Rightarrow cos\delta=\frac{7}{14}$$

$$\Rightarrow cos\delta=\frac{1}{2} \Rightarrow \delta=60°$$

δ = 2×(90°-θ) ⇨ 60°= 2×(90°-θ)

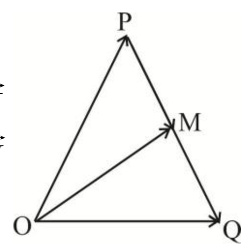
⇨ 30°= (90°-θ) ⇨ θ **=60°**

Q9. **(a)** $\overrightarrow{OQ}+\overrightarrow{OP}=2\overrightarrow{OM}$

(b) $\overrightarrow{OQ}-\overrightarrow{OP}=\overrightarrow{PQ}=2\overrightarrow{PM}$

(c) $\overrightarrow{OQ}-\overrightarrow{OM}=\overrightarrow{MQ}=\overrightarrow{PM}$

(s) $\overrightarrow{OP}+\overrightarrow{PM}=\overrightarrow{OM}$



Q10.

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(4i+5j+6k)-(3i+4j+5k)$$

$$=i+j+k$$

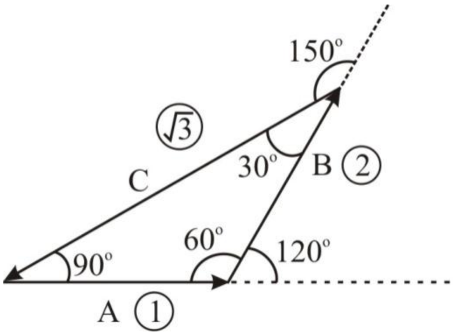
$$\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}=(4i+6j)-(7i+9j+3k)$$

$$=-3i-3j-3k$$

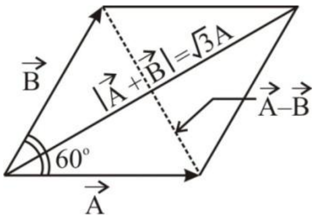
$$3\overrightarrow{AB}+\overrightarrow{CD}=3(i+j+k)+(-3i-3j-3k)$$

$$=3i+3j+3k-3i-3j-3k=0$$

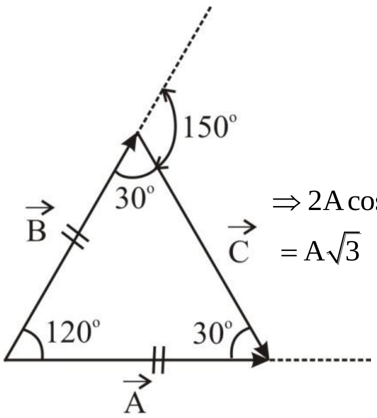
Q11.



Q12.



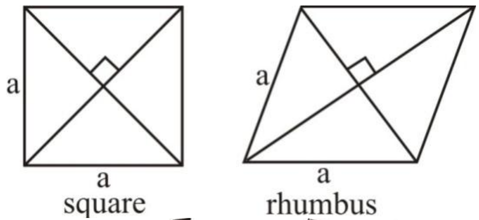
Q13.



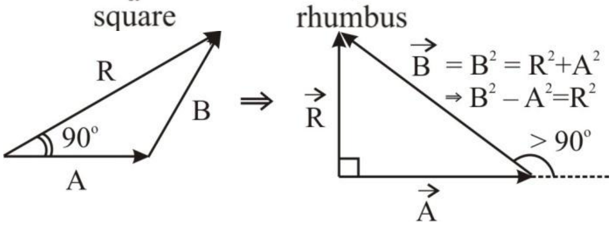
$\Rightarrow 2A\cos 30^o$

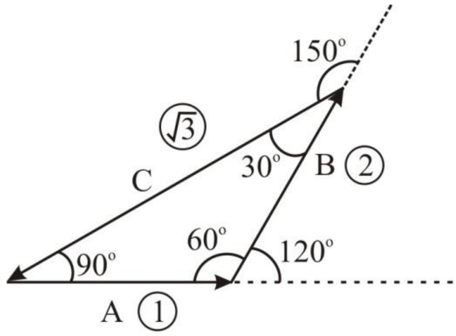
$= A\sqrt{3}$

Q14.



