

Er. Vinay Kumar's
dy/dx Tutorials

OMR ANSWER SHEET-JEE Adv-p-1

CANDIDATE ID

TEST ID
41103

SECTION-1
(a) (b) (c) (d)

SECTION-2
(a) (b) (c) (d)

SECTION-3

INSTRUCTIONS FOR FILLING THE SHEET

- This sheet should not be folded or crushed.
- Use only blue/black ball point pen to fill the circles.
- Use of pencil is strictly prohibited.
- Circles should be darkened completely and properly.
- Cutting and erasing on this sheet is not allowed.
- Do not use any stray marks on the sheet.
- Do not use marker or white fluid to hole the mark.

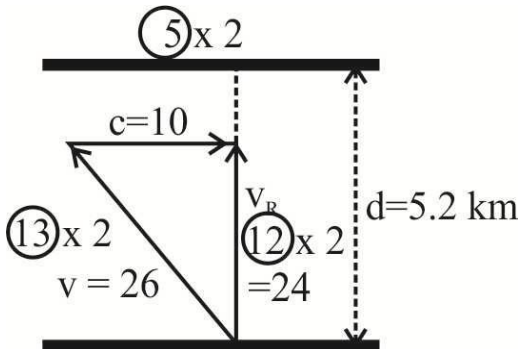
WRONG METHODS CORRECT METHOD

Candidate's Name : Solution-A

Father's Name :

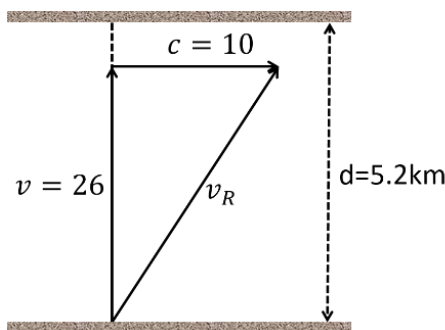
Batch : MWF / TTS Date : 010919

Q1. For boat A



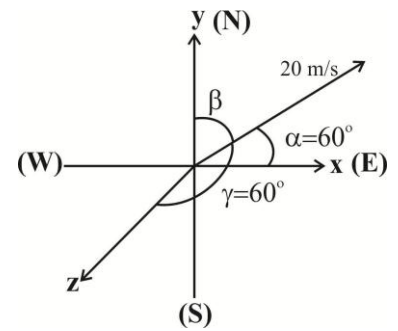
$$t_A = \frac{d}{V_R} = \frac{5.2}{24} = 0.217 \text{ hr}$$

For boat B



$$t_B = \frac{d}{v} = \frac{5.2}{26} = 0.2$$

$$t_A - t_B = 0.217 - 0.2 = 0.017 \text{ hr}$$



Q2. $\alpha = 60^\circ$
 $\beta = ?$
 $\gamma = 60^\circ$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\Rightarrow \cos^2 60^\circ + \cos^2 \beta + \cos^2 60^\circ = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \cos^2 \beta + \left(\frac{1}{2}\right)^2 = 1$$

$$\Rightarrow \cos^2 \beta = \frac{1}{2} \Rightarrow \cos \beta = \frac{1}{\sqrt{2}} \Rightarrow \beta = 45^\circ$$

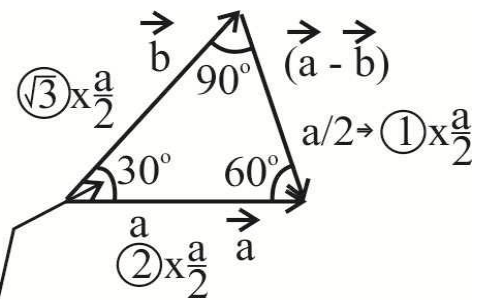
$$v_x = v \cos \alpha = 20 \cos 60^\circ = 20 \times \frac{1}{2} = 10$$

$$v_y = v \cos \beta = 20 \cos 45^\circ = 20 \times \frac{1}{\sqrt{2}} = 10\sqrt{2}$$

$$v_z = v \cos \gamma = 20 \cos 60^\circ = 20 \times \frac{1}{2} = 10$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \Rightarrow \vec{v} = 10\hat{i} + 10\sqrt{2}\hat{j} + 10\hat{k}$$

Q3.



