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# INDUSTRIAL APPLICATION OF GAS TURBINES COMMITTEE

## DYNAMIC PROPERTIES OF TILTING-PAD JOURNAL BEARINGS: ANALYSIS AND EXPERIMENT

**by**

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Waldemar M. Dmochowski studied mechanical engineering at the Technical University of Gdansk (Poland). After receiving his M.Sc. he worked at the Institute of Fluid Flow Machinery, Polish Academy of Sciences in Gdansk where he received his Ph.D. In 1987 he joined the National Research Council Canada and continued research in tribology, in particular on testing and modeling of the performance of journal bearings.

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## ABSTRACT

Rotors of high speed rotating machinery are usually supported in tilting-pad journal bearings (TPJB). These bearings offer inherent stability resulting from zero cross coupling dynamic coefficients. It has been found that the dynamic properties of TPJBs are frequency dependent. However, in engineering practice, analysis of rotor dynamics is based on the values determined for the frequency which corresponds to the shaft speed (synchronously reduced dynamic coefficients). These properties can be different at the system's natural frequency, or at frequencies of other excitations in the system.

The paper analyzes the results of bearing stiffness and damping from experimental and theoretical investigations. It has been found that the variations of tilting-pad bearings stiffness

and damping properties with frequency of excitation depend on the bearing operating conditions, and can be very significant.

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## 1.0 NOMENCLATURE

| | |
|---|---|
| $m_s$ | mass of rotor |
| $f_x$, $f_y$ | components of hydrodynamic force |
| $W_x$, $W_y$ | components of bearing load |
| $t$ | time |
| $x_j$,$y_j$ | shaft center coordinates in the rectangular system with the origin at the bearing equilibrium position |
| $k_{xx}$, $k_{xy}$, $k_{yx}$, $k_{yy}$ | bearing stiffness coefficients |

$c_{xx}$, $c_{xy}$, $c_{yx}$, $c_{yy}$ bearing damping coefficients

$m_p$ mass of pad

$\omega_{exc}$ frequency

$m_b$ bearing mass

$f_{d,x,}$, $f_{d,y}$ components of the dynamic force (excitation)

## 2.0 INTRODUCTION

Dynamics of high speed rotating machinery depend strongly on journal bearings. Currently, tilting pad journal bearings (TPJB) are dominant as shaft support in such machinery. This is mainly because of the following two characteristics of the TPJBs: 1) freedom from self-excited vibration, and 2) tolerance to misalignment.

Rotordynamic analysis is based on predicted bearing linear dynamic coefficients, which are determined assuming synchronous shaft vibrations (synchronously reduced stiffness and damping coefficients). However, some experimental and theoretical studies of tilting pad journal bearings indicated a certain effect of excitation frequency on the bearing stiffness and damping coefficients (1,2). Other reported results show rather limited dependency on frequency, or are inconclusive (3). If the bearing dynamic properties do depend on destabilizing frequency, it is important to also know the properties, which correspond to the first natural frequency. In addition, non-synchronous forces, such as those associated with internal flow, or due to magnetic effects in generators, may also affect rotor-bearing dynamics.

Obtaining a reliable estimate of the journal bearing dynamic properties has always been a challenging task. The literature documents relatively few cases of such attempts. Using external synchronous loads to excite the test bearing, Glienicke (4) evaluated stiffness and damping properties after measuring the applied load and the resulting shaft orbit. A similar technique was used by Morton (5), who used non-synchronous excitation in both the vertical and horizontal directions. Parkins (6), Brockwell and Dmochowski (7) used a more direct method of measuring the bearing dynamic properties of journal bearings by generating two distinct, straight line orbits of vibration. To obtain such orbits, the magnitude of two oscillating forces, as well as the phase difference between them, were carefully adjusted. The main advantage associated with this method was that it avoided the measurement of the phase angle between the vectors of shaft displacement and excitation force.

These methods represent time domain techniques. Unfortunately, it was found that a small error in triggering or in the measured phase angle could result in large errors in the calculated values. Frequency domain techniques overcome these problems. Burrows and Sahinkaya (8) and Rodriguez and Childs (1) used frequency domain algorithms for dynamic testing of bearings.

This paper analyzes the variations of the stiffness and damping coefficients for the tilting pad journal bearings with the frequency of excitation and describes the analytical and experimental techniques used to evaluate these properties.

