Cornell University®

Big Red Math Competition
Proof Round

Name: ______________________

Team ID: ______________________

10/28/2023 9:30–10:30am

INSTRUCTIONS — PLEASE READ THIS NOW

- **Show your work.** To receive full credit, your answers must be neatly written and logically organized.
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos\pi}{\frac{1}{2}}$ can be simplified to -2.
- **You have 1 hour to complete this exam.**
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources.

Please sign below to indicate that you have read and agree to these instructions.

Signature of Student: ______________________

Please don't mark anything below on the page, it's for official use only.

Grader 1: ______	Grader 2: ______	Grader 3: ______
1. ____ / 10 ____	1. ____ / 10 ____	1. ____ / 10 ____
2. ____ / 20 ____	2. ____ / 20 ____	2. ____ / 20 ____
3. ____ / 20 ____	3. ____ / 20 ____	3. ____ / 20 ____
4. ____ / 30 ____	4. ____ / 30 ____	4. ____ / 30 ____
Total: ____ / 80 ____	Total: ____ / 80 ____	Total: ____ / 80 ____

Question 1. (10 points) Let x, y, z be positive real numbers. Prove that

$$\sqrt{(z+x)(z+y)} - z \geq \sqrt{xy}.$$

Question 2. (20 points) This season, there are $3n+1$ teams in the MLS (Major League Soccer). As of now, each team has played exactly $n-1$ matches. Prove that there exist 4 teams such that none of the 4 teams have faced each other.

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Question 3. (20 points) Find all positive integer pairs (m, n) such that $m - n$ is a positive prime number and mn is a perfect square. Justify your answer.

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Question 4. (30 points) Let square $ABCD$ and circle Ω be on the same plane, and AA', BB', CC', DD' be tangents to Ω. Let $WXYZ$ be a convex quadrilateral with side lengths $WX = AA'$, $XY = BB'$, $YZ = CC'$ and $ZW = DD'$. If $WXYZ$ has an inscribed circle, prove that the diagonals WY and XZ are perpendicular to each other.

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