angle between
 \vec{a} & \vec{b}

or

$$(\vec{a} - \vec{b}) \perp \vec{b} \Rightarrow (\vec{a} - \vec{b}) \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{b} = 0 \Rightarrow ab \cos \theta - b^2 = 0$$

$$\Rightarrow ab \cos \theta = b^2 \Rightarrow c \cos \theta = \frac{b}{a} \dots (1)$$

$$|\vec{a} - \vec{b}| = \frac{1}{2} |\vec{a}| \Rightarrow \sqrt{a^2 + b^2 - 2ab \cos \theta} = \frac{1}{2} a$$

$$\Rightarrow a^2 + b^2 - 2ab \cos \theta = \frac{a^2}{4} \dots\dots(2)$$

$$\text{From (1) \& (2)} \Rightarrow a^2 + b^2 - 2ab \times \frac{b}{a} = \frac{a^2}{4}$$

$$\Rightarrow a^2 + b^2 - 2b^2 = \frac{a^2}{4} \Rightarrow a^2 - \frac{a^2}{4} = b^2$$

$$\Rightarrow \frac{3a^2}{4} = b^2 \Rightarrow \frac{b^2}{a^2} = \frac{3}{4}$$

$$\Rightarrow \frac{b}{a} = \frac{\sqrt{3}}{2} \dots\dots(3)$$

$$\text{From (1) \& (3)} \Rightarrow \cos \theta \frac{b}{a} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 30^\circ = \pi/6$$

Q4. $(\vec{a} + \vec{b}) \perp \vec{a} \Rightarrow (\vec{a} + \vec{b}) \cdot \vec{a} = 0$

$$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} = 0 \Rightarrow a^2 + ba \cos \theta = 0$$

$$\Rightarrow a(a + b \cos \theta) = 0 \Rightarrow a + b \cos \theta = 0$$

$$\Rightarrow \cos \theta = -\frac{a}{b} \dots\dots\dots(1)$$

$$(\vec{2a} + \vec{b}) \perp \vec{b} \Rightarrow 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = 0$$

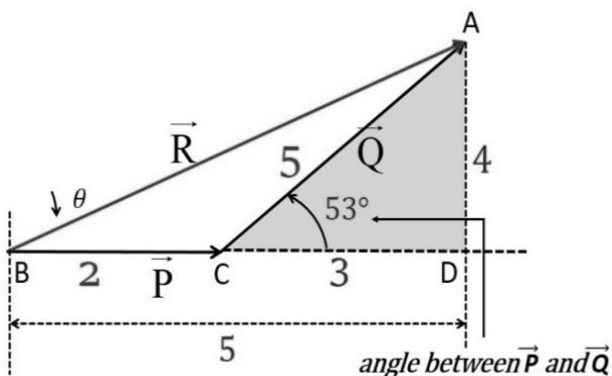
$$\Rightarrow 2ab \cos \theta + b^2 = 0$$

$$2a \cos \theta + b = 0 \Rightarrow \cos \theta = -\frac{b}{2a}$$

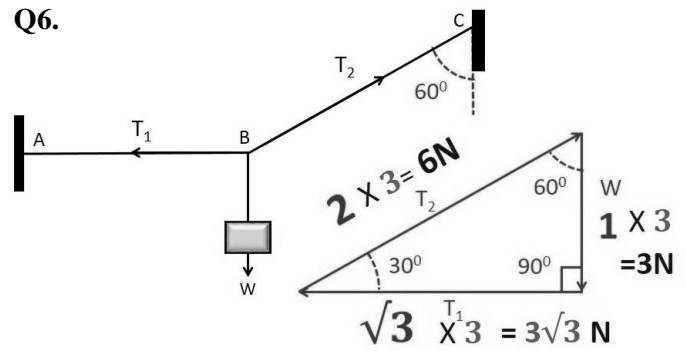
$$\text{Equating (1) \& (2)} \quad -\frac{a}{b} = -\frac{b}{2a}$$

$$\Rightarrow \frac{b^2}{a^2} = 2 \Rightarrow \frac{b}{a} = \sqrt{2}$$

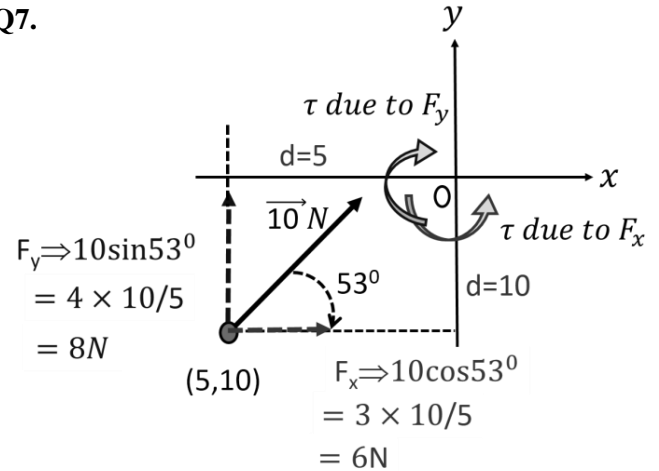
Q5. $\theta = \tan^{-1}\left(\frac{4}{5}\right) \Rightarrow \tan \theta = \left(\frac{4}{5}\right) = \left(\frac{AD}{BD}\right)$ in ΔABD



