Realistic fission transmission coefficients in the statistical Hauser-Feshbach compound-nucleus reaction theory

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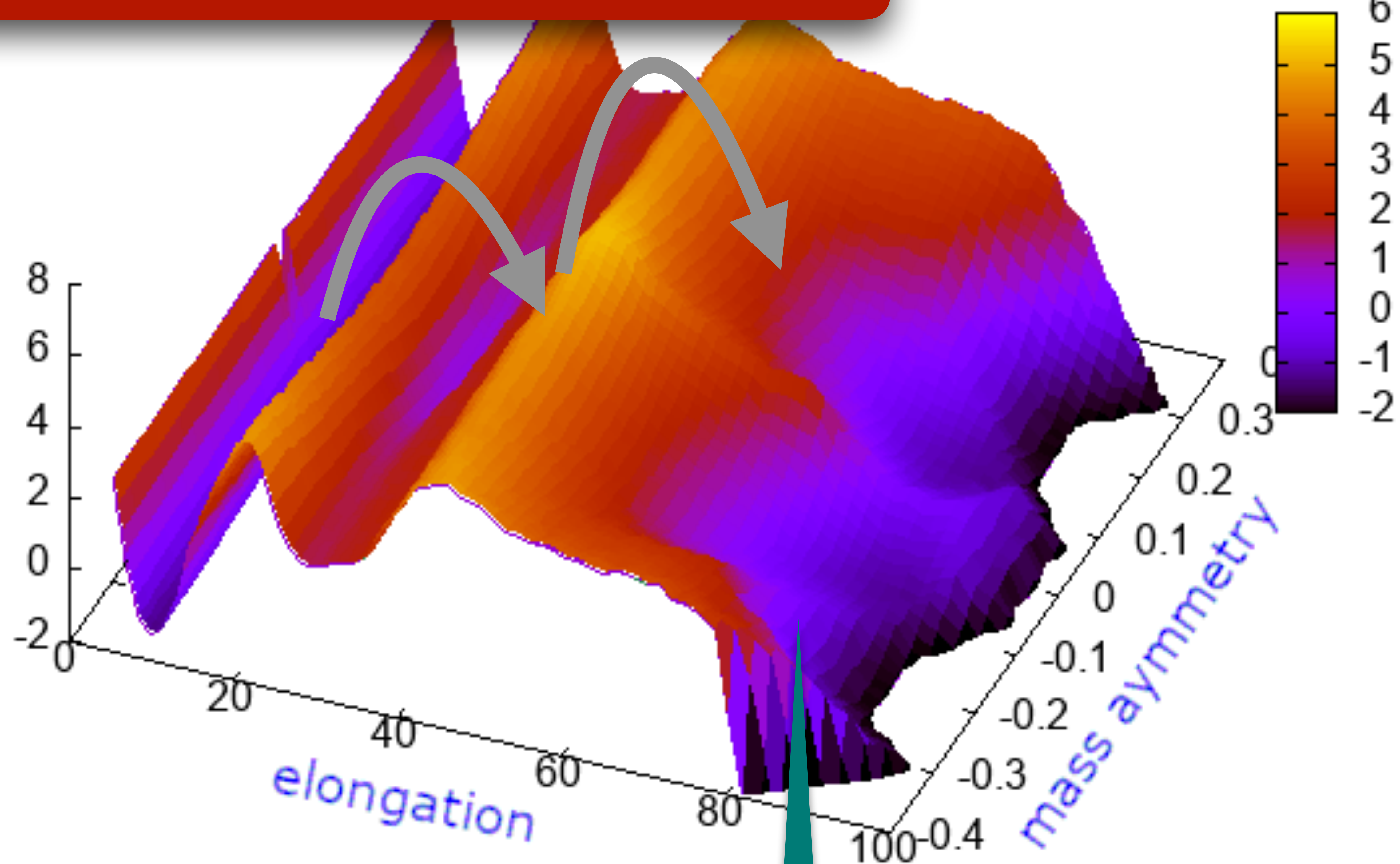
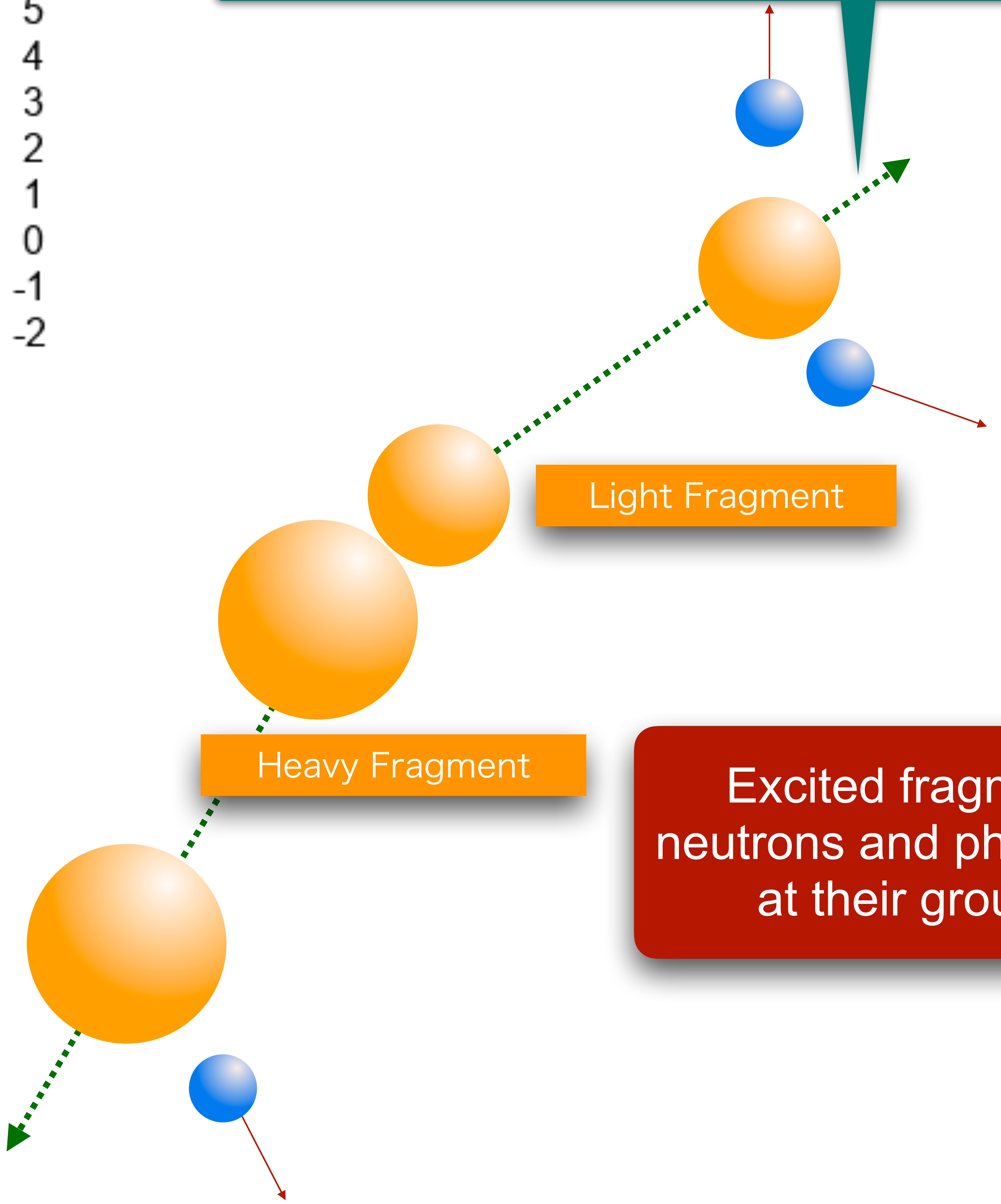
Pre and Post Fission Phenomena and Observables

Dynamical process toward scission



Penetration through potential energy surface
=
Fission Transmission Coefficient in HF

Fission product yields, prompt neutron and photons, etc are the fission observables



