

Knots, Shadow-spaces & Bracelets

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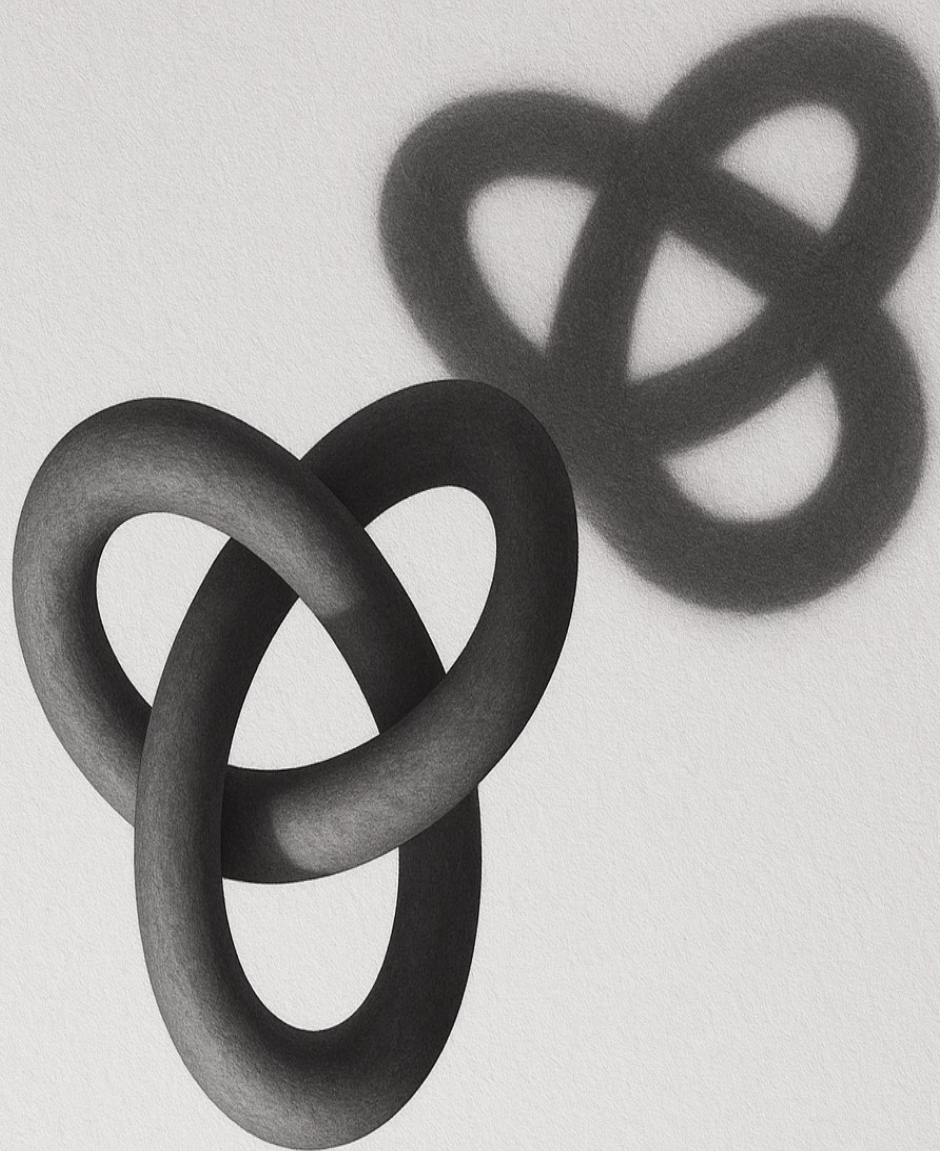


What are we looking at?

Shadow spaces- the sets of all resolutions of a knot shadow (projection). For a shadow with n crossings, each assignment of over/under information determines a resolution, encoded by a binary string in $\{0, 1\}^n$ yielding 2^n diagrams in total. To make classification feasible for large torus knots, we exploit diagram symmetries via group actions on these resolution strings.