Q15.



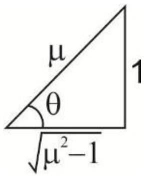


Q16. $\frac{dy}{dx}=\frac{1}{\tan\theta}$ ……..…….. (1)

-

$\mu\sin\theta \Rightarrow$ constant

$\Rightarrow 1\sin 90^o=\mu\sin\theta \Rightarrow \sin\theta=\frac{1}{\mu}$



$\Rightarrow \tan\theta=\frac{1}{\sqrt{\mu^2-1}}$ ……..…….. (2)

From (1) & (2) $\frac{dy}{dx}=\sqrt{\mu^2-1}=\sqrt{\left(\sqrt{1+y}\right)^2-1}$

$\Rightarrow \frac{dy}{dx}=\sqrt{1+y-1}$

$\Rightarrow \frac{dy}{dx}=y^{1/2} \Rightarrow y^{-\frac{1}{2}}dy=dx$

$\Rightarrow \int y^{-1/2}dy=\int dx \Rightarrow 2y^{1/2}=x+c$ …….. (3)

The curve satisfies origin $\Rightarrow$ at x= 0, y = 0 from eq. (3) c = 0

$2y^{1/2}=x \Rightarrow y=\frac{x^2}{4}$ ……….. (4)

For point P, y = 4,

$y=\frac{x^2}{4} \Rightarrow 4=\frac{x^2}{4} \Rightarrow x=4\text{cm}$

Q17. Ray enters the sphere at critical angle

$\sin C=\frac{1}{2} \Rightarrow C=30^o$

At surface PR

$2\sin 30^o=\mu\sin(90-C)$

$\Rightarrow 2\times\frac{1}{2}=\mu.\cos C \Rightarrow 1=\frac{1}{\sin C}\cdot\cos C$

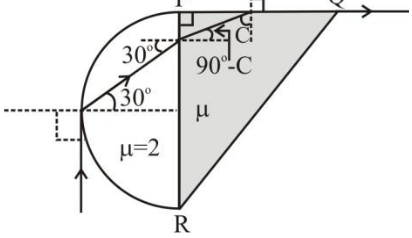
$\Rightarrow \tan C=1$

$\Rightarrow C=45^o$

$\Rightarrow \sin C=\frac{1}{\mu}$

$\Rightarrow \sin 45^o=\frac{1}{\mu}$

$\Rightarrow \mu=\sqrt{2}=\sqrt{n} \Rightarrow n=2$



Q18. **For concave mirror :** let the distance of the object from the concave mirror be d.

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \Rightarrow \frac{1}{-2}=\frac{1}{-(9-d)}+\frac{1}{-d}$$

$$\Rightarrow \frac{1}{2}=\frac{1}{9-d}+\frac{1}{d} \Rightarrow \frac{1}{2}=\frac{d+9-d}{(9-d)\,d}$$

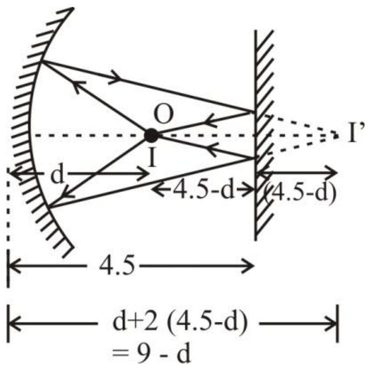
$$\Rightarrow (9-d)\,d=18 \Rightarrow 9d-d^2=18$$

