Random variables

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You should first know what random variable is and how to generate them. But let's define things first. Start with a simple **experiment, which is an activity with random outcomes,** like drawing a card from a deck of cards, or throwing a dice or coins. Let's say the experiment is throwing three coins.

The **sample space** is the set of all **possible** outcomes. Hence S = {HHH, HHT, … , TTT}. There should be 8 outcomes.

The **event** is a subset of the sample space. We could have an event A where the first coin is head and so A = {HHH, HHT, HTH, HTT} , or we could have B as the event where we get 1 tail, and so B={THH, THT, HTT}. You could always define your own event that interests you.

In simplest terms, **the random variable is just a way to assign numbers to events**. So for example, we define the random variable X = number of heads, then each outcome in the sample space would be assigned a number. Hence, HHH would be 3, HTH would be 2, TTT would be 0, and so on.

Now a random variable X = 1 would mean {HTT, THT, TTH}. Naturally we want to get the probability that a random variable would take some values. In the experiment where we toss 3 coins, P(X=0) = 1/8 since X=0 would be {TTT}. The table below illustrates the concept.

X	Outcomes	P(X)
0	TTT	1/8
1	HTT, THT, TTH	3/8
2	HHT, HTH, THH	3/8
3	HHH	1/8
		Σ = 1

Now the sum of all probabilities must be 1. The plot of X and P(X) is called probability mass function. I don't know if you did probability cumulative function or not but it would be best if you send me your notes on previous class so I can see what you are doing.

The example above is **discrete**. If we toss 3 coins and we define X = the # heads, then the possible values of X = {0,1,2,3}. These are all discrete or countable numbers and X is therefore a discrete random variable. As such, there is no way we get X = 1.15 in a three coin toss.

Other examples of discrete random variables would be the number of cars parked in the school garage, Y = 0, 1, …, n, which is the capacity of the garage}. We could have Z = number of accidents in the ABC avenue for the year, Z = {0, 1, …, infinity} since practically there is no limit to the # of accidents for the year.

Continuous random variables are that can take continuous (not only countable) numbers. Example: T = temperature of the room. My guess is that you are limited to discrete random variables since you have no calculus yet.

The harder part in here is counting. For simple experiments like the one above, it is easy to count and list the outcomes. We see that the probability that X=1 for example is just 3/8 since there are 3 outcomes that have 1 head and there are 8 possible outcomes when we toss 3 coins.

Hence we should know how to count. The basic principle is multiplication rule. Understanding this is the key. The permutation and combination formula comes from here.

Multiplication Rule: The number of ways of obtaining k objects where the first object has n1 possible choices, the 2nd object has n2 possible choices, … , and the kth object has nk possible choices is N = n1 *n2* …*nk. Note: * means times. I don't want to use x for times since x is used a lot as a variable.

Example:

You go to a restaurant and you want to eat. A meal consists of 1 entrée, 1 desert, and 1 beverage. There are 4 entrees to choose from, there are 6 deserts to choose from, and there are 7 drinks to choose from. In how many ways can you choose a meal.

Here k = 3 because you have 3 objects (entrée, desert, and drink). N1 = 4, n2 = 6, and n3 = 7. So the number of ways to choose a meal is N = n1*n2*n3 = 4*6*7 = 168. Simple, right?

Now sometimes arrangement matters. Permutation means arrangement. There is formula for that but I want you to see that multiplication rule is more powerful since that is the basis of the permutation formula. The following problem shows the intuition behind the formula.

Suppose you want to sit 5 people in a row of 5 seats drawn below for you to visualize it.

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All five people are standing right now. Pick any seat for the first seat (I would pick the leftmost, but it doesn't matter what I pick). In how many ways can you fill the first seat? Well there are 5 ways since there are 5 people standing to choose from.

Pick a 2nd seat. In how many ways can you fill the 2nd seat? Well there are 4 ways because there are only 4 people standing to choose from. Remember 1 person is already given a seat.

Pick a 3rd seat. In how many ways can you fill the 3rd seat? Well there are 3 ways because there are only 3 people standing to choose from. Remember 2 people are already given a seat.

And so on until the last seat. Hence we end up with 5*4*3*2*1, which is by definition simply 5! = 120.

Note that k! = k*(k-1)*(k-2)*…*2*1. Hence 4! = 4*3*2*1 , and 100! = 100*99*98*…*3*2*1. Note that 0! = 1.

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How many ways can you sit 10 people in a row of 4 seats?

All 10 people are standing right now. Pick any seat for the first seat (it doesn't matter what you pick). In how many ways can you fill the first seat? Well there are 10 ways since there are 10 people standing to choose from.

Pick a 2nd seat. In how many ways can you fill the 2nd seat? Well there are 9 ways because there are only 9 people standing to choose from. Remember 1 person is already given a seat.

Pick a 3rd seat. In how many ways can you fill the 3rd seat? Well there are 8 ways because there are only 8 people standing to choose from. Remember 2 people are already given a seat.

Pick the last seat. In how many ways can you fill the 4th seat? Well there are 7 ways because there are only 7 people standing to choose from. Remember 3 people are already given a seat.

Hence we end up with 10*9*8*7 = 5,040.

Your permutation formula says that the number of ways to arrange r objects from n objects is nPr = n!/(n-r)!.

The first problem is arranging 5 people in 5 seats so n=r = 5 and 5P5 = 5! / 0!   = 5!/1 = 120
The 2nd problem is 10P4 = 10! / (10-4)! =10! / 6! = 10*9*8*7*6*…*1 / (6*…*1) = 5,040 . Notice how 6! cancelled and you are left with 10*9*8*7 in the numerator. That is the essence of multiplication rule.

Next we do combination.