Introduction to vectors in plane

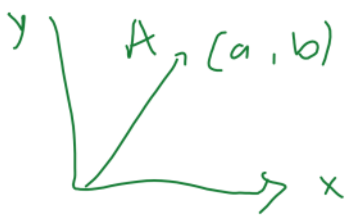
Thursday, December 14, 2017 11:17 PM

III. Two-Dimensional Motion
1. Vectors a. Graphical Solution b. Analytical Solution
2. Parallel and Perpendicular Components of Velocity and Acceleration
3. Relative Velocity
4. Projectile Motion
5. Application of Newton's Laws of Motion a. Systems b. Inclined Planes
6. Circular Motion a. Uniform Circular Motion b. Non-uniform Circular Motion

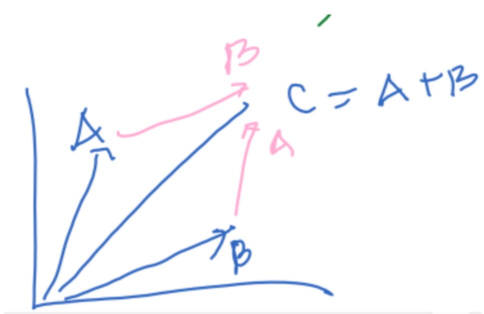
Vector vs Scalar
Vector - magnitude + direction (ex. Velocity)
Scalar - magnitude only (ex. Speed)

Notation:
A = vector , A = < a, b > or A = (a, b)
|A| = magnitude of A = sqrt(a^2 + b^2) = length of the arrow

Use Pythagorean theorem on right triangles.



Vector addition
Ex. A = (2,3) , B = (4,1) --> A+B = (2+4, 3+1) = (6, 4)



Scalar multiplication
Ex. A = (2,3), C = 2A = (2*2, 2*3) = (4,6)

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Components of single vector
|Ax| = |A |cos(Θ), |Ay| = |A| sin(Θ)

Again, by Pythagorean theorem --> |A|^2 = |Ax|^2 +|Ay|^2

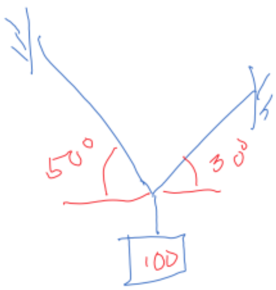
Unit vectors - vector with magnitude of 1. We can represent any vector as sum of vectors involving unit vectors. Let i= <1,0> , which is the unit vector in X -axis, and j = < 0, 1> , which is the unit vector in the y -axis.

Then Ax = |Ax|i and Ay = |Ay|j, and A = Ax + Ay = |Ax|i + |Ay|j

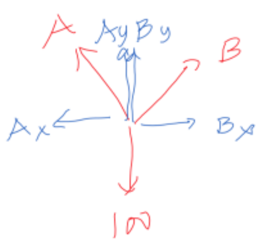
Hence the vector A = <2, 3> can now be written easily as A= 2i + 3j. Again, i and j are unit vectors in the x and y axes, respectively.

Typical illustrative problems would be like these:

The block weighs 100 kg. Find the tension in each chord (you can answer in kg as well, not Newtons).



Solution:



Recall that F = m*a. Hence if there is no motion (acceleration a = 0) F must be 0. So the sum of forces along the x axis must be 0 and the sum of forces along the y axis must also be zero.

ΣFx = 0 --> |Ax| - |Bx| = 0 --> |Ax| = |Bx| --> |A|cos50 = |B| cos 30 (1)

… I will be taking out the magnitude sign as it is very annoying to type.

ΣFy = 0 --> Ay + By - 100 = 0 --> Ay + By = 100 --> A sin 50 + B sin 30 = 100 (2)

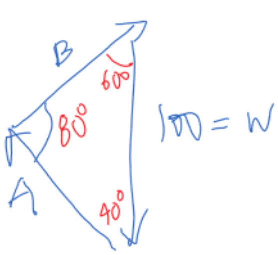
Here you have 2 equations and 2 unknowns so you can solve for A and B.

