Neural Computing and Applications
https://doi.org/10.1007/s00521-024-09960-z

REVIEW

Neuro-symbolic artificial intelligence: a survey

Bikram Pratim Bhuyan[1,2] · Amar Ramdane-Cherif [1] · Ravi Tomar [3] · T. P. Singh[4]

Received: 8 May 2023 / Accepted: 3 May 2024

Abstract

The goal of the growing discipline of neuro-symbolic artificial intelligence (AI) is to develop AI systems with more human-like reasoning capabilities by combining symbolic reasoning with connectionist learning. We survey the literature on neuro-symbolic AI during the last two decades, including books, monographs, review papers, contribution pieces, opinion articles, foundational workshops/talks, and related PhD theses. Four main features of neuro-symbolic AI are discussed, including representation, learning, reasoning, and decision-making. Finally, we discuss the many applications of neuro-symbolic AI, including question answering, robotics, computer vision, healthcare, and more. Scalability, explainability, and ethical considerations are also covered, as well as other difficulties and limits of neuro-symbolic AI. This study summarizes the current state of the art in neuro-symbolic artificial intelligence.

Keywords Neuro-symbolic artificial intelligence · Machine learning · Knowledge representation and reasoning · Spatial-temporal data · Neural networks · Artificial intelligence

1 Introduction

There have been several breakthroughs and innovations in the areas of artificial intelligence (AI) and deep learning (connectionist artificial intelligence) during the last decade [1]. The widespread use of AI and deep learning as cutting-edge technologies has been a significant recent development. Several industries, including healthcare, banking, transportation, agriculture, and arts, have profited from recent artificial intelligence and deep learning developments [2–4].

New technologies have advanced deep learning models in computer vision and natural language processing. Convolutional neural networks (CNNs) and transformers have improved sectors like image recognition and language translation [5]. Generative adversarial networks (GANs) and variational autoencoders (VAEs) may produce new data, images, and sounds [6]. Music production and design might leverage these models. Edge computing, another decade-old breakthrough, allows AI model installation on low-resource devices. Thus, AI and deep learning models may be applied on edge, closer to the data source, which is beneficial in constructing Internet of Things (IoT) devices [7].

Yet, connectionist AI is not without its caveats. One drawback is that training models properly usually require a lot of data (typically involving highly unstructured, perceptual data). These AI models may also lack the transparency and explainability of other forms of AI due to the complexity involved in understanding how they arrive at their predictions or choices [8].

Symbolic AI, commonly known as “good old-fashioned AI”, emerged as the foundation of AI research during the mid-twentieth century with notable figures such as Allen Newell and Herbert A. Simon [9–11]. Referred to as rule-based or expert systems, they were designed and implemented with a predefined set of explicit rules and logical reasoning mechanisms to address and resolve various problems. Ontologies were conceived as a means of representing and sharing knowledge [12]. Although symbolic

✉ Bikram Pratim Bhuyan
bikram-pratim.bhuyan@universite-paris-saclay.fr

1 LISV Laboratory, University of Paris Saclay, 10-12 Avenue of Europe, 78140 Velizy, France

2 School of Computer Science, University of Petroleum and Energy Studies, Bidholi, Dehradun, Uttarakhand 248006, India

3 Persistent Systems, Pune, Mumbai 411016, India

4 School of Computer Science Engineering and Technology, Bennett University, Greater Noida 201306, India

Published online: 06 June 2024

AI demonstrated proficiency in problem domains characterized by explicit rules and clear boundaries, they encountered difficulties when confronted with incomplete information [13]. Thus, the efficiency of these systems is hugely dependent on the completeness of the knowledge.

The drawbacks of both the fields individually in terms of 'Explainability', 'Efficiency', and 'Generalization' could be seen through Fig. 1. The efficiency of connectionist AI is typically considered high due to its ability to process vast amounts of data and learn complex patterns through neural networks. This efficiency stems from the processing capabilities of neural networks, which can handle and learn from high-dimensional data, making them particularly adept at tasks like image and speech recognition, where they can directly learn from raw inputs to outputs.

On the other hand, the efficiency of symbolic AI is often viewed as lower, particularly in the context of processing large datasets or handling perceptual tasks. Symbolic AI operates on explicit rules and logic, which can be computationally intensive and less flexible when dealing with nuanced or ambiguous data that does not fit neatly into predefined categories or rules. While symbolic AI excels in tasks that require clear, logical reasoning and interpretability, its rule-based nature can limit its efficiency in scenarios where learning from data or scaling to large problem spaces is essential.

However, it's crucial to contextualize these efficiency considerations within the specific domains and tasks to which each AI approach is applied. While connectionist AI may show higher efficiency in data-driven, pattern recognition tasks, symbolic AI can be more efficient in domains where clear reasoning, interpretability, and adherence to explicit knowledge or rules are paramount. This distinction underscores the complementary nature of these approaches, highlighting the potential of neuro-symbolic AI to leverage the strengths of both to achieve higher overall efficiency across a broader range of tasks.

The roots of neuro-symbolic (NeSy) AI may be traced all the way back to the 1950s and 1960s when the field of AI was getting its start [14]. In the past, artificial intelligence studies focused on creating rule- and symbol-based problem-solving machines. In the 1980s, however,

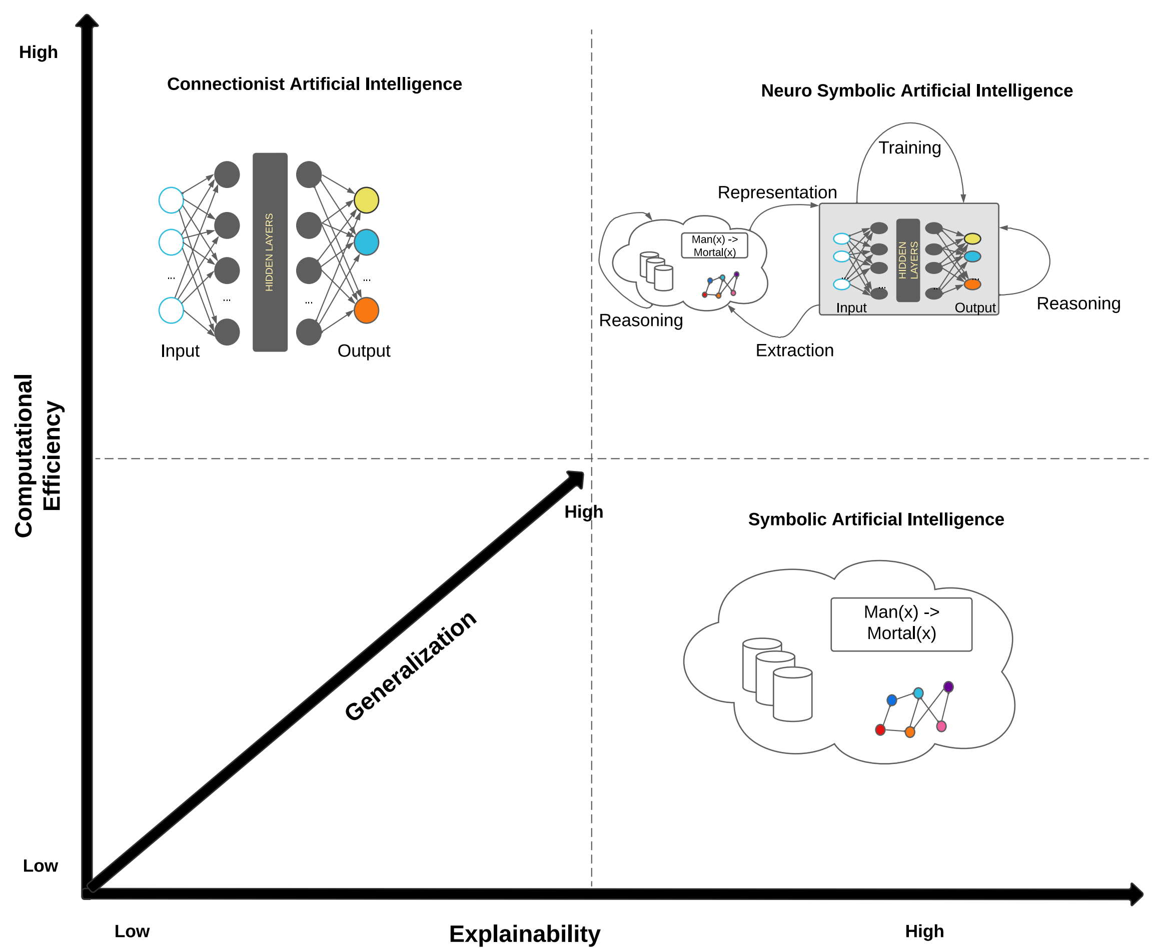

Fig. 1 The drawbacks of both the fields individually in terms of 'Explainability', 'Efficiency', and 'Generalization', when the fields merge together to form neuro-symbolic artificial intelligence, all three characteristics are high

scientists started to see the method's flaws. For example, natural language processing and vision were shown to be areas where symbolic AI systems faltered. Researchers began implementing neuroscientific principles into AI systems to address these shortcomings. In the early twenty-first century, scientists started looking at ways to combine the best features of the two methods. They came up with a new branch of AI, neuro-symbolic AI, which combines symbolic reasoning and representation with neural networks. It has been used in disparate fields such as healthcare, robotics, and natural language processing. One of the most exciting directions in artificial intelligence research today is neuro-symbolic AI, which aims to create intelligent systems that can learn and reason like humans. The growing interest in the field could be seen through the amount of literature published, as shown in Fig. 2. The literature contains books, monographs, thesis [15–23], review papers [20, 24–33], contributory articles [34–95], commentary articles [25, 39, 93, 96–143], and foundational workshops/talks [144–167]. It's worth noting that neuro-synthetic AI is a hot topic in both academia and industry because of its immense potential for artificial general intelligence.

Neuro-symbolic AI is a kind of AI that takes cues from the way the human brain processes information while also relying on symbolic logic to solve issues. The study of the brain and its functions serves as inspiration for the "neuro" component of neuro-symbolic AI [33]. The "neuro" component of this AI makes use of neural networks to learn from data and enhance its grasp of the environment, much like the way human brains process information and learn from experience. The "symbolic" component of neuro-symbolic AI uses symbolic representations and logical reasoning to accomplish its goals. This suggests that the AI can think logically and grasp notions like "if-then" statements. Knowledge may also be represented in a human-understandable form, for as via the use of words and symbols to stand in for real-world entities and abstract concepts.

Recent research on neural-symbolic integration, which seeks to combine the capabilities of symbolic AI with neural networks to produce more powerful and adaptable intelligent systems, is surveyed in the articles as shown in Table 1, and we base our classification method based on this with the objective of harnessing the complementing capabilities of the two paradigms [168]. The criteria for classification are taken from the Kautz's talk [169], which is even regarded as the turning point of the field [33].

All of the major developments over the last two decades are summarized in this survey article. It delves into the numerous aspects that have led to the hybridization of connectionist AI and symbolic AI. Its applications in many fields are also examined. The challenges are also being considered. Figure 3 depicts a conceptual map of the article. The organization of the survey is shown in Fig. 4.

2 Background and related work

2.1 Neuro-symbolic properties

We delve into the core components that define neuro-symbolic AI, encompassing representation, learning, reasoning, decision-making, knowledge, and logic. This exploration provides insight into how neuro-symbolic AI seeks to amalgamate the strengths of symbolic and neural approaches to overcome their limitations.

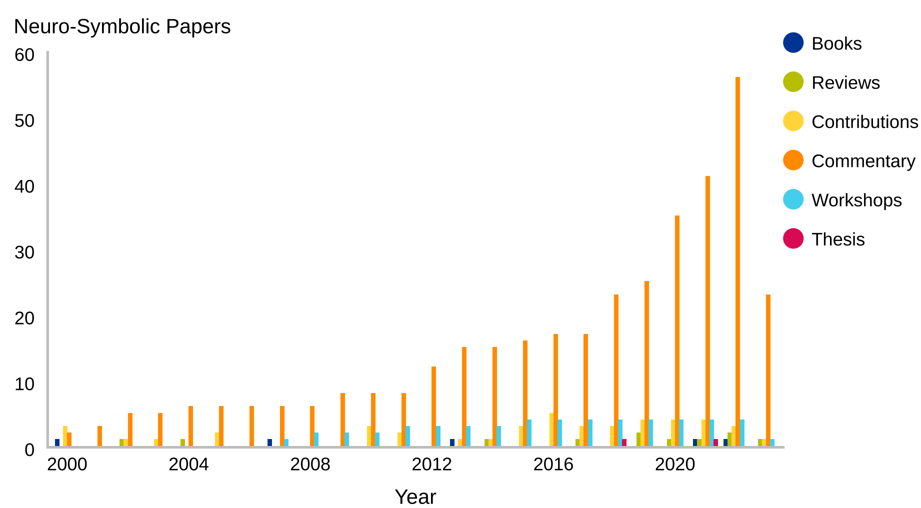

Fig. 2 Peer reviewed papers in the field of neuro-symbolic AI with keywords, 'neuro-symbolic', 'neural-symbolic', 'neuro symbolic', 'neural symbolic' and 'neurosymbolic'

Table 1 Review papers with the discussion upon the domain, properties, type of neural architecture and neuro-symbolic types represented by NS

		Properties							
Paper	Year	Domain	Representation	Learning	Reasoning	Decision making	Logic	Neural type	NS
Corchado et al. [24]	2002	Oceanography	–	–	–	–	–	✓	–
Hatzilygeroudis et al. [25]	2004	Expert Systems	–	–	–	–	–	–	–
Öztürk et al. [26]	2014	CBR	–	–	–	–	–	–	–
Besold et al. [27]	2017	General	–	–	–	–	✓	✓	–
Garnelo et al. [28]	2019	General	✓	–	–	–	–	–	–
Garcez et al. [29]	2019	General	–	✓	✓	–	✓	–	–
De et al. [30]	2020	General	–	–	–	–	✓	✓	–
Sarker et al. [31]	2021	General	–	✓	✓	-	✓	–	✓
Hitzler et al. [20]	2022	General	–	–	–	–	–	–	–
Wang et al. [32]	2022	General	–	✓	✓	✓	✓	–	–
Garcez et al. [33]	2023	General	✓	–	✓	–	-	–	✓
Our survey	2024	General	✓	✓	✓	✓	✓	✓	✓

2.1.1 Representations

When discussing symbolic AI, “localist representations” refer to using isolated symbols to stand in for abstract ideas or concrete objects [170]. Expert systems and rule-based systems are two examples of symbolic AI that extensively use localist representations [171]. As each sign represents a distinct idea that humans can readily grasp, they benefit from being interpretable and transparent.

In contrast to localist representations, distributed representations [170] have gained traction in recent years, particularly in the context of deep learning. Distinct dimensions of a vector of real-valued integers in distributed representations represent different features or aspects of a topic. This paves the way for more versatile and potent representations that encapsulate subtle but significant data linkages and patterns. The difference can be seen in Fig. 5.

Localist and distributed representation has their own benefits and drawbacks, as shown in Table 2.

Attention systems, graph neural networks, differentiable programming, variable grounding, symbol manipulation, and foundation model representation techniques make neuro-symbolic AI integration unique in the field.

Attention mechanisms in neuro-symbolic AI improve the model’s focus on relevant parts of the input data or internal representations. This is particularly used in tasks requiring sequential data processing, like natural language understanding by [91, 172], where the model needs to focus on relevant parts of the input sequence to make decisions or predictions.

Graph neural networks (GNNs) are pivotal in representing and processing data in graph form, which is inherently symbolic. GNNs can capture the complex relationships and structures within data, making them ideal for tasks that involve relational reasoning, knowledge graphs, and structured prediction. [173] surveys around this integration for encoding both entity attributes and the relationships between entities in a way that is amenable to neural network processing.

Differentiable programming extends the capabilities of neural networks by making them more flexible and capable of incorporating symbolic computation within the learning process. [174, 175] uses this approach to enable the integration of symbolic reasoning directly into the neural network’s architecture, allowing for the optimization of symbolic operations alongside standard neural network parameters, facilitating a tighter integration of symbolic and sub-symbolic AI components.

Variable grounding refers to the process of linking abstract symbols or concepts to concrete instances in data. In the context of neuro-symbolic AI, [176, 177] involves the identification and association of symbolic variables with relevant features or patterns learned by the neural network, enabling the system to reason about abstract concepts in a grounded, data-driven context.

Symbol manipulation in neuro-symbolic systems involves the use of operations on symbols that represent abstract concepts, akin to traditional symbolic AI. [178, 179] integrated these operations within a neural framework. Neuro-symbolic AI systems can perform symbolic reasoning, such as logical deduction and inference, while also benefiting from the adaptive learning capabilities of neural networks.

Finally, leveraging foundation models for representation can enhance performance in neuro-symbolic tasks, reduce data labeling, and minimize manual engineering, as

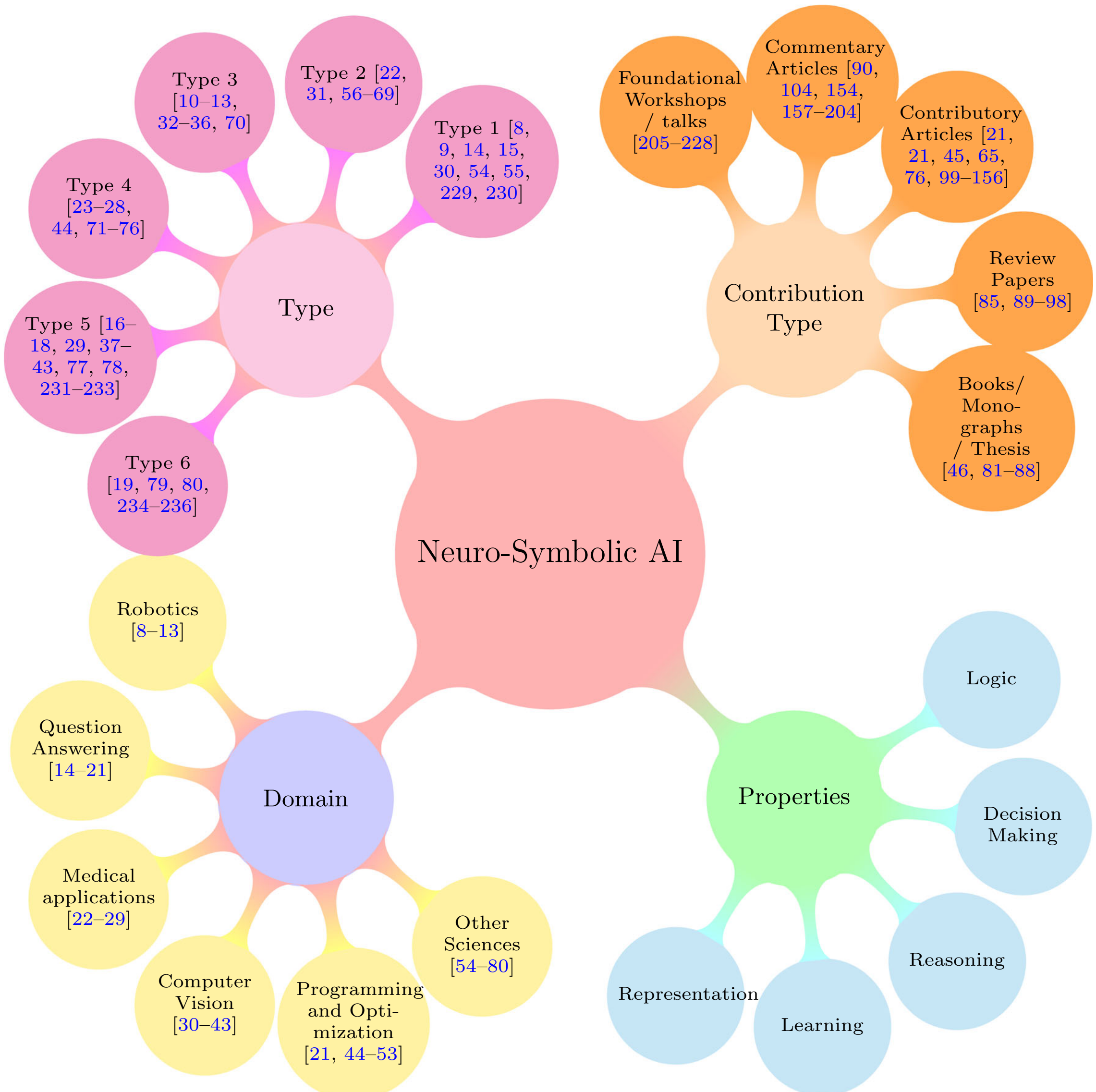

Fig. 3 A conceptual map of the survey, depicting the wide range of neuro-symbolic AI implementations, their respective type of integration, contribution kinds, and properties

demonstrated by the introduction of architectures like NeSyGPT [180].

2.1.2 Learning

Neuro-symbolic AI introduces a paradigm shift in how machines learn, blending the deductive, rule-based learning of symbolic AI with the inductive, pattern-recognizing capabilities of neural networks. This hybrid approach leverages the strengths of both domains to facilitate a more comprehensive learning methodology.

Traditional symbolic AI learns through logical deduction, inducing general rules from specific instances. Techniques like decision tree induction [181] and explanation-based learning [182] exemplify this, where new knowledge is systematically derived from existing rules and examples. However, this method's reliance on extensive manual curation of knowledge bases and datasets is a notable limitation [183].

In contrast, connectionist AI, particularly through deep learning, excels at learning representations from raw, unstructured data [184]. It employs various techniques (e.g., supervised, unsupervised, and reinforcement learning [185]) to adjust neural connections, enabling pattern recognition and decision-making. While powerful, this approach often lacks transparency and interoperability.

