

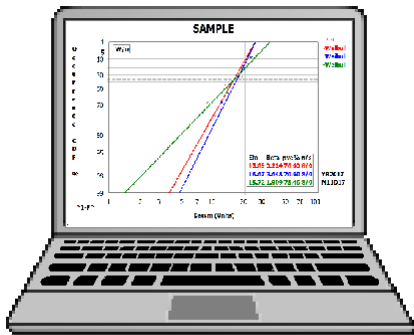
# *Play***TIME**™

With **SuperSMITH™** Software by Fulton Findings (TM)  
Tutorial for Either DEMO or FULL Versions

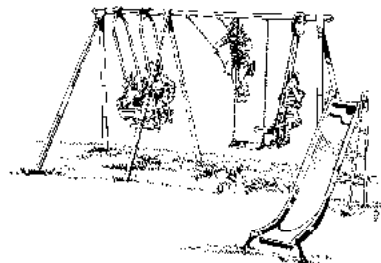
Author: Wes Fulton / Methodology: Dr. Bob Abernethy / Presentation: Paul Barringer

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Sixth Edition [With New Updates Released Periodically]



# A Computer Tutorial



## PlayTIME™ With SuperSMITH Software

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## Preface

This tutorial booklet is for use with SuperSMITH® software, both FULL and DEMO versions. The SuperSMITH DEMO software version contains the same code as the FULL version except for one difference. The DEMO includes a switch activated so all input data is randomly altered (some up and some down) before analysis, such that DEMO results provided are NOT for the original data . . . UNLESS a data file loaded was first saved by a FULL SuperSMITH version. The FULL version does not alter input data but can rearrange it. The tutorial can only work with DEMO software using provided files specifically designated herein. You **CANNOT** use the DEMO software to obtain useful results for any other input data.

The **SuperSMITH** software includes Wes Fulton's **SuperSMITH Weibull** and **SuperSMITH Visual** programs along with Carl Tarum's **SuperSMITH YBath™** program, all from Fulton Findings LLC. The reference book for all the material herein is the latest edition of Dr. Robert B. Abernethy's global Weibull Engineering (WE) standard The New Weibull Handbook©, "the Handbook" herein. The combination of Dr. Bob's handbook and this tutorial booklet along with the software is a college-level self-study course in WE analysis with the tools needed to use the expertise achieved.

SuperSMITH Weibull is a stand-alone menu-driven plotting/analysis program for complete best-practice Weibull analysis including many techniques not available elsewhere. This program has become the 'de facto' standard for life data analysis, prediction, and for evaluating variability and uncertainty in practically all industries. Capabilities of SuperSMITH Weibull include Weibull, Normal, Lognormal, and Gumbel distributions (lower and upper), and variations of these, as well as likelihood plotting, confidence plotting, distribution comparison, design comparison, probabilistic analysis, outlier detection, optimum replacement, failure forecasting, test substantiation planning, accelerated and step-stress testing, and mixture analysis. SuperSMITH Visual is a general plotting/analysis program with complimentary capability to SuperSMITH Weibull. Capabilities of SuperSMITH Visual include general function plotting, transforms, curve fitting, aggregate cumulative hazard (ACH) plotting, and Crow-AMSAA (Duane) reliability growth plotting. The new mixture analysis capability, particularly in Carl's YBath™ program, is significantly improved from that available in older software.

The SuperSMITH software may offer more useful options and capabilities than any other current life data analysis program. This wide-ranging capability is in response to the many applications in the industries and organizations we serve, including automotive, components, aerospace, medical, electronic, electric power, chemical, petrochemical, nuclear, rail transport, dental, materials analysis, foods, government, and others. This tutorial illustrates most of this capability and many of the applications. Hopefully, it is helpful for both novice and expert. The executable program (excluding associated data files) has a tiny footprint close to 0.01GB! Installation files are a tiny inner ring on the outdated CD formerly used for software distribution. The compact program size allows for easy transfer and use over networks.

We welcome suggestions and comments or questions. Extraordinary contributions are recognized. The case studies in this tutorial booklet are compiled from several hundreds of Weibull Engineering Workshops developed by Dr. Bob Abernethy and has been taught over the years by Dr. Bob since 1983 and then by Wes Fulton and Dr. Bob both separately and together. Ronald Schop has presented this workshop mostly in Europe. Carl Tarum has come on as another instructor. Dr. Bob and Wes are indebted to all those students who contributed case studies and case study solutions. Special thanks go to Paul Barringer for a great update to the visual layout herein and to Robert Silbernagel for improved PlayTIME organization. Thanks to Rich Unkle for recent improvements and clarifications.

Each new edition and printing of this tutorial will attempt to demonstrate all the options in the latest versions of SuperSMITH. Please contact the author(s) for latest software version information.

Call or write Wes Fulton or Carl Tarum for PlayTIME improvement, methodology questions, or questions regarding computing and software. Address and phone/FAX number information is included on the previous page. See the website mentioned where you can get updated to the latest life data technology and download a DEMO version of the software (it is amazingly quick and easy to do that). See the Barringer website mentioned (when up) for general introduction to reliability in practice, the latest Problem of the Month, and the downloadable DEMO software as well.

Dr. Bob Abernethy, Paul Barringer, Wes Fulton, Carl Tarum

# SPECIAL ACKNOWLEDGEMENT

We would like to especially acknowledge the following.

This SuperSMITH software tutorial and specifically the software itself would not be possible without methods and technology from **Dr. Robert B. Abernethy (Dr. Bob)**, one of the greatest technical minds of the 20<sup>th</sup> century and the father of Weibull Engineering (WE). His name is on the patent for the SR-71 Blackbird aircraft propulsion engines. This 1960's spy plane is still the fastest self-powered manned aircraft ever to fly (as far as we in the public know!) and helped maintain peace during the "Cold War". For over 30 years Dr. Bob has been the #1 authority on WE. For best WE guidance, see his standard reference, THE NEW WEIBULL HANDBOOK©, and the Weibull Workshop training he crafted. He was 91 when he succumbed to Alzheimer's in 2021. Dr. Bob's magnificent accomplishments endure.

**Paul Barringer** contributed largely to the methods, the look and feel, and the usability of the SuperSMITH Weibull software and to this tutorial booklet along with many more engineering contributions. His unexpected passing in December 2016 is a great loss to his many friends and a great loss to the technical community. Paul will be sorely missed, but his contributions live on.

**Dennis Keisic** helped in the past with Wes Fulton's presentation of Dr. Bob's Weibull Workshop on many occasions. His simple explanation of Weibull methods for Six-Sigma related techniques guided students working especially in manufacturing. He passed away too quickly in September 2017.



Dr. Bob (in tuxedo) and Paul at Wes Fulton's wedding in 2002



Dennis with his usual smile

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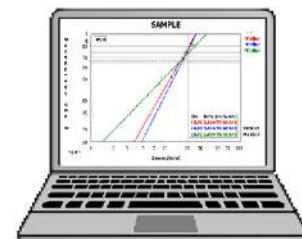
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### TIPS:

1. In Adobe Reader, click on a Table of Contents or Figure item to jump to that page.
2. There are several “Data” boxes like this: \_\_\_\_\_ where you can type in data, and save the Adobe file with that data. On the green bar at the top of Adobe Reader, click “Highlight Existing Fields” to make them visible like this: 
3. If you want to view a page at a time, press <ctrl> L. This is useful for paging through a document. To return to normal viewing, press <esc>.
4. On the first page, you can click on the Computer image to go to the Table of contents
5. Blue underlined Text like [TOC](#), will take you to a page in this document
6. Problem # lines end with [TOC](#). Click to go to the Table of Contents.





## Section I. Quick Start SuperSMITH (SS)

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We recommend you make a backup copy of any original installation files before starting.

**REMOTE INSTALL:** Available for networks ... contact Fulton Findings for info.

**DIRECT INSTALL:** Power up the PC (Windows or simulator). Have the SS **setup.exe** and other installation files available. Load onto the PC by running **setup.exe** and following the prompts. After install, within the SuperSMITH group, select the SuperSMITH Weibull program. This is the SMITHW.EXE file or the icon showing plotted data with fit and confidence lines. Initially select “Yes” to agree to usage conditions, and automatically advance to the Main Screen (Figure 1 below shows the former icon layout, Figure 2 on the next page shows the new icon layout).

*Example tutorial data files are included with program files in directory \SMITHDAT. If allowable, copy all files in the original \SMITHDAT subdirectory to C:\SMITHDAT on your computer for easiest access.*

The SuperSMITH programs, Weibull and Visual and YBath™, consume only about 10 Mb of storage space, a tiny memory footprint for such capabilities. Associated Windows system files and supporting files take another 2 Mb of space or so. PlayTIME™ tutorial data files consume another 1 Mb or so. Main programs in the software package:

- **SuperSMITH Weibull**—(SSW) probability plotting and analysis software for Windows
- **SuperSMITH Visual**—(SSV) visualization plotting and analysis software for Windows
- **SuperSMITH YBath** —(SSY) probability plotting of bathtub curves, BiWeibulls, & TriWeibulls.
- NOTE: YBath and Visual are “stand-alone” programs but also accessible from SuperSMITH Weibull.

Use single selecting (single touching for touch screens) mostly in SuperSMITH. Double selecting is discouraged to minimize hand stress. We will be using life data mostly for illustration, but any type of measured data with variability can be analyzed. **For this tutorial if using the DEMO software, all input data must only be loaded from a file previously saved by a FULL SuperSMITH software program (provided with the software).** If using the FULL software, this is not a concern but maybe easier to do. All file names shown here are on a single text line for easy copying. Reminder: Figure 1 shows the new SSW icon layout for standard pointing device use, and Figure 2 shows an optional larger icon layout with tabs for touch screen use.

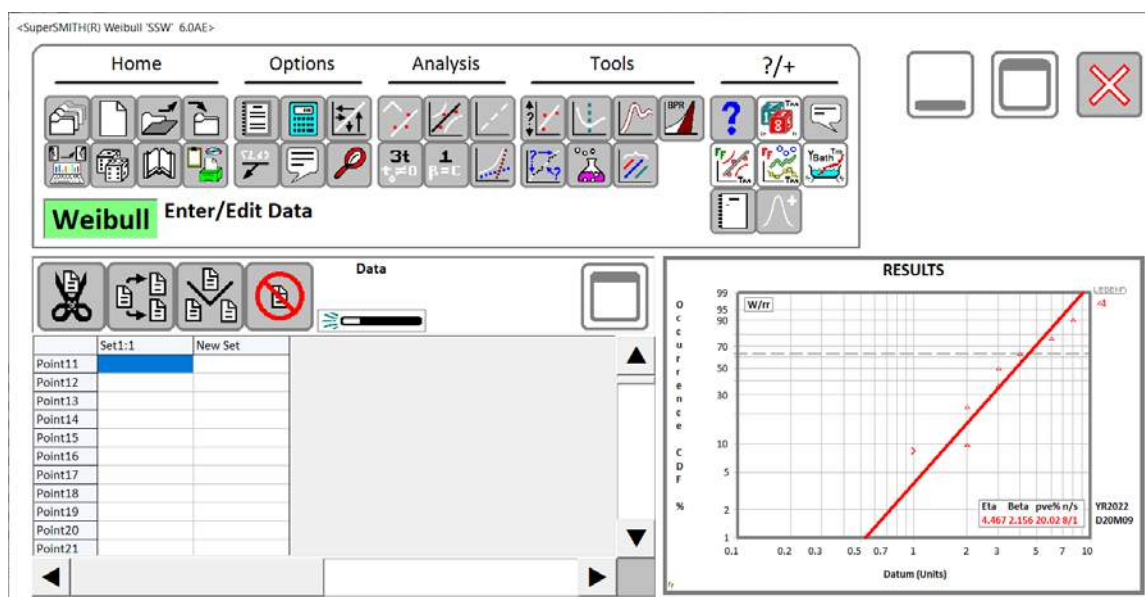
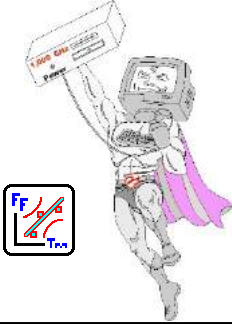


Figure 1: SSW 6.0 Layout (See Figure 2 For optional tabs layout, Figure 11 for New SSV/SSY Layout)

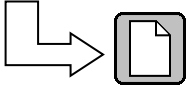


## Section II. Make Weibull Plots with SuperSMITH Weibull (SSW)!

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*NOTE: Images are provided on the side of the text to indicate the icon for selection.*



Start the SSW software by choosing Start from the Windows™ Main Screen, then the SuperSMITH group, then Weibull. On first use, select “Yes, I agree...” if you are asked and if you agree to the software use terms. You should see the Main Screen (Figure 2 below) with equation label (“Weibull” at first), some tabs, some icons, a data grid and a results box.

**DATA ENTRY, SOLUTION, MEANING:** Data for this 1<sup>st</sup> example = 8, 3, 6, 3, -1, 2, 4, 2.  
**NOTE:** For the DEMO, DO NOT enter this data by keyboard or by copy/paste as the DEMO software will alter any data entered that way. The only way to enter data into the DEMO without altering is to load a file previously saved from a FULL software version. Files indicated here have all been saved first by a FULL version and load with the data unaltered.

**A.** From the SuperSMITH Weibull Main Screen, select the Open / Import icon in the File group. Locate the **PlayTIME-A-Section-II-A.w** file and load it.

**B.** A probability plot should appear. Select the plot to enlarge. The plot should resemble the one in Figure 2. Your solution should indicate Eta=4.467, Beta=2.156. If not ... check the data and/or reset the method to Weibull “rank regression” standard. Select the full-window plot or use the <ESC> key to shrink the plot which will take you back to the Main Screen.

**C.** Exit software (Exit SSW), then start SuperSMITH Weibull again. Select Open / Import and load file **PlayTIME-A-Section-II-A.w** again (program remembers name of last file used).

**D.** At the Main Screen, select Open/Import. Load file **PlayTIME-A-Section-II-B.w**. For this 2<sup>nd</sup> data set (same data values as set 1), repeated values such as “3” and “2” are entered with only one data entry each by changing the “x1” to “x2”. Select the plot to enlarge. Your plot should resemble Figure 4 below. Same solution for both data sets! A shortcut way to enter repeated values is with the frequency value after the separator “x”. Select on the plot or use the <ESC> key to return to the Main Screen.

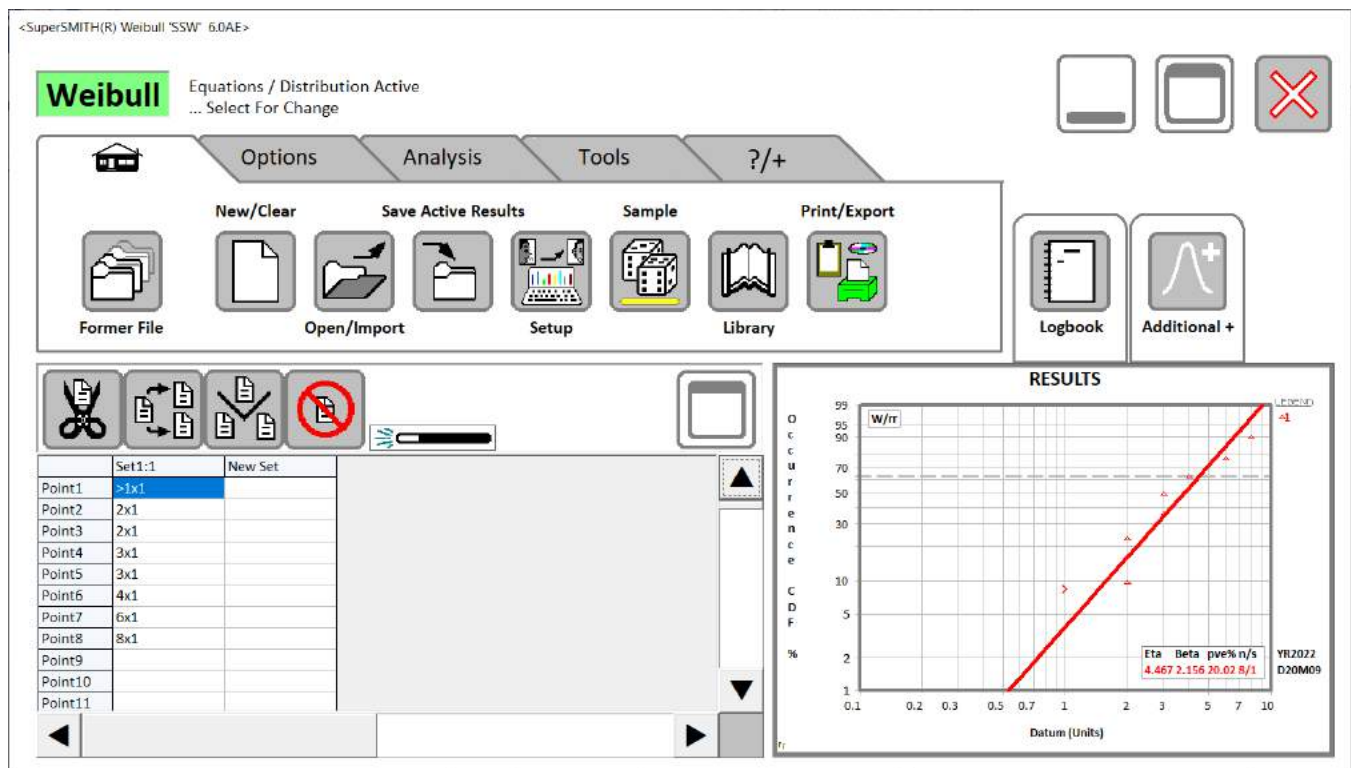


Figure 2: New SuperSMITH Weibull Main Screen Icon Layout.

This optional layout, using tabs for major topic areas, has larger icons for easier touch-screen operation.



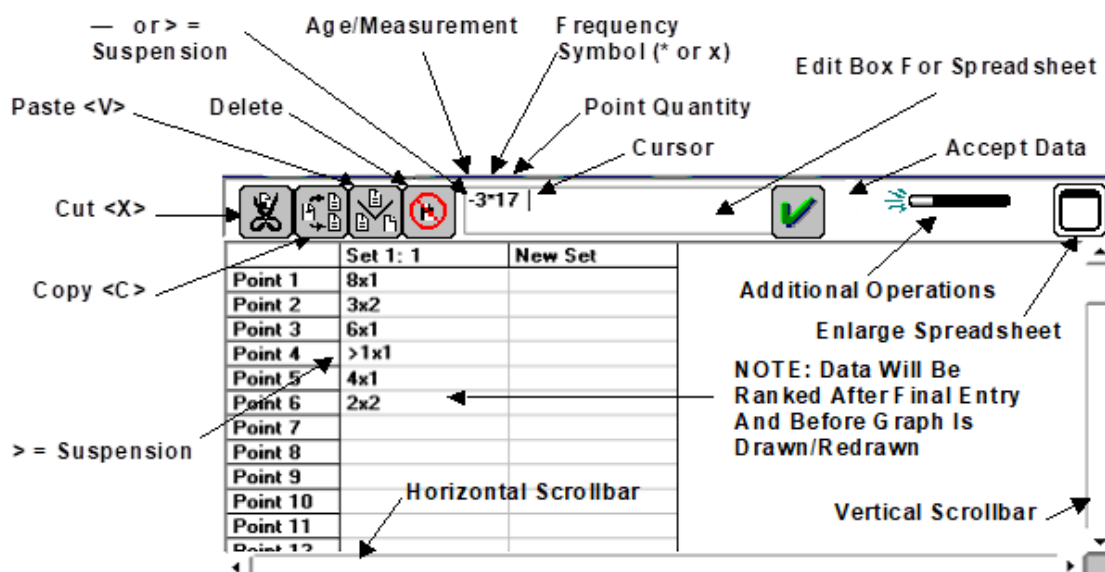


Figure 3: Same Data, Different Entry ... with Thumbnail Summary of Spreadsheet Operations

With the FULL software version (not DEMO) you can type in data or copy (or “cut”) data in text format from any source and paste (or load from file) into SSW’s data grid for fast data entry. Figure 3 describes details of this data grid. NOTE: The “>” sign in front of an entry value indicates suspension. If the plot is full screen, select the full screen plot to shrink back. Versions starting 15 January 2002 and later show pve%, which is an improved goodness-of-fit (GOF) measure based on Monte Carlo simulation, replacing  $r^2$ . The New Weibull Handbook© explains pve% in detail.

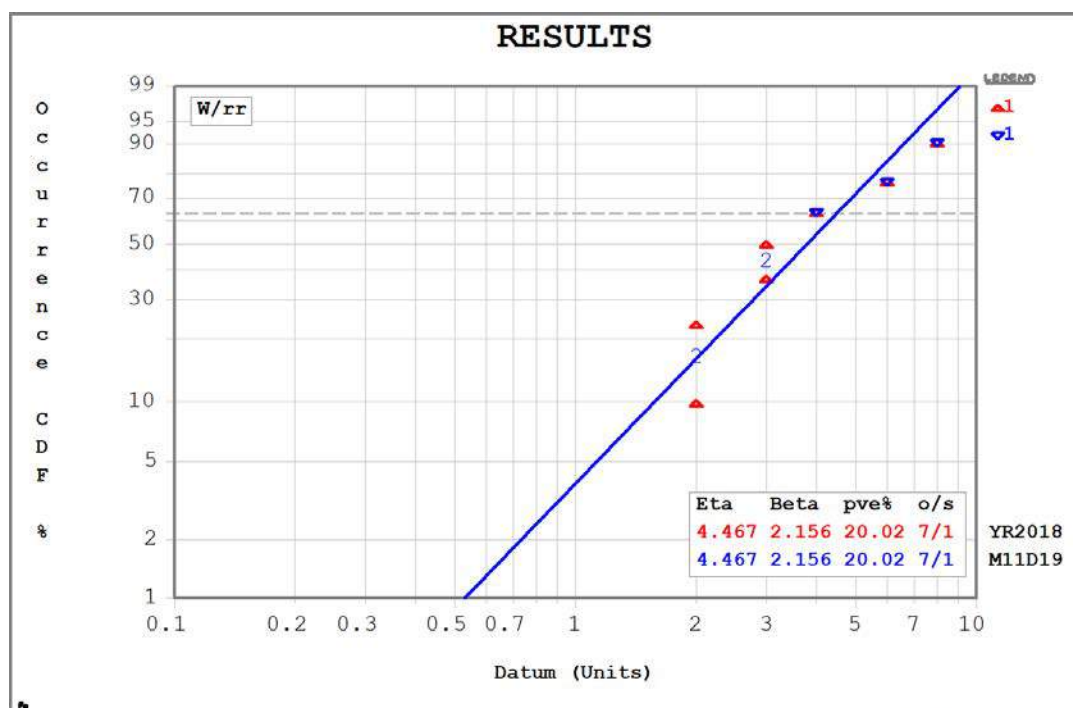


Figure 4: Weibull Plot Using “x2” After Data Values “2” and “3” for Set #2  
Note that Eta and Beta are the same as when each point was entered separately.

The following Figure 5 shows the icon layouts for the major topic area tab groups.