## 3.0 EVALUATION OF THE DYNAMIC COEFFICIENTS FOR THE TILTING-PAD JOURNAL BEARINGS

### 3.1 Bearing linear dynamic coefficients

For a rigid shaft we may write the following two equations of motion (Figure 1):

$$
\begin{aligned}
m_s \frac{d^2 x_j}{dt^2} + f_x(t) + W_x &= 0 \\
m_s \frac{d^2 y_j}{dt^2} + f_y(t) + W_y &= 0
\end{aligned}
\qquad (1)
$$

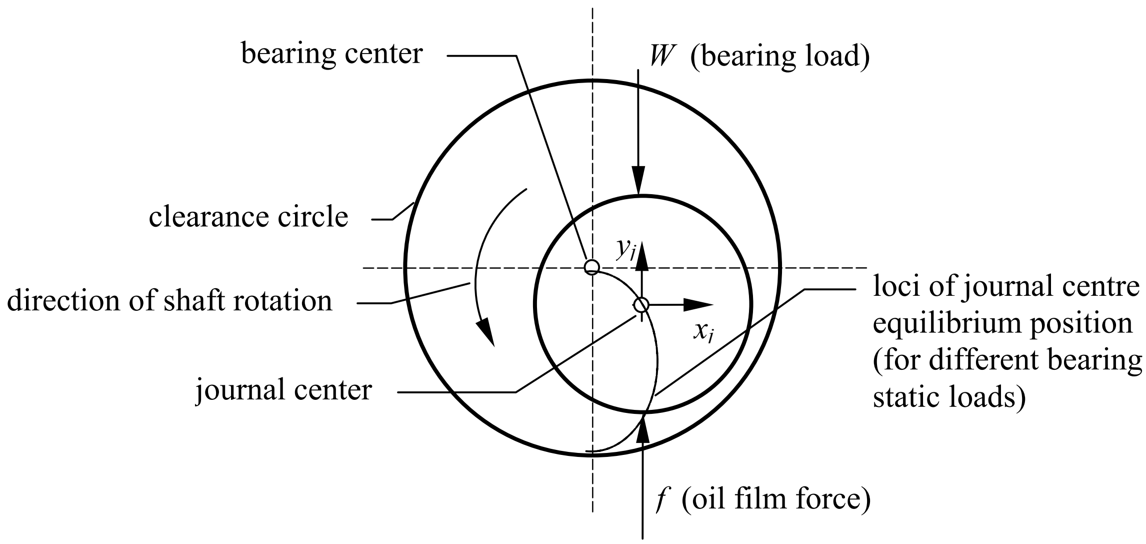



Fig. 1 Coordinate system for evaluation of journal bearing dynamic coefficients

For a small vibration around the shaft equilibrium position, we may assume that the resultant bearing force is linearly proportional to the journal displacements $x_j$ and $y_j$, and velocities $\dot{x}_j$ and $\dot{y}_j$. The equations (1) can then be written as follows:

$$
\begin{aligned}
&m_s \frac{d^2 x_j}{dt^2} + k_{xx} x_j + k_{xy} y_j + c_{xx} \dot{x}_j + c_{xy} \dot{y}_j = 0 \\
&m_s \frac{d^2 y_j}{dt^2} + k_{yx} x_j + k_{yy} y_j + c_{yx} \dot{x}_j + c_{yy} \dot{y} = 0
\end{aligned}
\qquad (2)
$$

The coefficients with the indices *xx* and *yy* are called direct stiffness and damping coefficients, while those with the indices *xy* and *yx* are referred to as cross-coupling coefficients. The latter represent a relationship of the hydrodynamic force component in the horizontal or vertical direction and the journal motion in the direction perpendicular to this component. Figure 1 illustrates the journal centre path resulting from changes to the bearing static load. The action of the cross-coupling stiffness coefficients is responsible for instability of hydrodynamic bearings (half-frequency whirl, oil whip).

For each of the pads of tilting-pad journal bearing its hydrodynamic force passes through the pivot line (Figure 2). If the bearing pads are symmetrically arranged, the bearing practically has zero cross-coupling stiffness, and thus is inherently stable.

