Dear Students,

This is your Math work for this summer. It consists of different skill sets ranging from Pre-Algebra to Geometry that are found on the ACT. There are example problems before each set of questions for you to use as a resource to complete the problems correctly.

It is possible to break up work and complete 2-3 questions daily over the entire summer or 4-5 questions daily for a month.

It will be graded for accuracy when we return. The due date is Friday, August 12th.

Please feel free to contact any of us Math teachers if you need assistance or have questions.

tara.doyle@shjhs.org

lauren.bastable@shjhs.org

kasha.fugate@shjhs.org

Have a great summer!

Sincerely,
Ms. Doyle, Ms. Bastable, Ms. Fugate

PRE-ALGEBRA
SKILL BUILDER ONE

Operations with Whole Numbers

If a numerical expression does not contain parentheses, first perform all multiplications and divisions, in order from left to right. Then perform all additions and subtractions, in order from left to right.

Example
What is the value of $6-5\times4+6\div3+4$?

Solution
Do the multiplications and divisions first, to give:

$$6-20+2+4$$

Then do the additions, left to right:

$$-14+2+4$$
$$-12+4$$
$$-8$$

If a numerical expression contains parentheses or brackets, roots, or powers, the order of operations is as follows. Perform all work within the parentheses/brackets first. Start with the innermost parentheses/brackets and work outward.

Example
What is the value of $[7+(3\times5)]$?

Solution
Simplify the parentheses and then add:

$$[7+15]$$
$$22$$

Example
Simplify $\sqrt{6(2)-(9-1)}$.

Solution

(Parentheses first) $\sqrt{12-8}$

(Then subtract) $\sqrt{4}$

(Simplify the root) 2

Multiplication and Division of Fractions

When you multiply fractions, the denominators need not be the same. Simply multiply the numerators, multiply the denominators, and reduce the resulting fractions to lowest terms. Mixed numbers can be converted to improper fractions. Canceling common factors can make the problem easier and allow you to do less reducing of your final answer. Whole numbers can be converted to fractions by placing them over the number 1.

Example
$1\frac{3}{8}\times1\frac{1}{3}=?$

Solution
Change to improper fractions and multiply:

$$\frac{11}{8}\times\frac{4}{3}=\frac{11}{\underset{2}{\cancel{8}}}\times\frac{\overset{1}{\cancel{4}}}{3}=\frac{11\times1}{2\times3}=\frac{11}{6}=1\frac{5}{6}$$

When dividing fractions, you do *not* need a common denominator. The easiest way to divide fractions is to change the division problem to a multiplication problem and then perform the same steps you would in multiplying fractions. To accomplish this conversion, find the reciprocal of the fraction you are dividing by and then multiply. An easier way to think of this is simply "invert" or turn the fraction you are dividing by upside down.

Example
$3\frac{2}{3}\div1\frac{1}{6}=?$

Solution

Change to improper fractions and multiply by the reciprocal of the divisor:

$$\frac{11}{3} \div \frac{7}{6} = \frac{11}{3} \times \frac{6}{7} =$$

$$= \frac{11}{\underset{1}{\cancel{3}}} \times \frac{\overset{2}{\cancel{6}}}{7} = \frac{11 \times 2}{1 \times 7} = \frac{22}{7} = 3\frac{1}{7}$$

Fractions, Decimals, and Percents

It is sometimes necessary to change a fraction to a decimal fraction. To do this, divide the numerator by the denominator, adding zeros after the decimal point in the numerator when they are needed.

Example

What is the decimal equivalent of $\frac{9}{20}$?

Solution

$\begin{array}{r} .45 \\ 20\overline{)9.00} \end{array}$ Therefore $\frac{9}{20} = .45$

Example

What is the decimal equivalent of $\frac{5}{8}$?

Solution

$\begin{array}{r} .625 \\ 8\overline{)5.000} \end{array}$ Therefore $\frac{5}{8} = .625$

To express a decimal as a percent, "move" the decimal point 2 places to the right and write the % sign.

Example

What is the percent equivalent of .35?

Solution

$.35 = \underset{\curvearrowright}{.35} = 35\%$

Example

What is the percent equivalent of .375?

Solution

$.357 = \underset{\curvearrowright}{.375} = 37.5\%$

Linear Equations with One Variable

There are standard methods of solving equations. The object in solving an equation is to isolate the variable on one side of the equation, keeping the original equation and the new equation equivalent.

Example

If $5x + 2 = 17$, then $x = ?$

Solution

Subtract 2 from both sides:

$$5x + 2 - 2 = 17 - 2$$
$$5x = 15$$

Divide both sides by 5:

$$\frac{5x}{5} = \frac{15}{5}$$
$$x = 3$$

Example

If $\frac{2}{3}y - 4 = -5$, then $y = ?$

Solution

Add 4 to both sides:

$$\frac{2}{3}y - 4 + 4 = -5 + 4$$
$$\frac{2}{3}y = -1$$

Multiply both sides by $\frac{3}{2}$:

$$\frac{3}{2} \times \frac{2}{3}y = -1 \times \frac{3}{2}$$
$$y = -\frac{3}{2}$$

Example

If $3x + 7 = 17 - 2x$, then $x = ?$

Solution

To collect all variables on the left side of the equation, add $2x$ to both sides:

$$3x + 2x + 7 = 17 - 2x + 2x$$
$$5x + 7 = 17$$

Subtract 7 from both sides:

$$5x + 7 - 7 = 17 - 7$$
$$5x = 10$$

Divide both sides by 5:

$$\frac{5x}{5} = \frac{10}{5}$$

$$x = 2$$

Example

If $3a + 1 + 2a = 6$, then $a = ?$

Solution

Combine like terms:

$$5a + 1 \ = 6$$

Subtract 1 from both sides:

$$5a + 1 - 1 = 6 - 1$$

$$5a = 5$$

Divide both sides by 5:

$$\frac{5a}{5} = \frac{5}{5}$$

$$a = 1$$

Example

If $3(x + 5) = 51$, then $x = ?$

Solution

Remove the parentheses (distributive law):

$$3x + 15 = 51$$

Subtract 15 from both sides:

$$3x + 15 - 15 = 51 - 15$$

$$3x = 36$$

Divide both sides by 3:

$$\frac{3x}{3} = \frac{36}{3}$$

$$x = 12$$

Practice Exercise 1

1. $[2 + (3 \times 2)] = ?$
   - A. 7
   - B. 8
   - C. 10
   - D. 12
   - E. 18

2. $16 + 20 \div 4 - 2 \times 2 = ?$
   - A. 5
   - B. 10
   - C. 14
   - D. 17
   - E. 38

3. $\frac{5}{6} + \frac{5}{8} \times \frac{2}{3} = ?$
   - A. $\frac{5}{9}$
   - B. $\frac{10}{21}$
   - C. $\frac{35}{36}$
   - D. $1\frac{1}{4}$
   - E. None of the above

4. $\dfrac{\frac{1}{2} + \frac{1}{5}}{\frac{1}{4}} = ?$
   - A. $\frac{1}{14}$
   - B. $\frac{3}{11}$
   - C. $\frac{7}{40}$
   - D. $1\frac{1}{7}$
   - E. $2\frac{4}{5}$

5. What is the percent equivalent of .06?
   - A. .006%
   - B. .06%
   - C. .6%
   - D. 6%
   - E. 60%

6. What is the decimal equivalent of $\frac{1}{12}$?
   - A. $.08\overline{3}$
   - B. $.008\overline{3}$
   - C. .12
   - D. .012
   - E. None of the above

7. If $7x + 6 = 27$, then $x = ?$
   - A. 2
   - B. 3
   - C. 9
   - D. 13
   - E. 21

8. If $3(y + 2) = 2(y + 4)$, then $y = ?$
   - A. $\frac{1}{2}$
   - B. 1
   - C. 2
   - D. 4
   - E. 8

9. Simplify: $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right) =$
   - A. $\frac{1}{5}$
   - B. $\frac{1}{2}$
   - C. 0
   - D. 1
   - E. 5

10. Solve for the variable: $\frac{5}{2x} = -\frac{1}{4}$
    - A. 20
    - B. -10
    - C. -5
    - D. -20
    - E. 10

SKILL BUILDER TWO

Averages

Example
John got a 75 on his first math test, 85 on his second, 80 on his third, and 88 on his fourth. What was his average grade on the four tests?

Solution
To compute the average of a series of numbers, first add the numbers. Then divide the sum by the total number of numbers added. In the example, the four test grades are added: $75 + 85 + 80 + 88 = 328$. Then divide the sum by the number of grades, 4, to compute the average: $328 \div 4 = 82$.

It is possible to use a known average to compute one of the values that made up that average.

To do this, multiply the number of values or items by their average cost or average value to get the total value. Then subtract the sum of the known values from the total to obtain the remaining value.

Example
The average of four temperature readings is 1,572°. Three of the actual readings are 1,425°, 1,583°, and 1,626°. What is the fourth reading?

Solution
Multiply the average (1,572°) by the number of readings (4):

$$1{,}572° \times 4 = 6{,}288°$$

Add the 3 given readings:

$$1{,}425° + 1{,}583° + 1{,}626° = 4{,}634°$$

Subtract this sum (4,634) from 6,288°:

$$6{,}288° - 4{,}634° = 1{,}654°$$

Probability

Look for the total number of possible occurrences. Then look for the number of occurrences that did or will take place.

The probability of a favorable result occurring may be computed by dividing the number of favorable results by the number of possible results if they are all equally likely.

Example
There are 5 cherry, 7 orange, and 4 grape lollipops in a box. If you pick a lollipop from the box without looking, what is the probability that it will be orange or grape?

Solution

$$\frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

$$\frac{7\text{ orange} + 4\text{ grape}}{(\text{Total})\,16} = \frac{7+4}{16} = \frac{11}{16}$$

Percentage Problems

Example
James bought a notebook for \$1.25, a pen for \$0.99, and a calculator for \$9.99. Including a sales tax of 6%, what was the total bill, rounded to the nearest penny?

Solution
To solve a problem involving sales tax, first find the total price. To find the sales tax on the total price, change the percent sales tax to a decimal by moving the decimal point in the number TWO places to the left. Next multiply the decimal by the total price to give the sales tax. Add the sales tax to the total price to find the total bill.

Total price:

$$\begin{array}{r} \$1.25 \\ .99 \\ \underline{+\ 9.99} \\ \$12.23 \end{array}$$

Change 6% to .06.Multiply the decimal by the total price to get the sales tax:

$$\begin{array}{r} \$12.23 \\ \underline{\times\ \ .06} \\ \$.7338 \end{array}$$

or $0.73, when it is rounded off.

Add the sales tax to the total price to find the total bill:

$$\begin{array}{r} \$12.23 \\ \underline{+\ 0.73} \\ \$12.96 \end{array}$$

Example
The price of a certain stock rose from $60 a share to $65 a share. What was the percent of increase?

Solution
To find the percent of increase, form the fraction:

$$\frac{\text{Amount of Increase}}{\text{Original Amount}}$$

$$\frac{65-60}{60}=\frac{5}{60}=\frac{1}{12}$$

$$1 \div 12 = 0.08\frac{1}{3} = 8\frac{1}{3}\%$$

Example
During the past five years the student enrollment at the local high school decreased from 1,250 to 1,000. What was the percent of decrease?

Solution
Form a fraction showing the *decrease* over the original enrollment.

$$\frac{1{,}250-1{,}000}{1{,}250}=\frac{250}{1{,}250}=\frac{1}{5}$$

$$5\overline{)1.00}^{\ .20} = 20\%$$

Word Problems Containing Fractions

Example
Jim completed $\frac{2}{5}$ of a job. The next day he completed $\frac{5}{8}$ of the remaining part of the job. What fractional part of the original job is left?

Solution
Jim completed $\frac{2}{5}$ of the job, so $\frac{3}{5}$ of the job remains. Then he completed $\frac{5}{8}$ of the remaining part of the job, or

$$\frac{5}{8}\times\frac{3}{5}=\frac{\overset{1}{\cancel{5}}}{8}\times\frac{3}{\underset{1}{5}}=\frac{3}{8}$$

Since Jim completed $\frac{2}{5}$ of the job the first day and $\frac{3}{8}$ of the job the next day,

$$\frac{2}{5}+\frac{3}{8}=\frac{16}{40}+\frac{15}{40}=\frac{31}{40}$$

of the original job was completed. The amount of the original job that is left is

$$1-\frac{31}{40} \text{ or } \frac{9}{40}$$

Word Problems Involving Money

Example
A local delivery service charges $1.80 for the first $\frac{2}{5}$ mile, $1.50 for the next $\frac{3}{5}$ mile, and $1.20 per mile thereafter. What is the cost to deliver a parcel to a company that is five miles away?

Solution

Cost for first $\frac{2}{5}$ mile = $1.80.

Cost for next $\frac{3}{5}$ mile = $1.50.

Since $\frac{2}{5}+\frac{3}{5}=\frac{5}{5}$ or 1 mile, the cost of the first mile = \$1.80 + 1.50 = \$3.30. The trip is five miles; therefore, the four additional miles cost 4 × \$1.20 = \$4.80. Total cost = \$3.30 + \$4.80 = \$8.10.

Word Problems Involving Proportions

A ratio is a comparison of numbers by division. A proportion is a statement that two ratios are equal.

Example

If Sarah can type 4 pages in 12 minutes, how long will it take her to type a 16-page report, working at the same rate?

Solution

To solve a problem involving proportions, set up a ratio that describes a rate or compares the first two terms. In the example, setting up a ratio with the first two terms gives:

$$\frac{4 \text{ pages}}{12 \text{ min.}}$$

Next, set up a second ratio that compares the third term and the missing term. Letting x stand for the missing term, the second ratio is:

$$\frac{16 \text{ pages}}{x \text{ min.}}$$

To solve for the missing term, set up a proportion where the ratios are equal:

$$\frac{4 \text{ pages}}{12 \text{ min.}} = \frac{16 \text{ pages}}{x \text{ min.}}$$

You may solve the proportion by cross multiplication.

$$\frac{4}{12} \times \frac{16}{x}$$

$$4x = 12 \bullet 16$$

$$4x = 192$$

$$x = 48$$

Sarah can type the report in 48 minutes.

Practice Exercise 2

1. The video store rented 42 videotapes on Monday, 35 on Tuesday, 51 on Wednesday, and 32 on Thursday. What was the average number of videotapes initially rented per day from Monday to Thursday if all were one-day rentals?
   A. 37
   B. 38
   C. 40
   D. 44
   E. 54

2. In the tournament, the Tigers scored 44, 56, and 47 points in the first three games. If their four-game tournament average score was 52 points, how much did they score in their final game?
   A. 52
   B. 55
   C. 58
   D. 61
   E. None of the above

3. Find the probability that a family with three children will have exactly two girls.
   A. $\frac{3}{4}$
   B. $\frac{2}{3}$
   C. $\frac{3}{8}$
   D. $\frac{1}{2}$
   E. $\frac{1}{3}$

4. There are 13 CDs in a box: 8 are hip-hop, 3 are country, and 2 are classical. Find the probability that a randomly selected CD will be country or classical.
   A. $\frac{1}{2}$
   B. $\frac{5}{8}$
   C. $\frac{5}{13}$
   D. $\frac{2}{3}$
   E. $\frac{3}{8}$

5. Tim bought a shirt for $10.99, a tie for $9.99, and a jacket for $59.00. Including a sales tax of 6%, what was the total bill?
   A. $68.74
   B. $74.68
   C. $78.48
   D. $79.98
   E. $84.78

6. Bill purchased six 6-packs of cola for $2.75 each. How much will this purchase cost including a 6% sales tax?
   A. $16.50
   B. $16.56
   C. $17.49
   D. $17.75
   E. $17.86

7. The price of gas at the pump recently rose from $2.95 to $3.04 in one week. This represents what percent increase?
   A. 0.0031%
   B. 0.031%
   C. 0.31%
   D. 3.1%
   E. 31%

8. The temperature dropped from 50° to 46°. What was the percent of decrease?
   A. 4%
   B. 8%
   C. 9%
   D. 10%
   E. None of the above

9. Betsy's softball team won 47 games, lost 15 games, and tied none. What fractional part of the games played did the team win?

A. $\frac{47}{15}$ B. $\frac{15}{47}$

C. $\frac{32}{47}$ D. $\frac{15}{62}$

E. $\frac{47}{62}$

10. Adam watches television for 3 hours each weekday and a total of 12 hours on the weekend. What fraction represents the amount of time he spends watching TV each week?

