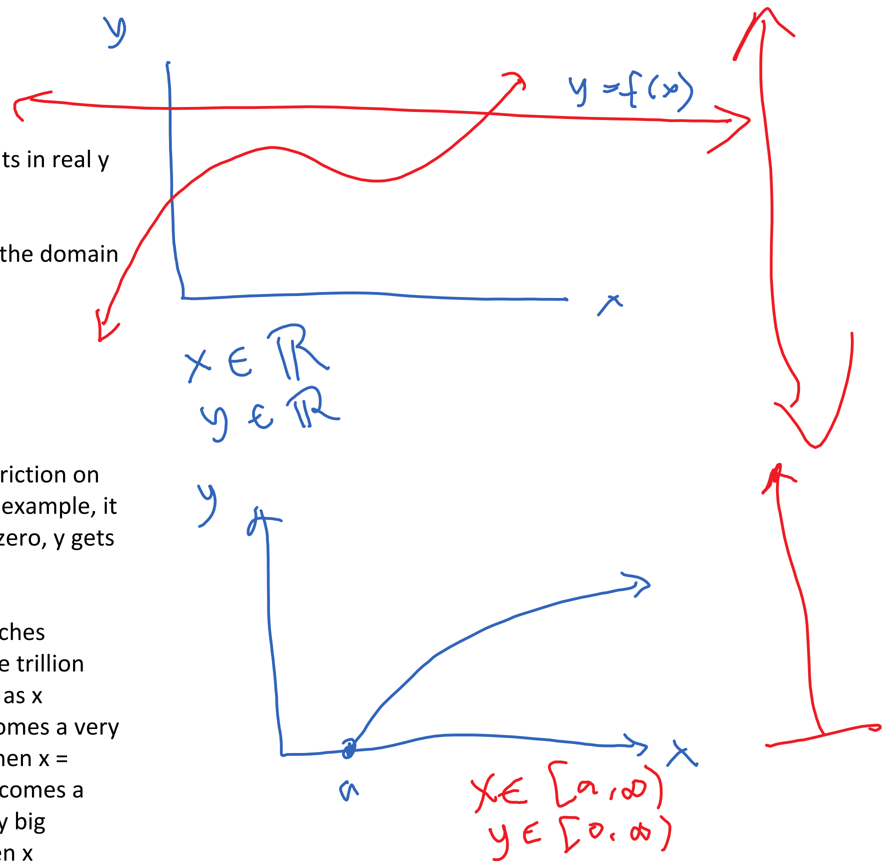
Graphing rational functions

Tuesday, January 9, 2018 8:05 AM

Goal:
Identify domain & range.
Identify asymptotes.
Graph the function.

Domain - set of all possible values of x that results in real y
Range - set of all resulting y

Look at the graphs to the right. See how we get the domain and range.



Let us develop your intuition first.

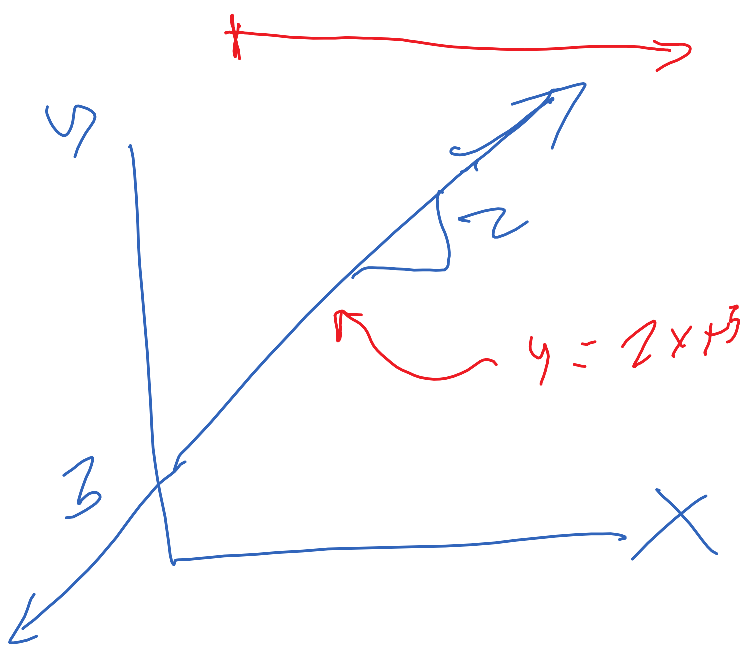
Consider y = 2x + 3.

Here, x can be any real number. There is no restriction on x. As a result y can also be any real number. For example, it is easy to see that as x gets closer and closer to zero, y gets closer and closer to 3.

