On the Convergence of Credit Risk in Current Consumer Automobile Loans

Jackson P. Lautier, FSA[1], Vladimir Pozdnyakov[1], Jun Yan[1]

[1]Department of Statistics, University of Connecticut

March 23, 2023

Table of Contents

UCONN
UNIVERSITY OF CONNECTICUT

A junior credit analyst's observation

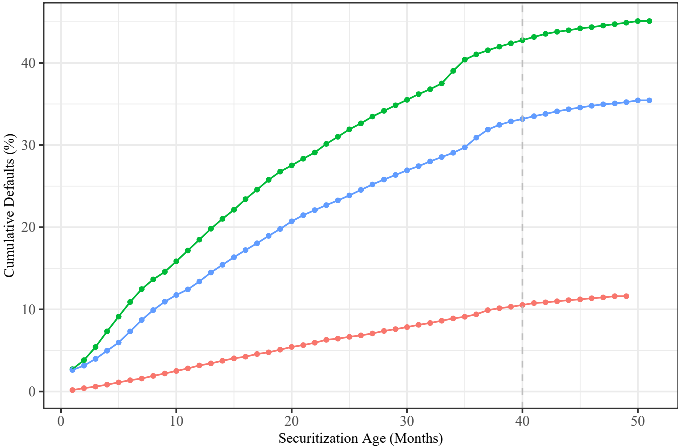


Figure: ABS loss curves all eventually flatten (see after vertical dashed line at age 40), despite being a subprime pool (top curves) or a more prime pool (bottom curve)

UCONN
UNIVERSITY OF CONNECTICUT

A tale of two papers

- Observation: After a sustained period of performance, current loans appear to stay current, regardless of the loan's initial risk classification. $\Longrightarrow$
- **Contribution 1**: A rigorous statistical demonstration of the concept of *credit risk convergence*: borrowers of secured consumer automobile loans in different credit risk bands that remain current eventually converge in default risk.
- Reflection: The cost of borrowing is positively associated with risk and is assigned at loan origination via risk-based pricing (e.g., Edelberg, 2006; Phillips, 2013). If current risky borrowers eventually become better credits, does this mean they eventually outperform their APR? $\Longrightarrow$
- **Contribution 2**: A study of the financial implications of credit risk convergence in light of the differences in APR due to risk-based pricing for both lenders and consumers.

How to classify this paper

At its heart, our work falls within the space of consumer finance, sharing genetic material with work in payday loans (e.g. Melzer, 2011; Bertrand and Morse, 2011), credit cards (e.g. Gross and Souleles, 2002; Agarwal et al., 2014), and automobile loans (e.g. Adams et al., 2009; Grunewald et al., 2020) to name but a few.

But, it requires statistical tools to empirically validate a current borrower's declining credit risk over time. Hence, we take effort to establish credit risk convergence, which then allows us to offer evidence of a possible inefficiency within consumer credit markets and opine on surprising borrower behavior.

We also reveal a rich source of publicly available consumer debt data that is currently underutilized in the literature.

UCONN
UNIVERSITY OF CONNECTICUT

Table of Contents

Regulation AB

The Securities and Exchange Commission (SEC) recently implemented changes to the rules governing the issuance of asset-backed securities (ABS) (Securities and Exchange Commission, 2014, 2016).

Notably, it requires public issuers of ABS to make freely available pertinent loan-level information and payment performance on a monthly basis beginning in 2017.

The data in `xml` format may be accessed via the Electronic Data Gathering, Analysis, and Retrieval (EDGAR) system operated by the SEC.

Selected Bonds

We wrote `Python` code to scrape SEC filings to amass over 275,000 consumer automobile loans from the ABS bonds:

- CarMax Auto Owner Trust 2017-2 (CarMax, 2017);
- Ally Auto Receivables Trust 2017-3 (Ally, 2017);
- Santander Drive Auto Receivables Trust 2017-2 (Santander, 2017b);
- Drive Auto Receivables Trust 2017-1 (Santander, 2017a).

These four bonds were selected because:

(i) Taken together, they span the full consumer credit profile;

(ii) No issuer is a subsidiary of an auto manufacturer;

(iii) The paying periods span the same macroeconomic environment (i.e., actively paying starting in March-April-May 2017 for 44-52 months).

UCONN
UNIVERSITY OF CONNECTICUT

Why auto loans?

- Subject matter expertise (Lautier former ABS auto analyst);
- Auto loans have a high *priority of payment* $\implies$ if default risk doesn't converge for auto loans, it probably doesn't converge.

  (unofficial consumer ABS credit analyst motto: "You can live in your car, but you can't drive your house to work.")

UCONN
UNIVERSITY OF CONNECTICUT

Loan selection

We filtered loans to be as comparable as possible:

- No co-borrowers;
- Income underwriting level: "stated not verified";
- No subvention;
- Used vehicles only;
- No loans in repossession status at ABS start;
- Loan age $\leq$ 18 months at ABS start;
- Loan term 72-73 months only;
- No unclear loan outcome (i.e., no default but total principle paid less than outstanding balance as of ABS start);

Number of loans left for analysis: 58,118. Largest geographic concentration (TX 13%); manufacturer (Nissan 13%).

Risk-Based Pricing

Following Phillips (2013), a borrower's interest rate in risk band a, r_a, is

$$r_a = r_c + m + l_a,$$

where r_c is the cost of capital, m is the added profit margin, and l_a is a factor that varies by risk band. More generally,

$$l_a \equiv f(\text{PTI}, \%\text{Down}, \text{Loan AMT}, \text{Vehicle Val.}, \text{Credit}, \text{etc.}).$$

That is, the interest rate is the market's reflection of a borrower's (i.e., loan's) risk profile.

Risk band assignment

Hence, we can defer to the market and assign borrowers to risk bands via interest rate. Specifically,

Risk Band	APR Range	Count	Default%[1]
deep subprime	20%+	21,630	52%
subprime	15-20%	21,332	37%
near prime	10-15%	6,677	21%
prime	5-10%	6,300	10%
super prime	0-5%	2,179	4%
		58,118	

Note: The terms “deep subprime”, “subprime”, etc. also correspond well to the traditional credit score ranges (Consumer Financial Protection Bureau, 2019); see next slide.

[1] We define 3 consecutive months of missed payments = default.