Excited fragments emit neutrons and photon to arrive at their ground state

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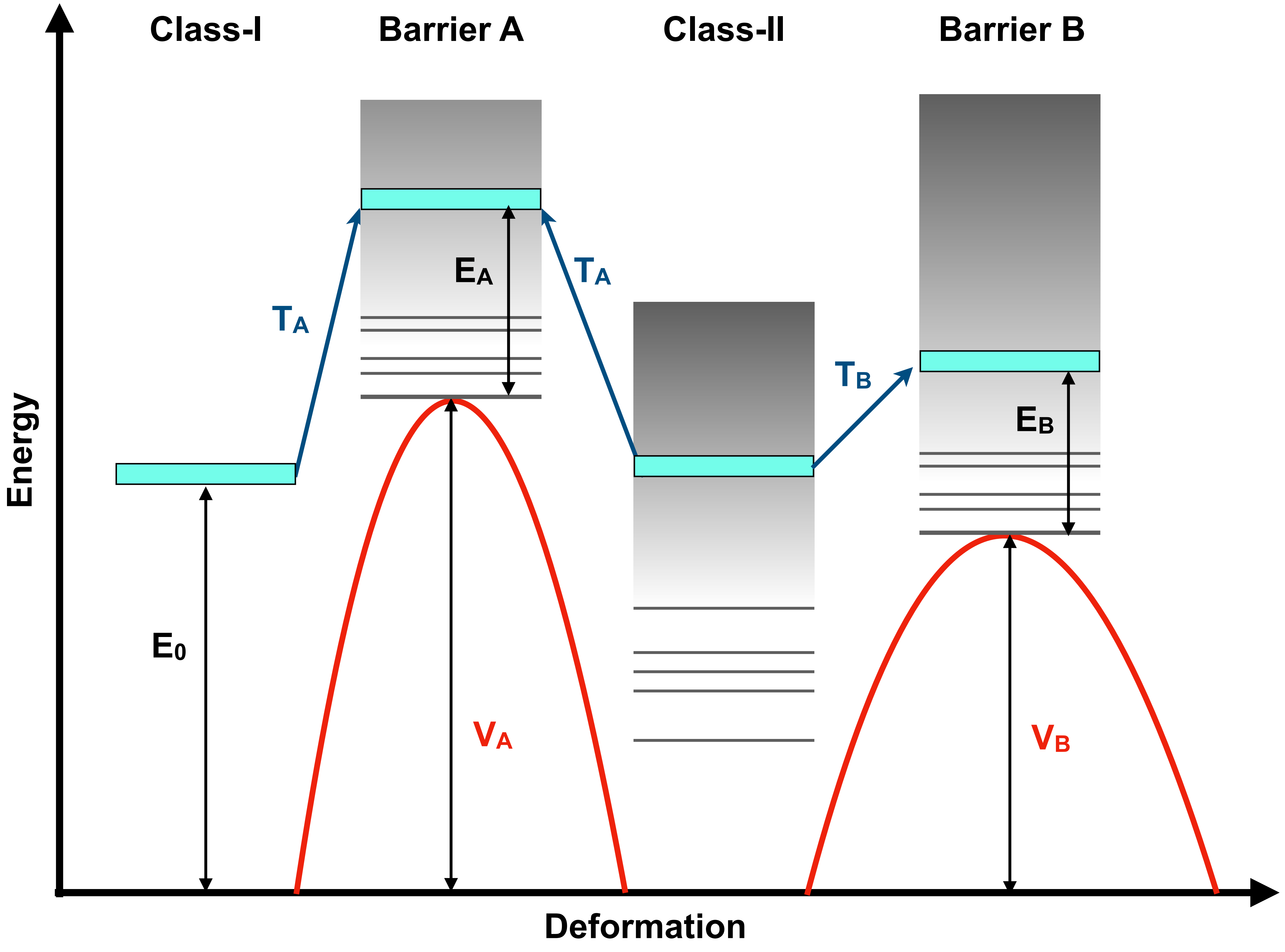
Fission Transmission Calculation - Conventional Method

- **Extremely simplified penetration model employed**
  - Hill and Wheeler expression gives an analytical expression by WKB approximation for Inverted parabolic barriers
  - Often double-humped barrier shape assumed, which are combined by

$$T_f = \frac{T_A T_B}{T_A + T_B}$$

  - Sometimes potential valleys considered (Class-II)
- **Because two barriers are fully decoupled, there is no actual fission path**
  - Number of fission channels determined by the level densities on top of each barrier
  - Includes fission level density enhancement

$$T_i(E) = \frac{1}{1 + \exp\left(2\pi \frac{V_i + E - E_0)}{C_i}\right)} \text{ , } i = A, B$$



Potential Issues in Conventional Fission Transmission Calculation

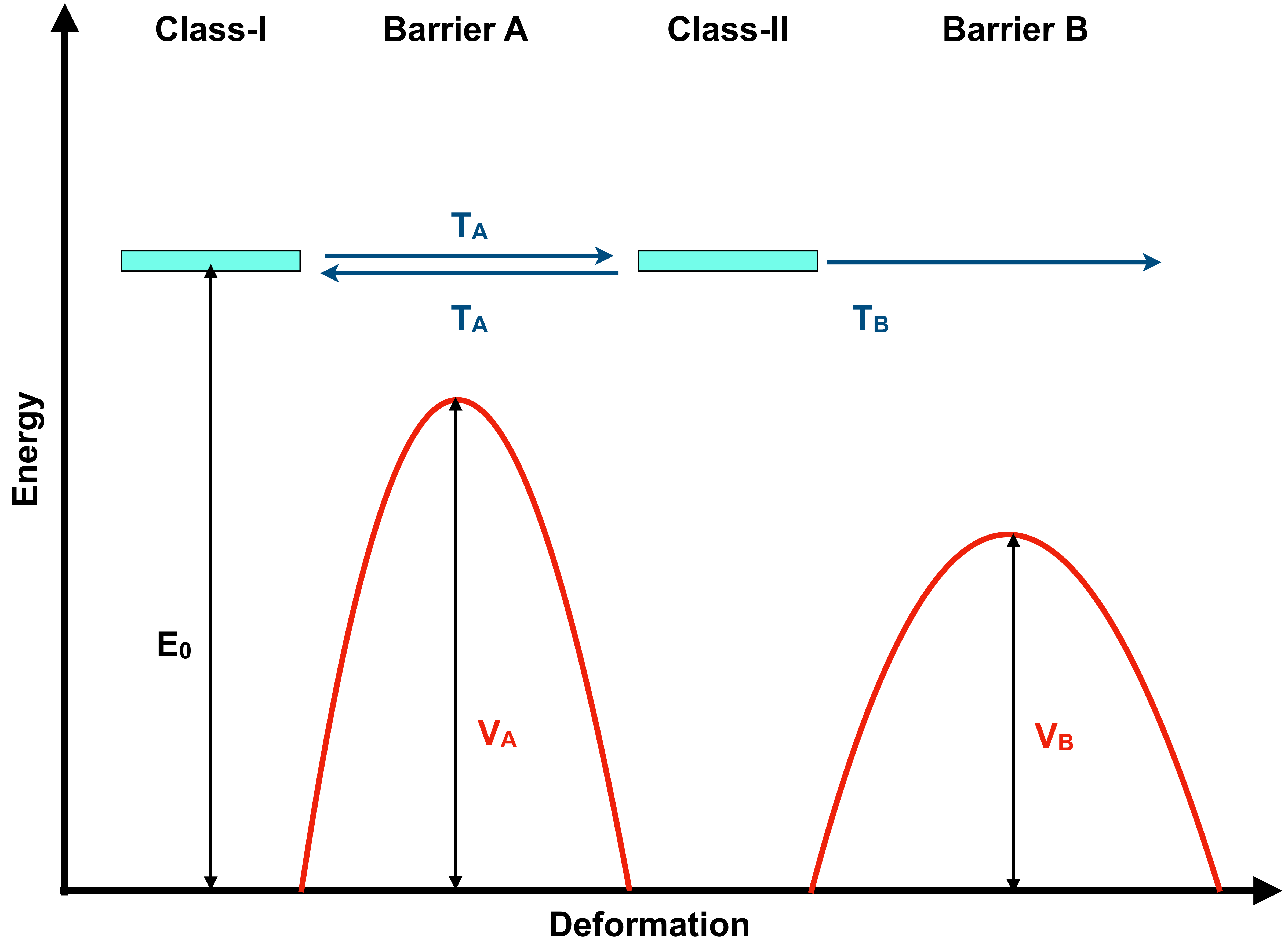
- **Lumped transmission coefficients first calculated by summing all the excited levels on top of each barrier**

$$T_i = \sum_k T_i(E_k) + \int_{E_c}^{\infty} T_i(E_x)\rho_i(E_x)dE_x \ , i = A, B$$

  - Nuclear structure other than the barrier tops is ignored
  - No fission trajectory for the excited states
  - Number of fission channels not well-defined
- **It assumes another compound nucleus at Class-II**
  - When the energy is above the all barriers

$$T_f = \frac{T_A T_B}{T_A + T_B} < 1$$

- **Fission level density uncertain**
  - Level density enhancement often phenomenological



Solving Schrödinger Equation for Fission Penetration

- **1-D Schroedinger Equation in the deformation coordinate**

$$\frac{d^2}{dx^2}\phi(x) + \frac{2\mu}{\hbar^2}\{E - (V(x) + iW(x))\}\phi(x) = 0$$

$$\frac{\mu}{\hbar^2} = 0.054A^{5/3} \quad \mathrm{MeV}^{-1}$$

Cramer, Nix PRC 2, 1048 (1970)