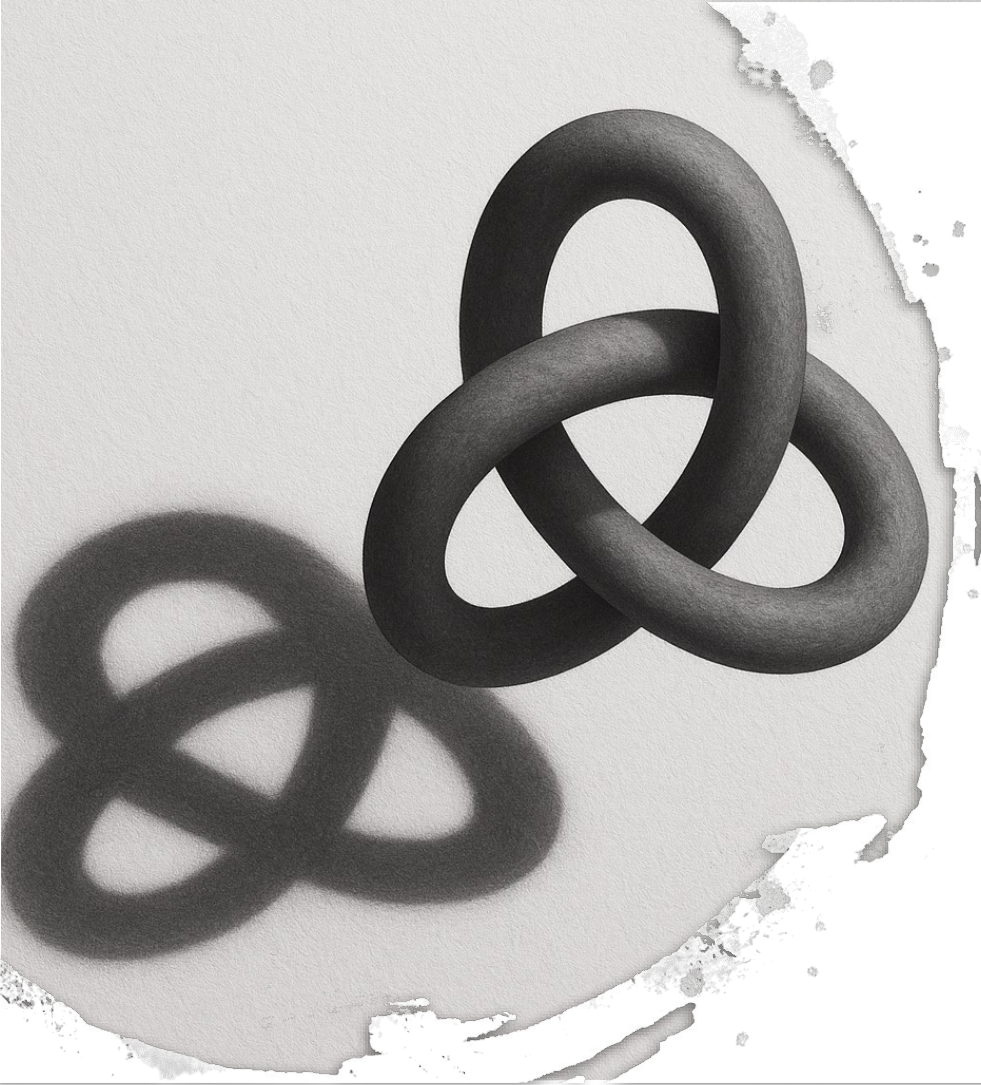
What Knots?

For the $(2,q)$ -torus shadow, the q -crossings lie on a regular q -gon, so dihedral symmetries act by rotation and reflection. Incorporating mirror identification corresponds to the complement map, giving an action of $\Gamma = D_q \times C_2$. By Burnside's Lemma, orbit representatives correspond to binary bracelets, leading to a closed formula $\beta(q)$ for the basis size in terms of bracelet counts $B(q)$.



