TVM

Single Cash Flow
Xo = Xn/(1+r)^n
Xn = Xo*(1+r)^n

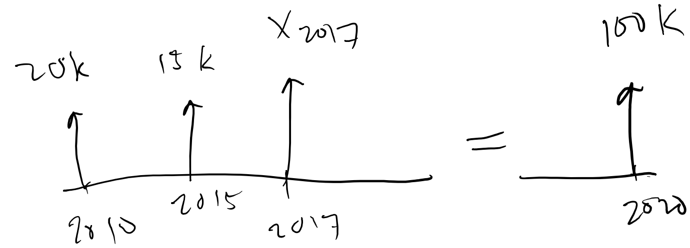
Multiple Cash Flow = sum of single CF

Perpetuity
Xo = X/r --> X = r*Xo --> r = Xo/P

Annuity
Xo = X/r (1 - 1/(1+r)^n)
Xn = X/r ((1+r)^n - 1)

Bonds:
coupon rate, n, r, par value, present value
DY = D/PV

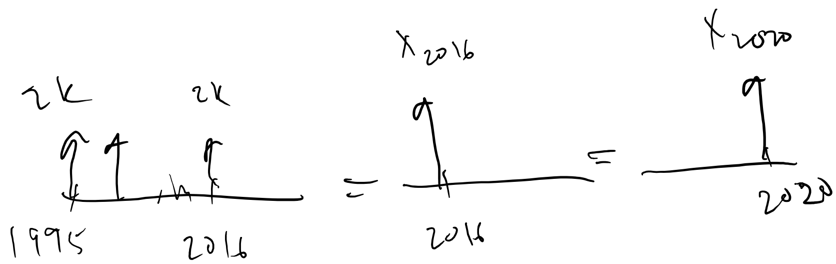
Ex1.



I invested 20k in 2010 and another 15k in 2015 at r=5%. How much would they be worth in 2020? If I want to have 100k in 2025, how much should I invest in 2017?

X2020 = 20 k*1.05^10 + 15k*1.05^5

100 = 20*1.05^15 + 15*1.05^10 + X*1.05^8
100 - (20*1.05^15 + 15*1.05^10) = X*1.05^8
33.988 = X*1.05^8
33.988 /1.05^8 = X
X = 23.0044 k --> X = 23,004.40



Ex2.

I deposited 2k per year starting end-1995 at r=10%. The last deposit was on end-2016. How much is it worth at the end of 2016? How much is it worth at the end of 2020?

X = 2k, n = 22, r = 10%
X2016 = X/r ((1+r)^n - 1) = 2000 / .10 * (1.10^22 -1) = 142,805.4988

X2020 = X2016*(1+r)^n = 142,805.4988*1.10^4 = 209,081.5308

Ex3.

Find the price of a bond that pays 5% coupon for 10 years if YTM = 10%.

n=10, r =10%, FV = 1,000 , C = 50

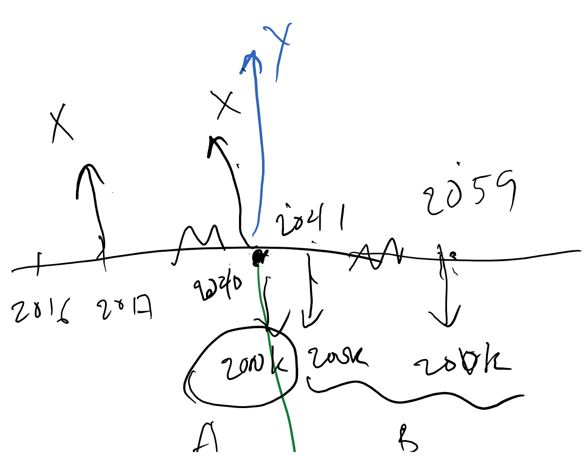
P = **C/r (1 - 1/(1+r)^n) + FV/(1+r)^n**
P = 50/.10* (1 - 1/1.10^10) + 1000/1.10^10 = 692.7716

Ex4
I plan to retire in end-2040 and I estimate that I would live for another 20 years. I also estimate that I would need 200k per year for 20 years. Today is end-2016 and I want to invest a fixed amount every year starting end-2017 at 3%. How much should I invest if my last investment is at end-2040?

Age at 2040 = 65
Age 2016 = 65 - (2040-2016) = 41

Let Y = value at 2040 of all $X deposits.
Let Z = value at 2040 of all 200k withdrawals.

--> Y = Z
Y = **X/r ((1+r)^n - 1), where r = 3%, n= 24**
Y = **X/.03* (1.03^24 - 1)**



Z = single CF + annuity for n-1 years instead of annuity due for n years!!!

Z = 200k + X/r (1 - 1/(1+r)^n) where r = 3%, n= 19

Z = 200k + 200k/.03 * (1 - 1/1.03^19) = 3,064.7598

Z = **3,064,759.80**

Y = Z

X/.03* (1.03^24 - 1) = 3,064,759.80
X = 3,064,759.80*.03 /(1.03^24 - 1)
X = 89,023.3527

Y = Z

X/.03* (1.03^24 - 1) = 3,064,759.80
X = 3,064,759.80*.03 /(1.03^24 - 1)
X = 89,023.3527

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If you start saving at age 28 and retires at 65 and lives for 20 more years, how much should be the saving per year? Same numbers as the previous question.

Z = single CF + annuity for n-1 years instead of annuity due for n years!!!

