Sunday, September 11, 2016 8:32 PM

Q6

$$\lim_{x\to 0} x^2 \sin\frac{1}{x} = \underbrace{\lim_{x\to 0} x}_{0} \lim_{x\to 0} \overbrace{\frac{\sin\frac{1}{x}}{1/x}}^{1} = 0$$

$$\boxed{\lim_{y\to\infty} \frac{\sin y}{y} = 0}$$

let $y = \frac{1}{x}$

as $x \to 0 \to y \to \infty$

$$\lim_{x\to 1} (x-1)\sin\left(\frac{\pi}{x-1}\right)$$

$$\lim_{x\to 1} \pi \cdot \underbrace{\frac{\sin(\pi/(x-1))}{\pi/(x-1)}} = \pi \cdot 0 = 0$$

Let $y = \pi/x-1$ as $x \to 1$, $y \to \infty$

$\Rightarrow \lim$

$$\lim_{y\to 0} \frac{\sin y}{y} = \lim_{y\to 0} \frac{\cos y}{1} = 1$$

$$\lim_{y \to 0} \frac{\sin(ay)}{by} = \lim_{y \to 0} \frac{a \cancel{\cos ay}}{b} = \frac{a}{b}$$

$$\lim_{t \to 0} (1+2t)^{3/t}$$

L'hopitals:

$$\frac{0}{0}, \frac{\infty}{\infty}, 1^{\infty},$$

Let $y = (1+2t)^{3/t}$

$$\ln y = \frac{3\ln(1+2t)}{t}$$

$$= \frac{3\left(\frac{1}{1+2t}\right) \cdot 2}{1}$$

$$\ln y = 6$$

$$\boxed{y = e^{6}}$$

$$\int \frac{-1}{x^2+7x-10}\,dx$$

$$= \int \frac{-1}{(x\quad)(x\quad)}\,dx$$

$x^2+7x-10=0$

$b^2-4ac = 49+40 = 89$

$x = \frac{-7 \pm \sqrt{89}}{2(1)} \Rightarrow a = \frac{-7+\sqrt{89}}{2}$

$b = \frac{-7-\sqrt{89}}{2}$

$$-\int \frac{dx}{(x-a)(x-b)}$$

$$\frac{1}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$$

$$1 = A(x-b) + B(x-a)$$

let $x=a \Rightarrow A = \frac{1}{a-b}$ ✓

let $x=b \Rightarrow B = \frac{1}{b-a}$ ✓

$$\int \frac{A}{x-a}\,dx + \int \frac{B}{x-b}\,dx$$

$$= A\ln(x-a) + B\ln(x-b) + C$$

8) $\int \frac{3x+2}{x^2-2x+1}\,dx$

$$\frac{3x+2}{x^2-2x+1} = \frac{A}{x-1} + \frac{Bx}{(x-1)^2}$$

$3x+2 = A(x-1) + Bx$

let $x = 0 \rightarrow A = -2$

let $x = 1 \rightarrow B = 5$

$$\int \frac{-2}{x-1}\,dx + \int \frac{5x}{(x-1)^2}\,dx$$

let $\boxed{u = x-1} \rightarrow du = dx$

$\rightarrow x = u+1$; $\frac{u+1}{u^2} = \frac{u}{u^2} + \frac{1}{u^2}$

$$5\int \frac{u+1}{u^2}\,du$$

$$5\left\{\int \frac{1}{u}\,du + \int \frac{1}{u^2}\,du\right\}$$

$$\int \frac{x^3-3x-4}{x^2-3x-4}\,dx$$

$(x-4)(x+1)$

$u = x^2-3x-4$

$du = (2x-3)\,dx$

$1 \quad 0 \quad -3 \quad -4$

$$\frac{x^2}{2} + \frac{3}{4}x - \frac{3}{8}$$

$$2x-3 \,\big)\, x^3 - 3x - 4$$

$$x^3 - \frac{3x^2}{2}$$

$$+\frac{3x^2}{2} - 3x$$

$$\frac{3x^2}{2} - \frac{9}{4}x$$

$\frac{9}{4} - \frac{12}{4}$

$-\frac{3}{4}x \quad -4$

$-\frac{3}{4}x + \frac{9}{8}$

$\boxed{\text{not } 0}$

$$\begin{array}{r} x + 3 \\ x^2-3x-4 \overline{)\, x^3 - 3x - 4} \\ x^3 - 3x^2 - 4x \\ \hline 3x^2 + x - 4 \\ -\quad 3x^2 - 9x - 12 \\ \hline 10x + 8 \end{array}$$

$$\int \left[(x+3) + \frac{10x+8}{x^2-3x-4} \right] dx$$

$$\int (x+3)\,dx + \int \frac{10x+8}{x^2-3x-4}\,dx$$

do by P.F.

$$\frac{10x+8}{x^2-3x-4} = \frac{A}{x-4} + \frac{Bx}{x+1}$$

$$10x+8 = A(x+1) + Bx(x-4)$$

$$\frac{1}{2}\int 2e^{2x}(1+e^{2x})^{-1}\,dx$$

$u = 1+e^{2x} \quad du = 2e^{2x}\,dx$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln\left|1+e^{2x}\right| + C$$

12) $\int 5\underbrace{(1+\sin x)}^{5}\underbrace{(1-\cos^2 x)}\,\underline{\cos x\,dx}$

12.) $\int 5(\underbrace{1+\sin x})\underbrace{(1-\cos^2 x)}_{\sin^2 x}$ ...

$u = 1+\sin x \rightarrow \sin x = u-1$

$\rightarrow \sin^2 x = (u-1)^2$

$du = \cos x \, dx$

$5\int u^5 (u-1)^2 \, du \Rightarrow$ power rule

13.) $5\int \frac{x+3}{x+5} \, dx$

$5\left[\int \frac{x}{x+5} \, dx + \int \frac{3}{x+5} \, dx\right]$

$u = x+5 \quad du = dx$

$x = u-5$

$\int \frac{u-5}{u} \, du$

$\downarrow$

$\int \left(1 - \frac{5}{u}\right) du$

$$\int \frac{x+3}{x+5}\,dx$$

$u = x+5 \Rightarrow du = dx$

$\Rightarrow x+3 = u-2$

$$\int \frac{u-2}{u}\,du$$

$$\int \left(1 - \frac{2}{u}\right) du$$

$u - 2\ln u$ $\leftarrow$ $u = x+5$