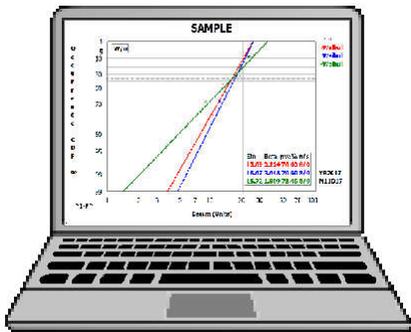


PlayTIME™

With SuperSMITH™ Software by Fulton Findings
Tutorial for Either DEMO or FULL Versions

Author: Wes Fulton, Copyrighted by Wes Fulton, Editor and Publisher
Fifth Edition May 2002 [With New Updates and Printings through Oct 2019]



A Computer Tutorial
A Computer Tutorial



PlayTIME™ With SuperSMITH, Fifth Edition

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Preface

The SuperSMITH DEMO software version contains the same code as the FULL version with one difference. The DEMO includes a switch activated so all input data is randomly altered (some up and some down) such that the results provided are NOT for the original data . . . UNLESS the data file loaded was first saved by a FULL software version. This DEMO tutorial booklet can only work with the DEMO software using provided files specifically designated herein. **DO NOT** use the DEMO software to obtain useful results for any other input data.

The **SuperSMITH** package includes this tutorial booklet, **PlayTIME™ with SuperSMITH**, and Dr. Bob Abernethy's standard Weibull reference, **The New Weibull Handbook©**, plus Wes Fulton's **SuperSMITH Weibull** and **SuperSMITH Visual** software programs from Fulton Findings LLC plus the **SuperSMITH Ybath™** program by Carl Tarum. This package is a college-level self-study course in Weibull Engineering (WE) analysis with the tools needed to use the expertise achieved.

SuperSMITH Weibull is a stand-alone menu-driven plotting/analysis program for complete best-practice Weibull analysis including many techniques not available elsewhere. This program has become the 'de facto' standard for life data analysis, prediction, and for evaluating variability and uncertainty in practically all industries. Capabilities of SuperSMITH Weibull include Weibull, normal, lognormal, and Gumbel distributions (lower and upper), and variations of these, as well as likelihood plotting, confidence plotting, distribution analysis, design comparison, probabilistic analysis, outlier detection, optimum replacement, failure forecasting, test substantiation planning, accelerated and step-stress testing, and mixture analysis. SuperSMITH Visual is a general plotting/analysis program with complimentary capability to SuperSMITH Weibull. Capabilities of SuperSMITH Visual include general function plotting, transforms, curve fitting, aggregate cumulative hazard (ACH) plotting, and Crow-AMSAA (Duane) reliability growth plotting. The primary functions of the older WeibullSMITH™, BiWeibullSMITH™, and MonteCarloSMITH™ programs (for MS-DOS) are included in SuperSMITH Weibull. The new mixture analysis capability in SuperSMITH Weibull and particularly in the YBath™ program from Fulton Findings LLC, created by Carl Tarum, is significantly improved from that available in the older software.

The SuperSMITH software may offer more useful options and capabilities than any other current life data analysis program, although it is extremely compact in size. This wide-ranging usefulness is due to the many applications in the industries we serve, including automotive, components, aerospace, medical, electronic, electric power, petrochemical, nuclear, rail transport, dental, materials analysis, foods, and others. The compact program size allows easy transfer/use over networks. This tutorial is intended to illustrate most of this capability and many of the applications. Hopefully, it is helpful for both novice and expert.

The reference book for all the material herein is the latest edition of **The New Weibull Handbook©**, authored and published by Dr. Robert B. Abernethy.

Each new edition and printing of this tutorial will attempt to demonstrate all the options in the latest versions of SuperSMITH. Please contact the author(s) for the latest version information.

We welcome suggestions and comments or questions. Extraordinary contributions are recognized. The case studies in this tutorial booklet are compiled from hundreds of Weibull/LogNormal Analysis Workshops in various locations around the world given to enthusiastic students both publicly and privately. The workshop was originally developed by Dr. Bob Abernethy and has been taught over the years by Wes Fulton and Dr. Bob both separately and together. Both Dr. Bob and Wes are indebted to all of those students who have contributed case studies or provided case study solutions. Special thanks go to Paul Barringer for a great update to the visual layout of this document and to Robert Silbernagel for improved PlayTIME organization.

Please call, FAX, or write Wes Fulton as the current coordinator for PlayTIME improvement. Contact either Wes or Carl for methodology questions. Wes or Carl should be contacted for questions regarding computing and software. The address and phone/FAX number information is included on the previous page. See the Wes Fulton website mentioned where you can get updated to the latest life data technology, download a **DEMO** version of the software (it is really quick and easy to do this). See the Paul Barringer website mentioned for general introduction to reliability in practice, the latest Problem of the Month, and the downloadable DEMO software as well.

Dr. Bob Abernethy, Paul Barringer, Carl Tarum, Wes Fulton

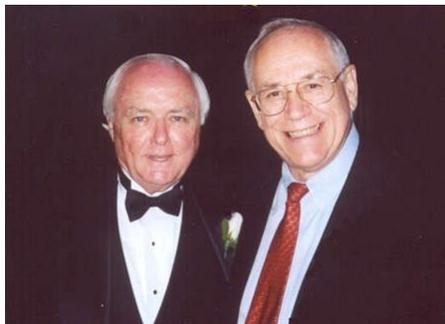
ACKNOWLEDGEMENT

Wes Fulton and Carl Tarum would like to acknowledge especially the following.

This tutorial for SuperSMITH software and specifically the software itself would not be possible without methods and technology from **Dr. Robert B. Abernethy (Dr. Bob)**. For more than thirty years, he has been the #1 authority on Weibull Engineering. For best guidance on Weibull applications, refer to his standard reference, THE NEW WEIBULL HANDBOOK©.

Paul Barringer contributed largely to the methods, the look and feel, and the usability of the SuperSMITH Weibull software and to this tutorial booklet along with many more engineering contributions. His unexpected passing in December 2016 is a great loss to his many friends and a great loss to the technical community. Paul will be sorely missed, but his contributions live on.

Dennis Keisic has helped with Wes Fulton's presentation of Dr. Bob's Weibull Workshop on many occasions. His simple explanation of Weibull methods related to Six-Sigma techniques guided students working in manufacturing. He passed away abruptly in September 2017.



Dr. Bob (in tuxedo) and Paul at Wes's wedding in 2002



Dennis with his usual smile

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Section I. Quick Start SuperSMITH

It is recommended to make a backup copy of any original installation media before starting. **REMOTE INSTALL:** Available for networks ... contact Fulton Findings for info. **DIRECT INSTALL:** Power up the PC (Windows or simulator). With CD ... Put the SuperSMITH disk into the PC, or from downloaded file, select the SETUP.EXE program. Run SETUP.EXE and follow the prompts. Within the SuperSMITH group, select the SuperSMITH Weibull program. This is the SMITHW.EXE file or the icon showing plotted data with fit and confidence lines. Initially select “Yes” to agree to usage conditions, and automatically advance to the main screen (Figure 1 below shows former icon layout, Figure 2 on the next page shows the new icon layout).

Data files for this tutorial are included with the program files in directory \SMITHDAT. If you cannot find these files already installed, then you can copy these data sets onto your PC from the original installation files. Copy all files in the original \SMITHDAT subdirectory to C:\SMITHDAT on your hard drive (if allowable ... this makes it easy to use these auxiliary example data files).

The standard SuperSMITH programs, Weibull and Visual, consume only about 5 Mb of disk space, very tiny for such capabilities. Another approximately 2 Mb of disk space is used for the Windows system files and supporting files. PlayTIME™ data files consume about 1 Mb of disk space. SuperSMITH Ybath™, for advanced mixture analysis, requires another 1 Mb of disk space. The DEMO software package includes:

- **SuperSMITH Weibull**—(SSW) probability plotting and analysis software for Windows
- **SuperSMITH Visual**—(SSV) visualization plotting and analysis software for Windows
- **SuperSMITH Ybath**—(SSY) probability plotting of bathtub curves, BiWeibulls, & TriWeibulls.

NOTE: Both Ybath and Visual are accessible from SuperSMITH Weibull.

Use single clicking (selecting) mostly in SuperSMITH; double-clicking is discouraged to minimize hand stress. We will be using life data mostly for illustration, but any type of measured data with variability will work. However, for this DEMO tutorial using the DEMO software, **all input data must only be loaded from a file previously saved by a FULL SuperSMITH software program.** These files are provided with the software. Figure 1 shows the former SuperSMITH Weibull icon layout and Figure 2 shows the new icon layout.

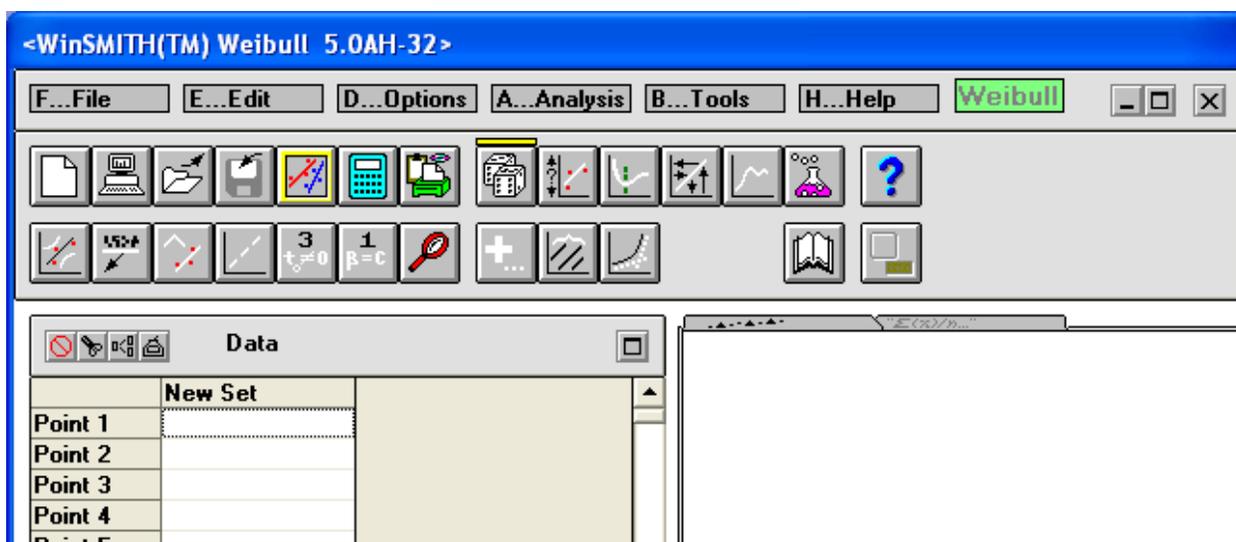


Figure 1: Old Main-Screen Layout (See Figure 2 For New SSW Layout and the last page for SSV)

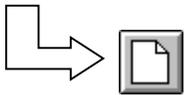
Section II. Make Weibull Plots with SuperSMITH Weibull (SSW)!



Start the SSW software by choosing Start from the Windows™ main screen and then selecting Programs (or All Programs), then SuperSMITH, then Weibull. Select “Yes, I agree...” to continue if you are asked and if you agree to software use terms. You should see the main screen with about 30 icons, some drop-down menus, a data grid, and a results box.

DATA ENTRY AND SOLUTION: Here is the data for this example: **8, 3, 6, 3, -1, 2, 4, and 2.** >>> **DO NOT** enter this data by keyboard or by copy/paste into the DEMO software as the DEMO software version will alter any data entered that way. The only way to enter data into the DEMO and not have it altered is to bring in a file previously saved from a FULL software version. Files indicated in this tutorial (and available with the software) have all been saved first by a FULL version. **For both FULL and DEMO versions do this . . .** From Home Screen (File), select Open / Import (open file icon). Locate the file **PlayTIME-A-Section-II-A.w** and load it.

NOTE: Images are provided on the side of the text to indicate the icon for selection.



- A probability plot should appear. Select the plot to enlarge. The plot should appear similar to the one in Figure 2. Your solution should indicate $\text{Eta}=4.467$, $\text{Beta}=2.156$. If not ... check the data and/or reset the method to Weibull rank regression standard. Select the full-window plot or use the <ESC> key to shrink the plot and go back to the main screen.
- Exit out of the software. Then start SuperSMITH Weibull again. This saves the path you used to access the data file, so you shouldn't have to relocate the path.
- From Home (File), select Open / Import. Locate the file with name **PlayTIME-A-Section-II-B.w**. For this new set (same data as set 1), repeated data entries such as “3” and “2” from data above are entered only once each by changing the “x1” to “x2”.
- Select the plot to enlarge. Your plot should resemble Figure 4. Select the plot or use <ESC> to return to the main screen. NOTE: A shortcut way to enter repeated values is with the frequency value after the “x”.

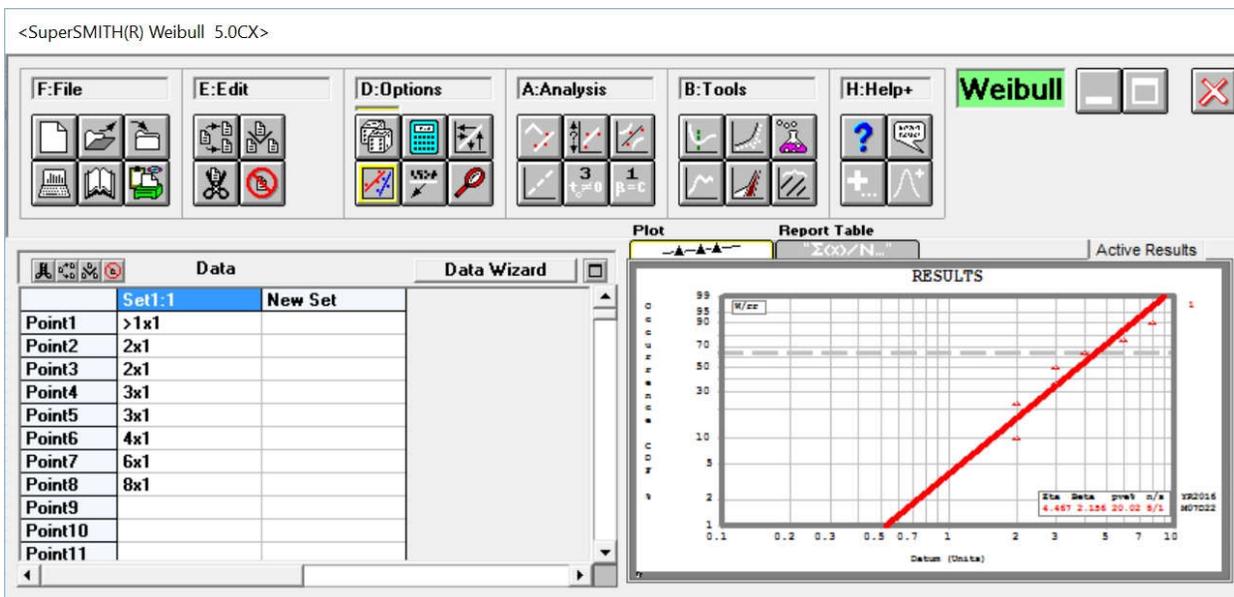


Figure 2: New SuperSMITH Weibull Main Screen Icon Layout with Solution Plot.

NOTE: This new icon layout has complete integration between the drop-down menus and the icons.

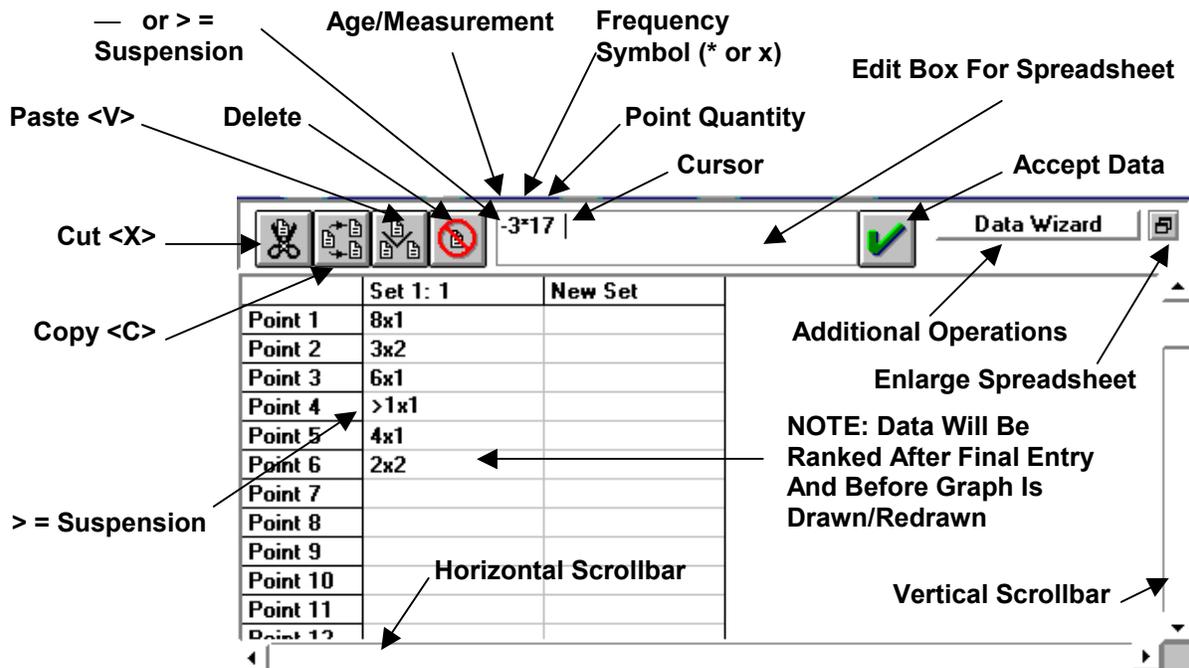


Figure 3: Same Data, Different Entry ... with Thumbnail Summary of Spreadsheet Operations

With the FULL software version (not the DEMO version) you can type in data or copy (or “cut”) data in text format from any source and paste (or load from file) into SSW’s data grid for fast data entry. Figure 3 describes details of this spreadsheet. Select the small-size results plot to enlarge — as shown in Figure 4. NOTE: The greater-than sign (“>”) display for suspension is an option. Select the full screen plot to shrink back. Versions starting in 15 January 2002 and later show pve%, which is an improved goodness-of-fit (GOF) measure replacing r² and Critical Correlation Coefficient (CCC). The New Weibull Handbook© explains this pve% metric based on Monte Carlo simulation.

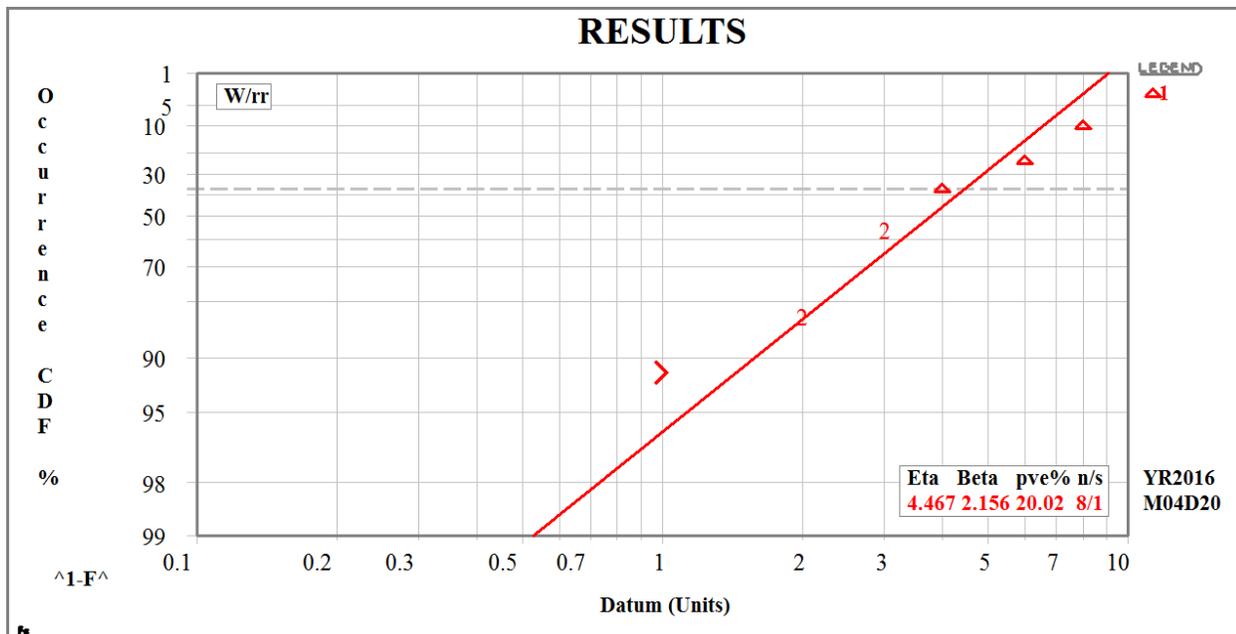


Figure 4: Weibull Probability Plot Appearance Using “x2” After Data Values “2” and “3”

Note that Eta and Beta are the same as when each point was entered separately.



Figure 5: Thumbnail Summary of SuperSMITH Weibull Main Screen Icons – New Icon Layout

NOTE: The new SuperSMITH Visual Main Screen icon layout is shown on the last page of this booklet. The new SuperSMITH Ybath™ Main Screen icon layout is shown in the section on Ybath mixture solution.

This section gives you a quick start using SuperSMITH Weibull. More details about the icons and features are given in “ICON MAP: Software Tour” in SuperSMITH Help. Use the icon tour to get more familiar. Now, let’s make another plot using **Monte Carlo Sampling available** within the Weibull software ...