A. $\frac{15}{24}$ B. $\frac{27}{148}$

C. $\frac{9}{56}$ D. $\frac{29}{168}$

E. $\frac{33}{168}$

11. Matt saves 20% on a $110 bowling ball but must pay 6% sales tax. What is the total amount that he must pay?

A. $ 88.00
B. $ 93.28
C. $ 96.00
D. $140.80
E. None of the above

12. In a metropolitan area, the assessed value of a house is calculated to be 60% of its current market value. The property tax is calculated to be 3.9% of the assessed value. What is the property tax on a house with a market value of $320,000?

A. $7,488
B. $3,744
C. $9,120
D. $3,022
E. $1,920

13. In the 9th grade, 7 of every 10 students are girls. If there are 200 students in the 9th grade, how many of the students are girls?

A. 120
B. 130
C. 140
D. 150
E. 160

14. In a factory, 15 of every 300 light bulbs tested were defective. At the same rate, if 1,200 bulbs are tested, how many of them would be defective?

A. 40
B. 50
C. 60
D. 80
E. 90

15. If $\frac{3}{5}$ of a job can be completed in 6 weeks, how long will it take to complete the entire job, working at the same rate?

A. 10 weeks
B. 12 weeks
C. 15 weeks
D. 20 weeks
E. 22 weeks

16. A telephone survey of registered voters was taken and the data were summarized in a table:

	Male	Female
Democrat	17	17
Republican	16	23
Independent	13	14

Find the probability that a randomly selected voter was female.

A. 17%
B. 54%
C. 34%
D. 63%
E. 50%

SKILL BUILDER THREE

Exponents

An exponent tells you how many times to write the base in a multiplication problem. For example, 3^2 is read as "three to the second power" or "three squared." The 3 is called the **base**. The 2 is called the **power** or **exponent**. When finding powers involving fractions and decimals, pay special attention to the fraction and decimal rules.

Example	**Solution**
$(.03)^2$	$.03^2$ means $.03 \times .03 = .0009$

Example	**Solution**
$\left(\frac{2}{3}\right)^2$	$\left(\frac{2}{3}\right)^2$ means $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

One raised to any power is 1.

Example	**Solution**
1^4	$1^4 = 1 \times 1 \times 1 \times 1 = 1$

Any number raised to the first power is that number.

Example	**Solution**
8^1	$8^1 = 8$

Any number (except zero) raised to the zero power is 1. This is a special case.

Example	**Solution**
9^0	$9^0 = 1$

Number Concepts

Commutative Law
Adding 6 + 4 or 4 + 6 yields an answer of 10. There is no change in the answer when adding any two integers. This is an example of the **commutative law** of addition. Similarly, 3 × 5 = 5 × 3 illustrates the commutative law for multiplication.

Associative Law
To add more than two integers, such as 8 + 7 + 5, add them together as (8 + 7) + 5, which is 15 + 5 = 20. You can also add them as 8 + (7 + 5) which is 8 + 12 = 20. In either case, the sum is the same. This is an illustration of the **associative law** of addition. Similarly, 2 × (3 × 4) = (2 × 3) × 4 illustrates the associative law for multiplication.

Distributive Law
Multiplying the sum of two numbers, 4 + 2, by another number, 6, is an example of the **distributive law** of multiplication over addition: 6(4 + 2). The value of this expression is 6(6) = 36 but the answer can also be found in the following way:

$$\begin{aligned} 6(4+2) &= 6(4) + 6(2) \\ &= 24 + 12 \\ &= 36 \end{aligned}$$

The integer 6 is said to be distributed over the sum of 4 and 2.

Even and Odd Numbers

A number that is divisible by 2 is an even number.

414, 3,050, 8,886

A number that is not even is an odd number.

1, 357, 5,129

Zero is an even integer.

Sums and Differences of Even and Odd Numbers
The sum of two even numbers or two odd numbers is always an even number.

$6 + 4 = 10 \qquad 13 + 15 = 28$

The sum of an even number and an odd number is always an odd number.

$8 + 7 = 15 \qquad 16 + 21 = 37$

The difference between two even numbers or two odd numbers is always an even number.

$12 - 8 = 4 \qquad 15 - 7 = 8$

The difference between an even number and an odd number is always an odd number.

$8 - 5 = 3 \qquad 25 - 18 = 7$

Products of Even and Odd Numbers

The product of two even numbers is always an even number.

$$4 \times 8 = 32$$

The product of two odd numbers is always an odd number.

$$3 \times 7 = 21$$

The product of an even number and an odd number is always an even number.

$$8 \times 9 = 72 \qquad 7 \times 12 = 84$$

Factors, Primes, and Factorials

Factors

When two or more whole numbers are multiplied, each is a factor of the product. The numbers 1, 2, 4, and 8 are factors of 8 because the product of both 1 and 8 and 2 and 4 is 8.

$$1 \times 8 = 8$$
$$2 \times 4 = 8$$

The positive factors of 8 are $\{1, 2, 4, 8\}$. If you divide 8 by each of the factors, the remainder is 0.

Example

What is the set of positive factors of each of the following numbers?

10 **Solution** $\{1, 2, 5, 10\}$
34 **Solution** $\{1, 2, 17, 34\}$
20 **Solution** $\{1, 2, 4, 5, 10, 20\}$
48 **Solution** $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

In arithmetic, a multiple of a number is a number that is the *product* of the given number and another *factor*. For example, the numbers 2 and 4 both have the number 2 as a factor. Therefore, 2 and 4 are multiples of 2. Also {6, 12, 18, 24, ...} represents the positive multiples of 6.

Some numbers have exactly two different positive factors: the number itself and 1. These numbers, such as 2, 3, 5, and 7, are called **prime** numbers. Numbers that have more than 2 different positive factors are called **composite** numbers. Examples of these numbers are 6, 8, 9, and 15. The number 1 is neither prime nor composite since it has only 1 positive factor. (1 and -1 are called **units.**)

Example

What are all the prime numbers from 1 to 40?

Solution

$\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$

If the factorization of a number contains only prime numbers it is called a **prime factorization** of that number. A prime factorization of 8 is 2 • 2 • 2, of 24 is 2 • 2 • 2 • 3, of 35 is 5 • 7.

Example

What is a prime factorization of 40?

Solution

How do you find the prime factorization of a composite number such as 40? Begin by finding any two factors of 40, say 8 and 5. Then express each factor as a product of two other factors.

$$40 = 5 \bullet 8$$
$$5 \bullet 2 \bullet 4$$
$$5 \bullet 2 \bullet 2 \bullet 2$$

or $2^3 \bullet 5$ in exponential form

Example

Which of the following statement(s) is (are) true?

I. 51 is not a prime number.
II. All composite numbers are even.
III. The product of two primes is always composite.

(A) I only
(B) III only
(C) I and II
(D) I and III
(E) II and III

Solution

Examine statement I. Attempt to prove the other statements false by supplying at least one counter example. Test different numbers in each statement.

I is true. 51 is not a prime number.
II is false, since 15 (3 × 5) is odd.
III is true, since any number that is a product of two primes will always have those two primes as factors. The answer is D.

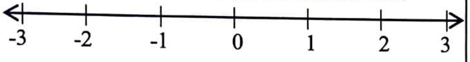
Factorials

For a positive integer *n*, the product of all the positive integers less than or equal to *n* is called a factorial. Factorial *n* is written *n*!

For example $1! = 1$
$2! = 2 \bullet 1 = 2$
$3! = 3 \bullet 2 \bullet 1 = 6$
$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24$
$5! = 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 120$, and
0! is defined as 1 (to make some mathematical formulas behave nicely)

Absolute Value, Square Roots, and Irrational Numbers

The absolute value of an integer is the distance the number is from zero on the number line.



Thus, the absolute value of 3 or of -3 is 3, since each number is 3 units from 0. This is written as $|3| = 3$ and read as "the absolute value of 3 equals 3." Similarly, $|-3| = 3$ is read as "the absolute value of -3 is 3."

Example
What is the value of $|-16| - |3|$?

Solution
$|-16| = 16$ $|3| = 3$
$|-16| - |3| = 16 - 3 = 13$

Example
Evaluate $|-4| + |8|$.

Solution
$|-4| = 4$ $|8| = 8$
$|-4| + |8| = 4 + 8 = 12$

Roots of Numbers
You know that $9 = 3^2$. Since 9 is 3 squared, it is said that the square root of 9 is 3. It is written as $\sqrt{9} = 3$. The **principal square root** (or positive square root) of 9 is 3. What is the positive square root of 25? Since $5^2 = 25$, then $\sqrt{25} = 5$.

Example
Simplify the following:
$\sqrt{49} = 7$
$\sqrt{100} = 10$
$-\sqrt{4} = -2$

Example
Simplify the following:
$\sqrt{25} + \sqrt{4} = 5 + 2 = 7$
$\sqrt{36} - \sqrt{16} = 6 - 4 = 2$
$(\sqrt{25})(\sqrt{36}) = 5 \bullet 6 = 30$

The square roots of numbers that are not perfect squares neither terminate nor repeat. For example, $\sqrt{5}$ is approximately equal to 2.2361 and belongs to the set of irrational numbers. Likewise, $\sqrt{2}$, $-\sqrt{3}$, $\sqrt{11}$, and $\sqrt[3]{5}$ are also irrational numbers.

Example
Simplify each expression by removing factors that form perfect squares.

$\sqrt{15}$: Since 15 does not contain a perfect square (other than 1) among its factors, it is said to be in simplified form.

$\sqrt{20}$: The factors of 20 are 1, 2, 4, 5, 10, and 20. Since 4 is a perfect square we can use (4)(5) = 20 as follows:

$$\sqrt{20} = \sqrt{(4)(5)} = (\sqrt{4})(\sqrt{5}) = 2\sqrt{5}.$$

$\frac{\sqrt{40}}{2}$: The factors of 40 are 1, 2, 4, 5, 8, 10, 20, and 40. Since 4 is a perfect square we can use (4)(10) = 40 as follows:

$$\frac{\sqrt{40}}{2} = \frac{\sqrt{(4)(10)}}{2} = \frac{(\sqrt{4})(\sqrt{10})}{2} = \frac{2\sqrt{10}}{2} = \sqrt{10}$$

$2\sqrt{72}$: The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72. The perfect squares are 1, 4, 9, and 36. To simplify, use the largest perfect square, which is 36:

$$2\sqrt{72} = 2\sqrt{(36)(2)} = 2(\sqrt{36})(\sqrt{2})$$
$$= 2(6)\sqrt{2} = 12\sqrt{2}$$

Practice Exercise 3

1. What is the value of $(.02)^2$?
   A. .4
   B. .04
   C. .004
   D. .0004
   E. .22

2. $4^3 \bullet 3^2 \bullet 2^3 = ?$
   A. 576
   B. 1,152
   C. 2,304
   D. 3,072
   E. 4,608

3. Which of the following numbers can be evenly divided by both 4 and 9?
   A. 1,350
   B. 2,268
   C. 4,700
   D. 5,756
   E. None of the above

4. Which of the following numbers are divisible by 3?
   (I) 242
   (II) 45,027
   (III) 804,597
   A. II only
   B. III only
   C. I and II
   D. II and III
   E. I, II, and III

5. $\frac{3^{14}}{27^4} = ?$
   A. $\frac{1}{9}$
   B. 1
   C. 3
   D. 9
   E. 27

6. Simplify: $4\sqrt{5} - \sqrt{80} =$
   A. $2\sqrt{5}$
   B. 0
   C. $\sqrt{80}$
   D. 1
   E. $4\sqrt{75}$

7. What is the prime factorization of 144?
   A. $1 \bullet 144$
   B. $2 \bullet 2 \bullet 36$
   C. $2 \bullet 2 \bullet 4 \bullet 9$
   D. $2^4 \bullet 3^2$
   E. $2^2 \bullet 3^4$

8. Evaluate $\frac{6!}{3!5!}$
   A. 0
   B. 1
   C. 48
   D. 90
   E. 720

9. What is the prime factorization of 210?
   A. $2 \bullet 5 \bullet 21$
   B. $3 \bullet 7 \bullet 11$
   C. $2 \bullet 3 \bullet 5 \bullet 7$
   D. $2 \bullet 3 \bullet 7 \bullet 11$
   E. $2 \bullet 105$

10. 108 is divisible by:
    A. 2, 3, 4, 7, and 9
    B. 2, 4, 6, and 8
    C. 2, 3, 4, 6, and 9
    D. 2, 3, 6, 8, and 9
    E. 2, 6, 9, and 14

ELEMENTARY ALGEBRA
SKILL BUILDER FOUR

Multiplication of Signed Numbers

Rules for Multiplying Two Signed Numbers

1. If the signs of the numbers are alike, the product is positive.
2. If the signs of the numbers are different, the product is negative.

Example
What is the product of (-6) and (-5)?
Solution
The signs are alike, the answer is positive.

$$(-6)(-5) = +30$$

Example
Find $(15)\left(-\frac{1}{5}\right)$.

Solution
The signs are different, the answer is negative.

$$\left(\frac{{}^{3}\cancel{15}}{1}\right)\left(-\frac{1}{\cancel{5}_{1}}\right) = -\frac{3}{1} = -3$$

Rules for Multiplying More Than Two Signed Numbers

1. If the problem contains an even number of minus signs, the product is positive.
2. If the problem contains an odd number of minus signs, the product is negative.

Example
(4)(-3)(8)(-2) = +192
The problem has two minus signs; since 2 is an even number, the product is positive.

Example
(-2)(-3)(4)(-5) = -120
The problem has three minus signs; since 3 is an odd number, the product is negative.

Evaluation of Algebraic Expressions

Example
Evaluate $5a + 3 - 3b + 7$ if $a = 4$ and $b = 5$.
Solution
Substitute the given number values for their respective letters:

$$5a + 3 - 3b + 7$$
$$5(4) + 3 - 3(5)+7$$

Perform the required operations, remembering to multiply before adding or subtracting:

$$20 + 3 - 15 + 7 = 15$$

Example
Evaluate $x^2 - y^2$ if $x = 5$ and $y = 4$.
Solution

(Substitute)	$5^2 - 4^2$
(Powers first)	$25 - 16$
(Subtract)	9

Writing Algebraic Expressions and Equations

In order to solve problems algebraically, it is necessary to express number relations by the use of symbols. English statements must be translated into mathematical symbols.

In the following examples, pay close attention to the underlined words that suggest which mathematical symbols to use. The letter "*n*" is used for the word "number" in these examples. Any letter can be used.

a. A number <u>increased by</u> 5
"increased by" indicates addition $n + 5$

b. Seven <u>less than</u> a number
"less than" indicates subtraction $n - 7$

c. Seven <u>decreased by</u> a number
"decreased by" indicates subtraction; in this case the number is being subtracted from 7 $7 - n$

d. The <u>product</u> of 3 and a number
"product" indicates multiplication $3n$

e. The sum *of* a number and one-fourth of the number
sum" indicates addition,
"of" indicates multiplication $n + \frac{1}{4}n$

f. A number divided by 6
Division is usually written as a fraction $\frac{n}{6}$

g. Five more than twice a number
"more than" indicates addition $5 + 2n$
or $2n + 5$

h. Half of a number decreased by 3
(only one order is correct) $\frac{1}{2}n - 3$

When writing equations from written statements, the verb suggests where to put the equal sign.

i. Ten more than 4 times a number is 46.
Replace the verb "is" with an equal sign.

$$4n + 10 = 46$$

j. Six times a number equals 21 more than 3 times the number.
The verb "equals" tells us where to put the = sign.

$$6n = 21 + 3n$$

Simplifying Algebraic Fractions

A fraction is in its simplest form when the numerator and denominator have no common integral factor except 1 or -1.

Example

Simplify $\frac{16xy}{24x^2}$.

Solution
Find the GCD (greatest common divisor) and reduce the fraction to its lowest terms. Here the GCD is $8x$.

$$\frac{16xy}{24x^2} = \frac{\overset{2}{\cancel{16}}\ \overset{1}{\cancel{x}y}}{\underset{3}{\cancel{24}}\ \underset{x}{\cancel{x^2}}} = \frac{2y}{3x}$$

When fractions contain polynomials, one method is to factor first and then reduce to lowest terms.