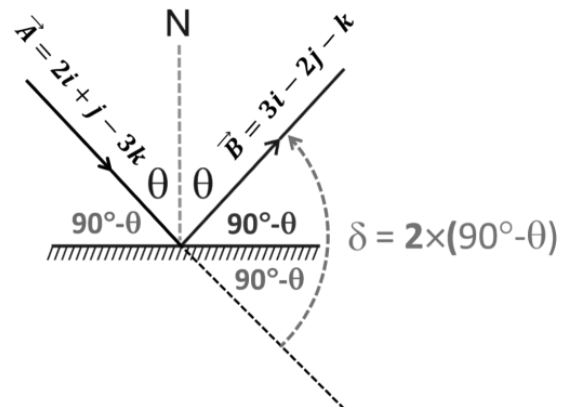
Q6.



Q7.



Q8.



$$\cos \delta = \frac{(2i + j - 3k)(3i - 2j - k)}{\sqrt{2^2 + 1^2 + 3^2} \cdot \sqrt{2^2 + 1^2 + 3^2}}$$

$$\Rightarrow \cos \delta = \frac{6 - 2 + 3}{\sqrt{14} \cdot \sqrt{14}} \Rightarrow \cos \delta = \frac{7}{14}$$

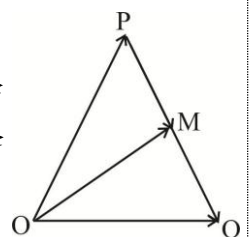
$$\Rightarrow \cos \delta = \frac{1}{2} \Rightarrow \delta = 60^\circ$$

$$\delta = 2 \times (90^\circ - \theta) \Rightarrow 60^\circ = 2 \times (90^\circ - \theta)$$

$$\Rightarrow 30^\circ = (90^\circ - \theta) \Rightarrow \theta = 60^\circ$$

Q9.

- (a) $\vec{OQ} + \vec{OP} = 2\vec{OM}$
- (b) $\vec{OQ} - \vec{OP} = \vec{PQ} = 2\vec{PM}$
- (c) $\vec{OQ} - \vec{OM} = \vec{MQ} = \vec{PM}$
- (s) $\vec{OP} + \vec{PM} = \vec{OM}$



Q10.

$$\vec{AB} = \vec{OB} - \vec{OA} = (4i + 5j + 6k) - (3i + 4j + 5k)$$

$$= i + j + k$$

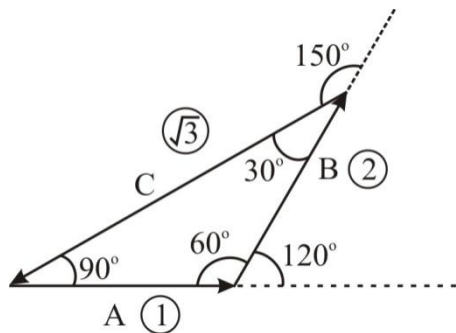
$$\vec{CD} = \vec{OD} - \vec{OC} = (4i + 6j) - (7i + 9j + 3k)$$

$$= -3i - 3j - 3k$$

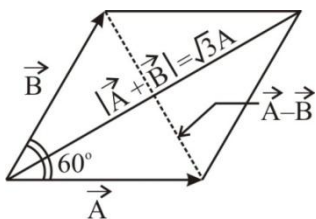
$$3\vec{AB} + \vec{CD} = 3(i + j + k) + (-3i - 3j - 3k)$$

$$= 3i + 3j + 3k - 3i - 3j - 3k = 0$$

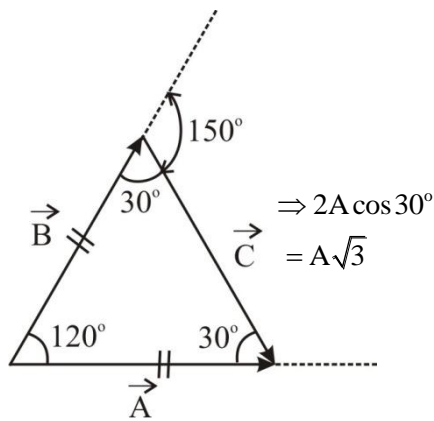
Q11.