- which satisfies the asymptotic solution

$$u^{(-)}(kx) - Su^{(+)}(kx) \quad x > x_{\mathrm{max}}$$

$$Au^{(-)}(kx) \quad x < x_{\mathrm{min}}$$

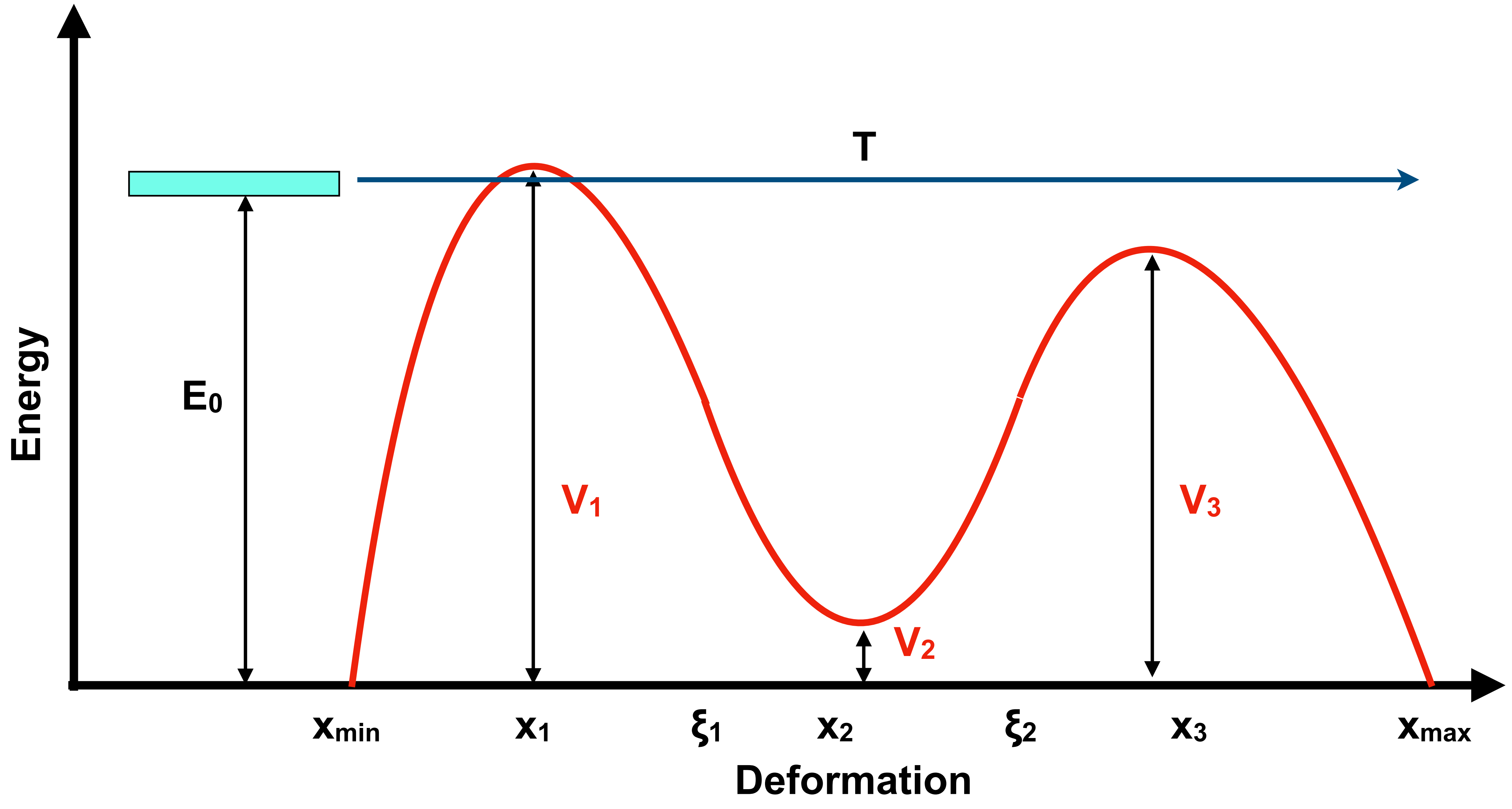
$$u^{(\pm)}(kx) = \cos(kx) \pm i\sin(kx)$$

- transmission coefficient (penetration) is given by

$$T = 1 - |S|^2$$

- when complex potential

$$T = A = \left.\frac{u^{(-)} - Su^{(+)}}{\phi}\right|_{x_m}$$



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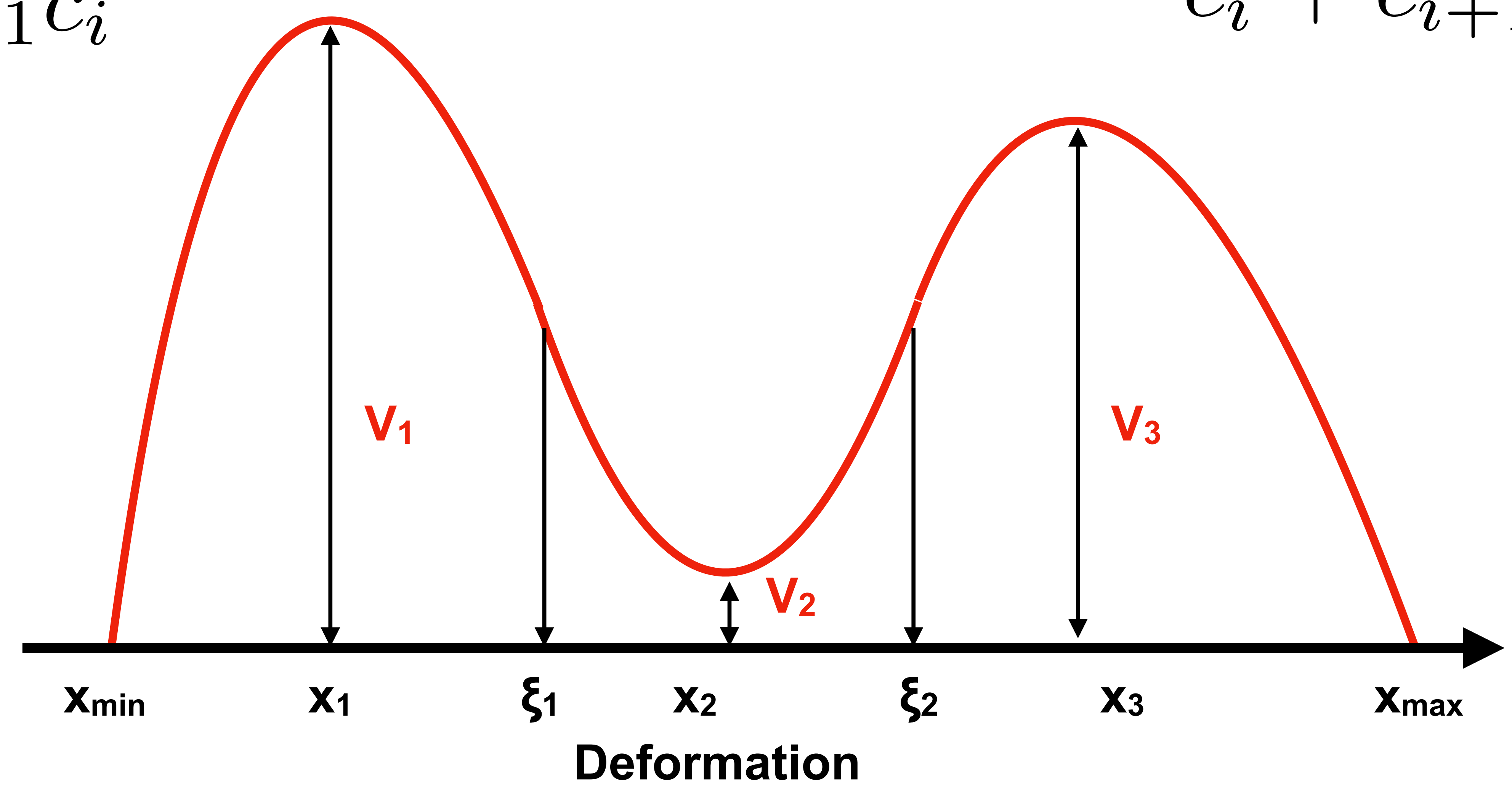
Concatenated Parabolas for Simplified Potential Shape

- **Although potential shape can be arbitrary, it is still convenient to use a segmented parameterization to characterize the potential shape**
  - V: height (depth) of barrier (valley)
  - C: curvature

$$V(i,x) = V_i + (-1)^i \frac{1}{2} c_i (x - x_i)^2, \quad i = 1, 2, \ldots \qquad c_i = \frac{\mu C_i^2}{\hbar^2}$$

$$x_i = x_{i-1} + \sqrt{\frac{2|V_{i-1} - V_i|(c_{i-1} + c_i)}{c_{i-1} c_i}} \qquad \xi_i = \frac{c_i x_i + c_{i+1} x_{i+1}}{c_i + c_{i+1}}$$

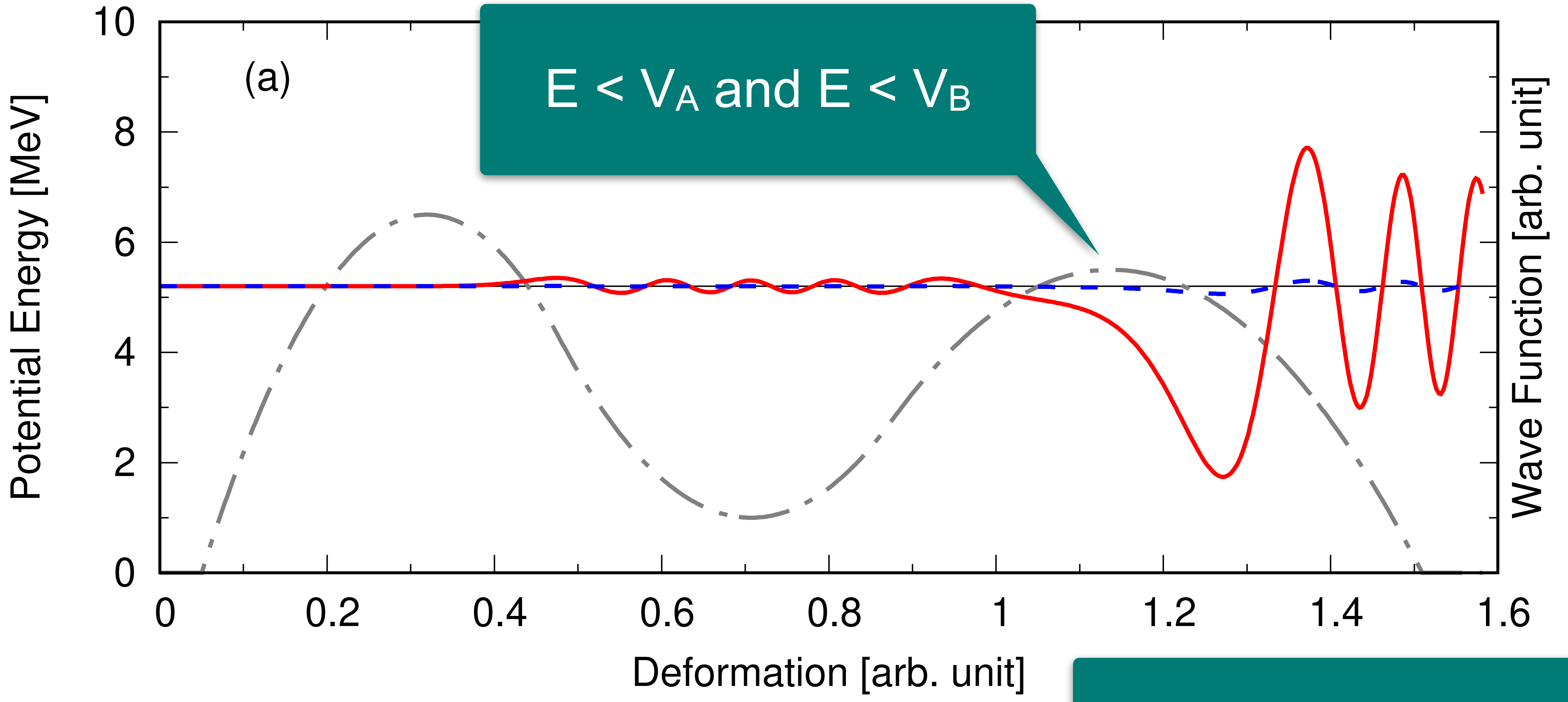
$$x_1 = x_{\min} + \sqrt{\frac{2V_1}{c_1}}$$