Summary of 58,118 loans

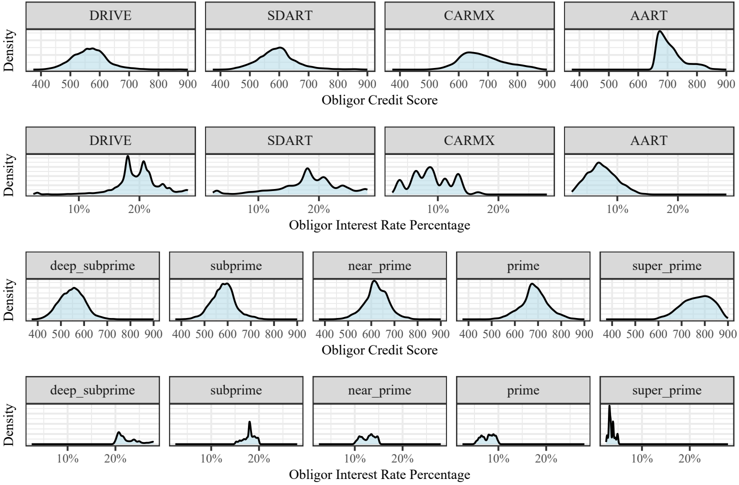


Figure: Details by bond, assigned risk band

Table of Contents

UCONN
UNIVERSITY OF CONNECTICUT

Survival Analysis 101

We are interested in the time-to-loan termination random variable (RV), X. The classical tool from survival analysis is the *hazard rate*,

$$\lambda(x) = \Pr(X = x \mid X \geq x) = \frac{\Pr(X = x)}{\Pr(X \geq x)}. \tag{1}$$

The probability in (1) is ideal for a current loan analysis. Further, a reliable estimate of (1) allows us to recover the distribution function of X via

$$1 - F(x-) = \Pr(X \geq x) = \begin{cases} 1, & x \leq x_{\min} \\ \prod_{x_{\min}}^{x-1} \{1 - \lambda(x)\} & x > x_{\min}, \end{cases}$$

where $x_{\min}$ is the lowest recoverable value of the lifetime X.

UCONN
UNIVERSITY OF CONNECTICUT

Statistical challenges of ABS data

When estimating the time-to-event distribution from loans sampled from a securitization pool, there are incomplete data challenges:

- Left-truncation (there is a delay from loan origination until the trust begins actively paying);
- Right-censoring (many loans will be known to be active but not yet terminated);
- Discrete-time (loan payments due monthly; assuming continuous time req. unrealistic assumptions (i.e., "no ties"))

The combination of left-truncation and discrete time has received surprisingly limited study in the statistical literature. See the papers Lautier et al. (2021) and Lautier et al. (2023) for a rigorous treatment in the context of ABS.

UCONN
UNIVERSITY OF CONNECTICUT

Generalization: Competing risks

We need more: to distinguish between a default and prepayment.

Let Z_x be a two-state RV with probabilities dependent on x (i.e., given time x, $Z_x \in \{1, 2\}$). This is a mulitstate process (e.g., Beyersmann et al., 2009). In survival analysis lingo:

- $\lambda_\tau^{01}(x)$: *cause-specific* hazard rate for default (event of interest).
- $\lambda_\tau^{02}(x)$: *cause-specific* hazard rate for prepayment (comp. event).
- $\lambda_\tau(x) = \lambda_\tau^{01}(x) + \lambda_\tau^{02}(x)$: *all-cause* hazard rate.

Formally,

$$\lambda_\tau^{0i}(x) = \Pr(X = x, Z_x = i \mid X \geq x) = \frac{\Pr(X = x, Z_x = i)}{\Pr(X \geq x)}, \quad i = 1, 2,$$

Goal: estimate λ_τ^{0i} by risk band.

UCONN
UNIVERSITY OF CONNECTICUT

Estimating λ_τ^{0i}

If we assume $Y \perp (X, Z_x)$ (reasonable for ABS) and define
$f_{*,\tau}^{0i}(x) = \Pr(X = x, X \leq C, Z_x = i \mid X \geq Y)$, $i = 1, 2$,
$C_\tau(x) = \Pr(Y \leq x \leq \min(X, C) \mid X \geq Y)$, then

$$\lambda_\tau^{0i}(x) = \frac{\Pr(X = x, Z_x = i)}{\Pr(X \geq x)} = \frac{f_{*,\tau}^{0i}(x)}{C_\tau(x)}. \tag{2}$$

Estimation of (2) follows naturally with

$$\hat{f}_{*,\tau,n}^{0i}(x) = \frac{1}{n} \sum_{j=1}^{n} \mathbf{1}_{X_j \leq C_j} \mathbf{1}_{Z_{X_j}=i} \mathbf{1}_{\min(X_j, C_j)=x},$$

$$\hat{C}_{\tau,n}(x) = \frac{1}{n} \sum_{j=1}^{n} \mathbf{1}_{Y_j \leq x \leq \min(X_j, C_j)};$$

that is, $\hat{\lambda}_{\tau,n}^{0i}(x) = \hat{f}_{*,\tau,n}^{0i}(x) / \hat{C}_{\tau,n}(x)$, $i = 1, 2$.

Note: the RVs Y and C correspond to censoring and truncation; see Lautier et al. (2023) for details.

UCONN
UNIVERSITY OF CONNECTICUT

Hypothesis test

We prove the estimator $\hat{\lambda}^{0i}_{\tau,n}(x)$ (itself a RV!) has attractive statistical properties (asymptotic normality & independence, see Proposition 1 in the Appendix). In combination with Lemma 1 (Appendix), this leads to a straightforward large sample hypothesis test.

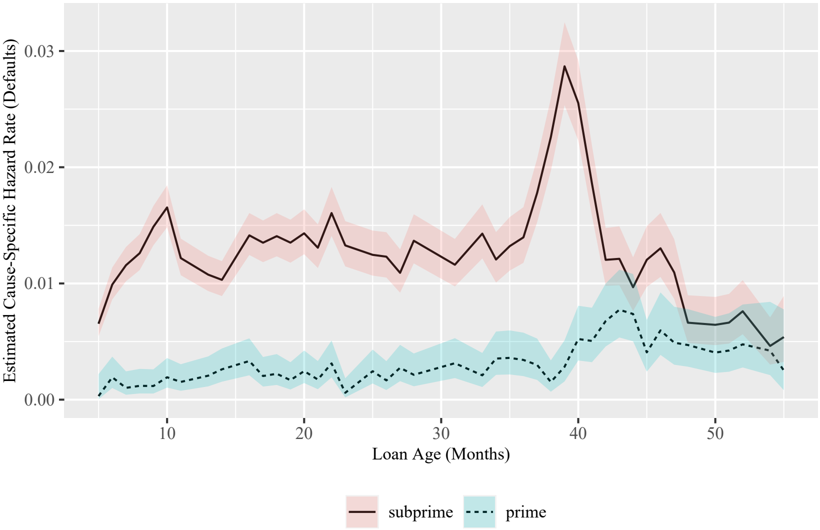
Specifically, let a, a' be two different risk bands (e.g., subprime vs. prime, etc.). Then we may test

$$H_0 : \lambda^{01}_{\tau,(a)} = \lambda^{01}_{\tau,(a')} \quad \text{vs.} \quad H_1 : \lambda^{01}_{\tau,(a)} \neq \lambda^{01}_{\tau,(a')},$$

for each age x by determining if the asymptotic confidence intervals in (3) overlap. Decision rule:

- Confidence intervals overlap $\implies$ fail to reject H_0 $\implies$ can't claim $\lambda^{01}_{\tau,(a)} \neq \lambda^{01}_{\tau,(a')}$ $\implies$ conditional default risk converged.
- Confidence intervals **do not** overlap $\implies$ reject H_0 $\implies$ accept $\lambda^{01}_{\tau,(a)} \neq \lambda^{01}_{\tau,(a')}$ $\implies$ conditional default risk has **not** converged.

Credit Risk Convergence Visualization

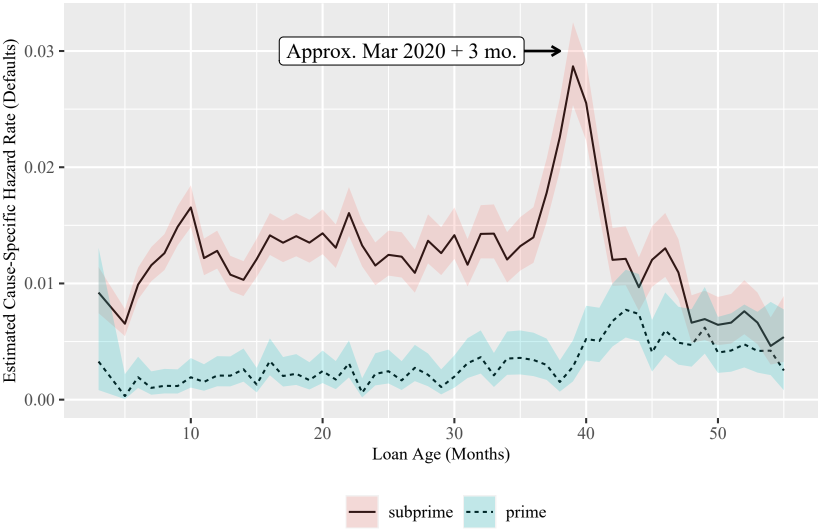


Credit risk convergence matrix (months)

	deep sub.	subprime	near-prime	prime	sup.-prime
deep sub.	10	36	50	50	52
subprime		10	41	42	48
near-prime			10	13	43
prime				10	10
sup.-prime					10

Note: The first of three consecutive months of confidence interval overlap after month 10.

A Digression on COVID-19



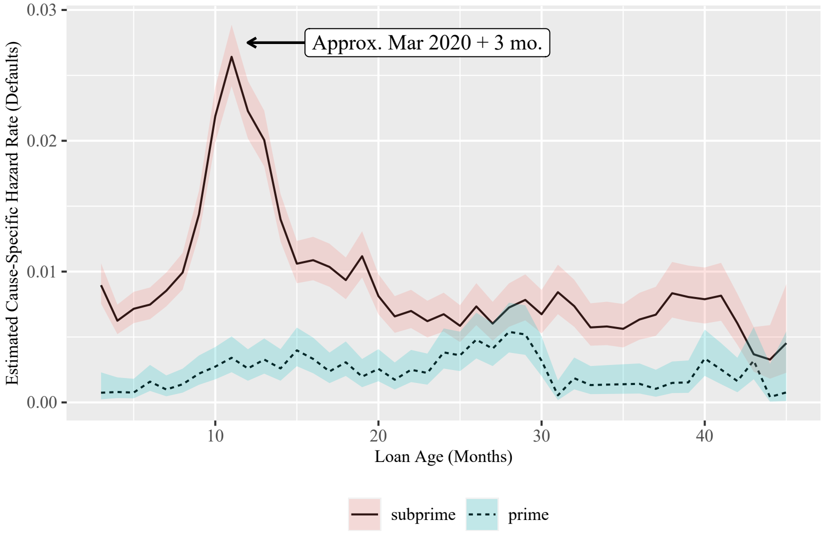
2019 Credit risk convergence matrix (months)

We repeat the 2017 analysis for the same bonds issued in late summer 2019 (SDART 2019-3, DRIVE 2019-4, CARMX 2019-4, AART 2019-3). While there is some evidence of earlier convergence, we see loan age also plays a role.

	deep sub.	subprime	near-prime	prime	sup.-prime
deep sub.	10	39	NA	NA	NA
subprime		10	23	24	NA
near-prime			10	15	15
prime				10	10
sup.-prime					10

Note: The first of three consecutive months of confidence interval overlap after month 10. The complete recoverable range of X is $4, 10 \leq X \leq 30, 35, 38 \leq x \leq 43$ for 72-73 month loans (i.e., "NA" $\implies$ no obs. conv. within recoverable range of X). The above consists of 65,892 total loans.

A Digression on COVID-19 (Cont.)



A Digression on COVID-19 (Summary)

Concern: the shock of the COVID-19 economic shutdown was so severe it "filtered" all the bad risks.

- We see some evidence of convergence prior to May 2020.
- 2019 analysis: COVID had impact but loan age also plays a role.
- Difficult to find a stretch of 72 consecutive months in the last 20 years without an economic shock (e.g., 9/11, global financial crisis, European sovereign debt crisis, COVID-19) $\Longrightarrow$ credit risk convergence perpetually present, even if partially driven by filtering effects of economic crisis.

Collateral Type & CarMax

All analysis has focused on used cars at the point of sale.

