Problem Set 4

1) [20 points] Suppose that you have used a Cepheid variable star as a “standard candle” to compute the distance to a particular galaxy. The distance you computed is r = 35 Mpc. Much to your embarrassment, you find that the Cepheid variable star has a luminosity L that is actually twice the luminosity you assumed when making your calculation. Is the galaxy closer or farther than you originally calculated? What is the true distance to the galaxy?

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2) [20 points] As we have seen, hydrogen has an absorption line at a wavelength λ0 = 656.3 nanometers (as long as the hydrogen is at rest). You observe a distant galaxy for which the same hydrogen absorption line has a wavelength λ = 715.4 nanometers. a) What is the redshift, z = (λ − λ0)/λ0, of the galaxy? b) What is the radial velocity of the galaxy, in kilometers per second? c) From Hubble's law, what is the distance to the galaxy? [Hint: assume H0 = 71 km/ sec/ Mpc.]

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3) [20 points] As we have seen in lecture, if the Hubble constant is H0 = 71 km/ sec/ Mpc, then the Hubble time is 1/H0 = 14 billion years. Edwin Hubble himself, because he grossly underestimated the distance to galaxies, believed that the Hubble constant was H0 = 500 km/ sec/ Mpc. For H0 = 500 km/ sec/ Mpc, what is 1/H0, in billions of years?

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4) [40 points] Suppose you found a star within our Milky Way Galaxy whose age was measured to be twice the Hubble time. Would this discovery make you abandon the Big Bang model for the universe? If the old star prompts you to abandon the Big Bang model, briefly explain an alternate model that would be consistent with Hubble's law ($v = H_0 d$). If you do not abandon the Big Bang model, briefly explain how a Big Bang universe can contain stars twice as old as the Hubble time ($1/H_0$).

Problem Set 4 - TA's Solution Sets

1) Suppose that you have used a Cepheid variable star as a "standard candle" to compute the distance to a particular galaxy. The distance you computed is $r = 35\,\text{Mpc}$. Much to your embarassment, you find that the Cepheid variable star has a luminosity L that is actually *twice* the luminosity you assumed when making your calculation. Is the galaxy *closer* or *farther* than you originally calculated? What is the true distance to the galaxy?

This question has two parts, both of which are a little tricky. With a little bit of thinking about the big picture, however, the answer should become clear. Before answering the question, some background explanation may prove useful.

Luminosity (L), flux (f, a.k.a. brightness), and distance (r or d) are intimately related concepts. Mathematically, the are related as follows:

$$f = \frac{L}{4\pi d^2} \quad \leftrightarrow \quad r = \sqrt{\frac{L}{4\pi f}}$$

Luminosity is an *intrinsic* property of an object; it's the amount of light (or energy or whatever) emitted per unit time (i.e., watts). Luminosity is an attribute that is **completely independent of any observer**. When you're in the hardware store and you see lightbulbs with different wattage ratings, those numbers tell you (more or less) the total energy that will be emitted by that bulb every second you leave it on (that's its luminosity). Importantly, the bulb does not care if you're nearby, far away, or maybe not even home! Its luminosity, i.e. the energy it emits while turned on, will be the same no matter where you are.

Unlike luminosity, distance (r) and flux (f) are not intrinsic properties of an object. Both of these quantities are only have meaning through the eyes (or camera) of some **observer**. Distance is self-explanatory; it's simply how far away you are from the light source in question. Flux is a fancy technical name for brightness. It's literally how bright some object looks. A light bulb that appears dim from down the hall would be extremely bright if it were just inches from your face! That's flux.

In practice, flux is what you measure and thus it's what you know. In Astronomy, flux is measured with telescopes and cameras. Given a measurement of flux, you can either (a) figure out the distance to some object and then *infer* the luminosity or, more likely, (b) figure out an object's luminosity and infer the distance to said object. Nature has provided us with a number of different "standard candles." These are objects whose luminosity can be determined without already knowing the distance. These are the astronomical equivalents the light bulb wattage written on the outside of the box.

One such beast is the Cepheid variable (the topic of this question). These stars pulsate, getting brighter and fainter over the course of days, weeks or months. It turns out that how long it takes them to pulsate is directly determined by their luminosity. As a result, we independently figure out their luminosities. Combining this with how bright they look (their flux), we can compute how far away they truly are.