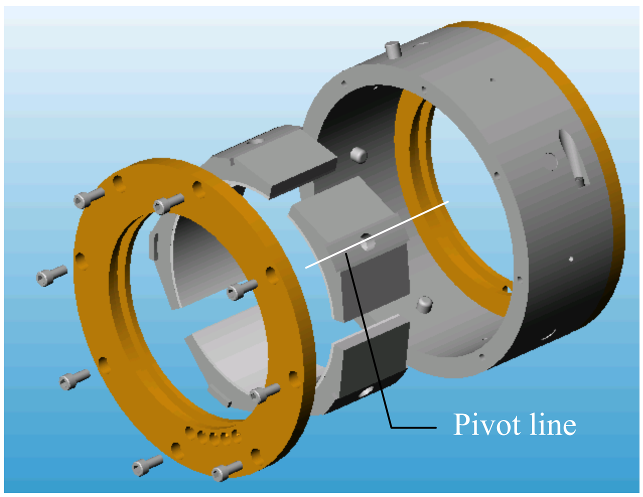



Fig. 2 Tilting-pad journal bearing

### 3.2 Computer model

Calculations of the bearing dynamic properties have been based on a three-dimensional model of tilting-pad journal bearings, though all the major equations are reduced to a two-dimensional form. The model has been described elsewhere (9). Here, only a brief description is given.

A finite length pressure equation (circumferential and axial directions) allows for viscosity variations in circumferential direction and across the oil film. Turbulent flow is also accounted for by including Reynolds number effect. The temperature and viscosity fields are obtained from a two-dimensional energy equation, which accounts for heat conduction in radial direction and heat convection circumferentially. Oil mixing in the bearing cavities as well as hot-oil carry over is also included in the analysis. Heat conducted through the pad is calculated from the Laplace equation, which accounts for heat conduction in circumferential and radial directions. The model also calculates both the thermal and elastic distortions of the individual pads.

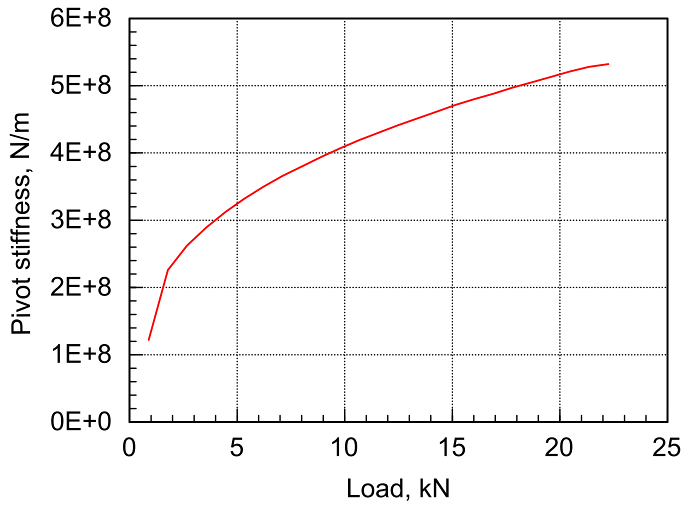



Fig. 3 Typical pivot stiffness of a 100 mm TPJB

The above hydrodynamic considerations allow for calculation of the stiffness and damping coefficients for fixed pads. They have been used to calculate the coefficients for tilting pads applying the technique described by Lund (10). This technique takes into considerations the mass and the excitation frequency of the tilting pad. Typically, calculated bearing dynamic properties are obtained after assuming that shaft mass can be neglected and that both shaft and pad motions are synchronous with shaft speed. Indeed, mass forces associated with tilting of the pads are negligible when compared with viscous forces. However, pivot stiffness can be of the same order of magnitude as the oil film stiffness, and thus can play an important role in the bearing dynamic properties. Figure 3 shows pivot stiffness for a typical 100 mm diameter bearing calculated using the formulae given by Kirk and Reedy (11). Thus, each pad can be represented by the mass, spring, and damper elements, as shown in Figure 4.