Neuro-symbolic AI (NeSy) aims to transcend these limitations by integrating the structured knowledge representation of symbolic AI with the adaptive learning mechanisms of neural networks. This integration enables

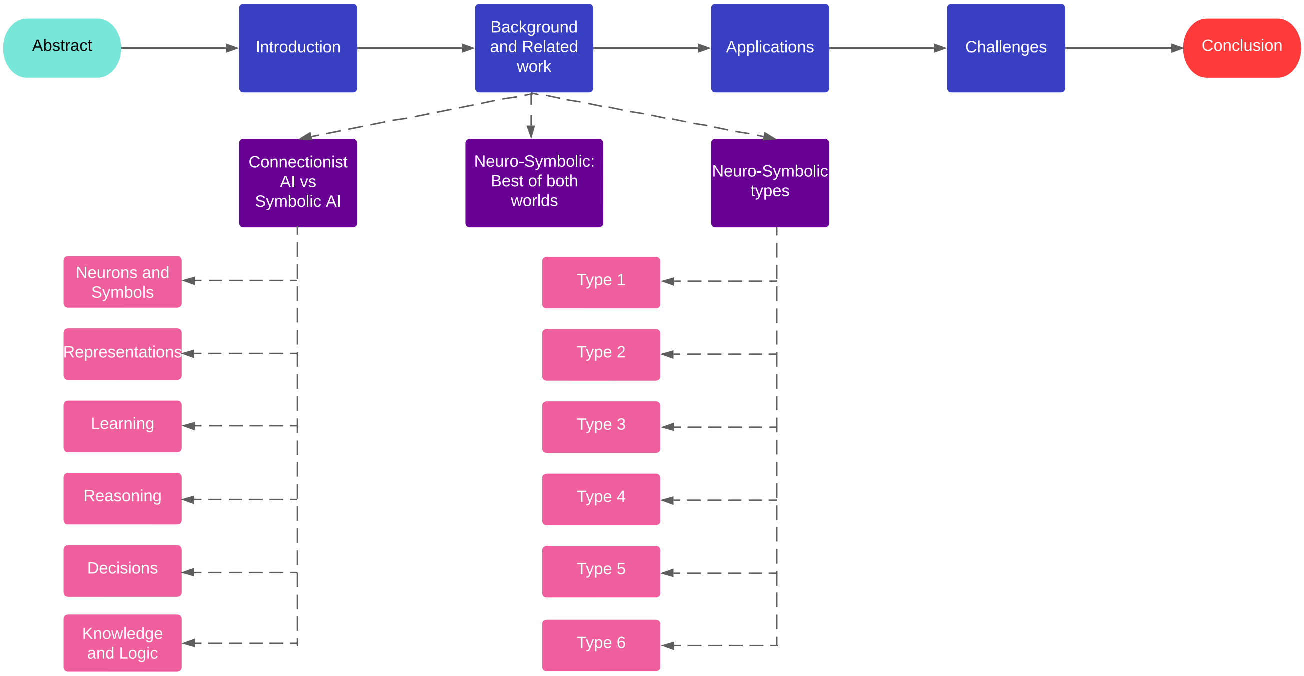

Fig. 4 Organization of the article as a flowchart

NeSy systems to: (a) learn from fewer examples by leveraging pre-existing symbolic knowledge, thus addressing the data-hungry nature of pure neural approaches; (b) enhance interpretability by grounding neural network outputs in symbolic representations, making the learning process and outcomes more understandable; (c) facilitate adaptable reasoning that combines the robustness of neural pattern recognition with the precision of symbolic logic; and (d) incorporate feedback loops where symbolic reasoning can guide neural learning and vice versa, enabling dynamic adaptation to new information or tasks. The comparison is shown in Table 3.

2.1.3 Reasoning

Reasoning, a fundamental aspect of intelligence, has been approached differently across the AI spectrum. The trade-off between learning and reasoning in symbolic AI and connectionist AI can be shown in Table 4. Symbolic AI, with its roots in formal logic and knowledge representation, traditionally employs deductive, inductive, and abductive reasoning [186]. These methods allow for deriving conclusions from known premises, generalizations from specific instances, and formulating plausible explanations from observations [186, 187]. While powerful in structured environments, symbolic reasoning struggles with ambiguity and the inherent uncertainty of real-world data.

In contrast, connectionist models, particularly neural networks, excel in pattern recognition and inference from vast datasets but cannot traditionally perform explicit, rule-based reasoning. However, there has been some recent work on developing reasoning tasks based on neural networks. For example, some researchers have explored using neural networks to understand natural language and answer questions [188]. Other researchers have looked into neural-symbolic integration, in which neural networks are used to learn representations of complex data, which are fed into symbolic reasoning systems to make logical inferences [189]. Even with all these efforts, making neural network-based approaches to reasoning tasks work well is still tough, especially when explicit rules or logic are needed. These challenges include how hard it is to encode symbolic information in a distributed representation, how fragile neural networks are when dealing with new inputs, and how little they can do abstract reasoning or figure out what information is missing.

Another important discussion is on combinatorial and common-sense reasoning [33]. Common-sense reasoning is a type of approximate reasoning that involves making assumptions or inferences based on general knowledge and experience rather than on explicit rules or algorithms. Problems in mathematics, computer science, and engineering are typically solved with the use of combinatorial reasoning methods, including counting principles, permutations, and combinations. The emergence of neuro-symbolic AI represents a paradigm shift, aiming to meld the structured reasoning capabilities of symbolic AI with the adaptive learning process of neural networks [136]. The various types of reasoning used are shown in Fig. 6.

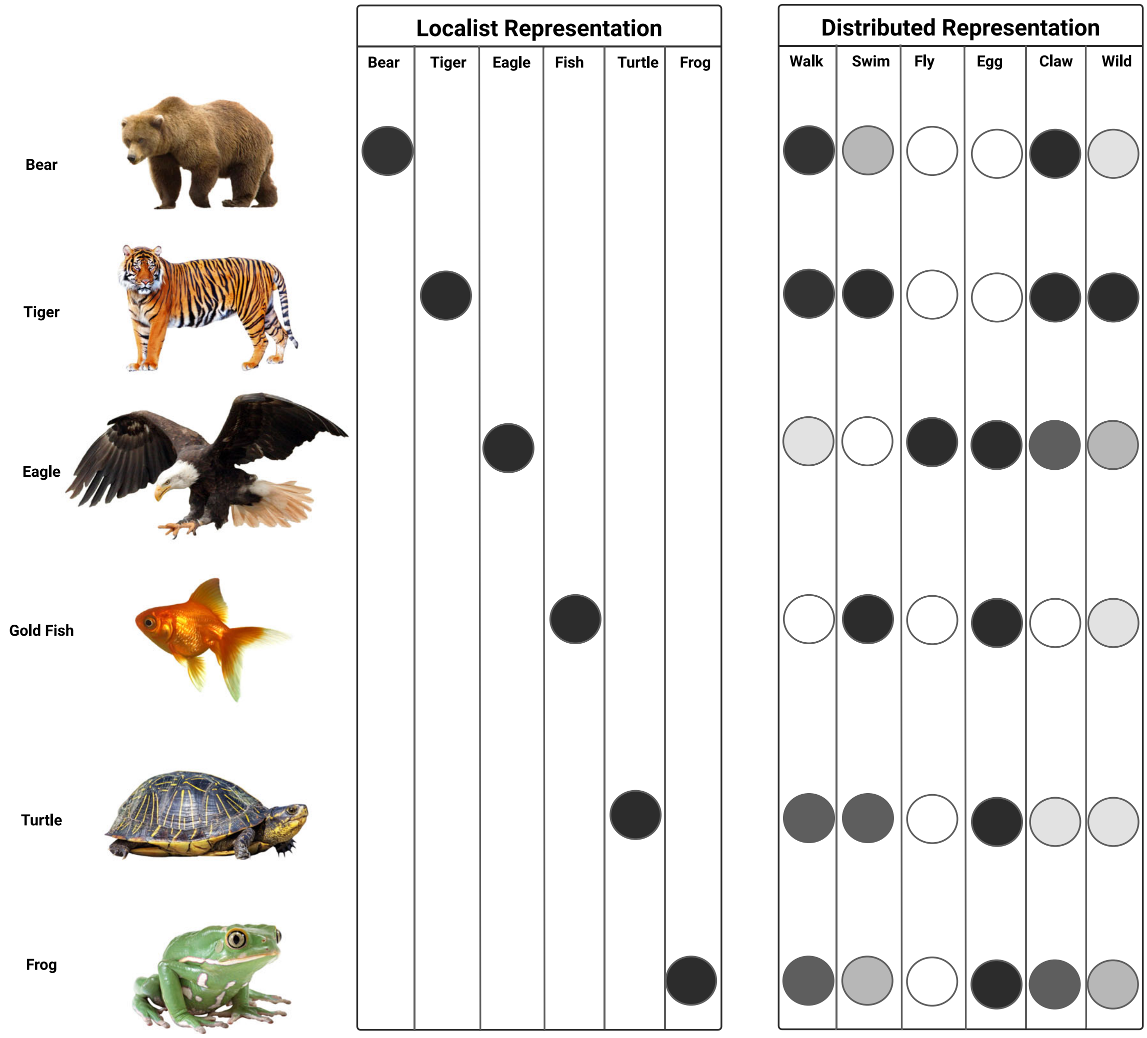

Fig. 5 Difference between localist and distributed representations

Table 2 Comparison of localist and distributed representations and integration in neuro-symbolic AI

Aspect	Localist representation	Distributed representation
Definition	Represents concepts with dedicated units or nodes in the network, where each unit represents a single concept or category	Represents concepts across many units, with each unit participating in the representation of multiple concepts, allowing for more nuanced representations
Benefits	High interpretability and transparency	Greater capacity for generalization
	Easier manipulation of individual concepts	Efficient use of network capacity
	Simplifies mapping of symbolic knowledge	Facilitates learning of complex patterns
Drawbacks	Limited scalability with the number of concepts	Reduced interpretability of individual units
	Less efficient in capturing complex patterns	Integration of explicit symbolic knowledge can be challenging
Neuro-symbolic AI integration	Neuro-symbolic AI leverages both approaches, utilizing localist representations for symbolic components and distributed methods for neural processing, enabling efficient integration of symbolic reasoning with neural learning	

Under NeSy, CTLK (Temporal-Epistemic Reasoning) [39, 40] exemplifies the application of deductive reasoning in neuro-symbolic systems, showcasing how neural networks can be employed to interpret and defend translations of non-classical logics, including temporal logic. CIL^2P [36, 37] (Connectionist Inductive Learning and

Table 3 Comparison of learning paradigms in neuro-symbolic AI

Learning paradigm	Characteristics	Neuro-symbolic integration
Symbolic learning	Involves logical deduction and induction to generate rules from data. Highly interpretable but requires extensive knowledge engineering	NeSy integrates symbolic rules with neural learning, allowing for the derivation of symbolic knowledge from neural representations, enhancing interpretability and leveraging pre-existing knowledge
Connectionist learning	Utilizes neural networks to learn patterns from large datasets. Excels in generalization but lacks transparency	NeSy harnesses neural networks for pattern recognition and generalization, while grounding the learned patterns in symbolic representations for improved transparency and reasoning
Hybrid learning	Aims to combine the strengths of symbolic and connectionist approaches, often using separate components for each	NeSy embodies true hybrid learning by deeply integrating symbolic and neural processes within a unified framework, enabling dynamic, bidirectional interaction between symbolic reasoning and neural learning
Reinforcement learning	Involves learning through interaction with an environment and receiving feedback in the form of rewards	NeSy applies reinforcement learning principles to both symbolic and neural components, enabling the system to refine its strategies and knowledge through experience
Unsupervised learning	Focuses on discovering hidden patterns or structures in unlabeled data	In NeSy, unsupervised learning techniques can be used to uncover latent symbolic structures within data, which can then be explicitly represented and manipulated

Table 4 Trade-off between learning and reasoning in symbolic AI and neural networks

Quantification	Symbolic AI		Neural network	
	Reasoning	Learning	Reasoning	Learning
Universal (∀)			Easy	Hard
Existential (∃)	Hard	Easy		

Logic Programming) serves as a prime example of inductive reasoning in neuro-symbolic AI, where a neural network is trained using propositional logic and then used to derive logical programs from the learned representations. MicroPsi [58, 59], CORGI (COmmonsense Reasoning by Instruction) and COMET (COMmonsense Transformers) [81, 82] stand out as a significant contribution toward modeling common-sense reasoning within a neuro-symbolic framework, focusing on cognitive architecture and autonomous motivation, which are essential for common-sense understanding and decision-making. DeepProbLog [75–77] integrates probabilistic logic programming with neural networks, offering a powerful approach to combinatorial reasoning where the system can reason over complex, structured data and learn from uncertain information, making it relevant for tasks that require combinatorial reasoning capabilities.

2.1.4 Decisions

Neuro-symbolic AI advances decision-making by integrating the rapid, intuitive processing akin to Kahneman's System 1 with the deliberate, logical reasoning of System 2 [190]. Table 5 summarizes the two types of decision-making in "Thinking, Fast and Slow" and their relationship to neuro-symbolic AI.

Neuro-symbolic models incorporate neural network components that mimic System 1 thinking by processing sensory data rapidly to produce intuitive responses. These components are adept at recognizing patterns and making quick predictions, similar to the fast and subconscious decision-making observed in humans. For instance, neural learning within NeSy can be trained on large datasets to swiftly identify patterns, akin to how humans rely on heuristics and past experiences for immediate decision-making.

Symbolic components within NeSy frameworks reflect System 2 thinking, employing logical rules and knowledge representation for reasoned analysis and decision-making. This aspect allows NeSy systems to handle complex, structured problems that require careful deliberation and logic. Techniques such as rule-based inference and symbolic manipulation enable NeSy models to perform tasks that necessitate a deep understanding of relationships and concepts, mirroring humans' slow, conscious decision-making process.

The logical neural networks (LNNs) developed by IBM Research [86] embody aspects of System 2 thinking by

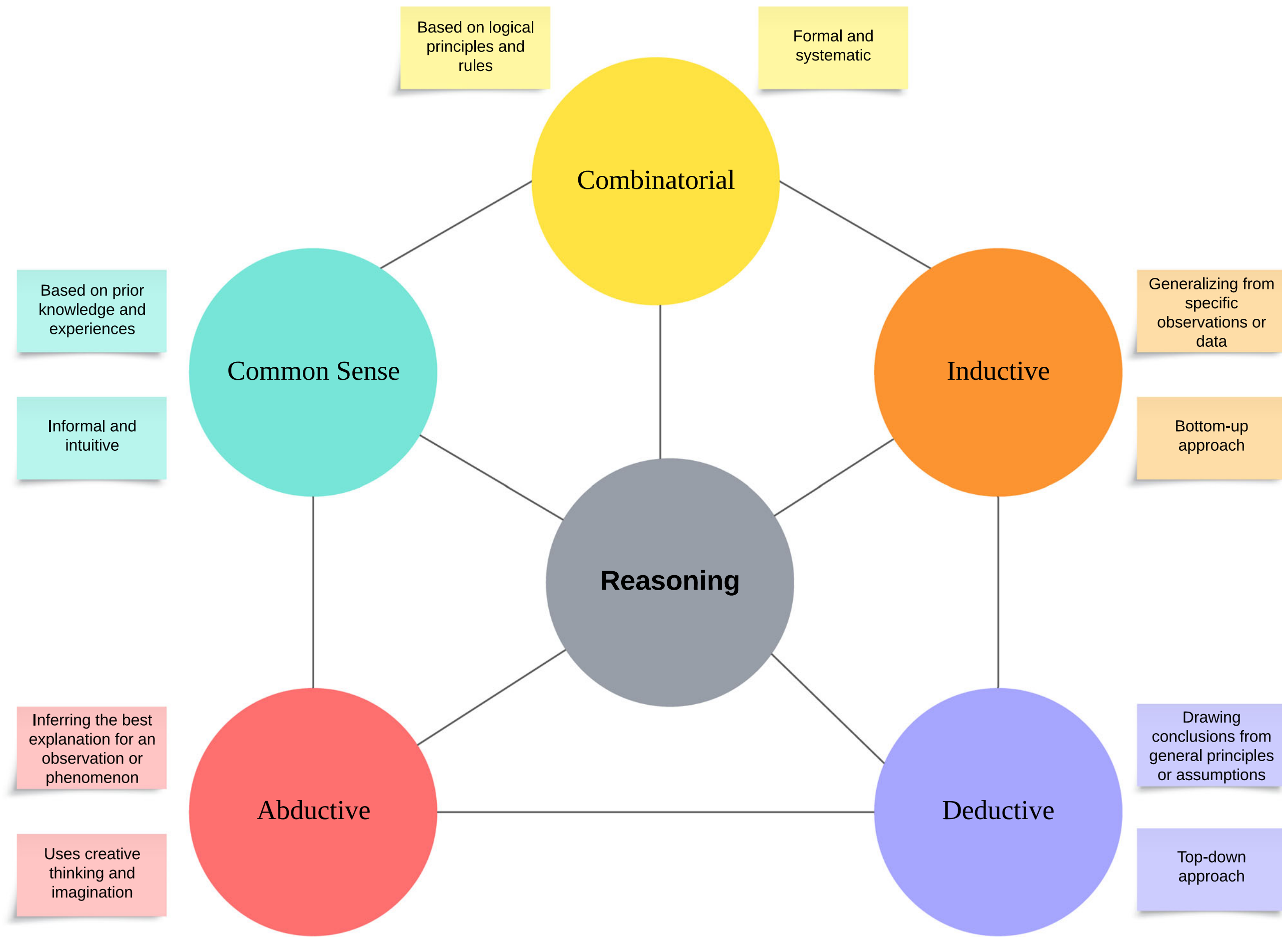

Fig. 6 Different types of reasoning which are not mutually exclusive and can often be used in combination with one another

Table 5 A table summarizing the two types of decision-making in "Thinking, Fast and Slow" and their relationship to neuro-symbolic AI

Type of decision-making	Description	Relationship to neuro-symbolic AI
System 1	Fast, automatic, subconscious decision-making based on heuristics and intuition	Similar to neural learning, where the system is trained on large amounts of data to quickly recognize patterns and make predictions.
System 2	Slow, deliberate, conscious decision-making based on reasoning, analysis, and logic	Similar to symbolic learning, where the system is provided with explicit logical rules and knowledge representation to reason about concepts and relationships.

supporting first-order logic, allowing for the representation of more complex kinds of knowledge in a way that's understandable and can represent uncertainty. LNNs improve predictive accuracy by representing the strengths of relationships between logical clauses via neural weights. They are tolerant of incomplete knowledge, unlike many AI approaches that make closed-world assumptions. This feature enables LNNs to operate under more realistic, open-world assumptions, accommodating incomplete knowledge robustly.

The Neuro-Symbolic Question Answering (NSQA) system [191] is another example where IBM Research has applied NeSy for knowledge-based question answering, requiring advanced reasoning such as multi-hop, quantitative, geographic, and temporal reasoning. The NSQA approach translates natural language questions into an abstract form that captures the conceptual meaning, allowing reasoning over existing knowledge to answer complex questions. This method provides interpretability, generalizability, and robustness, which are critical in enterprise natural language processing settings.

Implementations like Scallop [192], which supports differentiable logical and relational reasoning, and DeepProbLog [75–77], which combines neural networks with probabilistic reasoning, further illustrate the versatility and depth of NeSy approaches in bridging the gap between neural and symbolic architectures. These implementations showcase how NeSy can leverage large-scale learning and symbol manipulation for robust intelligence.

2.1.5 Knowledge and logic

Neuro-symbolic AI synergizes the structured expressiveness of logic with the adaptive learning capabilities of neural networks, fostering systems that excel in reasoning and knowledge representation. Figure 7 gives a pictorial view of such a framework's various kinds of logic.

NeSy architectures frequently employ propositional logic for its simplicity in representing binary relationships and decision processes. First-order logic (FOL), with its ability to quantify individuals, extends this capacity, allowing for more intricate representations of real-world scenarios. Integrating FOL in NeSy facilitates reasoning about entities and their relations, enhancing the system's ability to generalize from specific instances to broader concepts [20, 193].

Higher-order logic (HOL) further expands the expressive power of NeSy systems by enabling quantification over predicates and functions. This allows for the modeling of complex abstractions and relationships, which is pivotal for tasks requiring deep semantic understanding. However, the increased expressiveness of HOL comes with challenges in decidability and computational efficiency, necessitating innovative solutions within NeSy frameworks to harness its potential effectively [29, 194].

Logic is a foundational pillar for knowledge representation in NeSy, providing a formal structure for encoding domain-specific rules and relationships. By mapping logical constructs to neural representations, NeSy systems can leverage the robustness of neural learning while adhering to the precision of logical reasoning. This dual approach not only enhances the system's interpretability but also its adaptability to complex reasoning tasks [31, 195].

Knowledge graphs represent a pivotal component of NeSy, offering a structured and interconnected framework for representing complex knowledge bases. By encapsulating entities, concepts, and their relationships in a graph structure, knowledge graphs enable NeSy systems to perform sophisticated reasoning and inference, drawing on the rich semantic connections encoded within the graph [196, 197].

2.2 Neuro-symbolic: best of both worlds

Neuro-symbolic AI can build more powerful reasoning and learning systems by combining the strengths of deep learning-based methods and symbolic reasoning techniques. However, the key research questions (included in Wikipedia) asked [198] were:

A. What is the best way to integrate neural and symbolic architectures?
B. How should symbolic structures be represented within neural networks and extracted from them?
C. How should common-sense knowledge be learned and reasoned about?
D. How can abstract knowledge that is hard to encode logically be handled?