- A) Starting at the Main Screen, prepare for a new plot by Selecting the **New ... Clear Data <N>** icon (showing a blank page) to start a new graph. NOTE: The yellow bar over the **pair of dice** icon (for Demonstrate / Simulation / Sample / Special Data) indicates that the <Enter> key may be used to turn on this option if nothing else is active or being edited. This functionality is the same for the DEMO and FULL versions.
- B) Press the <Enter> key twice (this is equivalent to selecting the **pair of dice** icon and then accepting the dialog box choices by selecting the green check mark)—something should happen.
- C) Voila ... a plot! Note that every random sample will be different if this is repeated.
- D) Repeat pressing of the <Enter> key to get two or more different samples. You should have a plot like Figure 6 using this built-in Monte Carlo sampling routine. The Monte Carlo routine produces data sets with typical statistical scatter—later we’ll show the benefits of using this technique for serious work.

SuperSMITH Weibull is configured so that you can get productive results quickly. With just a few selections, you’ve automatically filled-out the spreadsheet and completed another plot. This is one way to get a quick start using the software to try different analysis features.

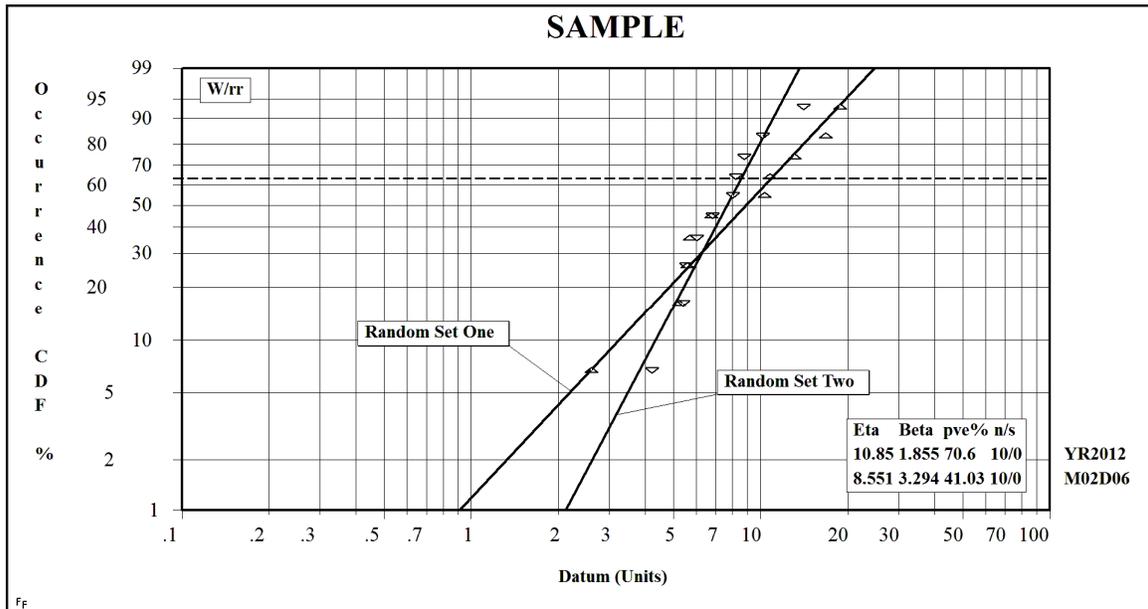
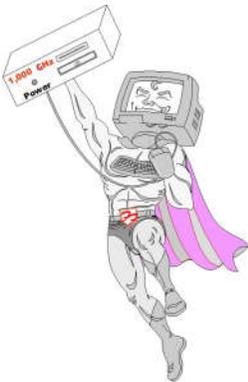


Figure 6: Quick Monte Carlo Plot

Dr. Bob's comments: How did SuperSMITH Weibull make this graph? Ten data points (n) and no suspensions (s) were drawn randomly from a Weibull with a true eta of 10 and a true beta of 2. (See Section 4.7 in The New Weibull Handbook for Monte Carlo theory.) The random samples were analyzed using rank regression (rr) on Weibull (W) probability paper as shown above. For the first random sample, the characteristic Weibull life is estimated to be 10.85 (Eta) and the shape factor defining the slope of the line is 1.855 (Beta), showing reasonable agreement with the true values. However, the parameters for the second set diverge more. The y-axis tells the % occurrence expected from this data. We expect each random sample to give somewhat different results.

Wes's comments: (FULL version only) Editing in the data grid, you can add a point label for each data entry by using an underline symbol ("_") at the right end of the entry with its text label after that. EXAMPLE: If you enter "4x1_SN523", the plot can show "SN523" in place of the standard symbol. This is useful for visualizing if serial number and failure age are related (maybe yes for a batch issue?).



Section III. Let's Solve Some Problems

Some people like to solve real problems in a tutorial and we believe it is the best way to learn how to use the SuperSMITH software. Each case study in this section will give experience with SuperSMITH Weibull (SSW) data entry, using the many options, as well as interpreting the results and plots produced. All the salient features in the SuperSMITH (SS) programs will be demonstrated. However, you may select just the problems that best relate to your current applications.

Each problem starts out with a few bits of information. The first few problems provide extensive coaching on entering the data. Later problems have less coaching as you become more familiar with the SSW software. The interpretation of the results is the hardest part of the problem after getting the data. Dr. Bob Abernethy and Wes Fulton have provided comments on the interpretation where appropriate. Please make your own interpretation before ... peeking. The following includes analysis of benchmark problems using data sets found in hallmark references on life data analysis. These examples validate SuperSMITH software allowing comparisons to other Weibull programs.

Problem 1. Data Input, Weibull “mle” versus “rr”



Learn how to enter data into SSW, input suspensions, and compare regression and likelihood solutions.



- At the SSW main screen select the **New ... Clear Data <N>** icon to start a new plot.
- If you are in the Weibull workshop you will remember day one, classwork problem 2-2 in the Handbook with the nasty adjustment for suspensions you did by hand. Let’s try it with SSW.
- From the Main Screen, select the Open icon in the File group. Locate the file **PlayTIME-Problem-1.w** and load it.

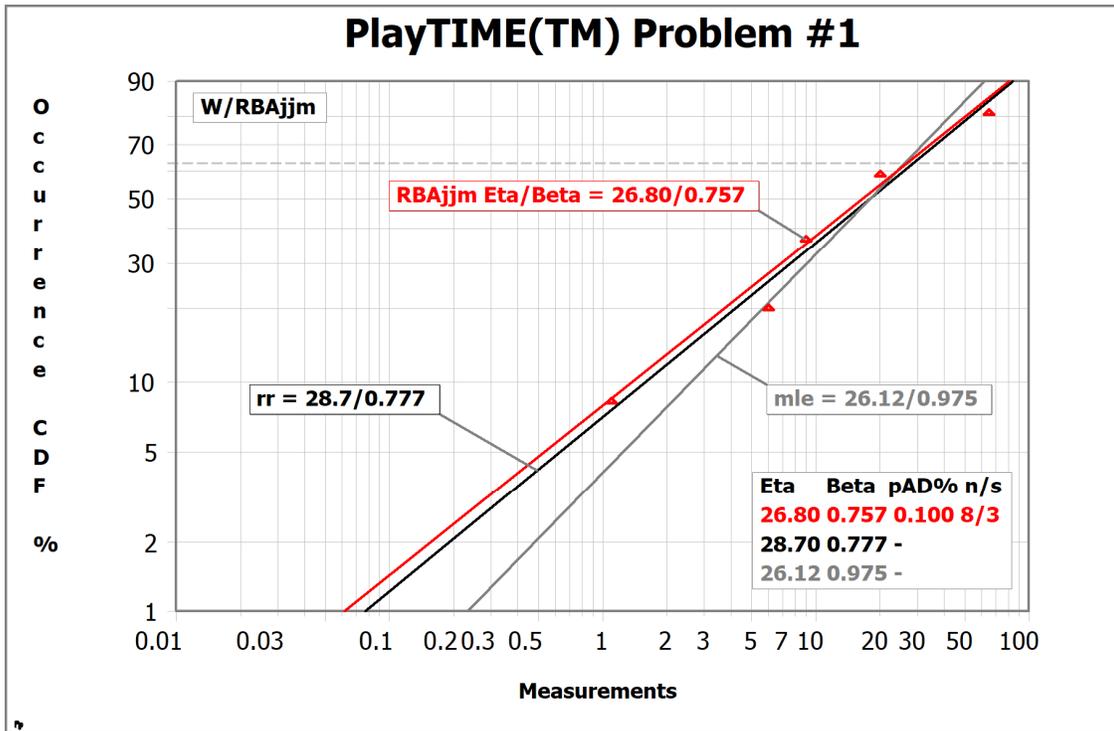


Figure 7: Comparing Rank Regression (rr), Maximum Likelihood Estimate (mle), & mle-RBA (RBAjmm)

Select the small plot to enlarge, it will look a little different than Figure 7. You should get **Eta=28.7, Beta=0.777**. Compared to hand calculations is SSW slower or faster?? Notice the median rank regression (rr) plot line fits the data well. Compare rr, mle, and modified mle (RBAjmm) solutions as follows:



- Select the full-size plot to make smaller. Select Analysis group, then Marker Line icon (dashed line). Select Set 1 from the menu to copy the rr line (Marker Line is hidden under the active fit).
- Select the “Method/Input” icon (with \wedge symbol and a line through points). Next select the mle icon (showing only the \wedge symbol and nothing else). NOTE: The mle line (**Eta=26.12, Beta=0.975**) does not visually match with the points, the slope is steeper than rr and there is no pve% for mle.
- Copy the mle line as follows. Select the Marker Line icon again, select Line 2, then select Set 1 (puts a Marker Line under the Set 1 active mle fit line).
- Get an RBAjmm modified mle solution as follows. From the Main Screen, select the Method/Input icon (showing \wedge ./; symbols), then select modified maximum likelihood icon (showing mle symbol + 2 arrowheads). Plot method should be “RBAjmm”. If not, return to Method/Input, select “H...Type Of mle Modified...”, select “1...Weibull...”, then “RBA Mindeguia”. This latest plot (Figure 7) should have RBAjmm fit (**Eta=26.80, Beta=0.757**) agreeing well with Line 1 (rr).

Dr. Bob’s comments: The mle vs. rr comparison shows the mle line is steeper as usual. This is the mle bias toward steeper betas with small samples. The modified mle solution (RBA factor) removes most of that bias. When rr and mle-RBA show reasonable agreement like this, it means we have a good Weibull. NOTE: There is no pve% with mle or mle-RBA, and no plot fit to the data, although SuperSMITH Weibull (SSW) software shows the points in the same median rank plot location. We’ll show you how to add labels on the plot later.

Problem 2. Copy, cut and paste data from a POSSIBLE mixture of failure modes

Reference [The New Weibull Handbook](#) Section 3.7.3.

BiWeibull data sets: A reliability consultant produced this interesting set of 12 data points separated by commas:

41.6, 2.8, 35.1, 4.8, 10, 13.5, 17.1, 18.8, 23.1, 26.8, 40.2, 45.3

The consultant suggests this is the classic BiWeibull or mixture situation, a low slope followed by a high slope. Usually this mixture of competitive failure modes is caused by a quality control mode with infant mortality or random slope, followed a wear out mode. The consultant says the data should be split (partitioned) into the lowest “k” data points for mechanism #1 and the highest 12-k data points for mechanism #2. Specifically, the consultant says the best split is first four and last eight. Partitioning the data into two groups is a popular solution recommended by many authors. Later in PlayTIME, we will use two better methods for this, SuperSMITH Ybath™ and the likelihood ratio test in SuperSMITH Weibull. However, for now, solve the problem using the partition method.



SOLUTION: FIRST! ... from the SSW main screen in the File group, select New Plot (blank page icon).



A. Reset the method to standard “rr” by selecting the method icon on the main screen and then selecting the reset icon (showing a U-turn arrow).

B. If “Results -... Save?” comes up, select “No Save ...”. Later in SSW if you want to reactivate a previous analysis plot shown on the main screen, just select it. (These are saved as temporary files.)

C. From the Main Screen, select the Open icon in the File group. Locate the file **PlayTIME-Problem-2.w** and load it. You should get a plot indicating $\eta=26.6$, $\beta=1.304$.

D. Record the pve% value... $pve\% = \underline{\hspace{2cm}}$.

E. From the main screen, in the File group, select “Save Active Data ...”, and save the results to a new file with file name “**BIW.W**” to analyze later.

F. Do we have enough data to support mixture analysis? No, we do not; you should have at least 30 occurrence points for a 4-parameter competing-risk model, but to illustrate the method we will proceed.

G. Select the midsize plot to maximize it. Look at the plot. By eye where would you partition the data? Where does the first data set end and the last begin? Select on the maximized plot to midsize it.

H. Select data "maximize" button (data grid upper right corner) to show a full data grid. Use the vertical scroll bar (right side of the data grid) to make all 12 points visible. Escape <ESC> to the Main Screen.

I. Repeat the file load with the same file (**PlayTIME-Problem-2.w**) as follows . . . Select the Open / Import icon (File group). Locate that same file again and load it. It should come in as Set # 2 with the same solution.

>>> **Let's try some data splits.** >>>

J. Starting with the 5th data point “17.1” in the 1st column (set #1), select down to the last data point “45.3” in the 1st column (highlighting those last 8 data points) in preparation to edit those.

K. Select the **Data Wizard** button on the data grid, then choose **Susp. To / From** to toggle occurrences to suspensions. This changes the last eight data points in set #1 to suspensions.

L. For set #2, select to highlight the first 4 data points, activate the Data Wizard, select **Susp. To / From** to similarly change those selected data values from occurrences to suspensions.

M. View the revised plot (NOTE: If the data grid is still full-screen, select the “midsize” button, upper right corner of the data grid, to return to the main screen). Select the small plot to enlarge. Again to midsize.

N. Write the two pve% values for set #1/set #2 in the “First 4/Last 8” blanks below.

O. For the 2nd split (“First 8/Last 4”) ... select the rows from point # 5 to point # 8 **for both sets at the same time**, then use the Data Wizard button as before and select Susp. To / From to toggle those points. Enter the resulting two new pve% values for “First 8/Last 4”.

P. For the 3rd split (“First 9/Last 3”) ... select the point # 9 row **for both data sets at the same time**. Then use Data Wizard, Susp. To / From to toggle. View the plot. Enter the two pve%'s for “First 9/Last 3”.

Enter pve%: First 4/Last 8= _____ / _____ First 8/Last 4= _____ / _____ First 9/Last 3= _____ / _____



Dr. Bob's comments: Although it may seem reasonable to assume all the points in one set precede all the points in the other set, we expect the sets might overlap. However, the statistical concept of choosing a "corner" to partition the data is popular for analyzing competitive failure modes with small effect in the corner vicinity. In this case the best partition appears to be the first 8/ last 4, the highest combination of pve% values.

Wes's Comments: Pve%'s are approximately 75/62, 95/85, and 98/38. Both Pve% values should be better than the original 87 to accept the more complex 4-parameter mixture model. No split has both pve%'s above the original.

Problem 3. Wire Fatigue Distribution Analysis

Learn how to do a distribution analysis and compare Weibull 2-parameter, Weibull 3-parameter, and lognormal; use pve% value or likelihood to measure goodness of fit. Handbook Ref: Section 3.2.1.

Pliers were used to rupture eight-stranded stainless-steel wire by clamping wire in needle nose pliers and hanging a 2-pound weight on it (increased to 2.6 pounds later). The pliers were rotated through 180 degrees, alternating clockwise and counterclockwise. The number of half twists to total rupture (failure) was recorded. The total rupture may require a t_0 correction as we do not expect total rupture on the first half twist, i.e., there may be a guaranteed failure free period corresponding to t zero. Alternatively, total rupture might be lognormal as strands fail progressively while stress increases on the remaining strands.

SOLUTION:

- A. Select for a "new plot" (blank page icon).
- B. Use Analysis group, Method/Input, Reset (U-turn arrow) for Weibull rr Point-By-Point Standard.
- C. Move the cursor over the computer icon, verify dimensions are at least 12x2000 (reset dimensions if needed by selecting the icon).
- D. WIRE-BREAK DATA: Do the following to use the Wire Fatigue Experiment data sheet on page 16. From the Main Screen, select the Open icon in the File group. Locate the file **PlayTIME-Problem-3.w** and load it. You should get $\text{Eta}(1)=35.82$, $\text{Beta}(1)=5.407$, $\text{Eta}(2)=16.89$, $\text{Beta}(2)=5.912$ for that data.
- E. Are there outliers? If you think so, go to the calculator icon and select Outlier...Extreme Point P-Value, look at the results (upper points in both sets show "Outlier" indications), then return to the main screen.
- F. NOTE: If some data is duplicated, then try selecting the Method/Input icon, and change to Inspect Option #1 by selecting the "Point-By-Point Inspect(1)" icon.
- G. Is there curvature? Good Weibulls? Or better lognormals? To find out do a rank regression (rr) distribution analysis using the distribution icon (showing a plot with a "?" along the vertical scale). Answer "No" if any "Change" question appears. The best distribution is the one with the largest pve%.
- H. Add check marks to the Distribution Analysis (Table 1 below) according to analysis results. How does this compare with expectations? Do you think there is a significant difference from increased weight?
- I. Save the data as **WIRE.W** using the Save Active Data icon on the main screen.
- J. Try a 3-parameter Weibull with the " t_0 " icon (showing "3"). Toggle "Find Optimum t_0 " to "Yes". Set the scale to As Recorded (AR). View the plot. Return to the "3" icon and select "None" to turn off.
- K. Shift to LogNormal by selecting the green "Weibull" label at the top of the main screen and changing to LogNormal. View the plot. Use the light blue "LogNormal" label at the top of the main screen or the setup icon (showing a computer) to change back to Weibull [2P] equations.
- L. Select Analysis group, Method / Input, and change Fit Method to maximum likelihood estimate (mle).
- M. Do another distribution analysis (Analysis group, Distribution Analysis). This time it's different. The software automatically selects the best 2-parameter distribution as the one with highest (least negative) log likelihood value. Then the 3-parameter solution log likelihood is compared to the 2-parameter winner with a likelihood ratio test. Complete the last columns in Table 1. The rr and mle solutions can disagree for small samples. New studies [Liu] confirm that distribution analysis requires at least 21 occurrences (failures for reliability) for reasonable statistical power. For smaller samples (less than 21 occurrences and a small quantity of suspensions) best practice is to start with two-parameter Weibull and rank regression.



Table 1. Distribution Analysis: $\sqrt{\text{Mark}}$ = Best Fit Distribution

	rr=Largest pve%		mle=Max Likelihood	
	Set 1	Set 2	Set 1	Set 2
2P Weibull				
3P Weibull				
LogNormal				



- N. Reset the method to standard Weibull/rr.
- O. Turn on label edit with the labels icon (showing characters underlined and an arrow pointing down).
- P. Select the Eta/Beta box, change its location, repeat to change back. Select the set labels and edit them (set 1 = “2 lbs”, set 2 = “2.6 lbs”) while moving each set label onto the plot pointing to correct set. Select the $\wedge F \wedge$ symbol in the lower left corner, then select **100 – Occurrence As %** to change to the reliability scale. Select again and reset to **Occurrence As % ... Standard**. Return to the main screen.
- Q. Save the final version again to WIRE.W so that later we can do more analysis of the wire data.
- R. Verify method is standard with **Method/Input icon, Reset To Standard icon (U-Turn Arrow)**

Dr. Bob’s comments: We expect that the 3-param. Weibull or the lognormal would be best for both weight levels. However, data scatter for small samples may cloud the issue. If you are in a workshop and this does not agree with your workshop results, remember who generated this data. (Not our fault!)

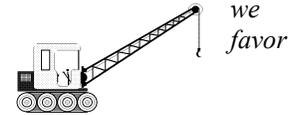
Problem 4. Goodness of Fit...Distribution analysis with a large sample

”FATIGUE.W” contains many points from previous class wire experiments. What’s the general pattern?



SOLUTION: At the SSW Main Screen, select the “blank page” icon to clear data and results. Select Open / Get / Import icon (open folder), retrieve **FATIGUE.W** and record the pve%’s here: _____/_____. Perform a distribution analysis. Which is the best fit (distribution with largest pve%) for each set? Note the vertical columns of points (coarse data). Select the Method/Input icon from the main screen and then change Input Data Format to Inspect(1) ... Point-By-Point. Does the pve% increase with the inspect option? Is the fit better?

Dr. Bob’s comments: We suspect both the three parameter Weibull [+t₀] and the lognormal may be appropriate because the data are curved downward on Weibull scaling. It would make physical sense to say there may be a minimum number of twists to failure or that the deterioration of braided wire is non-linear and progressive. To find out, compared 2 & 3 parameter Weibulls with lognormal plots using SuperSMITH. The results the Lognormal based on the pve% values. Same results with mle. Our class results are generally not as definitive because of smaller sample size.



Problem 4a. Compare Probability Density Function PDF with Cumulative Distribution Function CDF

SOLUTION: Compare the probability density representation with the cumulative as follows (NOTE: The Main Screen icon layout for SuperSMITH Visual is shown in the last figure at the end of this tutorial booklet) ...



- 1) Exit the SuperSMITH Weibull (SSW) software (no need to save anything at this time).
- 2) From the desktop, select START, PROGRAMS (or ALL PROGRAMS), SuperSMITH Group, then (SuperSMITH) **Visual** “SSV”. In later Windows versions this process should be at least as easy.
- 3) From the SSV main screen, select the open file icon (showing an open folder) and enter (or select) **FATIGUE.W** ... you should see a menu with item “PDF (= d[CDF] / dX) Probability Density”.
- 4) Select that PDF item ... then you should see the Visual plot with PDF curves and data points. SuperSMITH shows data points along the respective PDF curve. Most other programs can only show a histogram to represent the data. This is a special feature of SuperSMITH.
- 5) Density (PDF) is just a different way to view the solution. Exit the SSV program with no save.
- 6) Re-activate the SuperSMITH Weibull (SSW) program.