Armed with this information, we can now properly answer this question. The first part asks, if the Cepheid's luminosity L were twice as large as originally believed, would the true distance be closer or farther than the original guess? **The true distance must be farther**, for reasons that should become immediately apparent.

An important thing to remember: the flux f is unchanged! Recall that we *measured* the flux directly (with telescopes and cameras). A useful restatement of the above answer is: a more luminous object must be farther away in order to appear equally bright. If two objects appear to have the same brightness, the one that is intrinsically more luminous must be farther away.

The next (and more important) part of this question requires that you compute this new distance using the known relationships between flux, luminosity, and distance. This question was a bit tricky because you didn't have enough information to simply plug in numbers and get the answer. Instead, you're required to compute the new distance relative to the old guess. There are two ways to rigorously solve this and I'll go over both.

First, define your terms. There are two distinct situations (guesses, if you prefer). The old guess ($r = 35\,\text{Mpc}$) was incorrect. The new one hopefully won't be. For simplicity, call the two situations "old" and "new." We know: (1-2) the relationship between L, f, and d for both old and new, (3) the "old" distance, (4) the measured flux is the same in both cases, and (5) the "new" luminosity L_{new} is twice the old (L_{old}). All told, we have we then (mathematically) have:

$$r_{old} = \sqrt{\frac{L_{old}}{4\pi\, f_{old}}} \quad \leftrightarrow \quad f_{old} = \frac{L}{4\pi r_{old}^2} \tag{1}$$

$$r_{new} = \sqrt{\frac{L_{new}}{4\pi\, f_{new}}} \quad \leftrightarrow \quad f_{new} = \frac{L}{4\pi r_{new}^2} \tag{2}$$

$$r_{old} = 35\,\text{Mpc} \tag{3}$$

$$f_{old} = f_{new} \tag{4}$$

$$L_{new} = 2 \times L_{old} \tag{5}$$

These equations are enough to find the answer. The simplest method I found is as follows... Begin with equations 1 & 2 with flux in terms of luminosity and distance (on the right). Since the flux is the same for both old and new measurements (equation 4 above), you can combine these previous equations, finding:

$$\frac{L_{new}}{4\pi r_{new}^2} = f_{new} = f_{old} = \frac{L_{old}}{4\pi r_{old}^2} \quad \rightarrow \quad \frac{L_{new}}{4\pi r_{new}^2} = \frac{L_{old}}{4\pi r_{old}^2}$$

Next, cancel the 4π terms from both sides and rearrange to find:

$$r_{new}^2 = \frac{L_{new}}{L_{old}} \cdot r_{old}^2 \quad \rightarrow \quad r_{new} = r_{old} \cdot \sqrt{\frac{L_{new}}{L_{old}}}$$

Since $L_{new}/L_{old} = 2$ (equation 5 above) and $r_{old} = 35\,\text{Mpc}$, it follows that:

$$r_{new} = \sqrt{2} \cdot r_{old} \quad \rightarrow \quad r_{new} = \sqrt{2} \cdot 35\,\text{Mpc} = 49.5\,\text{Mpc}$$

Problem Set 4 - TA's Solution Sets

2) As we have seen, hydrogen has an absorption line at a wavelength $\lambda_0 = 656.3\,\text{nm}$ (as long as the hydrogen is at rest). You observe a distant galaxy for which the same hydrogen absorption line has a wavelength $\lambda = 715.4\,\text{nm}$.
a) What is the redshift, $z = (\lambda - \lambda_0)/\lambda_0$, of the galaxy?
b) What is the radial velocity of the galaxy, in kilometers per second?
c) From Hubble's law, what is the distance to the galaxy? [Hint: assume $H_0 = 71\ \text{km/s/Mpc}$.]

(a) This first part is straight plug and chug. From the question above, we know that $\lambda = 715.4\,\text{nm}$ and $\lambda_0 = 656.3\,\text{nm}$. Plugging these in to the formula provided, find:

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{715.4\,\text{nm} - 656.3\,\text{nm}}{656.3\,\text{nm}} = \frac{59.1\,\cancel{\text{nm}}}{656.3\,\cancel{\text{nm}}} = 0.09005 \approx 0.09$$

Please note that the units (nm) *have cancelled!* Redshift is a so-called dimensionless number, meaning it does not have any units. Since the wavelengths we started with each had 4 digits, it's OK to keep all 4 digits in your answer here (0.09005). However, rounding to 0.09 will make the math a bit easier in the coming parts.