We now try to find the solutions to these questions in the major algorithms/paradigms/language/frameworks developed for neuro-symbolic artificial intelligence integration

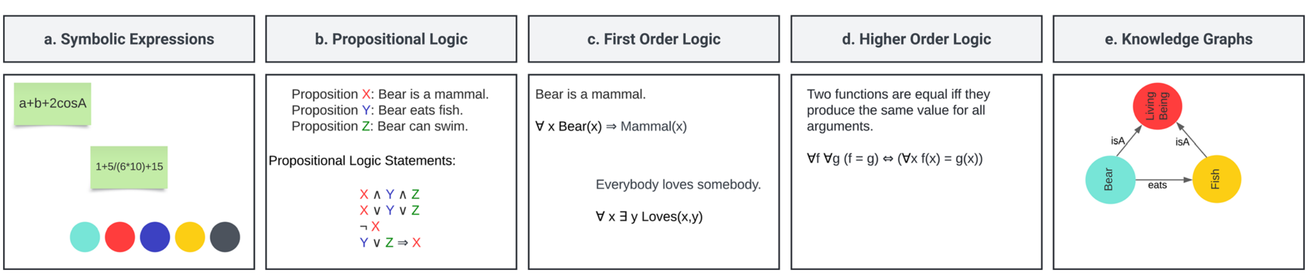

Fig. 7 Various disciplines of logic: a. Symbolic expressions—delving into the language of mathematics and logic, symbolic expressions use variables and operations to represent complex ideas succinctly. For example, 'a+b+2cosA' and '1+5/(6*10)+15' demonstrate how mathematical symbols and functions can encapsulate calculations or relationships. b. Propositional logic—this discipline focuses on forming and analyzing statements that can be either true or false. c. First-order logic—extends propositional logic by incorporating quantifiers and variables that can represent objects in a domain. d. Higher-order logic—builds on first-order logic by allowing functions and predicates to be inputs to other functions and predicates, facilitating more complex expressions of ideas. e. Knowledge graphs—representing complex networks of real-world information, knowledge graphs connect entities (such as individuals, places, and objects) through edges that represent their interrelations

during the last two decades. The summary of these frameworks is in Table 6. From Table 6, we can now cover some discussions based on the four questions posed.

The integration of neural and symbolic architectures has been approached in various innovative ways. Early methods like K_{BANN} [34] and Penalty Logic [35] laid the groundwork by mapping propositional logic and penalty systems onto neural networks, respectively. As the field evolved, more sophisticated frameworks like LTN [62, 66–68] and Tensor Networks [62] emerged, offering richer representations and interactions within neural networks through tensors and differentiable logical languages. More recent advancements like DeepLogic [92] and HRI [93] have focused on simultaneous learning of perception and reasoning, and hierarchical rule induction, showcasing the continuous evolution toward more seamless and efficient integration methods.

The representation and extraction of symbolic structures within neural networks have seen significant advancements. Early models like NSL [38] and CTLK [39, 40] introduced context-free languages and the capability to interpret non-classical logics, respectively. Over time, models like NTP [65] and DeepProbLog [75–77] have enhanced the representation of complex logical structures and probabilistic logic programming within neural networks. These developments highlight a trend toward more expressive and interpretable neuro-symbolic systems capable of embedding and reasoning with intricate symbolic information.

Learning and reasoning about common-sense knowledge have been central to neuro-symbolic AI's evolution. Initial approaches like CIL^2P [36] and SATyrus [41] focused on inductive learning and constraint processing. Later, models like NLM [78] and NSPS [62] demonstrated scalable learning from small to larger tasks and program synthesis, respectively, indicating a growing capability in common-sense reasoning. The introduction of models like CORGI [89] and NSFR [90], which engage in conversational reasoning and forward-chaining reasoning, respectively, showcases the field's progression toward more dynamic and interactive common-sense reasoning systems.

The handling of abstract knowledge has evolved from simpler logic mapping and penalty systems in models like K_{BANN} [34] and Penalty Logic [35] to more complex hierarchical and adaptive systems seen in HRI [93] and DeepLogic [92]. These recent developments demonstrate a significant advancement in neuro-symbolic AI's ability to process, reason, and learn from abstract concepts, moving closer to human-like reasoning capabilities.

2.3 Neuro-symbolic types

2.3.1 Type 1: symbolic neuro-symbolic

In the domain of type 1 neuro-symbolic AI, the interplay between neural networks and symbolic reasoning forms the cornerstone of representation, inference, and learning processes. Here, neural networks are harnessed for their powerful representational learning capabilities, enabling the extraction of nuanced patterns and features from complex data. This is particularly evident in natural language processing, where neural network-based vector embeddings, such as those developed by [199, 200], transform input symbols into rich, continuous vector spaces. These embeddings capture semantic and syntactic relationships inherent in the data, facilitating a broad spectrum of neural network-driven tasks like classification, prediction, and sequence generation.

Conversely, symbolic reasoning within type 1 systems is deployed to imbue these neural representations with structured, logical frameworks. This symbolic layer is pivotal for encoding knowledge, performing deductive reasoning, and ensuring the interpretability of the AI system's operations. It leverages symbols and formal logic to articulate rules and constraints, thereby guiding the decision-making processes in a transparent and explainable manner.

The fusion of neural networks and symbolic reasoning in type 1 neuro-symbolic AI endeavors to marry the adaptive, data-driven insights of neural networks with the clarity and rigor of symbolic logic. This hybrid approach not only enhances the system's ability to process and interpret complex, real-world data but also ensures that its operations remain grounded in logical principles that are comprehensible to human operators.

Figure 8 illustrates this synergistic relationship between neural representation and symbolic logic, highlighting how each contributes to the system's overall functionality. Sequential methodologies within this category, such as language translation or graph categorization, exemplify the application of neural networks for symbolic processing. However, as outlined in Table 7, despite their advancements, these integrations highlight the ongoing challenges in achieving the full potential of neuro-symbolic integration.

2.3.2 Type 2: symbolic [neuro]

Systems of type 2 neuro-symbolic AI employ neural networks as subroutines inside a broader symbolic problem solver; these systems are hybrid but are predominantly symbolic. Loose coupling between the symbolic and neural

Table 6 Major algorithms/paradigms/language/frameworks developed for neuro-symbolic artificial intelligence integration during the last two decades

Authors/Work (Ref)	Year	Question A: Best way to integrate	Question B: Representation and extraction of symbolic structures	Question C: Learning and reasoning about common-sense knowledge	Question D: Handling abstract knowledge
K_{BANN} [34]	1994	Hybrid learning system mapping domain theories onto neural networks	Propositional logic encoded within neural architectures	Utilizes past knowledge for generalization, aiding common-sense reasoning	Demonstrates superior generalization in molecular biology, indicating effective handling of abstract concepts
Penalty Logic [35]	1995	Penalty Logic as an alternative connectionist paradigm for integration	Embeds symbolic structures as penalties within neural networks	Addresses nonmonotonic reasoning and inconsistent beliefs, relevant to common-sense knowledge	Penalty system allows for approximation and reasoning about abstract knowledge
CIL^2P [36]	1999	CIL^2P model based on feed-forward ANN and logic programming	Utilizes a translational technique for embedding propositional logic	Inductive learning from examples and past knowledge supports common-sense reasoning	Logic programming aspect aids in handling abstract knowledge that is logically hard to encode
NSL [38]	2002	Integrates neural and symbolic systems via a context-free language embedded in neural networks	Employs weighted-sum nonlinear thresholded elements for symbolic representation	Facilitates common-sense reasoning through inductive learning and formal language structure	Addresses abstract knowledge using BNF formalism within a neural framework
CTLK [39, 40]	2003	Demonstrates artificial neural networks' capability to interpret and apply non-classical logics, including propositional temporal logic, showcasing an advanced integration method	Neural networks are employed to solve problems like the muddy-children puzzle, indicating a method for embedding and extracting complex logical structures	The ability to reason about new information suggests a pathway for learning and applying common-sense knowledge within neural frameworks	Addresses the challenge of encoding and processing abstract knowledge through the application of temporal-epistemic reasoning within neural networks
SATyrus [41]	2005	SATyrus showcases a neuro-symbolic approach for constraint processing by translating problems into energy functions, indicating a novel integration method	The architecture employs energy functions to represent symbolic constraints within neural networks, facilitating their extraction through global minima solutions	The model's ability to solve complex problems like the traveling salesman problem hints at its capacity for common-sense reasoning and problem-solving	Its approach to expressing problems as energy functions offers a unique way to handle abstract knowledge that is typically challenging to encode logically
NSBL [42]	2005	Neuro-symbolic language for robotics behavior modeling	Action-selection and inference mechanisms for symbolic representation	Adaptive behavior for common-sense reasoning in robotics	Modeling complex behaviors and navigation in robotics
Sathasivam et al. [44–46]	2010	Introduces the Pseudo inverse learning rule for enhancing Hopfield neural network logic programming	Demonstrates an effective method for representing logical functions within neural networks	Enhances the network's capability for inductive learning, relevant for common-sense reasoning	Compares with Hebb Rule and Direct learning rule, showcasing efficiency in handling complex logical constructs
Velik et al. [47]	2010	Introduces a neuro-symbolic network bridging neurological and symbolic levels, offering a unified approach to integration	Proposes neuro-symbolic coding to represent and process multimodal sensory information, facilitating the extraction of symbolic structures from neural data	Explores perceptual learning processes, suggesting a framework for common-sense knowledge acquisition and reasoning based on sensory inputs	Addresses the binding problem in perception, providing insights into handling abstract knowledge through neuro-symbolic interactions

Table 6 (continued)

Authors/Work (Ref)	Year	Question A: Best way to integrate	Question B: Representation and extraction of symbolic structures	Question C: Learning and reasoning about common-sense knowledge	Question D: Handling abstract knowledge
Komendantskaya et al. [48]	2010	Introduced neural networks capable of performing induction, presenting a novel approach to neuro-symbolic computation	Utilized symbol recognizers and recurrent connections for embedding and processing symbolic structures	Explored recursive computing for enhancing common-sense reasoning in neural networks	Demonstrated the neural network's ability to handle complex dependencies, contributing to the management of abstract knowledge
Neurule [49]	2011	Employs neurules derived from training examples or symbolic rule bases, showcasing a method for dynamic integration	Neurules enable efficient updates and interactive inference, illustrating advanced symbolic structure handling within neural frameworks	Enhances reasoning with case-based integration, indicating an approach for incorporating common-sense knowledge	Facilitates adaptive reasoning with diverse knowledge sources, addressing the challenge of managing abstract knowledge
SCTL [55]	2011	Utilizes sequences and counter-examples to integrate temporal logic rules into neural networks, offering a novel approach to neuro-symbolic integration	Employs a nonlinear recurrent network model to represent and extract temporal logic structures, enhancing symbolic representation within neural frameworks	The learning from sequences and system properties facilitates reasoning about common-sense knowledge, particularly in temporal domains	The adaptation of temporal logic rules and model checking into the neural network aids in managing abstract knowledge related to time and system behaviors
NTN [56]	2013	Introduces a method for entity vectors to interact through tensors, enhancing the integration of knowledge bases with neural networks	Employs tensors for rich representation and interaction of entity vectors, enabling the extraction of complex relational information	Utilizes knowledge base reasoning for predicting new entity relationships, indicating a capability for common-sense knowledge inference	Demonstrates high accuracy in classifying unseen relationships, showcasing the model's ability to manage abstract knowledge
Riveret et al. [57]	2015	Integrates probabilistic abstract argumentation with Boltzmann machines, offering a unique approach to neuro-symbolic reasoning	Enables alternative labeling within neural networks, facilitating the representation and extraction of argumentative structures	The probabilistic setup suggests a method for common-sense reasoning through argumentation	Demonstrates the handling of complex argument structures, contributing to the abstraction of knowledge within neural networks
MicroPsi [58]	2015	Explores neuro-symbolic cognitive architecture with a focus on autonomous motivation, bridging cognitive processes with symbolic reasoning	Models complex human-like behaviors and emotions, providing a framework for representing and extracting symbolic structures related to affective states	Utilizes polycyclic motivation and social demands to simulate common-sense reasoning and social interactions	Applies parameters and modulators to capture individual variance and personality traits, offering insights into abstract knowledge representation
Confidence Rules [63]	2016	Introduces a novel method for embedding quantitative ideas in neural networks using confidence criteria	Enhances the representation of deep networks through confidence-based layerwise extraction		Demonstrates the incorporation of historical data into training, suggesting a potential for abstract knowledge handling
Hu et al. [64]	2016	Provides a framework for enhancing neural networks with first-order logic, offering a novel integration approach	Utilizes iterative distillation to embed logic rules into network weights, improving symbolic structure representation		The technique's ability to infuse structured logical information into neural networks suggests a potential for handling abstract knowledge

Table 6 (continued)

Authors/Work (Ref)	Year	Question A: Best way to integrate	Question B: Representation and extraction of symbolic structures	Question C: Learning and reasoning about common-sense knowledge	Question D: Handling abstract knowledge
NTP [65]	2016	Utilizes differentiable backward chaining to integrate logical reasoning within neural networks	Enables the representation and learning of complex logical structures through replacement representations	The application of domain knowledge and canonical rules suggests a method for common-sense reasoning	Facilitates the handling of abstract knowledge by learning logical linkages from minimal data
LTN [66]	2016	Presents LTN as a framework combining neural networks with first-order logic for querying, learning, and reasoning	Utilizes Real Logic, a differentiable logical language, for representing and processing data and knowledge within neural networks	The framework's ability to handle rich data and abstract world knowledge suggests potential for common-sense reasoning applications	LTN's integration of first-order logic and neural computation offers a novel approach to managing abstract knowledge in AI tasks
Tensor networks [62]	2016	Introduces a Neuro-Symbolic Program Synthesis method, enabling autonomous code generation for replicating input–output pairs	Features two novel neural modules: a cross-correlation I/O network and R3NN for program synthesis	Demonstrates program synthesis capability, potentially applicable in learning common-sense reasoning patterns	Leverages context-free grammar rules for constructing parse trees, highlighting a novel approach to abstract knowledge representation
Wang et al. [69]	2017	Introduces DGCC, blending human cognition methods with machine learning for cognitive computing	Employs a multi-granularity approach to represent and process information, enhancing symbolic representation in neural networks		Proposes "hierarchical structuralism" as a new paradigm, potentially advancing the handling of abstract and complex knowledge
Tran et al. [70]	2017	Proposes a method to represent propositional formulas in Restricted Boltzmann Machines (RBMs), simplifying logical implications and Horn clauses representation	Enhances RBMs to handle symbolic structures through a new representation approach		Offers a less complex framework for integrating symbolic knowledge, suggesting potential in handling abstract knowledge
TPRN [72]	2018	Introduces TPRN for interpretable question answering using grammatical concepts without prior linguistic knowledge	Embeds discrete symbol structures within neural networks to represent and process linguistic information	Demonstrates learning of syntax/semantics through task performance, aligning with natural language acquisition theories	Enables deep learning systems to create representations encoding abstract grammatical concepts, bridging the gap between continuous numerical operations and discrete conceptual categories
δILP [73]	2018	Introduces δILP framework for robust logic programming against noisy data, extending beyond traditional ILP capabilities	Embeds logical structures within neural networks to enhance interpretability and reasoning capabilities	Facilitates learning from ambiguous data, suggesting an approach for common-sense knowledge acquisition	Supports data efficiency and generalization, addressing the challenge of encoding abstract knowledge that is hard to encode logically
DeepProbLog [75]	2018	Proposes DeepProbLog, integrating neural networks with probabilistic logic programming for enhanced reasoning	Combines symbolic and sub-symbolic representations, enabling complex logical reasoning within neural architectures	Aids in learning and reasoning with probabilistic models, contributing to the understanding of common-sense knowledge	Showcases the integration of logical reasoning and probabilistic modeling, offering new perspectives on handling abstract knowledge

Table 6 (continued)

Authors/Work (Ref)	Year	Question A: Best way to integrate	Question B: Representation and extraction of symbolic structures	Question C: Learning and reasoning about common-sense knowledge	Question D: Handling abstract knowledge
NLM [78]	2019	Introduces NLM for inductive reasoning and learning, employing logic programming alongside neural networks	Processes objects, attributes, and relations using logic programming within neural frameworks	Demonstrates scalability from small-scale tasks to larger applications, indicating potential for common-sense knowledge learning	Illustrates how neural networks can approximate complex functions, enhancing the handling of abstract knowledge
SGM [79]	2019	Combines deep generative models with neuro-symbolic programs, introducing a programmatic framework for structure expression	Enhances generative models by incorporating global structural expressions		Offers a new perspective on integrating programmatic frameworks with neural models, potentially advancing abstract knowledge representation
KENN [80]	2019	Develops KENN, adding logical constraints to neural network predictions through a Knowledge Enhancer layer	Integrates logical restrictions within neural networks to refine predictions		Facilitates the incorporation of learnable logical constraints, contributing to the discussion on abstract knowledge encoding
COMET [81]	2019	Adapts language models to generate new common-sense knowledge, validated against ATOMIC and ConceptNet databases	Enhances language models with common-sense reasoning capabilities	Demonstrates the generation of accurate common-sense knowledge	Addresses the integration of dynamic, contextually relevant common-sense knowledge into language models
PLANS [83]	2020	Applies hybrid systems to decode decision-making logic from visual narratives, introducing adaptive filtering for neurally inferred specifications	Integrates neural and rule-based reasoning for decision-making logic analysis	Reduces human oversight in understanding decision-making processes in complex scenarios	Innovates in combining neural and symbolic components efficiently for decision-making analysis
∇-FOL [84]	2020	Evaluates VQA models' reasoning using a differentiable first-order logic framework, independent of perception	Incorporates first-order logic for interpretability in reasoning processes		Facilitates the separation of reasoning from perception in VQA models, enhancing interpretability and analytical capabilities
MWS [85]	2020	Explores neuro-symbolic generative models using neural networks for both inference and symbolic data generation, capturing compositional structures	Introduces the MWS algorithm to enhance program induction within learning processes	Utilizes MWS to learn models in complex domains, suggesting an approach for acquiring common-sense knowledge	Focuses on explainability and compositional structure in generative modeling, contributing to abstract knowledge representation
LNN [86]	2020	Presents LNNs that evaluate logical equations, integrating predicate logic within neural frameworks	Enables neural networks to process logical predicates and equations, enhancing symbolic representation	Could facilitate logical reasoning and common-sense knowledge application through neural computation	Advances the field by embedding weighted logical systems within neural networks, addressing abstract reasoning challenges

Table 6 (continued)

Authors/Work (Ref)	Year	Question A: Best way to integrate	Question B: Representation and extraction of symbolic structures	Question C: Learning and reasoning about common-sense knowledge	Question D: Handling abstract knowledge
DLM [88]	2021	Proposes DLM for tackling ILP and RL problems using a neural-logic architecture	Utilizes predicates as weights, enabling a continuous representation of first-order logic programs within neural networks	Demonstrates the application in solving complex problems, implying potential for common-sense reasoning	Introduces a novel method for encoding and processing abstract logical knowledge through gradient descent, enhancing the neuro-symbolic AI domain
CORGI [89]	2021	Introduces a conversational approach for common-sense reasoning using a neuro-symbolic theorem prover	Engages in dialogue using a common-sense knowledge base, enhancing user interaction with AI	Demonstrates the evocation of common-sense knowledge through human speech, suggesting advancements in natural language understanding	Highlights the practical application of neuro-symbolic models in conversational AI, contributing to the field of common-sense reasoning
NSFR [90]	2021	Proposes a novel reasoning method using differentiable forward-chaining based on first-order logic	Transforms raw inputs into probabilistic ground atoms for reasoning, advancing symbolic representation in neural networks	Facilitates seamless deduction of new facts from existing knowledge, aligning with common-sense reasoning paradigms	Enhances the interpretability and flexibility of neuro-symbolic reasoning, pushing the boundaries of abstract knowledge handling
autoBOT [91]	2021	Explores autonomous development of text representations for explainable and efficient AI models	Evolves representations rather than learning them, offering a novel approach to handling symbolic structures		Contributes to the advancement of low-resource, explainable AI models, potentially impacting the representation of abstract knowledge
DeepLogic [92]	2022	Integrates neural perception and logical reasoning in a unified learning process	Utilizes a tree structure and logic operators for sophisticated logical formulations within neural networks	Optimizes mutual supervision signals for simultaneous learning of perception and reasoning	Describes first-order logical formulations, enhancing abstract knowledge handling
HRI [93]	2022	Solves ILP issues with a hierarchical rule induction approach, efficiently integrating neural and symbolic methods	Matches meta-rule facts with body predicates through learned embeddings, representing symbolic structures	Uses a set of generic meta-rules for common-sense knowledge reasoning	Employs controlled noise and interpretability-regularization for abstract knowledge
SenticNet 7 [94]	2022	Utilizes auto-regressive models and kernel methods for generating symbolic representations from text	Transforms real language into a proto-language for symbolic processing	Enhances sentiment analysis with unsupervised, repeatable, and interpretable models	Provides a trustworthy and explainable framework for abstract knowledge representation
ASL [95]	2023	Combines deep learning with abductive logical reasoning for subconcept learning and reasoning	Induces logical hypotheses for subconcept representation and detection in neural networks	Applies meta-interpretive learning for common-sense knowledge acquisition and reasoning	Reduces inconsistency in model outputs, advancing abstract knowledge handling through integrated learning

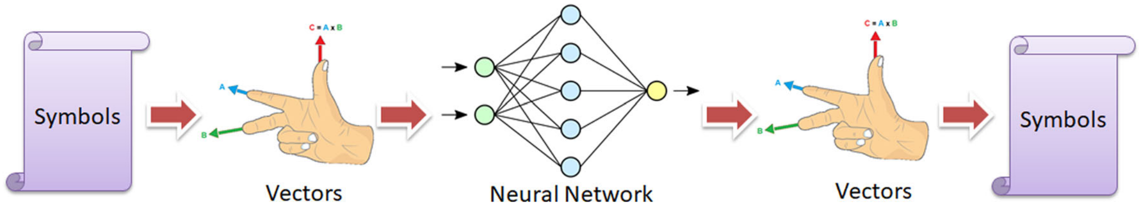

Fig. 8 Neuro-symbolic AI process flow in type 1 systems. Symbols are translated into vector representations, processed through neural networks to capture intricate patterns, and then converted back into symbolic outputs, integrating the adaptability of neural embeddings with the precision of symbolic logic

components is a hallmark of this integrated model type (Fig. 9). System types 2 include models, which use a symbolic stack machine to support recursion and sequence manipulation and a neural network to generate the execution trace. A notable instance of this hybrid approach is AlphaGo [208], which integrates Monte Carlo Tree Search (MCTS) [209] for problem-solving and a neural network for heuristic evaluations, thereby showcasing the potential of combining strategic decision-making processes with neural network-based insights. It's crucial to clarify that while AlphaGo exemplifies the innovative use of neural networks within a decision-making framework, its configuration primarily enhances decision strategies and may not fully encapsulate the traditional neural-symbolic integration aimed at combining deep semantic reasoning with neural computation. Another case in point is a rule-based system that leverages abstract notions recorded by a neural perception module as I/O requirements and is introduced for program synthesis from raw visual observations. The usefulness of combining the skills of symbolic thinking with brain processing for complicated problem-solving tasks is brought to light by type 2 systems. Table 8 shows the properties of some contributions.