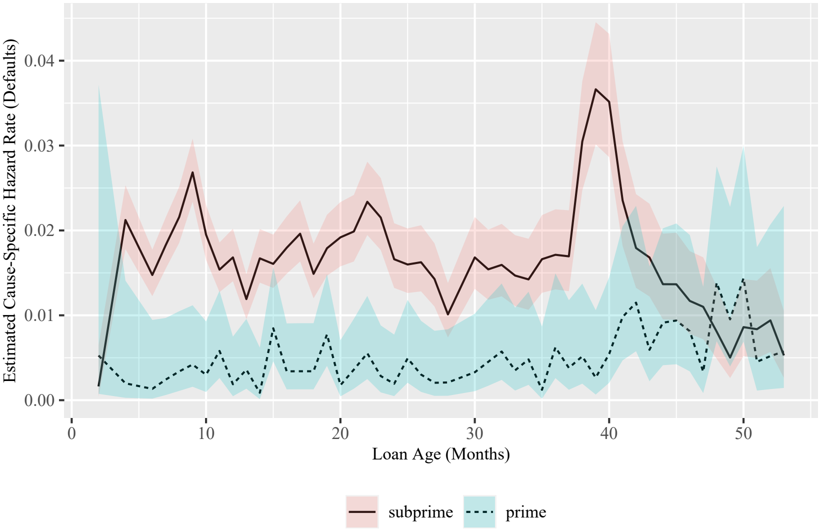
Additionally, the business model of CarMax differs than that of the other banks (Ally, Santander) issuing the bonds.

We repeated the 2017 analysis but this time filtering on new cars.

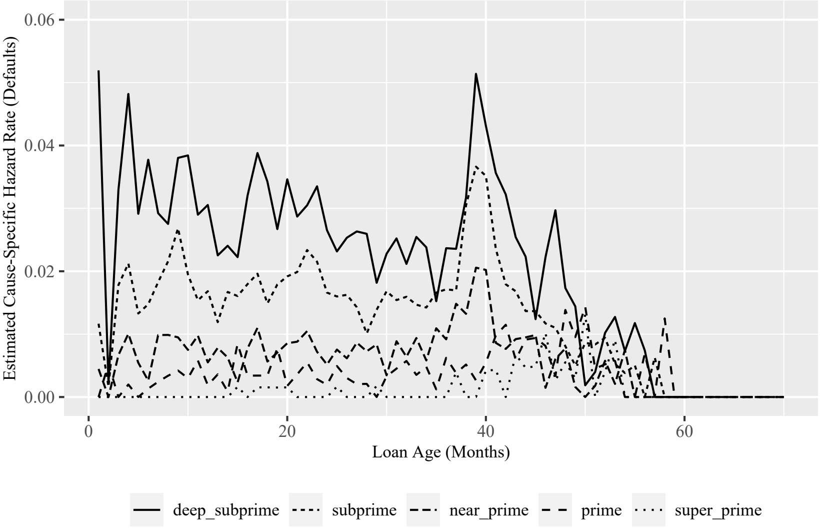
	DRIVE	SDART	ALLY	CMAX	Total
Count	7,692	7,369	1,342	9	16,412

	deep sub.	subprime	near-prime	prime	sup.-prime	total
Count	3,892	8,242	2,132	1,407	739	16,412

Robustness Check: New cars, no CarMax



Sometimes, a simple line plots suffices



Robustness Check: Loan type

Concern: These results for secured auto loans will not extend to other types of loans (e.g., credit-cards, peer-to-peer, unsecured, mortgages, etc.).

- Borrowers are slowly building an equity position, which likely acts as an incentive to keep making payments to own the vehicle outright (good sign for mortgages).
- Auto loans are a high priority of payment ("You can live in your car..."); may not hold for residential mortgages or vacation/income properties.
- We suggest caution before accepting these results for unsecured loan types, especially, and we leave this open as an area of further study.
- We postulate this "survivor bias" will extend too many other areas of finance, such as high-yield or junk bonds, but more research is needed.

Robustness Checks (Summary)

We see evidence of *credit risk convergence* with loans written on new cars and for a sample that does not draw meaningfully from CarMax.

There are fewer defaults, especially for new cars within prime and super-prime risk bands. This limits effectiveness of large sample statistics (i.e., obtaining the confidence intervals).

We suspect the results will generalize to other forms of debt, but more study is needed.

Table of Contents

UCONN
UNIVERSITY OF CONNECTICUT

Lender Profits "Backloaded"

Conventional wisdom is that the high-returns of high-risk loans that don't default help repay the lender for the loans that do default.

This is similar to insurance arrangements: the insured paying premiums that don't suffer claims help pay for the insureds that do suffer claims.

The loans we consider are sampled from securitization pools, however, and so the risk has already been transferred off the lender's books.

But, we are interested in estimating a month-by-month updated fair loan price at the individual loan level, as it remains current. We may use the probabilistic estimates of $\hat{\lambda}^{01}_{\tau,n}$ within an actuarial approach, which we now illustrate.

Let B_x denote the scheduled amortization balance at month x, P denote the scheduled payment, and R_x denote the assumed recovery of a defaulted consumer auto loan at month x.

For a given risk band a at each age x, we assume an investor purchases a one-month risky fixed-income asset for B_x that pays $B_{x+1} + P$ with probability $1 - \lambda_{\tau}^{01}(x)$ and pays R_{x+1} (defaults) with probability $\lambda_{\tau}^{01}(x)$.

$$
\begin{tikzcd}
 & B_{a|x+1} + P_a \\
B_{a|x} \arrow[ur, "1 - \lambda^{01}_{\tau,(a)}(x)"] \arrow[dr, "\lambda^{01}_{\tau,(a)}(x)"'] & \\
 & R_{x+1}
\end{tikzcd}
$$

Actuarial (Expected) Present Value

Hence, we find the rolling one-month risk-adjusted rate of return, $\tilde{r}_{a|x}$, such that

$$\text{EPV}^1_{a|x} = \lambda^{01}_a(x)\frac{R_{x+1}}{1+\tilde{r}_{a|x}} + (1-\lambda^{01}_a(x))\left[\frac{B_{a|x+1}+P_a}{1+\tilde{r}_{a|x}}\right] = B_{a|x}.$$

For simplicity, assume a common loan of \$100 amortized over 72 months with a payment that depends on the average APR of risk-band a.

We may simply replace λ^{01}_a with our earlier estimates $\hat{\lambda}^{01}_a$, but we need an estimate of the recoveries, R.