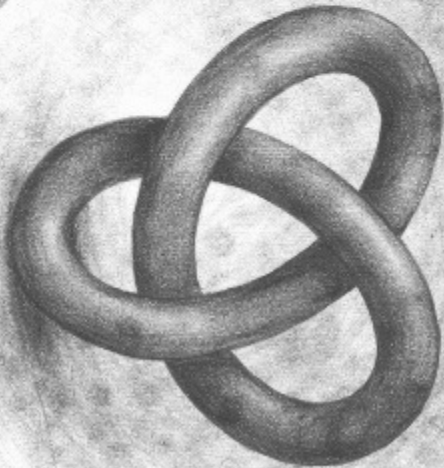
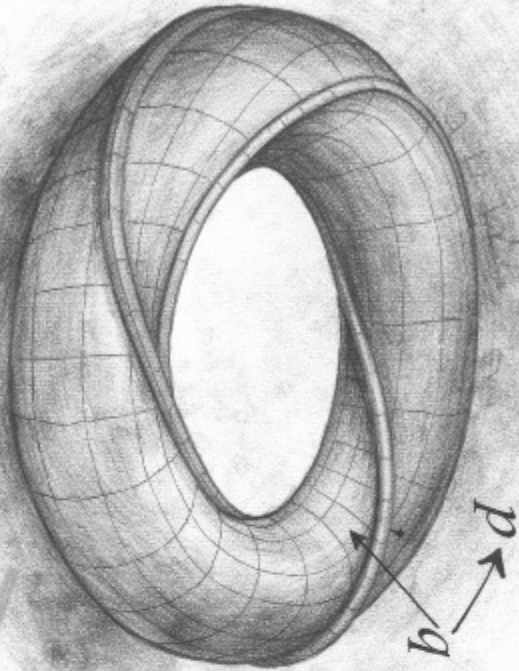
Why Knot?

We further prove an asymptotic efficiency result: the **symmetry based reduction** captures an overwhelming majority of redundancies, leaving only a small fraction of resolutions to be classified as the number of crossings grows. We conclude by outlining extensions to $(3,q)$ -torus knots, where product group actions produce rapid combinatorial growth and motivate broader algebraic generalizations.