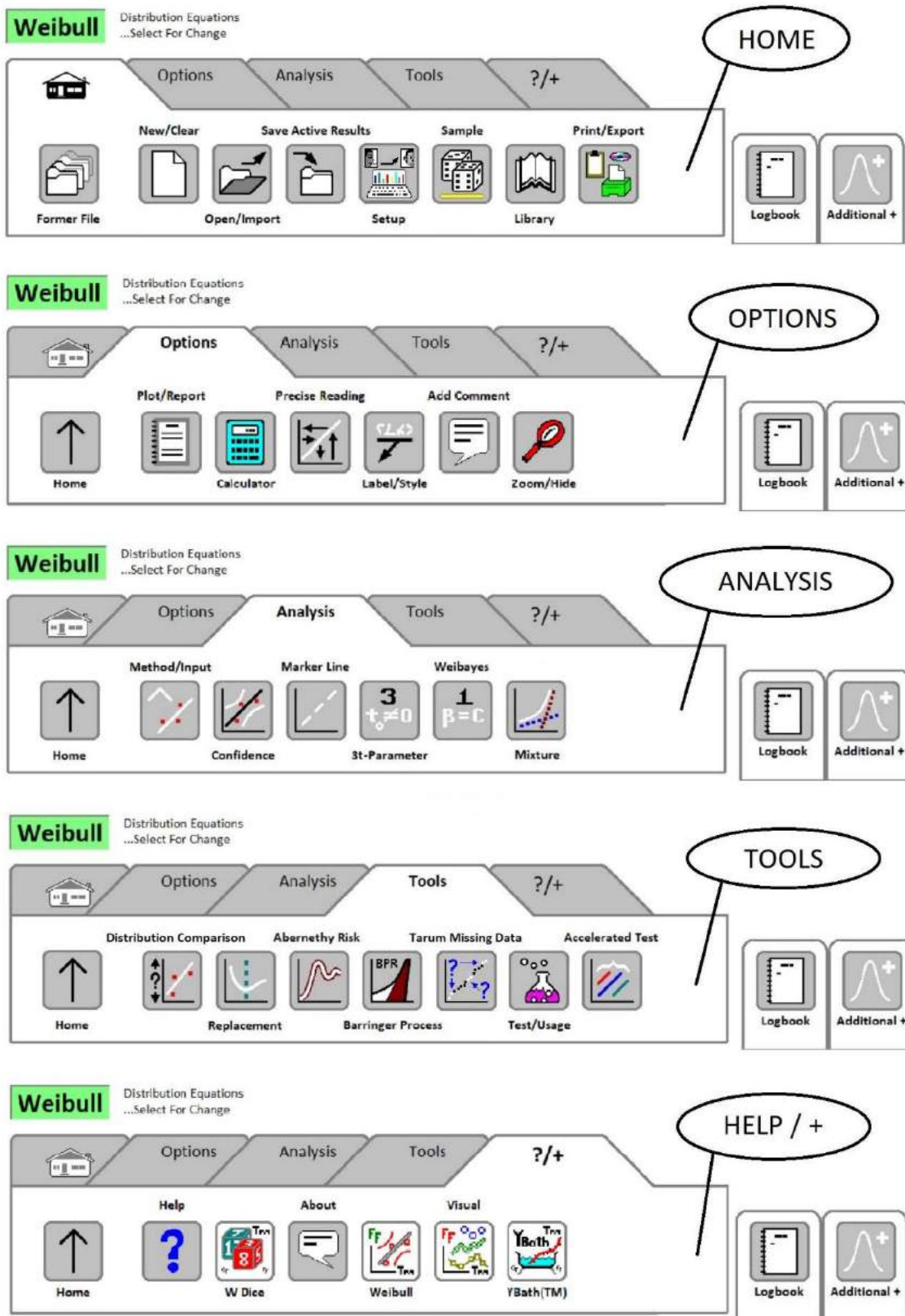


Figure 5: Thumbnail Summary of Main SuperSMITH Weibull Icons

Note: See Figure 11 for new [SuperSMITH Visual \(SSV\) and YBath™ \(SSY\) icon layout](#)

This current section gives you a quick start using SuperSMITH Weibull. More details about the icons and features are given in SuperSMITH Help, “ICON MAP: Software Tour”. Use the icon tour to get more familiar.

Now, let’s make another plot using **Monte Carlo Sampling available** within the Weibull software ...



- A) Exit software. Start SuperSMITH Weibull (SSW) again. Without change, the Home group should be visible.
- B) At the Main Screen (with Home group visible), select <Enter> key twice. **Something happens.**
- C) Voila ... a plot! A random sample. Note that every random sample will be a little different if this is repeated.
- D) With Home group still visible/active, repeat selecting <Enter> twice each time to get two or more different samples. You have a plot like Figure 6 showing sample scatter—later we’ll show the benefits of using this MC technique for serious work. SuperSMITH Weibull is configured for getting productive results quickly. In the Sample menu, you can change the model, change the quantity of data in a set, and change set quantity. With just a few selections, you’ve automatically filled the data grid and made a plot. This is one way to get a quick set of data for using the software, trying different analysis features, and getting familiar with software options.

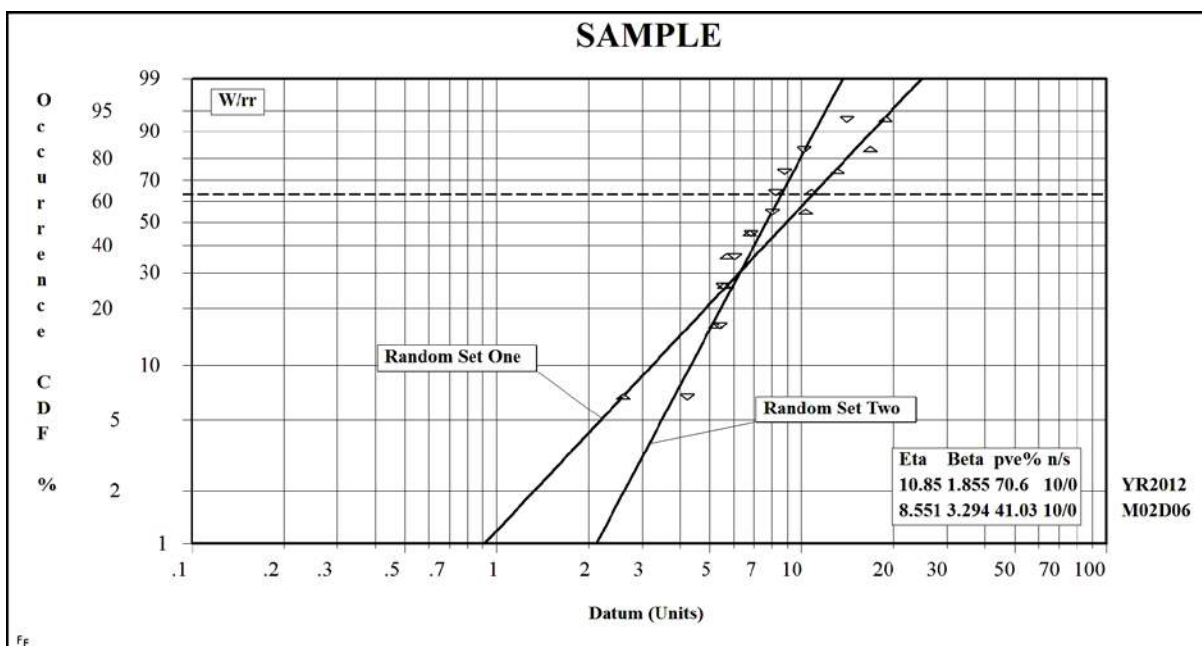


Figure 6: Quick Monte Carlo Plot

*Dr. Bob’s comments: How did SuperSMITH Weibull make this graph? Ten data points (n) and no suspensions (s) were drawn randomly from a Weibull with a true eta of 10 and a true beta of 2. (See Section 4.7 in The New Weibull Handbook for Monte Carlo theory.) The random samples were analyzed using rank regression (rr) on Weibull (W) probability paper as shown above. For the first random sample, the characteristic Weibull life is estimated to be 10.85 (Eta) and the shape factor defining the slope of the line is 1.855 (Beta), showing reasonable agreement with the true values. However, the parameters for the second set diverge more. The y-axis tells the % occurrence expected from this data. We expect each random sample to give somewhat different results.*

*Wes’s comments: (FULL version only) Editing in the data grid, you can add a point label for each data entry by using an underline symbol (“\_”) at the right end of the entry with its text label after that. EXAMPLE: If you enter “4x1\_SN523”, the plot can show “SN523” in place of the standard symbol. This is useful for visualizing if serial number and failure age are related (maybe yes for a batch issue?).*



### Section III. Let's Solve Some Problems

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Some people like to solve real problems in a tutorial, and we believe it is the best way to learn how to use the SuperSMITH software (not that useful for personal problems though!). Each case study in this section will give experience with SuperSMITH Weibull (SSW) data entry, using the many options, as well as interpreting the results and plots produced. All the important features in the SuperSMITH (SS) programs are demonstrated. However, you may select just the problems that best relate to your current applications.

Each problem starts out with a few bits of information. The first few problems provide extensive coaching on entering the data. Later problems have less coaching as you become more familiar with SSW software. The interpretation of results is the hardest part of the problem after getting the data. Wes Fulton and Dr. Bob Abemethy have provided comments on interpretation where appropriate. Please make your own interpretation before peeking at theirs. The following includes benchmark problems using data sets found in hallmark life data references. These examples validate SuperSMITH software allowing comparisons to other Weibull programs.

The 'Super Employee' above is amped up and ready to crunch some numbers on an ultra-fast computer. This person looks a little too happy. Not enough work to do! So, let's try something challenging.

#### Sample Intro Exercise. Making Widgets to the Drawing

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**The EZ-PZ-MFC metal-forming company has a potentially lucrative contract for supplying hinges to a door manufacturer. You are EZ-PZ-MFC's reliability technician. The mounting holes for the hinges should be consistently spaced within tight tolerances. This will allow the door manufacturer to focus on aligning and spacing your hinges and not bother with hole spacing issues within the hinge. These hinges, mounted robotically at the door factory, must have low rejections to make the customer happy and the lucrative potential a reality.**

You want to show your boss whether current production hinges meet specification. It requires the two farthest holes to be 7.875cm apart at tolerance of  $\pm 0.1$ cm with 90% reliability. What is the current reliability? How many rejections can EZ-PZ-MFC expect for every 5000 hinges made? File **EZ-PZ-MFC-data.w** has measurements of hole separation distance on 24 hinges randomly selected from an EZ-PZ-MFC trial production run.

**SOLUTION:** Start the SuperSMITH Weibull software if not already active.



- A) Select the Home Group, blank page icon, Open/Import icon. Bring in file **EZ-PZ-MFC-data.w** with the subject data. You should see a Weibull plot with Eta=7.979 and Beta=66.49.
- B) The customer specification demands 90% reliability. However, the specification is 2-sided with an upper and lower bound (7.775 – 7.975). When split evenly, that means only 5% maximum can be lower and only 5% maximum can be higher.
- C) The plot is hard to read due to the high fit-line slope. Use the following instructions to get the result directly from the fit-line equation using the SSW option for Precise Reading ...
- D) At the SSW Main Screen, select the Options group, Precise Reading, then select the menu item for Set1 with the Eta and Beta above. You should see another menu with several options.
- E) Select menu entry "A...Enter Largest Hole ... Get %" and enter value of 7.775. You should see then the result of "... (16.41%) Will Occur ..." This is the amount expected below the lower limit.
- F) Select Input Change.
- G) Input value of 7.975. You should see result of "... (37.9%) Will NOT Occur ...". This is the amount expected above the upper limit.
- H) Add the 2 percentages together. You get about 54.31% rejection (45.69% reliability). OUCH! ... not even close to customer expectation. Customer wants no more than 10% bad (at least 90% good).
- I) Multiply the rejection percent by 5000. You expect 2715 rejected for every 5000 hinges made. It's time for corrective action ... IMMEDIATELY! Return to the Main Screen.








### Problem 1. Failure Forecasting.

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Managers confronted with serious failures want to know how many failures will occur in the next month, six months, next year and so on. Here you will learn failure forecasting, also called “Abernethy Risk” analysis.

A heavy truck manufacturer is in trouble with steering failures. How many failures can we expect to come?

**SOLUTION:** Start the SuperSMITH Weibull software if not already active.

- 
- 
- 
- 
- 
- A) In SSW select Home group, New/Clear, to start a new analysis with new data.
  - B) Select Home group, Open/Import to load file **18WHEELR.W**. For “W/rr” solution, Eta should be 69362, Beta should be 3.257. Select small plot to go large. 10000 trucks could have failed but have not yet (suspensions).
  - C) Select the large plot to return to the Main Screen. In Tools Group, select **Abernethy Risk**.
  - D) Verify Risk Analysis Type = \*STANDARD FORECAST. Verify Output Format = \*CUMULATIVE.  
*See the help file for differences between STANDARD FORECAST, RBA1 Occurrence, and Event Probability.*
  - E) (Scroll down if needed) Select U...Usage, Single Rate, “1000” <Enter>, select Activate for output table. NOTE: “Next Event [Day]=+12” in the output means next failure is expected in 12 days.
  - F) Select Exit, return to the menu. Toggle R...Renewal ... = \*YES, select Activate. The Risk table should show about 5750 failures by month 60. Change Renewal to NO, to get 5535 at 60 mos. Exit back to menu.
  - G) Select U...Usage, Single Rate, 2000. At menu, select P...Production, Single Rate, 100, then select the green checkmark. Without Renewal, you should see 12930 at 60 mos. (with Renewal that goes to 19250).
  - H) Return to menu. Set Output Format = Interval (so the output is month-by-month instead of cumulative). Select the checkmark, 475 to 476 failures are expected in the 60<sup>th</sup> month alone with Renewal = YES.
  - I) View the **risk plot** as follows. While looking at the risk table output, select the Visual Plot option, so SuperSMITH Visual (with default installation) automatically activates and displays the **risk plot**. After viewing the risk plot, then exit SuperSMITH Visual (no save needed). In SuperSMITH Weibull, Abernethy Risk menu, activate analysis again (shows the risk table output again).
  - J) Select the Export XY Data ... Select SuperSMITH File for format, enter **18TRRSK.V** for name, and activate **Save** to save this for later. You should be returned to Abernethy Risk menu.
  - K) Set P...Production to NO (None), R...Renewal to YES, U...Usage to Single Rate, 3000. Activate to view output table. Select Visual Plot to activate SuperSMITH Visual (SSV) with **risk plot** of maintainability rhythm. To view plot, switch active program to SSV. After viewing, then exit out of SSV (no save).

*Wes’s Comments: We use “.W” extension for a SuperSMITH Weibull (SSW) file and “.V” for a Visual (SSV) file, but any extension will do. The software appropriately adds “.W” or “.V” if there is no extension.*




### Problem 2. Optimal Replacement Intervals

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**If the cost of an emergency replacement (unplanned) from a wear out failure mode is greater than the cost of a planned replacement, there will be an optimal replacement interval.**

The truck manufacture decides on replacing the steering links, until a new redesigned link can be produced for a recall program. A “planned” (preemptive) replacement (P) costs \$50 each and an unplanned replacement due to failure (U), “not planned”, costs significantly more at \$1000 each. What is the most cost-effective planned replacement mileage?

**SOLUTION:** (NOTE ... *If in SSV, then exit from SSV “Visual” and return to the SSW “Weibull” software first!*)

- 
- 
- 
- A) In SSW, return to the Main Screen, using same input data as the forecasting problem above, select Tools Group, **Optimum Replacement** (icon with plot of a dashed vertical line through the curve minimum).
  - B) Enter present cost values (P=50, U=1000). Confirm View Length (miles) is 60000. [Here always use a view length a little less than Eta and, if possible, divisible by 60.] Verify “Y...Cost Change” is “No”. Select Activate for output. RESULT: Planned replacement at 22000 miles gives lowest cost factor (~0.0033). **While still looking at the output table, select the Visual Plot option for a quick results plot in SSV.** After viewing this plot, then exit SSV, no save. This returns you to the Optimum Replacement menu. Select Activate again.
  - C) Back at this output table, select the Export XY Data option, then choose SuperSMITH file for format, enter **18TRREP.V** for the name, and activate **Save** to save the file. Exit to the SSW Main Screen.

### **Problem 2, Continued. Replacement Intervals, Warranty Limits, and Forecasting (Risk Analysis)** [TOC](#)

We just learned the lowest-cost planned replacement is every 22000 miles for our steering links. The network database indicates seasonal usage fluctuation. Expected usage is 0;500;1000;2000;3000;5000;5000;3000;2000;1000;500;0... in miles per truck each month for 12 months. With production of 100/month, How many failures are expected?

**SOLUTION:** Make sure you are in SSW (Weibull) and still have the 18Wheelr.w data in the data grid!



- At the Main Screen, the Weibull CDF plot should show Eta=69362, Beta=3.257. Select **Tools** Group, **Abernethy Risk** to get to the failure forecast menu. Verify Output format is \*INTERVAL.
- Select U...Usage, then Enter Variable Sequence, and enter the usage sequence as shown by the NOTE below with three period punctuation marks (...) after the last value. Then the usage sequence by interval should be displayed for your review. The pattern continues for the entire forecast. **Select OK to continue.**
- Back in the Abernethy Risk menu select P...Production, select 1...Enter Single Rate, and enter 100.
- Select S...Planned Replacement, 22000, 100% Affected, and F...For Quantity Expected.
- Select green check mark, view table. **Select the Visual Plot option for a quick plot in SSV (Visual).** View the resulting “up and down” forecast curve. Exit SSV, no save. (returns to SSW).
- How about warranty? Some units will exceed warranty of 36 months and become the responsibility of another department. For expected claims, in the Abernethy Risk menu, W...Warranty Limit in months and enter 36. Activate (check mark) to view output. **Select the Visual Plot option again for the updated SSV plot.** Activate SSV if not showing to view. Changes? The last 2 spikes are shorter! Exit SSV, no save. Exit Abernethy Risk to return to the SSW Main Screen.

NOTE: Entry for this variable usage should be as below with the three periods (...) at the end. Values are now separated by semicolons (“;”) instead of the previously used comma (“,”) to improve international use.

**0;500;1000;2000;3000;5000;5000;3000;2000;1000;500;0... [The “...” at the end means repeat this sequence.]**

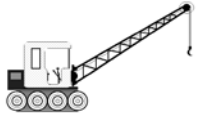
### **Problem 3. Goodness of Fit ... Distribution Comparison with a large sample**

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“FATIGUE.W” contains many points from previous wire experiments. What’s the general pattern?



**SOLUTION:** At SSW Main Screen, Home group, select New/Clear (“blank page”) icon. Select Home, Open/Import icon (open folder), load **FATIGUE.W**, and record pve%’s here >>> \_\_\_\_/\_\_\_\_. Select Tools group, Distribution Comparison (view Set 1, then select Set 2 to view it). Which is the best fit (distribution with largest pve%) for each set? Note stacked data. At Main Screen, select Analysis group, Method/Input, Inspect1. Does pve% increase? Is the fit better? Try Lognormal [2P] ‘L’ from the Equations Label. **THE FIT IS MUCH BETTER!**



*Dr. Bob’s comments: We suspect both the three parameter Weibull [ $+t_0$ ] and the Lognormal may be appropriate because the data are curved downward on Weibull scaling. It would make physical sense to say there may be a minimum number of twists to failure or that the deterioration of braided wire is non-linear and progressive. To find out, we compared 2 & 3 parameter Weibulls with Lognormal plots using SuperSMITH. The results favor the Lognormal based on the pve% values. Same results with mle. Our class results are generally not as definitive because of smaller sample size.*

### **Problem 3a. Compare Probability Density Function PDF with Cumulative Distribution Function CDF**

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**SOLUTION:** Compare the probability density representation with the cumulative as follows (NOTE: The Main Screen icon layout for SuperSMITH Visual is shown among the last figures at the end of this tutorial booklet) ...



- From SS Weibull (SSW), select “?/+” (Help+) group, then “Visual” to start SuperSMITH Visual (SSV).
- From the SSV Main Screen, select Home group, Open/Import (shows open folder) and enter (or select) **FATIGUE.W** for a new menu. Select item “PDF (= d[CDF] / dX) Probability Density”, to see a Visual plot with PDF curves and data points.
- Density (PDF) is just a different way to view the solution. SuperSMITH shows data points along the respective PDF curve. Most other programs can only show a histogram to represent the data. This is a special feature of SuperSMITH. Return to the SSV Main Screen. Exit the SSV program with No Save.
- View or Re-activate the SuperSMITH Weibull (SSW) program (Equation label may show “LogNormal”).



#### **Problem 4. Copy, cut and paste data from a POSSIBLE mixture of failure modes**

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Reference [The New Weibull Handbook](#) Section 3.7.3.

**BiWeibull data sets:** A reliability consultant produced this interesting set of 12 data points separated by commas:

**41.6, 2.8, 35.1, 4.8, 10, 13.5, 17.1, 18.8, 23.1, 26.8, 40.2, 45.3**

The consultant suggests this is the classic BiWeibull or mixture situation, a low slope followed by a high slope. Usually, this mixture of competitive failure modes is caused by a quality control mode with infant mortality or random slope, followed by a wear out mode. The consultant says the data should be split (partitioned) into the lowest “k” data points for mechanism #1 and the highest 12-k data points for mechanism #2. Specifically, the consultant says the best split is first four and last eight. Partitioning the data into two groups is a popular solution recommended by many authors. Later in PlayTIME, we will use two better methods for this (SuperSMITH YBath™ and the likelihood ratio ‘lr’ partition method test in SuperSMITH Weibull). However, for now, solve the problem using this simple partition method.



**SOLUTION:** >>> FIRST! ... from the SSW Main Screen, Home group, select New / Clear icon to start a new plot.

- A. In Analysis group, Method/Input icon, select Reset To Standard (icon shows U-turn arrow). YES to “Weibull” question if asked. This resets to Weibull with standard “rr” fit (rank regression) to start.
- B. If “Results -... Save?” comes up, select “No Save ...” Later in SSW if you want to reactivate a former analysis plot showing on the Main Screen, just select it. (These are saved as temporary files.)
- C. From the Main Screen, Home group, select the Open / Import icon. Load file **PlayTIME-Problem-4.w** (has data described above). You should get a plot showing Eta=26.60, Beta=1.304. We will analyze this data later.
- D. Record the pve% value... pve% = \_\_\_\_\_.
- E. Do we have enough data to support mixture analysis? No, we do not; you should at least about 40 occurrence points for a 4-parameter competing-risk model, but to illustrate the method we will proceed.
- F. Select the midsize plot to maximize it. Look at the plot. By eye where would you partition the data? Where does the first data set end and the second begin? Select on the maximized plot to midsize it.
- G. Repeat the file load procedure with the same data ... Home group, Open icon, file **PlayTIME-Problem-4.w**. It should come in as Set # 2 with the same solution. Use data grid vertical scroll if needed to see “Point1”. Note: If less than 12 points in a set show in the data grid, then you may need to increase your display resolution.

>>> **Let’s try some data splits.** >>>

- H. Starting with the 5<sup>th</sup> data point “17.1” in the 1st column (set #1), select down to the last data point “45.3” in the 1<sup>st</sup> column (highlighting the last 8 data points) in preparation to edit.
- I. Select the **Data Wizard** button (showing a magician’s wand) on the data grid, scroll down and choose **Susp. To / From** to toggle occurrences to suspensions. This changes the last eight data points in set #1 to suspensions.
- J. Scroll as needed to see “Point1”. For set #2, highlight the first 4 data points, activate the Data Wizard, scroll down and select **Susp. To / From** again to similarly change those selected data values from occurrences to suspensions.
- K. View the revised plot (NOTE: If the data grid is still full-screen, select the “midsize” button, upper right corner of the data grid, to return to the Main Screen). Select the small plot to enlarge. Select again to midsize.
- L. Write the two pve% values for set #1/set #2 in the “First 4/Last 8” blanks below (should be about 75 / 63).
- M. For the 2<sup>nd</sup> split (“First 8/Last 4”) ... **for both sets at the same time** select the rows from “Point5” to “Point8”, then use the Data Wizard button as before and select Susp. To / From to toggle those points. Enter in the appropriate blanks below the resulting two new pve% values for “First 8/Last 4” (should be about 96 / 85).
- N. For the 3<sup>rd</sup> split (“First 9/Last 3”) ... **for both data sets at the same time** select the “Point9” row. Then Susp. To / From as before. View the plot. Enter below the two pve%’s for “First 9/Last 3” (should be about 98 / 38). Enter in the appropriate blanks.

**Enter pve% ... First 4/Last 8=\_\_\_\_\_ / \_\_\_\_\_ First 8/Last 4=\_\_\_\_\_ / \_\_\_\_\_ First 9/Last 3=\_\_\_\_\_ / \_\_\_\_\_**

*Dr. Bob’s comments: Although it may seem reasonable to assume all the points in one set precede all the points in the other set, we expect the sets might overlap. However, the statistical concept of choosing a “corner” to partition the data is popular for analyzing competitive failure modes with small effect in the corner vicinity. In this case the best partition appears to be the first 8/ last 4, the highest combination of pve% values. Later we will present a more rigorous mixture solution.*

Wes's Comments: Pve%'s above are approximately 75/63, 96/85, and 98/38. Roughly, both pve% values should exceed the original 88 to accept the more complex 4-parameter mixture fit. No honest split has both pve%'s above 88, although close.

### Problem 5. Data Input, Weibull likelihood ("mle") solution versus graphical ("rr") solution

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Learn how to enter data into SSW, input suspensions, and compare regression and likelihood solutions.



- Back at the SSW Main Screen, Home group, select the **New / Clear <N>** icon to start a new analysis.
- If you are in the Weibull workshop you will remember day one, classwork problem 2-2 in the Handbook with the nasty adjustment for suspensions you did by hand. Let's try it with SSW.
- From the Main Screen, Home group, select Open/Import, and open the **PlayTIME-Problem-5.w** file. Select the small plot to enlarge, it should look similar to Figure 7, but with only one line plus the data points.