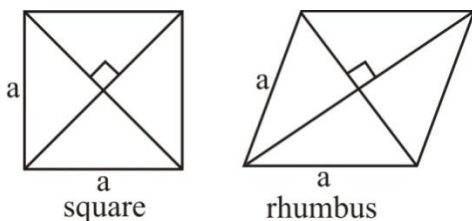
Q12.



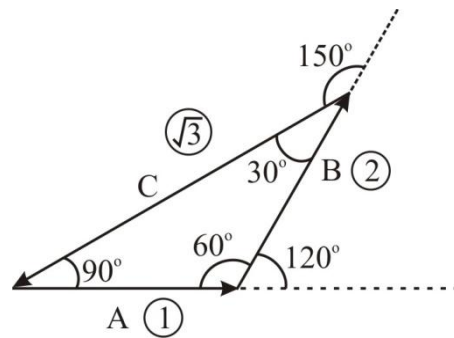
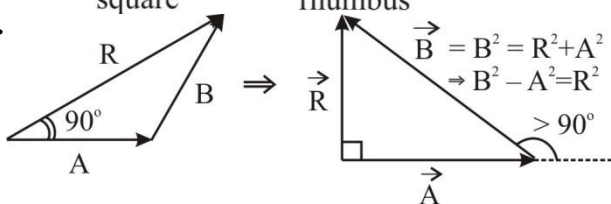
Q13.



Q14.



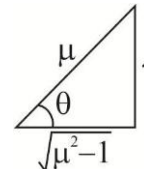
Q15.



Q16. $\frac{dy}{dx} = \frac{1}{\tan \theta}$ (1)

$\mu \sin \theta \Rightarrow \text{constant}$

$\Rightarrow 1 \sin 90^\circ = \mu \sin \theta \Rightarrow \sin \theta = \frac{1}{\mu}$



$\Rightarrow \tan \theta = \frac{1}{\sqrt{\mu^2 - 1}}$ (2)

From (1) & (2) $\frac{dy}{dx} = \sqrt{\mu^2 - 1} = \sqrt{(\sqrt{1+y})^2 - 1}$

$\Rightarrow \frac{dy}{dx} = \sqrt{1+y-1}$

$\Rightarrow \frac{dy}{dx} = y^{1/2} \Rightarrow y^{-1/2} dy = dx$

$\Rightarrow \int y^{-1/2} dy = \int dx \Rightarrow 2y^{1/2} = x + c$ (3)

The curve satisfies origin \Rightarrow at $x=0, y=0$ from eq. (3) $c=0$

$2y^{1/2} = x \Rightarrow y = \frac{x^2}{4}$ (4)

For point P, $y=4$,

$y = \frac{x^2}{4} \Rightarrow 4 = \frac{x^2}{4} \Rightarrow x = 4\text{cm}$

Q17. Ray enters the sphere at critical angle

$\sin C = \frac{1}{2} \Rightarrow C = 30^\circ$

At surface PR

$2 \sin 30^\circ = \mu \sin (90 - C)$

$\Rightarrow 2 \times \frac{1}{2} = \mu \cdot \cos C \Rightarrow 1 = \frac{1}{\sin C} \cdot \cos C$

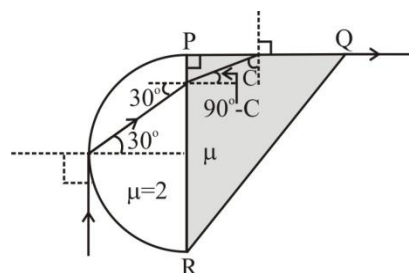
$\Rightarrow \tan C = 1$

$\Rightarrow C = 45^\circ$

$\Rightarrow \sin C = \frac{1}{\mu}$

$\Rightarrow \sin 45^\circ = \frac{1}{\mu}$

$\Rightarrow \mu = \sqrt{2} = \sqrt{n} \Rightarrow n = 2$



Q18. For concave mirror : let the distance of the object from the concave mirror be d .