UCONN
UNIVERSITY OF CONNECTICUT

Estimating recovery upon default (i.e., repo proceeds)

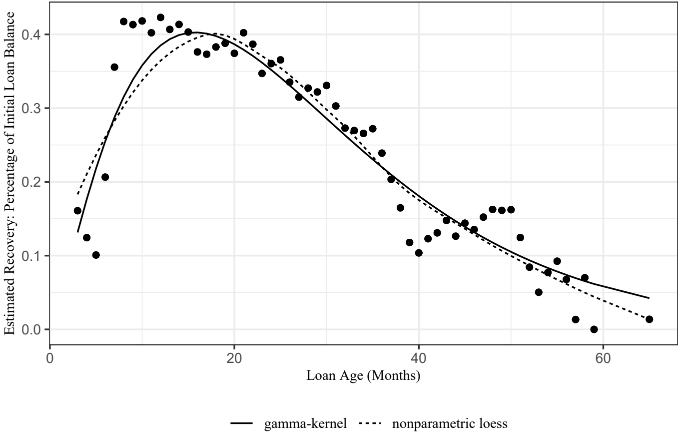


Figure: Estimation of recovery upon default assumption (based on 2017 sample of 58,118 loans)

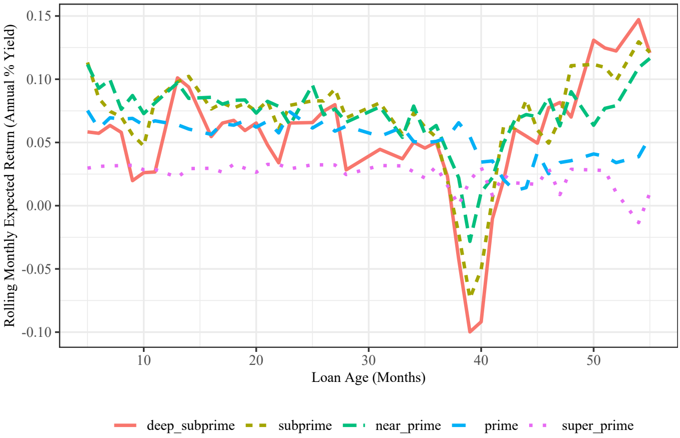


Figure: Est. conditional monthly risk-adjusted return (2017 issuance)

Lender Profitability (Summary)

Our risk-adjusted return calculations show that generally deep subprime, subprime, near-prime, and prime loans are closely clustered for approx. the first three years of the loan around 7.5%.

The negative impact of COVID is clear around month 40 (approx. Spring 2020).

Super prime loans are fairly stable around 2.5% (risk-adjusted).

After convergence, the returns of higher APR loans (deep subprime, subprime, near-prime) begins to accelerate (i.e., profits are "back-loaded").

A Consumer Perspective

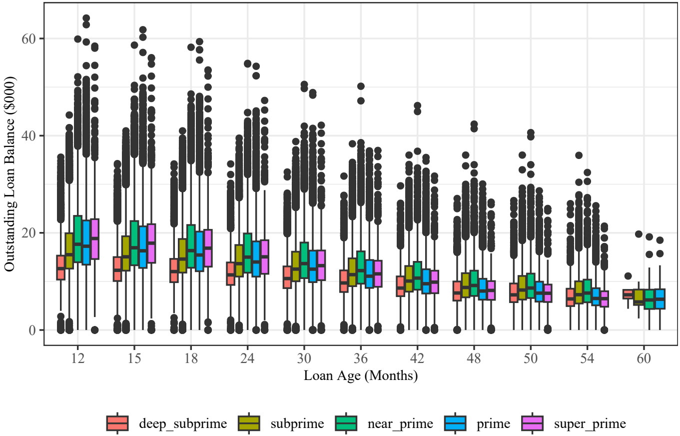


Figure: Outstanding balance by loan age, risk category

Estimated Savings (Deep Subprime Borrowers)

		Averages				Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	6,084	13,295	344	23.01	65								
15	5,622	12,887	343	22.99	62								
18	5,002	12,598	343	22.99	60								
24	4,082	11,976	341	22.92	55								
30	3,333	11,192	342	22.87	49								
36	2,766	10,245	341	22.85	43	16				577			
42	2,170	9,187	339	22.85	37	14				428			
48	1,782	8,237	342	22.83	32	16				386			
50	1,674	7,817	342	22.85	30	17	37	55		378	813	1,212	
54	1,062	6,897	338	22.81	26	16	33	49	62	287	599	888	1,117
60	4	7,493	348	21.34	27	11	30	45		136	364	541	

We find that deep subprime borrowers that remain current can maximize their savings by refinancing after about 48-50 months, when they converge in risk to prime/super prime borrowers.

Encouragingly, most current borrowers have prepaid by about loan age 60.

Estimated Savings (Subprime Borrowers)

	Averages					Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	18,261	16,693	395	17.97	64								
15	17,021	16,126	394	17.96	61								
18	15,487	15,619	393	17.95	59								
24	12,997	14,621	389	17.94	54								
30	11,021	13,420	388	17.94	48								
36	9,309	12,194	386	17.94	42								
42	7,481	10,835	384	17.93	37		29	54			857	1,616	
48	6,192	9,506	383	17.92	31		22	44	61		526	1,055	1,473
50	5,901	8,953	383	17.93	29		23	44	60		508	963	1,325
54	4,542	7,975	386	17.94	25		22	40	55		389	723	988
60	12	7,398	434	17.29	20		24	38			283	457	

We find that subprime borrowers that remain current can maximize their savings by refinancing after about 42 months, when they converge in risk to prime borrowers.

Again, most current borrowers have prepaid by about loan age 60.

Estimated Savings (Near-prime Borrowers)

		Averages				Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	5,807	19,111	411	12.79	64								
15	5,587	18,245	407	12.76	60			39				2,206	
18	5,315	17,617	405	12.74	58			40				2,158	
24	4,692	16,204	402	12.72	52			35				1,657	
30	4,146	14,694	400	12.71	47			37				1,546	
36	3,592	13,187	398	12.71	41			31				1,116	
42	3,041	11,446	394	12.67	35			28				847	
48	2,622	9,862	394	12.68	29			21	39			494	928
50	2,455	9,283	395	12.69	27			20	37			436	811
54	1,663	8,218	400	12.69	24			29	44			526	798
60	63	6,435	413	11.98	17			12				148	

We find that near-prime borrowers that remain current can maximize their savings by refinancing as soon as 15 months into the loan, when they converge in risk to prime borrowers.

Surprisingly, it appears many current near-prime borrowers follow a similar prepayment pattern as deep subprime, subprime borrowers (i.e., waiting until about loan age 60).