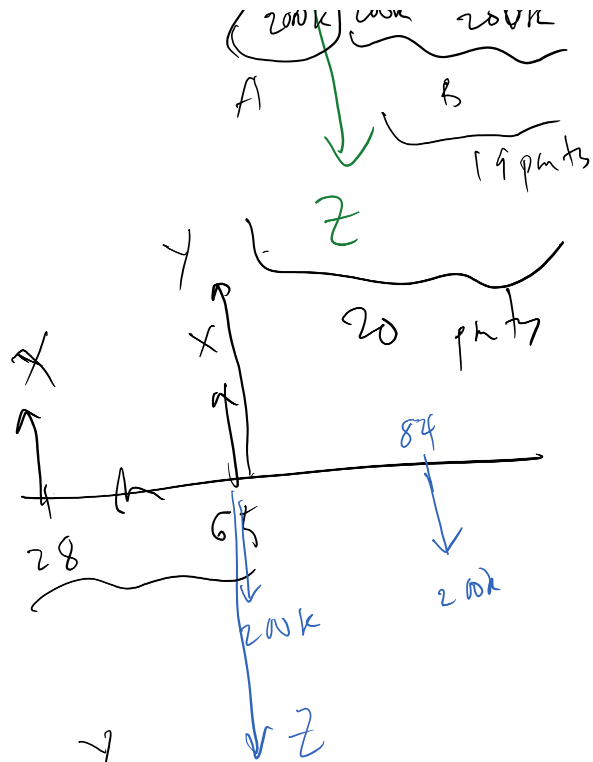
Z = 200k + X/r (1 - 1/(1+r)^n) where r = 3%, n= 19

Z = 200k + 200k/.03 * (1 - 1/1.03^19) = 3,064.7598

Z = **3,064,759.80**

Y = **X/r ((1+r)^n - 1), where r = 3%, n= 65-28+1 = 38**
Y = **X/.03* (1.03^38 - 1)**

Y = Z
X/.03* (1.03^38 - 1) = 3,064,759.80
X = 3,064,759.80*.03 /(1.03^38 - 1)
X = 44,314.4044



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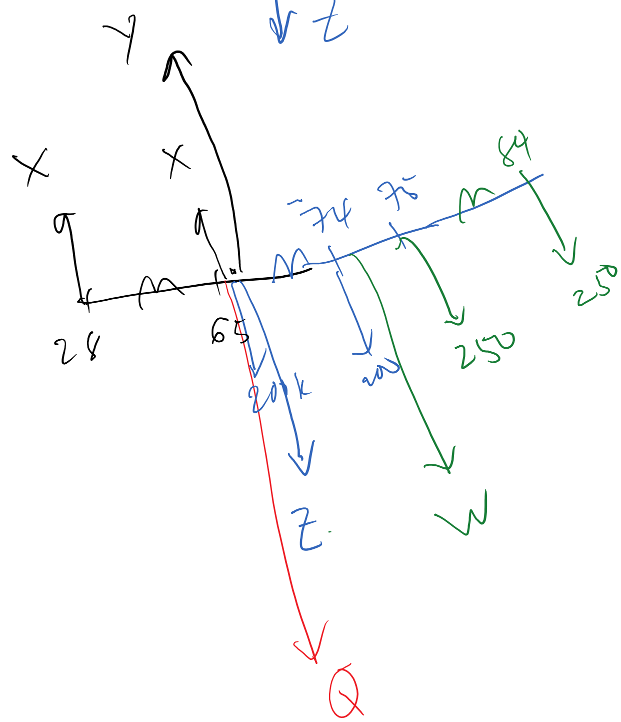
How would the answer change if instead of expenses of only 200k/year for 20 years, you need 200k/year for 10 years and 250k/yr for the next 10 years?



Let Y = value at age 65 of all $X deposits.
Let Z = value at age 65 of all 200k withdrawals.
Let W = value at age 74 of all 250k withdrawals.
Let Q = value of W at age 65

Y = Z + Q

Q = W/(1+r)^n , r =3%, n = 9
W = 250/r *(1 - 1/(1+r)^n) , r =3%, n = 10
W = 250/.03 *(1 - 1/1.03^10)
W = **2,132.5507 or 2,132,550.70**

Q = W/(1+r)^n = 2,132.55070 /1.03^9
Q = **1,634.42254 or 1,634,422.54**
--> The 250k payments from age75 to 84 is worth 1.6M at age 65.

Z = single CF + annuity for n-1 years instead of annuity due for n years!!!

Z = 200k + X/r (1 - 1/(1+r)^n) where r = 3%, n= 9
Z = 200 + 200/.03 *(1 - 1/1.03^9)
Z = **1,757.2218**
Z = **1,757,221.80**

Y = **X/r ((1+r)^n - 1), where r = 3%, n= 65-28+1 = 38**
Y = **X/.03* (1.03^38 - 1)**

--> Y = Z + Q

X/.03* (1.03^38 - 1) = 1,757,221.80 + 1,634,422.54
X = (1,757,221.80 + 1,634,422.54) *.03 / (1.03^38 - 1)
X = **49,040.9391**

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Annuity

$X_o = X/r\ (1 - 1/(1+r)^n)$
$X_n = X/r\ ((1+r)^n - 1)$

Lesson: Use annuity formulas + single CF to analyze bonds.
Price --> Xo, **Coupon --> X, maturity --> n , YTM --> r**

Perpetuity with no growth
$X_o = X/r$ --> $X = r*X_o$ --> $r = X_o/P$

Perpetuity with growth
$X_o = X/(r-g)$, $r > g$

Lesson: Use perpetuity formulas to analyze stocks. Dividends --> X
CAPM: $r = rf + Beta*MRP$