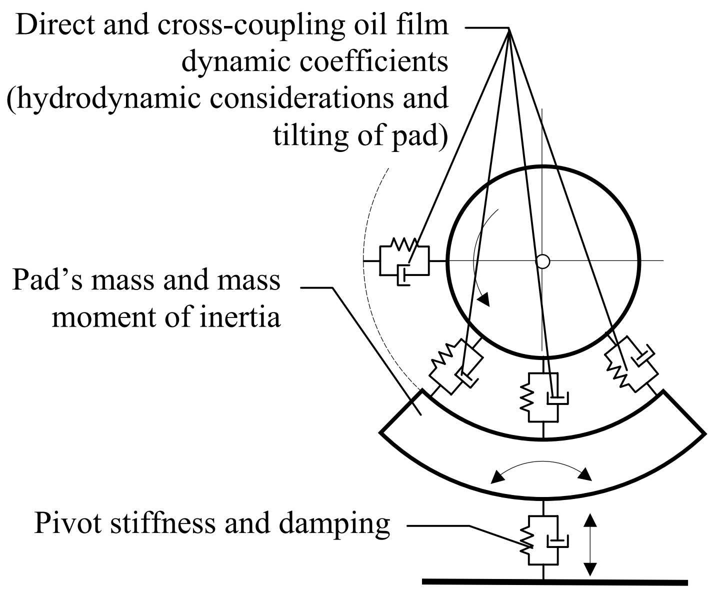



Fig. 4 Mass, spring, and damper elements for tilting pad

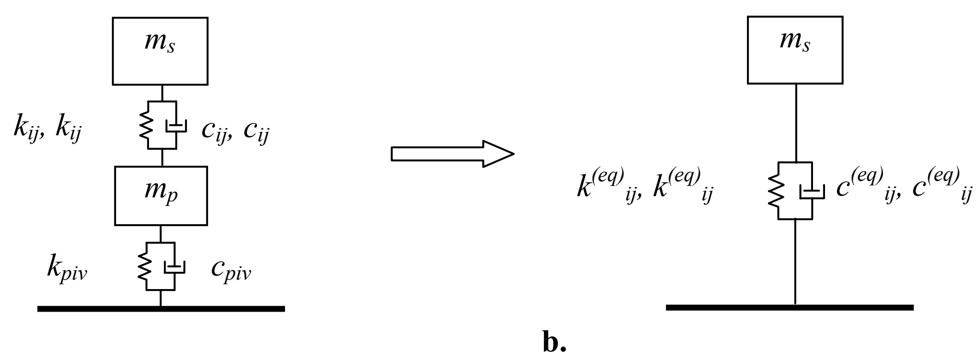



Fig. 5 Mass-spring-damper model for pads with flexible pivot (**a**) and equivalent bearing-shaft system (**b**)

The schematic shown in Figure 4 illustrates the mass-spring-damper system for the pad as well as the equivalent system to calculate the effective pad coefficients.

The shaft experiences the combined action of all the elements of the system represented by equivalent stiffness and damping coefficients. From the consideration of the systems shown in Figure 5 the equivalent coefficients can be evaluated from formulae (5).

$$k_{ij}^{(eq)} = \frac{k_{ij} k_z \left(k_{ij} + k_z\right) - \omega_{exc}^2 k_z c_{ij}^2}{\left(k_{ij} + k_z\right)^2 + \omega_{exc}^2 c_{ij}^2}$$

$$c_{ij}^{(eq)} = \frac{k_z^2 c_{ij}}{\left(k_{ij} + k_z\right)^2 + \omega_{exc}^2 c_{ij}^2} \tag{5}$$