UCONN
UNIVERSITY OF CONNECTICUT

Estimated Savings (Prime Borrowers)

	Averages					Mo Pmt Savings ($)				Total Savings ($)			
Age	#	Bal	Pmt	APR(%)	# Pmts	S	NP	P	SP	S	NP	P	SP
12	5,173	18,582	358	7.83	64				39				2,327
15	5,283	17,611	354	7.81	60				33				1,880
18	5,315	16,706	350	7.78	57				30				1,627
24	4,971	15,097	346	7.76	52				32				1,535
30	4,538	13,503	345	7.74	46				30				1,245
36	4,096	11,866	344	7.73	39				21				755
42	3,697	10,274	342	7.72	34				23				703
48	3,191	8,615	343	7.71	28				21				513
50	2,963	8,101	345	7.71	26				21				460
54	1,898	7,075	351	7.66	22				18				324
60	28	6,920	435	7.74	17								

We find that prime borrowers that remain current can maximize their savings by refinancing as soon as 12 months into the loan, when they converge in risk to super prime borrowers.

Surprisingly, it appears many current prime borrowers follow a similar prepayment pattern as deep subprime, subprime borrowers (i.e., waiting until about loan age 60).

Analyzing Consumer Behavior

We can use the sibling cause-specific hazard rate estimator for prepayments, $\hat{\lambda}^{02}_{\tau,n}$, to analyze prepayment behavior by risk band.

We also overlay the Manheim Used Vehicle Value Index (ticker: `MUVVI`) and timing of the Economic Impact Payments for the 2017 and 2019 issuance.

Analyzing Consumer Behavior (Cont.)

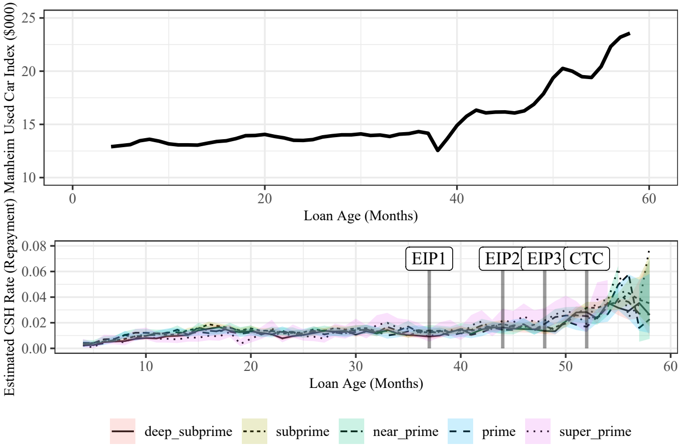


Figure: Conditional prepayment behavior by risk band (2017)

Analyzing Consumer Behavior (Cont.)

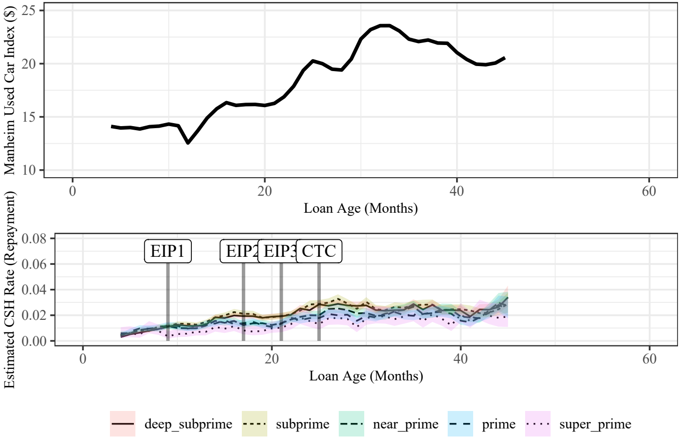


Figure: Conditional prepayment behavior by risk band (2019)

Analyzing Consumer Behavior (Summary)

All consumers are "too slow" to refinance, leaving potentially thousands of dollars on the table (these $ estimates increase for the 2019 credit risk convergence matrix).

In a surprise related to expectations of consumer sophistication and credit score, we estimate that it is actually prime and near-prime consumers that leave the most money on the table.

Prepayment behavior is consistent across all risk bands and appears to be partially driven by economic impact payments (EIP) and rising used auto values over the observation period rather than a borrower's understanding of their changing risk profile.

Table of Contents

Concluding remarks

Credit risk convergence: Regardless of a borrower's credit profile at contract signing, the longer a borrower remains active and paying, the lower the risk of default (intuitive: we contribute the method to measure, the when, and the economic implications).

Money on the table: Deep subprime, subprime, near-prime, and prime borrowers that don't refinance potentially overpay by thousands of dollars given an updated risk profile. In a surprise, it is actually the near-prime and prime borrowers that leave more money on the table.

Not alternative financing: We analyze secured auto loans from major financial institutions in a core economic lending space; such potentially large market inefficiencies are troubling.

Other consumer loan types(?): We find convergence between 2-5 years for 72-month auto loans. If convergence for traditional 30-year mortgages happens at a similar rate, the potential consumer inefficiencies could be substantial.

UCONN
UNIVERSITY OF CONNECTICUT

What can be done?

- *Consumerus Ignoramus?*: Consumers have a poor reputation in making financial decisions (e.g. Gross and Souleles, 2002; Stango and Zinman, 2011; Lusardi and de Bassa Scheresberg, 2013; Campbell, 2016; Heidhues and Kőszegi, 2016; Dobbie et al., 2021), but prepayments do accelerate as loans mature. Encourage borrowers to self-correct (questionable effectiveness (e.g., Keys et al., 2016; Agarwal et al., 2017)).
- *Financial innovation*: Lenders offer loans structured with a reducing payment based on good performance (may also act as an incentive to keep borrowers current).
- *Competition*: Competing lenders seek out these mature loans to offer refinancing (similar to SOFI with student loans). That is, borrower sloth possibly driven by perceived hassle, lack of options.
- *Regulation*: Require ongoing loans to be "re-underwritten" after a sustained period of good performance OR potentially offer borrowers cash rebates/larger trade-in values to refinance.

UCONN
UNIVERSITY OF CONNECTICUT

Thank you!