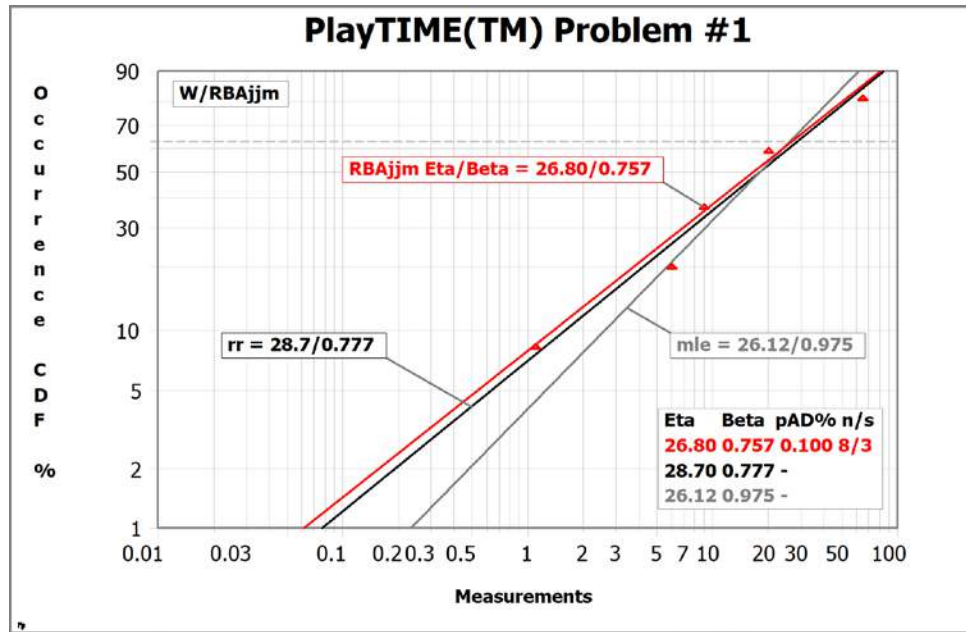


Figure 7: Comparing Rank Regression (rr), Maximum Likelihood Estimate (mle), & mle-RBA (RBAjmm)

You should get **Eta=28.7, Beta=0.777** at first. Compared to hand calcs is SSW slower or faster?? Notice the median rank regression (rr) plot line fits the data well. Follow below to compare rr, mle, and modified mle (RBAjmm) solutions:



- Select the full-size plot to make smaller. Select the Analysis group, then Marker Line icon (dashed line). Select Set 1 from the menu to copy the rr line (the new Marker Line is hidden under the active fit line).
- Select Analysis group, "Method/Input" icon (^ symbol and line through points). Next, select the mle icon (showing only the "^" symbol), to change method to mle. The plotted mle line (**Eta=26.12, Beta=0.975**) does not match well with the points, slope is steeper than rr solution, and there is no pve% for mle. With automatic Fit Comparison + Display selected, then "pAD%" is shown with negligible result ("---") instead.
- Copy the mle line as follows: From the Main Screen, select Analysis group, Marker Line, "Line 2", then select "Set 1" with different solution (hides a Marker Line underneath the Set 1 active mle fit line).
- Get "RBAjmm" (mmle) solution as follows: From the Main Screen, select Analysis group, Method/Input again, then select the modified maximum likelihood (mmle) icon (showing mle symbol + 2 arrowheads). "RBAjmm" should appear on the plot for solution method. If not, return to Method/Input, select "H...Type Of mle Modified...", select "1...Weibull...", then "RBA Mindeguia". This latest plot (Figure 7) shows RBAjmm fit (**Eta=26.80, Beta=0.757**) agreeing well with Marker Line # 1 ('rr').

Dr. Bob's comments: The mle vs. rr comparison shows the mle line is steeper as usual. This is the mle bias toward steeper betas with small samples. The modified mle solution (RBA factor) removes most of that bias. When rr and mle-RBA show reasonable agreement like this, it means we have a good Weibull. NOTE: There is no pve% with mle or mle-RBA, and the fit

is not exactly through the plotted points. SuperSMITH Weibull (SSW) software shows the plotted points for mle in the same vertical plot location as for graphical solution. We'll show you how to add labels on the plot later.

### **Problem 6. Wire Fatigue Distribution Comparison using separate SuperSMITH Dice program**

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**Learn how to do a Distribution Comparison and compare Weibull 2-parameter, Weibull 3t-parameter, and Lognormal; use pve% value or likelihood to measure goodness of fit.** Handbook Ref: Section 3.2.1.

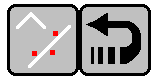
We clamped eight-stranded stainless-steel fishing-leader wire hanging a 2-pound weight on it, increased to 2.6 pounds later. The pliers were rotated back and forth through 180 degrees, clockwise and counterclockwise. We recorded the number of half twists to total rupture (failure). Total rupture may require a  $t_0$  correction as we do not expect it on the first half twist, i.e., a guaranteed failure free period corresponding to  $t$ -zero ( $t_0$ ) ... or ... total rupture might be Lognormal as tiny strands fail progressively while stress increases on remaining strands. The data was collected from over 500 presentations of Dr. Bob Abernethy's Weibull Workshop through 30 years with water bottles or lead weights. For easier air travel, Carl Tarum designed and manufactured green and red dice to simulate this data (green for lower 2-pound load and red for upper 2.6-pound load). Wes Fulton then made a virtual program to duplicate the data from Carl's dice, "SuperSMITH Dice".



#### **SOLUTION:**



- A. Select New / Clear for a "new plot" (blank page icon). Use Analysis group, Method/Input, Select Reset icon (U-turn arrow) for Weibull rr Point-By-Point Standard.
- B. In the Home group, select Setup (computer icon), verify Dimension menu item indicates Set Quantity is at least 12 and Point Quantity at least 1500 (reset dimensions if needed by selecting and changing).
- C. WIRE-BREAK DATA: For FULL software, use actual data if available from class. For the DEMO software, use stored data as follows. At Main Screen, select Home group, Open / Import, file **PlayTIME-Problem-6.w**. Upon loading, you should get Eta=35.82, Beta=5.407 for set 1, and Eta=16.89, Beta=5.912 for set 2.
- D. Are there outliers? If you think so, use Options group, Calculator, Outlier icon, and look at the results (for stored data, upper points in both sets show "Outlier"). Select OK to return to the Main Screen.
- E. NOTE: If some data is duplicated, try selecting from the Main Screen, Analysis group, Method/Input icon, and "Inspect 1" icon, Activate, for Inspect Option #1.
- F. View the plot. Is there curvature in the point pattern? To find out if the data sets are good Weibulls or better LogNormals, from the Main Screen, select Tools group, Distribution Comparison (icon shows plot with "?" on the vertical scale). If any "Change" question appears, just answer No. Use menu options to view results for set #1 and set #2. The best distribution using regression solution is the one with the largest pve%.
- G. Complete below Table 1 (Distribution Comparison) for rr. Agrees with expectations? Exit back to Main Screen.
- H. Select Main Screen, Analysis group, 3t-Parameter icon (showing "3t") for "t0" solution: Toggle S...Scale to AS RECORDED (AR). Toggle "Find Optimum  $t_0$ " to "YES". Select OK for plot. See curves? Return to Analysis, "3t" icon, and select "None" to turn off the 3t-parameter model and return to Weibull 2-parameter.
- I. Select the "Weibull" green equations label, change to LogNormal [2P] 'L'. View plot. Use blue "LogNormal" label on Main Screen to go back to standard by selecting Weibull [2P] 'W' for equations.
- J. Select Analysis group, Method / Input, mle icon to change the Fit Method to mle, Activate. View plot.
- K. Do another Distribution Comparison (Tools group, Distribution Comparison). This time it's different. The software automatically selects the best 2-parameter distribution with highest (least negative) log likelihood (LL). Then 3t-parameter Weibull LL is compared to 2-parameter with a likelihood ratio test. Complete all columns in Table 1. The rr and mle solutions can disagree for small samples. New studies [Liu] confirm that Distribution Comparison requires at least 21 occurrences (failures for reliability) for reasonable statistical power. For smaller samples (less than 21 occurrences, without many suspensions) best starting practice is two-parameter Weibull model and rank regression (rr) fit method. **Are the two sets significantly different? Does load value make a difference?** Later we'll see.



**Table 1. Distribution Comparison: ✓ Mark = Best Fit Distribution**

	rr=Largest pve%		mle=Max Likelihood	
	Set 1	Set 2	Set 1	Set 2
2P Weibull				
3t Weibull				
Lognormal				



- L. Go to Main Screen, select Analysis group, Method/Input, Reset To Standard. (Method changes to 'W/rr').
- M. Select Options group, Label/Style icon (characters underlined and down arrow). Plot with pink boxes appears.
- N. Select the Eta/Beta box, Change Location, repeat to change back. Select the set labels (top-right of plot) and edit them (set 1 = "2 lbs", set 2 = "2.6 lbs") while moving each label onto the plot pointing to correct set. Select the lower left " $\wedge F \wedge$ " symbol, then select **100 – Occurrence As %** to change to the reliability vertical scale. Select again and then **Occurrence As % ... Standard**. Select green checkmark or <ESC> key to return to the Main Screen. Later we will analyze this same type of wire data more.
- O. Verify method is standard with **Analysis group, Method/Input, Reset To Standard (U-Turn Arrow)**.

*Dr. Bob's comments: We expect that the 3t-param. Weibull or the Lognormal would be best for both weight levels. However, data scatter for small samples may cloud the issue. If you are in a workshop and this does not agree with your workshop results, remember who generated this data. (Not our fault!)*

### **Problem 7. Let's make some PDF and HDF Plots with SuperSMITH Visual (SSV)**

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You can get auxiliary plots showing results in different formats like the probability density function (PDF) format or the hazard distribution function (HDF) format or the cumulative distribution function (CDF) format plus others. SuperSMITH Weibull (SSW) and SuperSMITH Visual (SSV) share many common icons and functions. The **SOLUTION** is as follows:



- A) From the Main Screen, " $?/+$ " group, select the Visual icon to start **SuperSMITH Visual (SSV)**.
- B) In SSV, at the Main Screen, **Home** group, select **Open / Import**, input **18WHEELR.W** file, and choose C...CDF from the next menu to see the cumulative distribution function CDF but this time on linear scaling.
- C) Select Home group, New/Clear icon, load **18WHEELR.W** again, select P...PDF this time to view the probability density function. Use Zoom icon (magnifying glass) on plot to resize the plot to fit the function if needed.
- D) Repeat New/Clear, load **18WHEELR.W**, select H...HDF, to view the hazard function.
- E) Repeat New/Clear, retrieve **18TRRSK.V (saved earlier)**, then view the failure forecast (up and down wave).
- F) Repeat New/Clear, bring in **18TRREP.V (saved earlier)**, view optimum replacement plot (for minimum cost).
- G) From the Main Screen, Options group, select the Labels icon (underlined characters and an arrow pointing left and down). Select the set #1 label (top small box just off top-right of plot), enter a name like "Cost", select green check mark. A box should appear moving with the cursor ... move it onto the plot, select to drop in place, then select near low curve location to attach "leader" line. Green checkmark for Main Screen return.
- H) In Analysis group, select Plot Type, then Curve Type. Select Set 1 Type. Change from Line Only to Point Only then try again vice-versa (or to a "curve" when several points are not in a straight line. NOTE: "Contour" type is special for confidence likelihood and not useful here. Select green checkmark, Exit, return to Main Screen..
- I) At Home group, with peripheral attached if needed, for export, or output, select Print / Export icon. >>>>>>>
- J) **At Home, Select New / Clear to erase data and plot.** Exit SSV, no save, and return to the SSW program.
- K) From the SSW Main Screen, select Blank Page icon, then Sample (dice) icon, then the checkmark. View the plot.
- L) Back on the Main Screen, select " $?/+$ " group, icon showing a bell curve and a plus symbol ("Additional +").
- M) From the menu, choose P...PDF, a much faster way to get the PDF plot! Exit from SSV (no save). SSW should still be active. Return to the SSW Main Screen, if not already there.



# Wire Fatigue Experiment *\*Use ONLY if missing actual data from Workshop!*

Light 2 lbs	Half Twists to Rupture	Initials	Heavy 2.6 lbs	Half Twists to Rupture	Initials
1	37		1	10	
2	34		2	15	
3	27		3	18	
4	35		4	14	
5	28		5	17	
6	45		6	18	
7	30		7	24	
8	34		8	13	
9	21		9	17	
10	27		10	17	
11	24		11	19	
12	39		12	13	
13	27		13	15	
14	42		14	16	
15	37		15	17	
16	41		16	20	
17	32		17	15	
18	27		18	11	
19	51		19	14	
20	25		20	12	
21	35		21	13	
22	41		22	16	
23	24		23	20	
24	33		24	13	
25			25		
26			26		
27			27		
28			28		
29			29		

Probit for Problem #14. Format is Inspection Time (1/2 Twists) X Cum. Qty. Ruptured X Total Qty of Wires Inspected. Light Weight (2 lb.) Data Should Be Converted to Probit (Grouped) Here. Do Not Fill Cells Representing Zero Percent Occurred or 100 Percent Occurred or Where No Change Happened:

15 X _____ X 24	30 X _____ X 24	45 X _____ X 24	60 X _____ X 24
20 X _____ X 24	35 X _____ X 24	50 X _____ X 24	65 X _____ X 24
25 X _____ X 24	40 X _____ X 24	55 X _____ X 24	70 X _____ X 24



**Problem 8. Analyze Your Data (only with FULL software, the DEMO will alter your input data!)**

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If you have your own data and the FULL software, now is a good time to analyze. For help in class, call the instructor.

**Problem 9. WeiBayes with one failure versus mle.** Reference: Handbook 6.4.

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Five new-design parts are on test, and one has failed (data: -100, -200, **300**, -400, -500). Your Weibull Library indicates Beta of 3.5 on a similar failure mechanism from before. What is the Weibull for the new design?



**SOLUTION:** In SSW, Main Screen, Home group, select New/Clear, then load file **PlayTIME-Problem-9.w**. No solution fit line appears. Only a lonely data point is shown on the plot (plus possibly some suspension symbols). Select Analysis group, Weibayes (icon showing “1”) for the Weibayes menu. Input Beta = 3.5, verify Confidence For Marker Line = NO (mle). Eta output should be about 578. Verify menu item “W” is YES. Select OK to add a marker line (OK to data warning). Select Analysis group, Method/Input, choose **Fit Method** of mle, Activate. Compare (see comments below). Select Analysis, Method/Input. **Reset To Standard (rr)**. (Ignore warning messages)

*Wes's comments: We forced WeiBayes beta 3.5. The mle finds 2.926 Beta (613.85 Eta) . . . fair agreement.*

**Problem 10. WeiBayes without a Failure.** Ref: Handbook 6.3.

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**WeiBayes with no failures provides a lower one-sided confidence bound for the true unknown Weibull.**

Analysis says 300 is -300 (no failure). Select New / Clear, open **PlayTIME-Problem-10.w**. OK to any data warnings or “Y-Scale” messages. Select Analysis group, Weibayes, set Beta = 3.5, verify confidence approx. 63.2%. What is Eta?

*Dr. Bob's comments: The WeiBayes Eta was about 578 with or without a failure. When the data point was a failure, the WeiBayes line was an mle estimate, but with no failures, WeiBayes is lower 63.2% confidence taking one failure as imminent. For small failure quantity, WeiBayes is more accurate than Weibull **IF** we have a good estimate of Beta. The software provides WeiBayes confidence bounds with or without failures. With a good estimate of Beta, WeiBayes is most accurate.*

**Problem 11. How about a Dauser Shift?** Reference Handbook 5.7.

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**If suspension ages are not known, you can still estimate a Weibull line but with large uncertainties.**

**SOLUTION:** In SuperSMITH Weibull (SSW), Home group, select New/Clear, retrieve **GEARS.W** file. You should see the Weibull plot of 20 broken transmission gears from a European car (Eta=10670km, Beta=3.642). There are 2000 of these cars in operation. Average failure mileage is 9640 km.



- 1) From the Main Screen, Analysis group, select WeiBayes (showing a “1”). Then select the “Dauser” option.
- 2) **Input 1980** for suspension quantity. Verify text at the top shows “DS +1980S” with Eta near 34018 “.
- 3) Verify menu item W...Add Weibayes Marker Line = **YES** to add Marker Line on plot when activated.
- 4) Select the green checkmark to view the plot with the Dauser inspired Marker Line added to the right on the plot.
- 5) Use the Marker Line with Eta=34019 (approx.) for analysis and forecasting.

*Wes's comments: The Dauser Shift is a last resort. Suspension values should be in the range of the failures. We recommend an educated guess, engineering judgment, for the age of the units still in service as better practice. Still if you have **no idea** about the suspension ages, try the Dauser Shift.*

**Problem 12. Inspection Interval Data** Reference The New Weibull Handbook Chapter 5.8.

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**Inspection data is dirty because precise times-to-failure are unknown, all we know are the inspection times when we found the failures. Here we will use a special method to improve the accuracy of the inspection data analysis.**

Many analysts suggest that when there are benign failures found at inspection that only the top point in each series recorded at the same inspection should be considered in the least-squares analysis.

**SOLUTION:** In SSW, Home group, select New/Clear, then open file **INSPECT.W** for Sherwin's inspection data. Fit line look appropriate? Not so much. For a better solution: Select Analysis group, Marker Line, choose set # 1 to add a Marker Line there. Then select Analysis, Method/Input, menu item I...Input Data Format, and then Point-By-Point-Inspect(1). Better looking fit than Marker Line? Select Analysis, Method/Input, Reset To Standard.



*Dr. Bob & Wes's comments: There are three alternative methods. (1) If the number of pieces inspected at each interval is a constant, the Kaplan-Meier would be a good alternative. (2) If the inspection intervals vary from piece to piece, the mle interval option should be used. (3) If there is uncertainty about finding the failure even if it is there, the Probit method should be used, selecting no difference sample-to-sample for the Input Type (Method/Input Menu) for non-destructive testing. With larger samples, all of these should mostly agree.*

**Problem 13. Coarse Data** Reference: Handbook 5.8.

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**Coarse data collection produces “dirty data” like inspections.**

Recording data in months instead of days or hours or minutes produces columns of points. Inspection option #1 may help.

**SOLUTION:** In SSW, Home group, select New/Clear, retrieve **TWIST.W** produced by many students twisting wire until the 1<sup>st</sup> strand ruptures (pve% = 0.326). Select Analysis, 3t-Parameter, Set F...Find Optimum to NO. Select 1 (for Set 1), input 10.4 for t0, choose Activate, note pve% goes up slightly (to 0.789). Now select Analysis, Method/Input, I...Input Data Format, then Inspect(1). Much better pve%! Reset by selecting Analysis group, Method/Input, Reset to Standard.

**Problem 14. Inspection Probit Data** Reference The New Weibull Handbook Chapter 5.9.

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For inspection (grouped) data, regression is in the “y” direction as inspection times are more accurate than % failed. **Probit is best practice for grouped data from inspections.**

**SOLUTION:** In SSW, Home group, select New / Clear (blank page). See the **Wire Fatigue Experiment** data sheet in this tutorial a little after Problem #7. Suppose all were twisting wire simultaneously. The data record template for this grouped data (“probit”) is on the bottom of that data sheet. **NOTE: Operation B here skips the preparation in A.**

- A) Pick 5 for a convenient interval size in half-twists (bends) for the data in the table. Fill in the Probit section on the Wire Fatigue Experiment data sheet for the lightweight data values only, Set 1. The data format has 3 parts per entry: **inspection age** (X-Value = quantity of bends at inspection time), **cumulative quantity of failures**, and **quantity of pieces inspected** which is constant in this case at a value of 24. Do not enter data for no occurrences or where the last wires failed, since 0% and 100% cannot fit on standard probability scales. Zero change is also not entered.
- B) In the Home group, select Open/Import and load file **PlayTIME-Problem-6.w**. Delete Set # 2. What’s remains is the lower load Wire Fatigue Experiment data. Select Analysis group, Marker Line, and select Set 1 to add a Marker Line with same solution (Eta=35.82, Beta=5.407). This line marks the point-by-point data solution. Delete all the data leaving only the Marker Line. Bring in **PlayTIME-Problem-14.w**. The new solution represents the same data but collected from inspections every 5 half-twists and should agree with the entries in step A above (if you did that step). View the results, which are in fair agreement. Select Home group, New/Clear (blank page icon). Select Analysis, Method/Input, Reset To Standard.



*Wes's comments: Eta's agree, Beta's are a little different. Probit fits to actual percentages, not estimates.*

**\*NOTE: If you are in the second day of a workshop and you have finished with this problem (#14), then you have completed the computer work for the second day. Take all of your materials with you, go back to where you are staying, and complete the homework reading assignment. Then, get plenty of rest so you can be ready for the activities of the third day.**

### Problem 15. Better Methods for Solving the Classic BiWeibull

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[Problem 4](#) Continued: When we looked at this data earlier, we concluded the first 8, last 4 was the best split. There are other ways to analyze this mixture of two Weibulls in the same set of data. A simplistic method is to find the best partition for the data by maximizing the p-value of the likelihood ratio test iteratively for all possible “partitions” as follows.

**SOLUTION:** In SuperSMITH Weibull (SSW), select New / Clear. Then retrieve the **PlayTIME-Problem-4.w** file.



- A) Select Analysis group, Mixture. Select “1 ... Activate **SSW 4**” (4-Parameter Competing Risk mle Fit). The more different the two sets are, the more likely it is we have the correct partition. The plot shows the best split based on the modified mle likelihood ratio test in SuperSMITH Weibull. Log likelihood here is about -45.736.
- B) **BETTER:** Select Analysis group, Mixture. Select “F... Fit Method for SSY”, toggle until it is “mle”. Activate SSY 4P. This uses SuperSMITH YBath™ to optimize likelihood. Log likelihood here is -45.676 (less negative!).

*Wes's and Dr. Bob's Comments: Using the basic partition method with modified likelihood ratio test provides only a weak indication a Bi-Weibull solution is appropriate with partition between the first 9 and last 3 points. A likelihood ratio “lr” test (easy with Distribution Comparison in YBath software) shows only a 69% indication for mixture which is less than our standard 90% goal, so we stay with the simpler 2-parameter Weibull. You should always use caution with small samples like this. We recommend a minimum of 30 failures for useful 4-parameter mixture solution.*



Analyze directly in SuperSMITH YBath (SSY). From SSW, select the ?/+ group, then select YBath. YBath is prioritized for mixtures, so options are listed on the Analysis tab. Load **PlayTIME-Problem-4.w**. Try 5 and 7 parameter fits. You will get a message that mixture indication = small and analysis will revert to a simple Weibull fit. Select SSY4 again to get a mixture. Select Tools tab, then Distribution Comparison. This lists all fits run and shows  $r^2$  or  $\text{Ln}(L)$ . Notice the highest, or least negative,  $\text{Ln}(L)$ . The LR Test column compares the mixture fits to the 2 parameter fits (See chapter 7 in the handbook).

*YBath mle analysis provides results like SSW but better. YBath does not require a partition!*

### Problem 16. Are two data sets significantly different?

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Does changing load weight affect a wire's twist endurance? A very important question to answer is how to compare two or more data sets for significance difference, like for example comparing new design versus old design. Reference: Handbook Section 7.5. Let's use the wire experiment data from Problem 6 for this.

**SOLUTION:** In SuperSMITH Weibull (SSW), after New / Clear, retrieve the **PlayTIME-Problem-6.w** file data we analyzed before. Verify equations setting is either Weibull 2-parameter (no 3<sup>rd</sup> parameter t0) or Lognormal 2-parameter.



- A) Are the two sets significantly different at the 90% level... 95% level? To find out, Select Options, Calculator, and then select Set Comparison, L...Modified Likelihood Ratio Test. Is the pff-value shown above 90% (Significant Difference)?... above 95% (Extreme Difference)? This method using the Fulton Factor (pff value) unbiasing is more accurate for small samples than the standard likelihood ratio (lr) test.

- B) A complimentary test is whether likelihood contours intersect. Return to Main Screen, select Analysis group, Method / Input icon, then Modified mle (mmle) with reduced bias adjustment “RBAjmm”. Return to Main Screen, Select Options, Calculator, then Set Comparison again, but this time select C...Modified Likelihood Contour Test. Select View Plot With SSV, Yes, select 90%. You are now in SSV. Do the contours overlap? If the contours do not intersect, they are significantly different at least at the level selected (90% in this case). Exit out of SSV (no save) to return to SSW. **Select Analysis group, Confidence, Delete icon or “None” to delete confidence.**

NOTE: You can access Set Comparison in several places. **EXTRA EXERCISE:** If we generated two data sets from the same parent Weibull would they be different? Select New / Clear, then Sample icon (dice), OK, until two random sets of data from the same parent appear on the plot. Different at the 90% level? (The pff-value should exceed 90% only about 10% of the time in many trials.) Use the likelihood ratio test and the contour test as above to see.

### Problem 17. Accelerated Test Analysis (PFEV)

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If units are tested at two or more levels of load, what is the best practice to analyze the data? Reference [The New Weibull Handbook](#) Chapter 6. The simplest analysis is to make Weibull Eta a function of load. This will allow estimates for untested values of the acceleration parameter, like for example 0.5 LBS as demonstrated here.

**SOLUTION:** In SSW, select the New ... Clear Data icon, then load file **PlayTIME-Problem-6.w** into SSW as before.



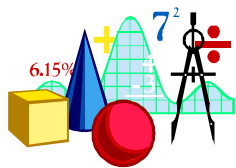
- A. Reset equations to standard Weibull 2-parameter, rr solution, then select Tools, Accelerated Test (showing a plot with parallel lines). From the menu select Standard PFEV, then select Find 1 Value For All, and enter LBS as the Engineering Variable Name.
- B. Enter 2 for 'Set 1 LBS' and 2.6 for 'Set 2 LBS'. Select PFEV Fit Type, then Power Fit. Scroll down and select Plot Line At New Variable Value, enter 0.5. Results for 0.5 are the Marker Line to the right on the plot.