2.3.3 Type 3: neuro | symbolic

Type 3 neuro-symbolic AI systems combine neural and symbolic components to improve both aspects' performance. In this setup, the relationship between the neurological and symbolic layers is more cooperative than strictly functional (Fig. 10). Some program synthesis algorithms, for instance, make use of deep learning to produce symbolic programs and rule systems that fulfill high-level task specifications; the interaction between the neural and symbolic components aids in the model's performance. To improve decision-making, symbolic planning is also included in RL in neural-symbolic RL. Similarly, NLProlog [188] and DeepProbLog [75–77] employ neural networks to calculate the probabilities of probabilistic facts and the inference mechanism of ProbLog to compute the required loss gradient, all of which are instances of type 3 systems. In general, type 3 neuro-symbolic AI systems combine the benefits of neural and symbolic techniques to solve difficult problems, as shown in Table 9.

2.3.4 Type 4: neuro-symbolic $\longrightarrow$ neuro

Systems of this fourth kind of integration include symbolic rules and information into the design or training of neural networks (Fig. 11). With the goal of seamlessly integrating symbolic domain information into connectionist architectures, this method has lately acquired traction. They also include tightly coupled but localist neuro-symbolic systems [237–242]. To teach a system in mathematics, for instance, one may use tree representations of equations and meaningful mathematical expressions [243]. Symbolic programs are produced and run by the neural network as completely differentiable operations in Visual Question Answering models [84]. Graph neural networks (GNNs) [244] are being used more recently to include external knowledge bases with entities and relationships. Though some critics claim GNNs' reasoning power is lacking, Kautz classifies such approaches as Type 4. Table 10 shows the properties of some contributions.

2.3.5 Type 5: neuro$_{Symbolic}$

In order to train a neural network, type 5 neuro-symbolic AI systems include symbolic information as soft restrictions into the loss function (tensors) (Fig. 12). The neural network is given the ability to reason with the information thanks to the incorporation of symbolic knowledge into the network weights. Logic tensor networks (LTNs) [62, 66–68] are an example of this method; they use fuzzy relations on real numbers to represent first-order logic equations in neural computing, enabling gradient-based sub-symbolic learning. To cope with approximate rather than accurate reasoning, LTNs soften Boolean first-order logic as soft fuzzy logic. End-to-end training of networks using symbolic knowledge is made possible by LTNs by including logic rules in the network learning aim. When designing classifiers, class hierarchies are used as both the classification targets and the background knowledge. The

Table 7 Collection of papers with neuro-symbolic type 1 and their properties

Paper	Year	Domain	Properties					
			Rep.	Learn.	Reason.	Dec. Mak.	Logic	Neural Typ.
Burattini et al. [201]	2001	Expert Sys.	Loc.	×	Comm.	×	×	×
Hitzler et al. [202]	2003	Logic Prog.	Dist.	Ded.	×	×	×	FF NN
Coraggio et al. [203]	2008	Robotics	Dist.	Ded.	×	×	×	FF NN
Staffa et al. [204]	2011	Robotics	Dist.	Diff. Evol. [205]	×	×	×	FF NN
Hasoon et al. [206]	2013	Op. Sys.	Dist.	Ded.	×	Rule B.	×	ANN
word2vec [199]	2013	QA	Dist.	Grad. Desc.	×	×	×	RNN
Glove [200]	2014	QA	Dist.	Grad. Desc.	×	×	×	RNN
Golovko et al. [207]	2020	Comp. Vis.	Dist.	Ded.	×	Rule B.	×	ANN

Rep. Representation, *Learn.* Learning, *Reason.* Reasoning, *Dec. Mak.* Decision Making, *Logic* Logic Type, *Neural Typ.* Neural Type, *Ded.* Deductive, *Dist.* Distributed, *Loc.* Localist, *FF NN* Feed Forward Neural Network, *ANN* Artificial Neural Network, *RNN* Recurrent Neural Network, *Comm.* Common-sense, *Rule B.* Rule Based, *Grad. Desc.* Gradient Descent

purpose of objective functions in training is to encourage consistency between predictions and the existing class structure. Additional training targets for hierarchical scene parsing are compositional relations over semantic hierarchies. Table 11 shows the properties of some contributions.

2.3.6 Type 6: neuro[symbolic]

Most experts agree that type 6 neuro-symbolic AI has the most promise for bringing together the best features of traditional symbolic AI with modern neural-based AI. A symbolic thinking engine is embedded directly into a neural engine, making this a completely integrated system (Fig. 13). Type 6 methods include a family of algorithms that mimics the logic of tensor calculus to train neural networks to carry out symbolic operations. Their capacity for logical thinking, however, remains low. Kautz argues that type 6 techniques should be able to do combinatorial reasoning since they are computer models of Kahneman's System 1 and System 2, although such a fully fledged system does not exist yet. According to Kautz, no current proper integration method comes close to matching the quality of a Type 6 system. Nevertheless, Type 6 systems could significantly advance AI by bringing together symbolic reasoning and neural networks. Table 12 shows the properties of some contributions claiming to be in type 6.

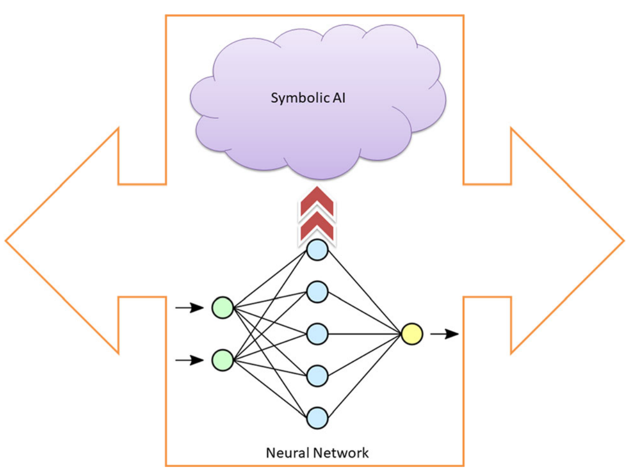

Fig. 9 Integration framework of type 2 neuro-symbolic AI. The diagram illustrates a neural network acting as an intermediary between input/output flows and a symbolic AI system. The neural components provide insight-driven inputs to the symbolic problem solver, characterizing the loosely coupled but predominantly symbolic nature of these systems

3 Applications

The rapid advancement of neuro-symbolic integration in recent years has paved the way for the emergence of a plethora of new applications. Here, we showcase several widely used applications in an effort to spark future innovation across a wider range of use cases.

3.1 Neuro-symbolic AI in robotics

Neuro-symbolic AI is significantly advancing robotics by enabling robots to perform complex tasks previously deemed unattainable, leveraging the fusion of neural network adaptability with the structured logic of symbolic AI. This synergy enhances robots' capabilities to perceive, reason, and act in intricate and unpredictable environments. Notable implementations include robots learning new skills from human demonstrations, translating these into symbolic plans, and reasoning about objects' physical properties and their environmental interactions.

Table 8 Collection of papers with neuro-symbolic type 2 and their properties

Paper	Year	Domain	Properties					Neural Typ.
			Rep.	Learn.	Reason.	Dec. Mak.	Logic	
Neuro-Data-Mine [210]	2000	Medical applications	Dist.	Unsup.	×	×	×	×
Corchado et al. [211]	2001	Oceanography	Dist.	Sup.	Case-B.	×	Prop.	Belief network
Riverola et al. [212]	2002	Oceanography	Dist.	Sup.	Case-B.	×	Prop.	RBF ANN
Neagu et al. [213]	2002	Air Quality	Dist.	Sup.	×	×	×	Basic ANN
Corchado et al. [214]	2003	Oceanography	Dist.	Sup.	Case-B.	×	×	Basic ANN
Fsfrt [215]	2003	Oceanography	Dist.	Sup.	Case-B.	×	Prop.	RBF ANN
Policastro et al. [216]	2003	Mechanics	Dist.	Sup.	Case-B.	×	Prop.	MLP
Fernandez et al. [217]	2004	Biology	Dist.	Unsup.	Case-B.	×	Fuzzy	×
Corchado et al. [218]	2005	Business	Dist.	Sup.	Case-B.	×	×	Basic ANN
Prentzas et al. [50, 219]	2008	UCI [220]	Dist.	Sup.	Case-B.	×	×	Basic ANN
Borrajo et al. [221]	2008	Business	Loc.	Sup.	Case-B.	Rule B.	Prop.	×
Hatzilygeroudis et al. [222, 223]	2011	Business	Loc.	Sup.	Case-B.	Rule B.	Prop.	×
Bach et al. [224]	2015	Minecraft	Dist.	Sup.	×	Rule B.	Prop.	×
Bologna et al. [225]	2017	Computer Vision	Dist.	Sup.	×	Rule B.	Prop.	Deep MLP

Rep. Representation, *Learn.* Learning, *Reason.* Reasoning, *Dec. Mak.* Decision Making, *Logic* Logic Type, *Neural Typ.* Neural Type, *Sup.* Supervised, *Unsup.* Unsupervised, *Case-B.* Case-Based, *Rule B.* Rule Based, *Prop.* Propositional, *Basic ANN* Basic Artificial Neural Network, *RBF ANN* Radial Basis Function Artificial Neural Network, *MLP* Multilayer Perceptron, *Deep MLP* Deep Multilayer Perceptron

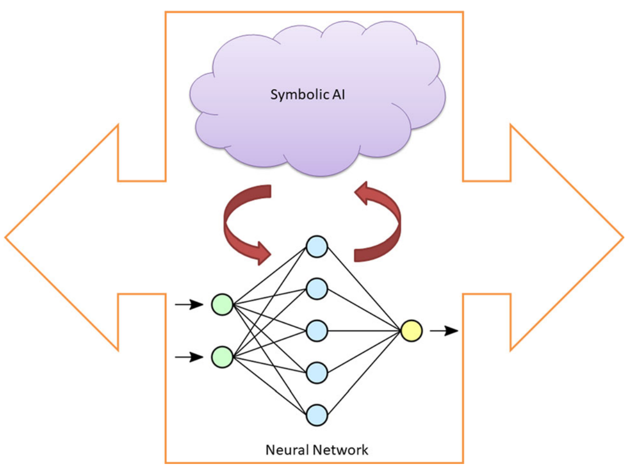

Fig. 10 Dynamic interplay in type 3 neuro-symbolic AI systems. The illustration depicts a cyclical interaction where a neural network and a symbolic AI system operate in a feedback loop, allowing for both procedural learning and logical inference. This structure supports complex tasks like program synthesis, as seen in systems that interpret visual data through neural perception and apply symbolic reasoning for output generation

Coraggio et al. [203] devised a neuro-symbolic system for robot self-localization in minimally sensor-equipped environments, utilizing natural environmental features as landmarks for navigation. This approach blends neural networks' perceptual strengths with symbolic AI's logical reasoning, enabling sophisticated decision-making processes based on landmark detection and encoding.

Staffa et al. [204] explored robotic control by tuning thresholds within a neuro-symbolic network, demonstrating enhanced adaptability and decision-making in behavior-based robotics. The dynamic adjustment of behavior in response to environmental changes showcases the potential of neuro-symbolic approaches in improving robotic autonomy and efficiency.

Coraggio and De Gregorio [229] developed a neuro-symbolic hybrid method for landmark recognition and robot localization, improving landmark detection robustness and robot navigation accuracy in complex settings. This method exemplifies the significant contributions of neuro-symbolic integration to the field of robotics, particularly in spatial awareness and adaptability applications.

An innovative approach to active video surveillance was presented in [230], integrating virtual neural sensors with BDI agents for enhanced system intelligence and reactivity. This integration yields a highly adaptive surveillance system capable of autonomous operation in dynamic environments, highlighting the benefits of combining neural networks' perceptual abilities with symbolic AI's reasoning capabilities.

Kraetzschmar et al. [226] utilized neuro-symbolic integration for environmental modeling in mobile robotics, enabling dynamic and efficient environment representation crucial for navigation and interaction. This approach underscores the importance of combining neural

Table 9 Collection of papers with neuro-symbolic type 3 and their properties

Paper	Year	Domain	Properties					Neural Typ.
			Rep.	Learn.	Reason.	Dec. Mak.	Logic	
Kraetzschmar et al.[226]	2000	Mobile Robotics	Dist.	Sup.	×	×	Prop.	Voronoi
WiSARD [227, 228]	2003	Computer Vision	Dist.	Sup.	×	×	F.O.	Basic ANN
Coraggio et al. [229]	2007	Robotics	Dist.	Sup.	×	×	F.O.	Basic ANN
De Gregorio et al. [230]	2008	Robotics	Dist.	Sup.	Ded.	×	F.O.	Basic ANN
Qadeer et al. [231]	2009	Home Care	Loc.	Sup.	Ded.	Ontology	Prop.	Basic ANN
Dietrich et al. [232]	2009	Robotics	Loc.	Sup.	Ded.	Ontology	Prop.	Basic ANN
Barbosa et al. [233, 234]	2017	Computer Vision	Dist.	Sup.	×	×	F.O.	Basic ANN
Yi et al. [235]	2018	Computer Vision	Dist.	Sup.	×	×	Symbolic	CNN
NLProlog [188]	2019	Question Answering	Dist.	ILP [236]	×	Rule B.	Symbolic	MLP

Rep. Representation, *Learn.* Learning, *Reason.* Reasoning, *Dec. Mak.* Decision Making, *Neural Typ.* Neural Type, *Sup.* Supervised, *Ded.* Deductive, *Prop.* Propositional, *F.O.* First Order, *ILP* Inductive Logic Programming, *CNN* Convolutional Neural Network, *MLP* Multilayer Perceptron

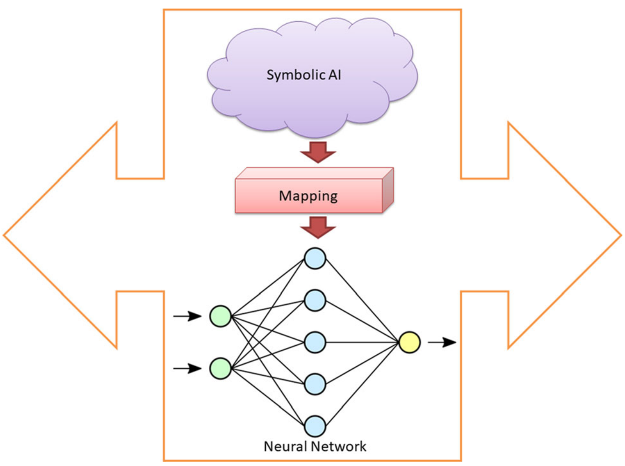

Fig. 11 Type 4 neuro-symbolic AI system with explicit mapping. This figure shows a structure where a distinct mapping layer explicitly connects the symbolic AI component with the neural network. This setup allows for direct translation of symbolic reasoning into neural operations and vice versa, facilitating complex tasks that require tight integration of both symbolic and sub-symbolic processes

adaptability with symbolic reasoning in enhancing robots' real-world operational effectiveness.

The research [131] conducted by Google Inc., ByteDance Inc., and Tsinghua University on the neuro-symbolic Neural Logic Machine (NLM) [78] has demonstrated state-of-the-art methods for solving general application tasks like array sorting, critical path finding, and more intricate tasks such as Blocks World. This approach allows for the application of generalized rules to achieve target results from randomized layouts, showcasing the potential of NeSy in enhancing robotic capabilities.

Moreover, the Neuro-Symbolic Concept Learner (NS-CL) model, designed for the CLEVR dataset [179], represents a significant advancement in the field. It adopts a quasi-symbolic approach, utilizing neural networks for inference and symbolic data for generating logical actions. This method provides a framework for common-sense knowledge acquisition and reasoning based on sensory inputs, thereby offering insights into handling abstract knowledge through neuro-symbolic interactions.

Furthermore, the development of the Neuro-Symbolic Dynamic Reasoning (NS-DR) model, tailored for the CLEVRER video reasoning dataset [280], introduces a neural dynamics predictor. This learned physics engine is crucial for accounting for causal relations in dynamic environments, making it particularly relevant for robotics applications where understanding and predicting physical interactions are key.

These are just a handful of the ways that neuro-symbolic AI is revolutionizing robotics. Several key viewpoints and limitations emerge that future researchers in the field of neuro-symbolic AI in robotics can address:

a. *Environmental complexity and dynamic adaptation* While neuro-symbolic systems like those developed by Coraggio et al. [203] and Staffa et al. [204] have shown promise in navigating and making decisions based on environmental features, the adaptability of these systems to rapidly changing or highly complex environments remains a challenge. Future research could focus on enhancing the robustness and flexibility of neuro-symbolic systems to better cope with unpredictable changes in the environment.

b. *Perception and landmark recognition* The work by Coraggio and De Gregorio [229] on landmark recognition for robot localization points to the need for improved perceptual accuracy and the ability to distinguish between similar features in the environment. Enhancing the perceptual capabilities of neuro-symbolic systems, possibly

Table 10 Collection of papers with neuro-symbolic type 4 and their properties

Paper	Year	Domain	Properties					Neural Typ.
			Rep.	Learn.	Reason.	Dec. Mak.	Logic	
NEURULES [237]	2000	Medical applications	Loc.	LMS	×	Rule B.	Prop.	×
INSS [238]	2001	Monk's Problem [243]	Loc.	Incr.	×	Rule B.	Prop.	Cascade correlation
Garcez et al. [239]	2001	Molecular Biology	Loc.	Ded.	×	Rule B.	Prop.	Basic NN
Prentzas et al. [240]	2002	Intelligent Tutoring	Loc.	Ded.	×	Rule B.	Prop.	Basic NN
Salgado et al. [241]	2003	Neurobiology	Loc.	Ded.	×	Rule B.	Prop.	Basic NN
Omlin et al. [245]	2003	Medical diagnosis	Dist.	Ind.	×	Rule B.	Prop.	Basic NN
Bologna et al. [246]	2003	Medical diagnosis	Dist.	Ind.	×	Rule B.	Prop.	MLP
Obot et al. [247]	2009	Medical diagnosis	Dist.	Sup.	C-B.	Rule B.	Prop.	MLP
Boulahia et al. [248]	2015	UCI [220]	Dist.	Sup.	C-B.	Rule B.	Prop.	Basic NN
Prentzas et al. [52]	2016	Life Insurance	Dist.	Sup.	Neurule	Rule B.	Prop.	Basic NN
Ghosh et al. [249]	2018	Medical applications	Dist.	Sup.	×	Rule B.	Prop.	Basic NN
Bhatia et al. [250]	2018	Code Correction	Dist.	Sup.	Constr.-based	Rule B.	×	RNN
Prentzas et al. [242]	2019	Medical diagnosis	Loc.	Ded.	×	Rule B.	Prop.	Basic NN

Rep. Representation, *Learn.* Learning, *Reason.* Reasoning, *Dec. Mak.* Decision Making, *Logic* Logic Type, *Neural Typ.* Neural Type, *Sup.* Supervised, *Unsup.* Unsupervised, *Case-B.* Case-Based, *Rule B.* Rule Based, *Prop.* Propositional, *Basic ANN* Basic Artificial Neural Network, *RBF ANN* Radial Basis Function Artificial Neural Network, *MLP* Multilayer Perceptron, *RNN* Recurrent Neural Network, *LMS* Least Mean Square, *Incr.* Incremental, *C-B.* Case-Based, *Constr.-based* Constraint-based

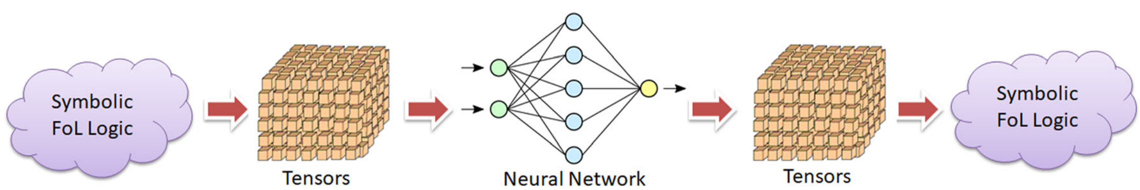

Fig. 12 Type 5 neuro-symbolic AI with tensor-based transformation. This visualization presents the conversion of symbolic first-order logic (FoL) into tensors, processed by a neural network, and then re-converted into symbolic FoL, highlighting a system where symbolic logic is seamlessly integrated with tensorial neural computation

through more advanced neural network architectures or more sophisticated symbolic reasoning mechanisms, could be a valuable area of exploration.

c. *Autonomy in surveillance systems* The integration of virtual neural sensors with BDI agents as explored in [230] highlights the potential for autonomous operation in surveillance systems. However, ensuring these systems can operate with minimal human intervention while making contextually appropriate decisions in dynamic scenarios is an ongoing challenge. Research could delve into optimizing the balance between neural network-driven perception and symbolic agent-driven decision-making to improve autonomy.

d. *Environmental modeling and interaction* Kraetzschmar et al.'s [226] work on environmental modeling underscores the importance of efficient and dynamic environment representation. Future efforts could focus on developing more sophisticated models that account for a wider range of environmental variables and enable more complex interactions between robots and their surroundings.

e. *Generalization and application of rules* The successes of the Neural Logic Machine (NLM) [78] and the Neuro-Symbolic Concept Learner (NS-CL) [179] in applying generalized rules to specific tasks suggest an area for further research in the generalization capabilities of neuro-symbolic systems. Investigating how these systems can learn and apply rules across a broader range of scenarios without significant retraining could enhance their applicability in robotics.

f. *Causal reasoning and physical interactions* The development of the Neuro-Symbolic Dynamic Reasoning (NS-DR) model [280] addresses the need for understanding causal relationships in dynamic environments, which is crucial for robotics. Expanding on this work to include more complex physical interactions and causal mechanisms could improve the predictive and reasoning capabilities of robotic systems.