Example

Simplify $\frac{x^2+4x-12}{5x-10}$.

Solution
First factor, and then reduce:

$$\frac{x^2+4x-12}{5x-10} = \frac{(x+6)\overset{1}{\cancel{(x-2)}}}{5\underset{1}{\cancel{(x-2)}}} = \frac{x+6}{5}$$

Example

Simplify $\frac{4-a}{a^2-16}$.

Solution
Factor the numerator, showing -1 as one of the factors:

$$\frac{4-a}{a^2-16} = \frac{-1\overset{1}{\cancel{(a-4)}}}{(a+4)\underset{1}{\cancel{(a-4)}}} = \frac{-1}{a+4}$$

Practice Exercise 4

1. What is the product of $\left(-\frac{3}{5}\right)$ and (-15)?

A. -25
B. -9
C. 9
D. $15\frac{3}{5}$
E. 25

2. What is the product of (-7), (+5), and (-1)?

A. -40
B. -35
C. -3
D. 13
E. 35

3. What is $\left(-\frac{1}{3}\right) \div \left(\frac{5}{9}\right)$?

A. $-\frac{3}{5}$
B. $-\frac{5}{18}$
C. $-\frac{2}{9}$
D. $-1\frac{2}{3}$
E. $-5\frac{2}{7}$

4. Barry weighs 20 pounds more than Will. Their combined weight is 370 pounds. How much does each weigh?

A. Barry weighs 185; Will weighs 165.
B. Barry weighs 204; Will weighs 185.
C. Barry weighs 175; Will weighs 155.
D. Barry weighs 195; Will weighs 175.
E. Will weighs 195; Barry weighs 175.

5. If $a = 5$ and $b = 4$, evaluate $10b - 5a$.

A. 5
B. 15
C. 25
D. 30
E. 70

6. If $e = 8$ and $f = 3$, evaluate $e^2 - f^3$.

A. 7
B. 37
C. 47
D. 49
E. 91

7. If $m = 12$ and $n = 4$, evaluate $\frac{2m+8}{n}$.

A. 1
B. 4
C. 8
D. 16
E. 24

8. What is the value of xy^2z^3 if $x = 2$, $y = -2$, and $z = 3$?

A. -216
B. -72
C. 72
D. 96
E. 216

9. "Five times a number *n* decreased by 10" can be written as:

A. $5 - 10n$
B. $10 - 5n$
C. $5n - 10$
D. $10n - 5$
E. $50n$

10. "Five less than 3 times a number *n*" can be expressed as:

A. $3n + 5$
B. $5 - 3n$
C. $5 + 3n$
D. $3n - 5$
E. $15n$

11. "The sum of 8 and a number *n* all divided by 5" is

A. $8 + \frac{n}{5}$
B. $\frac{8}{n} + 5$
C. $8n + 5$
D. $\frac{8n}{5}$
E. $\frac{8+n}{5}$

12. "When 4 times a number *n* is increased by 5, the result is the same as when 100 is decreased by the number *n*" can be written as:

A. $4n - 5 = n + 100$
B. $5n + 4 = 100 - n$
C. $5n - 4 = n - 100$
D. $4n + 5 = 100 - n$
E. $5 + 4n = n - 100$

13. Simplify $\frac{4r+20}{r+5}$, $r \neq -5$.

A. $\frac{1}{4}$
B. 4
C. $3r + 4$
D. 8
E. $4r + 4$

14. A twelve-foot board is cut into two pieces. One piece is three times as long as the other Find the length of each piece.

A. 2 and 6
B. 4 and 8
C. 3 and 6
D. 3 and 9
E. 6 and 8

SKILL BUILDER FIVE

Evaluating a Formula

A formula is an instruction written in the symbols of algebra. To use any formula, simply replace a value that is given in the problem for the appropriate letter. The order of operations rule must be followed.

Example
Changing Fahrenheit temperature to centigrade temperature is done by using the formula $C = \frac{5}{9}(F - 32)$. Find the centigrade temperature that corresponds to 41° Fahrenheit.

Solution

(Substitute 41 for *F*) $C = \frac{5}{9}(41 - 32)$

(Simplify the parentheses) $C = \frac{5}{9}(9)$

(Multiply) $C = \frac{5}{\cancel{9}_{1}} \bullet \frac{\overset{1}{\cancel{9}}}{1} = \frac{5}{1}$

$C = 5°$ centigrade

Example
$R = \frac{1}{2}wt^2$. What is the value of R when $w = 16$ and $t = 3$?

Solution

(Substitute) $R = \frac{1}{2}(16)(3)^2$

(Find the power) $R = \frac{1}{2}(16)(9)$

(Multiply) $R = \frac{1}{\cancel{2}_{1}} \bullet \frac{\overset{8}{\cancel{16}}}{1} \bullet \frac{9}{1} = \frac{72}{1} = 72$

Equations Containing Fractions

If an equation contains one or more fractions or decimals, clear it by multiplying each term by the LCD (lowest common denominator).

Example
Solve for x: $\frac{5x-7}{3} = x - 5$

Solution
Multiply each side by 3.

$$^{1}\cancel{3}\left(\frac{5x-7}{\cancel{3}_{1}}\right) = 3(x-5)$$

$5x - 7 = 3x - 15$

(To isolate the variable subtract $3x$ from both sides) $2x - 7 = -15$

(Add 7) $2x = -8$

(Divide by 2) $x = -4$

Factoring Quadratic Equations

To solve a quadratic equation:

- Write the equation in standard form (set it equal to zero).
- Factor 1 side of the equation.
- Set *each* factor equal to zero.
- Solve each equation.

Example
What is the smaller solution to the equation $3x^2 + 7x - 6 = 0$?

Solution
The equation is already in standard form. The simplest way to solve this problem is to find the two solutions and then choose the smaller one. The solutions can be found by factoring

$$3x^2 + 7x - 6 = 0$$
$$(3x - 2)(x + 3) = 0$$

Set each factor equal to zero

$$3x - 2 = 0 \qquad x + 3 = 0$$

$$3x = 2 \qquad x = -3$$

$$x = \frac{2}{3}$$

$x = -3$ is the smaller of the two solutions.

Miscellaneous Word Problems

Solving word problems

Step 1. Read the problem carefully.

Step 2. Select a suitable replacement for the unknown amount(s).

Step 3. Set up an equation using the information in the problem.

Step 4. Solve the equation.

Step 5. Answer the question in the problem.

Step 6. Check your result.

Example

The sum of three consecutive integers is 57. Find the numbers.

Solution

Step 1. Read carefully.

Step 2. Let x = first consecutive integer

$x + 1$ = second consecutive integer

$x + 2$ = third consecutive integer

Step 3. $x + (x + 1) + (x + 2) = 57$

$$3x + 3 = 57$$

$$3x = 54$$

Step 4. $x = 18$

Step 5. If $x = 18$, then $x + 1 = 19$, and $x + 2 = 20$.

Therefore, the 3 consecutive integers are 18, 19, and 20.

Step 6. 18, 19, and 20 are consecutive integers, and $18 + 19 + 20 = 57$.

Practice Exercise 5

1. Using the formula $P = 2(l + w)$, find P if l = 40 and $w = 20$.
   A. 60
   B. 80
   C. 100
   D. 120
   E. 140

2. In the formula $S = \frac{n}{2}(a + l)$, find S if $n = 5$, $a = 2$, and $l = 18$.
   A. 8
   B. $12\frac{1}{2}$
   C. $22\frac{1}{2}$
   D. 25
   E. 50

3. If $A = \frac{1}{2}h(b + c)$, find A if $h = 10$, $b = 8$, and $c = 12$.
   A. 25
   B. 50
   C. 100
   D. 200
   E. 400

4. The formula $c = 75 + 30(n - 5)$ is used to find the cost, c, of a taxi ride where n represents the number of $\frac{1}{5}$ miles of the ride. Find the cost of a taxi ride of $2\frac{2}{5}$ miles.
   A. \$2.10
   B. \$2.85
   C. \$4.35
   D. \$7.35
   E. None of the above

5. Andy was born on his mother's 32^{nd} birthday. Which expression best represents Andy's mother's age when Andy is n years old?
   A. $32 - n$
   B. $32n$
   C. $32 + n$
   D. $\frac{32}{n}$
   E. $32(n + 1)$

6. If $3a - 2b = 8$ and $a + 3b = 7$, what is the value of $4a + b$?
   A. -13
   B. -11
   C. 15
   D. 1
   E. 9

7. Solve for n: $4 - \frac{3}{n} = \frac{5}{2}$
   A. $\frac{1}{2}$
   B. $\frac{2}{3}$
   C. $\frac{3}{4}$
   D. 2
   E. None of the above

8. Solve for a: $1 + \frac{1}{a} = \frac{2}{a} + 2$
   A. -8
   B. -4
   C. -1
   D. 1
   E. No solution

9. John's test scores for this marking period are: 72, 84, 86, and 70. What score must John get on his next test to maintain an average of 80?
   A. 80
   B. 78
   C. 88
   D. 84
   E. 79

10. Which of the following is a factorization of the polynomial $x^2 - 7x - 18$?
    A. $(x - 18)(x + 1)$
    B. $(x + 9)(x - 2)$
    C. $(x - 9)(x - 2)$
    D. $(x - 9)(x + 2)$
    E. $(x - 6)(x - 3)$

11. Which of the following is not equal to the other three?
A. 25% of 80
B. $\frac{1}{5}$ of 100
C. $40 \div 0.5$
D. $2\sqrt{100}$
E. 20

12. Penny can knit 4 rows of a sweater in 5 minutes. How many *hours* will it take her to knit 300 rows?
A. 4
B. $6\frac{1}{4}$
C. $12\frac{1}{2}$
D. 240
E. 375

13. Consider the following list of new car prices:

Lexus	\$46,500	Eclipse	\$27,900
Infiniti	\$37,800	Jeep	\$21,300
Honda	\$18,900	Toyota	\$19,500

How much more is the Lexus than the mean of the other five cars?
A. \$8,700
B. \$17,850
C. \$24,210
D. \$21,420
E. \$13,750

14. Six times a number is 12 less than 10 times the number. What is the number?

A. 3
B. 6
C. 12
D. 18
E. 24

SKILL BUILDER SIX

Fundamental Operations with Monomials and Polynomials

Example
$(4x + 5y) + (2x - 11y) = ?$

Solution
Combine terms with the same variable:

$$4x + 5y + 2x - 11y$$
$$6x - 6y$$

Example
Simplify $(5a - 5b) - (3a - 7b)$.

Solution
The second polynomial is being subtracted. Change the signs of both terms in the second polynomial:

$$5a - 5b - 3a + 7b$$
$$2a + 2b$$

Example
Simplify $3a(4a^2 - 2a + 3)$.

Solution
Multiply ach term in the parentheses by $3a$:

$$3a(4a^2) + 3a(-2a) + 3a(3)$$
$$12a^3 - 6a^2 + 9a$$

Example
Multiply $(3x + 2)(x + 5)$.

Solution #1

$$3x + 2$$
$$x + 5$$
$$15x + 10$$
$$3x^2 + 2x$$
$$3x^2 + 17x + 10$$

Solution #2

$(3x + 2)(x + 5) = 3x^2$

$(3x + 2)(x + 5) = + 15x$

$(3x + 2)(x + 5) = + 2x$

$(3x + 2)(x + 5) = + 10$

$3x^2 + 17x + 10$

Factorization of Polynomials

Example
If $(x + a)(x + b) = x^2 + 7x + 12$ for all x, what is the value of $(a + b)$?

Solution
Factor $x^2 + 7x + 12$:

$$(x + 4)(x + 3)$$

Therefore $a = 4$ and $b = 3$, and the answer is the sum of 4 + 3 or 7.

Example
The length of a rectangle is $x + 3$ inches and the width is $x - 2$ inches. What is the area, in terms of x, of the rectangle?

Solution
Length of rectangle = $x + 3$
Width of rectangle = $x - 2$
The area of the rectangle is found by multiplying the length by the width.

$$(x + 3)(x - 2) = x^2 + x - 6$$

Practice Exercise 6

1. A rectangle has a length of $x + 7$ and a width of $2x - 3$. If its perimeter is 32, what is the value of $3x$?
   A. 4
   B. 12
   C. 14
   D. 36
   E. 42

2. The height of a triangle is 5 less than double its base, which is $\frac{13}{2}$. Find the triangle's area.
   A. 13 inches
   B. 26 inches
   C. 52 inches
   D. $11\frac{7}{2}$ inches
   E. $16\frac{1}{2}$ inches

3. $(x + 3) + (5x - 7) =$
   A. $6x - 4$
   B. $6x + 4$
   C. $5x + 10$
   D. $6x - 6$
   E. $4x - 6$

4. $(2x - 1)(3x^2 + 2x - 5) =$
   A. $6x^3 + x^2 - 12x + 5$
   B. $8x^3 + x^2 - 12x + 6$
   C. $12x^3 + x^2 - 6x + 5$
   D. $6x^3 + x^2 - 8x + 6$
   E. $3x^3 - 2x^2 + 6$

5. $(b - 2)^2 =$
   A. $b^2 - 4b + 4$
   B. $b^2 + 4b - 4$
   C. $b^2 - 4$
   D. $b^2 - 2$
   E. $b^2 - 2b - 4$

6. Factor completely: $x^2 - 6x + 5$
   A. $(x + 1)(x + 5)$
   B. $(x - 1)(x - 6)$
   C. $(x - 2)(x - 3)$
   D. $(x - 2)(x + 3)$
   E. $(x - 1)(x - 5)$

7. The limousine that you hired costs $400 plus $45 for each hour of service. If your total cost for the limousine is $670, how many hours did you have the vehicle?
   A. 8
   B. 7
   C. 5
   D. 6
   E. 9

8. Three times the sum of two consecutive integers is 69. The two integers are:
   A. 11 and 12
   B. 17 and 18
   C. 21 and 22
   D. 15 and 16
   E. 12 and 13

9. Factor completely: $y^2 + 15y + 56 =$
   A. $(y - 7)(y + 8)$
   B. $(y + 8)(y + 9)$
   C. $(y - 7)(y - 8)$
   D. $(y + 7)(y + 8)$
   E. $(y + 7)(y + 9)$

10. Solve for the variable: $(x - 3)^2 = 0$
    A. ±3
    B. -3 only
    C. 3 only
    D. 9 only
    E. ±9

INTERMEDIATE ALGEBRA
SKILL BUILDER SEVEN

Linear Inequalities in One Variable

Solving an algebraic inequality is similar to solving an equation. The only difference is that in solving an inequality, you must remember to reverse the inequality symbol when you multiply or divide by a negative number.

Example
Solve for a: $a - 5 > 18$

Solution
Add 5 to both sides $a > 23$

Example
Solve for b: $2b + 2 < -20$

Solution
Add -2 to both sides $2b < -22$
Divide by 2 $b < -11$

Example
Solve for x: $7 - 4x > 27$

Solution
Add -7 to both sides $-4x > 20$
Divide by -4 and reverse the inequality sign $x < -5$

Remember: If $a < b$, then $a + c < b + c$.
If $a < b$ and $c > 0$ (in other words c is positive) then $ac < bc$.
If $a < b$ and $c < 0$ (c is negative) then $ac \geq bc$.

Absolute Value Equations and Inequalities

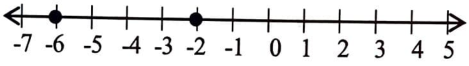
A combined sentence whose two parts are joined by the word *and* is called a **conjunction**. Its solution is the **intersection** of the solutions of its two component parts.

A combined sentence whose parts are joined by the word *or* is called a **disjunction**. Its solution is the **union** of its component parts.

In solving an equation or inequality involving **absolute value**, you should first rewrite the sentence as an equivalent conjunction or disjunction.

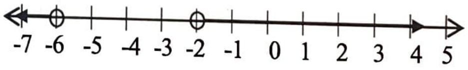
Example
Solve $|x + 4| = 2$.

Solution
$|x + 4| = 2$ is equivalent to a disjunction. It means $x + 4 = 2$ *or* $x + 4 = -2$.
Solve for x: $x = -2$ *or* $x = -6$



Example
Solve $|x + 4| > 2$.

Solution
$|x + 4| > 2$ is a disjunction. It means $x + 4 < -2$ *or* $x + 4 > 2$.
Solve for x: $x < -6$ *or* $x > -2$

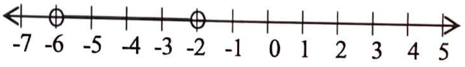


Example
Solve $|x + 4| < 2$.