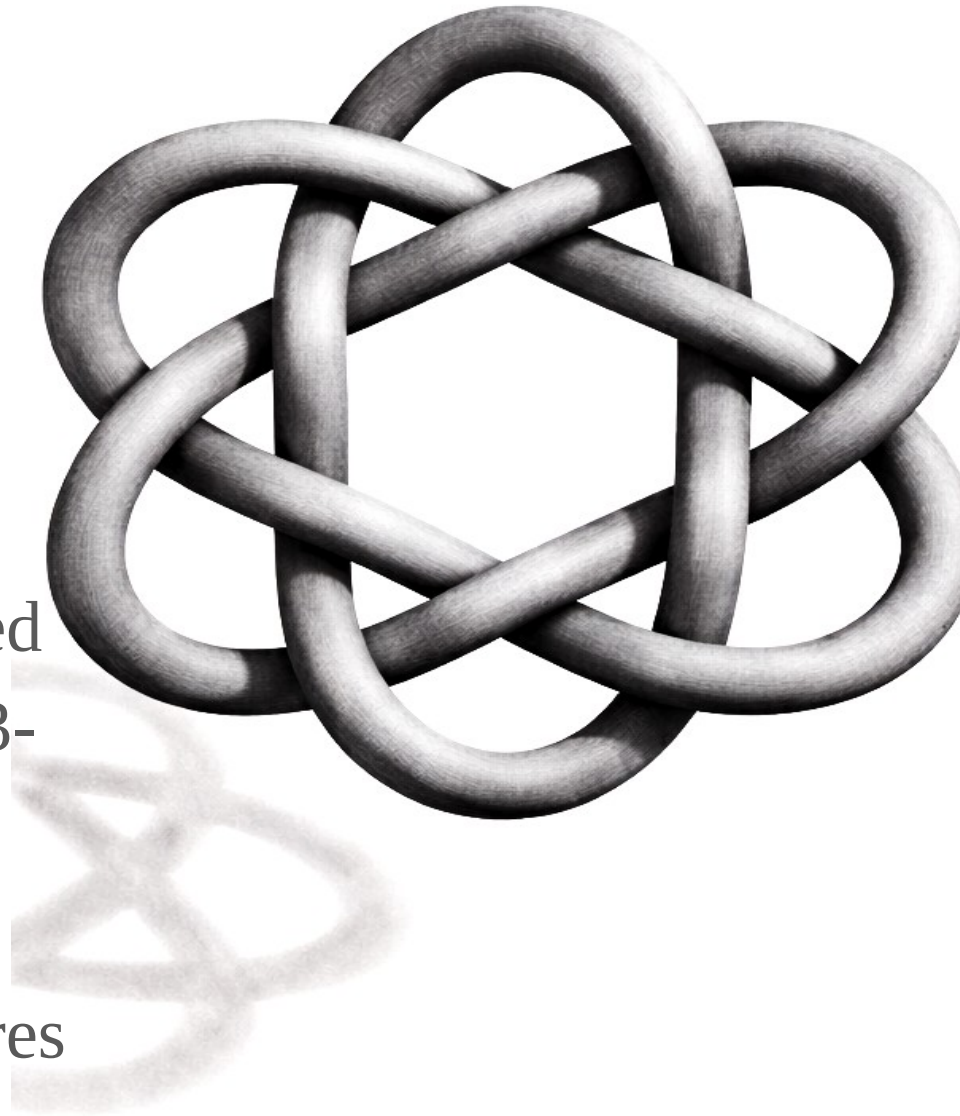
Research Approach

- Combinatorial approach to torus knots, via shadow spaces
- Shadow spaces remove over/under crossing data
- Ambient isotopies encoded using bracelet structures to represent knots
- Focus on shadow spaces, bases, and bracelet decompositions