Table 11 Collection of papers with neuro-symbolic type 5 and their properties

Paper	Year	Domain	Properties					Neural Typ.
			Rep.	Learn.	Reason.	Dec. Mak.	Logic	
Souici et al. [251]	2004	Text Recognition	Dist.	Ded.	Case-B.	Rule B.	Prop.	Basic ANN
Perrier et al. [252]	2005	Autonomous vehicles	Dist.	Sup.	Case-B.	Rule B.	Prop.	Basic ANN
Sanchez et al. [253]	2008	Textiles	Dist.	Incr.	Case-B.	Rule B.	-	Basic ANN
Velik et al. [254]	2010	Computer Vision	Dist.	Incr.	Ded.	×	Prop.	Basic ANN
SHERLOCK [255]	2011	×	Dist.	Ind.	Ded.	×	F.O.	Basic ANN
Saikia et al. [256]	2016	Optimization	Dist.	ILP	Ded.	×	F.O.	DBN
k-il [257]	2019	Medical	Dist.	Ind.	Knowledge Graph	Rule B.	F.O.	LSTM
Khan et al. [258]	2020	Computer Vision	Dist.	Sup.	Knowledge Graph	Rule B.	F.O.	DNN
Kapanipathi et al. [259]	2020	Question Answering	Dist.	Sup.	Knowledge Graph	Rule B.	F.O.	LNN
Neurasp [260]	2020	Computer Vision	Dist.	Unsup.	Common Sense	Rule B.	F.O.	Basic ANN
NSSE [261]	2021	Aircraft Maintenance	Dist.	Sup.	Knowledge Graph	Rule B.	F.O.	LSTM
Stammer et al. [262]	2021	Computer Vision	Dist.	Unsup.	Ded.	Rule B.	F.O.	CNN
Kimura et al. [263]	2021	Question Answering	Dist.	Sup.	Knowledge Graph	Rule B.	F.O.	LNN
Evans et al. [264]	2021	Computer Vision	Dist.	Unsup.	×	Rule B.	Prop.	LSTM
PIGLeT [177]	2021	Question Answering	Dist.	Unsup.	Common Sense	Rule B.	Prop.	LSTM
DUA [265]	2022	Optimization	Dist.	ILP	Inductive	Rule B.	F.O.	×

Rep. Representation, *Learn.* Learning, *Reason.* Reasoning, *Dec. Mak.* Decision Making, *Logic* Logic Type, *Neural Typ.* Neural Type, *Sup.* Supervised, *Ded.* Deductive, *Incr.* Incremental, *Case-B.* Case-Based, *Rule B.* Rule Based, *Prop.* Propositional, *Basic ANN* Basic Artificial Neural Network, *RBF ANN* Radial Basis Function Artificial Neural Network, *MLP* Multilayer Perceptron, *RNN* Recurrent Neural Network, *LMS* Least Mean Square, *ILP* Inductive Logic Programming, *DBN* Deep Belief Network, *LSTM* Long Short-Term Memory, *DNN* Deep Neural Network, *LNN* Logical Neural Network

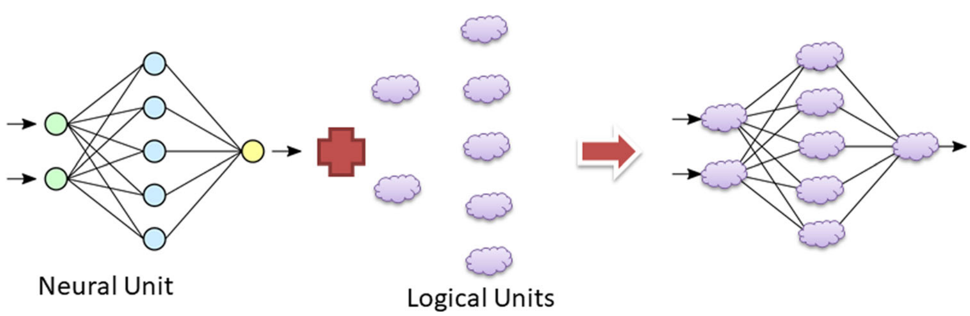

Fig. 13 Type 6 neuro-symbolic AI integration model. The process begins with a neural unit that feeds into a series of logical units, symbolizing the transition from sub-symbolic neural processing to higher-level logical reasoning. This represents an advanced form of integration where the neural network output is not just interpreted but also informs and shapes logical unit operations. This illustration conceptualizes the ideal of a fully integrated system, embedding a symbolic reasoning engine within a neural framework. As proposed by Kautz, it symbolizes the aspiration for a comprehensive AI model capable of both Kahneman's intuitive (System 1) and deliberate (System 2) thinking processes

Addressing these limitations and exploring these viewpoints could significantly advance the field of neuro-symbolic AI in robotics, leading to more capable, adaptable, and intelligent robotic systems.

3.2 Neuro-symbolic AI in question answering

The field of question answering (QA) has seen remarkable advancements through the integration of neuro-symbolic AI, blending the strengths of neural networks' data processing with symbolic AI's logical reasoning. Notably, models like Word2Vec and GloVe have revolutionized word representation, enabling AI systems to understand and process natural language queries more effectively. Mikolov et al.'s work on efficient word representations [199] and Pennington et al.'s development of GloVe [200] have set significant milestones in semantic understanding, essential for interpreting complex questions.

Table 12 Collection of papers with neuro-symbolic type 6 and their properties

Paper	Year	Domain	Properties					Neural Typ.
			Rep.	Learn.	Reason.	Dec. Mak.	Logic	
Alshahrani et al. [266]	2017	Biology	Dist.	Unsup.	K. Graph	Rule B.	F.O.	G. Embed. [267]
Agibetov et al. [268]	2018	Biology	Dist.	Unsup.	K. Graph	Rule B.	F.O.	G. Embed. [269]
Bianchi et al. [270]	2019	DBpedia	Dist.	Unsup.	K. Graph	Rule B.	F.O.	G. Embed. [271]
Oltramari et al. [272]	2019	Question Answering [273]	Dist.	Unsup.	K. Graph	Rule B.	F.O.	G. Embed. [274]
Doldy et al. [275]	2021	Edge Computing	Dist.	Unsup.	K. Graph	Rule B.	F.O.	G. Embed. [276]
Sun et al. [277]	2021	Table Understanding	Dist.	Unsup.	PSL [278]	Rule B.	F.O.	G. Embed. [279]

Rep. Representation, *Learn.* Learning, *Reason.* Reasoning, *Dec. Mak.* Decision Making, *Logic* Logic Type, *Neural Typ.* Neural Type, *Unsup.* Unsupervised, *K. Graph* Knowledge Graph, *Rule B.* Rule Based, *F.O.* First Order, *G. Embed.* Graph Embedding

Further enhancing QA systems, the Neuro-Symbolic Program Synthesis (NSPS) approach [62] exemplifies the seamless integration of symbolic knowledge into neural frameworks, enabling the execution of symbolic programs for query resolution. This method stands out for its performance on benchmark datasets like WikiTableQuestions and Spider, highlighting its efficacy in deriving accurate answers from structured data.

Innovations such as the PIGLeT model by Zellers et al. [177] introduce a novel dimension to QA by grounding language in a 3D world, merging physical common-sense with linguistic understanding. This dual approach, combining a physical dynamics model with a language model, allows for the prediction and verbalization of object interactions, showcasing the model's proficiency in neuro-symbolic interaction.

Research by Weber et al. [188], which integrates Prolog's reasoning with natural language processing, and the comparative study by Ma et al. [281] on common-sense QA, further illustrate the diversity of strategies employed to enhance question understanding and answer generation. These studies underscore the importance of knowledge base compatibility and the integration techniques' role in model performance, advocating for a hybrid approach that leverages both data-driven and knowledge-driven processes for superior reasoning and explainability in AI systems.

Through these pioneering works, the QA domain continues to evolve, with neuro-symbolic AI playing a pivotal role in developing more nuanced, context-aware systems capable of tackling the intricacies of human language and cognition. This fusion has led to more sophisticated natural language understanding and processing, essential for interpreting and responding to complex queries. Some key viewpoints in this domain can be:

a. *Semantic understanding and word representation* The development of models like Word2Vec and GloVe by Mikolov et al. [199] and Pennington et al. [200], respectively, has been instrumental in enhancing semantic understanding in QA systems. Future research could delve into further improving word representation models to capture nuanced linguistic features and contextual meanings, potentially through more advanced and higher dimensional integration of symbolic knowledge.

b. *Symbolic program execution for query resolution* The Neuro-Symbolic Program Synthesis (NSPS) approach introduced by Parisotto et al. [62] exemplifies the successful incorporation of symbolic knowledge into neural frameworks for query resolution. However, extending the applicability of such models to a broader range of natural language queries and diverse datasets remains a challenge, inviting further exploration into adaptable and scalable neuro-symbolic integration techniques.

c. *Grounding language in physical reality* The PIGLeT model by Zellers et al. [177] merges physical common-sense with linguistic understanding, a novel approach in QA. Expanding on this, future work could focus on enhancing the integration of physical dynamics models with language models to improve the prediction and verbalization of complex object interactions, moving toward more holistic neuro-symbolic systems that can reason about both the physical and linguistic aspects of queries.

d. *Knowledge base compatibility and reasoning* Studies such as those by Weber et al. [188] highlight the importance of integrating reasoning capabilities, like those in Prolog, with natural language processing for QA. Enhancing knowledge base compatibility and the techniques for integrating symbolic reasoning into neural models could lead to more accurate and explainable QA systems. Research could explore advanced methods for seamlessly merging data-driven insights with structured knowledge bases to improve reasoning and context-awareness in responses.

e. *Hybrid approaches for enhanced reasoning and explainability* The diversity of strategies employed in the QA domain underscores the potential of hybrid approaches that combine data-driven and knowledge-driven processes. Future research could investigate new methods for leveraging both neural network capabilities and symbolic AI's structured reasoning to create QA systems with superior reasoning, adaptability, and explainability.

Addressing these viewpoints and limitations could significantly advance the field of QA, leading to the development of AI systems that are not only more capable of handling complex queries but also more intuitive and aligned with human cognitive processes.

3.3 Neuro-symbolic AI in medical applications

The medical industry presents a promising landscape for the integration of neuro-symbolic AI, significantly advancing clinical decision support systems. By blending the analytical precision of symbolic AI with the adaptability of neural networks, neuro-symbolic reasoning (NSR) has been effectively employed for more accurate and personalized diagnoses. Research has demonstrated NSR's capability in accurately identifying acute abdominal pain, showcasing its potential in improving diagnostic accuracy [282].

Further, neuro-symbolic integration (NSI) has been applied to electronic health records analysis, combining deep learning with symbolic reasoning to extract actionable insights, potentially enhancing patient care [283]. The Neuro-Data-Mine framework by Ultsch [210] is notable for its efficient transformation of sub-symbolic to symbolic data, crucial for making high-dimensional medical data interpretable. This approach underlines the utility of neuro-symbolic methods in complex tasks like cerebrospinal fluid analysis, emphasizing their role in advancing precision medicine through improved data analysis and intelligibility.

Hybrid formalisms, such as those proposed by Hatzilygeroudis and Prentzas [237], integrate production rules with neural units to streamline knowledge bases, demonstrating improved inference efficiency in medical contexts like bone inflammation diagnosis. This approach highlights the effectiveness of neuro-symbolic systems in managing complex decision-making and pattern recognition tasks, offering superior performance compared to traditional methods.

Omlin and Snyders' work [245] on inductive bias in neural networks, tailored by prior knowledge, showcases the potential of neuro-symbolic approaches in medical analysis, such as breast tissue characterization from magnetic resonance spectroscopy. Bologna's development of the discretized interpretable multi-layer perceptron (DIMLP) [246] furthers the transparency of neural networks in medical diagnostics, enabling rule extraction that aligns with neural network responses and uncovering significant biomarkers for disease classification.

The framework by Obot and Uzoka [247] represents a comprehensive integration of case-based, rule-based, and neural network methodologies, overcoming individual limitations and providing a robust diagnostic tool. This hybrid system has shown strong correlations with conventional neural network results while offering additional explanatory insights, marking a significant step toward explainable and reliable medical AI applications.

The application of neuro-symbolic AI in the medical domain offers promising advancements, particularly in enhancing clinical decision support systems by merging the precision of symbolic AI with the adaptability of neural networks. This integration facilitates more accurate and personalized diagnoses, improving patient care through more insightful analyses of complex medical data. Some key viewpoint might be:

a. *Diagnostic accuracy and personalization:* The capability of neuro-symbolic reasoning (NSR) in precise medical diagnosis, such as the identification of acute abdominal pain, illustrates its potential in refining diagnostic processes [282]. Future research could focus on expanding the range of medical conditions NSR can accurately diagnose, ensuring broader applicability and personalization in patient care.

b. *Interpretability of high-dimensional data* The Neuro-Data-Mine framework by Ultsch [210] emphasizes the importance of transforming sub-symbolic data into a symbolic format to make complex medical data more interpretable. Enhancing these transformation techniques could further improve the clarity and usability of medical data, aiding in more nuanced data analysis and decision-making in healthcare.

c. *Efficiency in knowledge base management* The integration of production rules with neural units, as demonstrated by Hatzilygeroudis and Prentzas [237], showcases the potential for neuro-symbolic systems to streamline knowledge bases and improve inference efficiency in medical diagnostics. Research could explore advanced hybrid formalisms that further optimize knowledge base management and inference processes in medical applications.

d. *Transparency in medical diagnostics* The development of models like the discretized interpretable multi-layer perceptron (DIMLP) by Bologna [246] highlights the need for transparency in neural network-based medical diagnostics. Future efforts could aim at enhancing rule extraction techniques to align more closely with neural network responses, facilitating the identification of critical biomarkers and disease classifications with greater accuracy and interpretability.

e. *Comprehensive diagnostic tools* The comprehensive framework by Obot and Uzoka [247], which combines case-based, rule-based, and neural network methodologies, overcomes the limitations of individual approaches and offers a more robust diagnostic tool. Expanding this integration to incorporate the latest advancements in neural network architectures and symbolic reasoning methods could yield even more powerful and explainable medical diagnostic systems.

Addressing these aspects could significantly advance neuro-symbolic AI's contribution to the medical field, leading to the development of highly effective, transparent, and patient-centric clinical decision support systems.

3.4 Neuro-symbolic AI in computer vision

In the evolving landscape of computer vision, neuro-symbolic AI has emerged as a pivotal force, driving innovations across various domains including object recognition, scene interpretation, and image categorization. The integration of symbolic reasoning with deep learning models, facilitated by approaches like graph neural networks (GNNs) [244], has enabled the embedding of items and relations within external knowledge bases, such as ontologies or knowledge graphs, enhancing the interpretive capabilities of AI systems in understanding complex visual content.

A notable advancement in this field is the Neuro-Symbolic Concept Learner (NS-CL) framework [179], which leverages GNNs to encode the relationships between visual features and their corresponding concepts within a knowledge graph, thereby predicting potential concepts in new images. This framework exemplifies the fusion of sub-symbolic learning with symbolic knowledge, where logical principles are rendered into fuzzy relations using logic tensor networks (LTNs) [62, 66–68], offering a robust mechanism for interpreting visual scenes and reasoning about abstract ideas.

The application of neuro-symbolic AI in computer vision is vividly illustrated in the work of Golovko et al. [207], who developed an intelligent decision support system (IDSS) for enhancing product labeling quality control. This system epitomizes the synergy between deep neural networks, for image localization and recognition, and semantic networks, for intelligent data processing, demonstrating the efficacy of neuro-symbolic approaches in real-world manufacturing environments.

Further enriching the discourse, Bologna and Hayashi [225] explored the transparency of deep learning systems by characterizing symbolic rules within deep discretized interpretable multi-layer perceptrons (DIMLPs). Their work underscores the potential of deep learning models to maintain a balance between accuracy and interpretability, a crucial aspect in the application of AI in sensitive fields such as medical diagnostics.

In the realm of multimedia and language integration, Burattini et al. [227] and Grieco et al. [228] have contributed significantly by exploring the synergy between verbal and visual information and the concept of generating pattern examples from "mental" images, respectively. These studies highlight the multifaceted nature of neuro-symbolic AI in bridging the gap between cognitive reasoning and sensory perception, offering novel insights into pattern recognition and generation.

The neuro-symbolic approach has also been pivotal in spatial-temporal pattern analysis, as demonstrated by Barbosa et al. [233, 234] in their work on GPS trajectory classification. Their methodology exemplifies the integration of neural network adaptability with symbolic AI's structured logic, enhancing the interpretability and computational efficiency of trajectory analysis.

Moreover, the exploration of reasoning, vision, and language understanding by Yi et al. [235] through Neural-Symbolic Visual Question Answering (VQA) and the advancements in multimedia event processing by Khan and Curry [258] further underscore the breadth of neuro-symbolic AI's application in computer vision and beyond.

As the field continues to evolve, the focus on developing sophisticated neuro-symbolic architectures that seamlessly combine the learning process of neural networks with the structured knowledge representation of symbolic systems remains paramount. The future of computer vision lies in creating more adaptable and generalized models that not only mimic human visual capabilities but also encapsulate transparent and comprehensible reasoning processes, bridging the chasm between artificial intelligence and human cognition. Some key viewpoints in this domain can be:

a. *Enhancing interpretive capabilities* The integration of graph neural networks (GNNs) with symbolic reasoning has facilitated the embedding of visual elements and their relationships within external knowledge bases, improving AI systems' ability to understand intricate visual scenes. Future research could focus on refining these integrations to handle more complex, abstract visual concepts and their interrelations.

b. *Predicting concepts in images* The Neuro-Symbolic Concept Learner (NS-CL) framework represents a leap in encoding relationships between visual features and concepts within knowledge graphs. Expanding this framework to encompass a broader array of concepts and visual features could further enhance the predictive accuracy and applicability of neuro-symbolic systems in computer vision.

c. *Real-world application in manufacturing* The intelligent decision support system developed by Golovko et al. [207] exemplifies the practical application of neuro-symbolic AI in enhancing product labeling quality control. Research aimed at extending such systems to other manufacturing domains could revolutionize quality assurance processes across various industries.

d. *Balancing accuracy and interpretability* The work by Bologna and Hayashi [225] on characterizing symbolic rules within deep learning models highlights the importance of maintaining a balance between model accuracy and interpretability. Future efforts could explore novel methodologies to enhance the transparency and explainability of deep learning models without compromising their performance.

e. *Bridging cognitive reasoning and sensory perception* Studies by Burattini et al. [227] and Grieco et al. [228] underline the potential of neuro-symbolic AI in integrating verbal and visual information and generating pattern examples from "mental" images. Advancing these approaches could offer deeper insights into cognitive processes and sensory perception, facilitating more intuitive human-AI interactions.

f. *Spatial-temporal pattern analysis* The methodology employed by Barbosa et al. [233, 234] for GPS trajectory classification demonstrates the effectiveness of combining neural network adaptability with symbolic logic. Further research in this area could enhance the interpretability and efficiency of analyzing spatial-temporal patterns, with broad implications for navigation, urban planning, and environmental monitoring.

g. *Integrating reasoning, vision, and language* The exploration of neuro-symbolic approaches in tasks like Visual Question Answering (VQA) by Yi et al. [235] and multimedia event processing by Khan and Curry [258] showcases the vast potential of neuro-symbolic AI beyond traditional computer vision tasks. Expanding these methodologies to more complex, multimodal interactions could significantly advance AI's cognitive capabilities.

Addressing these aspects could propel the field of computer vision forward, leading to the development of AI systems that not only emulate human visual and cognitive abilities but also offer transparent and understandable reasoning processes, narrowing the gap between artificial intelligence and human-like cognition.

3.5 Neuro-symbolic AI in programming and optimization

The science of computer programming and optimization has greatly benefited from the integration of neuro-symbolic AI. The objective of program synthesis is to generate programs that fulfill a specified task, a challenge that remains to be fully automated. Neuro-symbolic AI techniques, which combine symbolic reasoning with neural network models, have shown promise in overcoming this challenge, enabling effective program synthesis for tasks such as sorting or searching algorithms [62]. Moreover, neuro-symbolic AI extends to enhancing software efficiency, where optimizations are discovered by blending symbolic reasoning with insights derived from neural network training on program execution patterns.

In the domain of programming and optimization, Bhatia, Kohli, and Singh [250] introduced a groundbreaking neuro-symbolic program corrector tailored for introductory programming assignments. This tool harnesses both neural networks and symbolic AI to identify and rectify errors in student-submitted code, providing an automated and intelligent feedback system that enhances the learning experience for programming novices. The neuro-symbolic approach not only detects syntactic errors but also grasps the semantic intent behind the code, ensuring corrections are accurate and contextually relevant.