(b) Using your answer to part (a), the radial velocity can be easily computed using the formula $v = c\,z$. Using $c = 3.0 \times 10^5$ km/s, I find:

$$v = c\,z = (3.0 \times 10^5\ \text{km/s})(0.09005) = 27,015\ \text{km/s} \approx 27,020\ \text{km/s}$$

As in part (a), I rounded this answer off after 4 digits. Using $z = 0.09$ instead, you would find $v = 27,000\ \text{km/s} = 2.7 \times 10^4\ \text{km/s}$ which you can easily do without a calculator (since $9 \times 3 = 27$).

(c) This last part provides one final plug and chug opportunity. Hubble's Law, written $v = H_0\,d$, relates the distance of galaxies to their recession (radial) velocity. Using $H_0 = 71$ km/s/Mpc and the answer from part (b), I find:

$$v = H_0\,d \;\rightarrow\; d = \frac{v}{H_0} = \frac{27,015\ \text{km/s}}{71\ \text{km/s/Mpc}}$$

It helps to be *extremely* careful with units to avoid mistakes, particularly with velocities! To be sure to get this correct, I rewrite my units as conventional fractions. The Hubble Constant 71 km/s/Mpc thus becomes $71\frac{\text{km}}{\text{s}\cdot\text{Mpc}}$. Plugging this back in, I have:

$$d = \frac{v}{H_0} = \left(27,015 \cdot \frac{\cancel{\text{km}}}{\cancel{\text{s}}}\right)\left(\frac{\cancel{\text{s}}\cdot\text{Mpc}}{71\,\cancel{\text{km}}}\right) = \frac{27,015}{71}\,\text{Mpc} = 380.5\,\text{Mpc}$$

Please notice that the km and s units cancel completely, leaving behind Mpc. It's always important to check whether or not the units make sense. Fortunately, Mpc is a measure of distance, which is what we're looking for.

Problem Set 4 - TA's Solution Sets

3) As we have seen in lecture, if the Hubble constant is $H_0 = 71$ km/s/Mpc, then the Hubble time is $1/H_0 = 14$ billion years. Edwin Hubble himself, because he grossly underestimated the distance to galaxies, believed that the Hubble constant was $H_0 = 500$ km/s/Mpc. For $H_0 = 500$ km/s/Mpc, what is $1/H_0$, in billions of years?

You can either calculate the new Hubble time directly with unit conversion, or you can solve for it by comparison to the actual Hubble time using ratios. Both methods give the same answer (i.e., they both work). If you opt to do the unit conversion directly, take great care to make sure that you cancel out your units properly!

As in the previous question, I'll rewrite the units in a more standard fraction form to avoid mistakes:

$$H_0 = 500 \text{ km/s/Mpc} = \frac{500 \text{ km}}{\text{s} \cdot \text{Mpc}}$$

Using conversion factors $1 \text{ Mpc} = 10^6 \text{ pc}$ and $1 \text{ pc} = 3.08 \times 10^{13} \text{ km}$, I find:

$$H_0 = \left(\frac{500 \cancel{\text{km}}}{\text{s} \cdot \cancel{\text{Mpc}}}\right)\left(\frac{1 \cancel{\text{Mpc}}}{10^6 \cancel{\text{pc}}}\right)\left(\frac{1 \cancel{\text{pc}}}{3.08 \times 10^{13} \cancel{\text{km}}}\right) = \frac{500}{3.08 \times 10^{19} \text{ s}} = 1.62 \times 10^{-17} \cdot \frac{1}{\text{s}}$$

Please note two things: firstly, nearly all of the units cancelled. Secondly, H_0 has units of *inverse* seconds (not seconds)! To compute the Hubble time, I take the reciprocal of the above expression. I find (taking care not to drop powers of 10!):

$$\frac{1}{H_0} = \frac{\text{s}}{1.62 \times 10^{-17}} = \frac{10^{17} \text{ s}}{1.62} = 6.173 \times 10^{16} \text{ s}$$