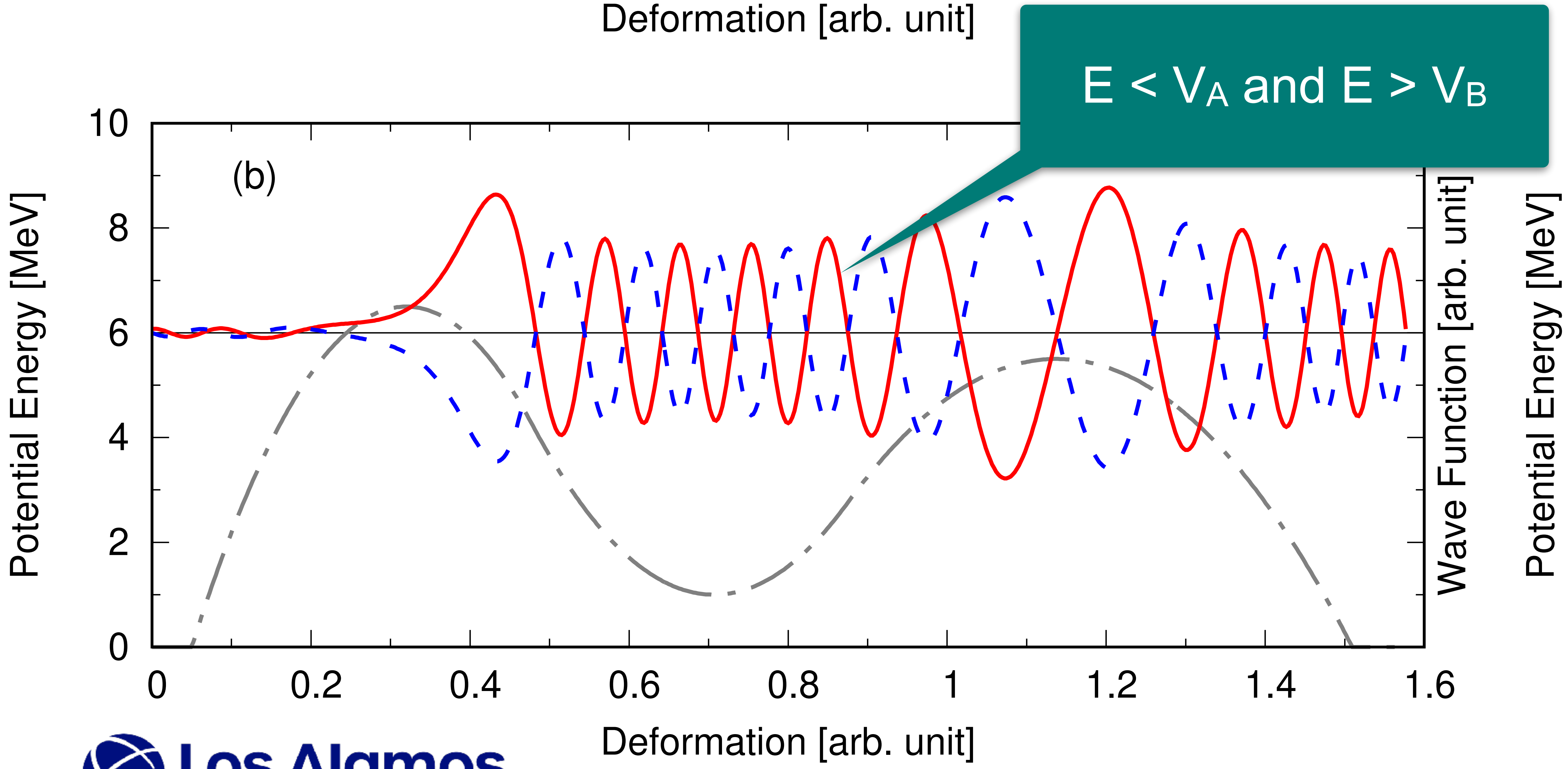


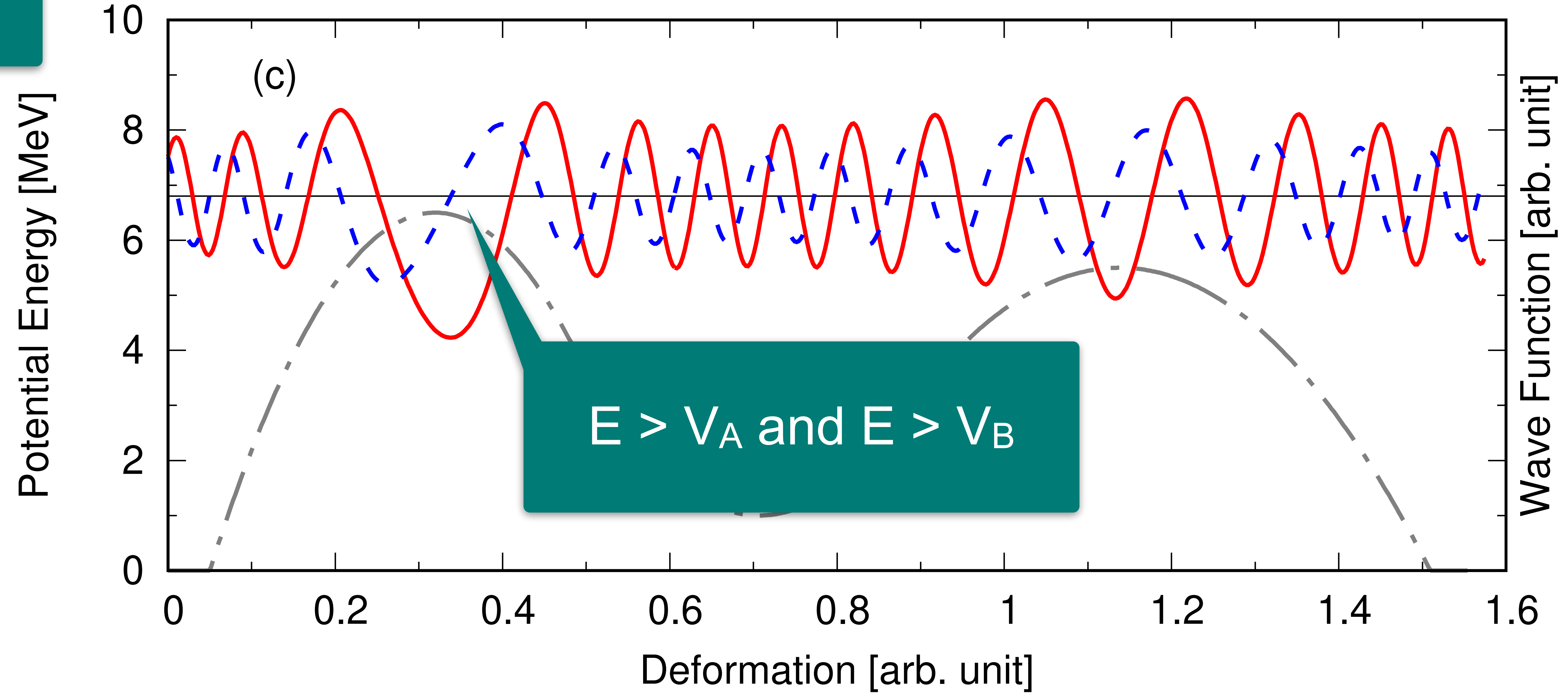
Solving Schroedinger Equation for 1-D Fission Potential

$$\frac{d^2}{dx^2}\phi(x) + \frac{2\mu}{\hbar^2}\left\{E - (V(x) + iW(x))\right\}\phi(x) = 0$$