Sen et al. [87] present a novel approach to inductive logic programming (ILP) by integrating it with logical neural networks (LNNs), offering a neuro-symbolic ILP framework that merges ILP's structured reasoning with the adaptability of LNNs. This combination facilitates the extraction and refinement of logical rules from data, marking a significant advancement in AI, particularly in programming and optimization.

Chaudhuri et al. [19] delve into neuro-symbolic programming, highlighting the fusion of neural networks with symbolic programming paradigms to address the limitations of purely data-driven or rule-based systems. This synthesis represents a pivotal shift toward creating more adaptable, interpretable, and robust AI systems in the programming and optimization domain.

Yin and Neubig [284] introduce a syntactic neural model for general-purpose code generation, leveraging structural patterns in programming languages to generate code from natural language descriptions. This advancement holds significant promise for automating coding tasks and bridging the gap between natural language processing and software engineering.

Ritchie et al. [285] explore the application of neuro-symbolic models in computer graphics, addressing the challenges of generating, rendering, and manipulating graphical content. This novel integration promises to revolutionize computer graphics by introducing more intelligent and adaptable systems.

Reddy and Balasubramanian [286] explore estimating treatment effects using Neuro-Symbolic Program Synthesis, offering a nuanced understanding of treatment efficacy and potentially transforming fields such as healthcare and policy analysis.

Li, Huang, and Naik [287] introduce "Scallop," a language designed for neuro-symbolic programming, aiming to bridge the gap between neural and symbolic computing paradigms and facilitate the development of neuro-symbolic applications.

Varela's doctoral dissertation [288] investigates the impact of hybrid neural networks on meta-learning objectives, shedding light on the potential of hybrid networks to enhance the efficiency and effectiveness of meta-learning processes.

Mundhenk et al. [289] explore symbolic regression via neural-guided genetic programming, aiming to enhance the efficiency and accuracy of symbolic regression tasks by leveraging the strengths of neural networks.

Chen et al. [290] embark on the symbolic discovery of optimization algorithms, signifying a pivotal shift toward automating the design of optimization algorithms and potentially accelerating the advancement of AI and computational sciences.

The infusion of neuro-symbolic AI into programming and optimization heralds a promising horizon, marked by enhanced learning tools, innovative problem-solving methodologies, and a deeper understanding of complex systems. While strides have been made, the journey toward fully realizing the potential of neuro-symbolic AI continues, with future research poised to tackle the remaining challenges of scalability, interpretability, and the seamless integration of neural and symbolic systems. The key points from the programming and optimization domain can be consolidated into broader themes to capture the essence of current achievements and future directions:

a. *Advancements in program synthesis and software optimization* The progress in automating program synthesis, exemplified by neuro-symbolic techniques [62], and the strides in enhancing software efficiency underscore the potential of neuro-symbolic AI in transforming software development practices. Future research could aim to extend these methodologies to more complex and diverse programming tasks, further automating and optimizing software development processes.

b. *Improving programming education and software development* Innovations such as the neuro-symbolic program corrector [250] highlight the potential for AI to significantly impact programming education by providing more nuanced error detection and correction. Extending these tools to accommodate a wider range of programming languages and complexities could revolutionize learning experiences and software development workflows.

c. *Expanding the scope of neuro-symbolic integration* The work in inductive logic programming [87], neuro-symbolic programming paradigms [19], and dedicated neuro-symbolic programming languages [287] demonstrates the evolving landscape of neuro-symbolic AI. Future efforts could focus on developing sophisticated frameworks and languages that ease the integration of neural and symbolic components, enhancing AI's adaptability and interpretability across various applications.

d. *Cross-disciplinary applications and innovations* The exploration of neuro-symbolic AI in fields such as computer graphics [285] and healthcare [286] illustrates its versatile applicability. Research aimed at exploring and expanding neuro-symbolic AI's capabilities in diverse domains could unlock new possibilities for innovative applications, from digital media to precision medicine.

e. *Automating the design of optimization algorithms* The initiative to automate the discovery of optimization algorithms [290] opens up new research avenues in making AI systems more efficient and autonomous. Investigating autonomous methods for identifying and implementing optimizations could lead to breakthroughs in computational efficiency and AI model performance.

By focusing on these consolidated themes, future research in neuro-symbolic AI within the programming and optimization domain can address existing challenges and unlock new potentials, paving the way for more intelligent, efficient, and user-friendly AI systems.

4 Challenges

The subject of neuro-symbolic AI is expanding quickly, thanks to its ability to integrate deep learning methods with symbolic reasoning to produce more robust and versatile AI systems. There are, however, obstacles that must be overcome before its full potential may be tapped. The following are some of the major obstacles facing neuro-symbolic AI:

Integration of deep learning and symbolic reasoning A critical challenge lies in the effective amalgamation of neural and symbolic components, a task that requires innovative architectural designs and learning paradigms. The question of how to seamlessly integrate these components without diluting their respective strengths remains open. Works like the Neuro-Symbolic Concept Learner (NS-CL) and Logical Tensor Networks (LTNs) offer promising directions, yet the quest for a universally efficient integration strategy continues. This challenge is compounded by the need for sophisticated representation schemes that can encapsulate symbolic structures within the fluidity of neural architectures, ensuring that the extracted symbolic knowledge retains its logical integrity and is amenable to rigorous reasoning processes.

Need of a spatial-temporal explainable learning and reasoning framework Developing frameworks that can interpret and reason about spatial-temporal data with transparency, as highlighted by the need for explainable neuro-symbolic AI in applications like smart city management and environmental monitoring, is paramount. Innovations such as CIL^2P [36] and NSL [38] showcase strides toward this goal, yet the quest for fully explainable and generalizable systems persists. The integration of graph neural networks (GNNs) with symbolic reasoning mechanisms offers a pathway to imbue AI systems with an enhanced understanding of spatial-temporal dynamics, pertinent to domains such as environmental modeling and autonomous navigation. The endeavor to refine these frameworks, extending their applicability and accuracy, stands as a crucial frontier in neuro-symbolic AI research.

Data quality and bias The quality and representativeness of training data are crucial across domains. Biases inherent in the data can lead to skewed AI models, making the development of comprehensive and unbiased datasets, as well as algorithms capable of identifying and correcting for bias, a universal challenge.

Human–machine collaboration Enhancing interfaces and methodologies to foster effective human-AI collaboration is vital. While frameworks like NSBL [42] and NTN [56] have made progress, creating systems that intuitively integrate human insights and AI capabilities remains a broad challenge.

Representation and handling of abstract knowledge The ability to represent and reason about abstract knowledge, a theme recurrent in works from neuro-symbolic cognitive architectures like MicroPsi [58] to logic-enhanced models like LTN [66], is a critical hurdle. Expanding AI's capacity to manage abstract concepts through novel neuro-symbolic integrations is essential for advancing AI's cognitive capabilities.

Ethical considerations As neuro-symbolic AI continues to evolve, it is imperative to address the ethical challenges that accompany its development and application. The integration of neural networks with symbolic reasoning introduces complex ethical dimensions that warrant careful consideration.

Neuro-symbolic AI systems, by leveraging the strengths of both neural networks and symbolic AI, have the potential to address complex problems with a high degree of interpretability and adaptability. However, the integration of these two paradigms introduces complexities in identifying and mitigating biases. Neural networks, known for their capacity to learn from vast datasets, may inadvertently encode and amplify existing biases within the data, leading to decisions that can perpetuate societal inequalities. Symbolic AI, while providing a framework for logical reasoning and interpretability, relies on the premises and rules defined by humans, which can also be a source of bias [291, 292].

The literature emphasizes the importance of transparency, fairness, and accountability in AI systems to address these challenges. For instance, the concept of "algorithmic auditing" has been proposed as a means to scrutinize and evaluate the ethical implications of AI algorithms, including those used in neuro-symbolic systems. This process involves a thorough examination of the algorithms' decision-making processes, data sources, and outcomes to identify potential biases and ensure that the systems operate within ethical boundaries [293].

Moreover, the development of interpretable models is advocated to enhance the transparency of AI systems, making it easier to understand how decisions are made and on what basis. This is particularly relevant for neuro-symbolic AI, where the rationale behind decisions should be accessible and understandable to users, especially in high-stakes domains such as healthcare, criminal justice, and public policy [292].

Addressing the ethical challenges of bias and fairness in neuro-symbolic AI also involves considering the broader societal impacts of these technologies. The potential for reinforcement of existing social inequalities through biased decision-making underscores the need for ethical frameworks that prioritize inclusivity, equity, and justice. Engaging with diverse perspectives and disciplines can provide a more comprehensive understanding of the social implications of neuro-symbolic AI and guide the development of more ethical and fair AI systems [294, 295].

Finally, the effects of neuro-symbolic AI on the labor market are a source of worry. Ethical concerns regarding the social effect and the necessity for retraining and education is raised as technology develops and threatens human jobs in specific sectors. Concerns about the morality of developing and deploying neuro-symbolic AI must be addressed if the technology is to be utilized for the greater good of society. "Neuro-symbolic AI should ensure transparency by making decision-making processes understandable, uphold accountability through clear delineation of responsibility for decisions, maintain fairness by actively mitigating biases in data and algorithms, protect privacy by safeguarding personal data, and adhere to non-maleficence by preventing harm and ensuring the benefits of AI applications outweigh potential risks."

As we navigate the future of neuro-symbolic AI, a multidisciplinary approach that amalgamates insights from cognitive science, computer science, and ethics is paramount. The exploration of novel integration strategies, advanced representation techniques, and ethical frameworks will be instrumental in realizing the full potential of neuro-symbolic AI across its diverse applications. The journey ahead, while fraught with challenges, holds the promise of transformative breakthroughs that could redefine the paradigms of artificial intelligence in an array of domains.

5 Conclusion

As this article has shown, neuro-symbolic AI is gaining traction in the area of AI as it seeks to integrate the best features of both symbolic reasoning and connectionist learning. Throughout this study, we have covered the representation, learning, reasoning, and decision-making aspects of neuro-symbolic AI. Robotics, question answering, healthcare, computer vision, and programming are just a few of the areas where neuro-symbolic AI has found success. The limits and difficulties of neuro-symbolic AI, including its scalability, explainability, and ethical implications, have also been examined. There is still a long way to go, but neuro-symbolic AI shows promise for creating AI systems with human-level intelligence and resemblance.

Acknowledgments This work is supported by the "ADI 2022" project funded by the IDEX Paris-Saclay, ANR-11-IDEX-0003-02.

Author contributions Conceptualisation, B.P.B., A.R.C., T.P.S. and R.T.; methodology, B.P.B., A.R.C. and R.T.; software, B.P.B., A.R.C. and R.T.; validation, B.P.B., A.R.C., T.P.S. and R.T.; formal analysis, B.P.B., A.R.C. and R.T.; investigation, B.P.B., A.R.C. and R.T.; resources, B.P.B., A.R.C. and R.T.; data curation, B.P.B., A.R.C. and R.T.; writing—original draft preparation, B.P.B.; writing—review and editing, B.P.B., T.P.S., A.R.C. and R.T.; visualisation, B.P.B.; supervision, A.R.C. and R.T.; project administration, B.P.B., A.R.C., T.P.S. and R.T.; funding acquisition, R.T. All authors have read and agreed to the published version of the manuscript.

Funding This research received no external funding.

Availability of data and materials Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Code availability Not applicable.

Declarations

Conflict of interest The authors declare no conflict of interest

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication The authors consent to the publication of this work.

References

1. Helm JM, Swiergosz AM, Haeberle HS, Karnuta JM, Schaffer JL, Krebs VE, Spitzer AI, Ramkumar PN (2020) Machine learning and artificial intelligence: definitions, applications, and future directions. Curr Rev Musculoskelet Med 13:69–76
2. Hassan AM, Rajesh A, Asaad M, Nelson JA, Coert JH, Mehrara BJ, Butler CE (2023) Artificial intelligence and machine learning in prediction of surgical complications: current state, applications, and implications. Am Surg 89(1):25–30
3. Novakovsky G, Dexter N, Libbrecht MW, Wasserman WW, Mostafavi S (2023) Obtaining genetics insights from deep learning via explainable artificial intelligence. Nat Rev Genet 24(2):125–137
4. Jebamikyous H, Li M, Suhas Y, Kashef R (2023) Leveraging machine learning and blockchain in e-commerce and beyond: benefits, models, and application. Discov Artif Intell 3(1):3
5. Rawat W, Wang Z (2017) Deep convolutional neural networks for image classification: a comprehensive review. Neural Comput 29(9):2352–2449
6. Bond-Taylor S, Leach A, Long Y, Willcocks CG (2021) Deep generative modelling: a comparative review of vaes, gans, normalizing flows, energy-based and autoregressive models. IEEE Trans Pattern Anal Mach Intell
7. Shakarami A, Ghobaei-Arani M, Shahidinejad A (2020) A survey on the computation offloading approaches in mobile edge computing: a machine learning-based perspective. Comput Netw 182:107496
8. Li B, Qi P, Liu B, Di S, Liu J, Pei J, Yi J, Zhou B (2023) Trustworthy ai: From principles to practices. ACM Comput Surv 55(9):1–46
9. Augusto LM (2021) From symbols to knowledge systems: A. Newell and Ha Simon's contribution to symbolic ai
10. Newell A (1980) Physical symbol systems. Cogn Sci 4(2):135–183
11. Newell A (1982) The knowledge level. Artif Intell 18(1):87–127
12. Uschold M, Gruninger M (1996) Ontologies: principles, methods and applications. knowl Eng Rev 11(2):93–136
13. Reed SK, Pease A (2017) Reasoning from imperfect knowledge. Cogn Syst Res 41:56–72
14. Youheng Z (2023) A historical review and philosophical examination of the two paradigms in artificial intelligence research. Eur J Artif Intell Mach Learn 2(2):24–32
15. Wermter S, Sun R An overview of hybrid neural systems. Subseries of Lecture Notes in Computer Science Edited by JG Carbonell and J. Siekmann, 1
16. Garcez ASd, Broda KB, Gabbay DM Neural-symbolic learning systems foundations and applications
17. Hammer B, Hitzler P (2007) Perspectives of neural-symbolic integration vol 77
18. Sun R, Alexandre F (2013) Connectionist-symbolic integration: from unified to hybrid approaches
19. Chaudhuri S, Ellis K, Polozov O, Singh R, Solar-Lezama A, Yue Y (2021) Neurosymbolic programming. Found Trends® Program Lang 7(3):158–243
20. Hitzler P, Eberhart A, Ebrahimi M, Sarker MK, Zhou L (2022) Neuro-symbolic approaches in artificial intelligence. Natl Sci Rev 9(6):035
21. Velik R (2008) A bionic model for human-like machine perception
22. Gallagher K (2018) Request confirmation networks: a cortically inspired approach to neuro-symbolic script execution. PhD thesis, Harvard University
23. Martin LJ (2021) Neurosymbolic automated story generation. PhD thesis, Georgia Institute of Technology
24. Corchado JM, Aiken J (2002) Hybrid artificial intelligence methods in oceanographic forecast models. IEEE Trans Syst Man Cybern Part C (Appl Rev) 32(4):307–313
25. Hatzilygeroudis I, Prentzas J (2004) Neuro-symbolic approaches for knowledge representation in expert systems. Int J Hybrid Intell Syst 1(3–4):111–126

26. Öztürk P, Tidemann A (2014) A review of case-based reasoning in cognition-action continuum: a step toward bridging symbolic and non-symbolic artificial intelligence. Knowl Eng Rev 29(1):51–77
27. Besold TR, Garcez Ad, Bader S, Bowman H, Domingos P, Hitzler P, Kühnberger K-U, Lamb LC, Lowd D, Lima PMV et al (2017) Neural-symbolic learning and reasoning: a survey and interpretation. arXiv preprint arXiv:1711.03902
28. Garnelo M, Shanahan M (2019) Reconciling deep learning with symbolic artificial intelligence: representing objects and relations. Curr Opin Behav Sci 29:17–23
29. Garcez Ad, Gori M, Lamb LC, Serafini L, Spranger M, Tran SN (2019) Neural-symbolic computing: an effective methodology for principled integration of machine learning and reasoning. arXiv preprint arXiv:1905.06088
30. De Raedt L, Dumančić S, Manhaeve R, Marra G (2020) From statistical relational to neuro-symbolic artificial intelligence. arXiv preprint arXiv:2003.08316
31. Sarker MK, Zhou L, Eberhart A, Hitzler P (2021) Neuro-symbolic artificial intelligence. AI Commun 34(3):197–209
32. Wang W, Yang Y (2022) Towards data-and knowledge-driven artificial intelligence: a survey on neuro-symbolic computing. arXiv preprint arXiv:2210.15889
33. Garcez Ad, Lamb LC (2023) Neurosymbolic ai: the 3rd wave. Artif Intell Rev 56:1–20
34. Towell GG, Shavlik JW (1994) Knowledge-based artificial neural networks. Artif intell 70(1–2):119–165
35. Pinkas G (1995) Reasoning, nonmonotonicity and learning in connectionist networks that capture propositional knowledge. Artif Intell 77(2):203–247
36. Avila Garcez AS, Zaverucha G (1999) The connectionist inductive learning and logic programming system. Appl Intell 11:59–77
37. França MV, Zaverucha G, Garcez AS (2014) Fast relational learning using bottom clause propositionalization with artificial neural networks. Mach Learn 94:81–104
38. Burattini E, De Gregorio M, Francesco A (2002) Nsl: a neuro-symbolic language for monotonic and non-monotonic logical inferences. In: SBRN, pp 256–261
39. Garcez A, Lamb L (2003) Reasoning about time and knowledge in neural symbolic learning systems. In: Advances in neural information processing systems, vol 16
40. Garcez ASd, Lamb LC (2006) A connectionist computational model for epistemic and temporal reasoning. Neural Comput 18(7):1711–1738
41. Lima PMV, Morveli-Espinoza MM, Pereira GC, Franga F (2005) Satyrus: a sat-based neuro-symbolic architecture for constraint processing. In: Fifth international conference on hybrid intelligent systems (HIS'05). IEEE, p 6
42. Burattini E, Datteri E, Tamburrini G (2005) Neuro-symbolic programs for robots. In: Proceedings of NeSy, vol 5
43. Burattini E, De Gregorio M, Rossi S (2010) An adaptive oscillatory neural architecture for controlling behavior based robotic systems. Neurocomputing 73(16–18):2829–2836
44. Sathasivam S, Velavan M (2010) Neuro symbolic integration using pseudo inverse rule. In: Annual international conference on advance topics in artificial intelligence, Phuket, Thailand
45. Sathasivam S (2011) Learning rules comparison in neuro-symbolicintegration. Int J Appl Phys Math 1(2):129
46. Sathasivam S (2012) Applying different learning rules in neuro-symbolic integration. In: Advanced materials research, vol 433. Trans Tech Publ, pp 716–720
47. Velik R (2010) The neuro-symbolic code of perception. J Cogn Sci 11(2):161–180
48. Komendantskaya E, Broda K, Garcez A (2010) Using inductive types for ensuring correctness of neuro-symbolic computations
49. Prentzas J, Hatzilygeroudis I (2011) Neurules-a type of neuro-symbolic rules: an overview. Springer, Berlin, pp 145–165
50. Prentzas J, Hatzilygeroudis I (2011) Efficiently merging symbolic rules into integrated rules
51. Hatzilygeroudis I, Prentzas J (2015) Symbolic-neural rule based reasoning and explanation. Expert Syst Appl 42(9):4595–4609
52. Prentzas J, Hatzilygeroudis I (2016) Assessment of life insurance applications: an approach integrating neuro-symbolic rule-based with case-based reasoning. Expert Syst 33(2):145–160
53. Sreelekha S (2018) Neurosymbolic integration with uncertainty. Ann Math Artif Intell 84(3–4):201–220
54. Prentzas J, Hatzilygeroudis I (2018) Using clustering algorithms to improve the production of symbolic-neural rule bases from empirical data. Int J Artif Intell Tools 27(02):1850002
55. Borges RV, Garcez Ad, Lamb LC (2011) Learning and representing temporal knowledge in recurrent networks. IEEE Trans Neural Netw 22(12):2409–2421
56. Socher R, Chen D, Manning CD, Ng A (2013) Reasoning with neural tensor networks for knowledge base completion. In: Advances in neural information processing systems, vol 26
57. Riveret R, Pitt JV, Korkinof D, Draief M (2015) Neuro-symbolic agents: Boltzmann machines and probabilistic abstract argumentation with sub-arguments. In: AAMAS, pp 1481–1489
58. Bach J (2015) Modeling motivation in micropsi 2. In: Artificial general intelligence: 8th international conference, AGI 2015, AGI 2015, Berlin, Germany, July 22-25, 2015, Proceedings 8. Springer, pp 3–13
59. Bach J (2009) Principles of synthetic intelligence psi: an architecture of motivated cognition, vol 4
60. Varadarajan KM, Vincze M (2015) Affordance and k-tr augmented alphabet based neuro-symbolic language-af-ktraans-a human-robot interaction meta-language. In: 2015 20th international conference on methods and models in automation and robotics (MMAR). IEEE, pp 394–399
61. Abubakar H, Masanawa SA, Yusuf S (2020) Neuro-symbolic integration of hopfield neural network for optimal maximum random ksatisfiability (maxrksat) representation. J Reliab Stat Stud 13:199–220
62. Parisotto E, Mohamed A-r, Singh R, Li L, Zhou D, Kohli P (2016) Neuro-symbolic program synthesis. arXiv preprint arXiv:1611.01855
63. Tran SN, Garcez ASd (2016) Deep logic networks: Inserting and extracting knowledge from deep belief networks. IEEE Trans Neural Netw Learn Syst 29(2):246–258
64. Hu Z, Ma X, Liu Z, Hovy E, Xing E (2016) Harnessing deep neural networks with logic rules. arXiv preprint arXiv:1603.06318
65. Rocktäschel T, Riedel S (2016) Learning knowledge base inference with neural theorem provers. In: Proceedings of the 5th workshop on automated knowledge base construction, pp 45–50
66. Serafini L, Garcez AS (2016) Learning and reasoning with logic tensor networks. In: AI* IA 2016 advances in artificial intelligence: XVth international conference of the Italian association for artificial intelligence, Genova, Italy, November 29–December 1, 2016, Proceedings XV. Springer, pp 334–348
67. Manigrasso F, Miro FD, Morra L, Lamberti F (2021) Faster-ltn: a neuro-symbolic, end-to-end object detection architecture. In: Artificial neural networks and machine learning–ICANN 2021: 30th international conference on artificial neural networks, Bratislava, Slovakia, September 14–17, 2021, Proceedings, Part II 30. Springer, pp 40–52
68. Badreddine S, Garcez Ad, Serafini L, Spranger M (2022) Logic tensor networks. Artif Intell 303:103649
69. Wang G (2017) Dgcc: data-driven granular cognitive computing. Granular Comput 2(4):343–355