Wire Fatigue Experiment *Use ONLY if missing actual data from Workshop!

Light 2 lbs	Half Twists to Rupture	Initials	Heavy 2.6 lbs	Half Twists to Rupture	Initials
1	37		1	10	
2	34		2	15	
3	27		3	18	
4	35		4	14	
5	28		5	17	
6	45		6	18	
7	30		7	24	
8	34		8	13	
9	21		9	17	
10	27		10	17	
11	24		11	19	
12	39		12	13	
13	27		13	15	
14	42		14	16	
15	37		15	17	
16	41		16	20	
17	32		17	15	
18	27		18	11	
19	51		19	14	
20	25		20	12	
21	35		21	13	
22	41		22	16	
23	24		23	20	
24	33		24	13	
25			25		
26			26		
27			27		
28			28		
29			29		
30			30		

Probit for Problem #14. Format is Inspection Time (1/2 Twists) X Cum. Qty. Ruptured X Total Qty of Wires Inspected. Light Weight (2 lb.) Data Should Be Converted to Probit (Grouped) Here. Do Not Fill Cells Representing Zero Percent Occurred or 100 Percent Occurred or Where No Change Happened:

15 X ____ X 24	30 X ____ X 24	45 X ____ X 24	60 X ____ X 24
20 X ____ X 24	35 X ____ X 24	50 X ____ X 24	65 X ____ X 24
25 X ____ X 24	40 X ____ X 24	55 X ____ X 24	70 X ____ X 24

Problem 5. Failure Forecasting

Managers confronted with serious failures want to know how many failures will occur in the next month, six months, and year. Here you will learn failure forecasting, also called Abernethy risk analysis.

A heavy truck manufacturer is in trouble with steering failures. How many failures can we expect?

SOLUTION:

- 
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- 
- 
- A) In SSW, select New Plot, and verify the method is standard Weibull, point-by-point / rr.
 - B) Use the “open folder” icon to load **18WHEELR.W** from your files. Eta should be 69362, Beta should be 3.257. Select small plot to go large. Note 10000 trucks have not failed (suspensions).
 - C) Select the large plot to return to the main screen. In Tools Group, select **Abernethy Risk**.
 - D) Set the Risk Analysis Type to Standard. Set Output Format to Cumulative.
 - E) Select U...Usage, Single Rate, “1000” <Enter>, then select green checkmark for the output table. NOTE: “Next Event [Day]=+15” in the output text means the next failure is expected in 15 days.
 - F) Select <ESC> to return to the menu. Select R...Renewal ... = Yes. The Risk table should show 5749 more failures by month 60. Change R...Renewal ... = No, you should see 5534 at 60 mos.
 - G) Select U...Usage, change to 2000, select P...Production, Single Rate, 100, and select the green checkmark. You should see 12929 at 60 mos. without Renewal and 19248 with Renewal.
 - H) Set Output Format = Interval (so the output is month-by-month instead of cumulative). Select the checkmark, 475.8 failures are expected in the 60th month.
 - I) View the **risk plot** as follows. While looking at the risk table output, select the bottom icon on the right-hand side of output. SuperSMITH Visual should (with default installation) automatically activate and display the **risk plot**. After viewing the **risk plot**, exit SuperSMITH Visual (“Save Active Data” not needed) to return looking at the SuperSMITH Weibull risk table output.
 - J) Next to the risk table output, select the icon just above the bottom icon. Then Select SuperSMITH file format, enter **18TRRSK.V** for name, and activate **Save** to save this file for later use.
 - K) Set P...Production to zero, set R...Renewal to Yes, set U...Usage to 3000. Activate to view the output table, then select the bottom icon to activate SuperSMITH Visual (SSV) with the **risk plot** of maintainability rhythm. After viewing the rhythm plot, exit out of Visual to return to SSW.

Wes’s Comments: We use a .W extension for a SuperSMITH Weibull file and .V for a SuperSMITH Visual file, but you may use your own file name convention. SuperSMITH software tries to load any named file. The software appropriately adds .W or .V if the name does not have an extension.

Problem 6. Optimal Replacement Intervals

If the cost of an emergency replacement (unplanned) from a wear out failure mode is greater than the cost of a planned replacement, there will be an optimal replacement interval.

The truck manufacture decides on replacing the steering links, until a new redesigned link can be produced for a recall program. If a “planned” replacement (P) costs \$50 and an unplanned replacement at failure (U), “not planned”, costs \$1000, what is the most cost-effective replacement mileage?

SOLUTION: (NOTE ... If in SSV, exit from SSV “Visual” and return to the SSW “Weibull” software first!)

- 
- 
- A) In SSW, start at the main screen, using same input data as the forecasting problem above, select Tools Group, **Optimal Replacement** (icon with a plot of a vertical line through the curve minimum).
 - B) Enter the present cost values above (P=50, U=1000) and confirm the View Length interval range is 60000 miles. [Use a view length a little less than Eta and, if possible, divisible by 60.] Verify “Cost Change” is “No”.
 - C) Select check mark for output. Lowest cost factor (~0.0033) is for planned replacement at 22000 miles.
 - D) **While still looking at the output table, select the bottom icon next to the output table for a quick results plot in SSV.** After viewing this plot, **exit SSV** (that will take you back to the output table).
 - E) Back at the output table, select the (X)(Y) icon on the right side just above the bottom icon, then choose SuperSMITH file format, enter **18TRREP.V** for the name, and activate **Save** to save the file.

Problem 8. Your Data

If you have your own data, now is a good time to do the analysis. If you need help, call the instructor over.

Problem 9. WeiBayes with one failure versus mle

Five new-design parts are on test, and one has failed (data: -100, -200, +300, -400, -500). Your Weibull Library indicates Beta of 3.5 on a similar failure mechanism from before. What is the Weibull for the new design?



SOLUTION: Close **SSV** if open. Return to **SSW**. Do a new plot, then load file **PlayTIME-Problem-9.w**. Use the icon with “1” (not the Equation Label) to get to the Weibayes menu. Input Beta = 3.5 and set confidence = no (mle). Eta should be 578. Select the Green Check Mark to add a marker line. Select the Method/Input icon and choose **Fit Method** of mle (Max. Likelihood). Compare. Reset to standard rr. Reference: Handbook 6.4.

Wes's comments: We forced the WeiBayes beta of 3.5. The mle finds beta of 2.926 (highest likelihood), which is fair agreement. With a good estimate of beta, WeiBayes is most accurate.

Problem 10. WeiBayes without a Failure

WeiBayes with no failures provides a lower one-sided confidence bound for the true unknown Weibull.

Analysis says +300 is -300 (no failure). Select **New Plot**, load **PlayTIME-Problem-10.w**. Ignore “Y-Scale” message. Select Weibayes icon, set Beta = 3.5, confidence at lower 63.2%. What is Eta? Ref: Handbook 6.3.

*Dr. Bob's comments: The WeiBayes line did not change with and without a failure, but note that when the data point was a failure, the WeiBayes line was an mle estimate of the true Weibull, but when there are no failures, WeiBayes is a lower 63.2% confidence bound, taking one failure as imminent. When the number of failures is small, WeiBayes will be much more accurate than Weibull **IF** we know beta from prior experience. SuperSMITH Weibull provides WeiBayes confidence bounds with or without failures.*

Problem 11. How about a Dauser Shift?

If suspension ages are not known, you can still estimate a Weibull line but with large uncertainties.

SOLUTION: In SuperSMITH Weibull (SSW), select New Plot (blank page icon), retrieve **GEARS.W** file. You should see the Weibull plot of 20 broken transmission gears from a European car (Eta=10670km, Beta=3.642). There are 2000 of these cars in operation. Average failure mileage 9640 km.

- 1) From the Main Screen select the WeiBayes icon (showing a “1”). Then select the “Dauser” option.
- 2) **Input 1980** for suspension quantity. Verify top of the menu shows “DS +1980S Eta=34018.6 . . .”.
- 3) Toggle item W...Add Weibayes Marker Line = **YES** to put Marker Line on plot when activated.
- 4) Select the green checkmark to view the plot.
- 5) Use the marker line plotted on the right for analysis and forecasting. Reference Handbook 5.7.

*Wes's comments: The Dauser Shift is a last resort. Suspension values should be in the range of the failures. We recommend an educated guess, engineering judgment, for the age of the units still in service as better practice. Still if you have **no idea** about the suspension ages, try the Dauser Shift.*

Problem 12. Inspection Interval Data

Inspection data is dirty because the times-to-failure are unknown, all we know are the inspection times when we found the failures. Here we will use a special method to improve the accuracy of the inspection data analysis. Reference The New Weibull Handbook Chapter 5.8.

Many analysts suggest that when there are benign failures found at inspection, that only the top point in each series recorded at the same inspection, should be considered in the least squares analysis.

SOLUTION: In SSW, select the blank page icon, then get data set **INSPECT.W**. Does the fit line look appropriate? There may be a better solution. Mark this solution with a Marker Line (dashed line icon - choose set # 1). Select the Method / Input icon, then menu item I...Input Data Format, and Point-By-Point-Inspect(1). Better looking fit? Compare with the standard solution Marker Line. Then reset the method to standard rr.

Dr. Bob & Wes's comments: There are three alternative methods. (1) If the number of pieces inspected at each interval is a constant, the Kaplan-Meier would be a good alternative. (2) If the inspection intervals vary from piece to piece, the mle interval option should be used. (3) If there is uncertainty about finding the failure even if it is there, the Probit method should be used, selecting no difference sample-to-sample for the Input Type (Method/Input Menu) for non-destructive testing.

Problem 13. Coarse Data

Collection produces similar “dirty data”. Reference: Handbook 5.8.

Recording failures in months instead of days or hours or minutes will produce vertical columns of points and the inspection option #1 may be useful.

SOLUTION: In SSW, select New Plot, retrieve **TWIST.W** produced by many students twisting wire until the 1st strand ruptures. Select the 3-parameter icon, choose 1 (Set 1), input 10.4 for t0. The pve% on the plot should increase. Now select the Method / Input icon, Option I, then Inspect(1). Better pve%? (... Yes!). Reset method.

Problem 14. Inspection Probit Data

For inspection (grouped) data, regression is in the “y” direction as inspection times are more accurate than % failed. **Probit is best practice for grouped data from inspections.** Do a “what-if” analysis by looking at the wire experiment data as if it came from periodic inspections of all twistors twisting simultaneously. Use your own data generated in a workshop where appropriate. The template for probit type data entry is on the bottom of the wire experiment data sheet (customize as needed). Reference The New Weibull Handbook Chapter 5.9.

SOLUTION: (For the DEMO software version, below is automated for the data on Page 16)

A) For the wire break data (p. 16) we picked a convenient interval size in half-twists (bends) of 5 as an example. **TRY THIS FIRST BEFORE GOING TO STEP B!** Using that interval size, fill in the probit section on the Wire Fatigue Experiment data sheet for the lightweight data values only (Set 1). The data format has 3 parts per entry: **inspection age** (X-Value = number of bends at inspection time), **cumulative number of failures**, and **number of pieces inspected** (constant in this case).

B) Check your input with this:

20x0x24, 25x4x24, 30x10x24, 35x16x24, 40x19x24, 45x23x24, 50x23x24, 55x24x24

NOTE: We cannot use 20x0x24 (=0%) or 55x24x24 (=100%) and we get a better solution without no-change data like 50x23x24 (previous entry also had 23 in the middle). So our Probit data is now **25x4x24, 30x10x24, 35x16x24, 40x19x24, 45x23x24.**

C) After New plot in SSW, load data file **PlayTIME-Problem-3.w**. Use the Marker Line icon to add a line for Set 1 solution Eta=35.82, Beta=5.407. Delete the data sets (leaving the marker line).

D) Load data file **Wire-PT-page16-Set1-Probit.w**. Compare. The Probit data solution (from file) agrees well with the Point-By-Point solution for the same original data. Note: For future reference, the Probit method can be selected with the Method/Input icon, then the Grouped/Probit(2,3) icon.



Wes's comments: Probit fits the Weibull to actual occurrence percentages, not estimates

***NOTE: If you are in the second day of a workshop and you have finished with this problem (#14), then you have completed the computer work for the second day. Take all of your materials with you, go back to where you are staying, and complete the homework reading assignment. Then, get plenty of rest so you can be ready for the activities of the third day.**

Problem 15. Better Methods for Solving the Classic BiWeibull

Problem 2 Continued: When we looked at this data earlier we concluded the first 8, last 4 was the best split. There are other ways to analyze this mixture of two Weibulls. First find the best partition for the data by maximizing the p-value of the likelihood ratio test iteratively for all possible “partitions” as follows.

SOLUTION: In SuperSMITH Weibull (SSW), select New Plot. Then retrieve the **BIW.W** file.



- A) Select the Mixture / Partial % Affected icon, and then Activate SSW Competing Risk mle Fit. The more different the two sets are, the more likely it is that we have found the correct partition. The plot shows the best split based on the modified mle likelihood ratio test.
- B) Select the Plot / Report / Table Options tab above the Active Results, then select “A...Activate Report” to see the p-values and page up and down as needed. We recommend the pff-value (from the modified test with the Fulton Factor). **The partition with the largest pff-value is the best partition.** Record the pff-value found. Is it greater than our standard 90% level for a strong mixture indication? Pff=_____



If you have SuperSMITH Ybath(TM) (SSY) installed and accessible, then return to the mixture menu and select 4...Activate Ybath 4-Parameter Fit. SSW will activate Ybath, get a solution, and return to SSW to display the results (for default installation of SuperSMITH). Switch to / from the Plot / Report as needed to see the plot.

Wes's and Dr. Bob's Comments: Using the modified likelihood ratio test provides a new and more rigorous approach, a quantitative measure, to determine the best mixture partition. There is only weak evidence that a Bi-Weibull solution is appropriate with the partition between the first 9 points and the last 3 points. The mixture pff-value for this data (approximately 69%) is less than our standard 90% minimum pff-value for a strong mixture indication, so we would reject the mixture solution and stay with the uniWeibull. Ybath mle analysis provides results that are very similar to the SSW likelihood ratio test. Ybath does not require a partition. Although the pff-value from SSW accounts for small sample size, you should always use caution with small samples like this one. A minimum of at least 20 failures are needed to lend creditability to partitioning mixture data.

Problem 16. Are two data sets significantly different?

Does changing load weight affect a wire's twist endurance? A very important question to answer is how to compare two or more data sets for significance difference, like for example comparing new design versus old design. Reference: Handbook Section 7.5. Let's use the wire experiment data from Problem 3 for this.

SOLUTION: In SuperSMITH Weibull (SSW), after New Plot, retrieve the **WIRE.W** file you saved earlier (from the first wire data problem). Verify equations setting is Weibull 2-parameter (no t0) or Lognormal.



- A) Are the two sets significantly different at the 90% level... 95% level? To find out, go to the calculator icon and select Set Comparison, then L...Modified Likelihood Ratio Test. Is the pff-value shown above 90% (Significant Difference)?... above 95% (Extreme Difference)? This method using the Fulton Factor (pff value) is more accurate for small samples than the standard likelihood ratio (LR) test.



- B) A complimentary test is to see if likelihood contours intersect. Return to the main screen. Select the Method / Input icon, then select the Modified mle with lower bias “RBA”. From the main screen, go to the calculator icon and select Set Comparison, C...Modified Likelihood Contour Test, View Plot With SSV, Yes to select, 90%. You are now in SSV (if your system allows automatic batch execution). Do the contours overlap? If the contours do not intersect, they are significantly different at least at the level selected (90% in this case). Exit out of SSV to return to SSW. **Turn confidence off.** NOTE: You can access Set Comparison also with the confidence icon. 



EXTRA EXERCISE: If we generated two data sets from the same parent Weibull would they be different? Do a New Plot, then select <Enter> until two random sets of data from the same parent appear on the plot. Are they significantly different at the 90% level? (The pff-value should be above 90% only approximately 10% of the time.) You can use the likelihood ratio test and the contour test as above to see.

Problem 17. Accelerated Test Analysis (PFEV)

If units are tested at two or more levels of load, what is the best practice to analyze the data? Reference [The New Weibull Handbook](#) Chapter 6. The simplest analysis is to make Eta a function of load. This will allow estimates for untested values of the acceleration parameter, like for example 0.5 LBS as demonstrated here.

SOLUTION: In SSW, select the New ... Clear Data icon, then load **WIRE.W** into SSW.

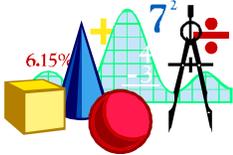
- A. Reset equations to standard Weibull 2-parameter, then select the Parameter as Function of Engineering Variable (PFEV) icon showing a plot with parallel lines. From the menu select Standard PFEV, then select Find 1 Value For All, and enter LBS as the Engineering Variable Name.
- B. Enter 2 for Set 1 LBS and then 2.6 for Set 2 LBS. Select PFEV Fit Type, set to Power. Select Plot Line At New Variable Value, enter 0.5. The new line should be to the right of the lower load data.

Wes's comment: You can solve for multiple load types, one less than the number of data sets (e.g. weight, temperature and humidity, if there are 4 data sets, etc.).

Problem 18. Crow-AMSAA (CA) Reliability Growth Modeling in Visual

Start SuperSMITH Visual (SSV). Perform a CA analysis of Bomber data (Handbook Example. 9-3)

SOLUTION: After New Plot in SSV (Visual), locate and load file **PlayTIME-Problem-18.v**. EFH are X, Cumulative Failures are Y. This is a raw data plot, not a Crow-AMSAA yet. Select the icon for Crow-AMSAA (CA). Input data will be "Cumulative". Set M...Method to IEC, select the check mark to view the plot (Lambda=.01521, Beta=.933). Note: If a message indicates data is not in order (low-to-high), select the Transform icon on the main screen, then Order By X-value before trying CA again. After you have a CA plot, return to the CA icon and select I...Instantaneous = Yes, and R...Rate Transform = Yes. Both cumulative and instantaneous failure rate lines are plotted. The instantaneous line is below the cumulative fit line and sloped down since the CA Beta is less than one (decreasing occurrence rate). **Exit out of SSV.**



Problem 19. Inspection Option #1 versus Kaplan-Meier (KM) for Warranty Data

Warranty claims in (months-of-age)x(quantity) format are... 1x27, 2x10, 3x13, 4x5, 5x4, 6x3, ... plus suspensions in similar format... -4x589, -5x787, -6x585, -7x1013, -8x464 ... for 3500 total units produced. Compare Inspect Option #1 and Kaplan-Meier.

SOLUTION:

- A) In SSW (Weibull), select the blank page icon. Confirm the equation label shows "Weibull".
- B) Load file **PlayTIME-Problem-19-Inspect1.w**.
- C) The data should show: **1x27, 2x10, 3x13, 4x5, >4x589, 5x4, >5x787, 6x3, >6x585, >7x1013, >8x464.**
- D) Plot should show Eta=13546 and Beta=0.512. What kind of component is this? (see comments below)
- E) Use the Marker Line icon to mark the Set 1 fit. Then, delete the data set (Marker Line should stay).
- F) Load file **PlayTIME-Problem-19-Kaplan-Meier.w**. Method will change to KM.
- G) The data should show: **1x27x3500, 2x10x3473, 3x13x3463, 4x5x3450, 5x4x2856, 6x3x2065.**
- H) The plot should show Eta=15640, Beta=0.503. Good agreement on plot! Return to the Main Screen.
- I) Select the Confidence icon. Set Method to Greenwood's (gw), Double, 90 %, then Activate.
- J) How do Inspect Option 1 and Kaplan-Meier compare? Which is best? (see below)

*Dr. Bob's comments: The Kaplan-Meier method has statistical rigor, best practice, for warranty data when a single component or single failure mode is the application. However, Inspect Option #1 is a widely-used good competitor. Here the two methods agree well. We ignored the KM actuarial correction. It is more rigorous and is illustrated in the warranty [matrix analysis](#) problem. If we had properly subtracted that correction, half the suspensions, the last three inputs would be: **4x5x3156, 5x4x2463, 6x3x1773**. For multiple repairable failure modes, you can use Wayne Nelson's recurrent events (RE) approach. Many people use KM even though it is not intended for repairable systems. For predicting total claims by calendar month (instead of age in months) Crow-AMSAA is best practice as shown later (warranty matrix).*

*Wes's comment: Inspect Option #1 solution is close to KM solution. **Beta < 1** indicates electronics?*

Problem 20. Review ... Are Two Weibull Data Sets Significantly Different?

There are many methods and assumptions. Which is best practice? Let us try methods recommended in the Handbook chapter on confidence calculation including how to compare data sets for difference.

SOLUTION: At the SSW main screen, select the blank-page icon, then move the cursor over the computer icon and set dimensions to 12x2000 (if not already at those values) ... return to main screen.