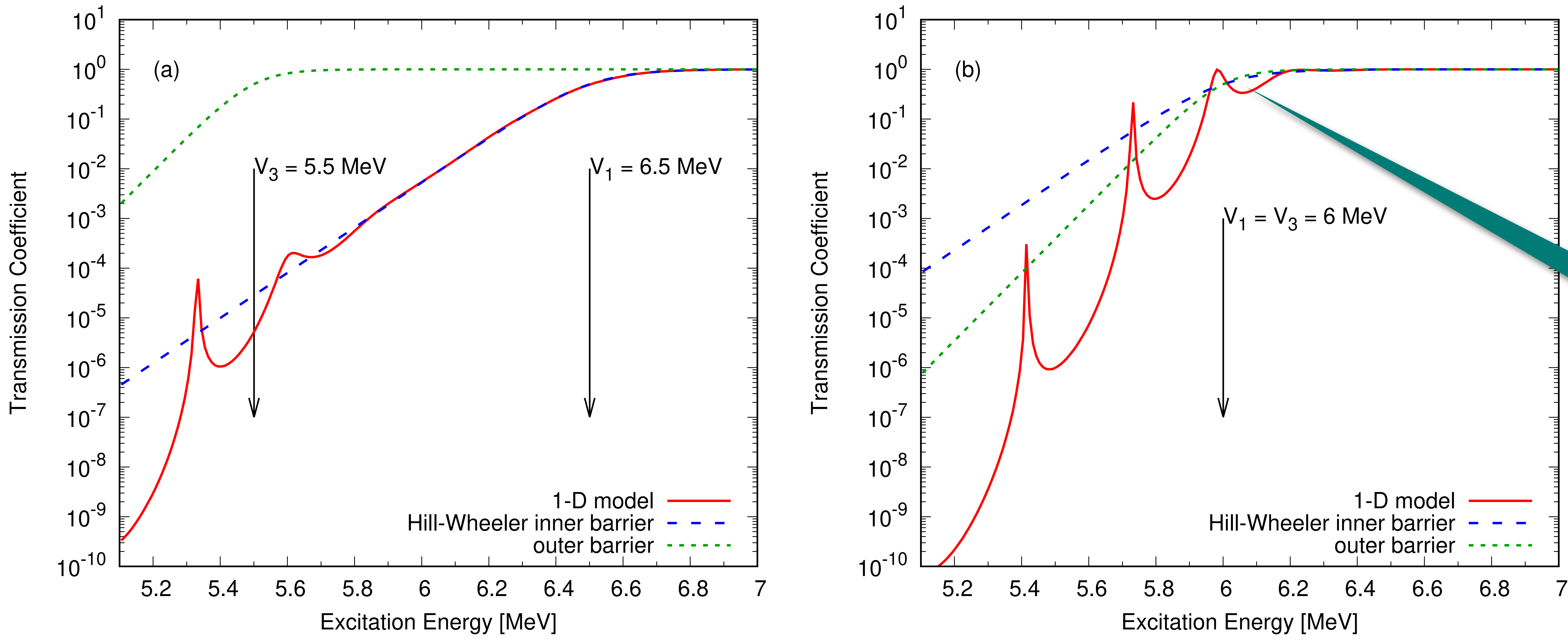
Transmission coefficient is given by the amplitude inside the barriers when unit wave in the exteror region is given





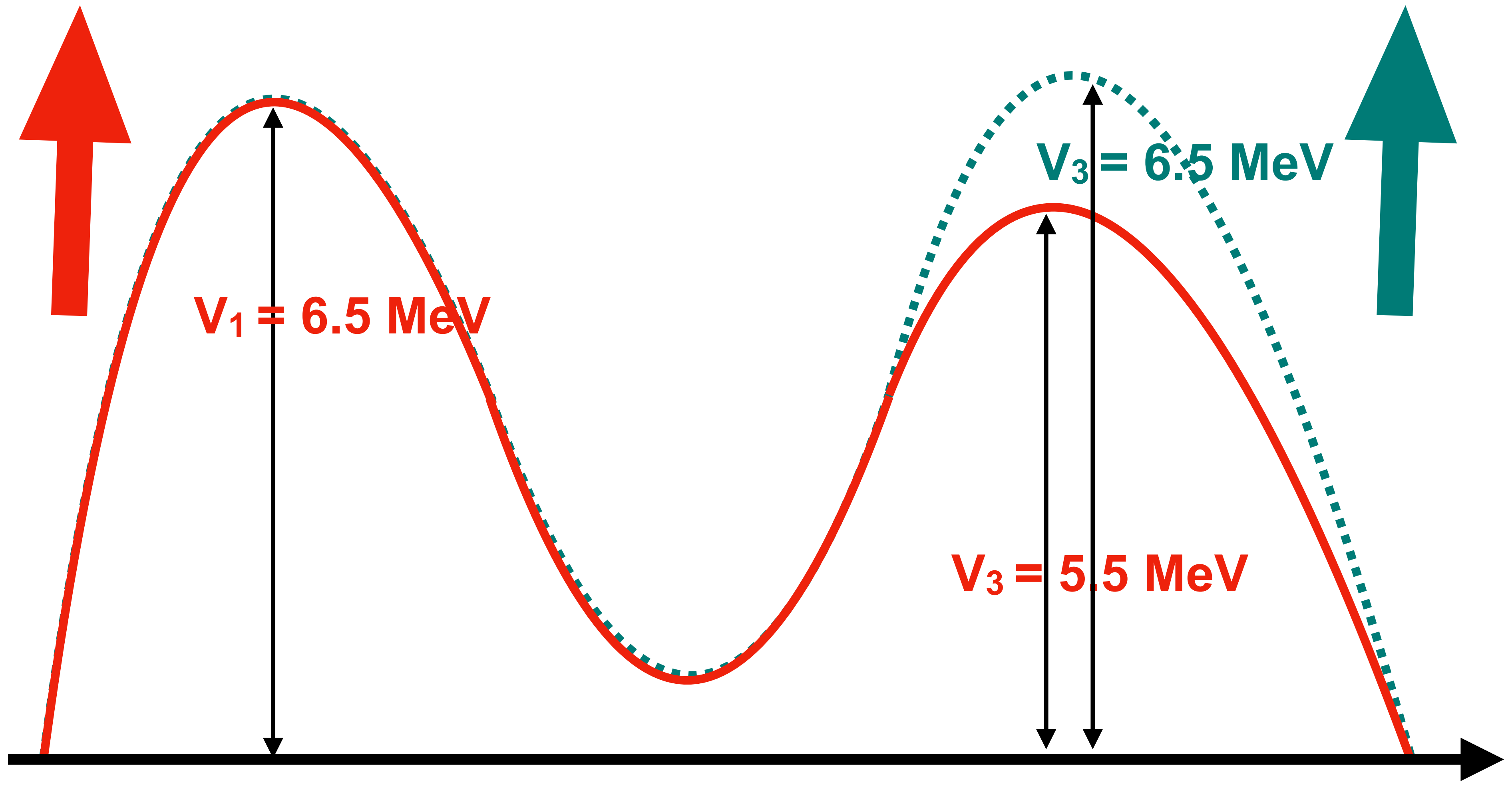


Fission Transmission Coefficient



Transmission coefficient is 1 when the system energy is higher than the all barriers

Fission cross section will be enhanced even the energy is below the fission barriers



- **Resonance-like structure appears**
  - not a CN resonance but nuclear shape effect
  - this happens when incoming and reflection waves are in phase

