Big Red Math Competition
Team Round

Team ID: ______________________

10/26/2024 2:15pm–2:45pm

INSTRUCTIONS — PLEASE READ THIS NOW

- **Write your Team ID. ONLY captain needs to turn in.**
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos\pi}{\frac{1}{2}}$ can be simplified to -2.
- **You have 30 minutes to complete this exam.**
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources. You CAN collaborate with your teammates.

Please sign below to indicate that you have read and agree to these instructions.

Signature of Captain, Print Name: ______________________

Answer Sheet:

1. ____________ 5. ____________ 9. ____________

2. ____________ 6. ____________ 10. ____________

3. ____________ 7. ____________

4. ____________ 8. ____________

Please don't mark anything below on the page, it's for official use only.

Grader 1 Name:	Grader 2 Name:	Grader 3 Name:
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Tally:	Tally:	Tally:
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Score:	Score:	Score:
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1. In an MLB division series, teams A and B face off in a series of games where the first team to win three games wins the series. If team A wins the series, how many possible scenarios are there for the series to play out? (For example, one possible scenario is for team A to win the first game, team B to win the second, and team A to win the third and fourth games.)

2. For each integer m between 1 and 2024! (inclusive) and each integer n between 1 and 2024 (inclusive), Bradley writes down the remainder of m when divided by n. Find the mean value of all $2024! \cdot 2024$ remainders that Bradley wrote down.

3. A farmer from Ithaca has a 30 acre farm and 22 days to plant bundles of crops where he plants one bundle at a time. The farmer can either plant a bundle of wheat which takes 3 acres and 1 day to plant, or he can plant a bundle of corn which takes 2 acres and 2 days to plant. Once a piece of land is used to plant a bundle of crop, it cannot be used again to plant another bundle of crop in the future. What is the maximum number of bundles of crops that the farmer can plant?

4. Let $\triangle ABC$ be a triangle with $AB = 2023$, $BC = 2024$, and $CA = 2025$. A point P is chosen uniformly at random on the interior of $\triangle ABC$. Compute the probability that the closest side of $\triangle ABC$ to P is AB.

5. Find the sum of all odd integers $k \geq 1$ such that the base-2 representation of $\frac{1}{k}$ is periodic with period exactly 16.

6. Alex has a tetrahedron $ABCD$ with side lengths $AB = AC = AD = 1$ and $BC = CD = DB = \sqrt{2}$. After each second, he moves each of the four vertices A, B, C, D *instantaneously* so that each vertex is moved to the centroid of the triangle formed by the other three vertices. Suppose after n seconds, vertex A is at point X_n, where we let X_0 denote the point that A originally starts at. Compute the length of X_0X_{2024}.

7. How many non-empty subsets S are there of $\{0, 1, 2, 3, 4, 5, 6\}$ such that for all $a, b \in S$ (where a and b are not necessarily distinct), the remainder when ab is divided by 7 is not in S?

8. Mikh independently samples 80 points $x_1, x_2, \ldots, x_{80}$ uniformly at random from the interval $[0, 1]$. For each pair of distinct integers $1 \leq i, j \leq 80$ such that the interval $[x_i, x_j]$ contains the point $\frac{1}{2}$, Mikh creates a new piece of string with length $x_j - x_i$. What is the expected combined length of all strings that Mikh creates?

9. Find the unique positive integer N that minimizes the ratio between N and the cube of the number of factors of N.

10. Let $n = 2024$. Define real numbers $0 < w_1, w_2, \ldots, w_n < 1$ such that $w_1 + w_2 + \cdots + w_n = 1$ and
$$\frac{w_1 + w_2 + \cdots + w_k}{w_1} = 2^{k-1}$$
for all $1 \leq k \leq n$. Given n points $X_1, X_2, \ldots, X_n \in \mathbb{R}^2$, define the *epicenter* of the n-tuple of points $(X_1, X_2, \ldots, X_n)$ to be the point
$$w_1X_1 + w_2X_2 + \cdots + w_nX_n \in \mathbb{R}^2.$$

Alex starts with $n+1$ points $A_0, A_1, \ldots, A_n \in \mathbb{R}^2$ such that $A_0A_1 \cdots A_n$ is a regular $(n+1)$-gon inscribed in the unit circle centered at the origin of $\mathbb{R}^2$. Starting with the 0th second, Alex begins moving points after each second where at the k-th second, he moves the point A_k to the epicenter of the n-tuple of points $(A_{k+1}, A_{k+2}, \ldots, A_{k+n})$, where indices are taken modulo $n+1$. Let d_k denote the distance between the origin of $\mathbb{R}^2$ and the point A_0 at the k-th second. Given that d_k converges to d as $k \to \infty$, compute d^{-1}.