Converting the above value to years, I get:

$$\frac{1}{H_0} = (6.173 \times 10^{16} \cancel{\text{s}})\left(\frac{1 \text{ yr}}{3.15 \times 10^7 \cancel{\text{s}}}\right) = \frac{6.173 \times 10^{16}}{3.15 \times 10^7} \text{ yr} = 1.96 \times 10^9 \text{ yr} = 1.96 \text{ billion years}$$

Alternatively, you could have noted that the new $1/H_0$ must be smaller than the true value by the ratio of H_0 values (if H_0 gets bigger, then $1/H_0$ must get smaller). Doing this, you would find:

$$\frac{1}{H_0} = \left(\frac{71 \cancel{\text{km/s/Mpc}}}{500 \cancel{\text{km/s/Mpc}}}\right) \cdot 14 \text{ billion years} = 0.142 \cdot 14 \text{ billion years} = 1.99 \text{ billion years}$$

Note that all the units in the H_0 ratio above cancelled immediately. Setting up the problem to intentionally cancel units is a useful way to avoid needless conversion mistakes.

Problem Set 4 - TA's Solution Sets

4) Suppose you found a star within our Milky Way Galaxy whose age was measured to be twice the Hubble time. Would this discovery make you abandon the Big Bang model for the universe? If the old star prompts you to abandon the Big Bang model, briefly explain an alternate model that would be consistent with Hubble's law ($v = H_0\, d$).
If you do not abandon the Big Bang model, briefly explain how a Big Bang universe can contain stars twice as old as the Hubble time ($1/H_0$).

This question was designed to make you think and to test your understanding of some of the core concepts of Cosmology, particularly the importance and physical meaning of H_0, the Hubble constant. As usual, however, it is important that you clearly state the problem and allow this to set up your response (typical essay stuff). Although not all of this would be required to get full credit, what follows is a fairly a comprehensive (but not exclusive) list of ideas relevant to this question.

Although it would be quite surprising, a star aged more than twice the Hubble time does not preclude a Big Bang universe. Hubble's constant (H_0) relates the measured distance of galaxies to their measured recession velocities. In effect, H_0 is the current expansion rate of the universe. Its reciprocal, $1/H_0$, is known as the Hubble time. If the expansion rate of the universe were constant for all time, then $1/H_0$ is how long ago all the observed galaxies would have been densely packed together (i.e., the Big Bang). Equivalently, if you ran time in reverse, $1/H_0$ is how long it would take before the observable universe crunched back together.

If our universe began at the time of the Big Bang, then $1/H_0$ is the age of our universe as long as the expansion rate has been constant since the beginning. As a result, a star twice as old as the Hubble time appears to pose a major problem. Simply put, you cannot have individual objects that are older than the universe within which they exist! Provided the age measurement of the star is not grossly wrong (assume it's ok), the Big Bang model has some explaining to do.

The key to resolving this apparent paradox is that the expansion rate of the universe need not be constant! If the expansion of the universe had accelerated significantly since this Big Bang, then the Hubble time could heavily underestimate the true age of our universe. The subtle point here is that the Hubble time extrapolates the ulinecurrent expansion rate back to zero separation but does not account for changes in said expansion that could have occurred in our past. As a result, the Hubble time isn't -really- the age of the universe. In fact, under certain circumstances, it could be highly inaccurate!

It's like trying to figure out the duration of a car trip by measuring how fast you're going at the very end. Say you had to travel 100 miles. Towards the end of the trip, you check your speedometer and see that you're going 50 miles-per-hour. Naively, this suggests that your trip took only two hours. What if, however, there was construction early on? You could have been stuck going 5 or 10 mph for a few hours before you actually achieved some useful speed. The whole trip might have taken 4 or 5 hours, but you wouldn't know that from the speed you had at the end. In the same way, the universe could have expanded much more slowly early on. This would give stars plenty of time to form.

Other explanations are OK too. If you abandoned the Big Bang theory, you could explain what we observe by invoking a universe that began expanding well after it was created! In such a model, the universe was never as compacted as it would be in the Big Bang scenario. However, a late start to expansion would nicely reproduce our observed Hubble law and allow for stars older than the Hubble time.