- A) Select Method/Input icon, select Reset-To-Standard Icon (u-turn arrow ... returns to main screen)
- B) Select Method/Input icon again, select Modified Maximum Likelihood icon (mle RBA ...).
- C) From the main screen, select the Demonstrate (dice) icon, change Point Quantity to five (5) and confirm Set Quantity is one (1). Set Sample Distribution to Weibull with Eta=100 and Beta = 3. Make sure the Sample Type is set to "Monte Carlo" (not set to "Precise"). Repeatedly, press the enter key to produce 10 data sets with associated fit lines on the plot.
- D) For each data set ... select the Data Wizard button on the Data Grid, then select Set Label, and enter "A" for Set 1, "B" for Set 2, "C" for Set 3, and so on for all data sets.
- E) By the plot, decide which two sets are most different (different etas or different betas or both). Keep those. Using the data spreadsheet, delete the other data sets (Select at the top of each of those data sets to highlight, then select Delete). Two data sets remain with the most difference (artificially created).
- F) Jim Lemke's method: Select the Confidence icon, choose the Fisher Matrix method (fm), and then select double-sided confidence intervals at 90%. Compare B10 values on the plot, if the upper confidence on the lower set is less than the lower confidence on the upper set then there is difference indicated at 95% confidence (the single bound level). Use this if only one vertical (%) position matters.
- G) At the SSW main screen, select the Calculator icon. Choose the Set Comparison / Difference Indication menu item. Select Modified Likelihood Contour Test (MLCT). Select View Plot With SSV option. Change Fit Type to mleRBA if asked. SSV should automatically start and display 2 contours (uncertainty regions for fit line solutions). If the contours overlap then no difference is indicated, otherwise difference is indicated (at the 90% level). The plot title should indicate pFF-Difference (%) using the Modified Likelihood Ratio Test (MLRT). MLCT and MLRT should agree within a few %.



Dr. Bob's comments. The modified likelihood contour test with the mle-RBA solution point and with Wes's justified likelihood function (JLF) for the similar modified likelihood ratio test are our best methods accounting for small sample bias. These two methods can work with more than two sets.

Problem 21. Probabilistic Design using Monte Carlo (MC) Simulation

The latest design has a predicted **life** distribution that is Weibull, eta = 2000, beta = 6. The **usage** distribution is a lognormal, Med = 500 (median) and SigF = 1.4 (Sigma Factor). Failure occurs if the usage is greater than the time-to-failure. Estimate p, the proportion that will fail.

SOLUTION:



- A) Select new plot from the SSW main screen, then move the cursor over the computer icon and verify dimensions are at least 12x2000. Verify method / input is Standard Point-By-Point (rr).
- B) Select the calculator icon, then select Interaction, then select Life-Usage to get the SSW calculator **Life-Usage** option.
- C) Set Life Distribution to Weibull, set Eta to 2000 and set Beta to 6.
- D) Set Usage Distribution to Lognormal, set Med to 500, and set SigF to 1.4. Select checkmark for results.
- E) Results may vary slightly with MC but should be about 0.18% failed with +/- 0.05 (5E-02%) precision.
- F) Select **OK** to return to menu.
- G) Select **Save Interaction To Data Set**, set the point quantity to 100 to see the resulting distribution.
- H) As there may be negative values that are not suspensions, select the computer icon from the main screen and shift to the normal distribution. If asked, answer yes to "negative values < 0". Note that Weibulls, lognormals, and other models may be convoluted here. **Change back to Weibull equations.**

Comments: Your results could be slightly different. These predictions are based on one design life. Results vary with each simulation when different starting random number seeds are used.

Problem 22. Fracture Mechanics

Based on the design of a new turbine blade and selected alloy, time-to-crack initiation is expected to be Weibull with $\eta=500$ and $\beta=3.4$. Tests of crack growth time to rupture show it is Weibull, $\eta=2000$ and $\beta=6$. Total life is the sum of these two. How is it distributed? What is R99% life?

SOLUTION: To get the answers in SuperSMITH Weibull (SSW) ...



- A) Use the calculator icon on the main screen, choose Interaction, and select Interaction: Random Point.
- B) Input the two distributions as given above. Toggle the "U" selection to Type=Sum. Select the green checkmark to activate Monte Carlo simulation.
- C) Results are stored in a data set and automatically plotted. Use the Distribution Analysis icon to find the best distribution and then switch to the best equations (accept if dialogue box appears offering change).
- D) Select the Precise Reading icon (showing four arrows) to find B1 life, select Set 1. **Reset to Weibull.**



Problem 23. The Binomial distribution can be evaluated with the calculator option. This may be useful for Quality Control Sampling inspection

A manufacturer wants to demonstrate that his percent defective for his part is less than or equal to 0.1% with 90% confidence.

SOLUTION: Return to SuperSMITH Weibull (SSW) ... with <ALT> <TAB> if needed.



- A) On the SSW main screen, select the calculator icon. Select binomial, trial quantity, probability of 1 event in 1 trial = 0.1%, event quantity = 0.0, and cumulative probability = 10% (C=90%).
- B) The answer: Trial Quantity = 2302 pieces as shown in Table 8.1 in the Handbook.

NOTE: Using the binomial (counting, not measuring) to demonstrate 99.9% reliability at 90% confidence with no failures, 2302 successful tests are required! Measuring instead requires much less.

Problem 24. The Poisson distribution applies to random events defined by an average rate

An electric utility averages 8 outages per month in its distribution network. What would a bad month produce, say at 90% confidence? What is the upper 90% one-sided confidence bound for outages per month?

SOLUTION: The 90% bound will be the number of outages that has Poisson cumulative probability just exceeding 90%.



- A) From the SSW Main Screen, select the Calculator icon, Poisson, Expected Quantity = 8
- B) Try different values of Event Quantity (>8) until the Cumulative Probability (shown in menu heading) just exceeds 0.90. See Example 8.8 in the Handbook

Comment: 12 events is the upper 90% one-sided confidence bound when expected quantity is 8.

Problem 25. Magic... Did Murphy really delete an important plot?

You just shut the SuperSMITH Weibull software down and turned off your computer. But WAIT! You forgot your boss wants a copy of the plot you were analyzing in SSW. But you didn't save anything! **UH OH!** Should you get a new resume ready? No need for that. Try the following ...

SOLUTION:

- A) Start the SSW program, then use the dice icon on the main screen to generate some data to get a plot.
- B) Exit entirely from SuperSMITH. Do not save active results (recreating the **UH OH** moment).
- C) Then go back in by starting the SSW program again.
- D) Select the drop-down menu item FILE and then FORMER FILE LIST.
- E) Select **Get Last .TMP Results** and load file **WS~0.TMP**. Magic!!!
- F) View the "lost" plot retrieved from previous results. Now, you can copy the plot for your boss!

Advanced Problems

Problem 26. “YF26” Systems Analysis for INDEPENDENT Failure Modes (Handbook section 4.6.1)

	Control	FOD	Bearing	Turbine Blades
ETA	30000	1200	700	600
BETA	0.5	1.0	2.0	6.0

With Weibulls for principal failure modes in a repairable system, can you do a failure forecast for the system?

SOLUTION: Start in SuperSMITH Weibull (SSW), confirm Weibull equations (green label), reset Method.

- 1) Select New. Load file **PlayTIME-Problem-26.w**. Ignore “Y-Scale” message. Data should show **>10x10, >20x10**. Move the cursor over the results box, you will see “Not Enough Data...WeiBayes?”
- 2) Select Abernethy Risk, verify Risk Analysis Type = Standard.
- 3) Set Format = Cumulative, set Renewal = Yes, select Usage, ... Single Rate ... , enter 25.
- 4) Select Data: Set #1... Distribution... , then Analysis (Weibull) Parameter, then Standard. Enter the Control values (above) for Eta and Beta, ... i.e. enter **30000** for Eta and **0.5** for Beta (leave t0 as zero).
- 5) Select the green check-mark to view the output. The 60th month cumulative risk should be about 4.2.
- 6) Select the bottom icon (plot of curvy line) on the side of the failure forecast table to view the failure forecast plot. Save this plot as **CONTROLCUM.RSK**. Exit SSV to return to SSW.
- 7) Repeat steps 2-6 above using appropriate Eta and Beta values for each of the remaining failure mechanisms, each time saving the forecast plot to a different appropriate filename (like **FODCUM.RSK**, **BRGCUM.RSK**, and **BLADECUM.RSK**).
- 8) View the 4 risk functions together as follows. Start the SSV (Visual) program, select New Plot, load the 4 files one after the other. They should be shown as separate data sets on the same plot.
- 9) We want to sum the 4 risk functions to obtain the system curve. So, from the Main Screen, select the transform icon [F(a)], answer “No” to save if asked. Choose transform # 6...Total Y-Value... then “Yes” to X-values the same... and view the resulting “Total” curve.
- 10) This newest curve is the sum of the risk for each independent failure mode, a system risk forecast.

*Dr. Bob’s comment: In SSV, after a New plot, retrieve the file **YF26TRSK.V**. This should match the plot you just did. The System Rhythm may be obtained by repeating the above process using **Format=Interval**. Alternatively, you can just clear with the “New” icon and retrieve **YF26RTHM.V** to see the rhythm. System optimum replacement may be convoluted the same way. Start with a New plot and then retrieve **YF26REP.V** to see the system optimum replacement.*

Problem 27. Warranty Claims Data Format and Forecasting with Weibull and Crow-AMSAA

Reference: Chapter 9 in the Handbook.

Background: There are two main types of data used for warranty forecasting of future claim quantity:

- 1) By calendar month in calendar time. Crow-AMSAA (C-A) is best for this ... only for near term.
- 2) By calendar month by equipment age. Weibull using “Inspect #1” fit option is best for this.

Let’s try a failure mode mixture example, the most difficult type of warranty issue. An automotive component has 4 Weibull failure modes, A) $\beta = 0.5, \eta = 250,000$ miles; B) $\beta = 1.0, \eta = 200,000$; C) $\beta = 2.0, \eta = 150,000$; and D) $\beta = 6.0, \eta = 80,000$. Monte Carlo simulation created a warranty claims array for the first 12 months of production (“MOP”). Results were saved in text format file **WAR12NR.WAR**, and shown in the triangular-shaped data table below. 100 units were produced each month (in the first row of the table). Warranty claims were reported at month intervals (in the body of the matrix). Rows 2 - 13 represent months in service (“MIS”) 1-12. The first month’s production of 100 units produced 9 claims in the 1st month, 2 claims in the 12th month. The most recent production (right-hand column) resulted in 4 claims in the 1st (and only) month of service. Data can be blocked, copied, and pasted from a spreadsheet program like Microsoft Excel (R), or loaded from a standard text file like **WAR12NR.WAR**.

Text in the WAR12NR.WAR file

100	100	100	100	100	100	100	100	100	100	100	100
9	5	12	7	5	3	6	6	10	9	6	4
5	4	3	1	6	3	0	3	1	3	3	
2	2	1	0	1	6	4	2	2	2		
2	3	4	2	1	2	3	3	1			
4	3	3	1	4	2	1	1				
1	1	2	1	1	1	1					
4	1	2	2	1	1						
2	4	0	4	3							
3	1	1	2								
2	3	1									
2	2										
2											



SOLUTION: At the SuperSMITH Weibull (SSW) main screen, Home group, select New (blank page icon). Select Analysis, Method/Input, then Reset (U-turn arrow) to reset equations.

- A) Enter the data (different for different software versions):
 - a. FULL version - Retrieve **WAR12NR.WAR** (no good for DEMO, as input is altered).
 - b. DEMO version – Select Sample (dice icon), Special Data Set, then item 10A Warranty.
- B) Verify Method For Warranty is Inspect insp1. Select W...Format Warranty to get the Warranty Import Data Format (WIDF) menu. At the WIDF menu, verify MOP vs. MIS (or vs. MOA) is selected. Then choose Activate. Verify Limit Correction, Renewal, and Warranty Limit all = No.
- C) Select OK to plot. You should get Eta=61.83, Beta=0.667. If not, change Method to Insp1. There are currently **227** failures. (“n/s” on plot shows 1200/973). Return to the Main Screen to see the data grid. The first entry is “1x82”, meaning 82 claims at 1 month of age. Four failures in the last month’s production leave 96 suspensions at 1 month of age which is the second entry (“>1x96”).
- D) Select Tools, Abernethy Risk. Reset To Start. Set Usage Rate = 1. Select OK. Claim quantity in 24 months should be about **329**. See Dr. Bob’s Comments below. **Exit SSW. Start SSV.**
- E) Enter the data as in operation A before, select option W, then A, again A to get the Crow-AMSAA (C-A) plot with C-A solution Lambda=0.246 and Beta=0.774.
- F) Enter a second data set as follows:
 - a. FULL version - Retrieve **WAR36NR.WAR** (no good for DEMO, as input is altered).
 - b. DEMO version – Select Sample (dice icon), Special Data Set, then item 11A Warranty.
- G) Results on the C-A plot for the second data set should be Lambda=0.431 and Beta=0.702.
- H) Select Tools, then Crow/AMSAA for the menu. Use Extrapolate Fit with Lower X=100 and Upper X=26064 (last x-value in data set #2). Set3 fit line’s last y-value is **644** (rounded).



*Dr. Bob’s Comments: Weibull Inspect#1 forecasts accurately **329** additional failures in the next two years. By inspecting the C-A data grid in SuperSMITH Visual and subtracting the 12-month cumulative claims (227) from the actual 36-month claims (=545), the true 24-month future claims are **318**. C-A didn’t do well in this case because of the cusp at 12 months. C-A over-predicts to **417** (= 644 – 227). This is just one comparison, so these results should not be generalized.*

With FULL software version only . . . For more warranty data analysis you can repeat this exercise using data with renewal. Try **WAR12R.WAR** and **WAR36R.WAR**. Kaplan-Meier and Inspection Option#1 are not intended for renewal. C-A is the best for renewal data, especially for near-term prediction (a few months ahead).

Problem 28. High Resolution on Batch Effects with ACH

David Nevell (per Geoff Cole's request) of Rolls-Royce developed a new graphical technique for evaluating model fits to the data called aggregate cumulative hazard (ACH) available in SuperSMITH.

SOLUTION: Start SSV (Visual). Bring the file **18WHEELER.W** into SuperSMITH Visual and select "A...Aggregate Cumulative Hazard". You will see the ACH plot. For rr fit the ACH line does not match with the final point. The ACH plot shows model to data compatibility. If the ACH fit line reasonably follows the data, the statistical model is appropriate. If there is significant deviation, the model might not be appropriate. The disparity with the rr solution may indicate a mixture or a batch (only a portion of the units can fail). Select New Plot. Bring in file **18WHEELER-mle.W**. Select "A..." again. For mle fit the ACH line always goes through the final data point. The mle solution in this case shows ACH line matching to the data but may not detect an existing batch issue. All of us (Geoff, David, Dr. Bob, and Wes) would be interested in any ACH applications you may find.

Problem 29. Barringer Process Reliability [by Paul Barringer]

At the SuperSMITH Weibull (SSW) main screen, select N...New. Then open data file **PR.W** with 365 days of production process output (in Mlbs). View the plot. The data looks like a mixture with several data groupings evident. The standard fit line does not fit well and does not relate to traditional failure modes, since data is from a business situation. What is the process reliability? Cutback losses? Crash-and-burn losses? If the process is capable of achieving a $\beta=75$, what is the nameplate rating? What is the demonstrated production quantity? What are the efficiency and utilization losses?

SOLUTION:



- A. The standard BPR vertical scale uses reliability (complement of failure probability). The upper reaches of this data follow a straight line down to the middle 50's [%]. From the main screen, in Tools, select Barringer Process Reliability (BPR) to get the BPR menu. **Set BPR Type to Barringer Standard.**
- B. Select Z...BPR Zoom from the menu and choose F...X-Range + Y-Range Lock: Select At Value + On Plot. Use the little arrowhead boxes on the axes to set horizontal plot limits 600 (x-lower) to 1200 (x-upper). In the same way set vertical reliability plot limits 85 [%] (y-lower) to 0.1 [%] (y-upper). Then use the <ESC> key or select the green check mark to return to the BPR menu.
- C. From the BPR menu choose option "E" for adding a Production Line to the plot. Then select G...Plot Line Drag. Move the mouse pointer (crosshairs) to where the upper data point trend first changes direction (approximately 53.4%, "Process Reliability" is defined at this point). Select (and hold) there, then drag a line up to fit the upper segment of data points, and release. The line drawn should indicate about $\beta=24.5$ and $\eta=1003$ representing the demonstrated "Production Line".
- D. In the BPR menu again, select F...Nameplate Line. Then select H...Plot Point Fit and drag a box around only the single highest data point (highest productivity). Enter 75 when asked for Beta value. This gives you $\beta=75$ and $\eta=1056$ for the "Nameplate Line" (the benchmark goal for production).
- E. From the BPR menu select Z...BPR Zoom, then select option F, and restore $x\text{-lower}=100$, $x\text{-upper}=1200$, $y\text{-lower}=99.9$, and $y\text{-upper}=0.1$. (*Notice the final slide to very low production occurs at 98.2% reliability. This identifies an important reliability point used for crash-and-burn calculations. A line through these low points would have $\eta=33158$ and $\beta=1$.*) Choose <ESC> or select the green checkmark to return to the BPR menu.
- F. At the BPR menu, choose item G...Set 1 Process Reliability for identifying change points in the data. From the next menu select G...On Plot, and then select the change points. The first one is the Process Reliability point #1 (at 53.4%), then point #2 (81%) where a change in the data occurs, and finally point #3 (98.2%) where the crash-and-burn segment begins. To exit the plot when you are finished entering change points, use the <ESC> key.
- G. In the BPR menu again, press the <ENTER> key or select the green check mark to get the BPR analysis results.

Paul Barringer's Comments: Production losses here from reliability issues are ~29,000 Mlbs in one year. There are three segments: 1) from loss of production reliability (point 1) to step at 81% (point 2) loss is ~12,000 Mlbs, 2) from step at 81% to beginning of crash-and-burn at 98.2% (point 3) loss is ~14,000 Mlbs, and 3) crash-and-burn losses (from 98.2% to 100%) are ~3,200 Mlbs. These are gaps between the Production Line and actual data. Efficiency and utilization losses between Nameplate and Production are ~24,000 Mlbs. The most important issue is reliability (29,000 > 24,400). Within reliability, the most important is from 81%-98.2% reliability (14,000 > 12,000 > 3200). Performance records with root cause information can help address these issues and improve control of process output.

Problem 30. Step – Stress Accelerated Testing and Analysis

Your boss wants a step-stress test on 15 items for quick results. The item’s useful life is 768 start-up cycles at a rated load of 10 ksi. The step-stress schedule has 10 steps starting at 10 ksi and increasing at 10 ksi intervals to 100 ksi at the 10th step. The time at the first step is 2/3 of the item’s useful life and every test duration length after the first step is one half the number of cycles of the step before. The reliability requirement for useful life is 99.9%. You also find out that the part will NEVER be used at rated load, only at half of rated load. You need to show your boss the plan and whether the item meets reliability or not. Your test has been completed and the resulting data (cycles) are as follows:

639	760	767	789	812	836	>840	858
888	912	914	930	932	>950	>950	

Note: The “>” sign indicates suspensions)



SOLUTION: Open SuperSMITH Weibull (SSW) software. Select New Plot. Load file **PlayTIME-Problem-30.w**. Select Method/Input icon, set fit-line solution method to rr (rank regression). Resulting plot should show Eta=907.57 and Beta=9.655 (*this Beta is too high before load sequence entry!*).



- A) Start at the main screen, and select the PFEV Accelerated Testing (A.T.) icon.
- B) Choose Step Load, enter the load name “ksi”, then option “K”, enter 10 for Step Quantity.
- C) Enter loading sequence ... load-level (Var. Value) vs. time-at-load as prompted one-by-one:

<u>Variable Value</u>	<u>Time</u>
10	512
20	256
30	128
40	64
50	32
60	16
70	8
80	4
90	2
100	1

- D) Verify sequence shown and select OK. On the PFEV dialogue box (with options for further analysis) results should indicate a Beta of approximately 2.5, V of about 360, and p about 2.3.
- E) Select Plot Line At New Variable Value, enter 10 (adds line for 10 ksi rated loading). Return to the PFEV A.T. icon, select L...Plot Line again to enter the actual operating load of 5 ksi. You will then have marker lines on the plot for both 10 ksi and 5 ksi. Line 2 is for 5 ksi.



- F) Return to the main screen, select Precise Reading, choose Line 2, and enter Datum of 768 cycles.
- G) Does the item meet 99.9% reliability required for useful life? ANSWER: Yes... the Precise Reading output indicates about 99.95%. BONUS: To graph the step load schedule, return to the main screen, select the PFEV A.T. icon again, and choose View Step Load Sequence to plot the schedule in SuperSMITH Visual (SSV).



- H) Exit SSV when done looking at the step load sequence.

Problem 31. Vasan-Fulton Former Usage To Test Ratio (FUTR). Your customer needs rapid development capability that can only be accomplished with accelerated development testing performed in-house. There are many pieces of equipment out in service without failure, good news! However, you need to get an acceleration factor for your in-house test. The acceleration factor is the life multiplier value you use to convert from in-house test results to in-service results. You decide to bring some of the in-service equipment in from the field for life testing. The equipment with one year of in-service operation fail at 2, 7, 7, 16, 22, 25, 25, 26, 28, 34, 49, 49, 57, and 59 hours (gives $\text{Eta}=33.11$, $\text{Beta}=1.229$), the equipment with two years of in-service operation fail at .2, 8, 14, 18, 20, 25, 27, 29, 31, 38, 39, 40, 52, and 85 hours (gives $\text{Eta}=35.45$, $\text{Beta}=.906$), and the equipment with three years of in-service operation fail at 6, 9, 15, 16, 22, 25, 28, 29, 30, 30, 32, 32, 42, and 49 hours (gives $\text{Eta}=29.88$, $\text{Beta}=2.013$). Your goal is to determine the acceleration factor. Try the following solution ...

SOLUTION:

- A) Starting in SuperSMITH Weibull, select New Plot (blank page icon) from the main screen, select the Method / Input icon, and reset the method to standard (Weibull/rr).
- B) Locate and load file **PlayTIME-Problem-31.w** to get three data sets as listed above with solutions as indicated.
- C) Select the Label/Style icon. Change X-Value Name to "Test Time". Change X-Value Units name to "Hours". Then return to the Main Screen.
- D) Select Accelerated Test / Degradation, then Vasan-Fulton Former Usage to Test Ratio (FUTR).
- E) Enter "Years" for Former Usage Units Name.
- F) Enter Former Usage Value as 1 for Set #1, 2 for Set #2, and 3 for Set #3.
- G) The analysis result FUTR is then shown on the screen. The acceleration factor is the FUTR normalized to remove dimensions. You should get close to 0.209 for the FUTR.