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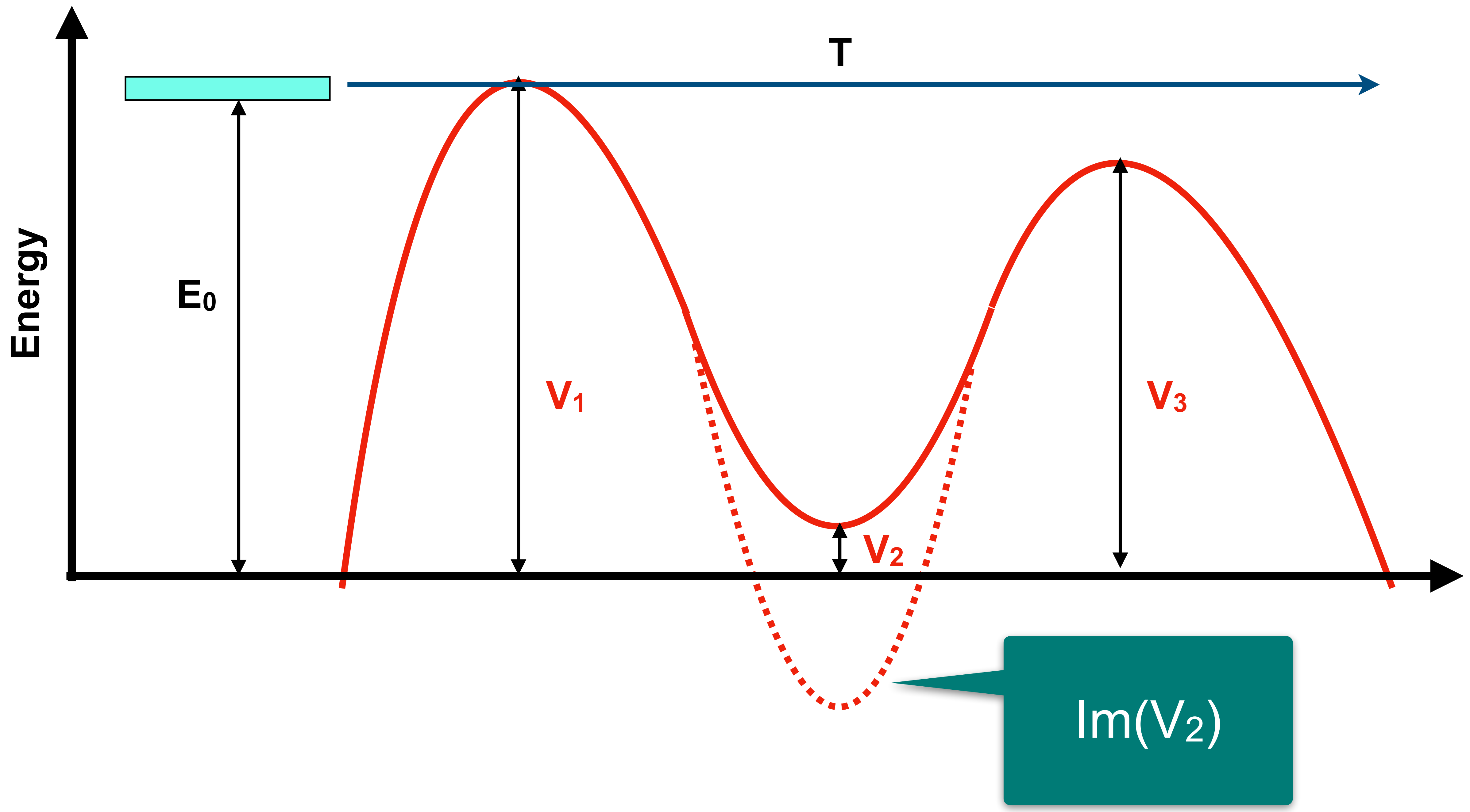
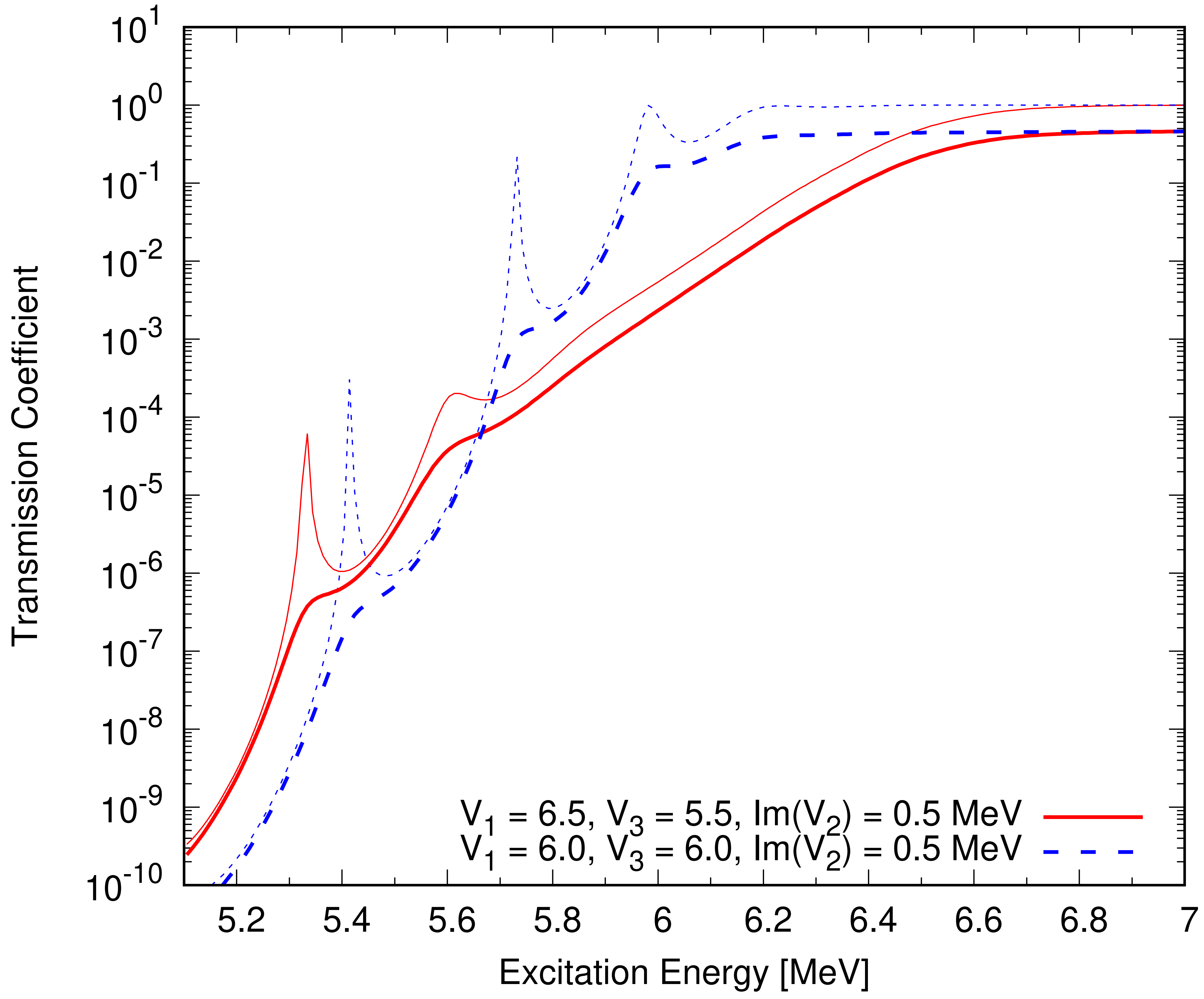
Fission Transmission for Complex Well

- **When Class-II well has an imaginary component**
  - Part of the flux toward the scission is absorbed by the potential
  - Transmission coefficients will be smaller
  - Physical interpretation not so straightforward





Penetration Through Excited States - Fission Trajectory Model

- **Penetration happen many excited states**
  - Fission paths must be defined for all of the possible trajectories
  - They are distorted by the nuclear deformation, mass asymmetry, pairing effect, etc
- **We employ a simplified trajectory compression model**

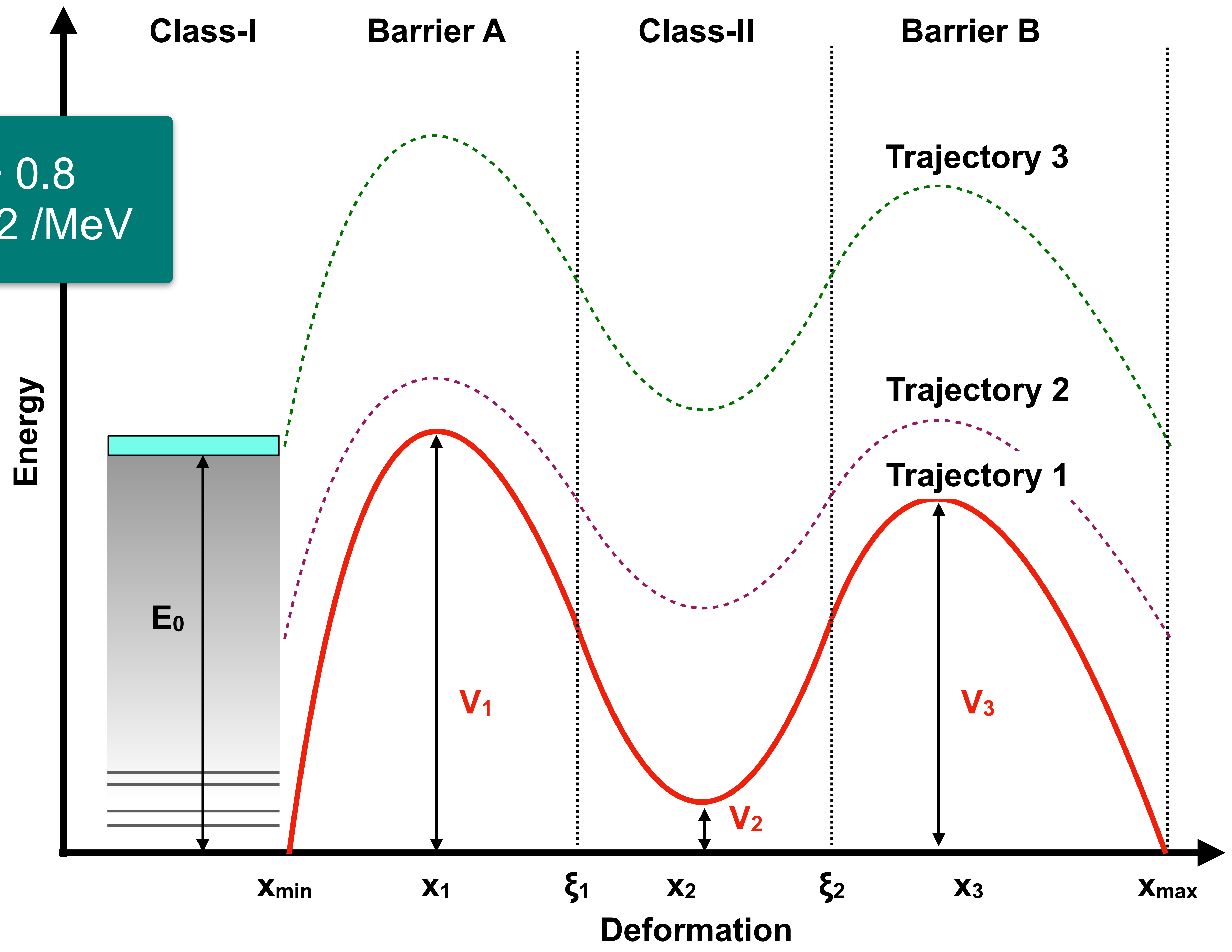
$f_0 \sim 0.8$
$f_1 \sim 0.2$ /MeV

$$\varepsilon_x = \left\{ f_0 + \left(1 - e^{-f_1 E_x}\right)(1 - f_0) \right\} E_x$$

$$V(E_x, x) = V_0(E_x, x) + \varepsilon_x$$

  - where f_0 and f_1 are parameters

Low-lying discrete levels are lowerd, and continuum at higher energy region asymptotes CN level density