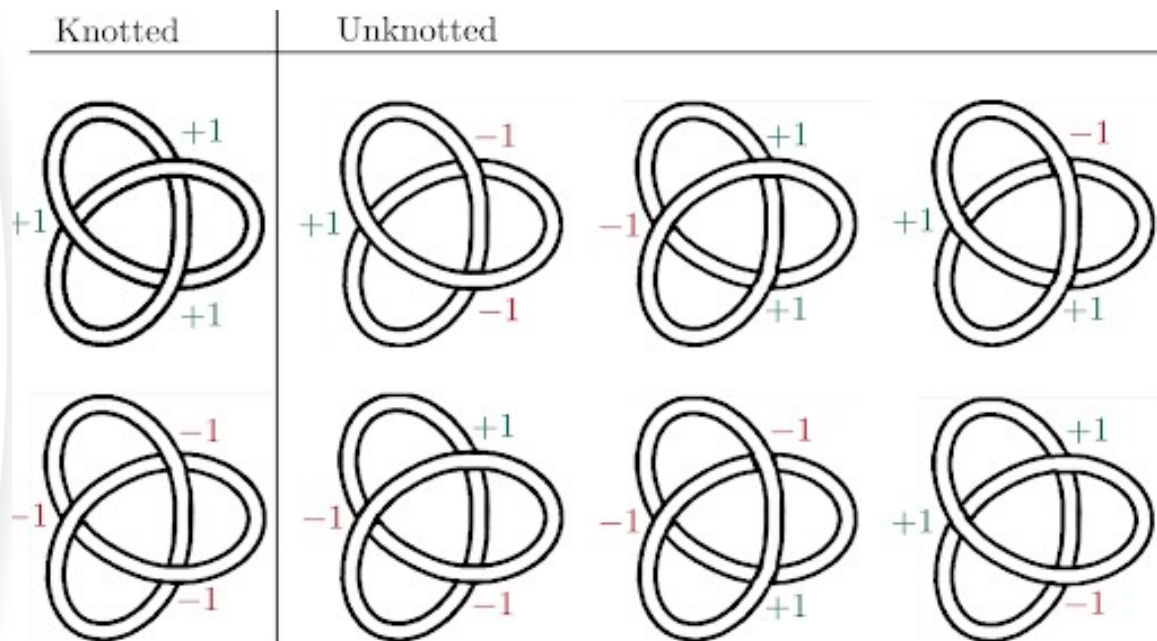
Structural Comparison

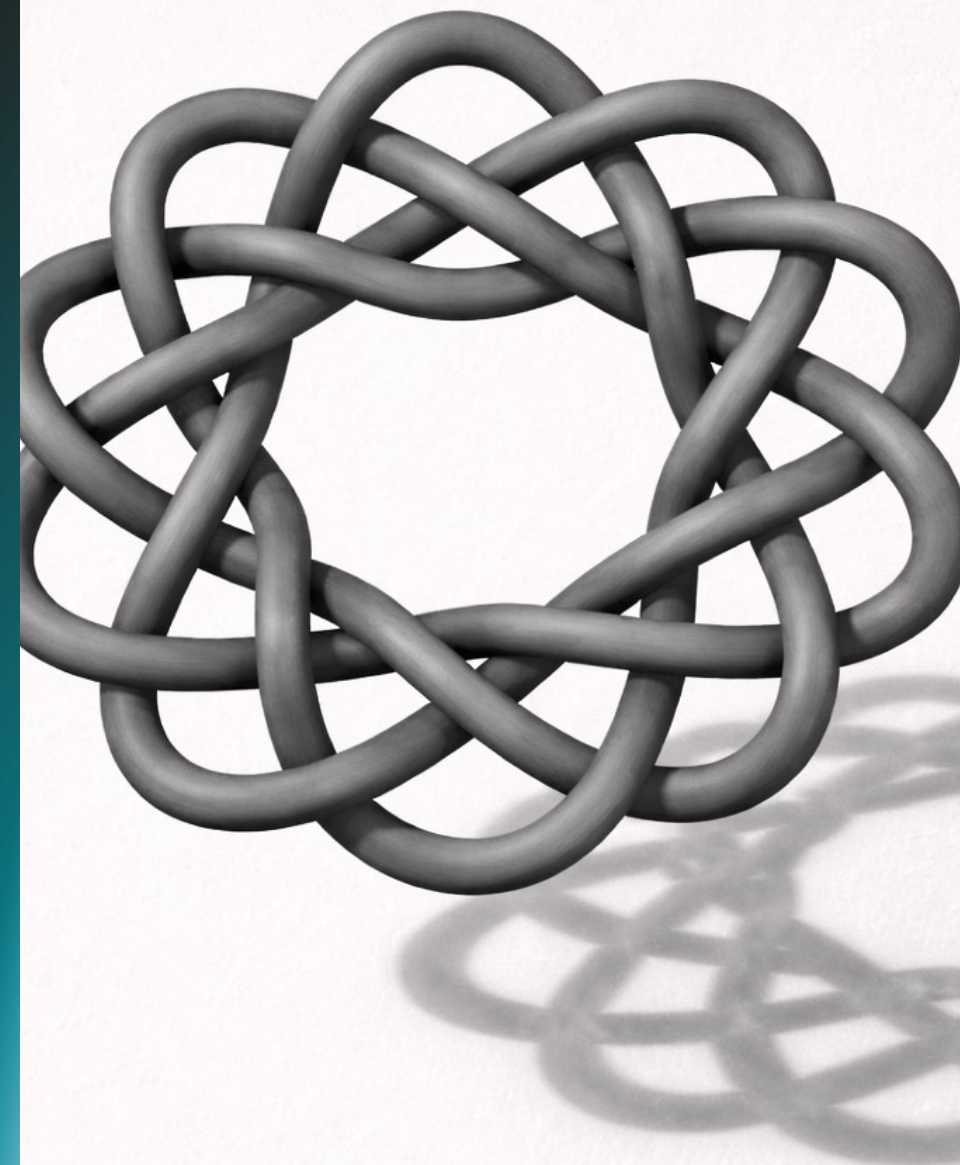
- Compare different bases and bracelet representations
- Analyze information preserved under dimensional collapse ($3\text{-D} \rightarrow 2\text{-D}$)
- Identify invariant vs non-invariant combinatorial features