Jackson P. Lautier, FSA
e: jackson.lautier@uconn.edu
w: www.jacksonlautier.com

Acknowledgements Contents of this presentation have benefited from seminar participants at Bentley University, the University of Connecticut (School of Business, Department of Finance), the University of Connecticut (Department of Economics) and personal correspondence with Brian Melzer, Jonathan A. Parker, and Jonathan Zinman. Any remaining errors are our own. Jackson P. Lautier was employed with Prudential Financial, Inc. from 2010-2019 but believes in good faith there are no conflicts of interest. This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. DHE 1747453.

Table of Contents

UCONN
UNIVERSITY OF CONNECTICUT

Incomplete data details

Let T be the random time of a loan origination. Let Y be the left-truncation random variable (it is a linear shift of T until the time the ABS starts paying).

Then we observe $X \iff X \geq Y$.

Define $C = Y + \tau$, where τ is a constant that depends on when the ABS transaction (observation window) ends.

Then we observe $X \iff (X \geq Y) \cap (X \leq C)$ or we observe $\min(X, C) \iff (X \geq Y) \cap (X > C)$.

We assume $Y \equiv f(T) \perp X$ (reasonable for ABS transactions, see Lautier et al. (2023, Section 4.3)).

UCONN
UNIVERSITY OF CONNECTICUT

Competing Risks & Defaults: Some references

There are many references on competing risks (e.g., Crowder, 2001; Pintilie, 2006; Kalbfleisch and Prentice, 2011) with some discrete-time specific (e.g., Tutz and Schmid, 2016; Lee et al., 2018; Schmid and Berger, 2021).

Our framing generalizes Lautier et al. (2023) with a multistate process (e.g., Andersen et al., 1993; Beyersmann et al., 2009), however, to avoid unrealistic assumptions, like independence between default and prepayment (e.g., Zhang et al., 2019).

Competing risks also has a long history in modeling loan defaults (e.g., Banasik et al., 1999; Stepanova and Thomas, 2002; Dirick et al., 2017; Frydman and Matuszyk, 2022), but none meet our ABS framework precisely.

UCONN UNIVERSITY OF CONNECTICUT

Proposition 1: Asymptotic Normality

For $i \in \{1,2\}$ and $x \in \{\Delta+1,\ldots,\xi\}$, define $\hat{\mathbf{\Lambda}}^{0i}_{\tau,n} = (\hat{\lambda}^{0i}_{\tau,n}(\Delta+1),\ldots,\hat{\lambda}^{0i}_{\tau,n}(\xi))^\top$. Then,

(i)

$$\hat{\mathbf{\Lambda}}^{0i}_{\tau,n} \xrightarrow{\mathcal{P}} \mathbf{\Lambda}^{0i}_{\tau}, \text{ as } n \to \infty;$$

(ii)

$$\sqrt{n}(\hat{\mathbf{\Lambda}}^{0i}_{\tau,n} - \mathbf{\Lambda}^{0i}_{\tau}) \xrightarrow{\mathcal{L}} N(\mathbf{0}, \mathbf{\Sigma}^{0i}), \text{ as } n \to \infty,$$

where $\mathbf{\Lambda}^{0i}_{\tau} = (\lambda^{0i}_{\tau}(\Delta+1),\ldots,\lambda^{0i}_{\tau}(\xi))^\top$ and

$$\mathbf{\Sigma}^{0i} = \text{diag}\left(\cdots, \frac{f^{0i}_{*,\tau}(x)\{C_\tau(x) - f^{0i}_{*,\tau}(x)\}}{C_\tau(x)^3}, \cdots\right).$$

That is, the cause-specific hazard rate estimators $\hat{\lambda}^{0i}_{\tau,n}(\Delta+1),\ldots,\hat{\lambda}^{0i}_{\tau,n}(\xi)$ are consistent, asymptotically normal, and independent.

Lemma 1: Asymptotic Confidence Intervals

The $(1-\theta)\%$ asymptotic confidence interval bounded within $(0,1)$ for $\lambda_{\tau}^{0i}(x)$, $x \in \{\Delta+1,\ldots,\xi\}$, $i=1,2$ is

$$\exp\left\{ \ln \hat{\lambda}_{\tau,n}^{0i}(x) \pm \mathcal{Z}_{(1-\theta/2)} \sqrt{\frac{\hat{C}_{\tau,n}(x) - \hat{f}_{*,\tau,n}^{0i}(x)}{n\hat{C}_{\tau,n}(x)\hat{f}_{*,\tau,n}^{0i}(x)}} \right\}, \tag{3}$$

where $\mathcal{Z}_{(1-\theta/2)}$ represents the $(1-\theta/2)$th percentile of the standard normal distribution.

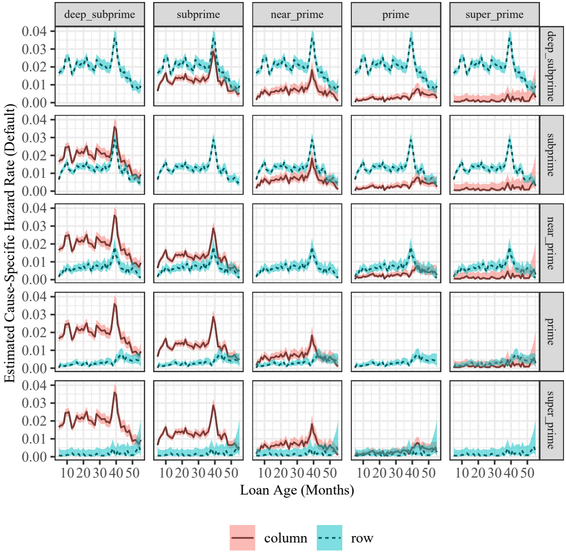


Figure: Credit Risk Convergence: Full 5x5 Comparison for 2017 Issuance

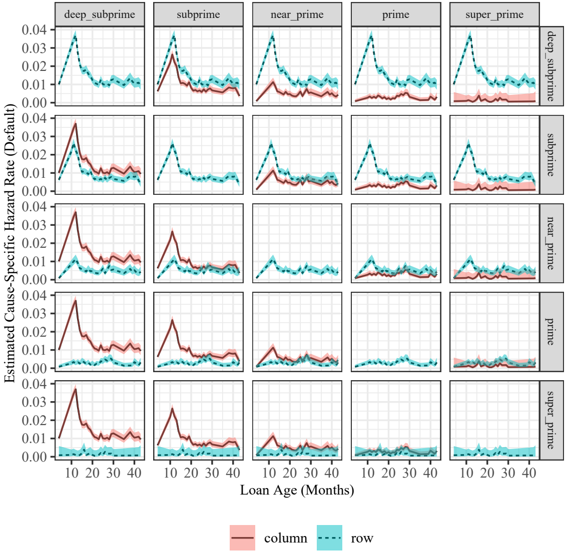


Figure: Credit Risk Convergence: Full 5x5 Comparison for 2019 Issuance

Adams, William, Liran Einav, and Jonathan Levin, 2009, Liquidity constraints and imperfect information in subprime lending, *American Economic Review* 99, 49–84.