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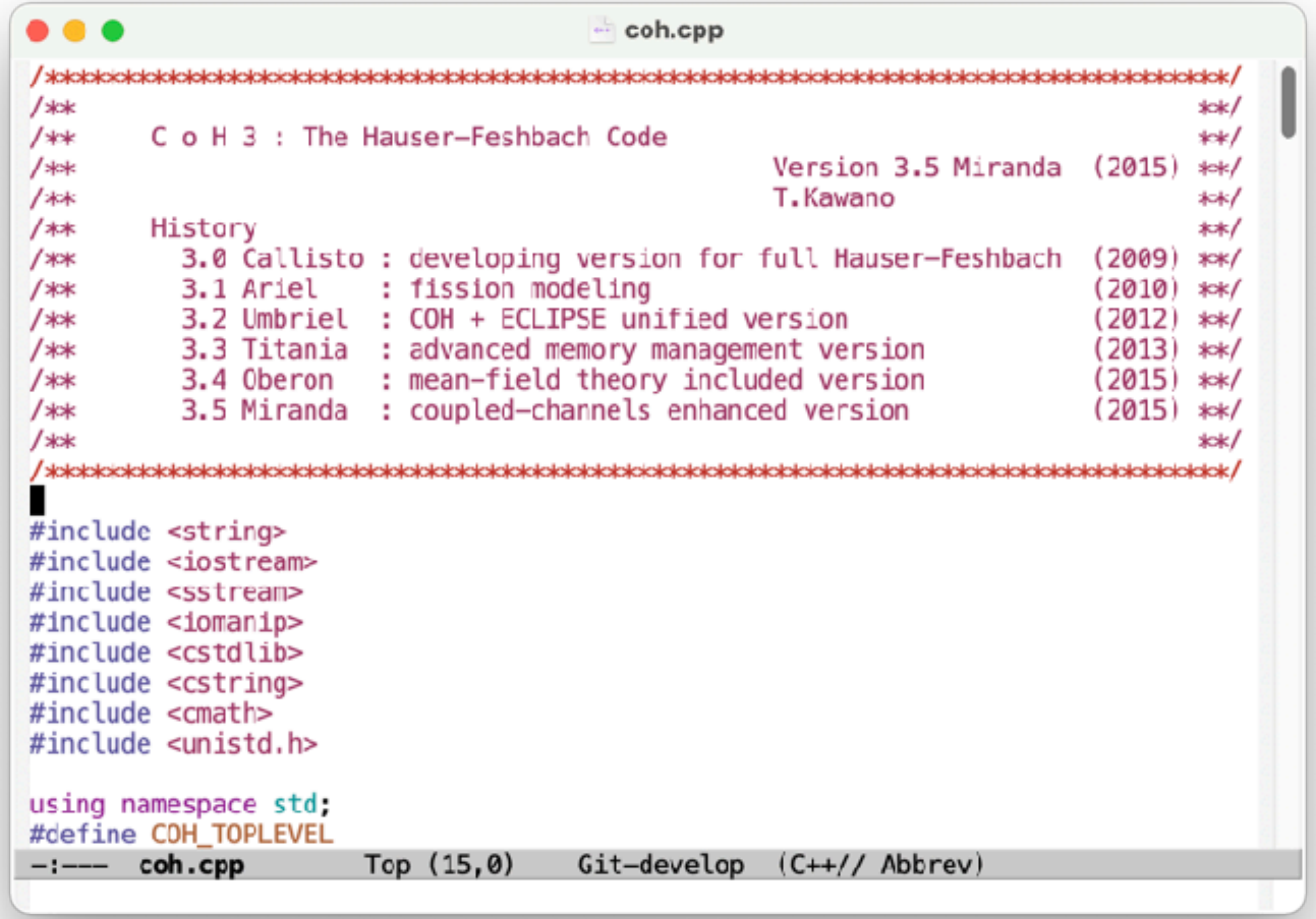
Coupled-Channels and Hauser-Feshbach Code CoH_3

- **Hauser-Feshbach-Moldauer theory for compound reaction**
  - 50k+ lines C++ code, including ~140 source and ~60 header files
  - written in OOP style, ~ 80 classes defined
  - GNU Autotools package for building
- **Some special features**
  - Internal optical model / coupled-channels solver
  - Unified description of coupled-channels and statistical model
  - Compound nucleus decay by deterministic or Monte Carlo method
  - Accurate exclusive reaction cross sections and spectra
  - Mean-field models included (FRDM, Hartree-Fock-BCS)
  - Subsidiary code BeoH
- **Open-source**
  - https://github.com/toshihikokawano/coh3



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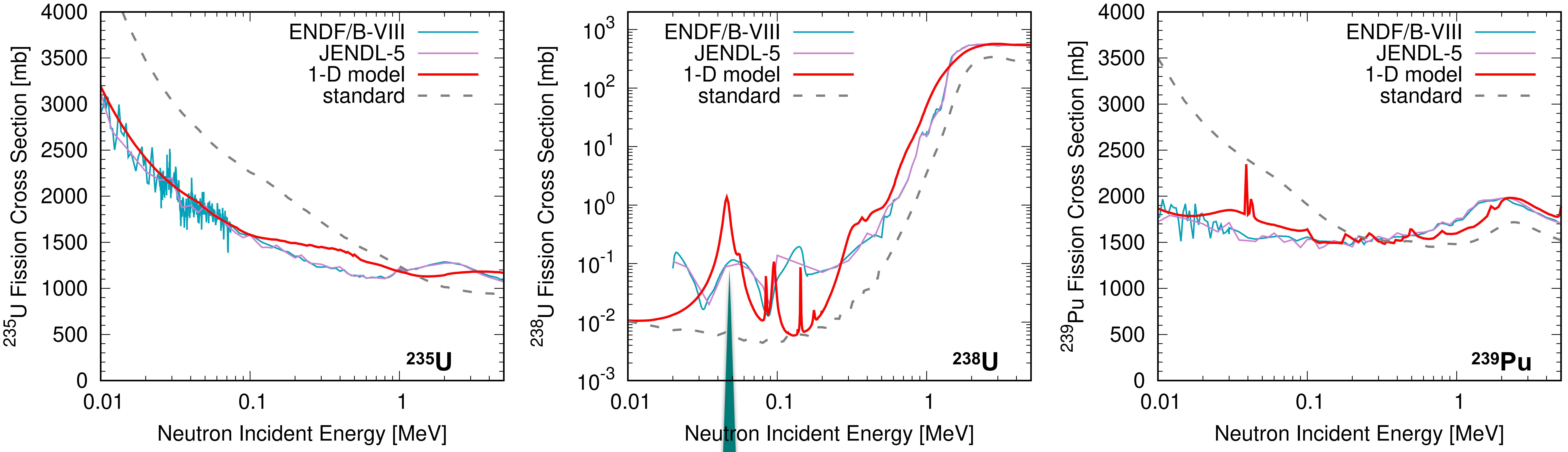
Calculated Fission Cross Sections for Major Actinides

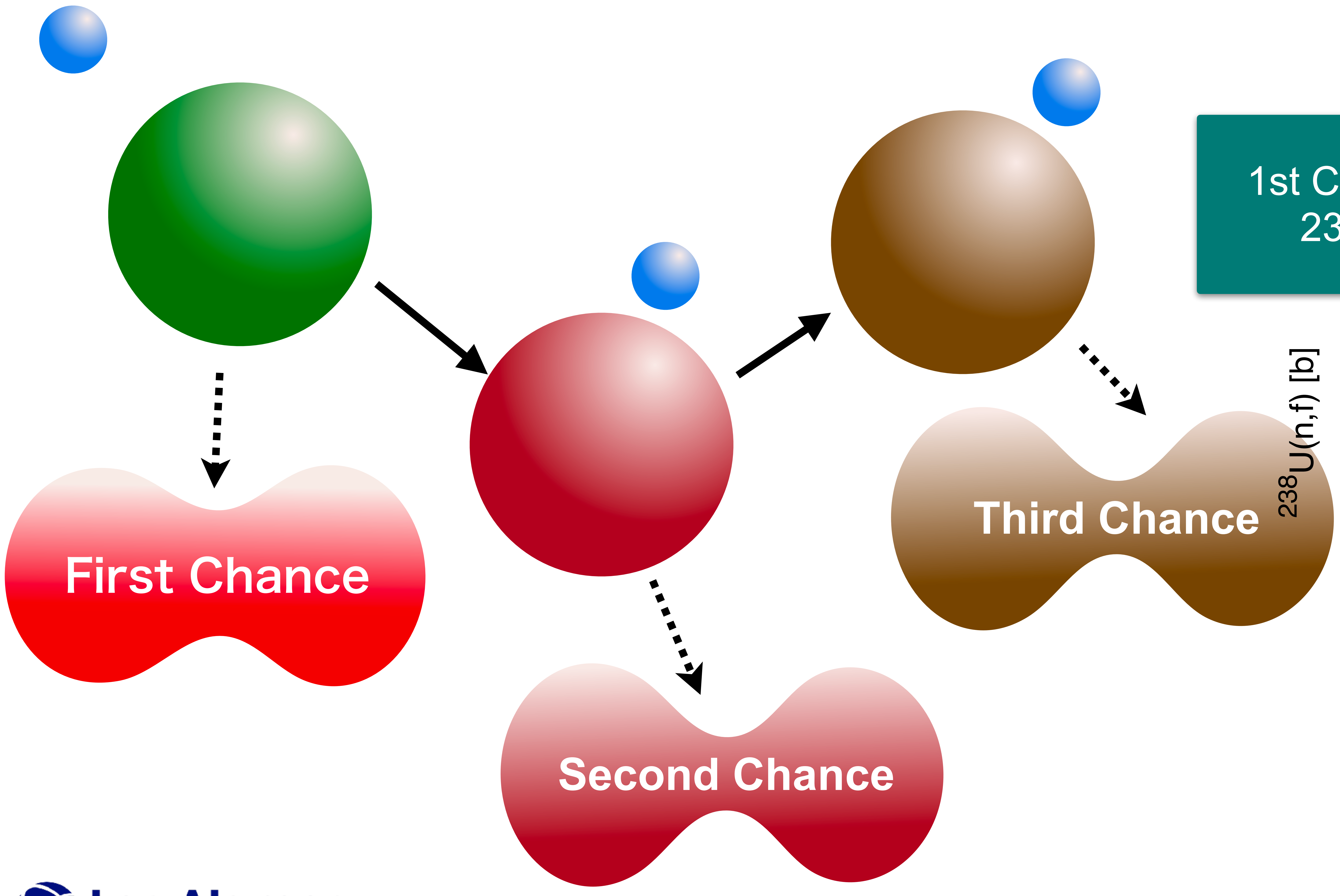
- **Calculated cross sections compared with "evaluated data"**
  - reveals resonance-like structure, remarkable for ^{238}U which is a sub-threshold fissioner
  - **there are only 4 parameters for each of targets, 2 barrier heights and 2 trajectory parameters**