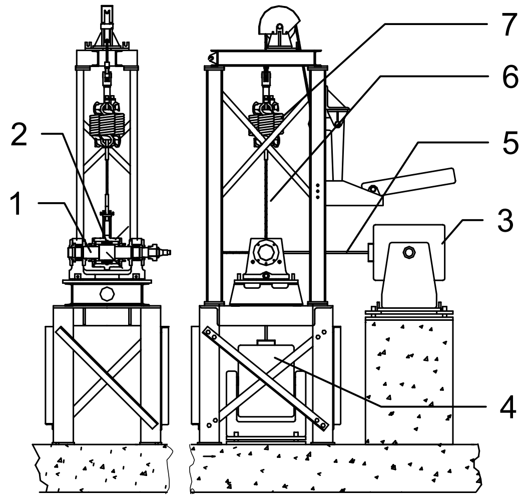
where

$$k_z = m_p \omega_{exc}^2 + k_{piv}$$

$i,j$ are x or y

### 3.3 Experimental investigation

The NRC's test rig, which is shown in Figure 6, utilizes the concept of a fixed rotating shaft (1) and a free vibrating test bearing (2). Two orthogonal electro-magnetic shakers (3) and (4) apply dynamic loads to the stator, and the bearing's response is measured. Each shaker is attached to the bearing housing through a steel rod and a flexible element assembly (5) that prevents any constraints of the housing in a direction perpendicular to the shaking force. The shaft is supported on high precision, angular ball bearings. A tensioned cable (6) applies a static load. Soft springs (7) minimize the effect of bearing vibration on the applied static load.




Side View Front view

Fig. 6 Journal bearing dynamic test rig

**Table 1 Rig specifications**

| | |
|---|---|
| Shaft speed | 0-16,500 rpm |
| Journal diameter | up to 0.09843 m (3.875 in) |
| Static load | up to 20 kN (4,500 lbf) |
| Lubricant flow | up to 0.44 $m^3/s$ (7 USgpm) |
| Lubricant inlet temperature | Up to 70°C (158 °F), |
| Power of the motor (electric ,variable speed) | 37 kW (50 HP) |

The shakers have been programmed to provide a multifrequency excitation. In the presence of external excitation, equations of motions (2) become

$$\begin{aligned} m_b\ddot{x}(t)+k_{xx}x(t)+k_{xy}y(t)+c_{xx}\dot{x}(t)+c_{xy}\dot{y}(t)&=f_{d,x}(t)\\ m_b\ddot{y}(t)+k_{yx}x(t)+k_{yy}y(t)+c_{yx}\dot{x}(t)+c_{yy}\dot{y}(t)&=f_{d,y}(t)\end{aligned} \tag{6}$$

The measurement of the bearing/shaft displacement within the bearing clearance is corrupted by noise. Collecting a certain number of records, and using their average can minimize this effect. However, averaging in time domain is not appropriate for random signals, and frequency domain techniques should be applied. Evaluation of the bearing dynamic properties using frequency domain techniques allows for minimization of noise effects and the errors associated with triggering.

After introducing Fourier transforms, equations (6) become

$$\begin{aligned} F_x(\omega) - m_b A_x(\omega) &= H_{xx} X(\omega) + H_{xy} Y(\omega) \\ F_y(\omega) - m_b A_y(\omega) &= H_{yx} X(\omega) + H_{yy} Y(\omega) \end{aligned} \tag{7}$$

where

$$H_{ij} = k_{ij} + i\omega c_{ij} \tag{8}$$

is the frequency response function, and

$F_x, F_y, A_x, A_y, X, Y$ are Fourier transforms of excitation forces in the horizontal and vertical directions, accelerations in the horizontal and vertical directions, and displacements in the horizontal and vertical directions, respectively.

The two Equations (7) contain four unknowns $H_{ij}$. Thus, an additional independent excitation is required to obtain two more equations, needed to calculate the bearing dynamic properties.

The Power Spectral Density method, which is described in (12), has been used to evaluate the bearing stiffness and damping coefficients. The bearing dynamic properties have been determined from 32 consecutive records. Each record consisted of 512 samples of each measured variable (6 channels), collected over a period of 0.1 s.

## 4.0 RESULTS AND DISCUSSION

In order to evaluate the frequency effects on the bearing dynamic properties calculations, an experimental investigation has been carried out for two operating conditions, which are typical for high speed rotating machinery. The bearing parameters and operating conditions are shown in Table 2. For both cases pivot stiffness is similar and its variation with load is shown in Figure 3.

**Table 2 Bearing parameters and test conditions**

| Parameter | Case 1 | Case 2 |
|---|---:|---:|
| Bearing type | 5 pad TPJB | 5 pad TPJB |
| Pad configuration | Load-between-pads (LBP) | Load-on-pad (LOP) |
| Nominal diameter | 98.5 mm (3.88 in.) | 100 mm (4.0 in) |
| Length/diameter ratio (L/d) | 0.4 | 1.0 |
| Preload (see ref. 9 for definition) | 0.3 | 0.3 |
| Bearing load | 4.0 kN (900 lbf.) | 4.45 kN (1000 lbf.) |
| Shaft speed | 9,000 rpm | 9,000 rpm |

Case 1 represents a moderately loaded bearing. Figure 7 illustrates the measured and calculated variations of the real part of the frequency response function, equation (6), which represents bearing stiffness. The results show a certain frequency effect on the bearing stiffness properties; the coefficients decrease with the excitation frequency. For the excitation frequencies up to that of the shaft rotation (150 Hz), coefficients of stiffness are relatively constant. This effect is particularly clear for the vertical bearing stiffness, $k_{yy}$=Re($H_{yy}$), at higher frequencies of excitation.