Solution
$|x + 4| < 2$ is a conjunction. It means $x + 4 > -2$ *and* $x + 4 < 2$.
Solve for x: $x > -6$ *and* $x < -2$

Another way of saying this is $x + 4$ is between -2 and 2.

$$-2 < x + 4 < 2$$
$$-6 < x < -2$$



Operations with Integer Exponents

Rule 1. The exponent of the *product* of two powers of the same base is the sum of the exponents of the two powers.

$a^m \bullet a^n = a^{m+n}$

$a^3 \bullet a^2 = a^{3+2} = a^5$

Rule 2. The power of a *product* equals the product of the powers.

$(ab)^m = a^m b^m$

$(ab)^3 = a^3 b^3$

$(3x^2y)^3 = 3^3 \bullet (x^2)^3 \bullet y^3 = 27x^6y^3$

Rule 3. The exponent of the quotient of two powers of the same base, when the power of the dividend is larger than the power of the divisor, equals the difference between the exponents of the two powers.

$a^m \div a^n = a^{m-n}$

$a^8 \div a^3 = a^{8-3} = a^5$

Rule 4. The exponent of a power of a power of the same base equals the product of the 2 exponents of the power.

$(a^m)^n = a^{mn}$

$(x^3)^3 = x^9$

$(3x^2)^4 = 3^4 \bullet (x^2)^4 = 81x^8$

Rule 5. *Zero as an exponent*

Any number (except zero itself) raised to the zero power equals 1. Example: $a^0 = 1$, $5^0 = 1$, $(5a)^0 = 1$, $3a^0 = 3 \bullet 1 = 3$.

Rule 6. *Negative integral exponents*

$a^{-n} = \dfrac{1}{a^n}$ is the definition of a negative exponent. Examples:

$$a^{-2} = \frac{1}{a^2}$$

$$\frac{x^2}{y^3} = x^2 \bullet y^{-3} = x^2y^{-3}$$

$$\frac{1}{100} = \frac{1}{10^2} = 10^{-2}$$

$$\frac{a^{-3}}{b^{-2}} = \frac{b^2}{a^3}$$

Fractional Exponents

Fractional exponents establish a link between radicals and powers. The general definition of a fractional exponent is

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Examples: $a^{\frac{1}{2}} = \sqrt[2]{a^1} = \sqrt{a}$

$a^{\frac{2}{5}} = \sqrt[5]{a^2}$

$27^{\frac{2}{3}} = \sqrt[3]{27^2} = 3^2 = 9$

$16^{\frac{5}{4}} = \sqrt[4]{16^5} = 2^5 = 32$

$x^{\frac{1}{2}}y^{\frac{1}{2}} = (xy)^{\frac{1}{2}} = \sqrt[2]{(xy)^1} = \sqrt{xy}$

Slope-Intercept Form of a Linear Equation

A linear equation in **slope-intercept form** is

$$y = mx + b$$

where m represents the slope and b represents the y-intercept.

Example

Find the slope and y-intercept of $2x + 3y = 3$.

Solution

The equation must be rewritten to the form $y = mx + b$. To do this, simplify. Solve the equation for y.

$$2x + 3y = 3$$

$$3y = -2x + 3$$

$$y = -\frac{2}{3}x + \frac{3}{3} \text{ or } y = -\frac{2}{3}x + 1$$

The slope and y-intercepts are $m = -\frac{2}{3}$ and $b = 1$.

Example

Find the slope and y-intercept of $y + 5 = 0$.

Solution

This is a special case.

$$y + 5 = 0$$

$$y = -5$$

or

$$y = 0 \bullet x - 5$$

The slope and y intercepts are $m = 0$ (horizontal line)

$$b = -5$$

Example
Find the slope and y-intercept of $x - 3 = 0$.

Solution
This is also is special case. Since the equation cannot be solved for y, solve it for x.

$$x - 3 = 0$$
$$x = 3$$

There is *no slope* since the equation cannot be solved for y and there is no y-intercept (vertical line). The only information we have, $x = 3$, tells us where the line crosses the x-axis (the x-intercept). NOTE: The graph is a vertical line.

Parallel and Perpendicular Lines

Parallel lines have the *same* slope.
Perpendicular lines have slopes that are *negative reciprocals* of each other.
When working with parallel and perpendicular lines, it is best to write your equations in the slope-intercept form ($y = mx + b$).

Example
Write the linear equation whose slope and y-intercept are 3 and -4, respectively.

Solution

$$y = mx + b$$

(Equation of a line where m = slope and b = y-intercept)

$$\boxed{y = 3x - 4}$$

Substituting $m = 3$ and $b = 4$

Example
Write a linear equation that is parallel to $y = 3x - 4$.

Solution

$$y = 3x - 4$$
$$y = mx + b$$

(Equation of a line)

$$\boxed{y = 3x + 2}$$

The slope of the parallel line equals the slope of the given line. The y-intercept may have any value, like 2, which is substituted in the formula. Same slope (3).

Example
Write a linear equation that is perpendicular to $y = 3x - 4$.

Solution

$y = mx + b$ (Equation of a line)

To write the equation of a line that is perpendicular to the given line, substitute the negative reciprocal of the slope of the equation for m. The y-intercept may have any value; in this case we kept -4.

$$y = 3x - 4$$
$$\boxed{y = -\frac{1}{3}x - 4}$$

Practice Exercise 7

1. Solve for x: $2 + 3(5 - x) < 8$
   A. $x < 3$
   B. $x > 3$
   C. $x > -9$
   D. $x < -3$
   E. $x > -3$

2. Which of the following inequalities is NOT true when r, s, and t are real numbers?
   A. If $r < 0$, then $\frac{1}{r} < 0$.
   B. If $r > s$, then $r + t > s + t$.
   C. If $r > s$ and $s > t$, then $r > t$.
   D. If $r < 0$, then $r^2 < 0$.
   E. If $r > 0$, then $-r < 0$.

3. Solve for x: $|3 - 2x| = 5$
   A. $\{-1, -4\}$
   B. $\{-1, 4\}$
   C. $\{1, 1\}$
   D. $\{1, -1\}$
   E. None of the above

4. The open sentence $|2x - 3| < 7$ is equivalent to which of the following graphs?

   A.


   B.


   C.


   D.


   E.


5. Simplify $-7x^4 \bullet 4x^2$.
   A. $11x^6$
   B. $-11x^8$
   C. $28x^8$
   D. $-28x^6$
   E. $-3x^6$

6. Simplify $\frac{36a^2b^6}{4ab^2}$.
   A. $9ab^3$
   B. $9a^3b^8$
   C. $9ab^4$
   D. $9a^2b^{12}$
   E. $40ab^8$

7. Simplify $(2a^3)^3$.
   A. $2a^6$
   B. $2a^9$
   C. $6a^6$
   D. $6a^9$
   E. $8a^9$

8. The expression $8^{-\frac{4}{3}}$ equals:
   A. $-\frac{1}{16}$
   B. $\frac{1}{16}$
   C. -8
   D. -16
   E. 16

9. The expression $\frac{1}{x^4} + \frac{2}{x^2y^2} + \frac{1}{y^4}$ is equivalent to:
   A. $\frac{1}{x^2} + \frac{1}{y^2}$
   B. $\left(\frac{1}{x} + \frac{1}{y}\right)^2$
   C. $\frac{1}{x^2} + \frac{\sqrt{2}}{xy} + \frac{1}{y^2}$
   D. $(x^{-2} + y^{-2})^2$
   E. None of the above

10. Find the y-intercept of the line with the equation $2x + y = 5$.

A. -5
B. -2
C. $-\frac{1}{2}$
D. 0
E. 5

11. The slope of a line $\frac{1}{2}y = x + 4$ is:

A. -1
B. $\frac{1}{2}$
C. 1
D. 2
E. 4

12. What is the slope of a line parallel to the line whose equation is $y = -\frac{1}{2}x + 3$?

A. -2
B. $-\frac{1}{2}$
C. $\frac{1}{2}$
D. 2
E. 3

13. What is the slope of a line perpendicular to the line whose equation is $3x - 2y = 0$?

A. $-\frac{3}{2}$

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{2}$

E. No slope

14. $b^{-2} =$

A. $-b^2$
B. $\frac{1}{b^2}$
C. $-2b$
D. $\frac{1}{b^{-2}}$
E. $\frac{1}{-b^2}$

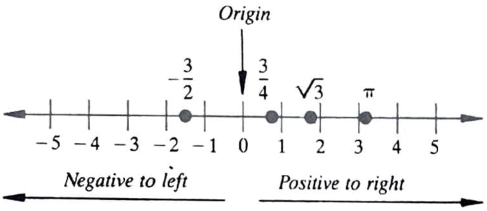
15. Which set best describes the graph below?

A. $x < -2$
B. $x > -2$
C. $x \leq -2$
D. $-2 < x < 2$
E. $x \geq -2$

COORDINATE GEOMETRY
SKILL BUILDER NINE

Graphing on the Number Line

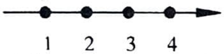
Every real number can be graphed as a point on a number line.



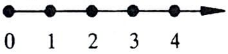
Familiarize yourself with these sets of numbers and their graphs.

Example

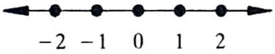
{natural numbers}



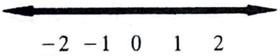
{whole numbers}



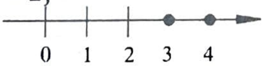
{integers}



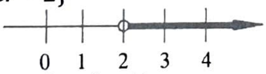
{real numbers}



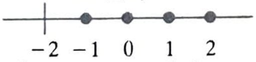
{any *integer* > 2}



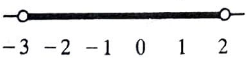
{any *number* > 2}



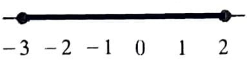
{any *integer* > -2 and ≤ 2}



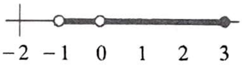
{any *number* between -3 and 2}



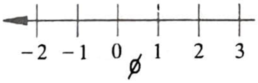
{any *number* between -3 and 2, inclusive}



{any *number* greater than -1 and less than or equal to 3, except 0}



{any integer between 1 and 2}



Graphs of Functions and Relations in the Standard Coordinate Plane

A **relation** is any set of ordered pairs of numbers. The set of first coordinates in the ordered pair is the **domain**, and the set of second coordinates is the **range**.

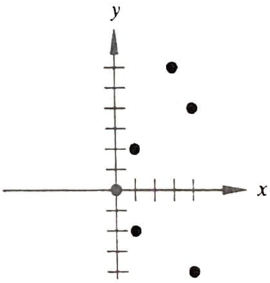
Example

Graph the relation.

{(0, 0), (1, 2), (1, -2), (4, 4), (4, -4), (3, 6)}

State the domain and range.

Solution



Domain = {0, 1, 3, 4}

Range = {-4, -2, 0, 2, 4, 6}

A **function** is a special kind of relation.

A **function** is a relation in which each element in the domain corresponds to a unique element in the range.

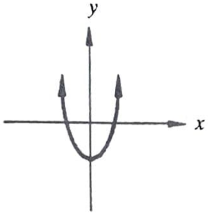
NOTE: Different number pairs never have the same first coordinate.

Example
{(1, 4), (4, 8), (5, 8), (8, 9)} is an example of a function.
Domain = {1, 4, 5, 8}
Range = {4, 8, 9}

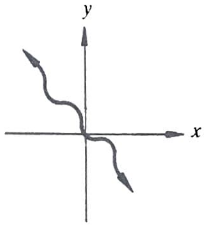
No vertical line intersects the graph of a function in more than 1 point.

Example
Are the following graphs of functions?

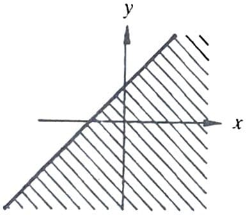
I. Function



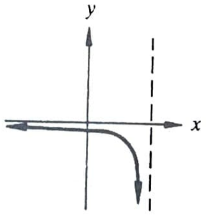
II. Function



III. Not a Function



IV. Function



III is not a function because a vertical line intersects at an infinite number of points within the shaded region. I, II, and IV all pass the vertical line test.

The notation $f(x) = x^2 + 5x + 6$ is also used to name a function. To find $f(0)$ means substitute 0 for x in $f(x)$.

Example
$f(x) = x^2 + 5x - 4$
$f(0) = 0^2 + 5 \bullet 0 - 4 = -4$
$f(2) = 2^2 + 5 \bullet 2 - 4 = 10$
$f(a) = a^2 + 5a - 4$

Example
If $f(x) = x^2 + 5$ and $g(x) = 1 + x$, find $g(f(2))$.

Solution
First find $f(2)$ $\quad f(2) = 2^2 + 5$
$f(2) = 4 + 5$
$f(2) = 9$

Next find $\quad g(f(2)) = g(9)$
$g(9) = 1 + 9$
$g(9) = 10$

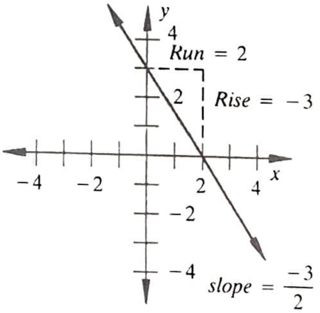
Slope of a Line

The slope of a line is its steepness or tilt. A vertical line is not tilted, therefore it has *no* slope. A horizontal line has a slope of zero.

$$\text{slope} = m = \frac{\text{Vertical change}}{\text{Horizontal change}} =$$

$$\frac{\text{difference of } y\text{'s}}{\text{difference of } x\text{'s}} = \frac{y_2 - y_1}{x_2 - x_1},\ (x_2 \neq x_1)$$

Example
If given the graph of a line the slope can be determined by finding $\frac{\text{rise}}{\text{run}}$.



Example
If given two points of a line the slope can be found by finding
(0, 3) (2, 0)

$$\text{slope} = \frac{3-0}{0-2} = \frac{3}{-2} = \frac{-3}{2}$$

$$\frac{\text{difference of } y\text{'s}}{\text{difference of } x\text{'s}} = \frac{y_2 - y_1}{x_2 - x_1},\ (x_2 \neq x_1)$$

Example
If given the equation of a line, transform it into the form $y = mx + b$ where m represents the slope.

$3x + 2y = 6$

$2y = -3x + 6$

$y = -\frac{3}{2}x + 3$

m (coefficient of x term) $= \frac{-3}{2}$

Distance Formula for Points in a Plane

The distance (d) between any two points A (x_1, y_1) and B (x_2, y_2) is found by using the formula:

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ or, in other words

$d = \sqrt{(\text{diff. of } x\text{'s})^2 + (\text{diff. of } y\text{'s})^2}$

Example
Find the distance between the points (7, 9) and (1, 1).

Solution

$$d = \sqrt{(7-1)^2 + (9-1)^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$= 10$$

Also important is the formula for finding the midpoint of a line segment.