Data Collection

- Generated families of $(2,q)$ torus knots
- Constructed shadow spaces as combinatorial datasets
- Enumerated all possible crossing assignments
- Each knot represented as a collection of bracelets





Equivalence Analysis

- Applied algebraic and combinatorial techniques
- Modeled ambient isotopy via Reidemeister moves
- Identified equivalent bracelets and bases

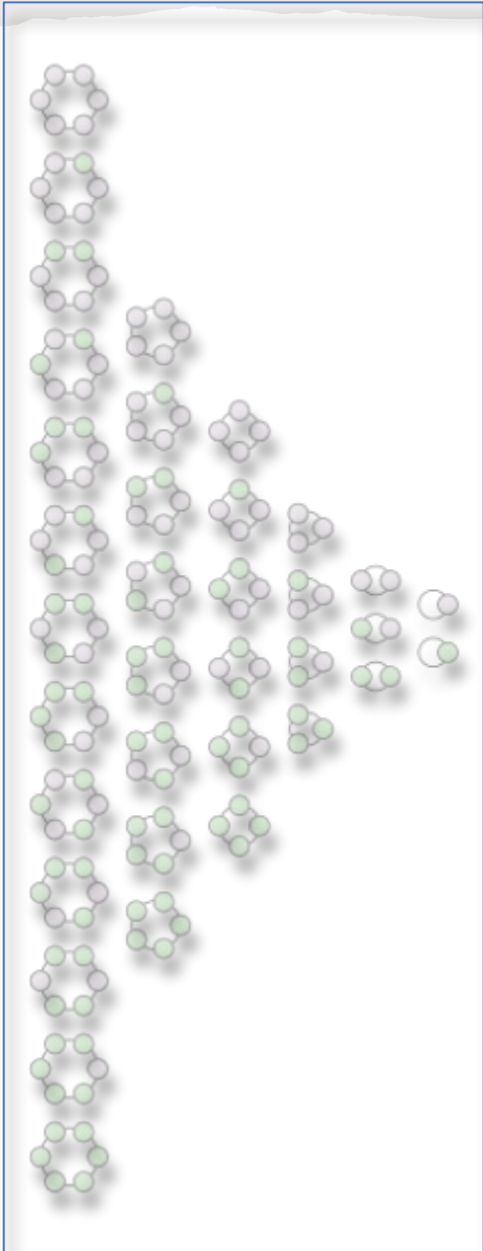


Area of Research

- Configuration space of torus knots
- Projection from S^3 to 2-D shadow spaces
- Focus on nontrivial bracelet structures