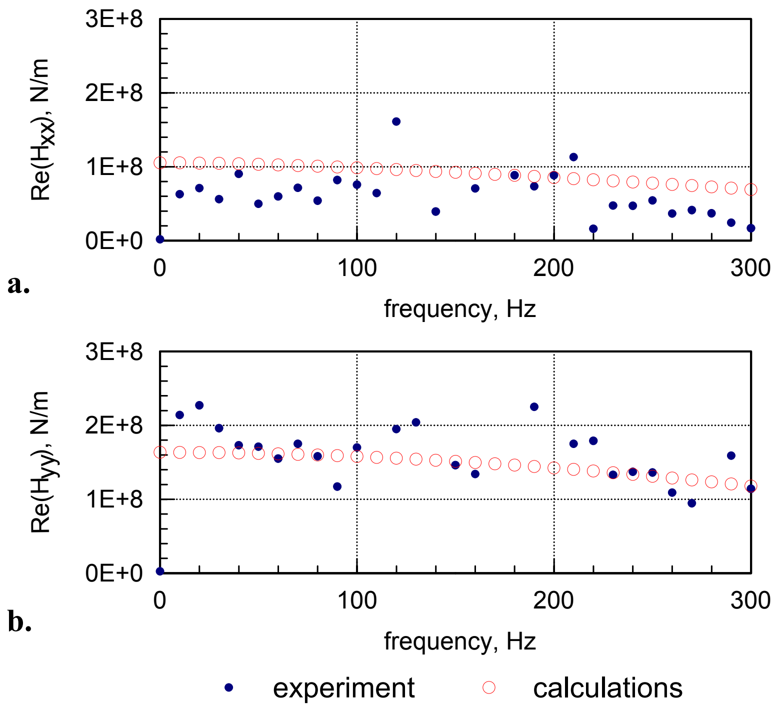



Fig. 7 Bearing direct stiffness coefficients: Case 1. **a.** horizontal **b.** vertical

The direct bearing damping coefficients for Case 1 are represented by the slope of the imaginary part of the frequency response function $H_{i,j}$ (Eq. 8) illustrated in Figure 8. Constant slope indicates negligible effects of both the pivot flexibility and the pad's mass.

Although the trends of the experimental results shown in Figures 7 and 8 are well defined, a certain spread of the results can be seen. Previous analysis has shown that the uncertainty for the stiffness and damping coefficients can exceed 10% and 15% respectively (13). Additionally, the results can be affected by the shaft flexibility.