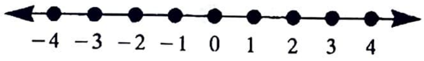
$$\text{Midpoint} = \left(\frac{\text{sum of the } x\text{'s}}{2}, \frac{\text{sum of the } y\text{'s}}{2}\right)$$

Example
The coordinates of the midpoint of the segment whose endpoints are (7, 9) and (1, 1).

$$\text{Midpoint} = \left(\frac{7+1}{2}, \frac{9+1}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{10}{2}\right)$$
$$= (4, 5)$$

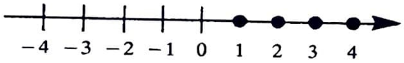
Practice Exercise 9

1. The figure represents the graph of what set of numbers?



A. {natural numbers}
B. {whole numbers}
C. {integers}
D. {rational numbers}
E. {real numbers}

2. The figure represents the graph of what set of numbers?



A. {natural numbers}
B. {whole numbers}
C. {integers}
D. {rational numbers}
E. {real numbers}

3. Which of the following represents the range of {(0, -5) (-1, 3) (1, 2) (2, 2) (3, -1) (-5, 3)}?
A. {-5, -1, 0, 1, 2, 3}
B. {-1, 0, 3}
C. {-5, -2, -1, 2}
D. {-5, -1, 2, 3}
E. {-5, -1, 0, 2, 3}

4. If $f(x) = x^3 - 1$, then $f(-2) = ?$

A. -27
B. -9
C. -7
D. -1
E. 9

5. The slope of the line whose equation is $4x + 5y = 20$ is ?

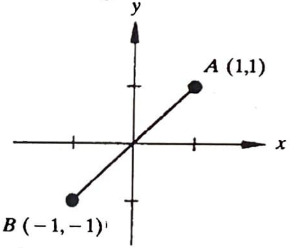
A. $\frac{4}{5}$
B. $-\frac{4}{5}$
C. 4
D. -4
E. $\frac{5}{4}$

6. What is the slope of the line joining (5, -2) and (3, -6)?
A. $-\frac{1}{4}$
B. 2
C. -2
D. -4
E. 4

7. What is the distance between the points (2, -4) and (-5, 3)?

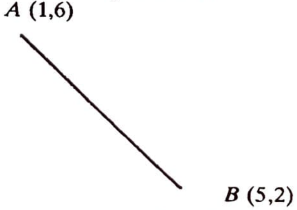
A. $7\sqrt{2}$
B. 5
C. $2\sqrt{7}$
D. 14
E. 98

8. What is the length of $\overline{AB}$?



A. $\sqrt{2}$
B. $\sqrt{3}$
C. $\sqrt{8}$
D. 1
E. 2

9. What is the midpoint of $\overline{AB}$?



A. (6, 8)
B. (3, 4)
C. (4, 3)
D. (8, 6)
E. (3, 1)

10. The distance between the points (4,2) and (-3,-2) is:
A. $2\sqrt{7}$
B. $\sqrt{77}$
C. $13\sqrt{5}$
D. 28
E. $\sqrt{65}$

PLANE GEOMETRY
SKILL BUILDER ELEVEN

Lines, Segments, and Rays

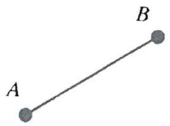
The following examples should help you distinguish between lines, segments, and rays. The three undefined terms in geometry are **point**, **line**, and **plane**.

• X represents point X—It has no size.

represents line *YZ*—It extends without end in both directions.

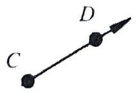
represents plane *W*—It has a flat surface that extends indefinitely in all directions.

Segment = A part of a line consisting of two endpoints and all the points between them.



Segment $\overline{AB}$ or segment $\overline{BA}$

Ray = A part of a line consisting of one endpoint and extending without end in the other direction.

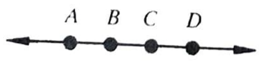


Ray *CD* (endpoint must be named first) or $\overrightarrow{CD}$

Collinear points are points that lie on the same line.

Non-collinear points are points that do not all lie on the same line.

NOTE:



$\overline{AD}$ and $\overline{DA}$ are the same segment.

But $\overrightarrow{AD}$ and $\overrightarrow{DA}$ are different rays (different endpoints).

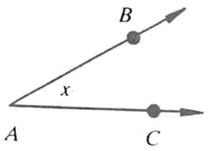
$\overleftrightarrow{AD}$ and $\overleftrightarrow{DA}$ are the same line.

$\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays (same endpoints).

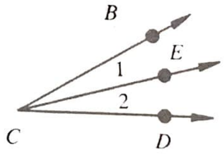
Measurement and Construction of Right, Acute, and Obtuse Angles

Naming Angles

An angle is formed by two rays having a common endpoint. This endpoint is called the **vertex** of the angle. Angles are measured in degrees.



Angles may be named in three different ways:
(1) by the letter at its vertex ($\angle A$).
(2) by three capital letters with the vertex letter in the center ($\angle BAC$ or $\angle CAB$).
(3) by a lower case letter or a number placed inside the angle ($\angle x$).

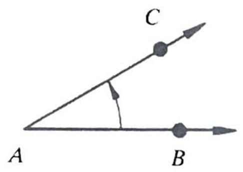


NOTE: Angle *C* cannot be the name for the angle because there are three angles that have the common vertex, *C*. They are angles *BCE*, *ECD*, and *BCD*. Angle *BCE* may also be named $\angle ECB$ or $\angle 1$. Angles 1 and 2 are adjacent angles since they share a common vertex, *C*, and a common side, *CE*, between them.

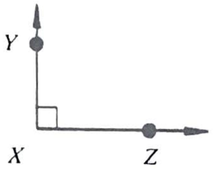
Classifying Angles

Angles are classified according to the number of degrees contained in the angle.

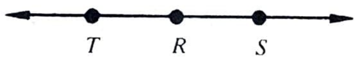
Type of Angle	Number of Degrees
acute angle	less than 90°
right angle	90°
obtuse angle	greater than 90° but less than 180°
straight angle	180°



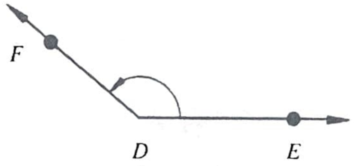
Acute $\angle BAC$



Right $\angle YXZ$



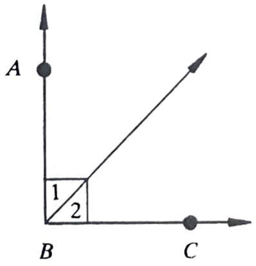
Straight $\angle TRS$



Obtuse $\angle EDF$

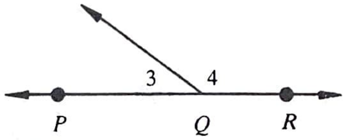
Complementary and Supplementary Angles

Complementary angles are two angles whose sum is 90°.



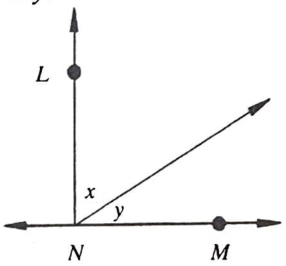
Since $\angle ABC$ measures 90°, angles 1 and 2 are complementary angles.

Supplementary angles are two angles whose sum is 180°.



Since $\angle PQR = 180°$, angles 3 and 4 are supplementary angles. This is also an example of a linear pair, adjacent angles such that two of the rays are opposite rays that form a linear pair.

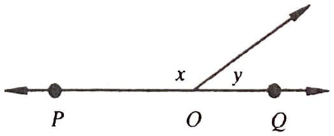
Example
If $LN \perp NM$, express the number of degrees in x in terms of y.



Solution
Since $LN \perp MN$, $\angle LNM$ measures 90°.

$$\begin{aligned} x + y &= 90^\circ \\ \underline{-y\qquad} &\underline{\qquad -y} \text{ (using the additive inverse)} \\ x &= 90^\circ - y \end{aligned}$$

Example
If PQ is a straight line, express y in terms of x.



Solution
A straight line forms a straight angle.
Therefore

y in terms of x

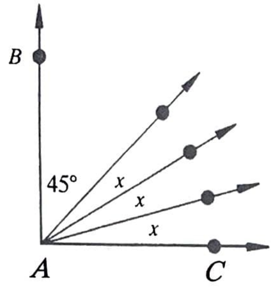
$$\begin{aligned} x + y &= 180^\circ \\ \underline{-x\qquad} &\underline{\qquad -x} \text{ (using the additive inverse)} \\ y &= 180^\circ - x \end{aligned}$$

x in terms of y

$$\begin{aligned} x + y &= 180^\circ \\ \underline{-y\qquad} &\underline{\qquad -y} \text{ (using the additive inverse)} \\ x &= 180^\circ - y \end{aligned}$$

Example

If $BA \perp AC$, find the number of degrees in angle x.



Solution

Since $BA \perp AC$, $\angle BAC = 90°$. Therefore,

$$x° + x° + x° + 45° = 90°$$

$$3x° + 45° = 90° \quad \text{(combining like terms)}$$

$$\underline{-45° - 45°} \quad \text{(using the additive inverse)}$$

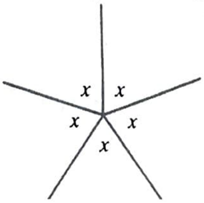
$$3x° = 45° \quad \text{(using the multiplicative inverse)}$$

$$\frac{1}{3}(3x°) = (45°)\frac{1}{3}$$

$$x° = 15°$$

Example

Find the number of degrees in angle x.



Solution

Since the five angles center about a point, their sum is 360°. Therefore,

$$x + x + x + x + x = 360° \quad \text{(combining like terms)}$$

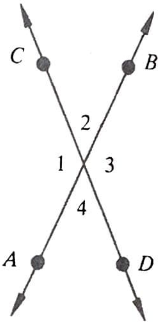
$$5x = 360°$$

$$\frac{1}{5}(5x) = (360°)\frac{1}{5} \quad \text{(using the multiplicative inverse)}$$

$$x = 72°$$

Vertical Angles

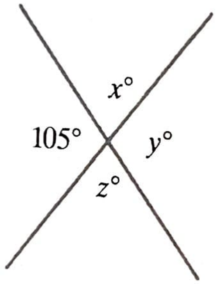
Vertical angles are the non-adjacent angles formed when two straight lines intersect.



Angles 1 and 3 are vertical angles. Angles 2 and 4 are also vertical angles. If $m\ \angle 2 = 50°$, then $m\ \angle 1 + m\ \angle 2 = 180°$ (straight $\angle AB$) and $\angle 1$ measures 130°. Also, $m\ \angle 1 + m\ \angle 2 = 180°$ (straight $\angle CD$), and $\angle 3$ measures 130°. Since supplements of the same angle are equal, vertical angles contain the same number of degrees. Thus, $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$.

Example

Find x, y, and z.



Solution

Since vertical angles are equal, $\angle\ y = 105°$. The same is true for x and z: $x = z$. Any two adjacent angles such as z and 105° are supplementary. Therefore,

$$z + 105 = 180$$

$$\underline{-105 \quad -105} \quad \text{(using the additive inverse)}$$

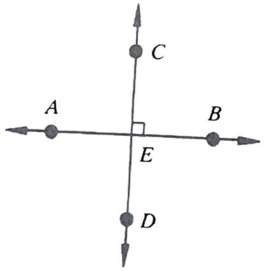
$$z = 75°$$

$$x = 75°$$

$$\text{and } y = 105°$$

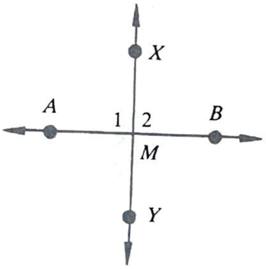
Perpendicular Lines

Perpendicular lines are lines that meet and form right angles. The symbol for perpendicular is ⊥.



$\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$
or
$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

If two intersecting lines form adjacent angles whose measures are equal, the lines are perpendicular.



If $m\angle 1 = m\angle 2$, then

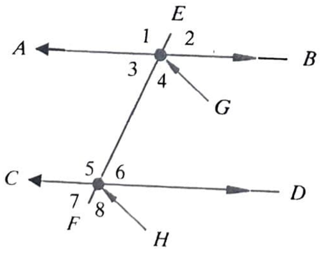
$\overleftrightarrow{AB} \perp \overleftrightarrow{XY}$

Perpendicular lines form four right angles.

Parallel Lines and Transversals

Angles Formed by Parallel Lines

The figure illustrates two parallel lines, *AB* and *CD*, and an intersecting line *EF*, called the **transversal**.



The arrows in the diagram indicate that the lines are parallel. The symbol || means "is parallel to": $\overline{AB} \parallel \overline{CD}$.

There are relationships between pairs of angles with which you are familiar from your previous studies. Angles 1 and 4 are vertical angles and congruent. Angles 5 and 7 are supplementary angles, and their measures add up to 180°.

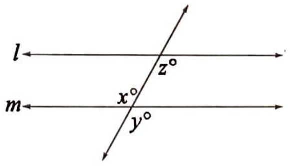
Corresponding angles are two angles that lie in corresponding positions in relation to the parallel lines and the transversal. For example, $\angle 1$ and $\angle 5$ are corresponding angles. So are $\angle 4$ and $\angle 8$. Other pairs of corresponding angles are $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$.

Angles 3 and 5 are interior angles on the same side of the transversal. These angles are supplementary. Angles 4 and 6 are also supplementary.

Alternate interior angles are two angles that lie on opposite (alternate) sides of the transversal and between the parallel lines. For example, $\angle 3$ and $\angle 6$ are alternate interior angles, as are $\angle 4$ and $\angle 5$.

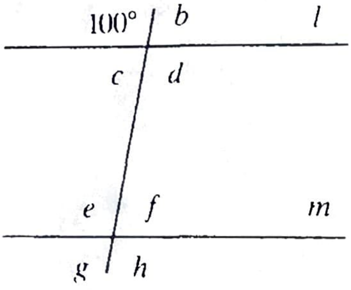
If two lines are parallel, corresponding angles are congruent or equal, and alternate interior angles are congruent or equal.

In the diagram below, if $l \parallel m$, the alternate interior angles are congruent, and $\angle x \cong \angle z$. Since corresponding angles are congruent, $\angle y \cong \angle z$. Therefore, $\angle x \cong \angle y \cong \angle z$.



Example

$l \parallel m$ and $m\angle a = 100°$. Find the number of degrees in angles b, c, d, e, f, g, and h.

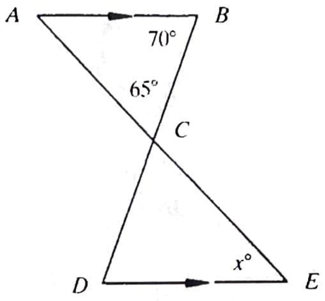


Solution

Because $\angle a$ and $\angle d$ are vertical angles, $\angle d$ measures 100°. Using supplementary angles, $m\angle b = 180° - 100° = 80°$ and $m\angle c = 180° - 100° = 80°$; therefore $m\angle b = m\angle c = 80°$. Use either property of parallel lines—corresponding angles or alternate interior angles—to obtain the remainder of the answers. For example, $m\angle b = m\angle f$ by corresponding angles or $m\angle c = m\angle f$ by alternate interior angles. Thus, $m\angle b = m\angle c = m\angle g = m\angle f = 80°$ and $m\angle e = m\angle h = m\angle d = 100°$.

Example

$AB \parallel ED$, $\angle B = 70°$ and $\angle ACB = 65°$. Find the number of degrees in x.



Solution

Knowing two angles of ΔABC, $\angle A = 45°$. Angles A and E are alternate interior angles. Therefore, $\angle A = \angle E$, since $AB \parallel DE$. Thus $\angle x = 45°$.

Practice Exercise 11

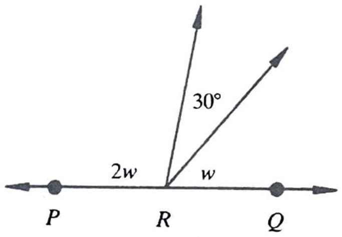
1. Three points, R, S, and T are collinear. Point S lies between R and T. If $RS = \frac{2}{3}RT$ and $RS = 48$, find $\frac{1}{2}RT$.

A. 72
B. 60
C. 48
D. 36
E. 24

2. Points E, F, and G are collinear. If $EF = 8$ and $EG = 12$, which point cannot lie between the other two?

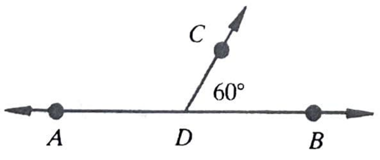
A. E
B. F
C. G
D. F and G
E. Cannot be determined

3. If PRQ is a straight line, find the number of degrees in $\angle w$.



A. 30
B. 50
C. 60
D. 70
E. 100

4. In the figure, if $\overleftrightarrow{AB}$ is a straight line and $m\angle CDB = 60°$, what is the measure of $\angle CDA$?