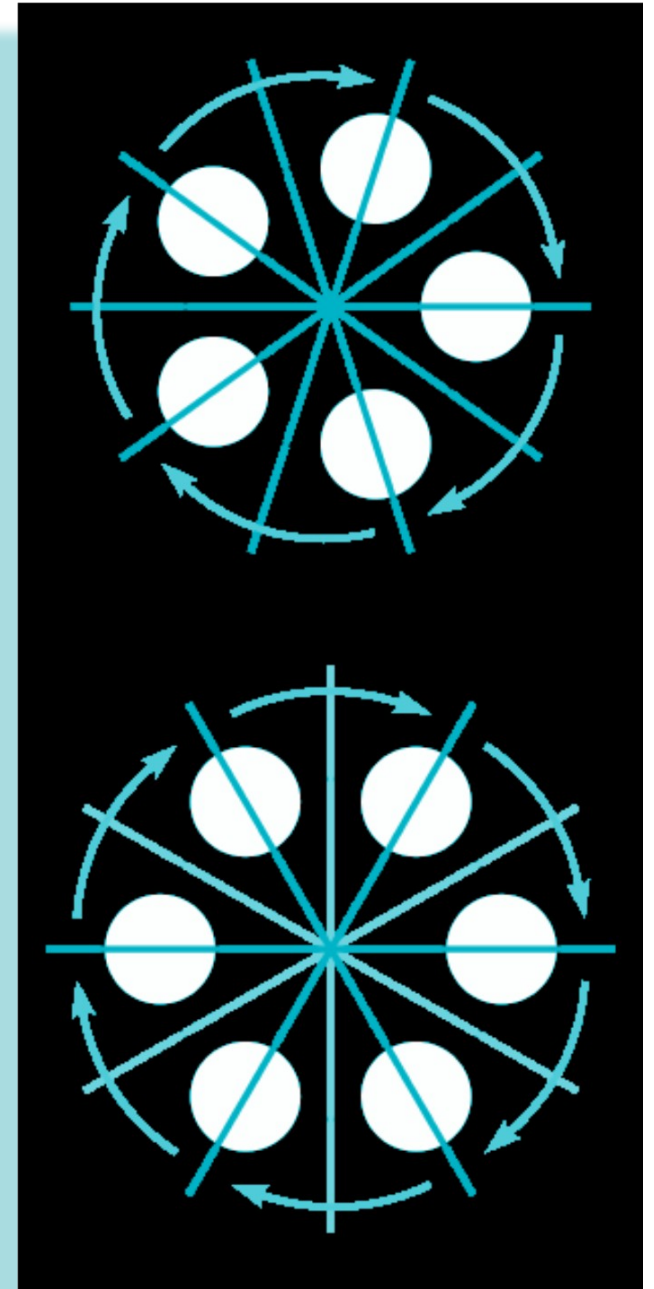
Interesting

- Systematic use of bracelet decomposition on shadow spaces
- Treats shadow spaces as structured combinatorial objects
- Organized framework for classifying ambient isotopy