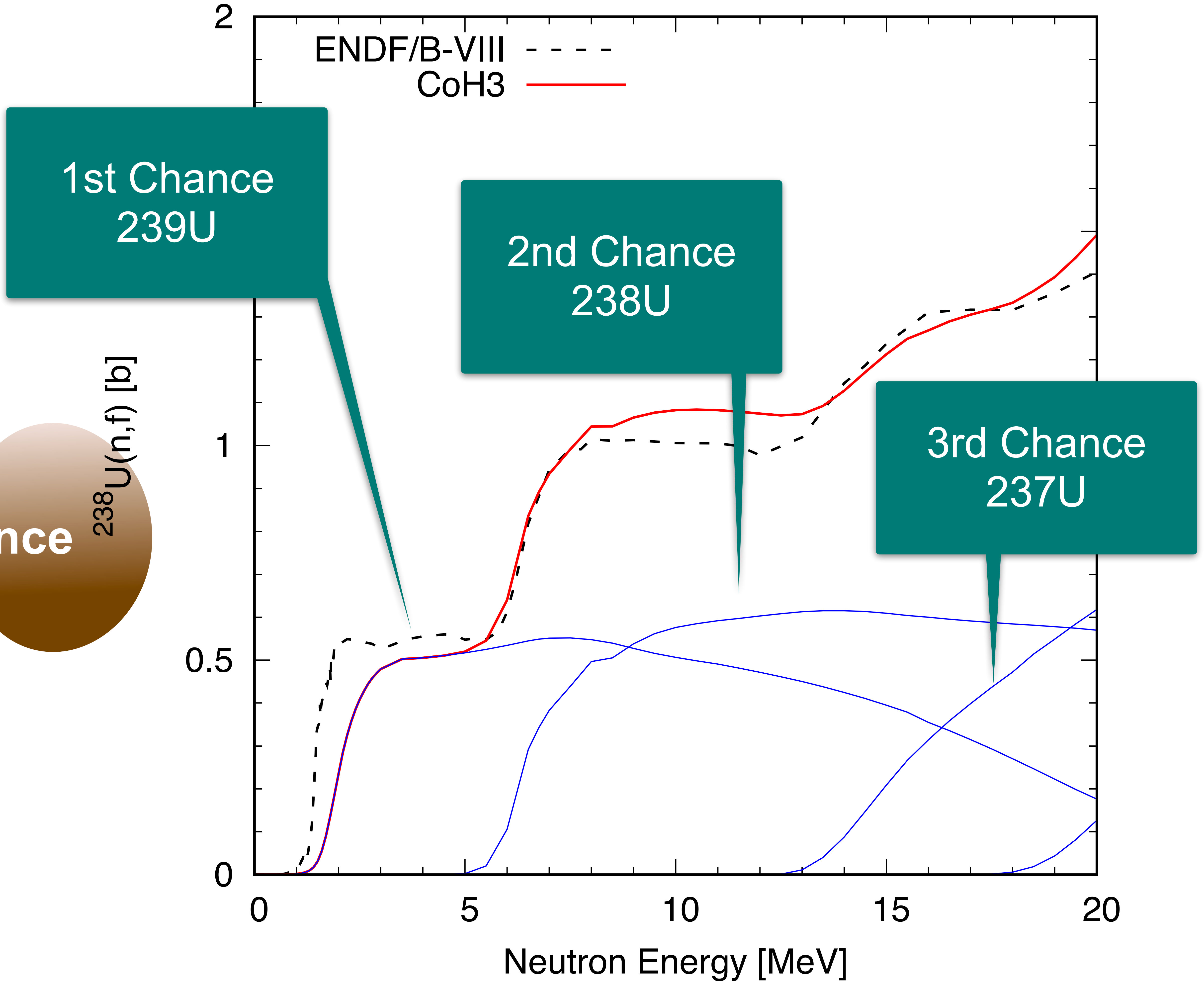


sub-threshold fission cross sections significantly increased

Multi-Chance Fission Phenomena

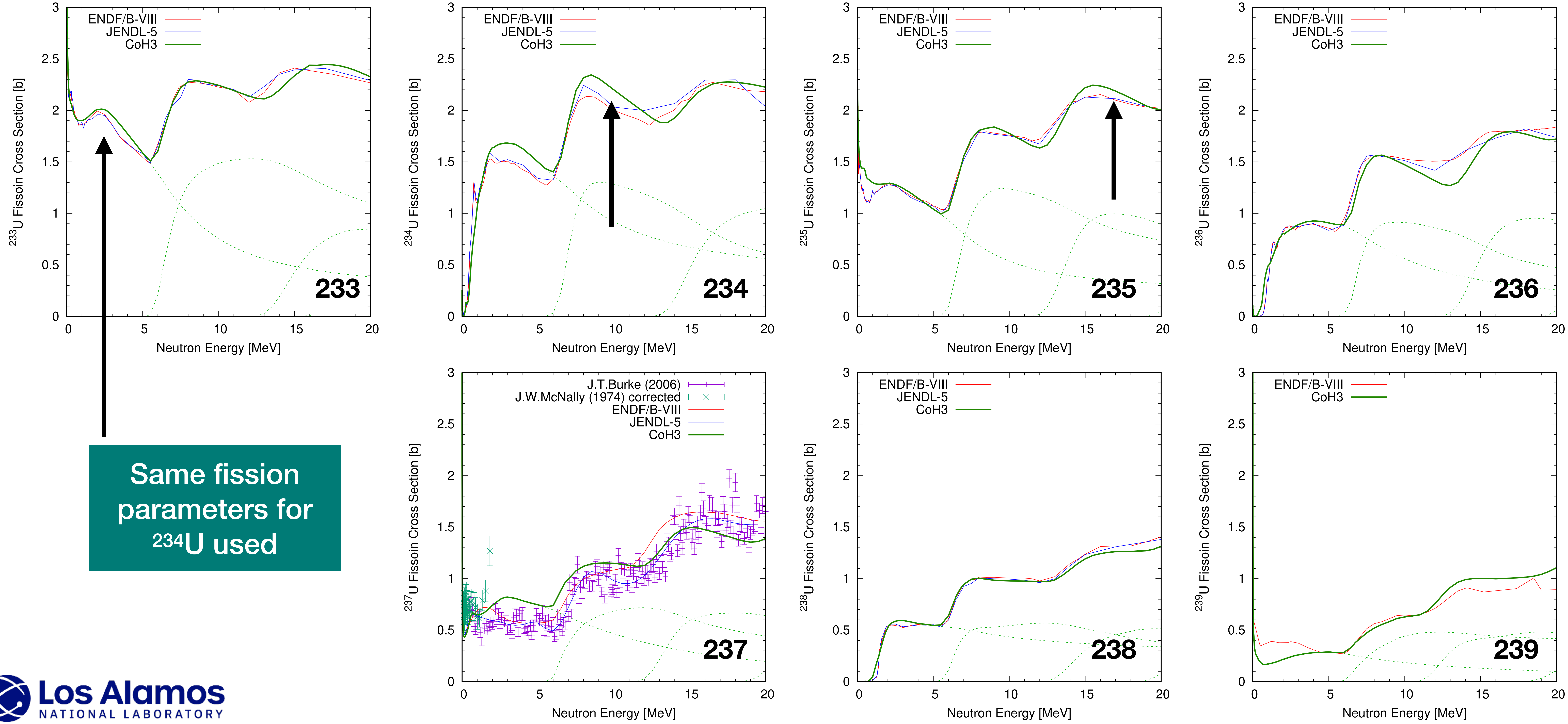
- At higher energies, an observed fission cross section may include contributions from several fissioning nuclei
- Fission parameters should be consistent for different targets when they appear in the multi-chance fission chain





Consistent Calculation of Fission Cross Sections (First Time!)

- **Our new fission model applied to U isotopes from A = 233 to 239**
  - There is no direct measurement of ^{237}U and ^{239}U fission cross sections



Concluding Remarks and Perspective

- **New fission penetration model incorporated into coupled-channels Hauser-Feshbach model**
  - Fission penetration (= fission transmission coefficients) is calculated for the arbitrary-shape 1-D potential by solving Schroedinger equation numerically
    - This model ensures that the penetrability is always unity when the system energy is above all the fission barriers
  - Fission paths through excited states expressed by the fission trajectory model
    - Although very crude at this moment, only limited number of parameters involved

- **Feasible (plausible) extensions**
  - Potential energy surface by more microscopic insights
  - Include different deformation coordinates: mass asymmetry and fragment deformation
  - Coupled-channels formalism for Schroedinger equation
  - Consistent calculation for both the fission cross section and the fission product yields (project started in 2024)