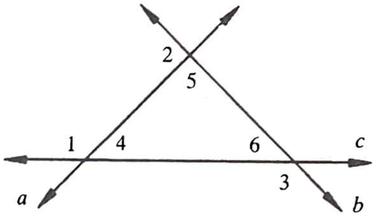


A. 15°
B. 30°
C. 60°
D. 90°
E. 120°

5. Line XY is perpendicular to line CD at D. Which conclusion can be drawn?

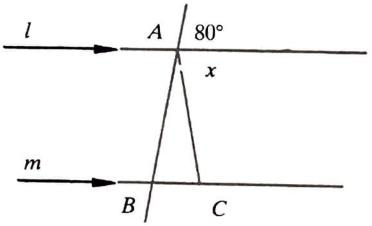
A. $XD = DY$
B. $XY = CD$
C. $m\angle XDC = 90°$
D. $m\angle XDC = 90°$ and $XD = DY$
E. All of the above

6. In the figure, a, b, and c are lines with $a \perp b$. Which angles are congruent?



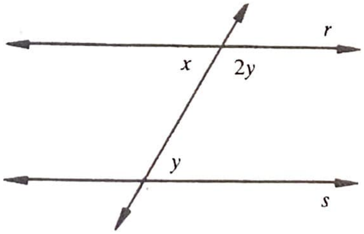
A. $\angle 4$, $\angle 5$
B. $\angle 4$, $\angle 6$
C. $\angle 4$, $\angle 3$
D. $\angle 2$, $\angle 5$
E. $\angle 2$, $\angle 6$

7. $l \parallel m$, and $AB = AC$. Find x.



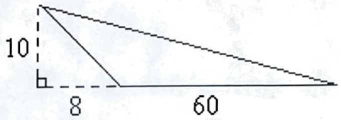
A. 40
B. 60
C. 80
D. 100
E. None of the above

8. In the figure, if lines r and s are parallel, what is the value of x?



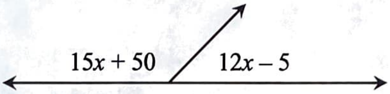
A. 30
B. 60
C. 90
D. 120
E. 150

9. The height of the triangle below is 10 units. What is its area?



A. 150
B. 300
C. 340
D. 600
E. 680

10. The measure of the smaller angle in figure below is:



A. 55°
B. 75°
C. 105°
D. 125°
E. 180°

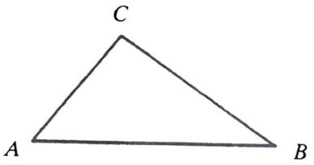
SKILL BUILDER TWELVE

Properties of Triangles

Classification by Sides

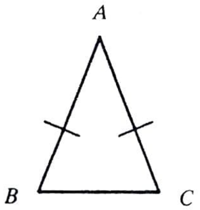
Triangles that are classified according to the lengths of their sides are equilateral, isosceles, or scalene.

Scalene Triangle: A triangle with no congruent sides.



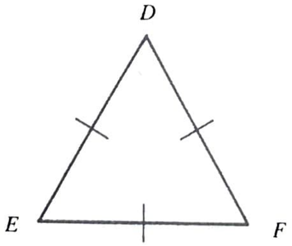
ΔABC is scalene.

Isosceles Triangle: A triangle with two or more congruent sides.



$\overline{AB} \cong \overline{AC}$, $\overline{BC}$ is called the base.
Angle A is the vertex angle,
$\angle B$ and $\angle C$ are the base angles.
$\angle B \cong \angle C$.

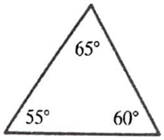
Equilateral Triangle: A triangle with three congruent sides.

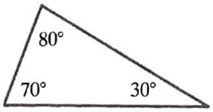


$\overline{DE} \cong \overline{DF} \cong \overline{EF}$
Each angle is congruent to the other angles.
$\angle D \cong \angle E \cong \angle F$.
Each has a degree measure of 60°.

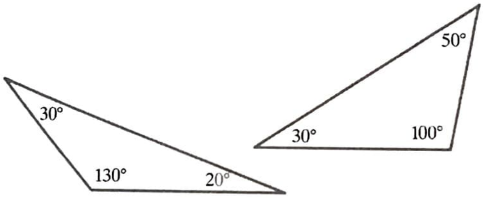
Classification by Angles

Triangles can be classified by their angles. An **acute triangle** is a triangle that has three **acute angles**. An acute angle is an angle whose measure is less than 90°.

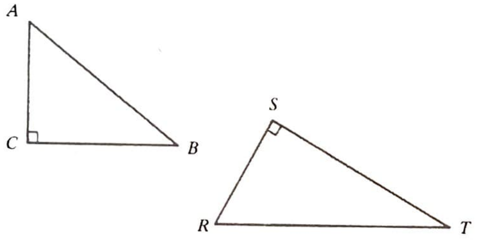




An **obtuse angle** is an angle whose degree measure is greater than 90° but less than 180°. An **obtuse triangle** has one obtuse angle.



A **right triangle** is a triangle that has one right angle. A right angle is an angle whose degree measure is 90°. The symbol for a right angle in a triangle is shown below.

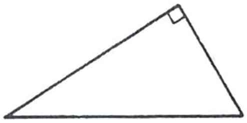


NOTE: In a right triangle, the sides have special names. The side opposite the 90° angle is the hypotenuse (the longest side of the right triangle). The two sides that form the 90° angle are called legs.

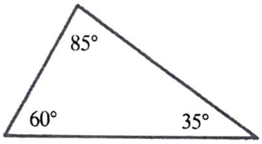
Thus, $\overline{AC}$ and $\overline{CB}$, $\overline{RS}$ and $\overline{ST}$ are legs. $\overline{AB}$ and $\overline{RT}$ are each called a hypotenuse.

Example
Classify each triangle pictured by the angles shown.

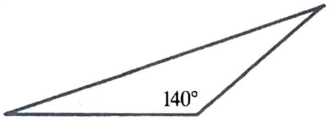
(A)



(B)



(C)

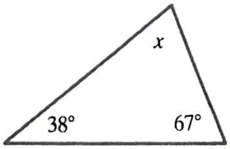


(A) The triangle has a right angle; therefore, it is a right triangle.
(B) The triangle is acute since each angle is acute.
(C) The obtuse angle identifies an obtuse triangle.

Sum of the Angles

The sum of the measure of the angles of any triangle is 180°.

Example
Find x.

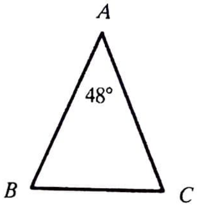


Solution

$x + 38 + 67 = 180$
$x + 105 = 180$ (combining similar terms)
$-105 \quad -105$ (using the additive inverse)
$x = 75$

Example
An isosceles triangle has a vertex angle whose degree measure is 48°. Find the degree measure of each base angle.

Solution



$\overline{AB} \cong \overline{AC}$ and vertex $m\angle A = 48°$.
Thus: $48° + m\angle B + m\angle C = 180°$

Let x represent the number of degrees in angles B and C

(by substitution) $48 + x + x = 180$
(combining similar terms) $2x + 48 = 180$
(using the additive inverse) $-48 \quad -48$
$2x = 132$

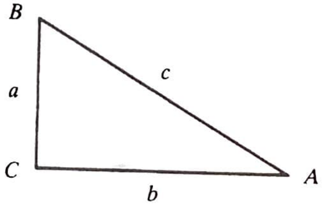
(using the multiplicative inverse) $\frac{1}{2}(2x) = (132)\ \frac{1}{2}$
$x = 66$

Therefore, $\angle B$ measures 66 and $\angle C$ measures 66°.

Perimeter of Triangles

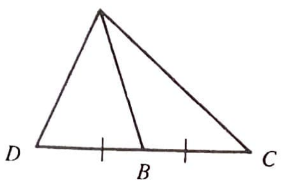
The perimeter of a triangle is the sum of the lengths of its sides.

Example
Find the perimeter of $\Delta\ ABC$.

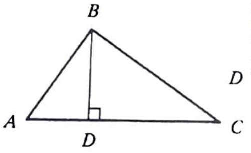


Solution
Perimeter $= a + b + c$

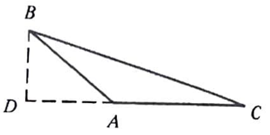
A **median** of a triangle is a segment from one vertex to the midpoint of the opposite side.



An **altitude** of a triangle is a segment from one vertex perpendicular to the opposite side.

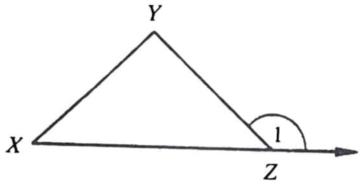


Altitude $\overline{BD}$



Altitude $\overline{BD}$

In ΔXYZ, $\angle 1$ is an exterior angle of the triangle because it forms a linear pair with an angle of the triangle.



Angles X and Y are called remote interior angles of the triangle with respect to $\angle 1$.

NOTE: $m\angle 1 = m\angle X + m\angle Y$.

Identification of Plane Geometric Figures

A polygon is a plane figure consisting of a certain number of sides. If the sides are equal, the figure is referred to as **regular**.

A triangle has 3 sides.
A quadrilateral has 4 sides.
A pentagon has 5 sides.
A hexagon has 6 sides.
A heptagon has 7 sides
An octagon has 8 sides.
A nonagon has 9 sides.
A decagon has 10 sides
A dodecagon has 12 sides.
An *n*-gon has *n* sides.

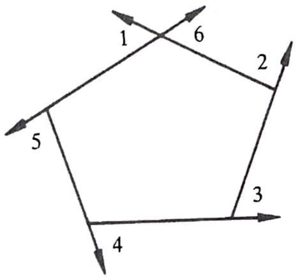
Angles of a Polygon

The sum of the measures of the angles of a triangle is 180°. The sum of the measures of the angles of a quadrilateral is 360°. The sum of the measures of the *interior* angles of a polygon is

$$S = 180(n - 2)$$

where n equals the number of sides.

An exterior angle of a polygon is an angle that forms a linear pair with one of the interior angles of the polygon.



NOTE: The sum of the measures of the exterior angles of any polygon is 360°.

Remember, triangles (polygons of three sides) are classified as:

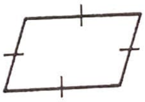
Acute—All three angles are acute angles
Obtuse—Contains one obtuse angle
Right—Contains one right angle
Equiangular—All three angles are equal
Scalene—No two sides equal
Isosceles—Two equal sides
Equilateral—All sides are equal

Quadrilaterals (polygons of four sides) are very important in geometry.

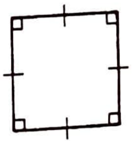
A parallelogram is a quadrilateral whose opposite sides are parallel.

A rectangle is a quadrilateral whose angles are right angles; it is a special kind of parallelogram.

A rhombus is a parallelogram with adjacent sides congruent.



A square is a four-sided figure with four right angles and four equal sides; it is a parallelogram.



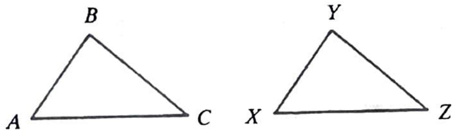
A trapezoid is a quadrilateral with exactly one pair of parallel sides.

An isosceles trapezoid is a trapezoid whose non-parallel sides are congruent.

Congruent and Similar Triangles

A correspondence between two triangles is a congruence if the corresponding angles and the corresponding sides are congruent. Triangles that have the same *shape* and *size* are congruent.

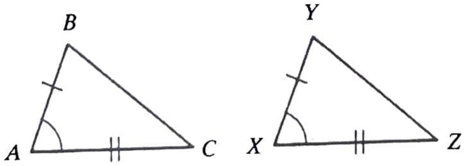
If $\angle A \cong \angle X$ $\quad \overline{AB} \cong \overline{XY}$
$\angle B \cong \angle Y$ $\quad \overline{BC} \cong \overline{YZ}$
$\angle C \cong \angle Z$ $\quad \overline{AC} \cong \overline{XZ}$



$\triangle ABC \cong \triangle XYZ$

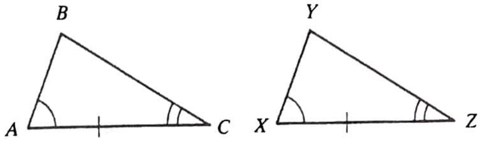
There are other ways to show triangles are congruent.
(A = Angle; S = Side)

SAS Postulate (Side-included angle-side)



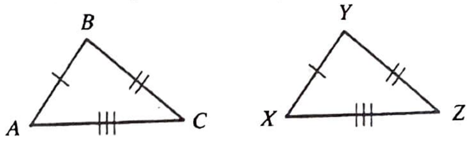
If $\overline{AB} \cong \overline{XY}$ (side)
$\angle A \cong \angle X$ (the *included* angle)
$\overline{AC} \cong \overline{XZ}$ (side)

ASA Postulate (Angle-included side-angle)



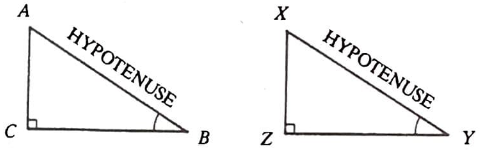
If $\angle A \cong \angle X$ (angle)
$\overline{AB} \cong \overline{XZ}$ (included side)
$\angle C \cong \angle Z$ (angle)

SSS Postulate (side-side-side)



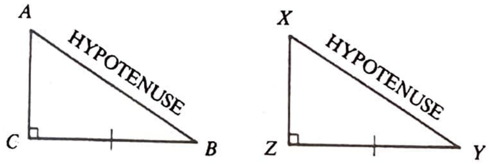
If $\overline{AB} \cong \overline{XY}$ (side)
$\overline{BC} \cong \overline{YZ}$ (side)
$\overline{AC} \cong \overline{XZ}$ (side)

If they are right triangles:



$\overline{AB} \cong \overline{XY}$

HA Postulate:
(hypotenuse-acute angle) If $\angle B \cong \angle Y$



$\overline{AB} \cong \overline{XY}$

HL Postulate
(hypotenuse-leg) If $\overline{CB} \cong \overline{ZY}$

Also, if two triangles have a side and two angles of one congruent to a side and two angles of the other, then the triangles are congruent (SAA).

Two triangles are similar if the corresponding angles are *congruent*.
AAA similarity theorem
or
AA similarity theorem

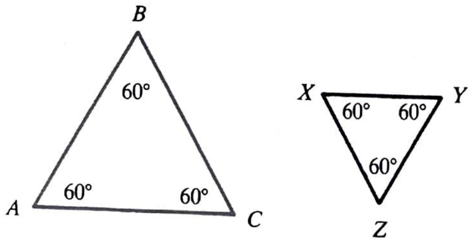
Other theorems used to show triangles similar are

SAS similarity theorem
SSS similarity theorem

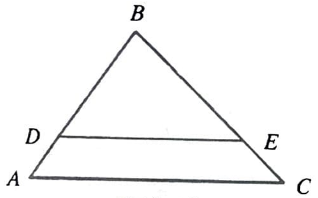
where the angles are congruent and the sides proportional.

NOTE: In addition, the perimeters, altitudes, and medians of similar triangles are proportional to any pair of corresponding sides.

The following diagrams are examples of similar triangles.



AAA Similarity Theorem

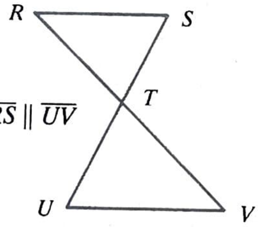


$\overline{DE} \parallel \overline{AC}$

$\angle A \cong \angle BDE$

$\angle C \cong \angle BCA$

AA Similarity Theorem



$\overline{RS} \parallel \overline{UV}$

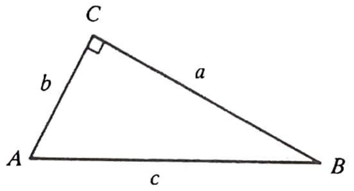
$\angle R \cong \angle V$ (alternate)

$\angle RTS \cong \angle VTU$ (vertical angles)

AA Similarity Theorem

Pythagorean Theorem

In a right triangle, the side opposite the right angle is called the **hypotenuse**, and the other two sides are called the **legs**.

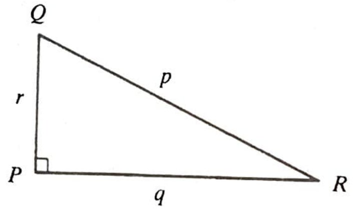


In the figure, $\overline{AB}$ is the hypotenuse whose measure is c. $\overline{AC}$ and $\overline{CB}$ are the legs whose measures are b and a, respectively.

In any right triangle, the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the legs. Thus: hypotenuse^2 = leg^2 + leg^2, or $c^2 = a^2 + b^2$, represents the Pythagorean Theorem.

This theorem can be used to find the measure of a third side of a right triangle if the measures of the other two sides are known.

Example
If $PQ = 5$ and $PR = 12$, find QR.