Now you ask yourself what happens if x approaches infinity, think of an unimaginably big number like trillion raised to the trillion, etc. You can say easily that as x becomes a very big **positive** number, y also becomes a very big **positive** number (ex. when x=100, y=203, when x = 1million, y = 2million +3, and so on); and as x becomes a very big **negative** number, y also becomes a very big negative number (ex. when x=-100, y=-203, when x = -1million, y = -2million +3, and so on).

Hence we formally say this:
The limit of y as x approaches positive infinity is positive infinity.

$$\lim_{x \to \infty} 2x+3 = \infty$$



The limit of y as x approaches negative infinity is negative infinity.

$$\lim_{x \to -\infty} 2x+3 = -\infty$$

Now consider $y = \sqrt{x+3}$

Here we have y as the positive square root of x+3.
The number inside the square root van be zero but cannot be negative so we say that x+3>= 0 --> x >= -3. When x = -3, y = 0.

Next we ask, what happens as x approaches infinity? Well, y also becomes bigger.

$$\lim \sqrt{x+3} = \infty$$

$$y = \sqrt{x+3}$$

$$\lim_{x \to \infty} \sqrt{x+3} = \infty$$

$$\lim_{x \to -3^+} \sqrt{x+3} = 0$$

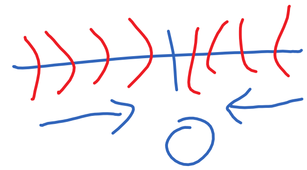
$$y = \sqrt{x+3} \geq 0$$

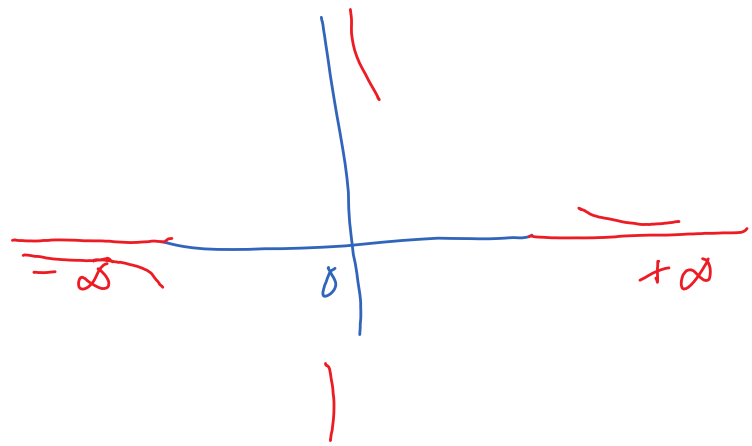
Hence the domain is x >= 3 or we can also write it as x = [-3, **infinity**), and y= [0, infinity).

Next, take y = 1/x.

$$\lim_{x \to 0^+} \frac{1}{x} = \infty \qquad \lim_{x \to \infty} \frac{1}{x} = 0$$

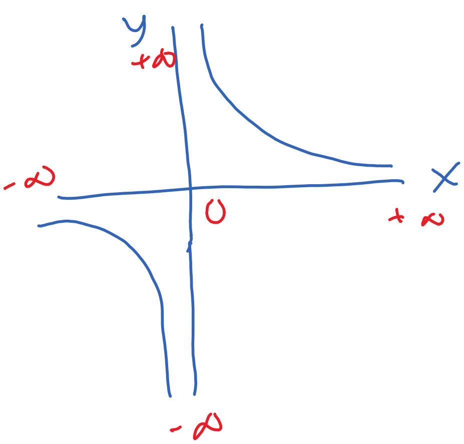
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty \qquad \lim_{x \to -\infty} \frac{1}{x} = 0$$





As x approaches zero from the right, 1/x becomes a big positive number. Imagine what happens as x = .01, .00001, .0000000001 but positive. You see that 1/x becomes gets bigger and bigger (1/x --> 1/.01 = 100.0 , 1/.00001 = 100,000.0).

Now imagine what happens to 1/x as x = -.01, -.00001, -.000000001. Here 1/x becomes big number but negative (1/x --> 1/-.01 = -100.0 , 1/-.00001 = -100,000.0).

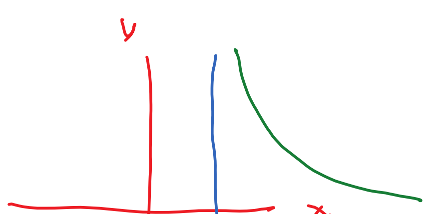


You can see that as x approaches 0 from the right, y goes to +infinity. As x approaches 0 from the left, it goes - infinity.

y= 1/(x-4)