*Wes's comment: You can solve for multiple load types, one less than the number of data sets (e.g. weight, temperature and humidity with 4 data sets).*

### Problem 18. Bomber Data Crow-AMSAA (CA) Reliability Growth in Visual (Handbook Example. 9-3).

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**SOLUTION:** From SSW, Main Screen, select “+/” group, and start SuperSMITH Visual (SSV). Select New / Clear. Load file **PlayTIME-Problem-18.v**. In the data grid, 1<sup>st</sup> column, Engine Flight Hours (EFH) are X (on the left), and Cumulative Failures are Y (on the right) with “x1” following. Plot is for raw data, not Crow-AMSAA (CA) yet. Select Tools, then Crow-AMSAA. Verify Data Input Type is “Already Cumulative”. Verify M...Method = \*IEC. Select Activate to view the plot (Lambda=.01521, Beta=.933). With CA plot active, return to Tools, then CA icon. Toggle I...Instantaneous = Yes, and R...Rate Transform = Yes. Plot has Cumulative and Instantaneous lines. Instantaneous line is below Cumulative and sloped down as CA Beta is less than one (decreasing occurrence rate). **Exit out of SSV (no save).**



### Problem 19. Inspection Option #1 versus Kaplan-Meier (KM) for Warranty Data

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Warranty claims in (months-of-age)x(quantity) format are... 1x27, 2x10, 3x13, 4x5, 5x4, 6x3, ... plus suspensions in similar format... -4x589, -5x787, -6x585, -7x1013, -8x464 ... for 3500 total units produced. Compare Inspect Option #1 (“Inspect1”) and Kaplan-Meier.

#### **SOLUTION:**



- A) In SSW (Weibull), Home group, select the blank page icon to clear and start. Confirm the equation label shows “Weibull”, and if not ... change to Weibull with Equations label. Load file **PlayTIME-Problem-19-Inspect1.w**.
- B) The data should show: **1x27, 2x10, 3x13, 4x5, >4x589, 5x4, >5x787, 6x3, >6x585, >7x1013, >8x464**.
- C) Plot should show Eta=13546 and Beta=0.512. What kind of component is this? (answer in comments below)
- D) Use Analysis, Marker Line, Set 1 to mark this “Inspect1” fit. Then, delete the data set (Marker Line should stay).
- E) Load file **PlayTIME-Problem-19-Kaplan-Meier.w**. Method will change to “km4n”.
- F) The data should show: **1x27x3500, 2x10x3473, 3x13x3463, 4x5x3450, 5x4x2856, 6x3x2065**.
- G) The plot should show Eta=15640, Beta=0.503. Good agreement with Marker Line! Return to the Main Screen.
- H) Select Analysis, then Confidence icon. Verify Method is Greenwood's (“gw”), Select C”...Type/Value, Double, 90 %, then ‘A’ or checkmark to Activate.
- I) How do Inspect Option 1 and Kaplan-Meier compare? Which is best? (see below)

*Dr. Bob's comments: The Kaplan-Meier method has statistical rigor, best practice, for warranty data when a single component or single failure mode is the application. However, Inspect Option #1 is a widely-used good competitor. Here the two methods agree well. We ignored the KM actuarial correction. It is more rigorous and is illustrated in the warranty matrix analysis problem. If we had properly subtracted that correction, half the suspensions, the last three inputs would be: **4x5x3156, 5x4x2463, 6x3x1773**. For multiple repairable failure modes, use Wayne Nelson's method for recurrent events (RE). Many people use KM even though it is not intended for repairable systems. For predicting total claims by calendar month (instead of age in months) Crow-AMSAA is best practice as shown later (warranty matrix).*

*Wes's comment: Inspect Option #1 solution is close to KM solution. **Beta < 1** very often indicates area studied is electronics.*

### **Problem 20. Review ... Are Two Weibull Data Sets Significantly Different?**

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There are many methods and assumptions for this. Which is best practice? Let us try methods recommended in the Handbook chapter on confidence calculation including how to compare data sets for difference.

**SOLUTION:** At the SSW Main Screen, Home group, select the blank-page icon (New / Clear), then select the Setup icon (computer w/ talking heads), verify dimensions are at least 12x1500 (reset if less than those) ... return to Main Screen.



- A) Select Analysis, Method/Input icon, Reset-To-Standard Icon (u-turn arrow).
- B) Select Analysis group again. In Method/Input, select Modified Maximum Likelihood icon (mmle ...).
- C) From Main Screen, Home group, select Sample (white dice). Verify Sample Distribution = Weibull. Select Sample Parameter ... set Eta=100, Beta = 3, then Activate/OK to continue. Set N...Point Quantity = 5. Verify U...Type = "Monte Carlo(MC)" (not "Precise" or "SFS"). Confirm S...Set Quantity = 1. Activate for plot. Select Sample icon twice each time and repeat until you have 10 sets with 10 fit lines on the plot.
- D) In the data grid, for each data set, select at the top (highlights entire data set), then use the Data Wizard button (magician wand), then Set Label to enter "A" for Set 1, "B" for Set 2, etc. for all sets up to "J" for set 10.
- E) Plot will show new set labels. By the plot, decide which two sets by Letter are most different (different etas or betas or both). Keep those. Using the data grid, delete the other data sets (Select at the top of each of those data sets to highlight, then select Delete, confirm). Two data sets remain with the most difference (artificially created).
- F) **Jim Lempke's method:** Select Analysis, then Confidence, choose Method and set to Modified Fisher Matrix (fm), Select Type/Value of Double, 90%, Activate. Compare B10 values on the plot, if the upper bound on the lower set is less than lower bound on the upper set (separation), difference is indicated at 95% confidence (single bound level corresponding to double-sided 90%). Use this method only if just one vertical (%) position matters.
- G) **Modified Likelihood Contour:** Return to Main Screen, Options, select Calculator then Set Comparison. Select C...Modified Likelihood Contour Test (MLCT). Select View Plot With SSV, 90%. Change Fit Type to RBA if asked. SSV should automatically start and display 2 contours (uncertainty regions for solution). If contours overlap then no difference is indicated, otherwise difference is indicated (at the selected confidence). The plot title should indicate the pFF-Difference (%) using the Modified Likelihood Ratio Test (MLRT). MLCT and MLRT results should not disagree more than a few %. Exit out of SSV (no save needed), back to SSW.

*Dr. Bob's comments. The modified likelihood contour test with the mle-RBA solution point and with Wes Fulton's justified likelihood function (JLF) for the similar modified likelihood ratio test are our two best methods accounting for small sample bias. Both can work with more than two sets.*

### **Problem 21. Probabilistic Design using Monte Carlo (MC) Simulation**

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The latest design has a predicted life capability distribution (age-to-failure) that is Weibull, eta = 2000, beta = 6. The usage distribution is Lognormal, Med = 500 (Median) and SigF = 1.4 (Sigma Factor). Failure occurs if the usage is greater than the age-to-failure. Estimate p, the proportion that will fail.

**SOLUTION: Start in SuperSMITH® Weibull (SSW) by selecting Home group.**



- A) Select New/Clear. Still in Home group, select Setup icon (computer with talking heads), verify dimensions are at least 12x1500. Return to Main Screen. In Analysis group, Method/Input, select Reset To Standard.
- B) From Main Screen, select Options, Calculator, Interaction, then Life-Usage to get the SSW calculator **Life-Usage** menu. Set Life Distribution to Weibull, Eta(1) to 2000, Beta(1) to 6. Set Usage Distribution to Lognormal, Med (2) to 500, SigF (2) to 1.4. Select checkmark (OK) or Activate for results (takes a few seconds). Results may vary slightly with each MC but should be about 0.2% failure with +/- 0.05 (5E-02%) precision. Select **OK**.
- C) In menu, select **Save Interaction To Data Set**, set the point quantity to 100 to see the resulting distribution.
- D) As there may be negative values that are not suspensions, from Main Screen, select Equation Label (green for Weibull) and shift to the Normal distribution. If asked, answer yes to "negative values < 0". Note that Weibulls, Lognormals, and other models may be convoluted here. **Change from Normal back to Weibull [2P ...]**.

*Comments: Your results could be slightly different. These predictions are based on one design life. Results vary with each simulation when using different starting random number seeds.*



### **Problem 22. Fracture Mechanics**

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Based on the design of a new turbine blade and selected alloy, time-to-crack initiation is expected to be Weibull with  $\eta=500$  and  $\beta=3.4$ . Tests of crack growth time to rupture show it is Weibull,  $\eta=2000$  and  $\beta=6$ . Total life is the sum of these two. How is total life distributed? What is R99% life of this turbine blade?

**SOLUTION:** To get the answers, start in SuperSMITH Weibull (SSW), Home group, New / Clear (blank page icon).

- A) At the Main Screen, select Options, Calculator, Interaction, and select Interaction: Random Point ... Sum.
- B) Enter Distribution 1 (Weibull,  $\eta=500$ ,  $\beta=3.4$ ), and Distribution 2 (Weibull,  $\eta=2000$ ,  $\beta=6$ ).
- C) Select OK (starts simulation). Results are stored in a data set and automatically plotted. Use Tools group, Distribution Comparison, to find the best model. Switch to the best equations (if menu item offers change). At Main Screen, select Options, Precise Reading (icon showing four arrows), Set 1, B1. "Datum" is B1 life. Answer should be near 1300 (add t0 if needed for 3P result). Select **Analysis group, Method/Input, Reset To Standard**.



### **Problem 23. The Binomial distribution can be evaluated with the Calculator option.**

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**This may be useful for Quality Control Sampling inspection**

A manufacturer wants to demonstrate that percent defective for his part is less than or equal to 0.1% with 90% confidence.

**SOLUTION:** In SuperSMITH Weibull (SSW), select Home group, New/Clear.

- A) On the SSW Main Screen, select Options, then the Calculator icon. Select Binomial, then in the "Binomial Find?" menu ... select Trial Quantity. Enter Probability of 1 Event With 1 Trial (in %) as 0.1 (so Reliability = 99.9%). Set the Event Quantity = 0. Verify the Cumulative Probability (%) is 10 (so Confidence = 90%).
- B) ANSWER: Output Trial Quantity = 2302 pieces as shown in Table 8.1 in the Handbook. Return to Main Screen.

**NOTE:** *Using the binomial (counting, not measuring) to demonstrate 99.9% reliability at 90% confidence with no failures, 2302 successful tests are required! Measuring instead requires much less effort in terms of testing.*



### **Problem 24. The Poisson distribution applies to random events defined by an average rate.**

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An electric utility averages 8 outages per month in its distribution network. What would a bad month produce, say at 90% confidence? In other words, what is the upper 90% one-sided confidence bound for outages per month?

**SOLUTION:** The 90% bound will be the number of outages that has Poisson cumulative probability just exceeding 90%.

- A) From the SSW Main Screen, select Options, then Calculator, Poisson, **Expected Quantity** = 8
- B) Try different values of **Event Quantity** (>8) until the Cumulative Probability (shown in menu header) just exceeds 0.90 (9 gives 0.716, 10 gives 0.816, etc.). See Example 8.8 in the Handbook. Return to Main Screen.

**Comment:** *12 events is the upper 90% one-sided confidence limit with average expected quantity = 8.*



### **Problem 25. Magic... Did Murphy really delete an important plot?**

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You just shut the SuperSMITH Weibull software down and turned off your computer. But WAIT! You forgot your boss wants a copy of the plot you were analyzing in SSW. But you didn't save anything! **UH OH!** Should you get a new resume ready? No need for that. Try the following...

**SOLUTION:**

- A) Start the SSW program, get a plot (put in data if needed). Remember the Beta value (for verification later).
- B) Exit entirely from SuperSMITH, but do not save anything (recreating the **UH OH** moment).
- C) Go back in by starting the SSW program again. Select Home Group, Former File.
- D) Select Get Last .tmp Results ... Magic!!! View the "lost" plot retrieved from previous results. Now, you can copy the plot for your boss! No new resume needed.



## Advanced Problems

### Problem 26. “YF26” Systems Analysis for INDEPENDENT Failure Modes (Handbook section 4.6.1)

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	Control	FOD	Bearing	Turbine Blades
ETA	30000	1200	700	600
BETA	0.5	1.0	2.0	6.0

With above Weibulls for principal independent failure modes in a repairable system, forecast failures for the system.



**SOLUTION:** In SSW, confirm Weibull equations (green label), Analysis group, Method/Input, Reset To Standard.



- 1) Home group, select New/Clear. Load file **PlayTIME-Problem-26.w**. (Ignore any “Not Enough ... “ and/or “Y-Scale” messages. Data should be **>10x10, >20x10** (all suspensions). No solution, but maybe a message or two.
- 2) Select Tools, Abernethy Risk (AR), verify Risk Analysis Type is \*STANDARD FORECAST.
- 3) Toggle so that **Format = CUMULATIVE**, **Renewal = \*YES**, then select Usage, Single Rate, enter 25.
- 4) Select D...Data, after verifying Data Set = #1, select Analysis (Weibull) Parameter, “No” to Select Present Fit, select Standard. Enter the **Control** values for Eta and Beta, i.e. **30000** for Eta and **0.5** for Beta (leave t0 as zero).
- 5) Select green check-mark, view output table. The 60<sup>th</sup> month **Control** cumulative risk should be about 4.8.
- 6) Select Visual Plot icon above forecast table, view failure forecast plot. Back at the Visual Main Screen, Save Active Results as **CONTROLCUM.RSK**. Exit SSV, return to the Abernethy Risk (AR) menu.
- 7) Repeat steps 4-6 above except using appropriate Eta and Beta values for each remaining failure mechanism (FOD, Bearing, Turbine Blades), each time saving the forecast plot to an appropriate filename (like **FODCUM.RSK**, **BRGCUM.RSK**, and **BLADECUM.RSK** respectively). View the 4 risk functions together as follows.
- 8) Back at the SSW Main Screen, select “?/+” group. Select the Visual icon to start SSV. Select Home group, New / Clear, then load the 4 files one after the other. They should come in as 4 separate data sets on the SSV plot.
- 9) We want to sum the 4 risk functions to obtain the system curve. So, from the SSV Main Screen, Analysis group, select the transform icon [“F(a)”], answer “No” to save if asked. Choose transform # 6...Total Y-Value... then “Yes” to X-values the same... then view the resulting “Total” curve in Set #5 (above the rest).
- 10) This newest curve is the sum of the risk for each independent failure mode, a system risk forecast.



*Dr. Bob’s comment: In SSV, after New / Clear, retrieve the file **YF26TRSK.V**. This should match the plot you just did. The System Rhythm for monthly intervals may be obtained by repeating the above process using **Format=Interval** ... or ... if that is a little too much work, the short cut is you can clear with the “New” icon and retrieve **YF26RTHM.V**. System optimum replacement may be convoluted the same way. Start with New / Clear and then retrieve **YF26TREP.V** to see the system optimum replacement.*

### Problem 27. Warranty Claims Data Format and Forecasting with Weibull and Crow-AMSAA

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Reference: Chapter 9 in the Handbook.

**Background:** There are two main types of data used for warranty forecasting of future claim quantity:

- 1) By calendar month in calendar time. Crow-AMSAA (C-A) is best for this ... only for near term.
- 2) By calendar month by equipment age. Weibull using “Inspect #1” fit option is best for this.

Let’s try a failure mode mixture example, the most difficult type of warranty issue. An automotive component has 4 Weibull failure modes, A)  $\beta = 0.5$ ,  $\eta = 250,000$  miles; B)  $\beta = 1.0$ ,  $\eta = 200,000$ ; C)  $\beta = 2.0$ ,  $\eta = 150,000$ ; and D)  $\beta = 6.0$ ,  $\eta = 80,000$ . Monte Carlo simulation created a warranty claims array for the first 12 months of production (“MOP”). Results were saved in text format file **WAR12NR.WAR**, and shown in the triangular-shaped data table below. 100 units were produced each month (shown in the first row of the table). Warranty claims were reported at month intervals (in the body of the matrix). Rows 2 - 13 represent months in service (“MIS”) 1-12. The first month’s production of 100 units produced 9 claims in the 1<sup>st</sup> month, 2 claims in the 12<sup>th</sup> month. The most recent production (right-hand column) resulted in 4 claims in the 1<sup>st</sup> (and only) month of service. Data can be blocked, copied, and pasted from a spreadsheet program like Microsoft Excel (R), or loaded from a standard text file like **WAR12NR.WAR**. Copy/Paste from external data source does not get altered with FULL version (DEMO alters).



## Text in the WAR12NR.WAR file

100	100	100	100	100	100	100	100	100	100	100	100
9	5	12	7	5	3	6	6	10	9	6	4
5	4	3	1	6	3	0	3	1	3	3	
2	2	1	0	1	6	4	2	2	2		
2	3	4	2	1	2	3	3	1			
4	3	3	1	4	2	1	1				
1	1	2	1	1	1	1					
4	1	2	2	1	1						
2	4	0	4	3							
3	1	1	2								
2	3	1									
2	2										
2											



**SOLUTION:** Exit from SSV if there. At the SuperSMITH Weibull (SSW) Main Screen, Home group, select New (blank page icon). Select Analysis, Method/Input, then Reset To Standard (U-turn arrow).

- A) Enter the data as follows. Home group, Sample icon (dice), Special Data Set, then item 10A Warranty.
- B) Verify Method For Warranty is Insp1. Select W...Format Warranty to get the Warranty Import Data Format (WIDF) menu. At the WIDF menu, verify MOP vs. MIS (or vs. MOA) is selected. Then choose Activate. Verify all “= No” for Limit Correction, Renewal, and Warranty Limit.
- C) Select OK to solve and plot. You should get Eta=61.83, Beta=0.667. If not, change Method to Insp1. “o/s” on the large plot shows 227 failures now (if “n/s” is on plot, it shows 1200/973). At the Main Screen view data grid. First entry is “1x82”, meaning 82 claims at 1 month of age. Four failures in the last month’s production leave 96 suspensions at 1 month of age which is the second entry (“>1x96”) ... and so on.
- D) Select Tools, Abernethy Risk. Select No Modified Options, verify with “Yes” if asked. Set Usage Rate (Single Rate Value) = 1. Select Activate. Claim quantity in 24 months should be about 329.
- E) **Return to SSW Main Screen. In “?/+” group, select Visual icon to start SSV.**
- F) Enter the data as in operation A before, select option W, then A, and again A to get the Crow-AMSAA (C-A) plot with C-A solution Lambda=0.246 and Beta=0.774. Enter a second data set as follows.
- G) From the SSV Home group, select Sample icon (dice), Special Data Set, then item 11A Warranty.
- H) Select option W...Format: Warranty, B... \*Format = MOP ... MIS (or MOA), A...Activate, A...Activate.
- I) Results on the C-A plot for the second data set should be Lambda=0.431 and Beta=0.702.
- J) Select Tools, then Crow/AMSAA for the C-A menu. Use Extrapolate Fit with Lower X=100 and Upper X=26064 (last x-value in data set #2). Set3 fit line’s last y-value is about 644. See Dr. Bob’s Comments below.

*Dr. Bob’s Comments: Weibull Insp1 forecasts accurately **329** additional failures in the next two years. By inspecting the C-A data grid in SuperSMITH Visual and subtracting the 12-month cumulative claims (227) from the actual 36-month claims (=545), the true 24-month future claims are **318**. C-A didn’t do well in this case because of the cusp at 12 months. C-A over-predicts to **417** (= 644 – 227). This is just one comparison, so these results should not be generalized.*

The following can only be done with the FULL software version, because the DEMO will alter text files entered without previously being saved from the FULL SuperSMITH(R) version . . .

For more warranty data analysis, you can repeat this exercise using data with renewal. Try **WAR12R.WAR** and **WAR36R.WAR**. Kaplan-Meier and Inspection Option#1 are not intended for renewal. C-A is the best for renewal data, especially for near-term prediction (only a few months into the future).

### **Problem 28. High Resolution on Batch Effects with ACH**

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David Nevell (working at times with Geoff Cole) of Rolls-Royce developed a new graphical technique for evaluating model fits to the data called aggregate cumulative hazard (ACH) available in SuperSMITH®.

**SOLUTION:** Start SSV (Visual) if not already there, in Home group, select New/Clear. Still in Home group, select Open/Import and bring in **18WHEELR.W** and select "A...Aggregate Cumulative Hazard". You will see the ACH plot. For rr fit the ACH line does not match with the final (top right) point. The ACH plot shows model to data compatibility. If the ACH fit line reasonably follows the data, the statistical model is appropriate. If there is significant deviation, the model might not be appropriate. The disparity with the rr solution may indicate a mixture or a batch (only a portion of the units can fail). In Home group again, select New / Clear. Load file **18WHEELR-mle.W**. Select ACH again. For mle fit, the ACH line always goes through the final data point. The mle solution in this case shows ACH line matching reasonably well to the data not indicating a batch issue. All of us (David, Geoff, Carl, and Wes) would be interested in any ACH applications you may find.

### **Problem 29. Barringer Process Reliability - BPR [Section written by Paul Barringer]**

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If in SuperSMITH Visual (SSV), exit (no save) and then start SuperSMITH Weibull (SSW). In SSW, at the Main Screen, Home group, New/Clear, then open data file **PR.W** containing 365 days of production process output (in Mlbs). Select and view the plot. The production data looks like a mixture of information with several different data groupings evident on the plot. The standard fit line does not fit the data well and clearly does not relate to traditional failure mode details, since the data represents a business situation. What is the reliability of the process? What are the cutback losses? What are the crash-and-burn losses? If the process is deemed capable of achieving a  $\beta=75$ , what is the nameplate rating? What is the demonstrated production quantity? What are the efficiency and utilization losses? Lots of questions! They can be answered with Barringer Process Reliability (BPR) analysis.

**SOLUTION:** The BPR plot ...



- A. The standard BPR vertical scale uses reliability (complement of failure probability). For the **PR.W** data set entered, the upper data values follow a straight-line trend down to the middle 50's [%]. Return to the Main Screen, select Tools group, Barringer Process icon to get the BPR menu.
- B. Select Z...BPR Zoom from the menu, then choose F...X-Range + Y-Range. Use the little arrowhead boxes (left and right boxes) on the axes to set horizontal plot limits 600 (x-lower) to 1200 (x-upper). In the same way use the top and bottom arrowhead boxes to set vertical reliability plot limits 85 [%] (y-lower) to 0.1 [%] (y-upper). Then use the <ESC> key or select the green check mark to return to the BPR menu.
- C. From the BPR menu choose "Set 1 Production Line", Plot Line Drag. Move the mouse pointer (crosshairs) to where the upper data point trend first changes direction (approximately 53.4%, "Process Reliability" is defined at this point). Select and hold, then drag a line up to fit the upper segment of data points, and release. The line drawn should indicate Eta near 1003 and Beta of about 24.5 representing the demonstrated "Production Line".
- D. In the BPR menu again, select Set 1 Nameplate Line". Then select H...Plot Point Fit and drag a box around only the single highest data point (highest productivity). Enter 75 when asked for Beta value. This gives you Eta=1056 and Beta=75 and for the "Nameplate Line" (the benchmark goal for production).
- E. From the BPR menu select Z...BPR Zoom, then select option F, and restore x-lower=100, x-upper=1200, y-lower=99.9, and y-upper=0.1. (*Notice the final slide to very low production occurs at 98.2% reliability. This identifies an important reliability point used for crash-and-burn calculations. A line through these low points would have eta=33158 and beta=1.*) Choose <ESC> or select the green checkmark to return to the BPR menu.
- F. At the BPR menu, choose item G...Set 1 Process Reliability for identifying change points in the data. From the next menu select G...On Plot, and then select the three major data trend change points. The first one is the Process Reliability point #1 (at 53.4%), then point #2 (81%) where a change in the data occurs, and finally point #3 (98.2%) where the crash-and-burn segment begins. To exit the plot when you are finished entering change points, use the <ESC> key. Results are in menu item "G".
- G. In the BPR menu again, press the <ENTER> key or select the green check mark to get the BPR analysis results.

*Paul Barringer's Comments: Production losses from reliability issues are ~29,000 Mlbs in one year. There are three segments: 1) from loss of production reliability (point 1) to step at 81% (point 2) loss is ~12,000 Mlbs, 2) from step at 81% to beginning of crash-and-burn at 98.2% (point 3) loss is ~14,000 Mlbs, and 3) crash-and-burn losses (from 98.2% to 100%) are ~3,200 Mlbs. These losses define the gaps between the Production Line and actual data values. Efficiency and utilization losses between the Nameplate Line and the Production Line are ~24,000 Mlbs. The most important issue is reliability (29,000 > 24,400). Within the reliability losses, the most important is from 81%-98.2% reliability (14,000 > 12,000 > 3200). Performance records with root cause can help address these issues and improve process control.*

### **Problem 30. Step – Stress Accelerated Testing and Analysis**

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Your boss wants a step-stress test on 15 items for quick results. The item's useful life is 768 start-up cycles at rated load of 10 ksi. The step-stress schedule has 10 steps starting at 10 ksi and increasing at 10 ksi intervals to 100 ksi at the 10<sup>th</sup> step. The time at the first step is 2/3 of the item's useful life and every test duration length after the first step is one half the number of cycles of the step before. Reliability required for useful life is 99.9%. But . . . the part will be used only at half rated load. Show the test plan and whether the item meets reliability. Your completed test gives the following results (cycles).