Solution

$p^2 = q^2 + r^2$

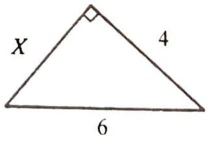
$p^2 = 12^2 + 5^2$

$p^2 = 144 + 25$

$p^2 = 169$

$p = \sqrt{169} = 13$

Example
Find X in its reduced form.



leg^2 + leg^2 = hyp^2 (Pythagorean Theorem)

Solution

$$\begin{aligned} x^2 + 4^2 &= 6^2 \\ x^2 + 16 &= 36 \\ x^2 &= 20 \\ x &= \sqrt{20} \\ x &= \sqrt{4}\sqrt{5} \\ x &= 2\sqrt{5} \end{aligned}$$

NOTE: There are sets of numbers that satisfy the Pythagorean Theorem. These sets of numbers are called Pythagorean triples.

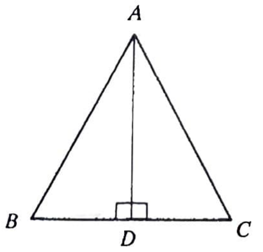
You should memorize the most common sets of Pythagorean triples

{3, 4, 5}
{6, 8, 10}
{5, 12, 13} ⟶ for example $5^2 + 12^2 = 13^2$
{8, 15, 17} $25 + 144 = 169$
{7, 24, 25} $169 = 169$

Ratio of Sides in 30°-60°-90° Triangles and 45°-45°-90° Triangles

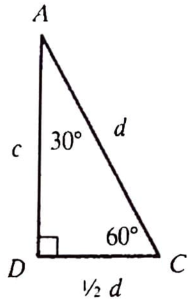
There are two special right triangles that deserve your attention. The special properties regarding these triangles can be found by using the Pythagorean Theorem.

30°-60°-90° Triangle: This special triangle is formed by starting with an equilateral triangle and drawing an altitude.



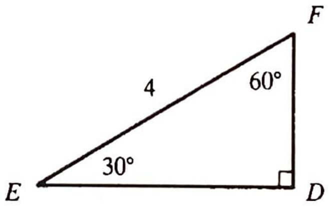
ΔABC is equilateral.
$\overline{AD}$ is an altitude.

The altitude drawn to the base of an isosceles triangle bisects the vertex angle and meets the base at the midpoint (or center).



Memorize this:
In a 30°-60°-90° right triangle, *the side opposite the 30° angle* is equal in length to one-half the hypotenuse. *The side opposite the 60° angle* is equal in length to one-half the hypotenuse times $\sqrt{3}$.

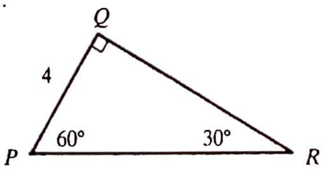
Example
ΔDEF is a 30°-60°-90° triangle, and $\overline{EF} = 4$. Find the measure of $\overline{DF}$ and $\overline{DE}$.



Solution
$\overline{DF}$ is the side opposite the 30° angle, so its measure is one-half the hypotenuse. Since $\overline{EF} = 4$, $\overline{DF} = 2$. $\overline{DE}$ is the side opposite the 60° angle, so its measure is one-half the hypotenuse times $\sqrt{3}$.

$$\overline{DE} = \frac{1}{2} \bullet 4 \bullet \sqrt{3} = 2\sqrt{3}.$$

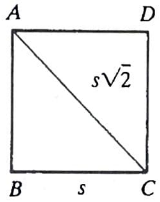
Example
ΔPQR is a 30°-60°-90° triangle and $PQ = 4$. Find QR and PR.



Solution
PQ is the side opposite the 30° angle and its measure is one-half the hypotenuse PR. Thus, PR = 8. QR, the side opposite the 60° angle, is one half the hypotenuse times $\sqrt{3}$.

$$QR = \frac{1}{2} \bullet 8 \bullet \sqrt{3} = 4\sqrt{3}$$

45°-45°-90° Triangle: This special right triangle is formed by drawing a diagonal in a square. A diagonal is a line drawn in the polygon that joins any two nonconsecutive vertices.



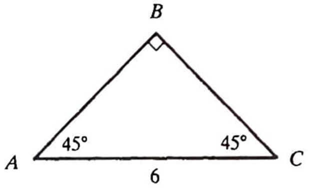
In the figure, $\overline{AC}$ is a diagonal. A diagonal of a square

- divides the square into two congruent isosceles triangles
- bisects two angles of the square
- forms two 45°-45°-90° right triangles

Memorize this:
In a 45°-45°-90° right isosceles triangle the hypotenuse or the length of the diagonal of the square is found by **multiplying** the length of the side of the square by $\sqrt{2}$.

To find the side of the square or one of the legs of the 45°-45°-90° triangle, *divide* the hypotenuse by $\sqrt{2}$ or take one-half the hypotenuse and multiply by $\sqrt{2}$.

Example
In the figure, ABC is a 45°-45°-90° triangle with AC = 6. Find AB and BC.



Solution
AB = 6 divided by $\sqrt{2}$

or

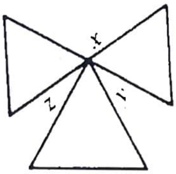
$\frac{1}{2}$ • hypotenuse • $\sqrt{2}$

$\frac{1}{2} \bullet 6\sqrt{2} = 3\sqrt{2}$

Since AB = BC, BC = $3\sqrt{2}$

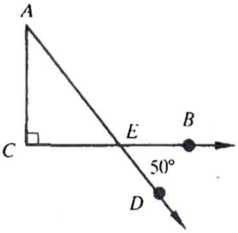
Practice Exercise 12

1. In the figure, the three triangles are equilateral and share a common vertex. Find the value of $x + y + z$.



A. 90°
B. 120°
C. 180°
D. 360°
E. Cannot be determined from the information given

2. In the figure, $\angle C$ measures 90°, $\overline{CB}$ and $\overline{AD}$ are straight line segments, and $\angle BED$ measures 50°. What is the measure of $\angle A$?

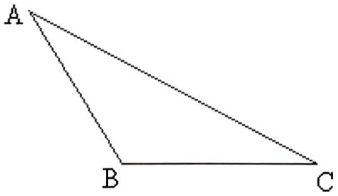


A. 40°
B. 50°
C. 90°
D. 100°
E. 130°

3. If a straight line is drawn from one vertex of a pentagon to another vertex, which of the following pairs of polygons could be produced?

A. Two triangles
B. Two quadrilaterals
C. A triangle and a quadrilateral
D. A quadrilateral and a pentagon
E. All of the above

4. The perimeter of the triangle below is 176 units. If sides AB and BC are the same length, and side AC is 56, what are the lengths of sides AB and BC?

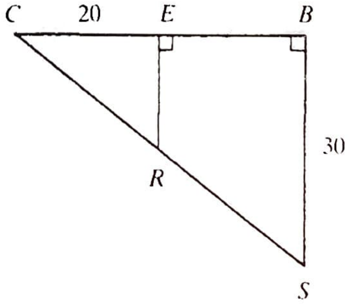


A. 56
B. 60
C. 68
D. 72
E. 90

5. In a triangle, the longest side is 8 units more than the shortest side, and the shortest side is half the remaining side. Find the length of the longest side if the triangle's perimeter is 32 units.

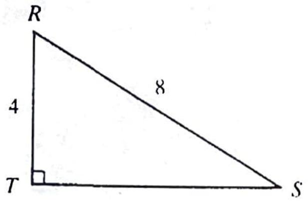
A. 18
B. 14
C. 12
D. 10
E. 6

6. In the figure below, E is the midpoint of $\overline{BC}$, and $\overline{RE}$ and $\overline{SB}$ are each perpendicular to $\overline{BC}$. If CE is 20 and SB is 30, how long is the perimeter of quadrilateral $REBS$?



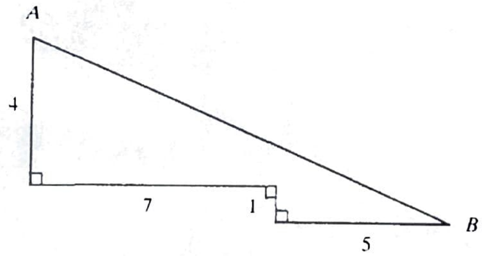
A. 50
B. 70
C. 90
D. 120
E. 220

7. In the figure, ΔRST is a right triangle. Hypotenuse $\overline{RS}$ is 8 units long, and side $\overline{RT}$ is 4 units long. How many units long is side $\overline{TS}$?



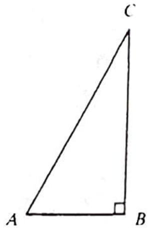
A. $2\sqrt{3}$
B. 4
C. $4\sqrt{3}$
D. 8
E. None of the above

8. In the figure below, what is the length of the segment AB?



A. 11
B. 13
C. $5 + \sqrt{66}$
D. 16
E. 17

9. In the figure below, $AB = \frac{1}{2}AC$ and $\angle ABC$ is a right angle. What is the measure of $\angle ACB$?



A. 25°
B. 30°
C. 40°
D. 45°
E. 60°

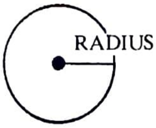
10. What is the length of a side of a square with a diagonal of length $5\sqrt{2}$ units?

A. $\frac{5\sqrt{2}}{2}$
B. 5
C. $\frac{10\sqrt{2}}{2}$
D. 10
E. $10\sqrt{2}$

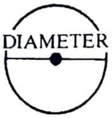
SKILL BUILDER THIRTEEN

Basic Properties of a Circle: Radius, Diameter, and Circumference

A radius is a line segment joining the center of a circle and a point on the circle.



A diameter is a straight line passing through the center of the circle and terminating at two points on the circumference. It measures the distance across a circle, and its measure is equal to twice the measure of the radius.

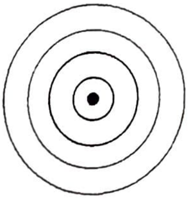


Circumference is the distance around a circle. It replaces the word perimeter in circles.

NOTE: No matter how large or small a circle is, the length of the diameter will always divide into the circumference the same number of times. This ratio is represented by the Greek letter π. Approximate values for π are 3.14 and $\frac{22}{7}$.

The formula for finding the circumference is $C = \pi d$, where d is the diameter.

Concentric circles are circles that lie in the same plane and have the same center and radii of different length.



Example
Approximate the circumference of a circle with a diameter of 14 inches. Use $\frac{22}{7}$ as an approximation for π.

Solution
$C = \pi d$
$C = \frac{22}{\underset{1}{\cancel{7}}} \bullet \overset{2}{\cancel{14}}$
$C = 44$ inches

Example
Approximate the circumference of a circle with a radius of 21 inches. Use $\frac{22}{7}$ as an approximation for π.

Solution
$C = \pi d$
$C = \frac{22}{\underset{1}{\cancel{7}}} \bullet \overset{6}{\cancel{42}}$
$C = 132$ inches

Example
The circumference of a circle measures 66 feet. Approximate the radius of the circle. Use $\frac{22}{7}$ as an approximation for π.

Solution

$$C = \pi d$$

$$66 = \frac{22}{7} \bullet d$$

$$\left(\frac{7}{\underset{1}{\cancel{22}}}\right)\overset{3}{\cancel{66}} = \frac{\overset{1}{\cancel{22}}}{\underset{1}{\cancel{7}}}\left(\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{22}}}\right) \bullet d$$

(Multiplying both sides by $\frac{22}{7}$)

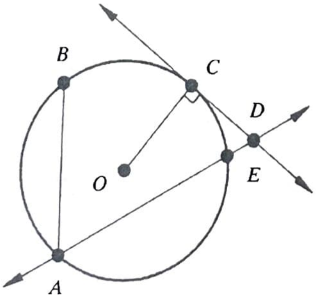
$$21 = d$$

The radius approximately equals $\frac{21}{2}$ or $10\frac{1}{2}$ feet.

Other important definitions in relation to a circle

Secant—A line drawn from a point outside a circle that intersects a circle in two points. See $\overrightarrow{AD}$ below.
Chord—A line segment joining any two points on the circle. (A diameter is a chord that passes through the center of the circle.) See $\overline{BA}$ below.
Tangent—a line that intersects a circle at one and *only* one point on the circumference. See $\overleftrightarrow{CD}$ below.

(If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of tangency.)
$\overline{OC}$ is a radius.
Line CD is perpendicular to $\overline{OC}$ at point C.



Circumference and Arc Length

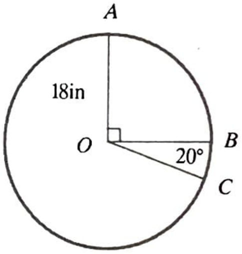
Circumference of a Circle

The circumference of a circle is the entire length of the arc of a circle or the distance around the circle. In all circles, regardless of the size, the ratio $\frac{\text{circumference}}{\text{diameter}}$ is constant. This constant value is π.

Example
Find the circumference of a circle if the length of its radius is 5". (Use $\pi = 3.14$).

Solution
Since radius = 5, diameter = 10
$C = \pi d$
$C = (3.14)(10)$
$C = 31.4$"

Example
Sometimes it is necessary to find the length of an arc, which is equivalent to finding a fractional part of the circumference. In the figure, how can the length of $\overset{\frown}{AB}$ and the length of $\overset{\frown}{BC}$ be found?



Solution
The first step is to find the circumference. If $r = 18$, then $d = 36$.
$C = \pi d$
$C = \pi \bullet 36$
$C = 36\pi$
The second step is to find the fractional part of the circumference contained in the length of the arc. Since $m\ \overset{\frown}{AB} = 90°$, it is $\frac{90}{360}$ or $\frac{1}{4}$ of the entire circle.

Therefore, $\overset{\frown}{AB} = \frac{1}{4} \bullet 36\pi = 9\pi$ inches.

Since $m\ \overset{\frown}{BC} = 20°$, it is $\frac{20}{360}$ or $\frac{1}{18}$ of the entire circle. $\overset{\frown}{BC} = \frac{1}{18} \bullet 36\pi = 2\pi$ inches.

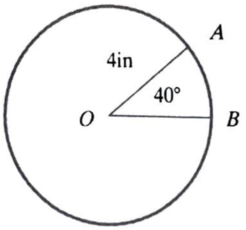
The length of an arc can be found using the above procedure, which results in the formula

$$\text{Length of arc} = \frac{n}{360} \bullet \pi d$$

where n = number of degrees in the arc.

Example

O is the center of the circle with a radius of 4”. Find the length of $\overset{\frown}{AB}$ intercepted by a central angle of 40°.



Solution

$\overset{\frown}{AB}$ contains 40° since it is intercepted by central $\angle AOB$, which measures 40°.

Using the formula

$$\text{length } \overset{\frown}{AB} = \frac{n}{360} \bullet \pi d$$
$$= \frac{40}{360} \bullet \pi \bullet 8$$
$$= \frac{8\pi}{9} \text{ inches}$$

Example

A wheel is rolled and makes five revolutions. If the diameter of the wheel is 3 feet, how far does the wheel travel?

Solution

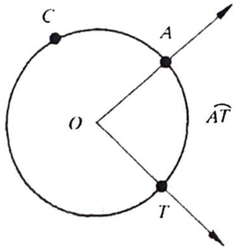
As the wheel makes one revolution, every point on the wheel touches the ground. The distance the wheel travels during one revolution is the distance around the wheel (the circumference). In this exercise,

$$C = \pi d$$
$$C = \pi(3)$$
$$C = 3\pi \text{ feet}$$

Since there are five revolutions, multiply 3π by 5. The answer is 15π feet.

Measurement of Arcs

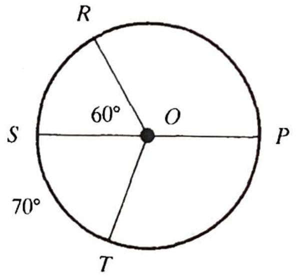
Central angle is an angle whose vertex is at the center of the circle.



O is the center of the circle
$\angle AOT$ is a central angle
$\overset{\frown}{AT}$ is a minor arc
$\overset{\frown}{ACT}$ is a major arc

IMPORTANT:
The measure of a minor arc equals the measure of its central angle.
The measure of a semicircle is 180°.
The measure of a major arc is 360° minus the measure of the central angle’s intercepted arc.