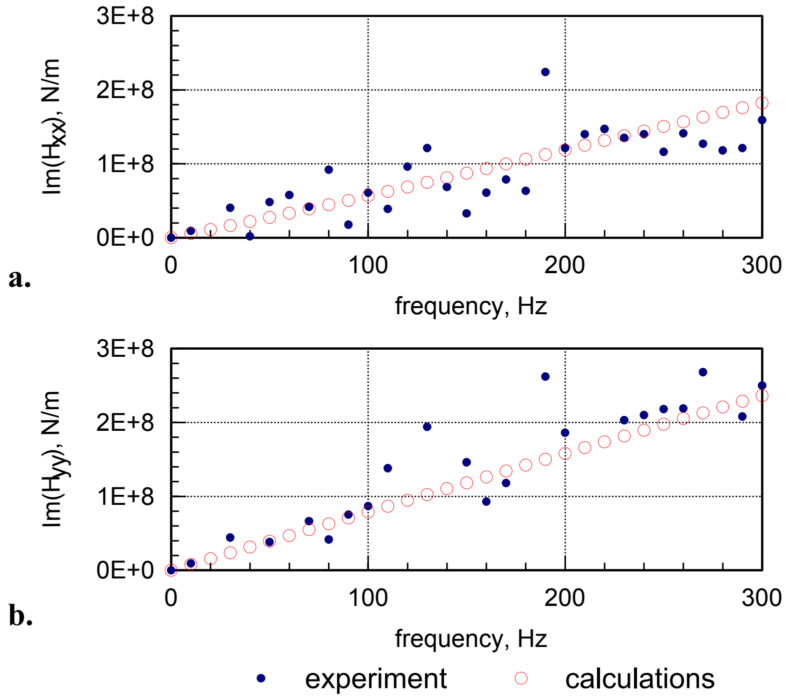



Fig. 8  Bearing direct damping properties: Case 1.  **a.** horizontal      **b.** vertical

With the validated model from Case 1, a different bearing dynamic behaviour has been observed in Case 2 (Table 2), which deals with similar bearing operating conditions and bearing geometry as those of Case 1, except for the bearing width.  With a L/d (length-to-diameter ratio) of 1.0, the bearing is considered to be lightly loaded, as the operating eccentricity was approximately 0.1 of the bearing radial clearance.  At such a low eccentricity, and with the preload of 0.3, each pad of the bearing operates under load.  In this situation, both the horizontal and vertical bearing direct stiffness coefficients increase with the frequency of excitation.

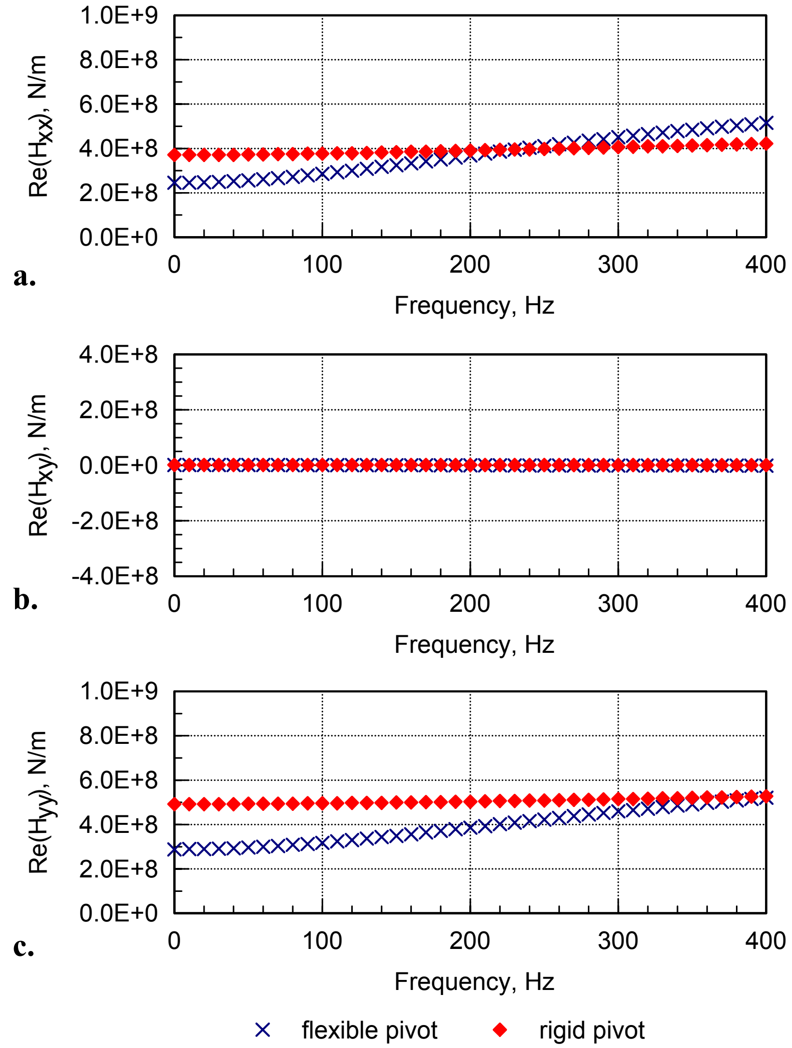



Fig. 9 Calculated bearing stiffness coefficients: Case 2.