Problem 32. Verify Improved Distribution Analysis With p-value Estimate (PVE%)

Correlation coefficient squared (r^2) indicates goodness of fit. However, it alone does not tell which distribution fits best. If r^2 is the same value for both 2-parameter and 3-parameter Weibull, the 2-parameter has an edge with fewer parameters. To gage goodness of fit, p-value is needed. P-value is the rank of a measurement within all possible values from Monte Carlo. Dr. Chi Chao Liu's research indicates p-value is one of the best goodness-of-fit indicators. Does p-value perform well?

SOLUTION: Select New Plot (on SSW main screen), then Method/Input, Reset To Standard (Weibull/rr)

- A. Select the Report tab (above Active Results box), set Fit Comparison + Display to Correlation P-Value Estimate (pve%). On the Main Screen, select the dice icon, generate 10 Weibull 2-parameter data sets ($t_0 = 0$), $N=30$ points each. Select the Distribution Analysis icon. Check "optimum distribution" for each data set. Most should indicate 2-parameter Weibull. What percent indicate different than that? Repeat a few times to verify. Exit to the Main Screen.
- B. Select New Plot, then use the dice icon to generate one data set with 30 points. Select the Confidence icon, then select Confidence Special Curve Type. Select "pv P-Value For r", then "No Save / No View". Select Confidence Type of Double, then 90% to get the plot with confidence bounds. Select the right-hand tab over the Active Results and Activate Report. There should be reasonable agreement between Prr-Value(%) from quick simulation and pve% from math modeling. The pve% is most accurate.



Problem 33. AIAG Heavy Vehicle/Equipment Usage Profiles

You have a fleet of 967 long-haul trucks in service for six months and there have been 10 failures so that ten trucks have been taken out of service. The failure data in terms of mileage is given in the file **18WHEELR.W**, but you only have general information about the suspensions. You decide to use the Automotive Industry Action Group (AIAG) usage profiles to estimate the suspensions.

SOLUTION: Starting in SuperSMITH Weibull at the main screen, select New Plot (blank page icon) to clear the data grid and the results window.

- 1) Select the Method/Input icon, then Reset-To-Standard icon (“u-turn” arrow) to reset to standard (Weibull/rr).
- 2) Bring in file **18WHEELR.W** from the sample data subdirectory. Delete suspensions only as follows. First, select at the column top to highlight the entire set. Select the Data Wizard button (or right mouse select). Select Edit Special, for Total Set, then Suspension Only Delete.
- 3) The resulting Weibull plot should show Eta of 68856 and Beta of 3.614. Save this data set as **18WFAIL.W** and select New Plot to clear the grid and results.
- 4) Select the dice icon to get the Sample menu. Select Special Data Set, and then select AIAG Usage Rate (Miles / Year) ... Line Haul to bring in the histogram of suspensions at different mileage values representing 1 year of usage. NOTE: Select OK anytime “Set 1 Not Enough Data ... WeiBayes?” or “Y-Scale Results Outside ... ” messages appear.
- 5) Select at the column top to highlight all data in Set 1. Then activate the Data Wizard again. There should be 191375 total suspensions indicated as being in the selected data.
- 6) Change this number to the correct quantity of 957 by selecting the Quantity $\sim(N*C)$ transform, then Yes to the “Total Set?” question if asked, and input 0.005 for the C Value multiplier.
- 7) Select at the column top to highlight all data in Set 1 again. Adjust miles (X-Values) by activating the Data Wizard, selecting X-Value Transform for total set, option M...(*C), then enter 0.5 for the C Value multiplier. This should give 142,500 for highest suspension mileage.
- 8) Bring in the **18WFAIL.W** (becomes Set #2). Select at top of data grid to highlight both sets. Use the Data Wizard to Merge Sets 1 and 2.
- 9) The resulting plot should show the better Weibull solution for the failures with the estimated unfailed units giving Eta=209952 and Beta=3.911. Load file **18WFAIL.W** again for Set #2.
- 10) Try the Dauser shift as follows. Select the WeiBayes icon (shows “1”), select Set #2 (with 3.614 Beta), then select S...Dauser, enter 957 for suspension quantity. Verify option W...Add Weibayes Marker Line = Yes. Select the green checkmark. The plot should show good agreement between actual and Dauser. (*We recommend estimating suspensions here however*)
- 11) Select Abernethy Risk icon and Yes to Select Present Fit. Verify option D...Data Set is set #1. Select option C, set % Estimate to 90, Lower. Set Usage Rate to single value, 10000 miles per month, and Renewal = No. Activate. In two years, approximately 876 more failures minimum are expected (without corrective action). With Renewal = Yes that goes up to 956. Compare between Risk Analysis Type = Standard or ... = RBA1 (with RBA1 the fractions go away).



Problem 34. Automatic Warranty Data Analysis and Time-To-Mileage Scale Conversion

As the CFO at your company, you are responsible for warranty cost on a new production project. The new project is similar to “Blue Edge” which launched 3 years ago. There is 36 months’ worth of Blue Edge warranty data from field service records in file WAR36NR.WAR formatted as month of production vs. month-in-service (MOP x MIS). Columns are month-by-month starting with earliest on the left. Rows are production quantities (in the 1st row) plus failure quantities by months in service starting with the first month at the top (in the 2nd row). You must determine if failures were subject to a wear-out, and if so, select the optimum replacement interval. The resulting Weibull has months of age on the horizontal scale, but we want it in terms of miles. The average accumulation is 850 miles per month estimated from the claims data only. What is the Weibull in terms of miles?

SOLUTION:

- A) Starting in SSW at the main screen, select Home group, New Plot (blank page icon).
- B) Bring in the data:
 - a. FULL version - Retrieve **WAR36NR.WAR** (no good for DEMO, as input is altered).
 - b. DEMO version – Select Sample (dice icon), Special Data Set, then item 11A Warranty.
- C) In the Import menu, verify “Method For Warranty Input” is “Inspect insp1” (toggle if needed).
- D) Choose the “Format: Warranty ...” menu item to proceed to the Warranty Import Data Format menu.
- E) Verify the “*” symbol is in front of the “MOP x MIS” (or x MOA) menu item (select it if not).
- F) Select Activate. Next menu should show Limit Correction, and Renewal, and Warranty Limit = No.
- G) Select OK (Activate) to get the Weibull plot. Select the small plot to enlarge. Eta should be 66.27. Beta should be 0.666. This is the Weibull of product life in units of months of service.
- H) Is this wear out? No, it is not (Beta < 1). No need for Optimum Replacement. Go to the next step.
- I) At the main screen, select at the top of the data set reading “Set 1” to highlight the data set. Select the Data Wizard button in the data grid, select “Edit Special”, answer “Yes” for Total Data Set if asked, then select “Occurrence Only Delete”. With no occurrences a message or messages will appear indicating standard solution cannot be done. Select Okay until your return to the main screen.
- J) Highlight the data again, select Data Wizard. Scroll down and select “X-Value Transform”. Answer “Yes” for the total data set question. Select “M...(* C)” and enter 850 (miles per month estimate) to convert months to miles. Messages appear again, since all the data values are suspensions.
- K) (For future reference, note that multiplying the Eta in months by 850 gives Eta in miles of 56329.5.)
- L) Return to the main screen. In the Home group, select the Sample icon (showing dice) and change Eta, Beta, and Point Quantity respectively to 56329.5, 0.666, and 545 (In a real situation use actual mileage for the failures from additional warranty data input). Select the green check mark to generate set # 2 with occurrences in miles. Messages about Set #1 still appear.
- M) Select and drag to highlight both data sets. Choose Data Wizard. Select “Merge”, then “Set Merge 1 – 2”. The resulting solution is an approximation for the Weibull in units of miles instead of months.



Problem 35. Total Uncertainty Interval Estimate – Accelerated Testing

Electronic circuit board switches are tested at 50°C and 60°C with lifetime in number of cycles as shown below. The temperature gage has not been calibrated. What is your estimate of the lower 95% **total uncertainty** bound on B10 life of the circuit board switch at its normal operating temperature of 30°C? (Kelvin = Centigrade + 273 degrees... in case you have forgotten). The 30-degree line, which we will add shortly, will have additional uncertainties due to the uncalibrated instrumentation and the extrapolation with Arrhenius Law to the lower temperature. The bias limit from these two sources might be +/- 5% in Dr. Bob’s experience.

Cycles to Failure	
50°C	60°C
21045	16551
25077	18935
31407	20996
33812	24363



SOLUTION: Start SuperSMITH Weibull, Select New Plot, reset Method/Input to Standard.

- A) Input file PlayTIME-Problem-35.w and add confidence as Pivotal (pv) one-sided lower 95%.
- B) Use the Additional Uncertainty on the confidence menu to input the value 5%.
- C) Get the **ratio** of 50°C life lower B10 life to its nominal B10 life. (should be about 0.37)
- D) Use PFEV Accelerated Test option to estimate the 30°C line (Input all temperatures in Kelvin . . . 323K and 333 K). Select PFEV Fit Type = Arrhenius. Plot Line at 303 K (30°C).
- E) Reduce the B10 life at 30°C by multiplying it by the **ratio** found earlier. This is your estimate of the lower 95% Uncertainty bound. You should get a value close to 15,000 cycles.



Problem 36. Total Uncertainty Estimates for Comparative Testing

You have carefully designed a comparative test using 8 units of the old design and 8 units of the new design. The cyclic counter has been calibrated to NIST standards of +/- .1%. The order of testing has been randomized such that old units and new units are tested in a random sequence. The definition of failure is clear and unambiguous. Failures will be obvious. The test objective is to determine if the new design is better than the old design. You plan to use the likelihood ratio and contour methods to test the new design data set to determine if it is significantly better at 90% confidence. At this point your boss steps in and criticizes your design as too expensive. He suggests that you just test the new design units and use the old design data from a year ago. You compliment the boss on his great cost saving suggestion, but you add that it will make it more difficult to prove they are significantly different, as the test uncertainty will increase. What steps would you take in the analysis to account for the fact that prior tests over several years on the old design show a spread of +/- 2%? How would you develop an operating characteristic curve to show the boss how much his cost savings will degrade the ability of the test to identify a real improvement?

SOLUTION:

- A) The closer the old design Weibull is to the new design Weibull the less likely there is a significant difference. In the worst case (false indication of improvement) the old design data might be low and the new design might be high. Therefore, copy the old design data set and multiply the values by 1.04.
- B) Now run the likelihood ratio and contour tests on the modified old design and the new design.
- C) How would you develop an operating characteristic curve to show the boss how much his cost savings will degrade the ability of the test to identify a real improvement?
- D) Look at eta improvements of 50%, 100% and 200% compared to the baseline. Use SuperSMITH Weibull to generate 40 pairs of random data sets for each level and test each pair to determine the percent that show a significant improvement.
- E) Now increase the base line eta by 4% and repeat.
- F) Plot the results against the eta ratio.

Problem 37. Total Uncertainty for Crow-AMSAA

Your Crow-AMSAA warranty forecast for the next 12 months is 1000 claims. Your Vice President asks, “What is the uncertainty on that prediction?” What is your answer?

SOLUTION: The best we can expect for 12 months ahead C-A forecasting is +/- 10% uncertainty. See [McGlone] in the Handbook references. Note that although 10% sounds large, the C-A model is the best predictor among the 27 different reliability growth models.

Problem 38. Bearing Probabilistic Design Function

Bearings failures form Weibull three parameter distributions (Reference: “Standard Handbook of Machine Design,” 2nd Edition, by Shigley and Mischke, McGraw-Hill, 1996, Section 27.3). SKF Industries used failure data from 2500 bearings to develop a bearing design function that predicts reliability as a function of load and B life for all new designs. The new bearing Weibull B10 life is determined at the specified load by design analysis. The design function is a three parameter Weibull, $\eta=4.439$, $\beta=1.483$, and $t_0=0.02$. Time (t) equals the ratio of B life to B10 life. What would the reliability be at a life of 10% of the B10 life? (B10 here is 1.0 and so B-Life/B10 is 0.1) Use “Precise Reading” to determine reliability as follows.

SOLUTION: Select the Precise Reading icon. Select “Q...Enter...”, then input beta and input eta as above. Select option “A...Enter Datum” and input the value $(t - t_0) = (t - 0.02) = (0.1 - 0.02) = 0.08$. View results.

Dr. Bob’s Comments: With loading at 10% of B10 life, predicted reliability is 99.74% (will not occur).

Problem 39. Tolerance Stackup Interference Prediction (Reference: “Technical Aids,” by Lloyd S. Nelson, ASQC, 1985)

Hole diameters have μ (mean) of 104 and σ (standard deviation) of 1.5; shaft diameters have a μ of 100 and σ of 1.0. {Both are normally distributed} Use the Calculator, Interaction, Random Point type inputting the hole parameters as distribution 1 (Type = Difference), shaft diameters as distribution 2. What is the probability that the shaft will NOT fit in the hole if the clearance must be (a) at least zero and (b) at least 0.4?

SOLUTION: For (b) change the shaft diameter to 100.4.

Dr. Bob’s comments: Lloyd Nelson assumed the dimensions were normally distributed. In reality this is almost never true unless the parts are machined under strict statistical process control. Most dimensions will be better fit by Weibulls or lognormals and the random set comparison will convolute all the combinations of normals, lognormals and Weibulls. Wes’s comments: Using “normal distribution algebra”, 1.325% and 2.292% respectively should not fit in the hole. Also, see the related “Six Sigma” problem in this tutorial on the use of the normal (standard) vs. the Weibull distribution.

Problem 40. Classic Mixture of Two Failure Modes

SOLUTION: Return to SuperSMITH Weibull and after New Plot retrieve **BWSOPAC.W**. Here are results from BiWeibullSMITH (we now have better methods). Select the Mixture Icon on the main screen. Select the Activate SSW Competing Risk option to get the SuperSMITH competing distribution likelihood ratio test. Delete confidence if asked. Write down the Etas and Betas for the 2 Weibulls. Check the pff-value on the plot. Are the two sets significantly different at 90% confidence? The high pff-value indicates **YES**. To estimate proportion of failures from infant mortality, enter Etas and Betas (low slope combination first) into the calculator icon random point comparison, Type = Difference. Compare the value with the plot. Does it make sense? Do a new plot and then load file **BWINDY92.W** for some 1992 Indianapolis motorcar race results.



Using SuperSMITH **YBath™** software: Start again as above except don’t select the SSW Competing Risk option, ... instead select Fit Method For YBath = rr, and then select Activate Ybath(TM) 4-Parameter Fit. The results should be similar with YBath being the more rigorous result.

Dr. Bob’s Comments: It is always better to separate the failures into separate failure modes by inspecting the broken parts, the source of the failures, the root cause of the failures, in other words, by engineering analysis. However, there are many cases where this cannot be done and we must rely on statistical methods. To estimate these many parameters larger samples are desirable. A problem is included to introduce you to some options in SuperSMITH YBath™

Problem 41. Mixtures of 2 or 3 Failure Modes with Batch Effects – Ybath™ [by Carl Tarum]

What can you do with a mixture of failure modes? What if infant-mortality, chance, and wear out failures exist, like the classic Bathtub curve? YBath can estimate the parameters for these mixtures.

Use judgment with the sample size. Suppose you have 10 data points and fit a model that says that 10 percent are infant mortality. This means that maybe only **ONE POINT** of your 10 data points is infant mortality. While the computer is happy with the numbers, it is absurd to justify a Weibull slope(b), Characteristic Life(h), **and** portion defective (u) based on one data point! Minimum recommendations are 25, 40, and 50 for 4, 5, and 7 parameter fits respectively. Consider the variation of the parameters and if the sub-population is large enough to justify the fit. If u=10% on 100 points, that indicates that 10 points are part of the infant mortality. But u=1% on 200 points indicates that just 2 points are defining the infant mortality. It is not reasonable to expect that 2 points can provide an accurate estimate of beta, eta, and the portion of the population.

Figure 8 below is a good example of a competing risk mixture. The steep slope followed by shallow indicates a batch effect; one of the failure modes only applies to a subset or batch of the entire sample. Therefore a (5 parameter) fit of the following equation may be appropriate:

$$F(t) = 1 - (u)\exp[-(t/h)^b - (t/\eta)^\beta] - (1 - u)\exp[-(t/\eta)^\beta]$$

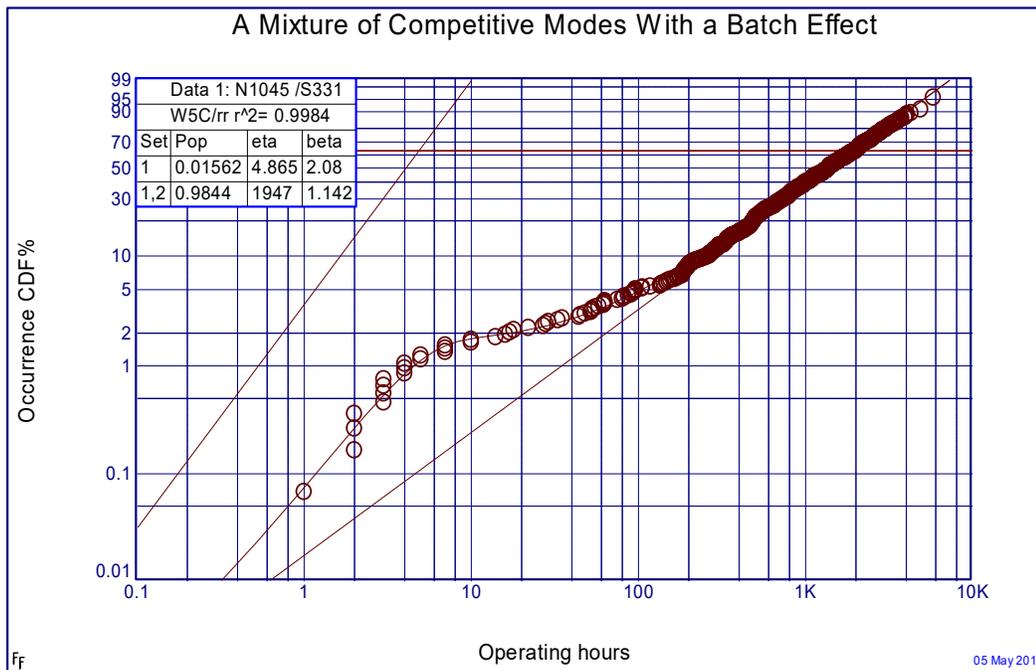


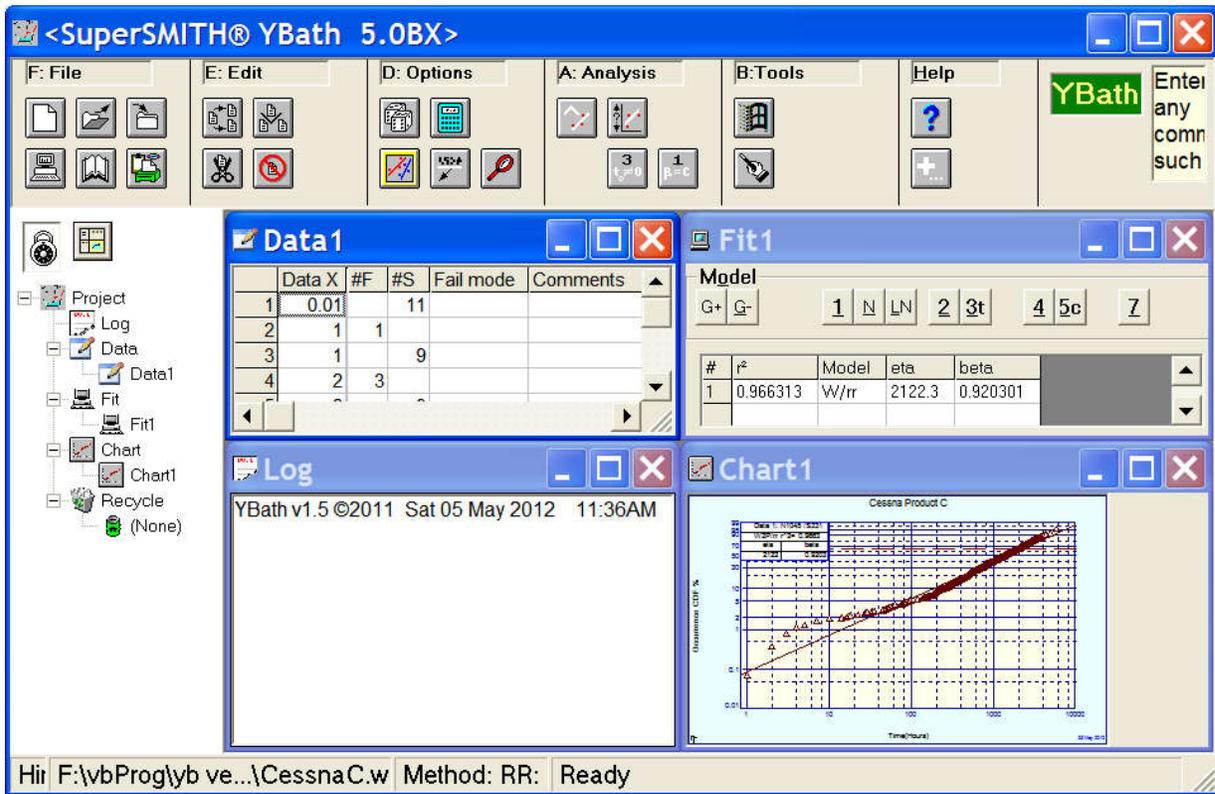
Figure 8: Mixture of Competitive Failure Modes

In SuperSMITH Weibull, open the data file **CESSNAC.W** and plot it with a 2 parameter Weibull (W2). There is some curvature at 10 hours. Note the PVE of only 0.55%. (Not 55%, but less than 1%!). A Distribution Analysis between W2, W3, and LN, shows no reasonable fit. With the obvious curvature, we should consider a mixture ... as below.