639	760	767	789	812	836	>840	858
888	912	914	930	932	>950	>950	

Note: The ">" sign indicates suspensions)



**SOLUTION:** In SuperSMITH Weibull (SSW). At the Main Screen, select Home group, New / Clear. Select Open / Import and load file **PlayTIME-Problem-30.w**. Select Analysis group, Method/Input, verify fit solution method is rr (rank regression). Resulting plot should show Eta=907.57 and Beta=9.655 (*this Beta is too high, unrealistic for a single load level, before load sequence entry!*).



- Start at the SSW Main Screen, Tools group, and select the Accelerated Test (A.T.) icon.
- Choose Step Load, enter the load name "ksi", then option "K", enter 10 for Step Quantity.
- Enter loading sequence ... load-level (Var. Value) vs. time-at-load as prompted one-by-one:

#### **Variable Value / Time**

10	512
20	256
30	128
40	64
50	32
60	16
70	8
80	4
90	2
100	1

- Verify sequence shown, then select OK (or select Change to edit) ... after some calculation delay, results are shown on the PFEV dialogue box (with options for further analysis). Results should indicate a Beta of approximately 2.5, V of about 300, and p of about 2.5 (answers depend on fit method selected).
- Select Plot Line At New Variable Value, enter 10 (adds lower slope line for 10 ksi). Return to Tools, Accelerated Test for the PFEV A.T. results. Select L...Plot Line again to enter the actual operating load of 5 ksi. You will then have marker lines on the plot for 10 ksi and 5 ksi. Line 2 (to the right) is for 5 ksi.
- Return to the Main Screen, Options, Precise Reading, choose Line 2, and enter Datum of 768 cycles.
- Does the item meet 99.9% reliability required? ANSWER: Yes... Precise Reading output indicates over 99.95% Will NOT Occur. BONUS: To view the step load schedule, return to the Main Screen, Tools, Accelerated Test icon again, choose View Step Load Sequence for SSV.
- Exit SSV (no save) when done looking at the step load sequence.



**Problem 31. Vasan-Fulton Former Usage To Test Ratio (FUTR).**[TOC](#)

Your customer needs rapid development capability that can only be accomplished with accelerated development testing performed in-house. There are many pieces of equipment out in service without failure, good news! However, you need to get an acceleration factor for your in-house test. The acceleration factor is the life multiplier value you use to convert from in-house test results to in-service results. You bring some of the surviving in-service equipment from the field into your lab for life testing to failure. The equipment with one year of in-service operation fail at 2, 7, 7, 16, 22, 25, 25, 26, 28, 34, 49, 49, 57, and 59 hours (gives  $\text{Eta}=33.11$ ,  $\text{Beta}=1.229$ ), the equipment with two years of in-service operation fail at .2, 8, 14, 18, 20, 25, 27, 29, 31, 38, 39, 40, 52, and 85 hours (gives  $\text{Eta}=35.45$ ,  $\text{Beta}=.906$ ), and the equipment with three years of in-service operation fail at 6, 9, 15, 16, 22, 25, 28, 29, 30, 30, 32, 32, 42, and 49 hours (gives  $\text{Eta}=29.88$ ,  $\text{Beta}=2.013$ ). Your goal is to determine the acceleration factor. Try the following solution ...

**SOLUTION:**

- A) Starting in SuperSMITH Weibull at the Main Screen, select Home group, New / Clear (blank page icon). Then select the Analysis, Method / Input, U-turn icon, Activate, to reset method to Weibull/rr.
- B) At the Main Screen, Home group, open file **PlayTIME-Problem-31.w** to get three data sets as listed above with solutions as indicated.
- C) Select Options, Label/Style. Then change X-Value Name from "Datum" to "Test Time". Change X-Value Units name from "Units" to "Hours". Return to the Main Screen.
- D) Select Tools, Accelerated Test, then Vasan-Fulton Former Usage to Test Ratio (FUTR).
- E) Enter "Years" for Former Usage Units Name.
- F) Enter Former Usage Value as 1 for Set #1, 2 for Set #2, and 3 for Set #3.
- G) The analysis result FUTR is then shown. The acceleration factor is the FUTR normalized to remove dimensions. Equivalency shown at top is about 0.209 usage years for each test hour (the FUTR).

**Problem 32. Verify Improved Distribution Comparison With p-value Estimate (PVE%)**[TOC](#)

Correlation coefficient squared ( $r^2$ ) indicates goodness of fit. However, it alone does not tell which distribution fits best. If  $r^2$  is the same value for both 2-parameter and 3t-parameter Weibull, the 2-parameter has an edge with fewer parameters. To gage goodness of fit, p-value is needed. P-value is the rank of a measurement within all possible values from Monte Carlo. Dr. Chi Chao Liu (now Allen Liu) performed research indicating p-value (pve%) is one of the best goodness-of-fit indicators. How well does pve% perform?

**SOLUTION: In SSW, Home, select New/Clear. From Analysis group, select Method/Input, Reset To Standard.**

- A. Starting on the Main Screen, select Analysis group, then Method/Input. Select Fit Comparison + Display, then select Correlation P-Value Estimate pve%. Back at the Main Screen, Home group, select Sample (dice), select 'W2' for distribution, Point Quantity = 5, Sample Parameter (Eta and Beta are your choice). Select green checkmark (repeat Sample operations 10 times to generate 10 Weibull 2-parameter data sets). At the Main Screen, Tools group, choose the Distribution Comparison icon. Check each set. What is the "optimum distribution" for each data set? Most should indicate 2-parameter Weibull (the true parent distribution). What percent of these sample data sets indicate different than that? Repeat a few times to verify. Exit to the Main Screen.
- B. At Main Screen, Home group, select New / Clear. At the Home group, use the Sample icon to generate 1 data set with 30 points. In the Analysis group, select Confidence, then Confidence Special Curve Type. Select "pv P-Value For r", then "No Save / No View" to get the "Select Type" menu. Choose "Yes". Select Confidence Type of Pivotal Monte Carlo "pv", Double, then 90 for confidence % to get the plot with confidence bounds after a few seconds. At Main Screen, Options group, Plot/Report icon, select Activate Report. The prr-Value(%) from quick simulation should reasonably agree with the pve% from extensive math modeling. The pve% is most accurate.

### Problem 33. AIAG Heavy Vehicle/Equipment Usage Profiles

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You have a fleet of 967 long-haul trucks in service for six months and there have been 10 failures so that ten trucks have been taken out of service (957 remain). Failure data values in terms of mileage are given in the file **18WHEELR.W**, but you only have general information about the suspensions. You decide to use the Automotive Industry Action Group (AIAG) usage profiles to estimate the suspensions.

**SOLUTION:** Starting at the SuperSMITH Weibull Main Screen, select Home, New / Clear (blank page icon).

- 1) Select the Analysis group, Method/Input icon, then Reset-To-Standard icon (“u-turn” arrow). This will reset to standard Weibull 2-parameter rank regression fit (Weibull/rr).
- 2) Back at the Main Screen, Home group, open file **18WHEELR.W**. Delete suspensions only as follows. **First, select at the Set 1 column top to highlight the entire set.** Select Data Wizard button (or right mouse select over the data grid). Select Edit Special, Yes for Total Set, then Suspension Only Delete.
- 3) The resulting Weibull plot should show Eta of 68856 and Beta of 3.614. From Home group, save Active Results to file **18WFAIL.W** for later use. Select **Home** group, **New / Clear icon** to clear data and results.
- 4) From the Home group, select the Sample icon to get the Sample menu. Select Special Data Set, scroll down then select AIAG Usage Rate (Miles / Year) Suspension Histogram **Linehaul**. **Select OK anytime “Set 1 Not Enough Data ... WeiBayes?” or “Y-Scale Results Outside ...” messages appear (this is expected when data are all suspensions).** You should get a large quantity of suspensions at different mileage values representing 1 year of usage.
- 5) Select at column top to highlight all data in Set 1. Then activate the Data Wizard again. Displayed “Point Total” should equal 76 representing “Quantity Total” 191375 entries (all suspensions).
- 6) Scroll to select **Quantity ~( $N \cdot C$ )**, answer Yes to “Total Set?”, input 0.005 for C Value. Ignore messages.
- 7) Back at the Main Screen, re-select the column top to highlight all data in Set 1 again. Activate the Data Wizard (with consolidation Point Total is now 44, Quantity Total is now 957). Scroll to select **X-Value Transform**, “Yes” for Total Set, select option  $M \cdot (*C)$ , and enter 0.5 for C Value multiplier. Ignore messages. The last, highest, suspension value in the data grid (Point44) should show “>142500x1”.
- 8) From the Main Screen, Home group, open file **18WFAIL.W** (becomes Set #2 and shows on plot). Select at top of data grid, highlight both Set 1 and Set 2. Use Data Wizard to Merge Sets 1 and 2.
- 9) The resulting plot should show the better Weibull solution for the failures including reasonably estimated suspensions with Eta=209952 and Beta=3.911. Load **18WFAIL.W** again for Set #2 (plots on the left).
- 10) Try Dauser: At Main Screen, select Analysis group, WeiBayes icon (showing “1”), select Set #2 (3.614 for Beta). Select S...Dauser Additional Suspension ..., enter 957 for suspension quantity. Verify option W...Add Weibayes Marker Line = YES. Select green checkmark. Plot should show agreement between Set 1 actual estimated and the Dauser Marker Line, but it is better to estimate the suspension values.
- 11) Select Tools group, Abernethy Risk icon, and Yes to Select Present Fit. Verify option D...Data Set is set #1. Select option C, set % Estimate to 90, Lower. Scroll down to select Usage, Enter Single Rate, 10000 (miles) per month, and Renewal = NO. OK to Activate. Result: In two years, approximately 876 more failures minimum are expected (without corrective action). Back in Risk menu, change to Renewal = Yes, Activate, and the 2-yr estimate goes up close to 956. Compare Risk Analysis Type = Standard Forecast to RBA1 Occurrence (with RBA1 Occurrence the fractions go away).



### Problem 34. Automatic Warranty Data Analysis and Time-To-Mileage Scale Conversion

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As the CFO at your company, you are responsible for warranty cost on a new production project. The new project is like “Blue Edge” which launched 3 years ago. There is 36 months’ worth of Blue Edge warranty data from field service records in file **WAR36NR.WAR** (view, but don’t import into SSW) formatted as month of production vs. month-in-service (MOP x MIS). Columns are month-by-month starting with earliest on the left. Rows are production quantities (in the 1<sup>st</sup> row) plus failure quantities by months in service starting with the first month at the top (in the 2<sup>nd</sup> row). You must determine if failures were subject to a wear-out, and if so, select the optimum replacement interval. The resulting Weibull has months of age on the horizontal scale, but we want it in terms of miles. The average accumulation is 850 miles per month estimated from the claims data only. What is the Weibull in terms of miles?



### SOLUTION:

- A) Starting in SSW at the Main Screen, select Home group, New / Clear (blank page icon)
- B) Select Sample icon (dice), Special Data Set, then select item 11A Warranty.
- C) In the following menu, verify “Method For Warranty Input” is “Inspect insp1” (toggle if needed).
- D) Choose the “W...Format: Warranty ...” menu item to proceed to the Warranty Import Data Format menu.
- E) For format, verify the “\*” symbol is in front of the “MOP x MIS” (or x MOA) menu item (select it if not).
- F) Select OK (Activate). Next menu should show “NO” for Limit Correction, Renewal, and Warranty Limit.
- G) Select OK to get the Weibull plot. Select small plot to enlarge. Solution should indicate Eta=66.27, Beta=0.666. This is the Weibull model of product life variability in months of service.
- H) Is this wear out? No ... Beta = 0.666 (< 1). No need for Optimum Replacement. Go to the next step.
- I) At the Main Screen, select at the top of the data set reading “Set 1” to highlight the entire data set. Select the Data Wizard button in the data grid, select “Edit Special”, answer “Yes” for Total Data Set if asked, then select “Occurrence Only Delete”. A message or messages will appear indicating standard solution cannot be done (no occurrences). Select Okay as needed and return to the Main Screen.
- J) Select at column top to highlight the entire data again, select Data Wizard. Scroll and select “X-Value Transform”. Answer “Yes” for the total data set question. Select “M...(\* C)” and enter 850 (miles per month estimate) to convert months to miles. Ignore messages again, since all the data values are suspensions.
- K) (For future reference, note that multiplying the Eta in months by 850 gives Eta in miles of 56329.5.)
- L) Return to Main Screen. In the Home group, select Sample icon (showing dice). Change Eta, Beta, and Point Quantity respectively to 562545, 0.666, and 545 (for real application, use actual from warranty input). Select the green check mark to generate set # 2 with occurrences in miles. Messages about Set #1 still appear.
- M) Select and drag at top of column to highlight both data sets. Choose Data Wizard. Select “Merge”, then “Set Merge 1 – 2”. Values close to Eta=667179, Beta=0.818 approximates the solution in miles instead of months.



### Problem 35. Total Uncertainty Interval Estimate – Accelerated Testing

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Electronic circuit board switches are tested at 50°C and 60°C with lifetime in cycles as shown below. The temperature gage has not been calibrated. What is your estimate of the lower 95% **total uncertainty** bound on circuit board switch B10 life at Normal operating temperature of 30°C? (Kelvin = Centigrade + 273 degrees... in case you have forgotten). The 30-degree line, which we will add shortly, will have additional uncertainties due to the uncalibrated instrumentation and the extrapolation with Arrhenius Law to the lower temperature. The bias limit from these two sources might be +/- 5% in Dr. Bob’s experience.

Cycles to Failure (50°C in Set # 1, 60°C in Set # 2)

50°C	60°C
21045	16551
25077	18935
31407	20996
33812	24363



**SOLUTION:** In SuperSMITH Weibull, Home group, select New/Clear, then Analysis group, Method/Input, Reset to Standard, Activate. Open file **PlayTIME-Problem-35.w**. In Analysis group, Confidence, select Lower icon, 95 (%). Change confidence Method to Pivotal Monte Carlo (‘pv’). Set Confidence Special Curve = NO.

- A) Activate to view the plot. What is B10 for Set # 1 with Eta=30271, Beta=4.727. By eye, it is about 19000.
- B) Return to Analysis, Confidence options, select Uncertainty Additional, and input “5” for the % (= +/- 5%).
- C) The lower bound B10 for Set #1 is about 7000. So, the 50°C lower B10 life **ratio** to nominal is about 0.37.
- D) Tools, Accelerated Test, Standard ... PFEV, Same Slope, “DegK” for Name, DegK values of 323 for Set1, 333 for Set2. Select PFEV Fit Type = Arrhenius-X. Plot Line At New Variable Value at 303 Deg K (30°C).
- E) Reduce the nominal B10 life at 30°C by multiplying it by the **ratio** found earlier. This estimates the lower 95% Uncertainty bound. You should get a value close to 15,000 cycles.





### **Problem 36. Total Uncertainty Estimates for Comparative Testing**

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You have carefully designed a comparative test using 8 units of the old design and 8 units of the new design. The cyclic counter has been calibrated to NIST standards of  $\pm .1\%$ . The order of testing has been randomized such that old units and new units are tested in a random sequence. The definition of failure is clear and unambiguous. Failures will be obvious. The test objective is to determine if the new design is better than the old design. You plan to use the likelihood ratio and contour methods to test the new design data set to determine if it is significantly better at 90% confidence. At this point your boss steps in and criticizes your design as too expensive. He suggests that you just test the new design units and use the old design data from a year ago. You compliment the boss on his great cost saving suggestion, but you add that it will make it more difficult to prove they are significantly different, as the test uncertainty will increase. What steps would you take in the analysis to account for the fact that prior tests over several years on the old design show a spread of  $\pm 2\%$ ? How would you develop an operating characteristic curve to show the boss how much his cost savings will degrade the ability of the test to identify a real improvement?

**SOLUTION: Note ... This example is a thought experiment (channel Albert Einstein)**

- A) The closer the old design Weibull is to the new design Weibull the less likely there is a significant difference. In the worst case (false indication of improvement) the old design data might be low and the new design might be high. Therefore, copy the old design data set and multiply the values by 1.04 (2% one way + 2% the other).
- B) Now run the likelihood ratio and contour tests on the modified old design and the new design.
- C) How would you develop an operating characteristic curve to show the boss how much his cost savings will degrade the ability of the test to identify a real improvement?
- D) Look at eta improvements of 50%, 100% and 200% compared to the baseline. Use SuperSMITH Weibull to generate 40 pairs of random data sets for each level and test each pair to determine the percent that show a significant improvement.
- E) Now increase the base line eta by 4% and repeat.
- F) Plot the results against the eta ratio.

### **Problem 37. Total Uncertainty for Crow-AMSAA**

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Your Crow-AMSAA warranty forecast for the next 12 months is 1000 claims. Your Vice President asks, “What is the uncertainty on that prediction?” What is your answer?

**SOLUTION: This example does not use the software!** The best we can expect for 12 months ahead C-A forecasting is  $\pm 10\%$  uncertainty. See [McGlone] in The New Weibull Handbook© references. Note that although 10% sounds large, the C-A model is the best predictor among the 27 different reliability growth models tested in a previous study.

### **Problem 38. Bearing Probabilistic Design Function (this one uses the software!)**

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**Bearings failures form Weibull three parameter distributions (Reference: “Standard Handbook of Machine Design,” 2<sup>nd</sup> Edition, by Shigley and Mischke, McGraw-Hill, 1996, Section 27.3).** SKF Industries used failure data from 2500 bearings to develop a bearing design function for reliability as a function of load and B life for all new designs. The new bearing Weibull B10 life is determined at the specified load by design analysis. The design function is a three parameter Weibull,  $\eta=4.439$ ,  $\beta=1.483$ , and  $t_0=0.02$ . Time (t) equals the ratio of B life to B10 life. What would the reliability be at a life of 10% of the B10 life? (B10 here is 1.0 and so B-Life/B10 is 0.1) . Use “Precise Reading” to determine reliability as follows.

**SOLUTION:** In SSW, select Options group, Precise Reading icon. Select “Q...Enter...”, then input Eta and Beta as above. Select option “A...Enter Datum” and input the value  $(t - t_0) = (t - 0.02) = (0.1 - 0.02) = \mathbf{0.08}$ . View results.

*Dr. Bob’s Comments: With loading at 10% of B10 life, predicted reliability is 99.74% (will not occur).*

### Problem 39. Tolerance Stackup Interference

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(Reference: “Technical Aids,” by Lloyd S. Nelson, ASQC, 1985)

Shaft and hole diameters are both normally distributed. Hole diameters have Mu (mean) of 104 and Sig (standard deviation) of 1.5. Shaft diameters have a Mu of 100 and Sig of 1.0. What is the probability that the shaft will NOT fit in the hole if the clearance must be (a) at least zero and (b) at least 0.4?

**SOLUTION:** Start in SSW. Use Options, Calculator, Interaction. Select Random Point ... Difference. Input hole parameters (both = Normal) as distribution 1, shaft diameters as distribution 2, then activate for simulation. For (b) change the shaft diameter to 100.4.

*Dr. Bob's comments: Lloyd Nelson assumed the dimensions were normally distributed. This is almost never true. Most dimensions will be better fit by Weibulls or Lognormals and the random set comparison will convolute all the combinations of normals, Lognormals, and Weibulls. Wes's comments: Using “Normal distribution algebra”, 1.325% and 2.292% respectively should not fit in the hole. Compare to the MC results. Also, see the related “Six Sigma” problem in this tutorial on the use of the Normal (standard) vs. the Weibull distribution.*

### Problem 40. Classic Mixture of Two Failure Modes

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**SOLUTION:** Start in SuperSMITH Weibull, Home group, select New / Clear, then open **BWSOPAC.W**. Select Analysis Group, Mixture. Then select SSW 4P ‘wm’ Competing Risk mle to get the SuperSMITH competing distribution likelihood ratio test results on the plot. Delete confidence if asked. Eta(1) = 8313.2 and Beta(1) = 0.603. Eta(2) = 64.73 and Beta(2) = 7.501. Back in Mixture menu, Fit Method For SSY, set to mle. Then check SSY 4. Then go to Main Screen, Tools group, and select Distribution Comparison. Look at the LR Ratio test results. Is there a significant indication of a mixture? **YES**, the LR test % is above 99! To estimate proportion of failures from infant mortality, return to the Main Screen, select Options, Calculator, and then Interaction. Select Random Point ... Difference (not Sum). Select Weibull for Distribution 1 and 2 choices, then enter the Weibull parameters as above. Select OK to activate simulation. Results should be close to 5% < 0 and 95% >= 0. This indicates that about 5% should be due to the low slope Weibull, and 95% should be due to the high slope Weibull. Compare this with the plot. Does this make sense? (bend in the curve is at about 5%)



You can repeat the analysis in SuperSMITH **YBath™** software (SSY).

*Dr. Bob's Comments: It is always better to separate the failures into separate failure modes by inspecting the broken parts, the source of the failures, the root cause of the failures, in other words, by engineering analysis. However, there are many cases where this cannot be done and we must rely on statistical methods. To estimate these many parameters larger samples are desirable. A following problem is included to introduce you to some options in SuperSMITH YBath™ (SSY).*

### Problem 41. Mixtures of 2 or 3 Failure Modes with Batch Effects – YBath™ [by Carl Tarum]

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What can you do with a mixture of failure modes? What if infant-mortality, chance, and wear out failures exist, like the classic Bathtub curve? YBath can estimate the parameters for these mixtures.

Use judgment with the sample size. Suppose you have 10 data points and fit a model that says that 10 percent are infant mortality. This means that maybe only **ONE POINT** of your 10 data points is infant mortality. While the computer is happy with the numbers, it is absurd to justify a Weibull slope(b), Characteristic Life(h), and portion defective (u) based on one data point! Minimum recommendations are 25, 40, and 50 for 4, 5, and 7 parameter fits respectively. Consider the variation of the parameters and if the sub-population is large enough to justify the fit. If u=10% on 100 points, that indicates that 10 points are part of the infant mortality. But u=1% on 200 points indicates that just 2 points are defining the infant mortality. It is not reasonable to expect that 2 points can provide an accurate estimate of beta, eta, and the portion of the population.

Figure 8 below is a good example of a competing risk mixture. The steep slope followed by shallow indicates a batch effect; one of the failure modes only applies to a subset or batch of the entire sample. Therefore a (5 parameter) fit of the following equation may be appropriate:

$$F(t) = 1 - (u)\exp[-(t/h)^b - (t/\eta)^\beta] - (1 - u)\exp[-(t/\eta)^\beta]$$

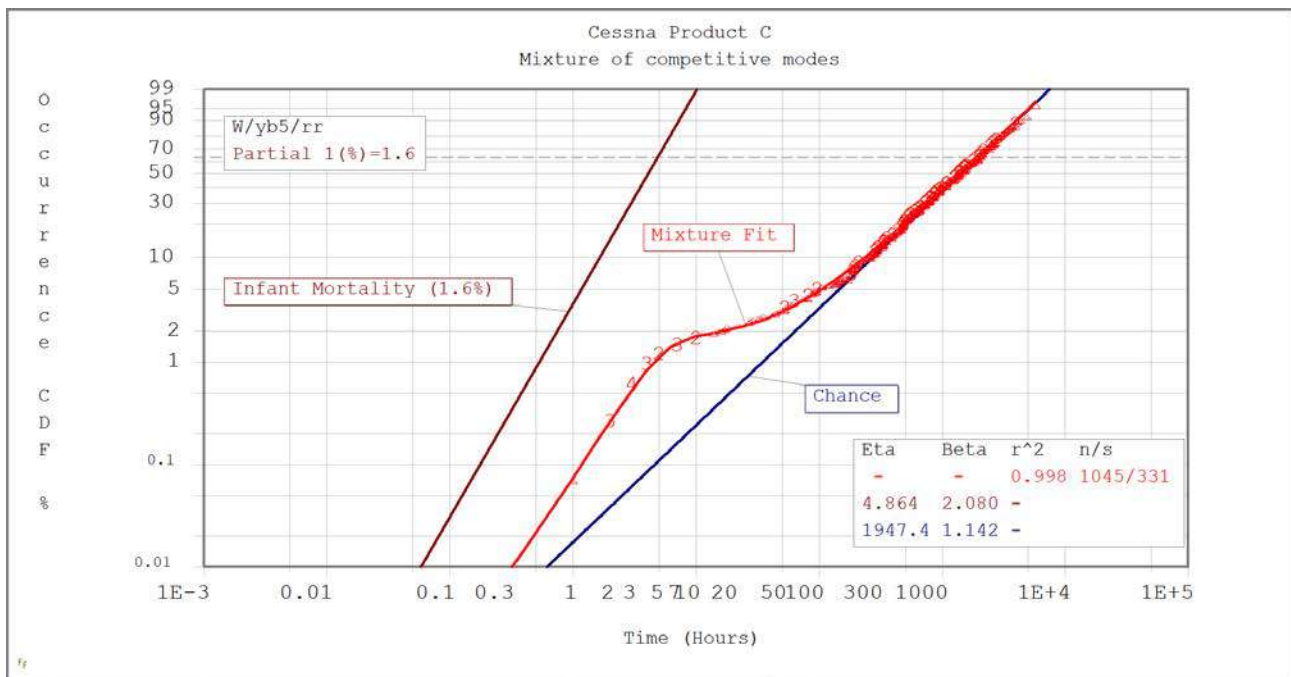


Figure 8: Mixture of Competitive Failure Modes

In SuperSMITH Weibull, select New / Clear, then open the data file **CESSNAC.W** and plot it with a 2 parameter Weibull (W2). There is some curvature at 10 hours. Note the PVE of only 0.55%. (Not 55%, but less than 1%!). A Distribution Comparison between W2, W3, and LN, shows no reasonable fit. With the obvious curvature, we should consider a mixture solution ... as below.



