Cornell University®

Big Red Math Competition
Team Round

Team ID: ____________________

10/28/2023 2:05–2:35pm

INSTRUCTIONS — PLEASE READ THIS NOW

- **Write your Team ID. ONLY captain needs to turn in.**
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos\pi}{\frac{1}{2}}$ can be simplified to -2.
- **You have 30 minutes to complete this exam.**
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources. You CAN collaborate with your teammates.

Please sign below to indicate that you have read and agree to these instructions.

Signature of Captain, Print Name: ____________________

Answer Sheet:

1. ________ 5. ________ 9. ________

2. ________ 6. ________ 10. ________

3. ________ 7. ________

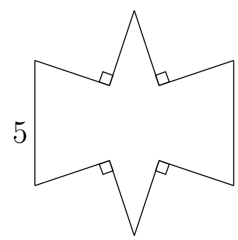
4. ________ 8. ________

Please don't mark anything below on the page, it's for official use only.

Grader 1 Name:	Grader 2 Name:	Grader 3 Name:
Tally:	Tally:	Tally:
Score:	Score:	Score:

1. Ben starts with an integer greater than 9 and subtracts the sum of its digits from it to get a new integer. He repeats this process with each new integer he gets until he gets a positive 1-digit integer. Find all possible 1-digit integers Ben can end with from this process.

2. The concave decagon shown below is embedded in the Cartesian coordinate plane such that all of its vertices have integer coordinates. Two opposite edges have length 5, whereas the remaining eight edges have length $\sqrt{10}$. Every pair of opposite edges is parallel. The sides of the decagon do not intersect each other, and the decagon has vertical and horizontal axes of symmetry. Find the area of the decagon.



3. Two buckets each have four balls; two red balls and two white balls in the first, and two red balls and two blue balls in the second. At first, a bucket is selected, then a ball in the bucket is selected, with both buckets and balls inside the selected bucket having equal probability of being chosen. Then, without replacement of the first ball, the process is repeated once more. Determine the probability that the first ball drawn being red if the second ball drawn was blue.

4. Alice, Bob, Carol, and David decide that they will share meals and that one of them will cook each night. Because David enjoys cooking, he will cook on 4 days of the week, while Alice, Bob, and Carol each pick a day of the week to cook on. If Alice, Bob, and Carol each choose the day they cook uniformly at random so as to avoid overlap, what is the probability that David does not cook on three consecutive days? For example, Monday, Tuesday and Wednesday are considered as three consecutive days, so are Saturday, Sunday and Monday.

5. The quadratic polynomial $f(x)$ has the expansion $2x^2 - 3x + r$. What is the largest real value of r for which the ranges of the functions $f(x)$ and $f(f(x))$ are the same set?

6. Find the sum of all positive divisors of 40081.

7. Among all ordered pairs of real numbers (a, b) satisfying $a^4 + 2a^2b + 2ab + b^2 = 960$, find the smallest possible value for a.

8. If r is real number sampled at random with uniform probability, find the probability that r is *strictly* closer to a multiple of 58 than it is to a multiple of 37.

9. Find the sum of all integers n such that $1 < n < 30$ and n divides
$$1 + \prod_{k=1}^{n-1} k^{2k}.$$

10. Let triangle ABC have side lengths $AB = 19$, $BC = 180$, and $AC = 181$, and angle measure $\angle ABC = 90$. Let the midpoints of AB and BC be denoted by M and N respectively. The circle centered at M and passing through point C intersects with the circle centered at the N and passing through point A at points D and E. If DE intersects AC at point P, find $\min(DP, EP)$.