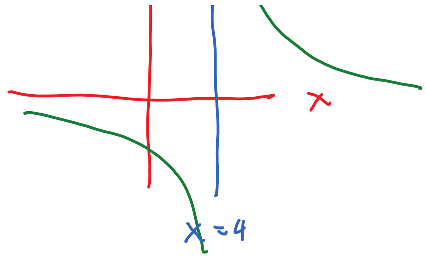
Domain: x - 4 ≠ 0 --> X ≠ 4
x= (-inf, 4) U (4, inf)

$$\lim_{x \to 4^+} \frac{1}{x-4} = \infty$$



$$\lim_{x \to 4^+} \frac{1}{x-4} = \infty$$

$$\lim_{x \to 4^-} \frac{1}{x-4} = -\infty$$



$$\lim_{x \to \infty} \frac{1}{x-4} = 0$$

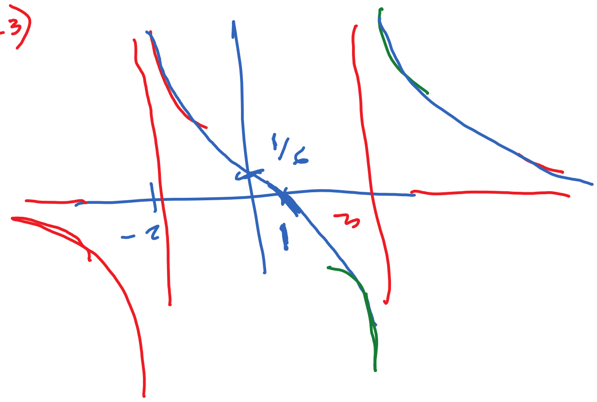
$$\lim_{x \to -\infty} \frac{1}{x-4} = 0$$

$$y = \frac{x-1}{x^2 - x - 6} = \frac{x-1}{(x+2)(x-3)}$$

$(x+2)(x-3) \neq 0$

$x+2 \neq 0 \to x \neq -2$

$x - 3 \neq 0 \to x \neq 3$



$$\lim_{x \to \infty} \frac{x-1}{x^2-x-6} = \lim_{x \to \infty} \frac{x^{+}}{x^2_{+}}$$

$$= \lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{x-1}{x^2-x-6} = \lim_{x \to -\infty} \frac{x^{(-)}}{x^2_{(+)}}$$

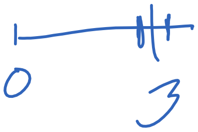
$$= \lim \frac{1}{x} = 0$$

$$\lim_{x \to -2^-} \frac{\overset{-}{x-1}}{\underset{-}{(x+2)}\underset{-}{(x-3)}} = -\infty$$

$$\lim_{x \to -2^+} \frac{x-1^{(-)}}{\underset{+}{(x+2)}\underset{-}{(x-3)}} = +\infty$$

$$\lim \frac{\overset{+\ +}{x-1}}{} = -\infty$$

$$\lim_{x \to 3^-} \frac{\overbrace{x-1}^{+}}{\underbrace{(x+2)}_{(+)}\underbrace{(x-3)}_{(-)}} = -\infty$$



$$\lim_{x \to 3^+} \frac{x-1}{(x+2)(x-3)} = +\infty$$

$y = 0 \to x = 1$

$x = 0 \to y = \frac{-1}{-6} = \frac{1}{6}$

$$y = \frac{x^3 + 2x^2 + 5x - 9}{x^2 - x + 1}$$

$x^2 - x + 1 \neq 0$

$b^2 - 4ac = 1 - 4 = -3$

$\to$ no real roots!!

$D: \ x \in \mathbb{R}$

$$\lim_{x \to \infty} y = \lim_{x \to \infty} \frac{x^3}{x^2} = \infty$$

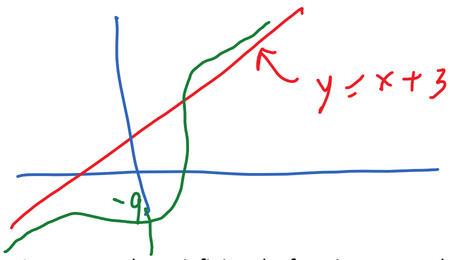
$$\lim_{x \to -\infty} y = -\infty$$

$$\begin{array}{r|l} & (x+3) + \frac{7x-12}{x^2-x+1} \\ \hline x^2-x+1 & x^3 + 2x^2 + 5x - 9 \\ - & x^3 - x^2 + x \\ \hline - & 3x^2 + 4x \\ & 3x^2 - 3x + 3 \\ \hline & 7x - 12 \end{array}$$

$$y = \underbrace{x+3}_{} + \frac{7x-12}{\underbrace{x^2-x+1}_{}}$$

$\lim_{x \to \infty} = 0$

$\to -\infty = 0$



As x approaches + infinity, the function approaches y = x+3.
As x approaches - infinity, the function approaches y = x+3.
When x = 0, y = -9, this is the y-intercept.
When y=0, x = ? To get where it crosses in the x-axis (x-intercept).