70. Tran SN (2017) Propositional knowledge representation and reasoning in restricted boltzmann machines. arXiv preprint arXiv:1705.10899
71. Cohen WW, Yang F, Mazaitis KR (2017) Tensorlog: Deep learning meets probabilistic dbs. arXiv preprint arXiv:1707.05390
72. Palangi H, Smolensky P, He X, Deng L (2018) Question-answering with grammatically-interpretable representations. In: Proceedings of the AAAI conference on artificial intelligence, vol 32
73. Evans R, Grefenstette E (2018) Learning explanatory rules from noisy data. J Artif Intell Res 61:1–64
74. Minervini P, Bošnjak M, Rocktäschel T, Riedel S, Grefenstette E (2020) Differentiable reasoning on large knowledge bases and natural language. In: Proceedings of the AAAI conference on artificial intelligence, vol 34, pp 5182–5190
75. Manhaeve R, Dumancic S, Kimmig A, Demeester T, De Raedt, L (2018) Deepproblog: neural probabilistic logic programming. In: Advances in neural information processing systems, vol 31
76. De Raedt L, Manhaeve R, Dumancic S, Demeester T, Kimmig A (2019) Neuro-symbolic= neural+ logical+ probabilistic. In: NeSy'19@ IJCAI, the 14th international workshop on neural-symbolic learning and reasoning
77. Manhaeve R, De Raedt L, Kimmig A, Dumancic S, Demeester T (2019) Deepproblog: integrating logic and learning through algebraic model counting. In: KR2ML Workshop@ Neurips'19, Vancouver, Canada
78. Dong H, Mao J, Lin T, Wang C, Li L, Zhou D (2019) Neural logic machines. arXiv preprint arXiv:1904.11694
79. Young H, Bastani O, Naik M (2019) Learning neurosymbolic generative models via program synthesis. In: International conference on machine learning. PMLR, pp 7144–7153
80. Daniele A, Serafini L (2019) Knowledge enhanced neural networks. In: PRICAI 2019: trends in artificial intelligence: 16th Pacific Rim international conference on artificial intelligence, Cuvu, Yanuca Island, Fiji, August 26–30, 2019, Proceedings, Part I 16. Springer, pp 542–554
81. Bosselut A, Rashkin H, Sap M, Malaviya C, Celikyilmaz A, Choi Y (2019) Comet: Commonsense transformers for automatic knowledge graph construction. arXiv preprint arXiv:1906.05317
82. Bosselut A, Le Bras R, Choi Y (2021) Dynamic neuro-symbolic knowledge graph construction for zero-shot commonsense question answering. In: Proceedings of the AAAI conference on artificial intelligence, vol 35, pp 4923–4931
83. Dang-Nhu R (2020) Plans: Neuro-symbolic program learning from videos. Adv Neural Inf Process Syst 33:22445–22455
84. Amizadeh S, Palangi H, Polozov A, Huang Y, Koishida K (2020) Neuro-symbolic visual reasoning: Disentangling. In: International conference on machine learning. PMLR, pp 279–290
85. Hewitt L, Le TA, Tenenbaum J (2020) Learning to learn generative programs with memoised wake-sleep. In: Conference on uncertainty in artificial intelligence. PMLR, pp 1278–1287
86. Riegel R, Gray A, Luus F, Khan N, Makondo N, Akhalwaya IY, Qian H, Fagin R, Barahona F, Sharma U, et al (2020) Logical neural networks. arXiv preprint arXiv:2006.13155
87. Sen P, Carvalho BW, Riegel R, Gray A (2022) Neuro-symbolic inductive logic programming with logical neural networks. In: Proceedings of the AAAI conference on artificial intelligence, vol. 36, pp 8212–8219
88. Zimmer M, Feng X, Glanois C, Jiang Z, Zhang J, Weng P, Dong L, Jianye H, Wulong L (2021) Differentiable logic machines. arXiv preprint arXiv:2102.11529
89. Arabshahi F, Lee J, Gawarecki M, Mazaitis K, Azaria A, Mitchell T (2021) Conversational neuro-symbolic commonsense reasoning. In: Proceedings of the AAAI conference on artificial intelligence, vol 35, pp 4902–4911
90. Shindo H, Dhami DS, Kersting K (2021) Neuro-symbolic forward reasoning. arXiv preprint arXiv:2110.09383
91. Škrlj B, Martinc M, Lavrač N, Pollak S (2021) autobot: evolving neuro-symbolic representations for explainable low resource text classification. Mach Learn 110:989–1028
92. Duan X, Wang X, Zhao P, Shen G, Zhu W (2022) Deeplogic: Joint learning of neural perception and logical reasoning. IEEE Trans Pattern Anal Mach Intell
93. Glanois C, Jiang Z, Feng X, Weng P, Zimmer M, Li D, Liu W, Hao J (2022) Neuro-symbolic hierarchical rule induction. In: International conference on machine learning, PMLR, pp 7583–7615
94. Cambria E, Liu Q, Decherchi S, Xing F, Kwok K (2022) Senticnet 7: A commonsense-based neurosymbolic ai framework for explainable sentiment analysis. In: Proceedings of the thirteenth language resources and evaluation conference, pp 3829–3839
95. Han Z, Cai L-W, Dai W-Z, Huang Y-X, Wei B, Wang W, Yin Y (2023) Abductive subconcept learning. Sci China Inf Sci 66(2):1–13
96. Wermter S, Sun R (2001) The present and the future of hybrid neural symbolic systems some reflections from the nips workshop. AI Mag 22(1):123–123
97. Kelley TD (2003) Symbolic and sub-symbolic representations in computational models of human cognition: what can be learned from biology? Theory Psychol 13(6):847–860
98. Rapaport WJ (2003) How to pass a turing test: Syntactic semantics, natural-language understanding, and first-person cognition. The Turing test: the elusive standard of artificial intelligence, 161–184
99. Bader S, Hitzler P, Hölldobler S (2004) The integration of connectionism and first-order knowledge representation and reasoning as a challenge for artificial intelligence. arXiv preprint cs/0408069
100. Pugeda TGS III (2005) Artificial intelligence and ethical reflections from the catholic church. Intelligence 26(4):53
101. Ray O, Garcez AS (2006) Towards the integration of abduction and induction in artificial neural networks. In: Proceedings of the ECAI, vol 6. Citeseer, pp 41–46
102. Rawbone P, Paor P, Ware JA, Barrett J (2006) Interactive causation: a neurosymbolic agent. In: IC-AI. Citeseer, pp 51–55
103. Velik R, Bruckner D (2008) euro-symbolic networks: introduction to a new information processing principle. In: 2008 6th IEEE international conference on industrial informatics. IEEE, pp 1042–1047
104. Kühnberger K-U, Gust H, Geibel P (2008) erspectives of neuro–symbolic integration–extended abstract–. In: Dagstuhl Seminar Proceedings. Schloss Dagstuhl-Leibniz-Zentrum für Informatik
105. Kühnberger K-U, Geibel P, Gust H, Krumnack U, Ovchinnikova E, Schwering A, Wandmacher T (2008) Learning from inconsistencies in an integrated cognitive architecture. Front Artif Intell Appl 171:212
106. Haikonen PO (2009) The role of associative processing in cognitive computing. Cogn Comput 1:42–49
107. Prentzas J, Hatzilygeroudis I (2009) Combinations of case-based reasoning with other intelligent methods. Int J Hybrid Intell Syst 6(4):189–209
108. Garcez AS (2010) eurons and symbols: a manifesto. In: Dagstuhl Seminar Proceedings. Schloss Dagstuhl-Leibniz-Zentrum fÃ1/4r Informatik
109. Velik R (2010) Why machines cannot feel. Mind Mach 20(1):1–18
110. Bruckner D, Velik R, Penya Y (2011) Machine perception in automation: a call to arms. EURASIP J Embed Syst 2011:1–9

111. POli R (2012) Discovery of symbolic, neuro-symbolic and neural networks with parallel. In: Artificial neural nets and genetic algorithms: proceedings of the international conference in Norwich, UK, 1997. Springer, p 419
112. Velik R (2013) Brain-like artificial intelligence for automation–foundations, concepts and implementation examples. BRAIN 4(1–4):26–54
113. Achler T (2013) Neural networks that perform recognition using generative error may help fill the "neuro-symbolic gap''. Biol Inspired Cogn Archit 3:6–12
114. Lima PM (2017) Q-satyrus: Mapping neuro-symbolic reasoning into an adiabatic quantum computer. In: NeSy
115. Shen S, Ramesh S, Shinde S, Roychoudhury A, Saxena P (2018) Neuro-symbolic execution: The feasibility of an inductive approach to symbolic execution. arXiv preprint arXiv:1807.00575
116. Lieto A, Lebiere C, Oltramari A (2018) The knowledge level in cognitive architectures: current limitations and possible developments. Cogn Syst Res 48:39–55
117. Wang P (2004) Toward a unified artificial intelligence. In: AAAI Technical Report (1), p 83
118. Hammer P (2019) Adaptive neuro-symbolic network agent. Springer, Berlin, pp 80–90
119. Sittón I, Alonso RS, Hernández-Nieves E, Rodríguez-Gonzalez S, Rivas A (2019) Neuro-symbolic hybrid systems for industry 4.0: a systematic mapping study. In: Knowledge management in organizations: 14th international conference, KMO 2019, Zamora, Spain, July 15–18, 2019, Proceedings 14. Springer, pp 455–465
120. Marcus G (2020) The next decade in ai: four steps towards robust artificial intelligence. arXiv preprint arXiv:2002.06177
121. Hameed HA (2020) Artificial intelligence: What it was, and what it should be? Int J Adv Comput Sci Appl 11(6)
122. Belle V (2020) Symbolic logic meets machine learning: a brief survey in infinite domains. In: Scalable uncertainty management: 14th international conference, SUM 2020, Bozen-Bolzano, Italy, September 23–25, 2020, Proceedings 14. Springer, pp 3–16
123. Tiddi I (2020) Directions for explainable knowledge-enabled systems. Knowledge Graphs for eXplainable Artificial intelligence: Foundations Applications and Challenges 47:245
124. Hanson D, Imran A, Vellanki A, Kanagaraj S (2020) A neuro-symbolic humanlike arm controller for sophia the robot. arXiv preprint arXiv:2010.13983
125. Franklin NT, Norman KA, Ranganath C, Zacks JM, Gershman SJ (2020) Structured event memory: a neuro-symbolic model of event cognition. Psychol Rev 127(3):327
126. Di Maio P (2020) Neurosymbolic knowledge representation for explainable and trustworthy ai
127. Anderson G, Verma A, Dillig I, Chaudhuri S (2020) Neurosymbolic reinforcement learning with formally verified exploration. Adv Neural Inf Process Syst 33:6172–6183
128. Gaur M, Kursuncu U, Sheth A, Wickramarachchi R, Yadav S (2020) Knowledge-infused deep learning. In: Proceedings of the 31st ACM conference on hypertext and social media, pp 309–310
129. Santoro A, Lampinen A, Mathewson K, Lillicrap T, Raposo D (2021) Symbolic behaviour in artificial intelligence. arXiv preprint arXiv:2102.03406
130. Ebrahimi M, Eberhart A, Bianchi F, Hitzler P (2021) Towards bridging the neuro-symbolic gap: deep deductive reasoners. Appl Intell 51:6326–6348
131. Susskind Z, Arden B, John LK, Stockton P, John EB (2021) Neuro-symbolic ai: An emerging class of ai workloads and their characterization. arXiv preprint arXiv:2109.06133
132. Alonso RS (2021) Deep symbolic learning and semantics for an explainable and ethical artificial intelligence. In: Ambient intelligence–software and applications: 11th international symposium on ambient intelligence. Springer, pp 272–278
133. Park K-W, Bu S-J, Cho S-B (2021) Evolutionary optimization of neuro-symbolic integration for phishing url detection. In: Hybrid artificial intelligent systems: 16th international conference, HAIS 2021, Bilbao, Spain, September 22–24, 2021, Proceedings 16. Springer, pp 88–100
134. Oltramari A, Francis J, Ilievski F, Ma K, Mirzaee R (2021) Generalizable neuro-symbolic systems for commonsense question answering, 294–310
135. Calvaresi D, Ciatto G, Najjar A, Aydoğan R, Torre L, Omicini A, Schumacher M (2021) Expectation: personalized explainable artificial intelligence for decentralized agents with heterogeneous knowledge. In: Explainable and transparent AI and multi-agent systems: third international workshop, EXTRAAMAS 2021, Virtual Event, May 3–7, 2021, Revised Selected Papers 3. Springer, pp 331–343
136. Nye M, Tessler M, Tenenbaum J, Lake BM (2021) Improving coherence and consistency in neural sequence models with dual-system, neuro-symbolic reasoning. Adv Neural Inf Process Syst 34:25192–25204
137. Gaur M, Gunaratna K, Bhatt S, Sheth A (2022) Knowledge-infused learning: a sweet spot in neuro-symbolic ai. IEEE Internet Comput 26(4):5–11
138. Samsonovich AV (2022) One possibility of a neuro-symbolic integration. In: Biologically inspired cognitive architectures 2021: proceedings of the 12th annual meeting of the BICA Society. Springer, pp 428–437
139. Dold D, Soler Garrido J, Caceres Chian V, Hildebrandt M, Runkler T (2022) Neuro-symbolic computing with spiking neural networks. In: Proceedings of the international conference on neuromorphic systems 2022, pp 1–4
140. Chitnis R, Silver T, Tenenbaum JB, Lozano-Perez T, Kaelbling LP (2022) Learning neuro-symbolic relational transition models for bilevel planning. In: 2022 IEEE/RSJ international conference on intelligent robots and systems (IROS). IEEE, pp 4166–4173
141. Kocoń J, Baran J, Gruza M, Janz A, Kajstura M, Kazienko P, Korczyński W, Miłkowski P, Piasecki M, Szołomicka J (2022) Neuro-symbolic models for sentiment analysis. In: Computational science–ICCS 2022: 22nd international conference, London, UK, June 21–23, 2022, Proceedings, Part II. Springer, pp 667–681
142. Alon U, Xu F, He J, Sengupta S, Roth D, Neubig G (2022) Neuro-symbolic language modeling with automaton-augmented retrieval. In: International conference on machine learning. PMLR, pp 468–485
143. Amado LR, Pereira RF, Meneguzzi FR (2023) Robust neuro-symbolic goal and plan recognition. In: Proceedings of the 37th AAAI conference on artificial intelligence (AAAI), 2023, Estados Unidos
144. Hitzler P, Roth-Berghofer T, Rudolph S (2007) Foundations of artificial intelligence faint-07 workshop at ki 2007. In: Workshop at KI, vol 2007. Citeseer
145. Garcez AS, Lamb LC, Gabbay DM (2008) Neural-symbolic cognitive reasoning
146. Komendantskaya E, Broda K, Garcez ASd (2010) Neuro-symbolic representation of logic programs defining infinite sets. ICANN (1) 6352:301–304
147. Andreasik J, Ciebiera A, Umpirowicz S, Speretta M, Gauch S, Lakkaraju P, Alessandrelli D, Pagano P, Nastasi C, Petracca M et al (2010) Hsi 2010 conference programme may 13
148. Barcelona CS, Garcez Ad, Lamb L Seventh international workshop on neural-symbolic learning and reasoning

149. Hatzilygeroudis I, Prentzas J (2011) Combinations of intelligent methods and applications. Springer, Berlin
150. Achler T (2012) Towards bridging the gap between pattern recognition and symbolic representation within neural networks. In: Workshop on neural-symbolic learning and reasoning, AAAI-2012. Citeseer
151. Garcez A, Gori M, Hitzler P, Lamb LC (2015) Neural-symbolic learning and reasoning (dagstuhl seminar 14381). In: Dagstuhl Reports, vol. 4. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik
152. Hatzilygeroudis I, Palade V (2016) 6thinternational workshop on combinations of intelligent methods and applications (cima 2016)
153. Hatzilygeroudis I, Palade V, Prentzas J (2017) Advances in combining intelligent methods
154. Hatzilygeroudis I, Palade V (2018) Advances in hybridization of intelligent methods
155. Hammer P, Agrawal P, Goertzel B, Iklé M (2019) Artificial general intelligence: 12th international conference, AGI 2019, Shenzhen, China, August 6–9, 2019, Proceedings, vol 11654. Springer
156. Shen S, Shinde S, Ramesh S, Roychoudhury A, Saxena P (2019) Neuro-symbolic execution: Augmenting symbolic execution with neural constraints. In: NDSS
157. Averkin A (2019) Hybrid intelligent systems based on fuzzy logic and deep learning. Artificial Intelligence: 5th RAAI Summer School, Dolgoprudny, Russia, July 4–7, 2019, Tutorial Lectures, 3–12
158. Pisano G, Ciatto G, Calegari R, Omicini A (2020) Neuro-symbolic computation for xai: Towards a unified model. In: WOA, vol 1613, p 101
159. Alam M, Groth P, Hitzler P, Paulheim H, Sack H, Tresp V (2020) Cssa'20: workshop on combining symbolic and sub-symbolic methods and their applications. In: Proceedings of the 29th ACM international conference on information & knowledge management, pp 3523–3524
160. Benzmüller C, Lomfeld B (2020) Reasonable machines: a research manifesto. In: KI 2020: advances in artificial intelligence: 43rd German conference on AI, Bamberg, Germany, September 21–25, 2020, Proceedings 43. Springer, pp 251–258
161. Ilkou E, Koutraki M (2020) Symbolic vs sub-symbolic ai methods: Friends or enemies? In: CIKM (Workshops)
162. Singh G, Mondal S, Bhatia S, Mutharaju R (2021) Neuro-symbolic techniques for description logic reasoning (student abstract). In: Proceedings of the AAAI conference on artificial intelligence, vol 35, pp 15891–15892
163. Branco R, Branco A, Silva JM, Rodrigues J (2021) Commonsense reasoning: how do neuro-symbolic and neuro-only approaches compare? In: CIKM Workshops
164. Basu K, Murugesan K, Atzeni M, Kapanipathi P, Talamadupula K, Klinger T, Campbell M, Sachan M, Gupta G (2021) A hybrid neuro-symbolic approach for text-based games using inductive logic programming. Combining learning and reasoning: programming languages, formalisms, and representations
165. Garcez Ad, Jiménez-Ruiz E (2021) Neural-symbolic learning and reasoning (nesy)
166. Saha A, Joty S, Hoi SC (2022) Weakly supervised neuro-symbolic module networks for numerical reasoning over text. In: Proceedings of the AAAI conference on artificial intelligence, vol 36, pp 11238–11247
167. Ahmed K, Teso S, Chang K-W, Broeck G, Vergari A (2022) Semantic probabilistic layers for neuro-symbolic learning. Adv Neural Inf Process Syst 35:29944–29959
168. Bader S, Hitzler P (2005) Dimensions of neural-symbolic integration—a structured survey. arXiv preprint arXiv:cs/0511042
169. Kautz H (2022) The third ai summer: Aaai Robert S. Engelmore memorial lecture. AI Mag 43(1):105–125
170. Browne A, Sun R (2001) Connectionist inference models. Neural Netw 14(10):1331–1355
171. Cloete I, Zurada JM (2000) Knowledge-based neurocomputing
172. Hamilton K, Nayak A, Božić B, Longo L (2022) Is neuro-symbolic ai meeting its promises in natural language processing? a structured review. Semantic Web (Preprint), 1–42
173. Yu D, Yang B, Liu D, Wang H, Pan S (2023) A survey on neural-symbolic learning systems. Neural Netw
174. Yang C, Chaudhuri S (2022) Safe neurosymbolic learning with differentiable symbolic execution. arXiv preprint arXiv:2203.07671
175. Shah A, Zhan E, Sun J, Verma A, Yue Y, Chaudhuri S (2020) Learning differentiable programs with admissible neural heuristics. Adv Neural Inf Process Syst 33:4940–4952
176. Barbin A, Cerutti F, Gerevini AE (2022) Addressing the symbol grounding problem with constraints in neuro-symbolic planning
177. Zellers R, Holtzman A, Peters M, Mottaghi R, Kembhavi A, Farhadi A, Choi Y (2021) Piglet: language grounding through neuro-symbolic interaction in a 3d world. arXiv preprint arXiv:2106.00188
178. Borghesani V, Piazza M (2017) The neuro-cognitive representations of symbols: the case of concrete words. Neuropsychologia 105:4–17
179. Mao J, Gan C, Kohli P, Tenenbaum JB, Wu J (2019) The neuro-symbolic concept learner: Interpreting scenes, words, and sentences from natural supervision. arXiv preprint arXiv:1904.12584
180. Cunnington D, Law M, Lobo J, Russo A (2024) The role of foundation models in neuro-symbolic learning and reasoning. arXiv preprint arXiv:2402.01889
181. De Mántaras RL (1991) A distance-based attribute selection measure for decision tree induction. Mach Learn 6:81–92
182. Valiant LG (1984) Deductive learning. Philos Trans R Soc Lond Ser A Math Phys Sci 312(1522):441–446
183. Tiddi I, Schlobach S (2022) Knowledge graphs as tools for explainable machine learning: a survey. Artif Intell 302:103627
184. Arulkumaran K, Deisenroth MP, Brundage M, Bharath AA (2017) Deep reinforcement learning: a brief survey. IEEE Signal Process Mag 34(6):26–38
185. Sutton RS, Barto AG (2018) Reinforcement learning: an introduction
186. Sætre AS, Ven A (2021) Generating theory by abduction. Acad Manag Rev 46(4):684–701
187. Al-Ajlan A (2015) The comparison between forward and backward chaining. Int J Mach Learn Comput 5(2):106
188. Weber L, Minervini P, Münchmeyer J, Leser U, Rocktäschel T (2019) Nlprolog: reasoning with weak unification for question answering in natural language. arXiv preprint arXiv:1906.06187
189. Zhang B, Zhu J, Su H (2023) Toward the third generation artificial intelligence. Sci China Inf Sci 66(2):1–19
190. SKahneman D (2013) Thinking, fast and slow
191. Kapanipathi P, Abdelaziz I, Ravishankar S, Roukos S, Gray A, Astudillo R, Chang M, Cornelio C, Dana S, Fokoue A, et al (2020) Leveraging abstract meaning representation for knowledge base question answering. arXiv preprint arXiv:2012.01707
192. Huang J, Li Z, Chen B, Samel K, Naik M, Song L, Si X (2021) Scallop: From probabilistic deductive databases to scalable differentiable reasoning. Adv Neural Inf Process Syst 34:25134–25145
193. Smullyan RM (1995) First-order logic
194. Andrews PB (2013) An introduction to mathematical logic and type theory: to truth through proof, vol 27
195. Garcez Ad, Bader S, Bowman H, Lamb LC, Penning L, Illuminoo B, Poon H, Zaverucha CG (2022) Neural-symbolic