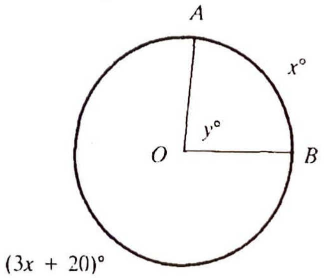
Example



If $m\angle ROS = 60°$
$m\ \overset{\frown}{ST} = 70°$
SP is a diameter
Then $m\ \overset{\frown}{RS} = 60°$
$m\ \overset{\frown}{RP} = 180° - 60° = 120°$
$m\ \overset{\frown}{RT} = 60° + 70° = 130°$
$m\ \overset{\frown}{STP} = 180°$
$m\ \overset{\frown}{RPT} = 360° - 130° = 230°$

Example

O is the center; a central angle intercepts a minor arc of $x°$ and a major arc of $3x + 20°$. Find y.

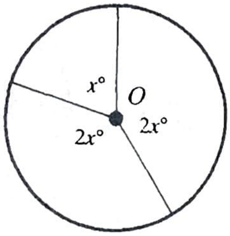


Solution
The sum of the measures of the minor and major arcs is 360°.

$$x + (3x + 20) = 360$$
$$x + 3x + 20 = 360 \quad \text{(simplifying terms)}$$
$$4x + 20 = 360 \quad \text{(combining similar terms)}$$
$$-20 \quad -20 \quad \text{(using the additive inverse)}$$
$$4x = 340$$
$$\frac{1}{4}(4x) = (340)\frac{1}{4} \quad \text{(using the multiplicative inverse)}$$
$$x = 85$$

Since ∠*AOB* is a central angle, $y = x = 85$.

Example
Given the three central angles, find the measure of minor arc *y*.

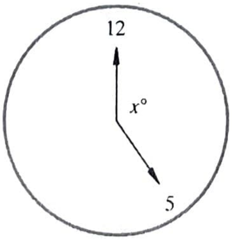


Solution
The three central angles intercept three arcs which form the entire circle = (360°). The equation would be:

$$x + 2x + 2x = 360$$
$$5x = 360 \quad \text{(combining similar terms)}$$
$$\frac{1}{5}(5x) = (360)\frac{1}{5} \quad \text{(using the multiplicative inverse)}$$
$$x = 72$$

Since $y = 2x$, $y = 144°$.

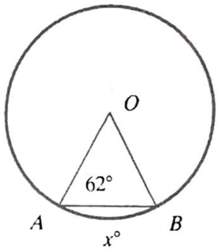
Example
What is the measure of the obtuse angle formed by the two hands of a clock at 5 P.M.?



Solution
x is $\frac{5}{12}$ of 360.

$$x = \frac{5}{\underset{1}{\cancel{12}}} \times \frac{\overset{30}{\cancel{360}}}{1}$$
$$x = 150°$$

Example
O is the center. The measure of ∠*A* is 62°. Find *x*.

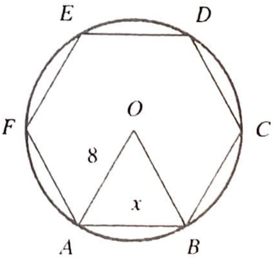


Solution
To find *x*, find the number of degrees in the central ∠*AOB*. *OA* and *OB* are radii in the same circle and are, therefore, equal. The angles opposite these equal sides are equal, so *m* ∠*B* = 62°.

$$m\angle AOB + 62° + 62° = 180° \quad \text{(combining similar terms)}$$
$$m\angle AOB + 124° = 180°$$
$$\underline{-124° \quad -124°} \quad \text{(using the additive inverse)}$$
$$m\angle AOB = 56°$$

Since central ∠*AOB* measures 56°, its intercepted arc measures 56°.

Example (special case)
O is the center; *ABCDEF* is a regular hexagon inscribed in the circle whose sides are eight units long. Find *x*.

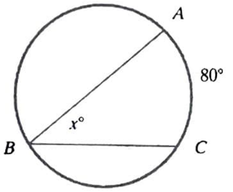


Solution

A regular hexagon is a polygon whose six sides and six angles are congruent. Its central angle measures 60° (360 ÷ 6 = 60°). Triangle AOB is isosceles and $\angle OAB = \angle OBA$. A regular hexagon contains 6 equilateral triangles.
Thus $x = 8$.

An **inscribed angle** is an angle whose vertex is on the circle and whose sides are chords of the circle. A chord is a line drawn within a circle touching two points on the circumference of the circle.

Example

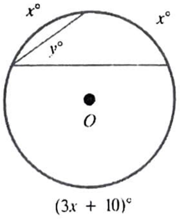


Angle ABC is an inscribed angle. The measure of an inscribed angle is equal to one-half the measure of its intercepted arc. If $m\ \widehat{AC} = 80$:

$\angle x$ measures $\frac{1}{2}(80)$; $\angle x$ measures 40°.

Example

If the three arcs of the circle measure $x°$, $x°$, and $(3x + 10)°$, find inscribed $\angle y$.



Solution

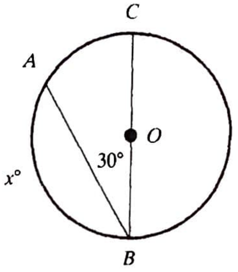
Find x by using the equation:

$x + x + (3x + 10) = 360$
$x + x + 3x + 10 = 360$ (simplifying terms)
$5x + 10 = 360$ (combining similar terms)
$\underline{-10 \quad -10}$ (using the additive inverse)
$5x = 350$
$\frac{1}{5}(5x) = (350)\frac{1}{5}$ (using the multiplicative inverse)
$x = 70°$

Since y is the measure of an inscribed angle, it measures $\frac{1}{2}(x) = 35°$

Example

O is the center. Find x.



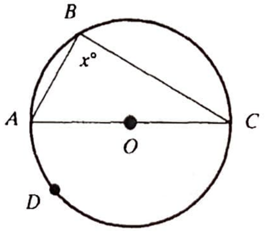
Solution

$\overline{BC}$ is a diameter (a chord that passes through the center of the circle forming 2 equal arcs of 180° each). Angle B is an inscribed angle. The measure of $\widehat{AC}$ is 60° and the measure of $\widehat{BAC}$ is 180°. Therefore:

$x + 60 = 180$
$\underline{-60 \quad -60}$ (using the additive inverse)
$x = 120$

Example

O is the center. Find x.



Solution

$\overline{AC}$ is a diameter and $\widehat{ADC}$ is a semi-circle whose measure is 180°.

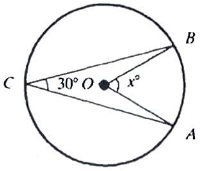
$$x = \frac{1}{2}\ m\ \widehat{ADC}$$

$$x = \frac{1}{2}(180)$$

$$x = 90°$$

Note: Any angle inscribed in a semi-circle is a right angle.

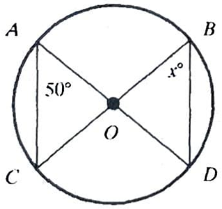
Example
O is the center. Chords $\overline{BC}$ and $\overline{AC}$ form an inscribed angle of 30°. Find the central angle whose measure is x.



Solution

$$\angle C = \frac{1}{2}\, m\ \overset{\frown}{AB}$$
$$30° = \frac{1}{2}\, m\ \overset{\frown}{AB}$$
$$2(30°) = (\frac{1}{2}\, m\,\overset{\frown}{AB})2 \text{ (using the multiplicative inverse)}$$
$$60° = m\ \overset{\frown}{AB}$$
$$x = m\ \overset{\frown}{AB}$$
$$x = 60°$$

Example
Inscribed $\angle CAD$ measures 50°. Find the inscribed angle whose measure is x.



Solution

$$m\ \angle A = \frac{1}{2}\, m\ \overset{\frown}{CD}$$
$$50° = \frac{1}{2}\, m\ \overset{\frown}{CD}$$
$$2(50°) = (\frac{1}{2}\, m\,\overset{\frown}{CD})2 \text{ (using the multiplicative inverse)}$$
$$100° = m\ \overset{\frown}{CD}$$
$$m\ \angle B = \frac{1}{2}\, m\ \overset{\frown}{CD}$$
$$m\ \angle B = \frac{1}{2}(100°)$$
$$m\ \angle B = 50°$$

Therefore, $x = 50°$

The measures of two inscribed angles are equal if they intercept the same arc.

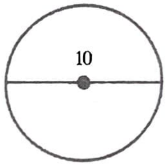
Areas of Circles, Triangles, Rectangles, Parallelograms, Trapezoids, and Other Figures with Formulas

Besides the circle, the most important polygons are the triangles and the quadrilaterals.

Following is a list of area formulas you should memorize with accompanying examples.

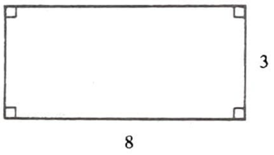
Circle

$$\text{Area } = \pi r^2$$
$$= \pi \bullet 5^2$$
$$= 25\pi$$



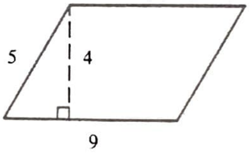
Rectangle

$$\text{Area } = bh$$
$$= 8 \bullet 3$$
$$= 24$$



Parallelogram

$$\text{Area } = bh$$
$$= 9 \bullet 4$$
$$= 36$$

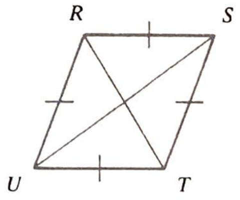


Rhombus

$$\text{Area } = \frac{1}{2} \text{ product of diagonals}$$
$$= \frac{1}{2} d_1 d_2, \text{ which is a special case of a parallelogram}$$

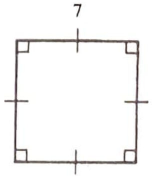
If $RT = 10$, $US = 24$

$$\text{Area } = \frac{1}{2} \bullet 10 \bullet 24$$
$$= 120$$

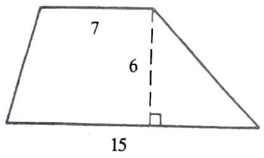


Square

$$\text{Area } = s^2$$
$$= 7^2$$
$$= 49$$



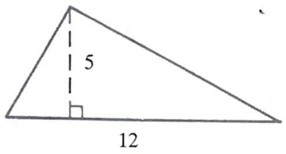
Trapezoid



Area $= \frac{1}{2}h(b_1 + b_2)$ where h = altitude and b_1 and b_2 are the lengths of the parallel bases

$$= \frac{1}{2} \bullet 6(15 + 7)$$
$$= \frac{1}{2} \bullet 6 \bullet 22$$
$$= 66$$

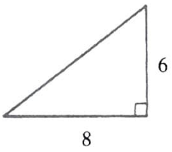
Triangle



$$\text{Area} = \frac{1}{2}bh$$
$$= \frac{1}{2} \bullet 12 \bullet 5$$
$$= 30$$

Right Triangle

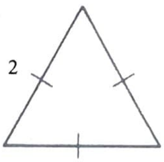
$$\text{Area} = \frac{1}{2}\text{product of legs}$$
$$= \frac{1}{2} \bullet 6 \bullet 8$$
$$= 24$$



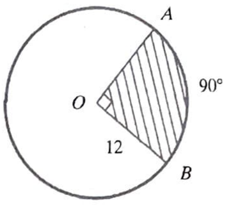
Equilateral Triangle

Area $= \frac{s^2\sqrt{3}}{4}$ where s = length of a side

$$= \frac{2^2\sqrt{3}}{4}$$
$$= \frac{4\sqrt{3}}{4}$$
$$= \sqrt{3}$$



A sector of a circle is the region bounded by an arc of the circle and two radii drawn to the endpoints of the arc.

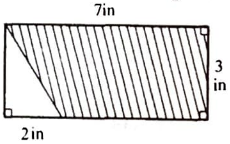


Area of sector $= \frac{n}{360} \bullet \pi r^2$ where n equals degree measure of the arc of the sector.

$$\text{Area of sector } OAB = \frac{n}{360} \bullet \pi r^2$$
$$= \frac{90}{360} \bullet \pi \bullet 12^2$$
$$= \frac{90}{360} \bullet \pi \bullet 12 \bullet 12$$
$$= 36\pi \text{ square units}$$

Computing the area of a specified region

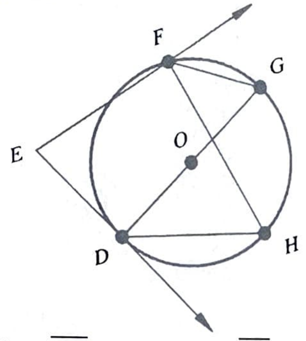
Example
What is the area of the shaded portion?



Solution
The area of the shaded portion equals the area of the rectangle minus the area of the triangle. Area of rectangle = length × width = 7 inches × 3 inches = 21 square inches. Area of triangle = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 2 inches × 3 inches = 3 square inches. Area of shaded region = 21 square inches − 3 square inches = 18 square inches. To solve this problem: (1) Find the area of the outside, larger figure; (2) Find the area of the smaller, undefined figure; (3) Subtract the area of the unshaded figure from the area of the outside larger figure.

Practice Exercise 13

1. In the diagram, which line segment is a diameter?



A. $\overline{DH}$ D. $\overline{DE}$

B. $\overline{DO}$ E. $\overline{FH}$

C. $\overline{DG}$

2. If a radius of a circle is doubled, what happens to the circumference of the new circle?

A. It remains the same.

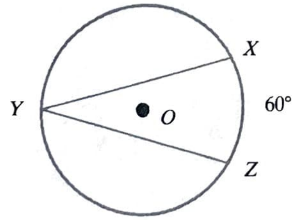
B. It is halved.

C. It is doubled.

D. It equals π.

E. It equals 2π.

3. In the figure below, $\angle XYZ$ is inscribed in circle O and $m\ \overset{\frown}{XZ} = 60°$. What is the measure of $\angle XYZ$?

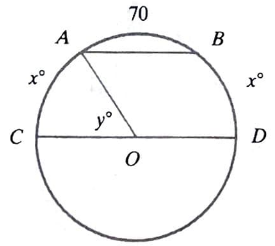


A. 20° D. 120°

B. 30° E. None of the above

C. 60°

4. O is the center. $\overline{AB} \parallel \overline{CD}$. Find y.

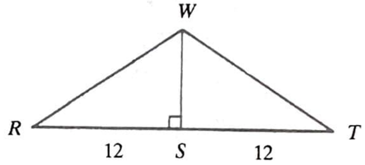


A. $24\frac{1}{2}°$ D. 110°

B. 55° E. None of the above

C. 60°

5. In the figure, points R, S, and T are on the same line, and $\overline{RS}$ and $\overline{ST}$ are each 12 units long. If the area of ΔRWT is 48 square units, how long is altitude $\overline{SW}$?

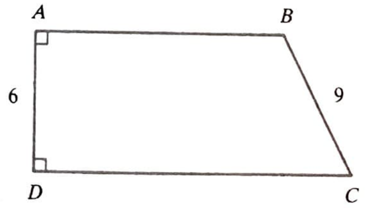


A. 2 D. 12

B. 4 E. 24

C. 8

6. In the trapezoid shown, the perimeter equals 45, $BC = 9$, and $AD = 6$. Find the area.

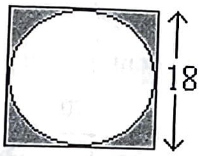


A. $7\frac{1}{2}$ D. 180

B. 18 E. $607\frac{1}{2}$

C. 90

7. The figure below is a circle inscribed within a square. The area of the shaded region is:

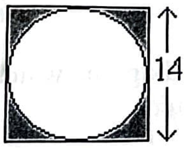


A. 243π
B. $381 - 24\pi$
C. 405π
D. $324 - 81\pi$
E. $18 - 9\pi$

8. The length of the diagonal of a 21 by 28 rectangle is:

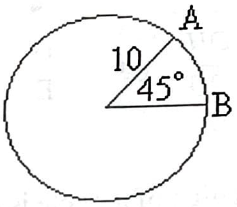
A. 49
B. $21\sqrt{3}$
C. 48
D. $28\sqrt{2}$
E. 35

9. The figure below is a circle inscribed within a square. The shaded area is:



A. $196 - 49\pi$
B. 147π
C. $28 - 7\pi$
D. $49 - 7\pi$
E. $149 - 96\pi$

10. In the circle below with radius equal 10, the length of arc *AB* is:



A. 2.5π
B. 5π
C. 25π
D. 10π
E. 100π