$$\Rightarrow d^2-9d+18=0$$

$$\Rightarrow d^2-6d-3d+18=0$$

$$\Rightarrow d\,(d-6)-3\,(d-6)=0$$

$$\Rightarrow (d-3)\,(d-6)=0 \quad \Rightarrow d=3,6$$



d= 6 can not be possible as the object is between the mirror (d < 4.5 cm)

$$\Rightarrow d=3$$

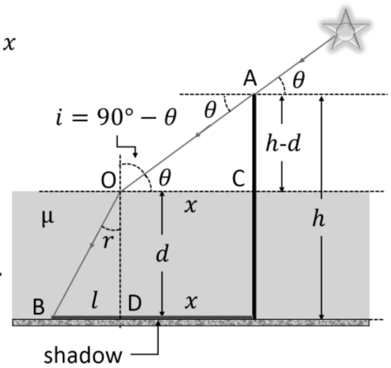
Q19.

Length of shadow = $l+x$

From ΔAOC

$\Rightarrow tan\theta=\frac{h-d}{x}$

$\Rightarrow x=\frac{h-d}{tan\theta}$ ......(1)



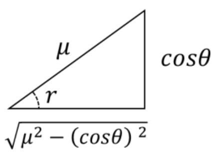
Snell's law:

$1.\sin(90°-\theta)=\mu sinr$

$cos\theta=\mu sinr$

$sinr=\frac{cos\theta}{\mu}$ ......(2)

$sinr=\frac{cos\theta}{\mu}$



From $\Delta OBD \Rightarrow tanr=\frac{l}{d} \quad \Rightarrow \frac{cos\theta}{\sqrt{\mu^2-(cos\theta)^2}}=\frac{l}{d}$

$\Rightarrow l=\frac{cos\theta}{\sqrt{\mu^2-(cos\theta)^2}}d$ ......(3)

From eq (1) and (3)$\Rightarrow$

Length of shadow = $l+x$

$$\Rightarrow l+x=\frac{cos\theta}{\sqrt{\mu^2-(cos\theta)^2}}d+\frac{h-d}{tan\theta}$$

$$=\frac{cos45°}{\sqrt{(\sqrt{2})^2-(cos45°)^2}}\sqrt{3}+\frac{2\sqrt{3}-\sqrt{3}}{tan45°}$$

$$=\frac{\frac{1}{\sqrt{2}}}{\sqrt{2-\frac{1}{2}}}\sqrt{3}+\frac{\sqrt{3}}{1} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{\frac{3}{2}}}\sqrt{3}+\sqrt{3} = 1+\sqrt{3}$$

$$= n+\sqrt{3} \Rightarrow \boldsymbol{n=1}$$

Q20. Since object is at large distance incident rays on the convex lens are parallel and the first image I_1 at focus of convex lens.

The image I_1 acts as object for concave lens whose optics axis is 60^o inclined

<u>For concave lens</u>

$$u=-(2f-f)\cos 60^o$$

$$u=-f\cos 60^o=-\frac{f}{2}$$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{-f}=\frac{1}{v}-\frac{1}{(-f/2)}$$

$$\Rightarrow -\frac{1}{f}=\frac{1}{v}+\frac{2}{f}$$

$$\Rightarrow \frac{1}{v}=-\frac{1}{f}-\frac{2}{f} \Rightarrow \frac{1}{v}=-\frac{3}{f} \Rightarrow v=-\frac{f}{3}$$

Final position of image I_2 on ox axis (from concave lens)

$$\Rightarrow \frac{v}{\cos 60^o}=\frac{f}{3}\times\frac{1}{1/2}=\frac{2f}{3}$$ on left of concave lens.

x-coordinate of final image

$$=2f-\frac{2f}{3}=\frac{6f-2f}{3}$$

$$=\frac{4f}{3}\left(=\frac{nf}{3}\right) \Rightarrow n=4$$