Agarwal, Sumit, Itzhak Ben-David, and Vincent Yao, 2017, Systematic mistakes in the mortgage market and lack of financial sophistication, *Journal of Financial Economics* 123, 42–58.

Agarwal, Sumit, Souphala Chomsisengphet, Neale Mahoney, and Johannes Stroebel, 2014, Regulating consumer financial products: Evidence from credit cards, *The Quarterly Journal of Economics* 130, 111–164.

Ally, 2017, Ally Auto Receivables Trust, Prospectus 2017-3, Ally Auto Assets LLC.

Andersen, Per Kragh, Ørnulf Borgan, Richard D. Gill, and Niels Keiding, 1993, *Statistical Models Based on Counting Processes* (Springer).

Banasik, John, Jonathan N. Crook, and L. C. Thomas, 1999, Not if but when will borrowers default, *Journal of the Operational Research Society* 50, 1185–1190.

Bertrand, Marianne, and Adair Morse, 2011, Information disclosure, cognitive biases, and payday borrowing, *The Journal of Finance* 66, 1865–1893.

Beyersmann, Jan, Aurélien Latouche, Anika Buchholz, and Martin Schumacher, 2009, Simulating competing risks data in survival analysis, *Statistics in Medicine* 28, 956–971.

Campbell, John Y., 2016, Restoring rational choice: The challenge of consumer financial regulation, *American Economic Review* 106, 1–30.

CarMax, 2017, CarMax Auto Owner Trust, Prospectus 2017-2, CarMax Business Services LLC.

Consumer Financial Protection Bureau, 2019, Borrower risk profiles, url: https://www.consumerfinance.gov/data-research/consumer-credit-trends/auto-loans/borrower-risk-profiles/ (Accessed: 2022-06-15).

Crowder, Martin J, 2001, *Classical Competing Risks* (Chapman and Hall/CRC).

Dirick, Lore, Gerda Claeskens, and Bart Baesens, 2017, Time to default in credit scoring using survival analysis: a benchmark study, *Journal of the Operational Research Society* 68, 652–665.

Dobbie, Will, Andres Liberman, Daniel Paravisini, and Vikram Pathania, 2021, Measuring bias in consumer lending, *The Review of Economic Studies* 88, 2799–2832.

Edelberg, Wendy, 2006, Risk-based pricing of interest rates for consumer loans, *Journal of Monetary Economics* 53, 2283–2298.

Frydman, Halina, and Anna Matuszyk, 2022, Random survival forest for competing credit risks, *Journal of the Operational Research Society* 73, 15–25.

Gross, David B., and Nicholas S. Souleles, 2002, Do liquidity constraints and interest rates matter for consumer behavior? Evidence from credit card data, *The Quarterly Journal of Economics* 117, 149–185.

Grunewald, Andreas, Jonathan A Lanning, David C Low, and Tobias Salz, 2020, Auto dealer loan intermediation: Consumer behavior and competitive effects, Working Paper 28136, National Bureau of Economic Research.

Heidhues, Paul, and Botond Kőszegi, 2016, Naïeté-based discrimination, *The Quarterly Journal of Economics* 132, 1019–1054.

Kalbfleisch, John D, and Ross L Prentice, 2011, *The Statistical Analysis of Failure Time Data* (John Wiley & Sons).

Keys, Benjamin J., Devin G. Pope, and Jaren C. Pope, 2016, Failure to refinance, *Journal of Financial Economics* 122, 482–499.

Lautier, Jackson, Vladimir Pozdnyakov, and Jun Yan, 2021, Estimating a distribution function for discrete data subject to random truncation with an application to structured finance, *arXiv* 1, 1–26.

Lautier, Jackson, Vladimir Pozdnyakov, and Jun Yan, 2023, Pricing time-to-event contingent cash flows: A discrete-time survival analysis approach, *Insurance: Mathematics and Economics* Accepted.

Lee, Minjung, Eric J. Feuer, and Jason P. Fine, 2018, On the analysis of discrete time competing risks data, *Biometrics* 74, 1468–1481.

Lusardi, Annamaria, and Carlo de Bassa Scheresberg, 2013, Financial literacy and high-cost borrowing in the United States, Working Paper 18969, National Bureau of Economic Research.

Melzer, Brian T., 2011, The real costs of credit access: Evidence from the payday lending market, *The Quarterly Journal of Economics* 126, 517–555.

Phillips, Robert, 2013, Optimizing prices for consumer credit, *Journal of Revenue & Pricing Management* 12.

Pintilie, Melania, 2006, *Competing Risks: A Practical Perspective* (John Wiley & Sons).

Santander, 2017a, Drive Auto Receivables Trust, Prospectus 2017-1, Santander Drive Auto Receivables LLC.

Santander, 2017b, Santander Drive Auto Receivables Trust, Prospectus 2017-2, Santander Drive Auto Receivables LLC.

Schmid, Matthias, and Moritz Berger, 2021, Competing risks analysis for discrete time-to-event data, *Wiley Interdisciplinary Reviews: Computational Statistics* 13, e1529.

Securities and Exchange Commission, 2014, 17 CFR Parts 229, 230, 232, 239, 240, 243, and 249 Asset-Backed Securities Disclosure and Registration.

Securities and Exchange Commission, 2016, 17 CFR §229.1125 (Item 1125) Schedule AL - Asset-level information.

UCONN
UNIVERSITY OF CONNECTICUT

Stango, Victor, and Jonathan Zinman, 2011, Fuzzy math, disclosure regulation, and market outcomes: Evidence from Truth-in-Lending reform, *The Review of Financial Studies* 24, 506–534.

Stepanova, Maria, and Lyn Thomas, 2002, Survival analysis methods for personal loan data, *Operations Research* 50, 277–289.

Tutz, Gerhard, and Matthias Schmid, 2016, *Modeling Discrete Time-to-Event Data* (Springer).

Zhang, Nailong, Qingyu Yang, Aidan Kelleher, and Wujun Si, 2019, A new mixture cure model under competing risks to score online consumer loans, *Quantitative Finance* 19, 1243–1253.