Players: C, F

Goods: X, Y

Endowment: $(X_C, Y_C) = (0, 4)$ ✓

$(X_F, Y_F) = (4, 0)$ ✓

U-fn: $U(X,Y) = 2X^{1/2} + Y^{1/2}$

$\Rightarrow U_C = 2X_C^{1/2} + Y_C^{1/2}$

$U_F = 2X_F^{1/2} + Y_F^{1/2}$

$MU_x = \frac{dU}{dX} = 2 \cdot \frac{1}{2} \cdot X^{-1/2} = X^{-1/2}$

$MU_y = \frac{dU}{dY} = 1 \cdot \frac{1}{2} Y^{-1/2} = \frac{Y^{-1/2}}{2}$

$MRS = \frac{MU_x}{MU_y} = \frac{X^{-1/2}}{Y^{-1/2}/2}$

$= X^{-1/2} \cdot \frac{2}{Y^{-1/2}} = \frac{2X^{-1/2}}{Y^{-1/2}}$

$MRS = \frac{2Y^{1/2}}{X^{1/2}} = 2\left(\frac{Y}{X}\right)^{1/2}$

(P) $= \frac{P_x}{P_y} \Rightarrow P_x = P \cdot P_y$

a) Income: $0 \cdot P_x + 4 \cdot P_y = I_C$

$I_C = 4 \cdot P_y$

Expenditure: $P_x X_C + P_y Y_C$

Budget line: $4P_y = P \cdot P_y X_C + P_y Y_C$

Slope of BL: $\frac{P_x}{P_y} = P$ → y, x

slope of BL: $\frac{P_x}{P_y} = P$ → (*)

$MRS = 2\left(\frac{Y_c}{X_c}\right)^{1/2}$

Optimum: MRS = slope of BL!!

$2\left(\frac{Y_c}{X_c}\right)^{1/2} = P$

⇒ Solve for Y_c:

$\left(\frac{Y_c}{X_c}\right)^{1/2} = \frac{P}{2}$

$X_c\left(\frac{Y_c}{X_c} = \frac{P^2}{4}\right)X_c$

$Y_c = \frac{X_c P^2}{4}$ → plug into BL

$qP_y = PP_y X_c + P_y Y_c$

BL $\left[q = pX_c + Y_c\right]$

$q = pX_c + \frac{X_c P^2}{4}$

$q = X_c\left[p + p^2/4\right]$

$\boxed{X_c = \frac{q}{p + p^2/4}}$ ← demand curve

Solve for X_c

$X_c = \frac{4Y_c}{P^2}$

BL: $q = pX_c + Y_c$

$q = p\cdot\left[\frac{4Y_c}{P^2}\right] + Y_c$

$+ Y_c$

$q = Y_c\left[\frac{4}{P} + 1\right]$

$\boxed{Y_c = \frac{q}{\frac{4}{P} + 1}}$

$$X_C = \frac{4}{p + p^2/4}$$ ← demand curve

Income Friday: $4P_X + 0P_y = \underline{4P_x}$

Expenditure: $\underline{P_x X_F + P_y Y_F}$

BL: $4P_x = P_x X_F + P_y Y_F$

$\$L$: $4\left(\frac{P_x}{P_y}\right) = \left(\frac{P_x}{P_y}\right) X_F + Y_F \; ; \; p = \frac{P_x}{P_y}$

$$BL: 4p = pX_F + Y_F$$

MRS = slope of BL

$$2\left(\frac{Y_F}{X_F}\right)^{1/2} = p$$

Solve for Y_F

$$\left(\frac{Y_F}{X_F}\right)^{1/2} = \frac{p}{2}$$

$$\frac{Y_F}{X_F} = \frac{p^2}{4}$$

$$Y_F = \frac{X_F p^2}{4}$$

Solve for X_F

$$X_F = \frac{4Y_F}{p^2}$$

$$4p = pX_F + Y_F$$

$$qp = pX_F + Y_F$$

$$qp = pX_F + \frac{X_F p^2}{4}$$

$$qp = X_F\left(p + p^2/4\right)$$

$$X_F = \frac{qp}{p + p^2/4}$$

$$qp = pX_F + Y_F$$

$$qp = p \cdot \frac{qY_F}{p^2} + Y_F$$

$$qp = \frac{qY_F}{p} + Y_F$$

$$qp = Y_F\left(\frac{q}{p} + 1\right)$$

$$Y_F = \frac{qp}{q/p + 1}$$

$$X_C = \frac{q}{p + p^2/4} \qquad Y_C = \frac{q}{q/p + 1}$$

Endowment C: $(0, 4)$; F $(4, 0)$

Market clearing for X:

$$q = X_F + X_C$$

$$4 = \frac{4p}{p + p^2/4} + \frac{4}{p + p^2/4}$$

$$\left[1 = \frac{p}{p + p^2/4} + \frac{1}{p + p^2/4}\right)\left(p + p^2/4\right)$$