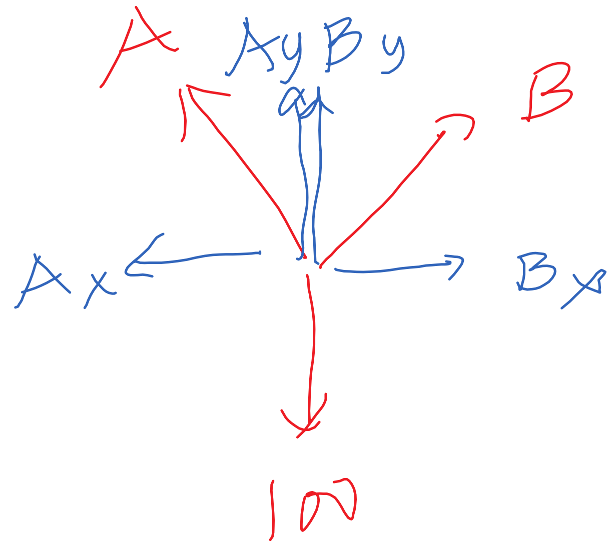
From (1), A = Bcos30/cos 50 . Plug into (2)
Bcos30/cos 50 sin 50 + B sin 30 = 100

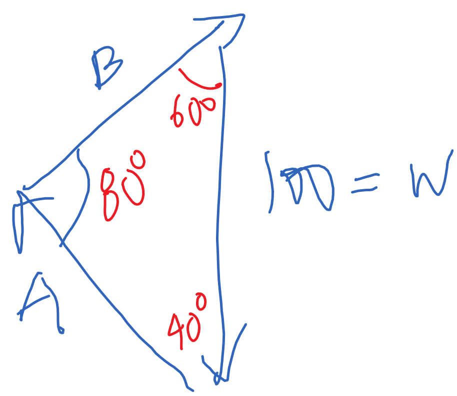
B (cos30/cos 50 *sin 50 + sin 30) = 100
B = 100 / (cos30/cos 50 *sin 50 + sin 30) = 65.27 kgs
A = Bcos30/cos 50 = 87.94 kgs

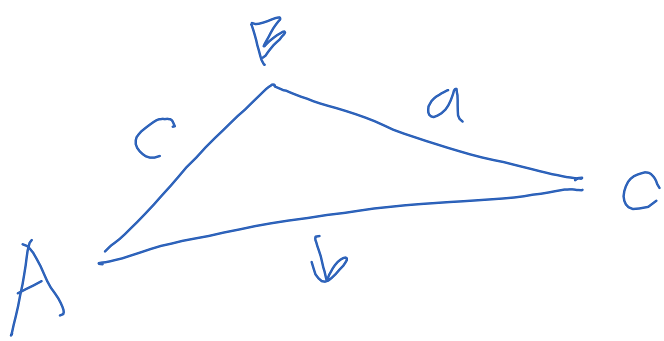
Solution #2
We know that the resultant force must be zero. Why again is this true? Because the object is static and a=0. So the vector sum of A, B and weight W must be zero. Be sure you know why the angles in the triangle are 40, 80, and 60 degrees.

A + B + W = 0

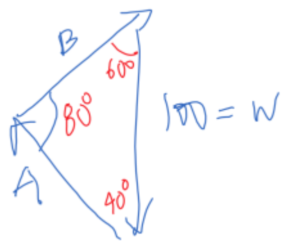


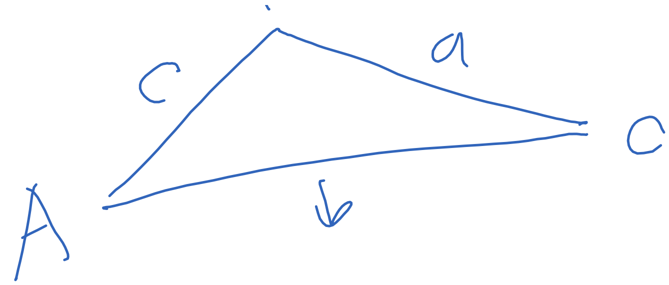




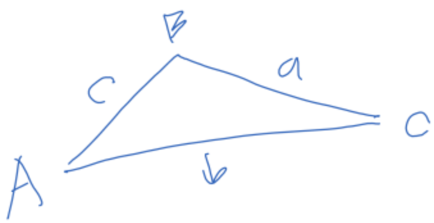


$A + B + W = 0$





You should know from trigonometry the sine law and the cosine law. Given the triangle with sides a, b, and c, and whose angles opposite the sides are A, B, and C, respectively, then we have:

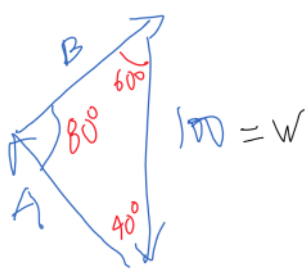


Sine law: a/sinA = b/sinB = c/sinC

Cosine law: c^2 = a^2 + b^2 -2abCosC
Of course, you can interchange letters and have b^2 = a^2 + c^2 -2acCosB instead.

The proof of these two laws are fairly easy to do and you should give them to me, along with the proof that the sum of the interior angles in the triangle is zero. A direct consequence of the cosine law is the Pythagorean theorem. With right triangle, C = 90 degrees and cos C = 0 and we have c^2 = a^2 + b^2, which is just the famous theorem.

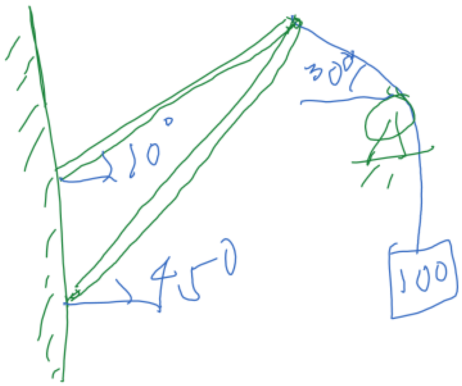
Now using sine law to solve for A and B in the original problem is so much easier. In fact it is so much easier than getting the components of each vector and summing the horizontal and vertical components. Also, this is hands down a more intelligent solution than the component method when we have 3 forces and whose resultant is zero.



Hence we have: W/sin 80 = A/sin60 = B/sin40

*--> A = 100*sin60/sin80 = 87.94 kgs*
*--> B = 100*sin40/sin80 = 65.27 kgs*

Find the force on the two bars.



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A boat went straight from one side of the bank to the opposite side at 4m/s. The downstream current is 3m/s. Find the velocity of the boat. If the river is straight and is 500 m wide, how long will the boat reach the other side?

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A plane is flying at 200 mile/hr and is headed N30E. A wind is blowing at 20 mile/hr in the direction N 20 W. What is the velocity of the plane?

Harder problems would involve more complicated versions like weight pulled by strings attached to 2 or more pulleys, etc.

Velocity & Acceleration

Define:
Speed = rate of change of the position (velocity, if you add the direction)
Acceleration = rate of change of speed and its direction.

Start by considering an object's position in only one axis.

Example: An object's position at any time t is given by X(t) = 2t^2 + 3t + 1 find the object's position at t = 0, and 1.

t	X(t) = 2t^2 + 3t + 1
0	X(0) = 2(0)^2 + 3(0) + 1 = 1
1	X(1) = 2(1)^2 + 3(1) + 1 = 6
2	X(2) = 15

Estimate the average velocity (rather speed since we are after for the magnitude only) in the time interval t=(0,1), t=(1,2) and t= (0,2).

V(t) = ΔX/t
V(0,1) = (6-1)/(1-0) = 5
V(1,2) = (15-6)/(2-1) = 9
V(0,2) = (15-1)/(2-0) = 7

Estimate the average acceleration in the time interval t=(1,2).
A(t) = ΔV/t = (9-7) / (2-1) = 2

Note there is a big difference between average velocity and **instantaneous** velocity. Unfortunately, you have no calculus yet at this point so instantaneous velocity will be "given" and you cannot solve for it yet.

If you have calculus, the above example would have velocity (rather speed) of v(t) = dX/dt = 4t + 3, and acceleration of A(t) = dv/dt = 4.

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A cannon was fired at an angle of 30 degrees from the horizontal. It has a muzzle velocity of 600m/s. If the gravity is 9.8m/s^2, what is the position of the cannon ball when it reaches the maximum height? How far will the cannon land, assuming that the ground is level?