SOLUTION: Open YBath either by selecting the YBath Icon, OR by selecting it from Start/Programs or from the File Manager, OR by choosing it under “Additional Analysis” in SuperSMITH

Figure 9: YBath Main Screen



YBath Screen Overview: Here are some of the main features/windows of Ybath ...

- Data: Enter time to failure, Number of failures and suspensions, the failure mode, and comments
- Fit: Choose the Fit model, and view the results
- Log: Review your work, and add comments
- Chart: The graph of your data. Right-select to format or copy
- Project Tree: Lists all of the windows for fast navigation
- Status: Shows method, and x, y plot position

Load file **CESSNAC.W** (on the SuperSMITH program disk) either using the open folder button, or by selecting File/Open from the menu. This data has a "bump" at about 1.5%, or 5 hours, indicating mixture.



Select the Distribution Analysis button for the input window (Figure 10). Select 2-, 4-, and 5-parameter fits. Select mle for method. Select the green "Go" button. This fits the models. View results in the "Log" window shown below (expand if necessary). It has the following columns:
= The Fit Number
r², Ln(L) = corr. coefficient (sq.) or nat. log of likelihood.
Model = Models compared (W/L/G-/G+, etc.) ...
 Parameters (2, 4..) ... and Method (rank regression, mle)
Sub-Pop = A Subpopulation number ...
u% = Portion belonging to sub population
eta, beta = Weibull parameters
LR Test = Likelihood Ratio Test (0-100% p-value, a higher percent gives stronger evidence for a mixture)

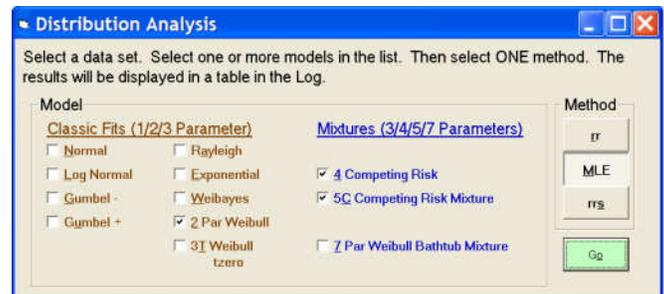


Figure 10. Distribution Analysis Input Window

“Log” window results for CESSNAC.W:

#	r ² , Ln(L)	Model	Sub-pop	u %	eta	beta	LR Test
1	0.96631	W/rr	1	100	2122.3	0.9203	-
2	-6123.33	W/mle	1	100	1950.44	1.0182	Base
3	-5717.78	W4P/mle	1	100	19244.5	0.5468	100.00%
			1	0	1531.12	1.5047	-
4	-5712.41	W5C/mle	1	2.2731	4.88656	1.8911	100.00%
			1,2	97.727	1275.5	1.2833	-

Fit #2 shows that a Weibull mle fit has Ln(Likelihood) of -6123.33. This is the basis for the likelihood ratio test. Fit #3 is a 4-Parameter fit and Ln(likelihood) of -5717.78. We have added 2 parameters and improved the likelihood by over 400 points. The likelihood ratio test is 100% (close to 100%, rounded within precision). Above 50% indicates better fit by the mixture. So, a 4- or 5-parameter fit is better than a 2-parameter fit.

Based on the shape of the curve (See the help file model section), we will choose the 5-parameter fit (last 2 rows in the “Log” window) as the best fit. This indicates that 2.3% of the population has an eta of 4.9 and beta of 1.8, while the rest of the population has a life of 1275.5 and a slope of 1.28. This data set has 1045 points: 714 Failures and 331 suspensions. 2.3% of 714 points is 16. Sixteen points is adequate in this case to estimate the infant mortality population and parameters.

Other Fits: You can view the fits that you have selected in the Fit Window. You can select one of the buttons to plot a different fit.

Formatting the Chart: Under the Option menu (or right selecting the chart), you can change the chart symbols, colors, formats and size.

NOTE: See Figure 13 for details of mixture analysis.

Problem 42. Distribution of Beta (Weibull Slope)

NOTE: Some problems on **Monte Carlo Simulation** with SuperSMITH require later versions (4.0WV or higher). The precision error may be judged by running repeated cases. Variation from case to case is statistical and may be reduced by running more trials. The improvement in precision is a function of the reciprocal of the square root of the number of trials. For real problems use at least several thousand trials. More trials are better. Additional trials require more time, not an issue with fast computers.

For WeiBayes, it’s common practice to average betas from prior data sets for a nominal value. One of your staff experts says that’s wrong. The expert thinks the distribution of beta is more lognormal than normal, and therefore the geometric mean or median would be better. The rest of the staff is skeptical. Is the expert correct?



SOLUTION: Start SuperSMITH Weibull (SSW) & select Setup (computer icon) from the main screen.

- A) Select Dimension, verify Set Quantity [12] and Point Quantity [2000], and return to main screen.
- B) Select blank page icon. Select Method/Input icon. Then choose Reset To Standard (Weibull/rr).
- C) Select the Demonstrate / Simulation (dice) icon, then select Special Monte Carlo Simulation.
- D) Select Monte Carlo Type, then select Model Parameter Value [MPV], set B-Value Select = None.
- E) Select “C...Sample From”, choose Weibull, set Beta = 4, Eta = 100, t0 = 0, then select “OK”.
- F) Set the Fit Method = rr, Monte Carlo Trial Quantity = 1000, Point Quantity = 10, % Suspension = 0.
- G) Select “A...Activate”, and after simulation select All Monte Carlo Results To File, and save as X.TXT.
- H) Return to the main screen, and open X.TXT (choose import data option New Column=New Set). Ignore “... Slope Too Big” messages. What’s the best distribution for Set #3?



Comments: Best fit for beta variability is usually lognormal. Lognormal is skewed, so the median value of x is a better typical value than the mean. The staff expert is correct. With the lognormal distribution, the antilog-of-the-mean-of-log-x (“Med”) is the median estimate of x (also called “B50”).

Problem 43. How Good Are the Answers?

Management was pleased you provided quantitative uncertainty bounds but they are nervous about this Monte Carlo solution. How good are your uncertainty bounds?

SOLUTION: Start in SSW at the main screen and select New (blank page) icon. Continue with <ENTER>, <ENTER> and View the Monte Carlo data and solution. The clock provides the seed for random numbers so each set of trials give slightly different results. How close are they to previous results? The differences are a measure of statistical precision. (You could generate 10 or more tables of results and calculate the mean and standard deviation of the variability.) More trials improve precision by the square root of the number of trials. Monte Carlo pivotal (pv) confidence bounds are an automatic way to evaluate confidence with simulation, and this option is located under the confidence icon. Every time you run a Monte Carlo, the answers are slightly different reflecting the uncertainty in the statistical process. To avoid this scatter in the SSW pv confidence bounds, you may “lock” at a particular seed number and get repeatable results. Or you can "unlock" with a time-based seed number (=0) to produce variability. Dick Rudy of Chrysler suggested this locking option.

Problem 44. Statistical Uncertainty of Important Results!

Your latest Weibull analysis of the #1 issue in the top ten has received upper management attention. Although results are in reasonable agreement with engineering analysis, management wants some measure of the uncertainties associated with this new technique. Your Weibull analysis is based on a sample of 18 failures with eta = 165 and beta = 4.3. How can you respond?

SOLUTION: Determine 90% two-sided “uncertainty” bounds for eta, beta, r, B1, and B5 life.

- A) Start SuperSMITH Weibull (SSW) and go to the main screen. Select New (blank page) icon. Select Method/Input icon. Reset Method/Input to standard (Weibull/rr). Select Setup/Dimension and verify Dimension says at least Set Quantity = 12 and at least Point Quantity = 2000
- B) Select the Demonstrate/Simulation (dice) icon, Select 2...Special Monte Carlo Simulation.
- C) Select M...Monte Carlo Type = Model Parameter Value [MPV]. Select B...B-Value Select. Enter 1 and then 5. Select C...Sample From, enter Weibull, Beta = 4.3, Eta = 165, verify t0 = 0, OK. Verify F...Fit Method = rr. N...Monte Carlo Trials = 1000, P...Point Quantity = 18, Q...% Suspension = 0.
- D) Select A...Activate to start, then after simulation select 5...All Monte Carlo Results To File.
- E) Save to file **MCX.TXT** and return to the main screen (use <ESC> key twice). NOTE: When using the DEMO version software, we cannot input data like it is saved in this kind of text file. However, later you can bring the MCX.TXT file into a spreadsheet program like Excel from Microsoft to view. The resulting data from a previous run-through of this example is saved in file PlayTIME-Problem-44.w.
- F) At the Main Screen, locate and load file **PlayTIME-Problem-44.w** saved previously.
- G) Open the **MCX.TXT** file, select A...New Column = New Set (X-Value)
- H) You should have 6 data sets in the data grid (the al-zero data set for t0 is automatically removed). Page down to the 50th point.
- I) Record values from Set 2 (Eta), Set 3(Beta), Set 5(r), Set 6(B1), and Set 7(B5). _____
- J) Page down to the 500th point and record Eta, Beta, r, B1, and B5 as above. _____
- K) Page down to the 950th point and record Eta, Beta, r, B1, and B5 as above. _____

Compare your results with the values below. (“~” indicates approximation, **BOLD** here indicates true value). The difference between the True Value and simulation Median Estimate measures the bias in the method.

	<u>Eta</u>	<u>Beta</u>	<u>r</u>	<u>B1</u>	<u>B5</u>
50 th Point (-95% Conf.) =	~150.5	~3.03	~.944	~34.8	~59.5
500 th Point (Median) =	165	4.3	~.9816	56.61	82.7
950 th Point (+95% Conf.) =	~178.1	~5.85	~.9935	~76.1	~101.1

Select New Plot. Reset to Weibull equations with Equation Label (White for Normal, Green for Weibull).

Problem 45. Accelerated Testing on Light Bulbs, Covariate Analysis, and Viewing S/N Curves

Ed Ramsey and **Paul Reisinger** performed an accelerated test demonstration for the benefit of the instructor, Wes Fulton, during a Weibull/Lognormal Engineering Workshop held in Elyria, Ohio. Their test fixture illuminated four 6-volt automotive bulbs simultaneously at a selected voltage with digital timers indicating seconds of bulb operation for each bulb. The mechanism of interest, open circuit, stopped the timer for the failed bulb. One set of bulbs was tested at 13 volts, another at 14 volts, two sets at 15 volts, and a set at 10 volts. The 10-volt test took overnight to complete. While waiting for 10-volt results, we could analyze the first three load levels for what to expect at 10 volts. The lists below give age to failure (seconds) for the first three test voltages.

Set 1: (13V)	Set 2: (14V)	Set 3: (15V)
3304	1367	570 861
2628	1409	972 664
3387	1460	713 788
2951	1949	906 666

SOLUTION: In SuperSMITH Weibull (SSW) start at the main screen and select the New / Clear icon (showing a blank page) to start a new analysis.



- A) Select the Method / Input icon and then select the Reset To Standard icon (u-turn arrow).
- B) Locate and load file **PlayTIME-Problem-45-A.w** for input data. Standard rr Weibull solutions are Eta=3222, Beta=8.809 for set 1, Eta=1631, Beta=7.498 for set 2, and Eta=822.6, Beta=6.146 for set 3.

NOTE: Low pve% for 14 Volts is unusual (only 1 in 65 chance with good sampling) but possible.



- C) Select PFEV Accelerated Test / Degradation / Covariate, and enter 1...Standard. Respond "No" if requested to change to mle. Set Engineering Variable Quantity to 1, and Find 1 Value for Slope. Engineering Variable Name is "Volts". Volts entry is 13 (set 1), 14 (set 2), and 15 (set 3).
- D) If just starting, the software asks you to select PFEV Fit Type. Select that menu item and then Exponential. The solution (top of the dialogue box) should show common slope (Beta) of about 7.067.

S/N CURVE: If SuperSMITH Visual (SSV) software is available on your computer, try the following ...

- E) Still in the SSW PFEV menu, select View XY Data Or Function, then Plot Of XY Data ... (S/N Curve) to see the "S/N" curve of life vs. load in SSV. **Return to SSW by exiting SSV.** From the SSW main screen, select the Accelerated Test / Degradation icon to return to the PFEV menu dialogue box.
- F) Select the Plot Line At New Variable Value choice, enter 10 Volts, and view the projected Weibull line for the untested value of 10 Volts. Select the PFEV icon to return to the PFEV menu dialogue box.
- G) Select Fit Type as Power, as above ... view the S/N Curve and get the projected line for 10 Volts.
- H) Select Fit Type as Arrhenius, as above ... view the S/N Curve and get the projected line for 10 Volts.

Later, we got actual data for these bulbs at 10 Volts. Let's compare the model with reality as follows.

- I) **IMPORTANT**, turn off the active solution by selecting the accelerated test icon from the main screen, then None ... Reset / Clear PFEV to return to the main screen (NOTE: Marker lines remain visible).
- J) Locate and load file **PlayTIME-Problem-45-B.w**, then compare actual to the projected Marker lines.

Set 4: (10V)	
	62101
	70369
	86358
	82363

Which model seems to forecast best? Why do you think that model works best with this data?

Wes's Comments: The stress (load) on the bulb is voltage, higher voltage is higher energy which is higher temperature. The Arrhenius model is best for effects of temperature on product life in general (Reference: Accelerated Testing by Dr. Wayne Nelson listed in The New Weibull Handbook reference section).

Problem 46. Accelerated Testing For Mars Encounter Probe (2007 Gosset Challenge Problem 1)

You are the program engineer for the new Mars Encounter Probe that will scout the Mars terrain just before Donald Trump suits up and attempts to land there (funded by Hillary Clinton). The design requires small emergency power components inside the craft only to be used for emergencies. Your test temperature chamber was first set to **320 Kelvin**, and the **24** power supply assemblies originally in the oven were cycled on/off until failure was sensed in each one. The assembly is simple, and these all fail in the same way in this test. The last of these original items failed 2 weeks ago and all were removed and replaced with **24** new power supply assemblies (same configuration) with the temperature increased to **330 Kelvin**. This group completed testing 1 week ago and another **24** were tested at **340 Kelvin**. The final data set is in a file (**WUC-2007-GOSSETCH-1.w**) including the last of these replacements which just failed, so now you have cycle count data for all three temperatures. You decide to keep the fit line solution method simple by using rank regression. **Which age-to-failure variability model fits best?** You use your engineering knowledge to guess that since temperature is involved, solution with a common value of slope using the Arrhenius model for PFEV is best. **Does the cycle capability at R99C50 exceed 55 cycles, if operation of the equipment is at the 300 Kelvin normal operating temperature? What is the standard error of fit for Arrhenius? What does the related S/N curve look like for this analysis?**

SOLUTION: Start SSW, select Method / Input, and then Reset To Standard to get Weibull rr. (Note that you should always use absolute temperature values for a covariate analysis such as an Arrhenius fit.)

- A) Open the **WUC-2007-GOSSETCH-1.w** file. You should get Eta=125.8, 82.31, 55.10, and Beta=1.991, 2.443, 2.051 for Sets 1, 2, 3 respectively.
-  B) Select the Distribution Analysis icon to find best fit model (Select No for change each time). For Set 1 LogNormal is best, also for Set 2, and for Set 3. At Main Screen change Equation Label to LogNormal and you should get Med=95.52, 65.75, 42.17 and SigF=1.911, 1.686, 1.852.
-  C) Select the Accelerated Test icon, and select Standard PFEV. Enter 1 for Engineering Variable Quantity (temperature only changing between data sets). Select Find 1 Value for Slope. Enter Kelvin for Engineering Variable Name. Enter respective temperatures ... 320 for Set 1, 330 for Set 2, 340 for Set 3. You should see the PFEV dialogue box (no results yet).
- D) Select F...Fit Type and enter C...Arrhenius to get results of 2.63Units (Cycles) for Med standard error. Select L...Plot Line At New Variable Value and enter 300 (normal operating temperature, Kelvin). You should see the LogNormal plot with SigF=1.814 for all fit lines and a Marker Line with Med=244.8 and SigF=1.814.
-  E) Select the large plot, return to the main screen. Select the precise reading icon and Line 1 (Marker Line). Select B1 to get 61.26 cycles capability (no associated confidence). The capability would be different if mle-RBAmu was used instead of rank regression. Yes, capability exceeds 55 cycles.
-  F) Use the <ESC> key to return to the main screen, select the PFEV icon and then Y...View/Save XY Data Or Function With SSV (S/N Curve), and select P...Plot Of XY Data to view the related S/N curve. Exit out of the SSV program to return to SSW when finished looking at the S/N curve.

Problem 47. Six Sigma Quality Control for a Precision Pump (2007 Gosset Challenge Problem 2)

Your boss comes from General Electric. He wants you to use your **Six Sigma** background to evaluate plating thickness on a precision pump component used in thousands of pumps. The measurements [in.] of plating thickness are given in file **WUC-2007-GOSSETCH-2.w**, critical thickness tolerance is **.019 to .036** inches. Requirement is less than **3.4 parts per million (PPM)** rejection for process design. **Are you okay?**

SOLUTION: Start SSW, and Reset Method/Input to standard (Weibull with rr fit line solution).

- 
- A) Open the **WUC-2007-GOSSETCH-2.w** file. With rr fit you get Weibull solution of 39 points with $\text{Eta}=0.02976$ and $\text{Beta}=24.31$. High Beta is expected with this material property variability.
 - B) Return to the Main Screen. Plating thickness cannot be negative, so you stick to options of only Weibull 2-parameter (Weibull-2p), Weibull 3-p (+ t0 only), and LogNormal distributions, which cannot go negative.
 - C) Select the Distribution Analysis icon to compare between Weibull-2p, Weibull-3p, and LogNormal. The best pve% for Weibull-2p so no change is required.
 - D) Return to the Main Screen. Select the Plot / Report / Table Options tab (over Active Results) and then choose V... View Additional Probability Plot With SSV.
 - E) Select Q...QLC Quality Limit Comparison For Ppk or PPM. After selecting D...Double Limit, enter 0.019 for Lower Limit and 0.036 for Upper Limit. Make sure the "PPM Type Of Results is set to "Yes". Select "A...Activate". You should get a plot indicating 18 PPM are expected to be lower than the low limit and practically none (" $>UL=9E-30$ ") expected higher than the high limit.
 - F) The 18 PPM exceeds the 3.4 maximum requirement. Tell the boss plating thickness is outside tolerance on the low side using the Weibull distribution.
 - G) Exit out of SSV, return to SSW. You should see the Probability Plot menu (if not then repeat steps A – D). If you want to see Ppk, select QLC again from the Probability Plot menu, change the Ppk Type Of Results (either with the 1.5 shift or without), then Activate in the QLC menu to get the plot with Ppk results.

*Wes's Comments: Using the generally accepted normal distribution suggests practically none are expected below the low limit and less than 1 PPM are expected to exceed the high limit. These incorrect results indicate that plating thickness is acceptable when it is not. The normal distribution allows negative values, which cannot happen with plating thickness, shaft run-out, pipe wall thickness, etc. Better results come from using a more appropriate distribution like Weibull. Thanks go to **Dennis Keisic** for this insight with respect to Six Sigma.*

NOTE: See Figure 12 for the Weibull to Six Sigma conversion chart from Dennis Keisic.

Problem 48. Life Remaining on a Relief Valve (2007 Gosset Challenge Problem 3)

Money and schedule are tight. There are many new projects underway at your chemical plant and you want to stretch your assets as far as possible. You have ten special-order high-temperature relief valves in a variety of different service applications throughout the plant (not in the same system). It takes many months before a new order can be filled. The Weibull model fitted to the laboratory time-to-failure data for these valves indicates **Eta = 72 months and Beta = 2.6**. The recommended **scheduled replacement interval is 24 months** of service. They all currently have **24 months of service and none has failed**. The reliability for 24 months of service is **94.41 percent** using the original Weibull model. You want minimum **reliability of 95 percent** (no worse than original requirement) for continued operation. **How much longer can you expect to keep these valves in service still meeting 95 percent reliability? How much longer can you expect to run them in a second extra service if they all survive the first extra service?**

SOLUTION: Start the SSW program.

- 
- A) Select the Calculator icon.
 - B) Choose the Life Remaining option.
 - C) Select Distribution, set Distribution Equations to Weibull, set Eta to 72, and set Beta to 2.6. Choose "Activate" from this menu to return to the Life Remaining menu.
 - D) Select Present Age and set it to 24.
 - E) Select Allowable Failure Probability [%] and set it to 5 (so Reliability will be 95%).

- F) **Result #1:** Expected Remaining Life Usage is somewhat more than 6 months.
- G) After 6 months of additional zero-failure service, all of them will have 30 months of age. Select Present Age again and set it to 30.
- H) Reliability changes (default dependent variable). Select Probability... and set it again to 5%.
- I) **Result #2:** Expected Remaining Life Usage after 30 no-failure months is an additional 5 months.

Wes's Comments: Using equipment past that equipment's rated lifetime is often needed. When operators find parts operated already to their useful life, those people may be reluctant to use them further. In this example we can see that such operation is acceptable given sufficient background knowledge.

Problem 49. Nelson Recurrent Events (RE) Analysis – Simplest Data Format

The SuperSMITH Visual program has a simple implementation of Nelson RE analysis developed by Dr. Wayne Nelson. It is included in the Visual program, and not the Weibull program, because the resulting plot is not a probability plot. Standard RE analysis is non-parametric and does not require special probability scaling.

