

Source of Oil	Cost per Gallon	Daily Gallons Available
Texas well	0.30	12,000
Oklahoma well	0.40	20,000
California well	0.48	24,000

i	Xi (# gallons from well i to produce Blend A)	Yi (# gallons from well i to produce Blend B)
1	X1	Y1
2	X2	Y2
3	X3	Y3

i	BlendA	BlendB	
1	35%	20%	
2	50%	30%	
3	15%	40%	
Price	3.10	3.20	
Requirement	20000	20000	

Problem: We want to know how many gallons from well i is used to produce each type of oil blend that maximizes profit.

Decision variable:

Xi = # gallons from well i used in blend A
 Yi = # gallons from well i used in blend B

Define:

A = # gallons of blend A that is produced
 B = # gallons of blend B that is produced

$$\text{Revenue} = 3.10 \cdot A + 3.20 \cdot B$$

$$\text{Cost} = .30 \cdot (X1 + Y1) + .4 \cdot (X2 + Y2) + .48 \cdot (X3 + Y3)$$

$$\begin{aligned} X1 + Y1 &= \text{\# gallons from well \# 1} \\ X2 + Y2 &= \text{\# gallons from well \# 2} \\ X3 + Y3 &= \text{\# gallons from well \# 3} \end{aligned}$$

Objective function:

$$\text{Max Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Profit} = 3.10 \cdot A + 3.20 \cdot B - [.30 \cdot (X1 + Y1) + .4 \cdot (X2 + Y2) + .48 \cdot (X3 + Y3)]$$

$$\begin{aligned} A &= X1 + X2 + X3 \\ B &= Y1 + Y2 + Y3 \end{aligned}$$

$$\text{Max Profit} = 3.10 \cdot (X_1 + X_2 + X_3) + 3.20 \cdot (Y_1 + Y_2 + Y_3) - [.30 \cdot (X_1 + Y_1) + .4 \cdot (X_2 + Y_2) + .48 \cdot (X_3 + Y_3)]$$

By choosing $X_1, X_2, X_3, Y_1, Y_2, Y_3$

Subject to:

$$X_1 + Y_1 \leq 12000 \text{ (TX supply constraint)}$$

$$X_2 + Y_2 \leq 20000 \text{ (OK supply constraint)}$$

$$X_3 + Y_3 \leq 24000 \text{ (CA supply constraint)}$$

$$.35 / .65 = 0.5385$$

$$.15 / .85 = 0.1765$$

$$X_1 / A \geq 35\% \rightarrow X_1 \geq .35 \cdot (X_1 + X_2 + X_3) \rightarrow X_1 \geq .35X_1 + .35X_2 + .35X_3$$

$$\rightarrow .65X_1 \geq .35X_2 + .35X_3 \rightarrow$$

$$\rightarrow \mathbf{X_1 \geq 0.5385 X_2 + 0.5385 X_3}$$

$$.3 / .7 = 0.4286$$

$$X_2 / A \leq 50\% \rightarrow X_2 \leq .5 \cdot (X_1 + X_2 + X_3) \rightarrow .5X_2 \leq .5X_1 + .5X_3$$

$$\rightarrow \mathbf{X_2 \leq X_1 + X_3}$$

$$X_3 / A \geq 15\% \rightarrow X_3 \geq .15 \cdot (X_1 + X_2 + X_3) \rightarrow .85 X_3 \geq .15X_1 + .15 X_2$$

$$\rightarrow \mathbf{X_3 \geq .1765 X_1 + .1765 X_2}$$

$$Y_1 / B \geq 20\% \rightarrow Y_1 \geq .20 \cdot (Y_1 + Y_2 + Y_3) \rightarrow .80 Y_1 \geq .20Y_2 + .20 Y_3$$

$$\rightarrow \mathbf{Y_1 \geq .25 Y_2 + .25 Y_3}$$

$$Y_2 / B \geq 30\% \rightarrow Y_2 \geq .30 \cdot (Y_1 + Y_2 + Y_3)$$

$$\rightarrow Y_2 \geq .4286 Y_1 + .4286 Y_3$$

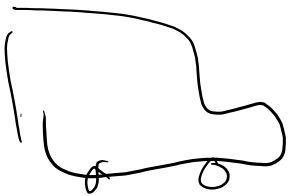
$$Y_3 / B \leq 40\% \rightarrow Y_3 \leq .40 \cdot (Y_1 + Y_2 + Y_3)$$

$$\rightarrow \mathbf{Y_3 \leq .6666 Y_1 + .6666 Y_2}$$

$$A \geq 20000 \rightarrow X_1 + X_2 + X_3 \geq 20000$$

$$B \geq 20000 \rightarrow Y_1 + Y_2 + Y_3 \geq 20000$$

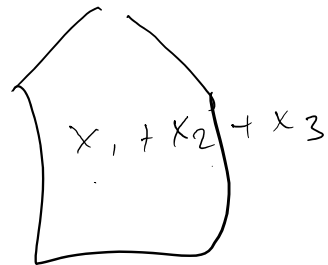
$$X_i, Y_i \geq 0$$



X_1



X_2



A