**a.** horizontal **b.** cross-coupling **c.** vertical

As opposed to Case 1, frequency of excitation very strongly affected the bearing damping properties. Figure 10 shows that the imaginary part of the frequency response function levels out, which means a significant decrease in bearing damping, even at subsynchronous (with respect to the shaft rotational frequency) frequencies.

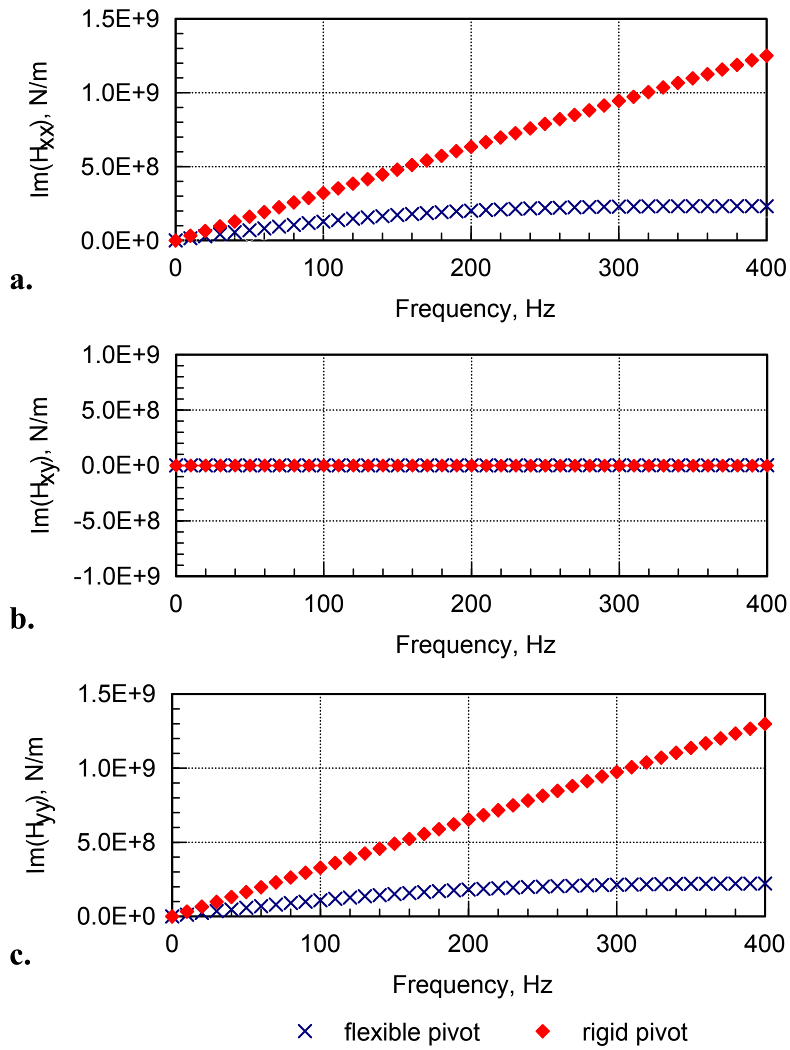



Fig. 10  Calculated bearing damping properties: Case 2.

**a.** horizontal  **b.** cross-coupling  **c.** vertical

Figures 9 and 10 also compare the effect of pad inertia in tilting and radial motion (enabled by pivot flexibility). It is the pad radial movement and pivot stiffness that lead to variations of the dynamic properties with frequency of excitation. Bearings with rigid pivots have constant stiffness and damping coefficients for the entire range of considered frequencies.

Figures 9b and 10b show that the pad flexibility and associated inertia forces do not affect the cross coupling coefficients for tilting-pad journal bearings.

## 5.0 CONCLUDING REMARKS

1. An inadequate knowledge of the bearing dynamic properties is behind many of the vibration problems in rotating machinery. This study points at frequency effects on the stiffness and damping coefficients of the tilting-pad journal bearings as one of the potential issues in rotordynamic stability analysis.
2. Pivot flexibility can have a significant effect on the TPJB's dynamic properties, in particular at higher frequencies of excitations.
3. As a result of pivot flexibility, the bearing stiffness coefficients can increase or decrease with the frequency of excitation, depending on operating conditions and bearing design.
4. In the presence of pivot flexibility, an increase in frequency of excitation can lead to a significant decrease in bearing damping in the horizontal as well as in the vertical directions.

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