**SOLUTION:** Open YBath either by selecting the YBath Icon, OR by selecting it from Start/Programs or from the File Manager, OR by choosing it from the “HELP plus” group in SuperSMITH. Open the file **CESSNAC.W**. In the Analysis group, select Method/Input, and select MLE. While this appears to be a 5 Parameter fit, select 4 Parameter, then 5 Parameter. Note the status of the solution in the top of the screen. You can hit <esc> to pause/cancel the fit. When done, go to the Tools group, and select Distribution Comparison. (Results shown on Next page)

Note the Ln(L) of the different distributions in Figure 9. The note at the bottom notes that yb5 is the best option. The LR test is above 90%, so this indicates that yb5 is a better choice than the 2 Parameter fit. Near the top, it is noted that there are 714 Occurrences and 331 Suspensions. IF you had selected rank regression and turned on Inspection Option 1, there would be 605 points.

A yb7 usually gives a response that the mixture indication is small, with the LR Test  $\leq 1\%$ . If you try rank regression and Inspection, you may get a “solution”, but the 3<sup>rd</sup> line is just parallel to the second line and shifted, so it doesn’t impact the fit. If you get the “LR Test  $\leq 1\%$ ” message, you will need to select a valid mixture before doing the distribution analysis.



You can click on the icons at the top of the results page to copy the text to the clipboard, the logbook, or the printer. Copy the results to the Logbook. Close the results, then click on the Logbook on the Main Screen. You can see the results, add notes, or highlight results that you want to emphasize.



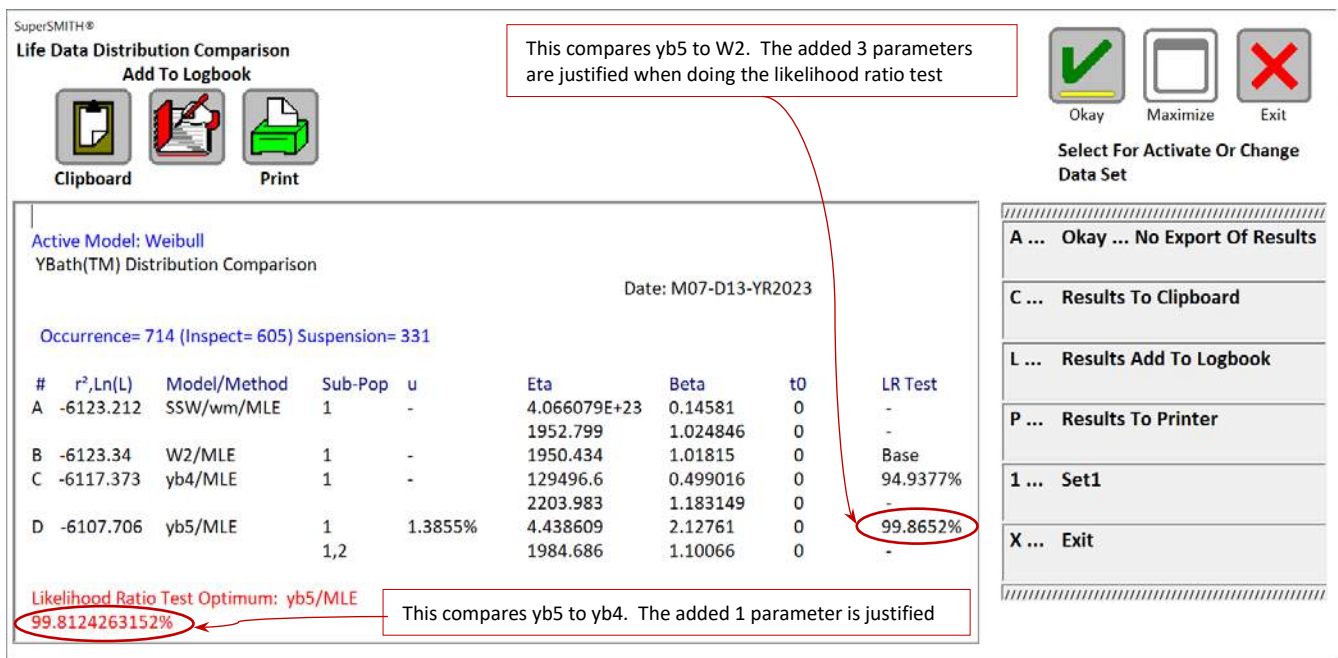
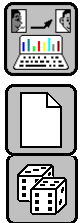


Figure 9: YBath Distribution Comparison

## Problem 42. Distribution of Beta (Weibull Slope)

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For WeiBayes, it's common practice to average betas from prior data sets for a nominal value. One of your staff experts says that's wrong. The expert thinks the distribution of beta is Lognormal, so geometric mean or median is better. Is the expert correct? Precision error with MC simulation is judged by repeating cases. Precision is related to the reciprocal of the square root of trial quantity. For real problems use at least several thousand trials. More trials are better.



**SOLUTION:** Start SuperSMITH Weibull (SSW). At Home group, select Setup (computer icon).

- Verify Dimension capability at least Set Quantity [12], Point Quantity [1500], return to Main Screen.
- In Home group, select New / Clear. In Analysis, select Method/Input, Reset To Standard, Activate.
- In Home group, select Sample (dice) icon, then Special Monte Carlo Simulation.
- Select Monte Carlo Type, then Model Parameter Value [MPV], set B-Value Select = None.
- Select "C...Sample From", choose Weibull, set Beta = 4, Eta = 100, t0 = 0, then select "OK".
- Set the Fit Method = rr, Monte Carlo Trial Quantity = 1000, Point Quantity = 10, Suspension [%] = 0.
- Select "A...Activate". Then after simulation select All Monte Carlo Results To File, and save as X.TXT.
- NOTE: For the DEMO software, we cannot input data like it is saved in this kind of text file. However, later you can bring this X.TXT file into a spreadsheet to view. For FULL version . . .
- Return to the Main Screen, activate New / Clear, open X.TXT (choose import data option New Column=New Set). Ignore "Slope Too Big" messages, select "OK". What's the best distribution for Set #3 (Beta)?



*Comments: Best fit for beta variability is usually Lognormal. Lognormal is skewed, so the median value of x is a better typical value than the mean. The staff expert is correct. With the Lognormal distribution, the antilog-of-the-mean-of-log-x ("Med") is the median estimate of x (median is the same as "B50").*

### **Problem 43. How Good Are the Answers?**

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Management was pleased you provided quantitative uncertainty bounds but they are nervous about this Monte Carlo (MC) solution. How good are your uncertainty bounds?

**SOLUTION:** Start in SSW at the Main Screen, Home group, select New (blank page) icon. Also in the Home group, select the Sample icon (showing a pair of white dice), then select OK several times to generate a few Monte Carlo (MC) samples. View the simulated data and solutions. The clock provides the seed for random numbers so each set of trials give slightly different results. How close are sets to each other? The differences are a measure of statistical precision. (You could generate 10 or more tables of results and calculate the mean and standard deviation of the variability.) More trials improve precision by the square root of the number of trials. MC pivotal (pv) confidence bounds are an automatic way to evaluate confidence with simulation, and this option is located under the confidence icon. For every MC, the answers are slightly different reflecting the uncertainty in the statistical process. To avoid this scatter in the SSW pv confidence bounds, you may “lock” MC confidence at a particular seed number and get repeatable results. Or you can “unlock” with a time-based seed number = 0 to produce variability. Dick Rudy of Chrysler suggested this locking option.

### **Problem 44. Statistical Uncertainty of Important Results!**

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Your latest Weibull analysis of the #1 issue has received upper management attention. Although results agree with engineering analysis, management wants a measure of the uncertainties associated with this new technique. Your Weibull analysis is based on a sample of 18 failures with eta = 165 and beta = 4.3. How can you respond?

**SOLUTION:** Determine 90% two-sided “uncertainty” bounds for eta, beta, r, B1, and B5 life.

- A) Start SuperSMITH Weibull (SSW). In Home group, select New / Clear (blank page). Select Analysis Group, Method/Input icon. Reset Method/Input to standard (Weibull/rr). Select Home Group, Setup, and verify Dimension Maximum says at least Set Quantity = 12 and at least Point Quantity = 1500
- B) At Main Screen, Home group, select Sample (dice) icon, then choose Special Monte Carlo Simulation.
- C) Select M...Monte Carlo Type = Model Parameter Value [MPV]. Select B...B-Value Select. Enter 1 and then 5. Select C...Sample From, select Weibull, Beta = 4.3, Eta = 165, verify t0 = 0, Select OK. Verify F...Fit Method = rr. Monte Carlo Trial Quantity = 1000, P...Point Quantity = 18, Q...% Suspension = 0.
- D) Select A...Activate to start, then after simulation select F...All Monte Carlo Results To File .
- E) Save to file **MCX.TXT**, then exit to the Main Screen. Select as follows for your type of software . . .
  - a. FULL: Open the **MCX.TXT** file, select A...New Column = New Set (X-Value). Ignore messages.
  - b. DEMO: At the Main Screen, select New / Clear, open file **PlayTIME-Problem-44.w**.
- F) Should have 6 data sets (one not plotted, and t0 set automatically removed). Scroll down to the 50<sup>th</sup> point.
- G) Record values from Set 2 (Eta), Set 3(Beta), Set 4(r), Set 5(B1), and Set 6(B5)
- H) Page down to the 500<sup>th</sup> point and record Eta, Beta, r, B1, and B5 as above.
- I) Page down to the 950<sup>th</sup> point and record Eta, Beta, r, B1, and B5 as above.

Compare your results with the values below. (“~” indicates approximation, **BOLD** here indicates true value). The difference between the True Value and simulation Median Estimate measures the bias in the method.

	<u>Eta</u>	<u>Beta</u>	<u>r</u>	<u>B1</u>	<u>B5</u>
50 <sup>th</sup> Point (-95% Conf.) =	~150.5	~3.03	~.944	~34.8	~59.5
500 <sup>th</sup> Point (Median) =	<b>165</b>	<b>4.3</b>	~.9816	<b>56.61</b>	<b>82.7</b>
950 <sup>th</sup> Point (+95% Conf.) =	~178.1	~5.85	~.9935	~76.1	~101.1

Select New / Clear. Select Equation Label and choose Weibull 2P.



**Ed Ramsey** and **Paul Reisinger** performed an accelerated test demonstration for the benefit of the instructor, Wes Fulton, during a Weibull/Lognormal Engineering Workshop held in Elyria, Ohio. Their test fixture illuminated four 6-volt automotive bulbs simultaneously at a selected voltage with digital timers indicating seconds of bulb operation for each bulb. The mechanism of interest, open circuit, stopped the timer for the failed bulb. One set of bulbs was tested at 13 volts, another at 14 volts, two sets at 15 volts, and a set at 10 volts. The 10-volt test took overnight to complete. While waiting for 10-volt results, we could analyze the first three load levels for what to expect at 10 volts. The lists below give age to failure (seconds) for the first three test voltages.

Set 1: (13V)	Set 2: (14V)	Set 3: (15V)
<b>3304</b>	<b>1367</b>	<b>570    861</b>
<b>2628</b>	<b>1409</b>	<b>972    664</b>
<b>3387</b>	<b>1460</b>	<b>713    788</b>
<b>2951</b>	<b>1949</b>	<b>906    666</b>

**SOLUTION:** In SuperSMITH Weibull (SSW) start at the Main Screen and select the New / Clear icon (showing a blank page) to start a new analysis.



- A) Select Analysis, then Method / Input, and Reset To Standard icon (U-turn arrow). Verify Weibull.
- B) Open file **PlayTIME-Problem-45-A.w** for input data. Standard rr Weibull solutions are Eta=3222, Beta=8.809 for set 1, Eta=1631, Beta=7.498 for set 2, and Eta=822.6, Beta=6.146 for set 3.

NOTE: Low pve% for 14 Volts is unusual (only 1 in 65 chance with good sampling) but possible.



- C) In Tools, select Accelerated Test, and enter 1...Standard PFEV. Respond "No" if requested to change to mle. Verify Engineering Variable Quantity = 1, then select Find 1 Value for All. Then input the Engineering Variable Name as "Volts". Next is Volts value input. Volts entry is 13 (set 1), 14 (set 2), and 15 (set 3).
- D) If just starting, the software asks you to select PFEV Fit Type. Select that menu item and then Exponential. The solution (top of the dialogue box) should show common slope (Beta) of about 7.

S/N CURVE: Try the following ...

- E) Still in the SSW PFEV menu, scroll down, select View/Save XY Data Or Function, then Plot Of XY Data (S/N Curve) to see the "S/N" curve of life vs. load shown in SSV. **Return to SSW by exiting SSV (no save).**
- F) At the SSW Main Screen, select Tools, Accelerated Test to return to the PFEV menu dialogue box.
- G) Select Plot Line At New Variable, enter 10 Volts, view the projected Weibull line for that as-yet untested value. **Select Tools, Accelerated Test to return to PFEV menu dialogue box (do this after each item below).**
- H) Select Fit Type, this time choose Power, view S/N Curve, return to PFEV, get the projected line for 10 Volts.
- I) Select Fit Type, this time Arrhenius-X, view S/N Curve, return to PFEV, get the projected line for 10 Volts.
- J) Plot now should have 6 fit lines: 3 through data sets, and 3 Marker Lines for different models at 10 Volts.

Later, we got actual data for these bulbs at 10 Volts ...

Set 4: (10V) data = **62101, 70369, 86358, 82363.**

>>> Let's compare the model with reality as follows.

- K) IMPORTANT, first turn off the active solution as follows:
  - a. From the Main Screen, select Tools, then Accelerated Test, then None ... Reset / Clear PFEV
  - b. You should arrive at the Main Screen with plot (NOTE: Marker lines remain visible).
- L) Open file **PlayTIME-Problem-45-B.w** (10V data above), compare actual to the projected Marker lines.

Which model seems to forecast best? Why do you think that model works best with this data?

*Wes's Comments: The stress (load) on the bulb is voltage, higher voltage is higher energy which is higher temperature. The Arrhenius model is best for effects of temperature on product life in general (Reference: Accelerated Testing by Dr. Wayne Nelson listed in The New Weibull Handbook reference section).*

**Problem 46. Accelerated Testing For Mars Encounter Probe (2007 Gosset Challenge Problem 1)**[TOC](#)

You are the program engineer for the new Mars Encounter Probe that will scout the Mars terrain just before Donald Trump suits up and attempts to land there (funded by Hillary Clinton). The design requires small emergency power components inside the craft only to be used for emergencies. Your test temperature chamber was first set to **320 Kelvin**, and the **24** power supply assemblies originally in the oven were cycled on/off until failure was sensed in each one. The assembly is simple, and these all fail in the same way in this test. The last of these original items failed 2 weeks ago, and all were removed and replaced with **24** new power supply assemblies (same configuration) with the temperature increased to **330 Kelvin**. This group completed testing 1 week ago, and another **24** were tested at **340 Kelvin**. The last of these replacements just failed, and now you have cycle count data for all three temperatures (data in **WUC-2007-GOSSETCH-1.w**). You decide to keep the fit line solution method simple by using rank regression. **Which age-to-failure variability model fits best?** You use your technical knowledge to guess that since temperature is involved, solution with a common value of slope using the Arrhenius model for PFEV is best. **Does the cycle capability at R99C50 exceed 55 cycles, if operation of the equipment is at the 300 Kelvin Normal operating temperature?** What is the standard error of fit for Arrhenius? What does the related S/N curve look like for this analysis?

**SOLUTION:** Start SSW, New/Clear, select Analysis, Method / Input, Reset To Standard to get Weibull rr.

(Note that you should always use absolute temperature values for a covariate analysis such as this one)

- A) With no data in the data grid and no results, open the **WUC-2007-GOSSETCH-1.w** file. You should get Eta=125.8, 82.31, 55.10, and Beta=1.991, 2.443, 2.051 for Sets 1, 2, 3 respectively.
- B) Choose the Tools group, Distribution Comparison to find best fit model (Select “No” for change each time). For Set 1 Lognormal is best, also for Set 2, and for Set 3. At the Main Screen, change Equation Label to Lognormal, and you should get Med=95.52, 65.75, 42.17 and SigF=1.911, 1.686, 1.852.
- C) Select Tools, Accelerated Test, and select Standard PFEV. Enter 1 for Engineering Variable Quantity (temperature only changing between data sets). Select Find 1 Value for All. Enter “Kelvin” for Engineering Variable Name. Enter respective temperatures ... 320 for Set 1, 330 for Set 2, 340 for Set 3. You should see the PFEV dialogue box (no results yet).
- D) Set “Lognormal Fit” to rr. If asked, select options as in operation before. Returns to PFEV menu.
- E) Select F...PFEV Fit Type and enter Arrhenius. You should get 2.63 Units (Cycles) for Med standard error. Select L...Plot Line At New Variable Value and enter 300 (Normal operating temperature, Kelvin). You should see the Lognormal plot with SigF=1.814 for all fit lines and a Marker Line with Med=244.8 and SigF=1.814. Select the small plot to enlarge.
- F) Select the large plot, returns to the Main Screen. Select Options, Precise Reading icon and Line 1 (Marker Line). Select B1 to get 61.26 cycles capability (no associated confidence). The capability would be different if mle-RBAmu was used instead of rank regression. Yes, capability exceeds 55 cycles.
- G) Use the <ESC> key (twice) to return to the Main Screen, select Tools, then the Accelerated Test icon, and then scroll down to select View/Save XY Data Or Function With SSV (S/N Curve), and select P...Plot Of XY Data to view the related S/N curve with the associated input data points. Exit out of the SSV program to return to SSW when finished looking at the S/N curve. Use Home, New/Clear to clear data and results.

**Problem 47. Six Sigma Quality Control for a Precision Pump (2007 Gosset Challenge Problem 2)**[TOC](#)

Your boss comes from General Electric and wants you to use your **Six Sigma** background to evaluate plating thickness on a precision pump component used in thousands of pumps. The measurements [cm] of plating thickness are given in file **WUC-2007-GOSSETCH-2.w**, critical thickness tolerance is **.019 to .036** cm. Requirement is less than **3.4 parts per million** (PPM) rejection for process design. **Are you okay?**

**SOLUTION:** Start SSW, select Analysis, Method/Input, then Reset To Standard, “Yes” to Weibull (Weibull/rr).

- A) Select New/Clear. Open the **WUC-2007-GOSSETCH-2.w** file. With rr fit you get Weibull solution of 39 points with Eta=0.02976 and Beta=24.31. High Beta is expected with material property variability.
- B) You know plating thickness cannot be negative, so you stick to options of only Weibull 2-parameter (Weibull-2p), Weibull 3t-p (+ t0 only), and Lognormal distributions, which cannot go negative.



- C) At the Main Screen, select Tools group, Distribution Comparison to compare between Weibull-2p, Weibull-3t, and Lognormal. The best pve% is for Weibull-2p, so no change is required.
- D) Return to the Main Screen. Select Options, Plot/Report, scroll down and select V...Additional Plot Options With SSV, then select Q...QLC Quality Limit Comparison.
- E) After selecting D...Double Limit from menu, enter 0.019 for Lower Limit and 0.036 for Upper Limit. Make sure the "PPM Type Of Results is set to "YES". Select "A...Activate". You should get a SSV plot indicating on upper right of plot that 18 PPM are expected to be lower than the low limit and practically none (" $>UL=9E-30$ " ... near zero) expected higher than the high limit.
- F) The 18 PPM exceeds the 3.4 maximum requirement. Tell the boss plating thickness is outside tolerance on the low side using the Weibull distribution. Exit out of SSV to return to SSW (no save needed).
- G) Easier access: From SSW Main Screen, Select "Additional +" icon, choose menu item for QLC. Keep Double Limit values the same. This time Select Ppk Type Of Results (either with the 1.5 shift or without). Activate to get the plot with Ppk results. Exit from SSV with no save needed. NOTE: Ppk is a measure over all groups of same-type data with higher value indicating less variability, better quality.
- H) See the **Weibull to/from Six Sigma Conversion Table** at the back of this tutorial booklet.

*Wes's Comments: Using the standard Normal, there are practically none below the low limit and less than 1 PPM above the high limit. These incorrect results indicate plating thickness is acceptable when not. Normal allows negative values, not possible with most manufacturing. Better to use an appropriate distribution like Weibull. Thanks to **Dennis Keisic** for this insight with respect to Six Sigma. Dennis also developed the Six Sigma Conversion Table mentioned above.*

#### **Problem 48. Life Remaining on a Relief Valve (2007 Gosset Challenge Problem 3)**

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There are many new projects underway at your chemical plant and you want to stretch your assets as far as possible. You have ten special-order high-temperature relief valves in a variety of different service applications throughout the plant (not in the same system). It takes many months before a new order can be filled. The Weibull model previously fitted to the laboratory time-to-failure data (same duty-cycle, same environment) for these valves indicates **Eta = 72 months and Beta = 2.6**. The recommended **scheduled replacement interval is 24 months** of service. They all currently have **24 months of service with none failed**. Reliability for 24 months of service is **94.41 percent** using the original Weibull. You want minimum **reliability of 95 percent** (no worse than original requirement) for continued operation. **How much longer can you expect to keep these valves in service still meeting 95 percent reliability? How much longer can you expect to run them in a second extra service if they all survive the first extra service?**

**SOLUTION:** In the SSW program, Home group, select New/Clear. Reset method to Standard (rr) if needed.



- A) Select Options Group, then the Calculator icon. Choose the "Life Remaining" option.
- B) Select Distribution, set Distribution Equations to Weibull, set Beta to 2.6, and set Eta to 72. Choose Activate ... OK from this menu to return to the Life Remaining menu with current results at the top.
- C) Select Present Age and set it to 24.
- D) Select Minimum Reliability [%] and set it to 95%.
- E) **Result #1:** Additional Expected Life is somewhat more than 6 months.
- F) After 6 months of additional zero-failure service, all of them will have 30 months of age. So select Present Age again and set it to 30.
- G) Reliability changes (= default dependent variable). So, select Minimum Reliability, set it again to 95%.
- H) **Result #2:** Additional Life after 30 no-failure months is an additional 5 months.

*Wes's Comments: Using equipment past that equipment's rated lifetime is often needed. When operators find parts operated already to their useful life, they may be reluctant to use them further. In this example we can see that such operation is acceptable with proper limitations given sufficient background knowledge (from your Weibull Library!).*

## **Problem 49. Nelson Recurrent Events (RE) Analysis – Simplest Data Format**

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The SuperSMITH Visual program has a simple implementation of Nelson RE analysis developed by Dr. Wayne Nelson. It is included in the Visual program, not the Weibull program, because the resulting plot is not a probability plot. Standard RE analysis is non-parametric and does not require special probability scaling.

Suppose you have 3 pieces of equipment with serial number identification SN007, SN008, and SN015. Two repairs occurred in **SN007** at ages [hours] **391** and **643**, and it is **967 hours old now**. Two repairs occurred in **SN008** almost together at age **622**, and it is **636 hours old now**. One repair occurred in **SN015** at **485** hours, and it is **923 hours old now**. **What is the mean cumulative function (MCF) of repairs vs. age?**

As background, for standard Nelson RE every data value must also be identified with a piece of equipment ... “equipment” may be a person or a project in the broader sense. The Nelson RE analysis is age-based and so the data must be put into overall age order before numbers are crunched. The software can put the data in order for you if it is not already in age order. In SuperSMITH software, the first number (X-Value) in the data entry is ALWAYS time or age when working with Weibull-Engineering type data (age) or Crow-AMSAA type data (operating time) or Nelson RE data (age). The second number (after the “x” used as a delimiter) for SSV is the Y-Value. For Nelson RE that Y-Value represents the number of events occurring at precisely that age value (usually 1 for one repair). The Nelson RE input Y-Value is NON-CUMULATIVE data, but the resulting analysis will be in CUMULATIVE form! Also, the Y-Value could be cost at that age value when performing cost analysis. Every individual equipment piece represented by Nelson RE analysis MUST have one suspension entry at the last known age for that equipment. Before suspension that same equipment may have any number of event (or cost) data entries. To enter a suspension, use zero (0) as the Y-Value or put a “>” sign in front of the age. The identifier text is put at the right-hand side using the underline symbol (“\_”) as the delimiter.