Suppose you have 3 pieces of equipment with serial number identification SN007, SN008, and SN015. In **SN007** two repairs have occurred at ages [hours] **391** and **643**, and it is **967 hours old now**. In **SN008** two repairs occurred **almost together** at age **622**, and it is **636 hours old now**. In **SN015** one repair occurred at **485** hours, and it is **923 hours old now**. **What is the mean cumulative function (MCF) of repairs vs. age?**

As background, for standard Nelson RE every data value must also be identified with a piece of equipment ... "equipment" may be a person or a project in the broader sense. The Nelson RE analysis is age-based and so the data must be put into overall age order before numbers are crunched. The software can put the data in order for you if it is not already in age order. In SuperSMITH software, the first number (X-Value) in the data entry is ALWAYS time or age when working with Weibull-Engineering type data (age) or Crow-AMSAA type data (operating time) or Nelson RE data (age). The second number (after the "x" used as a delimiter) for SSV is the Y-Value. For Nelson RE that Y-Value represents the number of events occurring at precisely that age value (usually 1 for one repair). The Nelson RE input Y-Value is NON-CUMULATIVE data, but the resulting analysis will be in CUMULATIVE form! Also, the Y-Value could be cost at that age value when performing cost analysis. Every individual equipment piece represented by Nelson RE analysis MUST have one suspension entry at the last known age for that equipment. Before suspension that same equipment may have any number of event (or cost) data entries. To enter a suspension, use zero (0) as the Y-Value or put a ">" sign in front of the age. The identifier text is put at the right-hand side using the underline symbol ("_") as the delimiter. The above data for entry into the software without putting in overall age order is:

```
391x1_SN007
643x1_SN007
967x0_SN007
622x2_SN008
636x0_SN008
485x1_SN015
923x0_SN015
```

SOLUTION: Start the SSV (Visual not Weibull) program, and select the blank page icon to reset for a new analysis.

- A) Locate and load **PlayTIME-Problem-49.v** file to get the plot.
- B) NOTE: This is **not yet** a Nelson RE analysis. This is a plot of the beginning input data.
- C) Select the Nelson Recurrent Event (RE) Analysis icon on the main screen to get the RE menu.
- D) Verify that Plot Display Type is set to STEP FUNCTION for both Confidence and MCF.
- E) Select Activate Analysis or select the green check mark ... a message about order should appear.
- F) Select Order By X-Value to put the data entries into overall age sequence and solve.
- G) You should get a plot indicating MCF value (on vertical scale) of about 1.833 total repairs/equipment starting at age 643 (on horizontal scale).



Problem 50. Nelson Recurrent Events (RE) Analysis – Another Data Format

There are many different formats possible for Nelson RE data. The data below from Dr. Wayne Nelson is provided in the SuperSMITH HELP text file **SMITHHLP-BENCHMARKING.TXT**, as Benchmark 9V (Format #2). This file is in the SMITHDAT subdirectory with the original program files,. The identification for this input data is now on the left and values afterward are mileage at repair. The trailing “+” indicates suspension. What is the mean cumulative function (MCF)?

Car#	Mi(1)	Mi(2)	Mi(3)
024	7068	26744+	
026	28	13809+	
027	48	1440	29834+
029	530	25660+	
031	21762+		
032	14235+		
034	1388	21133+	
035	21401+		
098	21876+		
107	5094	18228+	
108	21691+		
109	20890+		
110	22486+		
111	19321+		
112	21585+		
113	18676+		
114	23520+		
115	17955+		
116	19507+		
117	24177+		
118	22854+		
119	17844+		
120	22637+		
121	375	19607+	
122	19403+		
123	20997+		
124	19175+		
125	20425+		
126	22149+		
129	21144+		
130	21237+		
131	14281+		
132	8250	21974+	
133	19250	21888+	

SOLUTION: Start the SSV (Visual) program...

- A) Select the blank page icon to reset for a new analysis.
- B) Select the dice icon for Special Data. Select Special Data Set. Choose “9V – Nelson Recurrent Event Plot”.
- C) Result: MCF plot indicating about 0.303 total repairs/equipment starting at age 19250.
- D) NOTE: The Nelson RE icon is selected automatically for you.



Problem 51a. Test Substantiation / Test Planning

The Weibull Workshop introduction video includes an example of test planning for a bearing. Data from this test-planning example is also presented in the handbook and referenced again here to show how software provides the answer.

Reference The New Weibull Handbook Section 6.9. on “Weibayes” Substantiation Test Plans

Suppose a ball and roller bearing follows a Weibull distribution for age to failure with respect to an unbalance failure mode, with Weibull slope (Beta) = 2, and Weibull characteristic life (Eta) = 500 hours. A new design has been initiated, and three (3) redesigned units are available for testing. Now the question is: **How many hours should each system be tested to demonstrate a minimum 500 hours for its characteristic life (Eta)?** You should try both a zero-failure test plan and a zero-or-one-failure test plan (allowing one failure).

SOLUTION: Start the SSW (Weibull) program...

- 
- A) From the main screen, select the Test / Usage Requirement icon (showing a beaker).
 - B) Confirm that Test Type is “Time Or Life Or Age”. Then select “Find Test Time Length”.
 - C) Enter Weibull Slope Beta of 2, and afterward you should be taken to the test options menu.
 - D) Verify that Allowable Occurrence (Failure) Quantity is set to zero (0) to start.
 - E) Select Confidence [%] and set it to 90.
 - F) Select Demonstrate Minimum Character. Value. (B-Life and Reliability automatically reset)
 - G) Select G...Character. Value Goal and set it to 500.
 - H) Select N...Test Item Quantity and set it to 3.
 - I) You should get the result of approximately 0.876 for *Test Time Ratio.
 - J) You should get the result of approximately 438 for Test Time For Each Item.
 - K) Change Allowable Occurrence Quantity to 1.
 - L) You should get about 1.27 Time Length Ratio and 638.5 Test Time For Each Item.

Problem 51b (51a Continued). Consumer Risk and Producer Risk

For the same bearing test as above, we want to evaluate the consumer risk and producer risk. The consumer risk is the probability the buyer might accept an unacceptable product. The producer risk is the probability the seller might reject an acceptable product. Suppose you estimate that the actual Weibull characteristic life of the new design is twice that of the old design. **What are the consumer risk and the producer risk values for this test design?**

SOLUTION: In the SSW program.

- 
- A) From the main screen, select the Test / Usage Requirement icon (showing a beaker).
 - B) Use the same inputs as in Problem 51a above (but for zero failure testing ... no failures allowed).
 - C) The C...Confidence menu item indicates CONSUMER RISK [%] = 10.
 - D) Select the Q... menu item and enter 2 for r, the ratio of actual equipment Eta to the Eta goal.
 - E) The Q... menu item indicates PRODUCER RISK (r of 2) [%] = 43.77.
 - F) Now try allowing 1 failure and compare the difference.

Wes’s comments: Allowing a failure increases the required test time for each test unit. However, it is possible in the case of highly reliable equipment to actually end up with a shorter test AND lower producer risk by selecting a zero-or-one failure test. If test items are tested in sequence and all the test items before the last one pass the test, then testing of the last one is not necessary. With respect to this bearing test the consumer risk is the same (10%) for the zero-or-one failure test, but the producer risk significantly lowers (and improves) to only 26.12 percent.

Problem 52. Weibull Library

The most important thing! You want to start and maintain a Weibull Library, so you can use the powerful Weibayes, 1-parameter Weibull, technique for small samples. There is capability in the SuperSMITH software to help. Get started by generating a library database item for the **18WHEELR.W** data file.

The device under analysis is a steering link used in a vehicle steering system. The root cause failure mechanism is fatigue fracture. The usage is heavy duty and the environment is normal temperature. You also want to keep the item part number (P/N 822347) with the data base item.

You are going to add each new Weibull analysis summary item to an existing file after it is started. Each stored item should have the item labels stored with it each time. If you have an analysis with multiple data sets, you want each data set analysis to be on its own separate line (row). The n/s information about total data quantity and total suspension quantity is automatically included. Use **WLIBRARY.TXT** for the filename of the Library Total Output file.

NOTE: The files **LIBDEVCE.TXT**, **LIBEQUIP.TXT**, **LIBCAUSE.TXT**, **LIBUSAGE.TXT**, and **LIBENVIR.TXT** all should be available in a system subdirectory to allow for drop-down selection of some of the library entries. If you installed the setup data example files earlier (18WHEELR.W, FATIGUE.W, etc.), then the LIB*.TXT files should have been installed also. They are available on the program CD in the SMITHDAT subdirectory. Edit these LIB*.TXT files to customize them as needed for your possible library item entries.

SOLUTION: Start the SSW program...



- 1) At the main screen, select Open / Get / Import (open file folder icon), bring in **18WHEELR.W**
- 2) Select the Library Save icon (showing a book) and you should see the library setup options
- 3) Select B...Weibull Library Item Save, and toggle it so it reads ... = To File
- 4) Select C...Save Library Label Row, and toggle it so it reads ... = Yes
- 5) Select D...Save Type, and toggle it so it reads ... = Add To Library Output
- 6) Select J...Each Set On Same Item Line, and toggle it so it reads ... = No
- 7) Select T...Library File Name Input to get the Library File Name input menu
- 8) Select U...Library Total Output File Name and set it to **WLIBRARY.TXT**
- 9) Select A...OK ... Activate to return to the library setup options menu
- 10) Select A...OK ...Activate To File and you should return to the main screen



- 11) Select the Save Active Data / Results icon (closed folder) and save to **18WHEELR.W**
- 12) Select Yes to Delete Former File ... and you will be shown the library item input options
- 13) Select B...Type Of Device, enter Linkage in the text box, and select the green checkmark
- 14) Select C...Type Of Equipment, enter Steering System (or select from drop-down options)
- 15) Select D...Type Of Occurrence mechanism, enter Fracture (or select from drop-down)
- 16) Select E...Type Of Utilization, enter Heavy Duty (or select from drop-down)
- 17) Select F...Type Of Environment, enter Temperature – Normal (or select from drop-down)
- 18) Select I...Comment #1, #2, #3, #4 Input to get the comment input menu
- 19) Select I...Comment, and enter P/N 822347
- 20) Select L...Library Comment Title Edit, Select 1, enter Part Number, select A...OK, A...OK
- 21) Back at the library item input options menu, select A...OK to activate
- 22) At the Save Library Item To dialogue box, you should see **WLIBRARY.TXT** as the default file name for library item storage ... select OK or SAVE ... your item should be saved. **Select the Book icon on the Main Screen, then select N...None ... Reset (to clear Library Save).**

To view the file, import it into a spreadsheet program like EXCEL™ by Microsoft®. Start your own Weibull library in a similar way and add to it for each analysis you do. To de-activate the library item generator, simply select the Library Item Save icon (book) from the main screen and use the B... option to turn off. Remember to back-up the library file so that a software or hardware issue will not destroy all of your hard-earned work! Further saves to the library are easier, your previous inputs are saved.

Problem 53. Forecasting: Abernethy Risk vs. Crow-AMSAA, With Same Warranty Data Input

Compare forecasting with Abernethy Risk and forecasting with Crow-AMSAA (CA). Let's use Abernethy Risk, then CA on the same data. Warranty Data Format allows dual analysis like this. It has enough information to do either a Weibull or a CA model (the units of age/time will be in equipment months if no adjustment is made). We want to see the difference between results from these two models.

SOLUTION: Start the SSW program, and at the main screen, select Home group, New / Clear.

- 1) Verify the Equations Label, on the main screen, shows "Weibull" (on green background)
- 2) Select Sample (dice icon), Special Data Set, 10A – Warranty Format For PT 27.
- 3) Verify M...Method For Warranty Import = Inspct insp1. Select W...Format: Warranty.
- 4) In the next menu, Select "B...Format = MOP x MIS" (or x MOA) with asterisk symbol in front.
- 5) Select "A...Activate" here for the Warranty Input menu. Verify that L...Limit Correction = **No**, and Renewal = **Yes**, and Warranty Limit For Input Age = **No**.
- 6) Select "A...Activate" for the plot. Eta / Beta should be 101.9 / 0.629. Return to the main screen.
- 7) Select Tools, Abernethy Risk. Select icon "Reset To Start". Select U...Usage, 1...Enter Single Rate, and enter the value of 1 (for 1 month per calendar month). Set R...Renewal = ***YES**.
- 8) Select OK (checkmark) to generate the forecast table showing you should expect to accumulate about **400** additional occurrences from now until 24 months from now.
- 9) Select the bottom icon on the right-hand side of the output table to view results plotted in SSV.
- 10) Return to the SSV main screen, select the Save Active Results icon and enter **FORECASTRISK.V** for the filename.
- 11) Exit SSV and also exit SSW, then Start SSV again. Select Home group, New / Clear.
- 12) Select Sample (dice icon), Special Data Set, 10A – Warranty Format For PT 27.
- 13) Select W...Format: Warranty here, and you will be given a new menu with choice of format.
- 14) Select "B...Format = MOP x MIS" (or x MOA) with asterisk symbol in front.
- 15) Select "A...Activate" here to get the Warranty Input menu. Verify options L...Limit Correction = **No**, and Renewal = **Yes**. Select A...Activate to generate the Crow-AMSAA (C-A) plot for this warranty-format data. Verify C-A lambda = 0.285 and C-A Beta = 0.745 with default **IEC** method and **Automatic** equation selection ("Crow/IEC/A" should display on the plot).
- 16) From the SSV main screen, Tools, select the Fulton Risk icon (same as the SSW Abernethy Risk icon). Select menu item N...Present Quantity Active and enter 1200. Set R...Renewal = **YES**. Select menu item U...Usage, choose 1...Enter Single Rate, and enter 1.
- 17) Select A...Activate to generate the C-A forecast table showing that you should expect to accumulate about **492** additional occurrences from now until 24 months from now.
- 18) Select the bottom icon on the right-hand side of the output table (View Results With SuperSMITH Visual Plot) to see a plot of accumulated occurrences vs. months-into-the-future.
- 19) Select the plot to return to the SSV main screen, select Home group, Save Active Results icon and enter **FORECASTCA.V** for the filename.
- 20) Close all SSV open instances (use task bar if needed). Start SSV again. Select New / Clear icon.
- 21) Bring in files **FORECASTRISK.V** (Weibull) and **FORECASTCA.V** (C-A) so they overlay on the same plot. Note that the C-A forecast is somewhat higher than Weibull. Which one do you think is more accurate? After you are finished, exit the SSV program without saving.

*Wes's comments: Extrapolating **Crow-AMSAA** this way cannot account for subtle effects like maintainability rhythm. Weibull analysis can. Additional options for Weibull **Abernethy Risk** account for increasing/decreasing population, renewal, planned replacement, and so on. Typically, a proper Weibull forecast can be much more detailed as compared to a Crow-AMSAA forecast.*

Problem 54. Which Distribution Model Fits Best For 2-Dimensional Manufacturing Miss-Distance?

Want some exercise going back and forth between SuperSMITH Weibull and SuperSMITH Visual? We knew you would...

What is the distribution of radial error when trying to locate a feature like a drilled hole? Radial error is the shortest distance from the finished hole center to the ideal hole center (on the drawing) without regard to error in the displacement angle. Suppose you have a robotic manufacturing system for making the holes and suppose the “N-S” axis error and the “E-W” axis error are each described well with the normal distribution. If your standard deviation of single axis error is 0.005 (inches or centimeters as long as you are consistent), what is the distribution of 2D radial hole-placement error?

SOLUTION: Start SSW, at the main screen select Home group, select New / Clear (blank page icon).

- 1) Select the Equations Label (Green for Weibull), scroll down to select Z...Additional [Dimension Data Equations], and then select N...Normal. Equations label will change to Normal, and the Equations Label background will turn white.
- 2) Select Sample icon (showing a pair of dice). Verify U... Type = Monte Carlo.
- 3) Select D...Distribution Parameter. Enter zero (0) for Mean (Mu) and .005 for Std. Dev. (Sig),
- 4) Select N...Point Quantity and set to 500.
- 5) Select the Sample To File Only (Not Sorted) icon (file folder above sample menu), and save simulation results to filename **X.NOR** ... select Yes if asked to Delete Former File + Save New.
- 6) Repeat steps 2-5 **EXCEPT** this time save simulation results to filename **Y.NOR**.
- 7) From the SSW main screen, select Method / Input, choose Reset To Standard, answer “Yes” if asked to Change To Weibull. The Equations Label background should turn green.
- 8) Select the “dice” icon again and change the Eta and Beta and Point Quantity values to reasonable numbers (e.g. like Eta of about 10, Beta of about 2, and N...Point Quantity to 10 or so).
- 9) Exit from SSW (no need to Save Active Data ...) and then start the SSV program.
- 10) From the SSV main screen, Home group, open file **X.NOR**, select A...As X-Value, and you should see a horizontal line of points along the X-axis on the X-Y plot.
- 11) Open file **Y.NOR**, select A...As X-Value, enter 2 for Paste Into Set to put into set # 2.
- 12) Select the Analysis group, Transform icon showing a curved arrow pointing to F(a), and enter No to “Save Present Data Before ...?” question if asked.
- 13) Choose option S...Swap Axis, the plot should display the data vertically.
- 14) Select Analysis and the Transform icon again (enter “No” to “Save Data” question if asked).
- 15) Select option 5...New X For Set B = Y From Set A, and enter Yes (‘A’ X-Value Same As ‘B’ X-Value), enter 1 (Set # For A), then enter 2 (Set # For B) ... you should a lot of randomly crossing lines in the center of the resulting plot (this is expected).
- 16) Select Analysis and the Transform icon again (enter No to Save Data question if asked).
- 17) Choose option A...Rectangular To Polar (this plots vertical Radius vs. horizontal Angle).
- 18) Select Analysis and the Transform icon again (enter No to Save Data question if asked).
- 19) Choose option S...Swap Axis, and Radius and Angle should swap places (NOTE: Analysis by SSW of a SSV file only evaluates X-Value variability, and we want to analyze radial error).
- 20) From the SSV main screen, select Save Active Data, then save to filename **RADIUS.V** (answer Yes if asked to Delete Former and Save New).
- 21) **Exit from SSV** and then **start the SSW program**. Select Home group, New / Clear icon.
- 22) Open the file **RADIUS.V** just saved, and select the small plot to enlarge it. Does the fit look good? How does this match with your expectations (if any)? We expect a Weibull Beta near 2.

Wes's comments: Theoretical analysis indicates the Rayleigh distribution should fit (Weibull with Beta of 2). Our results are a little different due to the MC simulation variability but the fit should be mostly okay (pve% >10), and Beta should be reasonably close to 2. In this exercise, we just performed a Monte Carlo simulation, my favorite technique, to get those “miss-distance” values.

Problem 55. mle Interval Solution

Reference The New Weibull Handbook Section 5.8.5.

From reading Dr. Bob's handbook and from using this PlayTIME™ tutorial, you should be already familiar with some of the following data types (described for use in reliability analysis):

“Occurrences” – age values for items that have failed by the failure mechanism of interest (FMOI).

“Suspensions” – age values for items that could fail by the FMOI but have not failed yet, or that have failed for a different reason.

“Discoveries” – age values for items failed by the FMOI found at the very first inspection. No sensory information indicates the exact failure time. An example of this would be a housing crack that does not degrade performance or cause leakage, so is considered failed and must be fixed before further usage.

“Intervals” - age values for items failed by the FMOI but found at a later inspection (not the first).

Some data sets may have all of those mixed together. Let's use the inspection data originally from the INSPECT.W file, but summarized roughly into these four different data types for Interval mle solution. NOTE: Commas separate the data entries below with age before “x” and quantity after “x”:

<1x4, 1x1, <2x2, 2x1, 3x1, 5x1, >5x1, >6x1, 8x1, >9x1, >13<16x2, 25x1, 28x1, 61x1

DATA SOURCE: Noisy failure in a new unit (one month of age) startles the equipment operator. The operator notifies quality control (QC). The QC supervisor orders an inspection of other units with age of one. Upon inspection, four more one-month-old units already had failed but without sensory indication. These four dormant failures occurred before age one (“<1x4”), and were discovered by the inspection ordered due to the failure at age one (“1x1”). So the first data entry above represents four “Discoveries”. Shortly after that, one of three new two-month-old units also failed noisily. Upon inspection, two other two-month-old units exhibited similar but dormant failure (<2x2). These were found only due to the noisy failure at age two (2x1). More data gathering produces failures (3x1, 5x1, 8x1, 25x1, 28x1, 61x1) and suspensions (>5x1, >6x1, >9x1) plus two other failures (>13<16x2). This last “Interval” data entry means that two units were both inspected at 13 and found to be failure-free, but at the next inspection, 16, both were found failed. No noisy indication happened in those. We only know that failure occurred sometime between 13 months and 16 months in both of them. How do we get a Weibull plot?

SOLUTION: In the SSW program ...

- 
- A) Select New (blank page icon) to clear the data grid and the results.
 - B) Select Method/Input and choose Reset To Standard for Weibull rank-regression (W/tr).
 - C) Locate and load the **PlayTIME-Problem-55.w** file.
 - D) You should get the plot with Method automatically changed to interval mle solution.
 - E) The solution shown on the plot should indicate Eta of 8.53 and Beta of 0.553. NOTE: If Beta is different, select Method/Input and change Fit Method from mle-RBA (modified) to just “mle”.
 - F) The plot should show “W/mle/intv5” for the solution method and “.../3/6/2” in the parameter box representing data quantity for .../suspensions/discoveries/intervals.
 - G) Select the Marker Line icon and choose Set 1 (current solution) to return to the plot.
 - H) Delete the Set 1 data column. (Only the Marker Line should remain on the plot).
 - I) Select the Method/Input icon then select Reset To Standard again.
 - J) Retrieve the **INSPECT.W** file for regression solution, and notice the fit-line difference.
 - K) Select the Method/Input icon, and change Input Data Format + Inspect Type to “...Inspect(1)...”.