$$\left[p + p^2/4 = p + 1\right) 4$$

$$\left[4p + p^2 = 4p + 4\right]$$

$$p^2 = 4$$

$$p^2 = 4$$

$$p = 2$$

$$\Rightarrow X_F = \frac{4p}{p + p^2/4} = \frac{8}{2 + 2^2/4} = \frac{8}{3}$$

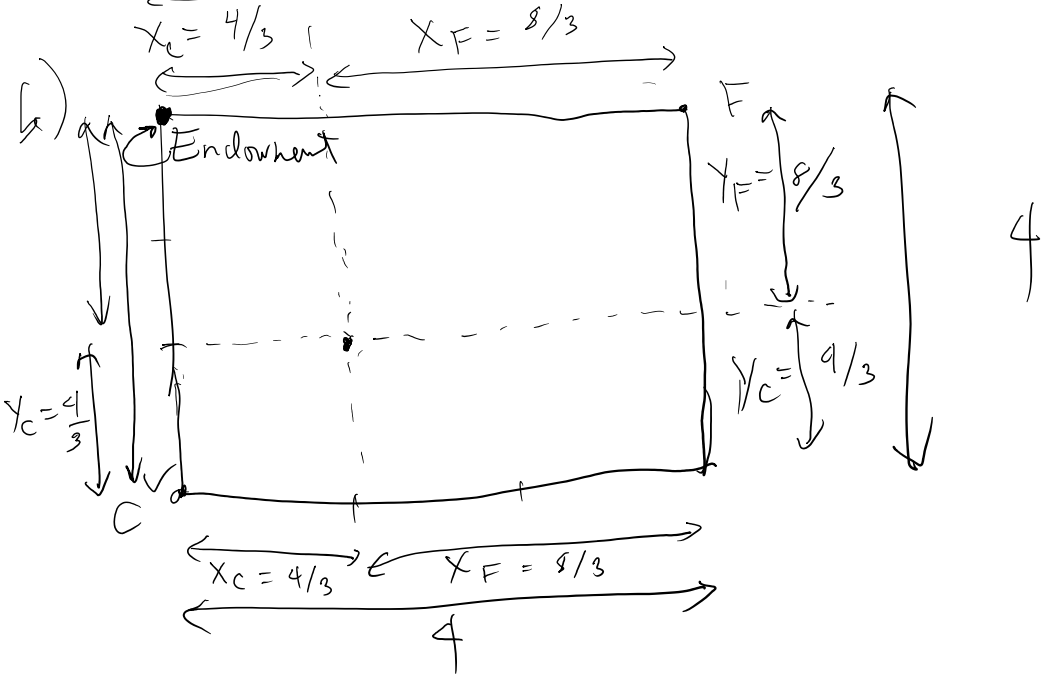
$$X_C = \frac{4}{p + p^2/4} = \frac{4}{3}$$

Check $X_F + X_C = \frac{8}{3} + \frac{4}{3} = \frac{12}{3} = 4$

$$Y_F = \frac{4p}{4/p + 1} = \frac{8}{4/2 + 1} = 8/3$$

$$Y_C = \frac{4}{4/p + 1} = \frac{4}{3}$$

$Y_F + Y_C = \frac{8}{3} + \frac{4}{3} = 4$



c) Autarky: $X_F = 4$, $Y_F = 0$ $\Rightarrow U_F = 2X_F^{1/2} + Y_F^{1/2}$

(no trade)

$U_F = 2 \cdot 4^{1/2} + 0^{1/2}$

$U_F = 4$ ✓

(no trade)

$U_F = 2 \cdot 4 + 0$

$U_F = 4$ ✓

$X_C = 0, Y_C = 4 \Rightarrow U_C = 2X_C^{1/2} + Y_C^{1/2}$

$U_C = 2 \cdot 0^{1/2} + 4^{1/2}$

$U_C = 2$ ✓

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with trade: $X_F = \frac{8}{3}, Y_F = \frac{8}{3}$.

$U_F = 2X_F^{1/2} + Y_F^{1/2}$

$= 2\left(\frac{8}{3}\right)^{1/2} + \left(\frac{8}{3}\right)^{1/2}$

$U_F = 4.90 > 4.00$

$X_C = 4/3, Y_C = 4/3$

$U_C = 2X_C^{1/2} + Y_C^{1/2}$

$= 2\left(\frac{4}{3}\right)^{1/2} + \left(\frac{4}{3}\right)^{1/2}$

$U_C = 3.46 > 2$

with $p = 2$, $\frac{4}{3}$ of X is imported by C & exported by F.

& $\frac{8}{3}$ of Y is imported by F from C

& exports by -.

If $P = 1.77$

$MRS = P$

$$2\left(\frac{Y_c}{X_c}\right)^{1/2} = 1$$

$$\left(\frac{Y_c}{X_c}\right)^{1/2} = \frac{1}{2}$$

$$\frac{Y_c}{X_c} = \frac{1}{4}$$

$$Y_c = \frac{X_c}{4}$$

$$X_c = 4Y_c$$

BL: $4 = X_c + Y_c$

$$4 = X_c + \frac{X_c}{4}$$

$$4 = X_c\left(1 + \frac{1}{4}\right)$$

$$4 = X_c + Y_c$$

$$4 = 4Y_c + Y_c$$

$$4 = Y_c(4 + 1)$$

$$Y_c = \frac{4}{}$$

$$x_C = \frac{4}{1.25}$$

$$x_C = 3.2$$

$$y_C = \frac{4}{5}$$

$$y_C = .8$$

$p = 1$

$$MRS = p$$

F: $$2\left(\frac{y_F}{x_F}\right)^{1/2} = 1$$

$$y_F = \frac{x_F}{4}$$

$$x_F = 4y_F$$

BL: $$4 = x_F + y_F$$

$$4 = x_F + \frac{x_F}{4}$$

$$4 = x_F + y_F$$

$$4 = 4y_F + y_F$$

$$4 = 5y_F$$

$$4 = X_F\left(1 + \frac{1}{4}\right)^4$$

$$X_F = \frac{4}{1.25}$$

$$4 = 5^{-Y_F}$$

$$Y_F = \frac{4}{5}$$

$$\rightarrow X_F = 3.2$$

$$\rightarrow X_C = 3.2$$

$$Y_F = .8$$

$$Y_F = .8$$

Shortage of X

surplus of Y

$\Rightarrow$ No eqm at $p = 2.11$

$\Rightarrow$ Trade is <u>not</u> possible when $p = 1$.

$\Rightarrow$ No trade $\Rightarrow$ Autarky

U_F & U_C under autarky