The above data values for entry into the software without putting in overall age order (data labels at the end) are:

391x1\_SN007  
643x1\_SN007  
967x0\_SN007  
622x2\_SN008  
636x0\_SN008  
485x1\_SN015  
923x0\_SN015

**SOLUTION:** Start the SSV program (**this is Visual ... do not do this in Weibull!**) ...

- A) From the Main Screen, Home group, select the blank page icon to reset for a new analysis.
- B) Open file **PlayTIME-Problem-49.v** file to get the plot. (data labels scattered around plot)
- C) NOTE: This is not yet a Nelson RE analysis. This is a plot of the beginning input data.
- D) Select Tools group, Nelson Recurrent Event (RE) Analysis to get the RE menu.
- E) Verify that Plot Display Type is set to STEP FUNCTION for both Confidence and MCF.
- F) Select Activate Analysis or select green check mark ... message w/ “Not By Order” should appear.
- G) Select Order By X-Value to put the data entries into overall age sequence and to solve and plot.
- H) You should get a plot indicating ending MCF value of about 1.833 total repairs/equipment (on vertical scale) starting at age 643 (on horizontal scale).



*Wes's Comments: Nelson RE is a method giving rigorous results for maintainability analysis. For best RE solution, the data collection should include identification of equipment for each maintenance event (which individual piece of equipment was the one requiring maintenance). Weibull does not require such detail in the input data, although such information can be helpful for Weibull as well. Sometimes having the extra sequencing information provides useful insights. Bottom line ... for each event (failure for reliability, repair or refurbishment for maintainability) put as much detail in the collected data as feasible. It can help big-time later.*

**Problem 50. Nelson Recurrent Event (RE) Analysis – Another Data Format**[TOC](#)

Nelson RE has several different input formats. The data below comes from Dr. Wayne Nelson. Each piece of equipment needs identification. Car# in this case is on the left and values to the right are mileage at repair. A trailing “+” indicates suspension. What is the mean cumulative function (MCF)?

Car#	Mi(1)	Mi(2)	Mi(3)	Car#	Mi(1)	Mi(2)	Mi(3)
24	7068	26744+		115	17955+		
26	28	13809+		116	19507+		
27	48	1440	29834+	117	24177+		
29	530	25660+		118	22854+		
31	21762+			119	17844+		
32	14235+			120	22637+		
34	1388	21133+		121	375	19607+	
35	21401+			122	19403+		
98	21876+			123	20997+		
107	5094	18228+		124	19175+		
108	21691+			125	20425+		
109	20890+			126	22149+		
110	22486+			129	21144+		
111	19321+			130	21237+		
112	21585+			131	14281+		
113	18676+			132	8250	21974+	
114	23520+			133	19250	21888+	

**SOLUTION:** Start the SSV program (Visual) ...

- A) In the Home group, select New / Clear to reset for a new analysis.
- B) Select Sample icon (dice) to get the Sample (Simulate) menu.
- C) Select Special Data Set for some choices of reference data sets for benchmarking.
- D) Select 9V – Nelson Recurrent Event Plot for the data, analyzed with Nelson RE, and resulting plot.
- E) Result: MCF plot indicating about 0.303 total repairs/equipment starting at age 19250.
- F) Select Tools group, Nelson RE, for the Nelson Recurrent Events menu.
- G) Change Plot Display Type For Confidence to Point By Point
- H) Change Plot Display Type For MCF to Point By Point.
- I) Select Confidence and enter 95 for the Confidence [%] Double value.
- J) Select OK (Activate Analysis). The MCF plot shows the nominal data points with point symbols and the upper and lower confidence limits with “-“ symbols.
- K) Exit out of SuperSMITH(R) Visual (SSV).





### **Problem 51a. Test Substantiation / Test Planning**

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The Weibull Workshop introduction video includes an example of test planning for a bearing. Data from this test-planning example is also presented in the Handbook and referenced here again to show how software provides the answer. **Reference The New Weibull Handbook Section 6.9. “Weibayes” Substantiation Test Plans. >>>**

Suppose a ball and roller bearing follows a Weibull distribution for age to failure with respect to an unbalance failure mode, with Weibull slope (Beta) = 2, and Weibull characteristic life (Eta) = 500 hours. A new design has been initiated, and three (3) redesigned units are available for testing. Now the question is: **How many hours should each system be tested to demonstrate a minimum 500 hours for its characteristic life (Eta)?** You should try both a zero-failure test plan and a zero-or-one-failure test plan (allowing one failure).

**SOLUTION:** In the SSW (Weibull) program...



- A) From the Main Screen, Tools group, select the Test / Usage icon (showing a beaker).
- B) Confirm that Test Type is “Time Or Life Or Age”. Then select “Find Test Time Length”.
- C) Enter Weibull Slope Beta of 2, and you should be taken to the full test options menu.
- D) Select N...Test Item Quantity and set it to 3.
- E) A...Allowable Occurrence (Failure) Quantity should be 0 (zero).
- F) Select C...Confidence [%] and set it to 90.
- G) Select D...Demonstrate Minimum Character. Value. (Eta), to auto-reset B-Life and Reliability.
- H) Select G...Life Requirement Goal For Each Test Item and set it to 500.
- I) >>> The \* Ratio Of Test Time result should be approximately 0.876.
- J) >>> The \* Requirement Of Test Time For Each Item should be about 438 (hours, if Eta is in hours).
- K) Scroll up to the top of the menu. Change A...Allowable Occurrence Quantity to 1.
- L) Ratio of Test Time should change to ~1.27, and Test Time For Each Item should be ~ 638.5.
- M) Don't leave this menu, so it is easy to do the next problem (#51b).

### **Problem 51b (51a Continued). Consumer Risk and Producer Risk**

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For the same bearing test as above, we want to evaluate the consumer risk and producer risk. The consumer risk is the probability the buyer might accept an unacceptable product. The producer risk is the probability the seller might reject an acceptable product. Suppose you estimate that the actual Weibull characteristic life of the new design is twice that of the old design. **What are the consumer risk and the producer risk values for this test design?**

**SOLUTION:** In the SSW program.



- A) From the Main Screen, select the Test / Usage Requirement icon (showing a beaker).
  - B) Use the same inputs as in Problem 51a above (but for zero failure testing ... no failures allowed).
- NOTE: START BELOW HERE IF ALREADY IN TEST REQUIREMENT MENU ...**
- C) Set A...Allowable Occurrence to zero.
  - D) The C...Confidence item indicates CONSUMER RISK [%] = 10.
  - E) Select the Q... menu item and enter 2 for r, the ratio of actual equipment Eta to the Eta goal.
  - F) With no occurrences allowed, the Q... menu item indicates PRODUCER RISK (r of 2) [%] = 43.77.
  - G) Now try allowing 1 failure and compare the difference.

*Wes's comments: Allowing a failure increases required test time. However, it is possible in the case of highly reliable equipment to actually end up with a shorter test AND lower producer risk by selecting a zero-or-one failure test (1 failure allowed). If test items are tested in sequence and all test items before the last one pass, then testing the last one is not necessary. Allowing 1 failure, the consumer risk is the same (10%) for the zero-or-one failure test, but the producer risk significantly lowers (and improves) to only 26.12 percent.*

## **Problem 52. Weibull Library.** The most important thing!

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You want to start and maintain a Weibull Library, so you can use the powerful Weibayes (1-parameter Weibull) technique for small samples. There is capability in the SuperSMITH software to help. We will generating a library database item for the **18WHEELR.W** data file. This is about a steering link for a vehicle steering system.

**SOLUTION:** Start the SSW program, select Home group, New / Clear, to start a new analysis ...



- 1) At the Main Screen, Home group, select Open (open file folder icon), bring in file **18WHEELR.W**.
- 2) Select Home group, Library icon (showing a book) and you should see the library setup options.
- 3) Select H...Header, and toggle it so it reads ... \*YES. Note the following format on the menu:
  - a. Letter....Gray background typical menu item
  - b. “•”, Green background: A choice is already made
  - c. “ø”, Blue background: Value taken from solution/plot (such as beta)
  - d. “o”, Yellow background. Ready for a choice to be made
- 4) Scroll to Select Device, select **Connecting Rod**
- 5) Select Equipment, select **Steering System**
- 6) Select Cause, select **M3 Mech: Fatigue**
- 7) Select Duty, Select **Heavy Duty**
- 8) Select Environment, select **Temperature: Normal**
- 9) As you change these items, the menu scrolls to show the next entry. After environment, there are several items set by the analysis, such as Model, Method, and Confidence. You don't change them here. If you want to change them, you change them at the Weibull Main Screen. The next four items are “Comments” These are free fields, with some suggestions
- 10) Select Comment 1. Click on **Part #:** This puts the reference into the text box. Now enter 822347 in the text box above and click on “OK”. The comment is now green with a solid bullet indicating that you have made an entry
- 11) Here's how to save this to the library file. Click the Icon with the arrow pointing to the library book. This will save it to the file **WL\_ClassData2021.csv** in your c:\Smithdat directory



### **Using a Reference File:**



There is a brief reference file that comes with the Smithdat directory. Suppose you wanted information on Bearings. In the text box under the Find icon (binoculars), enter “**be**” (without the quotes), then click the Find icon. You should see several rows. It also shows 2 plots with “Steep **Betas** hide problems.. So change the search to bear, and click the search button.



For convenience while searching, you may wish to make the library form full screen, and make the reference table fit the form. Two buttons near the upper right will toggle the form between full screen and mid-size, or the reference table mid-size or full form (hiding the menu items). You can now scroll to the column marked Variability, and see the Weibull slope was 2.617 on one data set, and 8.317 on another. If you click the 2.617, and then click the copy icon, it will copy that value to the clipboard.



To restore all the rows, click the Reset icon. You can sort the visible rows by clicking on the Heading. If you click on “Title 1” all the rows are sorted alphabetically. To undo the sort, you can click the Reset icon, or double click on the “Row” header.



Suppose we wanted to look at the 18 Wheeler data we just saved. Toggle the Midsize button at the top right so that the menu can be seen.

1. Select R... Load Reference file, and select WL\_ClassData2021.csv in the c:\smithdat directory. It will only have one line. As you get more data, you can search and sort this file as well

To restore the original reference file, you can repeat this step, but load WLR\_Smithdat.csv, or you can Exit and restart the library form.

Compare forecasting with Abernethy Risk and forecasting with Crow-AMSAA (CA). Let's use Abernethy Risk, then CA on the same data. Warranty Data Format allows dual analysis like this. It has enough information to do either a Weibull or a CA model (the units of age/time will be in equipment months if no adjustment is made). We want to see the difference between results from these two models.

**SOLUTION:** Start the SSW program, and at the Main Screen, select Home group, New / Clear.

- 1) Verify the Equations Label, on the Main Screen, shows "Weibull" (on green background)
- 2) Select Home group, Sample icon (dice), Special Data Set, choose "10A – Warranty Format For PT 27".
- 3) Toggle M...Method For Warranty Import = INSPECT insp1 (/ Kaplan...). Select W...Format: Warranty.
- 4) In the next menu, Select "B...Format = MOP x MIS" (or x MOA) so the "\*" symbol is in front.
- 5) Select "A...Activate" here for the Warranty Input menu. Verify that L...Limit Correction = **No**, and Renewal = **Yes**, and Warranty Limit For Input Age = **No**.
- 6) Select "A...Activate" for the plot. Eta / Beta should be about 101.9 / 0.629. View the Main Screen.
- 7) Select Tools, Abernethy Risk, "No Modified Options" icon, then "Yes" if asked, U...Usage, Enter Single Rate, and enter the value of 1 (for 1 month per calendar month). Toggle R...Renewal = \***YES**.
- 8) Select OK (checkmark) to generate the forecast table showing you should expect to accumulate about **401** additional occurrences from now until 24 months from now.
- 9) Select the icon on top of the output table with the wavy line to view Visual Plot in SSV.
- 10) At the SSV Visual Main Screen, Home group, select the Save Active Results and enter **FORECASTRISK.V** for the filename, and save.
- 11) Exit SSV and exit SSW completely (no save), then Start SSV again. Select Home group, New / Clear.
- 12) Select Home group, Sample icon (dice), Special Data Set, "10A – Warranty Format For PT 27".
- 13) Select W...Format: Warranty. You will be given a new menu with choice of format.
- 14) Select "B...Format = MOP x MIS" (or x MOA) to put "\*" symbol in front.
- 15) Select "A...Activate" here to get the Warranty Input menu. Verify options L...Limit Correction = **No**, and Renewal = **Yes**. Select A...Activate to generate the Crow-AMSAA (C-A) plot for this warranty-format data. Verify C-A lambda = 0.285 and C-A Beta = 0.745 with default **IEC** method and **Automatic** equation selection ("Crow/IEC/A" should display in small box on the plot).
- 16) From the SSV Main Screen, Tools, select the Fulton Risk icon (wavy line). Select menu item N...Present Quantity Active and enter 1200. Toggle R...Renewal = **YES**. Select menu item U...Usage, choose 1...Enter Single Rate, and enter 1.
- 17) Select A...Activate to generate the C-A forecast table showing that you should expect to accumulate about **492** additional occurrences from now until 24 months from now.
- 18) Select the icon on top of the output table with the wavy line to view results in SSV. Select the small plot to enlarge for the large plot accumulated occurrences vs. months-into-the-future.
- 19) Select the plot to reduce plot size and return to the SSV Main Screen, select Home group, Save Active Results icon and enter **FORECASTCA.V** for the filename, save.
- 20) Close all SSV open instances (use task bar if needed), no save. Start SSV again. Select New / Clear icon.
- 21) Bring in file **FORECASTRISK.V** (from Weibull). Bring in file **FORECASTCA.V** (from C-A), so they overlay on the same plot. Note that the C-A forecast (Set 2) is somewhat higher than Weibull (Set 1). Which one do you think is more accurate? After you are finished, exit the SSV program without saving.

*Wes's comments: Extrapolating **Crow-AMSAA** (C-A) this way does not account for subtle effects like maintainability rhythm. Weibull analysis does. Additional options for Weibull **Abernethy Risk** account for increasing/decreasing population, renewal, planned replacement, and so on. Typically, a proper Weibull forecast is much more detailed compared to a C-A forecast. However, a good C-A forecast automatically accounts for a subpopulation issue ("batch problem") out to a few months into the future and is easy to do. Abernethy Risk forecasting, without proper batch consideration, over-predicts for such an issue.*

## Problem 54. Which Distribution Model Fits Best For 2-Dimensional Manufacturing Miss-Distance?

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Want some exercise going back and forth between SuperSMITH Weibull and SuperSMITH Visual? We knew you would...

What is the distribution of radial error when trying to locate a feature like a drilled hole? Radial error is the shortest distance from the finished hole center to the ideal hole center (on the drawing) without regard to error in the displacement angle. Suppose you have a robotic manufacturing system for making the holes and suppose the “N-S” axis error and the “E-W” axis error are each described well with the Normal distribution. If your standard deviation of single axis error is 0.005 (inches or centimeters, doesn’t matter as long as you are consistent), what is the distribution of 2D radial hole-placement error?

**SOLUTION:** Start SSW, at the Main Screen select Home group, select New / Clear (blank page icon).

- 1) Select the Equations Label (green background for Weibull), then select N...Normal. The Equations label will change to “Normal”, and the Equations Label background will turn light gray.
- 2) Select Home group, Sample icon (dice). Verify U... Type = \* MONTE CARLO.
- 3) Select D...Sample Distribution Parameter. Enter zero (0) for Mean (Mu), 0.005 for Std. Dev. (Sig).
- 4) OK to return to Sample menu. Select N...Point Quantity and set to 500.
- 5) Select V... Sample To File Only (Not Sorted), and enter filename **X.NOR** ... select Yes if asked to Delete Former File + Save New.
- 6) Repeat steps 2-5 above **EXCEPT** this time save simulation results to filename **Y.NOR**.
- 7) From the SSW Main Screen, Analysis group, select Method / Input, choose Reset To Standard, answer “Yes” if asked to Change To Weibull, Activate. The Equations Label background should turn green.
- 8) Select Home group, “dice” icon again, and change the Eta and Beta and Point Quantity values to reasonable numbers (e.g. like Eta of 10, Beta of 2, and N...Point Quantity to 10).
- 9) From Main Screen, select “?/+” group, select “Visual” icon to start the SSV program.
- 10) From the SSV Main Screen, Home group, open file **X.NOR**, select A...As X-Value, and you should see a horizontal line of points along the X-axis on the X-Y plot.
- 11) From Home group, Open file **Y.NOR**, select A again, enter 2 for “Paste To Set” to put into set # 2.
- 12) Select the Analysis group, Transform icon showing a curved arrow pointing to F(a), and enter No to “Save Present Data Before ...?” question if asked.
- 13) Choose option S...Swap Axis, and the plot should display the data vertically.
- 14) Select Analysis group, Transform icon again (enter “No” to “Save Data” question if asked).
- 15) Select option 5...New X For Set B = Y From Set A, and enter Yes (‘A’ X-Value Same As ‘B’ X-Value), enter 1 (Set # For A), then enter 2 (Set # For B) ... WOW! Plot has a lot of randomly crossing lines in the center of the resulting plot (this is expected when plotting lines instead of just the data points).
- 16) Select Analysis group, Transform icon again (enter No to Save Data question if asked).
- 17) Choose option A...Rectangular To Polar (this plots vertical Radius vs. horizontal Angle).
- 18) Select Analysis group, Transform icon again (enter No to Save Data question if asked).
- 19) Choose option S...Swap Axis, and Radius and Angle should swap (NOTE: Analysis by SSW of a SSV file only evaluates X-Value variability, and we want to analyze radial error with “Radius” on the horizontal).
- 20) NOTE: The result of this exercise is already saved from a FULL version to file **RADIUS.V** for analysis by the DEMO. If you are in the FULL version, you can save your current result as **RADIUS.V**. Choose OK if message about Marker Line appears.
- 21) **Exit from SSV. Start the SSW program if not active.** Select Home group, New / Clear icon.
- 22) Open the file **RADIUS.V** and select the small plot to enlarge it. Does the fit look good? How does this match with your expectations (if any)? We expect a Weibull Beta near 2.

*Wes's comments: Theoretical analysis indicates the Rayleigh distribution should fit the radius data (Weibull with Beta of 2). Our results are a little different due to the MC simulation variability, but the fit should be mostly okay (pve% >10), and Beta should be reasonably close to 2. In this exercise, we just performed a Monte Carlo simulation, my favorite technique, to get those “miss-distance” values.*

Reference [The New Weibull Handbook](#) Section 5.8.5.

From reading Dr. Bob’s handbook and from using this PlayTIME™ tutorial, you should be already familiar with some of the following data types (described for use in reliability analysis):

“Occurrences” – failure age values for items that have failed by the failure mechanism of interest (FMOI).

“Suspensions” – age values for items that could fail by the FMOI but have not so far, or that have failed for a root cause mechanism that is different than the FMOI.

“Discoveries” – age values for items failed by the FMOI found at the very first inspection age. No sensory information indicated the exact failure time. An example of this would be the discovery of a housing crack that does not degrade performance or cause leakage, but is still considered failed and must be fixed before further usage.

“Intervals” – age values for items failed by the FMOI but found at a later inspection age (not the first inspection).

Some data sets may have some or all of those mixed together. Let’s use the inspection data originally from the **INSPECT.W** file but summarized roughly into these four different data types for Interval mle solution. NOTE: Commas separate the data entries below with age before “x” and quantity after “x”:

**<1x4, 1x1, <2x2, 2x1, 3x1, 5x1, >5x1, >6x1, 8x1, >9x1, >13<16x2, 25x1, 28x1, 61x1**

**DATA SOURCE:** Noisy failure in a new unit (one month of age) startles the equipment operator. The operator notifies quality control (QC). The QC supervisor orders an inspection of other units with age of one. Upon inspection, four more one-month-old units already had failed but without sensory indication. These four dormant failures occurred before age one (“<1x4”), and were discovered by the inspection ordered due to the failure at age one (“1x1”). So the first data entry above represents four “Discoveries”. Shortly after that, one of three new two-month-old units also failed noisily. Upon inspection, two other two-month-old units exhibited similar but dormant failure (<2x2). These were found only due to the noisy failure at age two (2x1). More data gathering produces failures (3x1, 5x1, 8x1, 25x1, 28x1, 61x1) and suspensions (>5x1, >6x1, >9x1) plus two other failures (>13<16x2). This last “Interval” data entry means that two units were both inspected at 13 and found to be failure-free, but at the next inspection, 16, both were found failed. No noisy indication happened in those. We only know that failure occurred sometime between 13 months and 16 months in both. How do we get a Weibull plot?



**SOLUTION:** In the SSW program ...



- A) Select Home group, New / Clear (blank page icon), to clear the data grid and the results.
- B) Select Analysis, Method/Input, Reset To Standard, Activate, for Weibull rank-regression (W/rr).
- C) Open the **PlayTIME-Problem-55.w** file. You should get the plot with Method automatically changed to interval mle solution (“W/mle/intv5”). The solution shown on the plot should indicate Eta of 8.53 and Beta of 0.553. NOTE: If Beta is different, select Method/Input and change Fit Method to “mle” (pure likelihood solution, not modified).
- D) The “.../3/6/2” in the parameter box represents data quantity for .../suspensions/discoveries/intervals.
- E) Select Analysis, Marker Line icon, and choose Set 1 (current solution) to return to the Main Screen.
- F) In the Data Grid, delete the Set 1 data column. (Only the Marker Line should remain on the plot).
- G) Select Analysis, Method/Input icon, then select Reset To Standard, Activate, again.
- H) In the Home group, Open/Import, retrieve file **INSPECT.W** for rr solution of similar data. Notice the fit-line difference.
- I) In Analysis group, Method/Input, change Input Data Format + Inspect Type to “...Inspect(1)...”.



*Wes’s comments: Using Inspect Option #1 there is good agreement between the regression and the mle/intv5 likelihood Marker Line solution. Dr. Todd Marquart has shown reasonable agreement between Inspect Option #1, Probit, and Interval mle solution methods for the same original data. We’ve obtained good solutions in the past using the Inspect Option #1 regression method on inspection data. However, the Interval mle solution is best practice when the original data includes discovery and/or interval data.*



## Problem 56. Degradation (Deterioration) Modeling

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The PFEV (parameter as function of engineering variable) model in the SSW software for accelerated testing data analysis can model degradation data as well. The math is the same. It is also called “Covariate” analysis; different data sets represent measurements grouped by different values of some variable. This type of study is called “Degradation” analysis when measurements get worse with time. Let’s take an example where inspections provide data on storage vessel wall thickness.

For liquid storage vessels the wall thickness is crucial. Vessels can lose wall material when they store caustic agents or liquids at high temperature. Here are storage vessel wall thickness readings (mils) on the bottom of the same vessel at 12 locations for different inspection times:




Inspection at 1000 Days: 265, 310, 321, 350, 289, 292, 274, 342, 266, 284, 298, 307

Inspection at 2000 Days: 232, 285, 291, 341, 272, 274, 253, 316, 237, 263, 280, 284

Inspection at 2500 Days: 183, 229, 230, 242, 216, 217, 204, 235, 184, 214, 221, 228

Minimum acceptable thickness is 120 mils. Find the longest inspection time (Days) where we expect 99.9% chance minimum of acceptable thickness (same as 0.1% probability of unacceptable thickness)?

**SOLUTION:** In the SSW program, starting at the Main Screen, Home group ...

- 
- A) Select the blank page icon. Then select Analysis, Method / Input, and choose Reset To Standard.
  - B) Open data file **PlayTIME-Problem-56.w** to get the Lognormal cumulative distribution plot.
  - C) The Lognormal plot solution parameter values Median / SigmaFactor (Med SigF) should be “298.7” / “1.101”, “275.8” / “1.123”, and “216.1” / “1.096” respectively (with **rr** fit method). Return to the Main Screen if not there.
  - D) Select Options, Label/Style icon, and change the horizontal scale Units (in parentheses) to “mils”. Return to the Main Screen. In the Tools group, select Accelerated Test, Standard ... PFEV.
  - E) Enter 1 for Engineering Variable Quantity. Select “Find 1 Value For All” for Slope Results Type. Enter “Days” for Units. For Variable Value (Days), enter 1000 (Set 1), 2000 (Set 2), and 2500 (Set 3).
  - F) You should see the PFEV dialog box. Select menu item PFEV Fit Type and choose Exponential to get the solution at the top of the dialog box. Med Standard Error should be about 27.9 mils. The common SigF for this rr fit type should be about 1.1. Let’s look at results for different inspection times (in Days).
  - G) In this PFEV menu, select Plot Line At New Variable Value and enter 3800 for Engineering Variable 1 Days. A Marker Line will be added left of the data fit lines for the expectation at that inspection time. Repeat this operation (must select Tools, Accelerated Test from Main Screen each time) instead entering 4000 and 4200 and 4400 in turn for Engineering Variable. **RESULT:** You should have 4 Marker Lines showing on the plot with higher values of inspection time giving smaller wall thickness (mils) each time. Change Marker Line color if needed to better identify which lines go with which potential future inspection.
  - H) Select Options, Precise Reading, Line 1, then choose Enter Datum ... Get %, input 120, get expected occurrence probability, select Logbook icon to put text into the Logbook. Return to Main Screen.
  - I) Repeat the solution item just above except substitute sequentially Line2, Line3, and Line4. Return to main screen and open the logbook. Which inspection time shows closest to 0.1% will occur?
- 
- 

*Wes’s Comments: The Line 3 solution (4200 Day inspection time) gives about 0.1% probability for 120 mils thickness or less (99.9% probability of having 120 mils or more thickness). There are two general ways data can be organized for degradation analysis. This case study involves data from inspections. However, you can encounter different degradation-type data. Another format for degradation analysis takes data from sensors. If you had wall-thickness sensors generating data when a specific thickness limit is reached, then you would solve degradation differently from the approach used for this case study. For an analysis using sensors instead of inspection, each data set represents the variability in number of days it takes to reach a specified thickness. The software can then find the relationship between the thickness variability and how it migrates to lower values with time.*

### **Problem 57. One-Shot Devices Like Rockets, Missiles, and Munitions**

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You could use the binomial model for one-shot devices, however the requirements with the binomial lead to larger and more expensive tests. A better option when available might be the Crow-AMSAA (CA) model for trending the results. Here is an example of doing it using the SuperSMITH™ Visual software.