learning and reasoning: a survey and interpretation. Neuro-Symb Artif Intell State Art 342(1):327
196. Ehrlinger L, Wöß W (2016) Towards a definition of knowledge graphs. SEMANTiCS (Posters, Demos, SuCCESS) 48(1–4):2
197. Ji S, Pan S, Cambria E, Marttinen P, Philip SY (2021) A survey on knowledge graphs: representation, acquisition, and applications. IEEE Trans Neural Netw Learn Syst 33(2):494–514
198. Sun R (2002) Hybrid systems and connectionist implementationalism. Encyclop Cogn Sci 1:697–703
199. Mikolov T, Chen K, Corrado G, Dean J (2013) Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781
200. Pennington J, Socher R, Manning CD (2014) Glove: Global vectors for word representation. In: Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP), pp 1532–1543
201. Burattini E, De Gregorio M, Tamburrin G (1999) Pictorial and verbal components in artificial intelligence explanations. In: Vision: the approach of biophysics and neurosciences: proceedings of the international school of biophysics, Casamicciola, Napoli, Italy, 11-16 October 1999, vol 11, p 471
202. Hitzler P, Seda AK (2003) Continuity of semantic operators in logic programming and their approximation by artificial neural networks. In: KI 2003: advances in artificial intelligence: 26th annual German conference on AI, KI 2003, Hamburg, Germany, September 15-18, 2003. Proceedings 26. Springer, pp 355–369
203. Coraggio P, De Gregorio M, Forastiere M (2008) Robot navigation based on neurosymbolic reasoning over landmarks. Int J Pattern Recognit Artif Intell 22(05):1001–1014
204. Staffa M, Rossi S, De Gregorio M, Burattini E (2011) Thresholds tuning of a neuro-symbolic net controlling a behavior-based robotic system. In: ESANN
205. Price KV (2013) Differential evolution. Handbook of Optimization: From Classical to Modern Approach, 187–214
206. Hasoon SO, Jasim YA (2013) Diagnosis windows problems based on hybrid intelligence systems. J Eng Sci Technol 8(5):566–578
207. Golovko V, Kroshchanka A, Kovalev M, Taberko V, Ivaniuk D (2020) Neuro-symbolic artificial intelligence: application for control the quality of product labeling. In: Open semantic technologies for intelligent system: 10th international conference, OSTIS 2020, Minsk, Belarus, February 19–22, 2020, Revised Selected Papers. Springer, pp 81–101
208. Wang F-Y, Zhang JJ, Zheng X, Wang X, Yuan Y, Dai X, Zhang J, Yang L (2016) Where does alphago go: from church-turing thesis to alphago thesis and beyond. IEEE/CAA J Autom Sin 3(2):113–120
209. Świechowski M, Godlewski K, Sawicki B, Mańdziuk J (2023) Monte Carlo tree search: a review of recent modifications and applications. Artif Intell Rev 56(3):2497–2562
210. Ultsch A (2000) The neuro-data-mine. In: Symposia on neural computation (NC'2000), Berlin, Germany
211. Corchado JM, Lees B (2001) Adaptation of cases for case based forecasting with neural network support. In: Soft computing in case based reasoning, pp 293–319
212. Fdez-Riverola F, Corchado JM, Torres JM (2002) Neuro-symbolic system for forecasting red tides. In: Artificial intelligence and cognitive science: 13th Irish conference, AICS 2002 Limerick, Ireland, September 12–13, 2002 Proceedings. Springer, pp 45–52
213. Neagu C-D, Avouris N, Kalapanidas E, Palade V (2002) Neural and neuro-fuzzy integration in a knowledge-based system for air quality prediction. Appl Intell 17(2):141
214. Corchado Rodríguez JM, Aiken J, Rees N et al (2003) Neuro-symbolic reasoning system for modeling complex behaviours
215. Fdez-Riverola F, Corchado JM (2003) Fsfrt: Forecasting system for red tides: a hybrid autonomous ai model. Appl Artif Intell 17(10):955–982
216. Policastro CA, Carvalho AC, Delbem AC (2003) Hybrid approaches for case retrieval and adaptation. In: KI 2003: Advances in Artificial Intelligence: 26th Annual German Conference on AI, KI 2003, Hamburg, Germany, September 15-18, 2003. Proceedings 26. Springer, pp 297–311
217. Fernández-Riverola F, Corchado JM (2004) Employing tsk fuzzy models to automate the revision stage of a cbr system. In: Current topics in artificial intelligence: 10th conference of the Spanish association for artificial intelligence, CAEPIA 2003, and 5th Conference on Technology Transfer, TTIA 2003, San Sebastian, Spain, November 12-14, 2003. Revised Selected Papers. Springer, pp 302–311
218. Corchado JM, Borrajo ML, Pellicer MA, Yáñez JC (2005) Neuro-symbolic system for business internal control. In: Advances in data mining: applications in image mining, medicine and biotechnology, management and environmental control, and telecommunications; 4th industrial conference on data mining, ICDM 2004, Leipzig, Germany, July 4-7, 2004, Revised Selected Papers 4. Springer, pp 1–10
219. Prentzas J, Hatzilygeroudis I, Michail O (2008) Improving the accuracy of neuro-symbolic rules with case-based reasoning. In: Proceedings of the first international workshop on combinations of intelligent methods and applications in conjunction with 18th European conference on artificial intelligence, pp 49–54
220. Newman CBD (1998) Uci repository of machine learning databases. http://www.ics.uci.edu/mlearn/MLRepository.html
221. Borrajo ML, Laza R, Corchado JM (2008) A complex case-based advisor. Appl Artif Intell 22(5):377–406
222. Prentzas J, Hatzilygeroudis I (2011) Case-based reasoning integrations: Approaches and applications. Case-based reasoning: processes, suitability and applications, 1–28
223. Hatzilygeroudis I, Prentzas J (2013) Fuzzy and neuro-symbolic approaches in personal credit scoring: assessment of bank loan applicants. In: Innovations in Intelligent Machines-4, p 319
224. Bach J, Herger P (2015) Request confirmation networks for neuro-symbolic script execution. In: CoCo@ NIPS
225. Bologna G, Hayashi Y (2017) Characterization of symbolic rules embedded in deep dimlp networks: a challenge to transparency of deep learning. J Artif Intell Soft Comput Res 7(4):265–286
226. Kraetzschmar G, Sablatnög S, Enderle S, Palm G (2000) Application of neurosymbolic integration for environment modelling in mobile robots. In: Hybrid neural systems. Springer, pp 387–401
227. Burattini E, Coraggio P, De Gregorio M, Ripa B (2003) Agent wisard: go and catch that image. In: Proc. First IAPR TC3 Workshop, Florence, Italy, vol 89, p 95
228. Grieco BP, Lima PM, De Gregorio M, França FM (2010) Producing pattern examples from "mental'' images. Neurocomputing 73(7–9):1057–1064
229. Coraggio P, De Gregorio M (2007) A neurosymbolic hybrid approach for landmark recognition and robot localization. In: Advances in brain, vision, and artificial intelligence: second international symposium, BVAI 2007, Naples, Italy, October 10-12, 2007. Proceedings 2. Springer, pp 566–575
230. De Gregorio M (2008) An intelligent active video surveillance system based on the integration of virtual neural sensors and bdi agents. IEICE Trans Inf Syst 91(7):1914–1921
231. Qadeer N, Velik R, Zucker G, Boley H (2009) Knowledge representation for a neuro-symbolic network in home care risk identification. In: 2009 7th IEEE international conference on industrial informatics. IEEE, pp 277–282

232. Dietrich D, Bruckner D, Zucker G, Muller B, Tmej A (2009) Psychoanalytical model for automation and robotics. In: AFRICON 2009. IEEE, pp 1–8
233. Barbosa R, Cardoso DO, Carvalho D, França FM (2017) A neuro-symbolic approach to gps trajectory classification. ESANN
234. Barbosa R, Cardoso DO, Carvalho D, Franca FM (2018) Weightless neuro-symbolic gps trajectory classification. Neurocomputing 298:100–108
235. Yi K, Wu J, Gan C, Torralba A, Kohli P, Tenenbaum J (2018) Neural-symbolic vqa: Disentangling reasoning from vision and language understanding. In: Advances in neural information processing systems, vol 31
236. Lavrac N, Dzeroski S (1994) Inductive logic programming. In: WLP. Springer, pp 146–160
237. Hatzilygeroudis I, Prentzas J (2000) Neurules: improving the performance of symbolic rules. Int J Artif Intell Tools 9(01):113–130
238. Osório F, Amy B, Cechin A (2001) Hybrid machine learning tools: Inss-a neuro-symbolic system for constructive machine learning. Deep fusion of computational and symbolic processing, 121–144
239. Garcez Ad, Broda K, Gabbay DM (2001) Symbolic knowledge extraction from trained neural networks: a sound approach. Artif Intell 125(1–2):155–207
240. Prentzas J, Hatzilygeroudis I, Garofalakis J (2002) A web-based intelligent tutoring system using hybrid rules as its representational basis. In: Intelligent tutoring systems: 6th international conference, ITS 2002 Biarritz, France and San Sebastian, Spain, June 2–7, 2002 Proceedings 6. Springer, pp 119–128
241. Salgado GR, Amy B (2003) Neuro-symbolic hybrid system for treatment of gradual rules. Neural Information Processing—Letters and Reviews 1(2)
242. Prentzas N, Nicolaides A, Kyriacou E, Kakas A, Pattichis C (2019) Integrating machine learning with symbolic reasoning to build an explainable ai model for stroke prediction. In: 2019 IEEE 19th international conference on bioinformatics and bioengineering (BIBE). IEEE, pp 817–821
243. Thrun SB, Bala JW, Bloedorn E, Bratko I, Cestnik B, Cheng J, De Jong KA, Dzeroski S, Fisher DH, Fahlman SE, et al (1991) The monk's problems: A performance comparison of different learning algorithms. Technical report
244. Zhou J, Cui G, Hu S, Zhang Z, Yang C, Liu Z, Wang L, Li C, Sun M (2020) Graph neural networks: a review of methods and applications. AI Open 1:57–81
245. Omlin CW, Snyders S (2003) Inductive bias strength in knowledge-based neural networks: application to magnetic resonance spectroscopy of breast tissues. Artif Intell Med 28(2):121–140
246. Bologna G (2003) A model for single and multiple knowledge based networks. Artif Intell Med 28(2):141–163
247. Obot OU, Uzoka F-ME (2009) A framework for application of neuro-case-rule base hybridization in medical diagnosis. Appl Soft Comput 9(1):245–253
248. Boulahia J, Smirani L, KSA MA (2015) Experiments of a neuro symbolic hybrid learning system with incomplete data
249. Ghosh J, Taha I (2018) A neuro-symbolic hybrid intelligent architecture with. In: Recent advances in artificial neural networks, 1
250. Bhatia S, Kohli P, Singh R (2018) Neuro-symbolic program corrector for introductory programming assignments. In: Proceedings of the 40th international conference on software engineering, pp 60–70
251. Souici-Meslati L, Sellami M (2004) A hybrid approach for arabic literal amounts recognition. Arab J Sci Eng 29
252. Perrier M, Kalwa J (2005) Intelligent diagnosis for autonomous underwater vehicles using a neuro-symbolic system in a distributed architecture. In: Europe Oceans 2005, vol 1. IEEE, pp 350–355
253. Sánchez VGC, Villegas OOV, Salgado GR, Dominguez H (2008) Quality inspection of textile artificial textures using a neuro-symbolic hybrid system methodology. WSEAS Trans Comput 12:1899–1905
254. Velik R, Boley H (2010) Neurosymbolic alerting rules. IEEE Trans Ind Electron 57(11):3661–3668
255. Komendantskaya E, Zhang Q (2011) Sherlock-a neural network software for automated problem solving. In: Proceedings of seventh international workshop on neural-symbolic learning and reasoning
256. Saikia S, Vig L, Srinivasan A, Shroff G, Agarwal P, Rawat R (2016) Neuro-symbolic eda-based optimisation using ilp-enhanced dbns. arXiv preprint arXiv:1612.06528
257. Kursuncu U, Gaur M, Sheth A (2019) Knowledge infused learning (k-il): Towards deep incorporation of knowledge in deep learning. arXiv preprint arXiv:1912.00512
258. Khan MJ, Curry E (2020) Neuro-symbolic visual reasoning for multimedia event processing: Overview, prospects and challenges. In: CIKM (Workshops)
259. Kapanipathi P, Abdelaziz I, Ravishankar S, Roukos S, Gray A, Astudillo R, Chang M, Cornelio C, Dana S, Fokoue A, et al (2020) Question answering over knowledge bases by leveraging semantic parsing and neuro-symbolic reasoning. arXiv preprint arXiv:2012.01707
260. Yang Z, Ishay A, Lee J (2020) Neurasp: embracing neural networks into answer set programming. In: 29th international joint conference on artificial intelligence (IJCAI 2020)
261. Siyaev A, Jo G-S (2021) Neuro-symbolic speech understanding in aircraft maintenance metaverse. IEEE Access 9:154484–154499
262. Stammer W, Schramowski P, Kersting K (2021) Right for the right concept: revising neuro-symbolic concepts by interacting with their explanations. In: Proceedings of the IEEE/CVF conference on computer vision and pattern recognition, pp 3619–3629
263. Kimura D, Ono M, Chaudhury S, Kohita R, Wachi A, Agravante DJ, Tatsubori M, Munawar A, Gray A (2021) Neuro-symbolic reinforcement learning with first-order logic. arXiv preprint arXiv:2110.10963
264. Evans R, Bošnjak M, Buesing L, Ellis K, Pfau D, Kohli P, Sergot M (2021) Making sense of raw input. Artif Intell 299:103521
265. Mitchener L, Tuckey D, Crosby M, Russo A (2022) Detect, understand, act: a neuro-symbolic hierarchical reinforcement learning framework. Mach Learn 111(4):1523–1549
266. Alshahrani M, Khan MA, Maddouri O, Kinjo AR, Queralt-Rosinach N, Hoehndorf R (2017) Neuro-symbolic representation learning on biological knowledge graphs. Bioinformatics 33(17):2723–2730
267. Perozzi B, Al-Rfou R, Skiena S (2014) Deepwalk: Online learning of social representations. In: Proceedings of the 20th ACM SIGKDD international conference on knowledge discovery and data mining, pp 701–710
268. Agibetov A, Samwald M (2018) Fast and scalable learning of neuro-symbolic representations of biomedical knowledge. arXiv preprint arXiv:1804.11105
269. Wu L, Fisch A, Chopra S, Adams K, Bordes A, Weston J (2018) Starspace: Embed all the things! In: Proceedings of the AAAI conference on artificial intelligence, vol 32
270. Bianchi F, Palmonari M, Hitzler P, Serafini L (2019) Complementing logical reasoning with sub-symbolic commonsense. In: Rules and reasoning: third international joint conference,

RuleML+ RR 2019, Bolzano, Italy, September 16–19, 2019, Proceedings 3. Springer, pp 161–170
271. Bianchi F, Palmonari M, Nozza D (2018) Towards encoding time in text-based entity embeddings. In: The semantic web–ISWC 2018: 17th international semantic web conference, Monterey, CA, USA, October 8–12, 2018, Proceedings, Part I 17. Springer, pp 56–71
272. Oltramari A, Francis J, Henson C, Ma K, Wickramarachchi R (2020) Neuro-symbolic architectures for context understanding. arXiv preprint arXiv:2003.04707
273. Singh P, Lin T, Mueller ET, Lim G, Perkins T, Li Zhu W (2002) Open mind common sense: knowledge acquisition from the general public. In: On the move to meaningful internet systems 2002: CoopIS, DOA, and ODBASE: confederated international conferences CoopIS, DOA, and ODBASE 2002 Proceedings. Springer, pp 1223–1237
274. Wang Q, Mao Z, Wang B, Guo L (2017) Knowledge graph embedding: a survey of approaches and applications. IEEE Trans Knowl Data Eng 29(12):2724–2743
275. Doldy D, Garridoy JS (2021) An energy-based model for neuro-symbolic reasoning on knowledge graphs. In: 2021 20th IEEE international conference on machine learning and applications (ICMLA). IEEE, pp 916–921
276. Nickel M, Tresp V, Kriegel H-P (2011) A three-way model for collective learning on multi-relational data. In: Icml, vol 11, pp 3104482–3104584
277. Sun K, Rayudu H, Pujara J (2021) A hybrid probabilistic approach for table understanding. In: Proceedings of the AAAI conference on artificial intelligence, vol 35, pp 4366–4374
278. Kimmig A, Bach S, Broecheler M, Huang B, Getoor L (2012) A short introduction to probabilistic soft logic. In: Proceedings of the NIPS workshop on probabilistic programming: foundations and applications, pp 1–4
279. Gol MG, Pujara J, Szekely P (2019) Tabular cell classification using pre-trained cell embeddings. In: 2019 IEEE international conference on data mining (ICDM). IEEE, pp 230–239
280. Ding M, Chen Z, Du T, Luo P, Tenenbaum J, Gan C (2021) Dynamic visual reasoning by learning differentiable physics models from video and language. Adv Neural Inf Process Syst 34:887–899
281. Ma K, Francis J, Lu Q, Nyberg E, Oltramari A (2019) Towards generalizable neuro-symbolic systems for commonsense question answering. arXiv preprint arXiv:1910.14087
282. Sundar LKS, Muzik O, Buvat I, Bidaut L, Beyer T (2021) Potentials and caveats of ai in hybrid imaging. Methods 188:4–19
283. Kang T, Turfah A, Kim J, Perotte A, Weng C (2021) A neuro-symbolic method for understanding free-text medical evidence. J Am Med Inform Assoc 28(8):1703–1711
284. Yin P, Neubig G (2017) A syntactic neural model for general-purpose code generation. arXiv preprint arXiv:1704.01696
285. Ritchie D, Guerrero P, Jones RK, Mitra NJ, Schulz A, Willis KD, Wu J (2023) Neurosymbolic models for computer graphics. In: Computer graphics forum, vol 42. Wiley Online Library, pp 545–568
286. Reddy AG, Balasubramanian VN (2022) Estimating treatment effects using neurosymbolic program synthesis. arXiv preprint arXiv:2211.04370
287. Li Z, Huang J, Naik M (2023) Scallop: A language for neurosymbolic programming. Proceedings of the ACM on Programming Languages 7(PLDI):1463–1487
288. Varela FA (2022) The effects of hybrid neural networks on meta-learning objectives. PhD thesis
289. Mundhenk TN, Landajuela M, Glatt R, Santiago CP, Faissol DM, Petersen BK (2021) Symbolic regression via neural-guided genetic programming population seeding. arXiv preprint arXiv:2111.00053
290. Chen X, Liang C, Huang D, Real E, Wang K, Pham H, Dong X, Luong T, Hsieh C-J, Lu Y et al (2024) Symbolic discovery of optimization algorithms. In: Advances in neural information processing systems, vol 36
291. Mittelstadt BD, Allo P, Taddeo M, Wachter S, Floridi L (2016) The ethics of algorithms: mapping the debate. Big Data Soc 3(2):2053951716679679
292. Rudin C (2019) Stop explaining black box machine learning models for high stakes decisions and use interpretable models instead. Nature Mach Intell 1(5):206–215
293. Kazim E, Denny DMT, Koshiyama A (2021) Ai auditing and impact assessment: according to the UK information commissioner's office. AI Ethics 1:301–310
294. Jobin A, Ienca M, Vayena E (2019) The global landscape of ai ethics guidelines. Nat Mach Intell 1(9):389–399
295. Tamang MD, Shukla VK, Anwar S, Punhani R (2021) Improving business intelligence through machine learning algorithms. In: 2021 2nd International conference on intelligent engineering and management (ICIEM). IEEE, pp 63–68