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-2} = \frac{1}{-(9-d)} + \frac{1}{-d}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{9-d} + \frac{1}{d} \Rightarrow \frac{1}{2} = \frac{d+9-d}{(9-d)d}$$

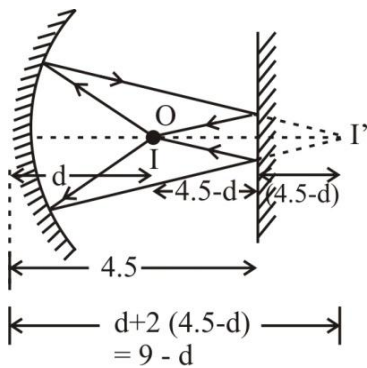
$$\Rightarrow (9-d)d = 18 \Rightarrow 9d - d^2 = 18$$

$$\Rightarrow d^2 - 9d + 18 = 0$$

$$\Rightarrow d^2 - 6d - 3d + 18 = 0$$

$$\Rightarrow d(d-6) - 3(d-6) = 0$$

$$\Rightarrow (d-3)(d-6) = 0 \Rightarrow d = 3, 6$$



$d = 6$ can not be possible as the object is between the mirror ($d < 4.5$ cm)

$$\Rightarrow d = 3$$

Q19. Length of shadow = $l + x$

From ΔAOC

$$\Rightarrow \tan \theta = \frac{h-d}{x}$$

$$\Rightarrow x = \frac{h-d}{\tan \theta} \dots (1)$$

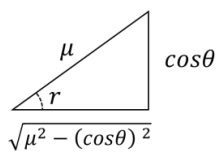
Snell's law:

$$1. \sin(90^\circ - \theta) = \mu \sin r$$

$$\cos \theta = \mu \sin r$$

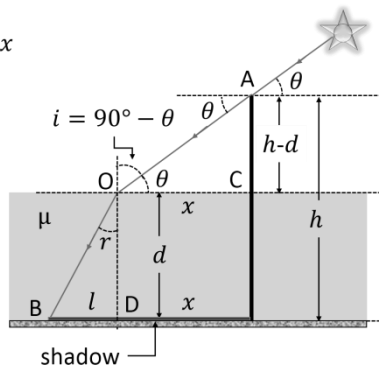
$$\sin r = \frac{\cos \theta}{\mu} \dots (2)$$

$$\sin r = \frac{\cos \theta}{\mu}$$



$$\text{From } \Delta OBD \Rightarrow \tan r = \frac{l}{d} \Rightarrow \frac{\cos \theta}{\sqrt{\mu^2 - (\cos \theta)^2}} = \frac{l}{d}$$

$$\Rightarrow l = \frac{\cos \theta}{\sqrt{\mu^2 - (\cos \theta)^2}} d \dots (3)$$



From eq (1) and (3) \Rightarrow

Length of shadow = $l + x$

$$\Rightarrow l + x = \frac{\cos \theta}{\sqrt{\mu^2 - (\cos \theta)^2}} d + \frac{h-d}{\tan \theta}$$

$$= \frac{\cos 45^\circ}{\sqrt{(\sqrt{2})^2 - (\cos 45^\circ)^2}} \sqrt{3} + \frac{2\sqrt{3} - \sqrt{3}}{\tan 45^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\sqrt{2 - \frac{1}{2}}} \sqrt{3} + \frac{\sqrt{3}}{1} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{\frac{3}{2}}} \sqrt{3} + \sqrt{3} = 1 + \sqrt{3} = n + \sqrt{3} \Rightarrow n = 1$$

Q20. Since object is at large distance incident rays on the convex lens are parallel and the first image I_1 at focus of convex lens.

The image I_1 acts as object for concave lens whose optics axis is 60° inclined

For concave lens

$$u = -(2f - f) \cos 60^\circ$$

$$u = -f \cos 60^\circ = -\frac{f}{2}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{-f} = \frac{1}{v} - \frac{1}{(-f/2)}$$

$$\Rightarrow -\frac{1}{f} = \frac{1}{v} + \frac{2}{f}$$

$$\Rightarrow \frac{1}{v} = -\frac{1}{f} - \frac{2}{f} \Rightarrow \frac{1}{v} = -\frac{3}{f} \Rightarrow v = -\frac{f}{3}$$

Final position of image I_2 on ox axis (from concave lens)

$$\Rightarrow \frac{v}{\cos 60^\circ} = \frac{f}{3} \times \frac{1}{1/2} = \frac{2f}{3} \text{ on left of concave lens.}$$

x -coordinate of final image

$$= 2f - \frac{2f}{3} = \frac{6f - 2f}{3}$$

$$= \frac{4f}{3} \left(= \frac{nf}{3} \right) \Rightarrow n = 4$$