Suppose you have rocket firing data like this:

2x1  
5x2  
15x3  
18x4  
26x5  
33x6  
36x7  
62x8  
70x9

The number on the left in each data entry above is the accumulated quantity of firings. The number after the “x” is the accumulated quantity of unsuccessful firings (or “occurrences” in SuperSMITH terms). The definition of an unsuccessful firing is subjective but could indicate either that the launch was not completely satisfactory or instead it could be that the mission was a total failure or somewhere in between. Defining what is considered an occurrence is the job of the program manager with possible direction from a customer. **We want to show a nominal 90% reliability (50% confidence).** Let’s put the data into the SSV (Visual) program.

**SOLUTION:** In SSV (Visual, NOT Weibull!) software, starting at the Main Screen, Home group ...

- 1) Select the Open / Import icon and bring **ONE-SHOT-DEVICES.v** from the provided data files.
- 2) In the Tools group, select the Crow-AMSAA icon to get the CA menu. Make sure that the Method = IEC, and that no special options are selected.
- 3) Select the green checkmark to activate the analysis and get the CA plot.
- 4) The CA Lambda value should be close to 0.696 with CA Beta close to 0.602.
- 5) Return to the CA menu (Tools group, CA icon), then select the “S...SURV Transform”.
- 6) You should see a plot of SURV (survival probability, or launch reliability) vs. cumulative firing quantity.
- 7) Note that the reliability is increasing because the CA Beta is less than one. Read the cumulative reliability by looking at the last plotted point (the Y-value of the last data entry = 0.87, that is 87% reliability). This does not meet our goal of 90% ... but wait. Since reliability has been increasing all along, then our actual current (instantaneous reliability) should be better than what the cumulative shows. Let’s look at the instantaneous reliability.
- 8) Return to the CA menu (Tools group, Crow-AMSAA). Toggle the “I...Instantaneous Line: ... “ option so it reads “ \* YES”, then select the green checkmark to see the CA plot with the added instantaneous line (Set # 3) above the cumulative fit line.
- 9) Read the last Y-value in Set # 3. The “0.9225587...” shows 92% reliability minimum, meeting our goal.
- 10) EXTRA CREDIT --- Try the Binomial option in the SSV calculator to Find “N...Trial Quantity” needed to show 90% reliability with 50% confidence. Here’s how to do that ...
  - a. Select the Options group, Calculator icon. Select Binomial from the menu.
  - b. Select “N...Trial Quantity” for what to find. The binomial calculator menu appears.
  - c. In the binomial calculator menu, select “C...Cumulative Probability” and set the % to “50”. This will give 100%-50% for the final results confidence of 50%.
  - d. In the same menu, select “E...Event Quantity” and set to “9” (same as the data above).
  - e. In the same menu, select “P...Probability Of 1 Event ... “, set % to “10” for R = 90%.
  - f. The menu then shows “ \* Trial Quantity = 97” so that the trial quantity needed is 97 test firings using the binomial allowing 9 failures maximum, but our CA method only needed 70 test firings to show the same results. The upward CA trend in reliability helped our cause.

## Problem 58. Missing Data Estimates

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Ref: “Estimating Quantity of Missing Data in Tails of Distribution”, Tarum & Fulton, RAMS 2021

Sometimes you know you don’t have all the data, but don’t know how much is missing. In SSW, open file **FIG3-11.W**. These were cables buried in 1971/72/73, but tracking wasn’t started until 1976. Does it really make sense that the 1971 cables were failure free for 5 years? The t0 solution implies this. We will look at this with a “Missing Data” view.

### SOLUTION:

1. Start in SSW (Weibull). Select New/Clear for a new analysis.
2. After opening file **FIG3-11.W**, then select the Tools group, then “Tarum Missing Data” Icon.
3. Select Set #1, 1971. A “Fast Estimate” will be shown. There may be points missing at the lower end, in the middle, and/or points missing at the upper end. The estimate indicates that it is likely that data are missing.



On the menu, check that Lower Range Analysis = YES, and Middle Range Analysis = NO. Then click Activate. Shortly you will see this screen.

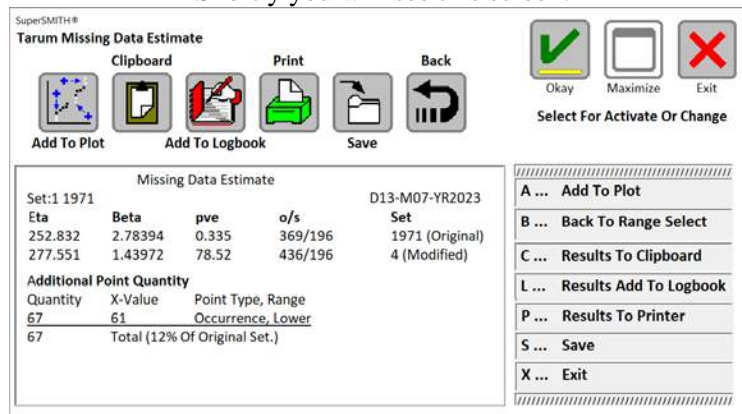
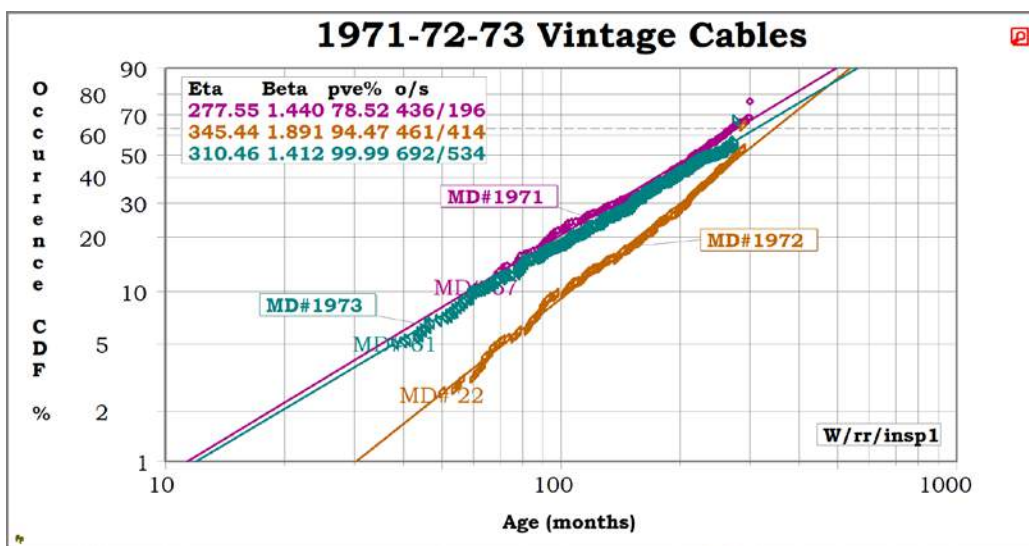


Figure 10: Missing Data Estimate

4. The analysis indicates that adding 67 points at time 61 months will improve the pve from 0.335 to 78.5%. Click the “Add to Plot” icon. This will make a new set with the 67 points added at the first point, labeled “MD#67”. You can repeat for 1972 and 1973. You can use the Report on the Options group to compare B lives. Note that with Missing Data estimates, the B lives are lower. Also note that 1971 and 1973 data are very similar. This isn’t obvious from the t0 analysis unless you select “Not As Recorded” from the 3 parameter options. Below is the result for all years.



## **Section IV - Benchmark-Baseline Problems**

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Accuracy and uncertainty with good and bad data are covered in detail by Dr. Abernethy's standard reference, "The New Weibull Handbook©. " Many software programs and textbooks do not recommend best practices among all possible alternative methods and do not include benchmark case studies to verify their methods are correct. SuperSMITH and The New Weibull Handbook do both. In the SuperSMITH software, select the **HELP** icon (shows "?" in blue) to display HELP. Then select **Product Information** and then **Benchmarking**. Follow the instructions in that section to review several hallmark life data case studies.

Many organizations compare SuperSMITH to their own in-house Weibull program or to other Weibull programs. SuperSMITH can validate other software. You can identify deficiencies in other software such as incorrect regression method, incorrect guidance to use only maximum likelihood for every data set, blind advocacy of Weibull results based on very small samples, and obsolete confidence. These validity checks are appropriate since some corrective actions make heavy use of SuperSMITH results. Confidence must be established that the software is accurate. To assist in this effort, baseline problems were taken from hallmark textbooks in life data analysis (referenced in The New Weibull Handbook) and made available in the software. SuperSMITH HELP describes these. Some of the data sets are available under the dice (sampling) icon with selection "Special Data Set ...". Where there is any question about differences between SuperSMITH and other computer programs, these baseline problems should be run on both programs to provide comparison. If differences are observed and cannot be explained, please contact the author. Figure 11 below illustrates the SuperSMITH Visual Main Screen. Visual is a standalone companion to the Weibull software for general auxiliary plotting. Crow-AMSAA trending is one of the main capabilities of Visual along with function viewing, transforms, and contour generation. SuperSMITH HELP presents a description of each selection on this Main Screen. A screen-shot of YBath <sup>TM</sup> is provided in the section on YBath advanced mixture analysis.

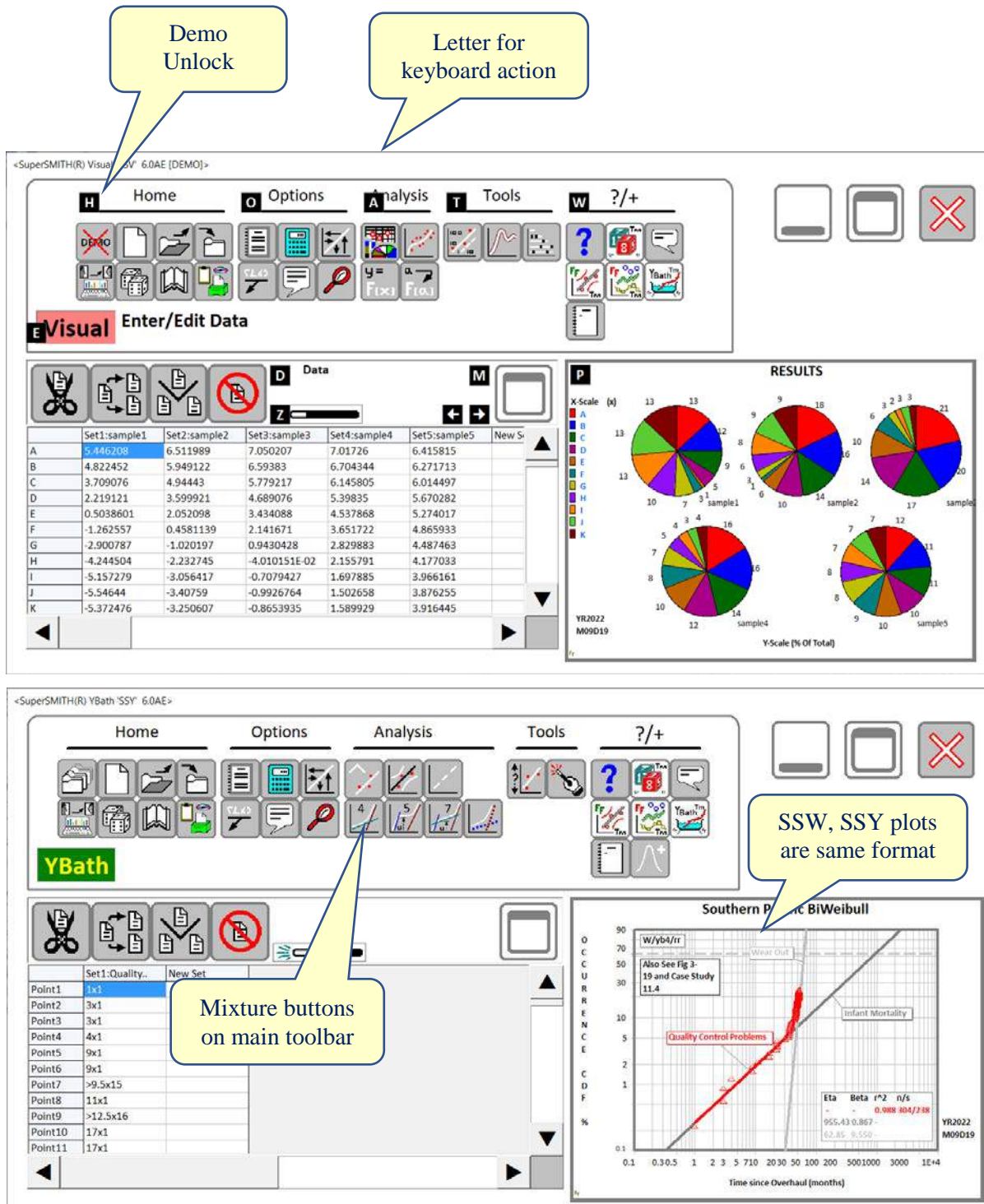


Figure 11: New SuperSMITH Visual and YBath Main Screen Layout.

These layouts show the “Small Icon” option available under Setup, while Figure 2 shows the tabbed layout version that is friendlier for touch-screen. SS Visual shows the Demo unlock button at upper left. The **White on Black letters** show available keyboard commands when the <alt> key is pressed and released. SS YBath shows a sample plot and the direct selection buttons for 4, 5, and 7 parameter Weibull.



Figure 12 below is a Weibull to/from Six Sigma Conversion table to work with other models and report in Six Sigma terminology. Figure 13 below describes the math of advanced mixture models in YBath™.

Normal Distribution										Weibull	
Double Sided				Single Sided							
Short Term Capability (actual Z tables)		Double Sided		PPM / DPMO	Short Term Capability		Long Term Capability with Shift of 1.5 Std Dev		Single Sided (BXX)		
Sigma	Pp	% Non-Coforming	PPM		Sigma	Pp	Sigma (with 1.5)	Pp (with 1.5)	Lower	Upper	
-0.5	-	-	-	* 691,462 *	- 0.5	-	1.0	0.33	-	-	
0.0	0	-	-	500,000	0	0	1.5	0.50	50	50	
0.3	0.10	76.4177	764,177	382,089	0.3	0.10	1.8	0.60	38	62	
0.5	0.17	61.7076	617,076	308,538	0.5	0.17	2.0	0.67	31	69	
0.6	0.20	54.8506	548,506	274,253	0.6	0.20	2.1	0.70	27	73	
0.9	0.30	36.8120	368,120	184,060	0.9	0.30	2.4	0.80	18	82	
1.0	0.33	31.7300	317,300	158,650	1.0	0.33	2.5	0.83	16	84	
1.2	0.40	23.0139	230,139	115,070	1.2	0.40	2.7	0.90	12	88	
1.5	0.50	13.3614	133,614	66,807	1.5	0.50	3.0	1.00	7	93	
1.8	0.60	7.1860	71,860	35,930	1.8	0.60	3.3	1.10	3.6	96.4	
2.0	0.67	4.5500	45,500	22,750	2.0	0.67	3.5	1.17	2.3	97.7	
2.05	0.68	4.0000	40,000	20,000	2.05	0.68	3.6	1.18	2.00	98.00	
2.1	0.70	3.5729	35,729	17,865	2.1	0.70	3.6	1.20	1.79	98.21	
2.32	0.77	2.0000	20,000	10,000	2.3	0.77	3.8	1.27	1.00	99.00	
2.4	0.80	1.6395	16,395	8,198	2.4	0.80	3.9	1.30	0.82	99.18	
2.5	0.83	1.2420	12,420	6,210	2.5	0.83	4.0	1.33	0.62	99.38	
2.7	0.90	0.6934	6,934	3,467	2.7	0.90	4.2	1.40	0.35	99.65	
3.0	1.00	0.2700	2,700	1,350	3.0	1.00	4.5	1.50	0.135	99.865	
3.09	1.03	0.2000	2,000	1,000	3.09	1.03	4.6	1.53	0.100	99.900	
3.3	1.10	9.669E-02	967	483	3.3	1.10	4.8	1.60	0.048	99.952	
3.5	1.17	4.600E-02	460	233	3.5	1.17	5.0	1.67	0.023	99.977	
3.72	1.24	2.000E-02	200	100	3.72	1.24	5.2	1.74	0.010	99.990	
3.9	1.30	9.619E-03	96	48	3.9	1.30	5.4	1.80	0.0048	99.995	
4.0	1.33	6.334E-03	63	32	4.0	1.33	5.5	1.83	0.0032	99.997	
4.2	1.40	2.669E-03	26.7	13	4.2	1.40	5.7	1.90	0.0013	99.999	
4.27	1.42	2.000E-03	20.0	10	4.27	1.42	5.8	1.92	0.0010	99.999	
4.5	1.50	6.795E-04	6.8	3.4	4.5	1.50	6.0	2.00	0.00034		
4.7	1.57	2.600E-04	2.6	1.3	4.7	1.57	6.2	2.07	0.0001		
4.8	1.60	1.587E-04	1.6	0.8	4.8	1.60	6.3	2.10			
5.0	1.67	5.733E-05	0.573	0.287	5.0	1.66	6.5	2.17			
5.5	1.83	3.798E-06	0.038	0.019	5.4	1.80	7.0	2.33			
6.0	2.00	1.973E-07	0.002	0.001	6.0	2.00	7.5	2.50			
7.0	2.33	2.560E-10	0.000003	0.0000013	7.0	2.33	8.5	2.83			
8.0	2.670	1.244E-13	0.000000	0.0000000006	8.0	2.670	9.5	3.17			
9.0	3.00	2.258E-17	0.000000	0.0000000000001	9.0	3.00	10.5	3.50			

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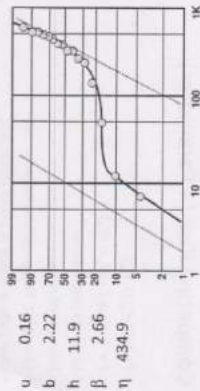
8/16/2017

Figure 12: Weibull to/from Six Sigma Conversion Table ("Keisic Table")

<p>Classification and analysis of Weibull Mixtures</p> <p>Carl D. Tarum, PE ctweb@att.net Ph 989.780.5088 (cell)</p>	<p>Weibull Mixtures</p> <ul style="list-style-type: none"> <li>If possible, separate data and analyze <ul style="list-style-type: none"> <li>Better predictions</li> <li>But don't arbitrarily assign to a sub-set</li> </ul> </li> <li>Simple (Exclusive OR) Mixture <ul style="list-style-type: none"> <li>Batch mixture if no 2<sup>nd</sup> failure mode</li> </ul> </li> <li>Competing Risk Mixture</li> <li>Partial Competing Risk Mixture</li> <li>Compound Competing Risk Mixture</li> <li>Note: Some authors use other definitions. Check the equations to see what is being described.</li> </ul>	<p>Equation &amp; Nomenclature</p> <ul style="list-style-type: none"> <li>Weibull: <math>F(t) = 1 - \exp[-W]</math> <ul style="list-style-type: none"> <li>where <math>W = (t/\eta)^\beta</math></li> </ul> </li> <li>General Weibull Mixture: <math display="block">F(t) = 1 - (u_j \exp[-W_j - W_2 - \dots] - (u_j \exp[-W_j - W_2 - \dots] - u_2 \exp[-W_2 - \dots] - \dots - (1 - u_1 - u_2 - \dots - u_n) \exp[-W_n - W_2 - \dots])</math> <ul style="list-style-type: none"> <li><math>u_n</math> = subset of population</li> </ul> </li> </ul>
<p>Simple (Exclusive OR) Mixture</p> <ul style="list-style-type: none"> <li><math>F = u \cdot F_1 + (1 - u) \cdot F_2</math></li> <li><math>F = 1 - (u_j \exp[-W_j]) - (1 - u_j) \exp[-W_2]</math></li> <li>Each population has own failure mode</li> <li>Example: Thermal issues in trucks sold in <ul style="list-style-type: none"> <li>Michigan or Alaska</li> <li>Truck is in one place or the other</li> </ul> </li> </ul>	<p>Simple (Exclusive OR) Mixture</p> $F = 1 - (u_j \exp[-W_j]) - (1 - u_j) \exp[-W_2]$ <p>u 0.16 b 2.22 h 11.9 <math>\beta</math> 2.66 <math>\eta</math> 434.9</p> <p>Data from: One Failure Assurance Technologies</p>	<p>Subset (Batch) Mixture</p> $F = 1 - (u_j \exp[-W_j])$ <p>4 early failures, many suspensions beyond 1K</p>
<p>Competing Risk (4 Parameter)</p> <ul style="list-style-type: none"> <li><math>F = 1 - (1 - F_1)(1 - F_2)</math></li> <li><math>F = 1 - \exp[-W_1 - W_2]</math></li> <li>Uniform population</li> <li>Mixture of risks</li> <li>Example: Tire puncture or wear out</li> <li>One Failure mode is often dominant</li> </ul>	<p>Competing Risk (4 Parameter)</p> <p>u --- b 0.56 h 1283.9 <math>\beta</math> 5.8 <math>\eta</math> 490.7</p>	<p>Competing Risk Batch Mixture</p> <ul style="list-style-type: none"> <li><math>F = u \cdot (1 - F_1) + (1 - u) \cdot F_2</math></li> <li><math>F = 1 - (u_j \exp[-W_j - W_2] - (1 - u_j) \exp[-W_2])</math></li> <li>First subset has additional failure mode</li> <li>Example: Tire with <math>u_j</math> out-of-round <ul style="list-style-type: none"> <li>First subset may have replacement or puncture</li> <li>Remainder has only puncture risk</li> <li>Out of round does not prevent puncture!</li> </ul> </li> </ul>

Figure 13: YBath™ Advanced Mixture Models (Page 1 of 2) From Carl Tarum.

### Competing Risk Batch Mixture



### Other Models Possible

- Double partial Competing Risk
- Triple Competing Risk Mixture
- (Define your own)
- Literature available on Web
  - Search "Weibull" AND "Mixture"
  - [www.harrington1.com](http://www.harrington1.com)
  - [www.weibullllnw.com](http://www.weibullllnw.com)

### Analysis of Parameters

- Simple Weibull
  - $\beta < 1$  implies infant mortality
- Weibull Mixtures
  - $\beta < 1$  implies ???
- Consider a polynomial
  - $Y = ax^{c1} + bx^{c2}$
  - Many values of  $a$  &  $b$  give similar results.

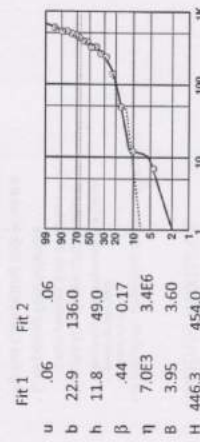
### WeiBath (Weibull Bathtub) (Compound Competing Risk Mixture)

- $F = u[1 - (1-F_1)(1-F_2)]$
- $+ (1-u)[1 - (1-F_1)(1-F_2)]$
- $F = 1 - (u_1 \exp[-W_1 - W_2] - (1 - u_1) \exp[-W_1 - W_2])$
- First subset has additional failure mode
- Example: Tire with
  - a lot of round, b/puncture, c/wear out

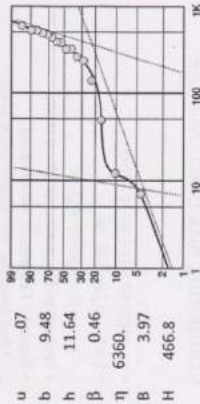
### Solution Methods

- Rank Regression or MLE fits
- Excel Solver or MathCad can be used
- SuperSmith/ YBath will solve
  - Faster convergence (seconds vs hours)
  - Graphs

Two possible fits, different parameters.  
(Same data set, two different fits)



### Compound Competing Risk Mixture WeiBath



### Model Selection

- Good fit doesn't mean correct fit.
- High order models should be chosen based on theory, not apparent fit.

### Conclusions

- Mixtures can be modeled.
  - Best to separate failures, if possible
  - Models predict fit if modes are lumped
- Several choices for 2 failure modes.
  - Competing Risk Mixture
  - Partial Competing Risk Mixture
- Don't interpret individual parameters.
- Techniques apply to other distributions

Figure 14: YBath™ Advanced Mixture Models (Page 2 of 2) from Carl Tarum.



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