Wes's comments: With Inspect Option #1, there is very good agreement between the regression solution and the mle/intv5 Marker-Line solution. Dr. Todd Marquart has shown reasonable agreement between Inspect Option #1, Probit, and Interval mle solution methods for the same original data. We've obtained good solutions in the past by using the Inspect Option #1 regression method on inspection data. However, the Interval mle solution is best practice when the original data includes discovery and/or interval types.

Problem 56. Degradation (Deterioration) Modeling

The PFEV (parameter as function of engineering variable) model in the SSW software for accelerated testing data analysis can model degradation data as well. The math is the same. It is also called “Covariate” analysis; different data sets represent measurements grouped by different values of some variable. This type of study is called “Degradation” analysis when measurements get worse with time. Let’s take an example where inspections provide data on storage vessel wall thickness.

For liquid storage vessels the wall thickness is crucial. Vessels can lose wall material when they store caustic agents or liquids at high temperature. Here are storage vessel wall thickness readings (mils) on the same equipment at 12 locations for different inspection times:

Inspection at 1000 Days: 265, 310, 321, 350, 289, 292, 274, 342, 266, 284, 298, 307

Inspection at 2000 Days: 232, 285, 291, 341, 272, 274, 253, 316, 237, 263, 280, 284

Inspection at 2500 Days: 183, 229, 230, 242, 216, 217, 204, 235, 184, 214, 221, 228

Minimum acceptable thickness is 120 mils. Find the longest inspection time (Days) where we expect 99.9% chance minimum of acceptable thickness (same as 0.1% probability of unacceptable thickness)?

SOLUTION: In the SSW program, starting at the main screen ...



- A) Select the blank page icon. Then select the Method / Input icon, and choose Reset To Standard.
- B) Select the Equations Label showing “Weibull” on a green background; change to “LogNormal”.
- C) Locate and enter the **PlayTIME-Problem-56.w** data file to get the probability distribution plot.
- D) The lognormal plot solution values Median:SigmaFactor (“Med:SigF”) should be 298.7:1.101, 275.8:1.123, and 216.1:1.096 respectively (with **rr** fit method).
- E) Select the Label/Style icon and change the horizontal scale units (in parentheses) to “mils”.
- F) In the Tools group, select the icon for PFEV Accelerated Test, Degradation, and CoVariate Analysis.
- G) Choose Standard ... PFEV; enter 1 for Engineering Variable Quantity.
- H) For this example, select Slope Results Type as “Find 1 Value For All”. For Unites enter “Days”..
- I) For Variable Value enter 1000 for Set 1, enter 2000 for Set 2, and 2500 for Set 3.
- J) You should see the PFEV dialog box.
- K) Select PFEV Fit Type from the dialog box menu items, and choose Exponential to get the solution at the top of the dialog box. The Med Standard Error for this fit should be about 27.9 mils. The common SigF for this rr fit type should be 1.106575.
- L) Let’s look at results for different inspection times (in Days). Select menu item Plot Line At New Variable Value and enter 3800 for Engineering Variable 1. A Marker Line will be added on the plot for the expectation at that inspection time. Select the PFEV icon again to return to the PFEV dialog box. Repeat this operation entering 4000 and 4200 and 4400 in turn. You should have 4 Marker Lines showing on the plot with higher values of inspection time showing smaller thickness. Change Marker Line color if needed to better identify which lines go with which potential future inspection.
- M) For each Marker Line in turn, use the Precise Reading icon, choose Enter Datum ... Get %, and input 120 to find the expected occurrence probability. Which of the four inspection times (3800, 4000, 4200, 4400) indicates only about 0.1% will occur?

Wes’s Comments: The Line 3 solution (4200 Day inspection time) gives about 0.1% probability for 120 mils thickness or less (99.9% probability of having 120 mils or more thickness). There are two general ways data can be organized for degradation analysis. This case study involves data from inspections. However, you can encounter different degradation-type data. Another format for degradation analysis takes data from sensors. If you had wall-thickness sensors generating data when a particular thickness limit is reached, then you would plot degradation differently from the approach used for this case study. For an analysis using sensors instead of inspection, each data set represents the variability in number of days it takes to reach a specified thickness. The software can then find the relationship between the thickness variability and how it migrates to lower values with time.

Problem 57. One-Shot Devices Like Rockets, Missiles, and Munition

You could use the binomial model for one-shot devices, however the requirements with the binomial lead to larger and more expensive tests. A better option when available might be the Crow-AMSAA (CA) model for trending the results. Here is an example of doing that using the SuperSMITH™ Visual software.

Suppose you have rocket firing data like this:

2x1
5x2
15x3
18x4
26x5
33x6
36x7
62x8
70x9

The number on the left in each data entry above is the accumulated quantity of firings. The number after the “x” is the accumulated quantity of unsuccessful firings (or “occurrences” in SuperSMITH terms). The definition of an unsuccessful firing is subjective but could indicate either that the launch was not completely satisfactory or instead it could be that the mission was a total failure or somewhere in between. Defining what is considered an occurrence is the job of the program manager with possible direction from a customer. **We want to show a nominal 90% reliability (50% confidence).** Let’s put the data into the SSV (Visual) program.

SOLUTION: In the SSV program, starting at the main screen, Home group ...

- 1) Select the Open / Import icon and bring **ONE-SHOT-DEVICES.v** from the provided data files.
- 2) In the Tools group, select the Crow-AMSAA icon to get the CA menu. Make sure that the Method = IEC, and that no special options are selected.
- 3) Select the green checkmark to activate the analysis and get the CA plot.
- 4) The CA Lambda value should be close to 0.696 and the CA Beta should be close to 0.602.
- 5) Return to the CA menu (Tools group, CA icon), then select the “S...SURV Transform”.
- 6) You should see a plot of SURV (launch reliability in this case) vs. the cumulative firing quantity
- 7) Note that the reliability is increasing because the CA Beta is less than one. Read the cumulative reliability by looking at the last plotted point (the Y-value = 0.87, that is 87% reliability). This does not meet our goal of 90% ... but wait. Since reliability has been increasing all along, then our actual current (instantaneous reliability) should be better than what the cumulative shows. Let’s look at the instantaneous reliability.
- 8) Return to the CA menu. Toggle the “I...Instantaneous Line: ...” option so it reads “*YES”, then select the green checkmark to see the CA plot with the added instantaneous line (Set # 3) above the cumulative fit line.
- 9) Read the last Y-value in Set # 3. The “0.92...” indicates 92% reliability, meeting our goal.
- 10) EXTRA CREDIT --- Try the Binomial option in the SSV calculator to Find “N...Trial Quantity” needed to show 90% reliability with 50% confidence. Here’s how to do that ...
 - a. Select the Options group, Calculator icon.
 - b. Select “N...Trial Quantity” for what to find. The binomial calculator menu appears.
 - c. In the binomial calculator menu, select “C...Cumulative Probability” and set the % to “50”. This will give 100%-50% for the results confidence of 50%.
 - d. In the same menu, select “E...Event Quantity” and set to “9” (same as the data above).
 - e. In the same menu, select “P...Probability Of 1 Event ...” and set % to “10” for R = 90%.
 - f. The menu then shows that the trial quantity needed is 97 test firings using the binomial if we allow 9 failures maximum, but our CA method only needed 70 test firings to show the same results. The upward CA trend in reliability helped our cause.

Section IV - Benchmark-Baseline Problems

Accuracy and uncertainty with good and bad data are covered in detail by Dr. Abernethy's standard reference, "The New Weibull Handbook©." Many software programs and textbooks do not recommend best practices among all of the possible alternative methods and do not benchmark case studies to verify that their methods are correct. SuperSMITH and The New Weibull Handbook do both. In the software, select the **HELP** icon (shows "?" in blue) to display the HELP area of SuperSMITH. Then select **Product Information** and then **Benchmarking**. Follow the instructions in that section to review several hallmark life data case studies.

Many organizations and businesses compare SuperSMITH results to their own in-house Weibull program or they compare SuperSMITH to other commercial Weibull computer programs. SuperSMITH is often used to validate other software. Baseline details can identify deficiencies in software such as incorrect choice of regression method, incorrect guidance to use only pure maximum likelihood for every data set, blind advocacy of Weibull estimates based on very small samples, and obsolete confidence interval estimates. These comparisons and validity checks are appropriate since some recommendations and corrective action can heavily make use of SuperSMITH results.

Confidence must be established that the software provides accurate answers. To assist in this effort, the baseline problems were taken from hallmark textbooks in life data analysis referenced in The New Weibull Handbook and made available in the software. SuperSMITH Help describes these baseline problems. Some of the data sets are available under the dice (sampling) icon with selection "Special Data Set ...". Where there is any question about differences between SuperSMITH and other computer programs, these baseline problems should be run on both programs to provide comparison. If differences are observed and cannot be explained please call Wes Fulton asking for assistance.

Figure 11 below illustrates the icon layout for the SuperSMITH Visual main screen. The Visual program is a standalone companion to the Weibull software for general auxiliary plotting purposes. Crow-AMSAA trending analysis is one of the main capabilities of the Visual program along with function viewing, transforms, and contour generation. The SuperSMITH HELP file presents a description of each selection on this main screen. A screen-shot of the Ybath™ main screen is provided in the section on Ybath advanced mixture analysis.

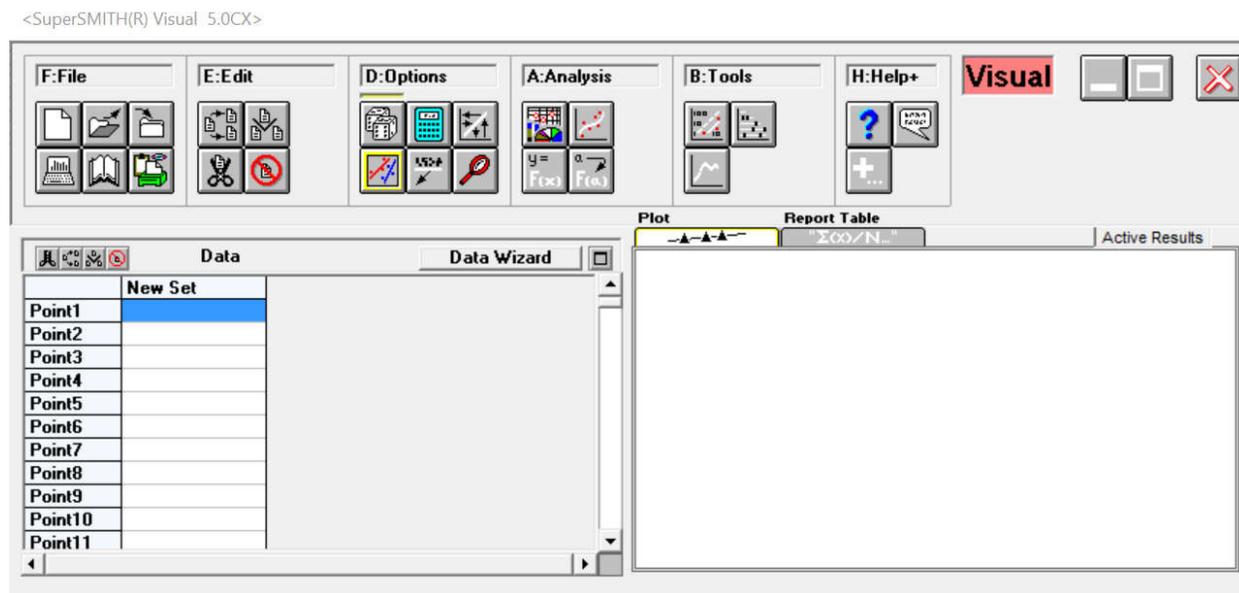


Figure 11: New SuperSMITH Visual Main Screen Layout

Normal Distribution									Weibull	
Double Sided				PPM / DPMO	Single Sided				Single Sided (BXX)	
Short Term Capability (actual Z tables)		Double Sided			Short Term Capability		Long Term Capability with Shift of 1.5 Std Dev			
Sigma	Pp	% Non- Coforming	PPM		Sigma	Pp	Sigma (with 1.5)	Pp (with 1.5)	Lower	Upper
-0.5	-	-	-	* 691,462 *	- 0.5	-	1.0	0.33	-	-
0.0	0	-	-	500,000	0	0	1.5	0.50	50	50
0.3	0.10	76.4177	764,177	382,089	0.3	0.10	1.8	0.60	38	62
0.5	0.17	61.7076	617,076	308,538	0.5	0.17	2.0	0.67	31	69
0.6	0.20	54.8506	548,506	274,253	0.6	0.20	2.1	0.70	27	73
0.9	0.30	36.8120	368,120	184,060	0.9	0.30	2.4	0.80	18	82
1.0	0.33	31.7300	317,300	158,650	1.0	0.33	2.5	0.83	16	84
1.2	0.40	23.0139	230,139	115,070	1.2	0.40	2.7	0.90	12	88
1.5	0.50	13.3614	133,614	66,807	1.5	0.50	3.0	1.00	7	93
1.8	0.60	7.1860	71,860	35,930	1.8	0.60	3.3	1.10	3.6	96.4
2.0	0.67	4.5500	45,500	22,750	2.0	0.67	3.5	1.17	2.3	97.7
2.05	0.68	4.0000	40,000	20,000	2.05	0.68	3.6	1.18	2.00	98.00
2.1	0.70	3.5729	35,729	17,865	2.1	0.70	3.6	1.20	1.79	98.21
2.32	0.77	2.0000	20,000	10,000	2.3	0.77	3.8	1.27	1.00	99.00
2.4	0.80	1.6395	16,395	8,198	2.4	0.80	3.9	1.30	0.82	99.18
2.5	0.83	1.2420	12,420	6,210	2.5	0.83	4.0	1.33	0.62	99.38
2.7	0.90	0.6934	6,934	3,467	2.7	0.90	4.2	1.40	0.35	99.65
3.0	1.00	0.2700	2,700	1,350	3.0	1.00	4.5	1.50	0.135	99.865
3.09	1.03	0.2000	2,000	1,000	3.09	1.03	4.6	1.53	0.100	99.900
3.3	1.10	9.669E-02	967	483	3.3	1.10	4.8	1.60	0.048	99.952
3.5	1.17	4.600E-02	460	233	3.5	1.17	5.0	1.67	0.023	99.977
3.72	1.24	2.000E-02	200	100	3.72	1.24	5.2	1.74	0.010	99.990
3.9	1.30	9.619E-03	96	48	3.9	1.30	5.4	1.80	0.0048	99.995
4.0	1.33	6.334E-03	63	32	4.0	1.33	5.5	1.83	0.0032	99.997
4.2	1.40	2.669E-03	26.7	13	4.2	1.40	5.7	1.90	0.0013	99.999
4.27	1.42	2.000E-03	20.0	10	4.27	1.42	5.8	1.92	0.0010	99.999
4.5	1.50	6.795E-04	6.8	3.4	4.5	1.50	6.0	2.00	0.00034	
4.7	1.57	2.600E-04	2.6	1.3	4.7	1.57	6.2	2.07	0.0001	
4.8	1.60	1.587E-04	1.6	0.8	4.8	1.60	6.3	2.10		
5.0	1.67	5.733E-05	0.573	0.287	5.0	1.66	6.5	2.17		
5.5	1.83	3.798E-06	0.038	0.019	5.4	1.80	7.0	2.33		
6.0	2.00	1.973E-07	0.002	0.001	6.0	2.00	7.5	2.50		
7.0	2.33	2.560E-10	0.000003	0.0000013	7.0	2.33	8.5	2.83		
8.0	2.670	1.244E-13	0.000000	0.00000000006	8.0	2.670	9.5	3.17		
9.0	3.00	2.258E-17	0.000000	0.00000000000001	9.0	3.00	10.5	3.50		

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Figure 12. Weibull to Six-Sigma Conversion Chart

8/16/2017

Classification and analysis of Weibull Mixtures

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Weibull Mixtures

- If possible, separate data and analyze
 - Better algorithms
 - Better predictions
 - But don't arbitrarily assign to a sub-set
- Simple (Exclusive OR) Mixture
 - Batch mixture if no 2nd failure mode
- Competing Risk Mixture
- Partial Competing Risk Mixture
- Compound Competing Risk Mixture
- Note: Some authors use other definitions. Check the equations to see what is being described.

Equation & Nomenclature

- Weibull: $F(t) = 1 - \exp[-W]$
 - where $W = (t/\eta)^b$
- General Weibull Mixture:

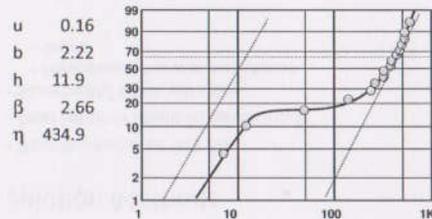
$$F(t) = 1 - (u_a)\exp[-W_a - W_b - \dots] - (u_b)\exp[-W_b - W_c - \dots] - \dots - (1 - u_a - u_b - \dots) \exp[-W_d - W_e - \dots]$$
 - $u_a =$ subset of population

Simple (Exclusive OR) Mixture

- $F = u \cdot F_1 + (1 - u) \cdot F_2$
- $F = 1 - (u_a)\exp[-W_a] - (1 - u_a)\exp[-W_b]$
- Each population has own failure mode
- Example: Thermal issues in trucks sold in
 - Michigan or Alaska
 - Truck is in one place or the other

Simple (Exclusive OR) Mixture

$$F = 1 - (u_a)\exp[-W_a] - (1 - u_a)\exp[-W_b]$$

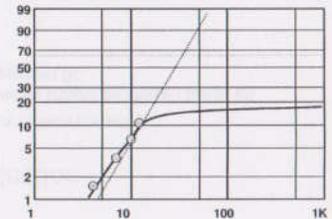


Data from: Dev Raheja, Assurance Technologies

Subset (Batch) Mixture

$$F = 1 - (u_a)\exp[-W_a]$$

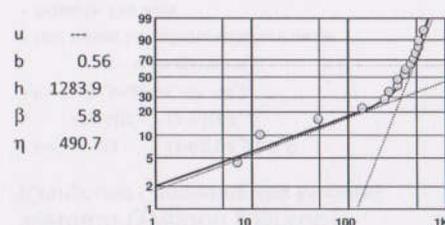
4 early failures, many suspensions beyond 1K



Competing Risk (4 Parameter)

- $F = 1 - (1 - F_a)(1 - F_b)$
- $F = 1 - \exp[-W_a - W_b]$
- Uniform population
- Mixture of risks
- Example: Tire puncture or wear out
- One Failure mode is often dominant

Competing Risk (4 Parameter)

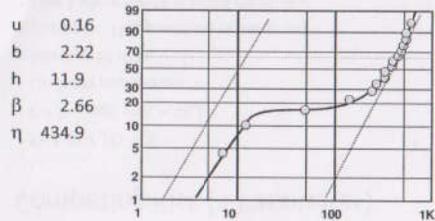


Competing Risk Batch Mixture

- $F = u \cdot [1 - (1 - F_a) \cdot (1 - F_b)] + (1 - u) \cdot F_1$
- $F = 1 - (u_a)\exp[-W_a - W_b] - (1 - u_a)\exp[-W_b]$
- First subset has additional failure mode
- Example: Tire with u_a out-of round
 - First subset may have replacement or puncture
 - Remainder has only puncture risk
 - Out of round does not prevent puncture!

Figure 13. SUMMARY: Advanced Mixture Analysis with Ybath™ Page 2 of 2

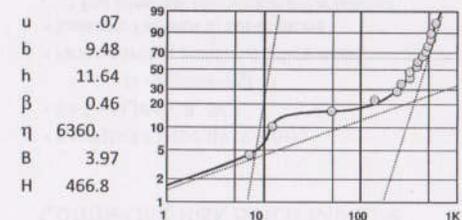
Competing Risk Batch Mixture



WeiBath (Weibull Bathtub) (Compound Competing Risk Mixture)

- $F = u[1 - (1-F_i)(1-F_c)(1-F_w)] + (1-u)[1 - (1-F_i)(1-F_w)]$
- $F = 1 - (u_a)\exp[-W_a - W_b - W_c] - (1 - u_a)\exp[-W_b - W_c]$
- First subset has additional failure mode
- Example: Tire with
a)out-of-round, b)puncture, c)wear out

Compound Competing Risk Mixture WeiBath



Other Models Possible

- Double partial Competing Risk
- Triple Competing Risk Mixture
- (Define your own)
- Literature available on Web
 - Search "Weibull" AND "Mixture"
 - www.barringer1.com
 - www.weibullnews.com

Solution Methods

- Rank Regression or MLE fits
- Excel Solver or MathCad can be used
- SuperSmith/ YBath will solve
 - Faster convergence (seconds vs hours)
 - Graphs

Model Selection

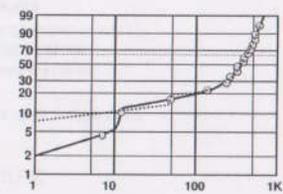
- Good fit doesn't mean correct fit.
- High order models should be chosen based on theory, not apparent fit.

Analysis of Parameters

- Simple Weibull
 - $\beta < 1$ implies infant mortality
- Weibull Mixtures
 - $\beta < 1$ implies ???
- Consider a polynomial
 - $Y = aX^{1.1} + bX^{1.12}$
 - Many values of a & b give similar results.

Two possible fits, different parameters. (Same data set, two different fits)

	Fit 1	Fit 2
u	.06	.06
b	22.9	136.0
h	11.8	49.0
β	.44	0.17
η	7.0E3	3.4E6
B	3.95	3.60
H	446.3	454.0



Conclusions

- Mixtures can be modeled.
 - Best to separate failures, if possible
 - Models predict fit if modes are lumped
- Several choices for 2 failure modes.
 - Competing Risk Mixture
 - Partial Competing Risk Mixture
- Don't interpret individual parameters.
- Techniques apply to other distributions

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