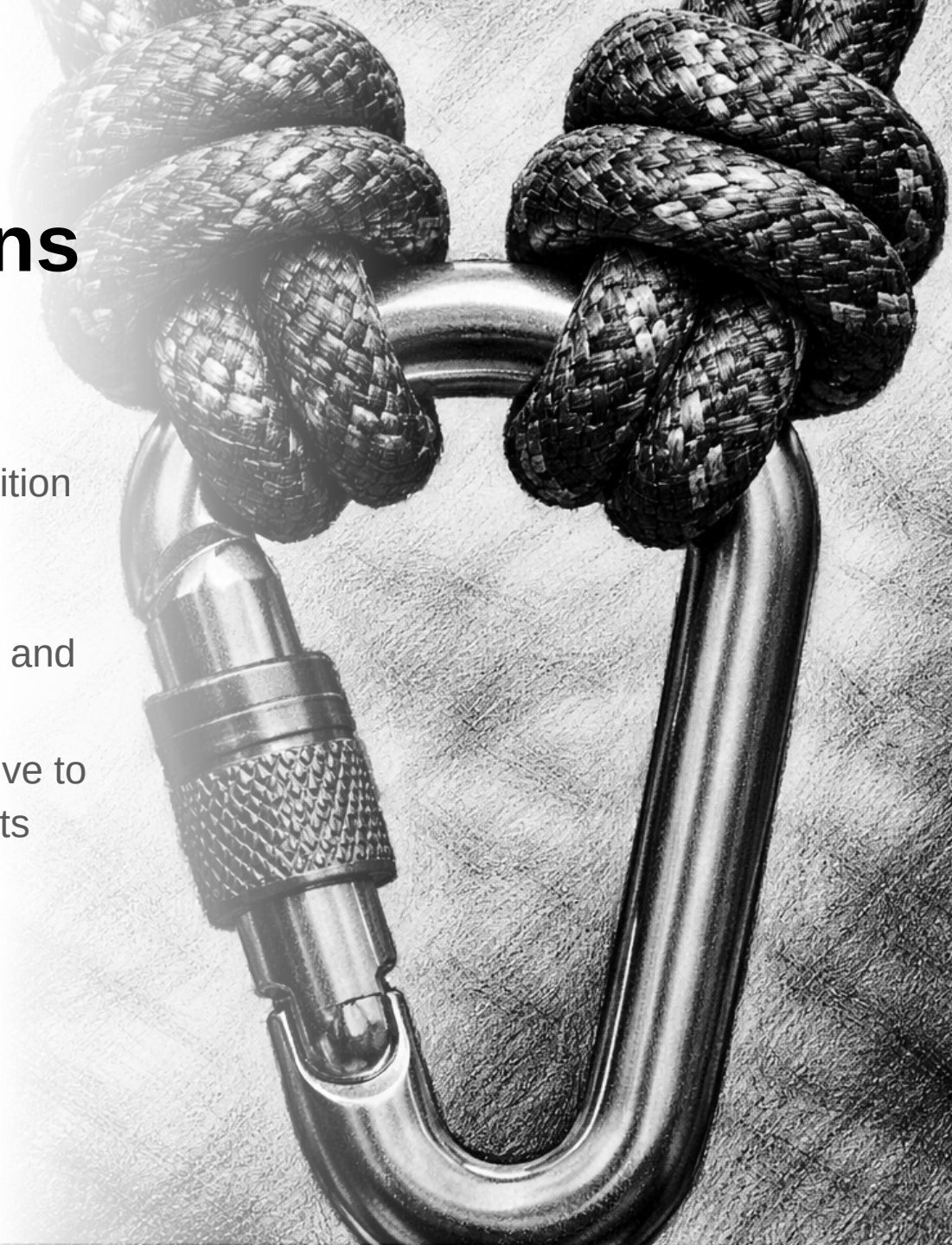
Findings

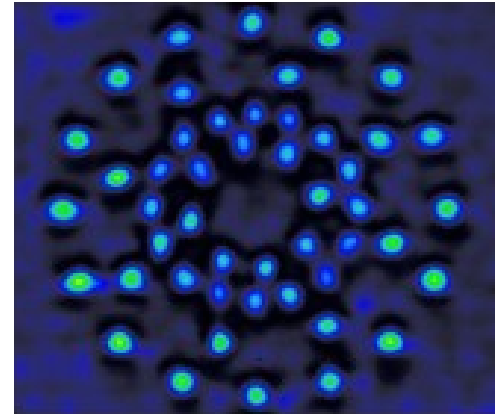
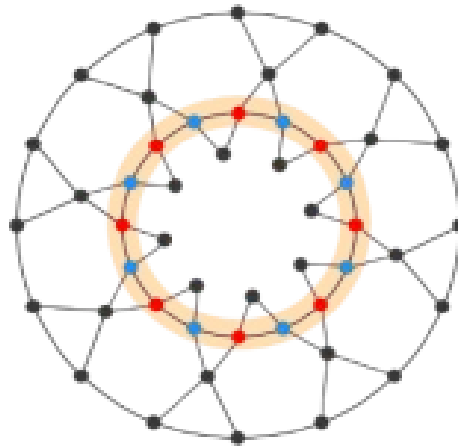
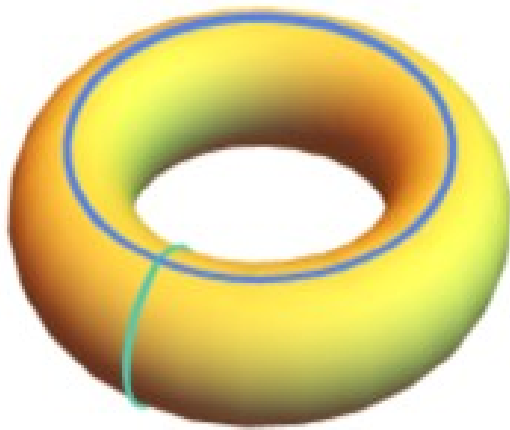
- Relationship between crossings, bracelets, and basis size
- Burnside's Lemma computes basis size
- Determines cardinality, not explicit generators



Implications

- Bracelet decomposition as a simpler knot invariant
- Easier visualization and computation
- Accessible alternative to polynomial invariants

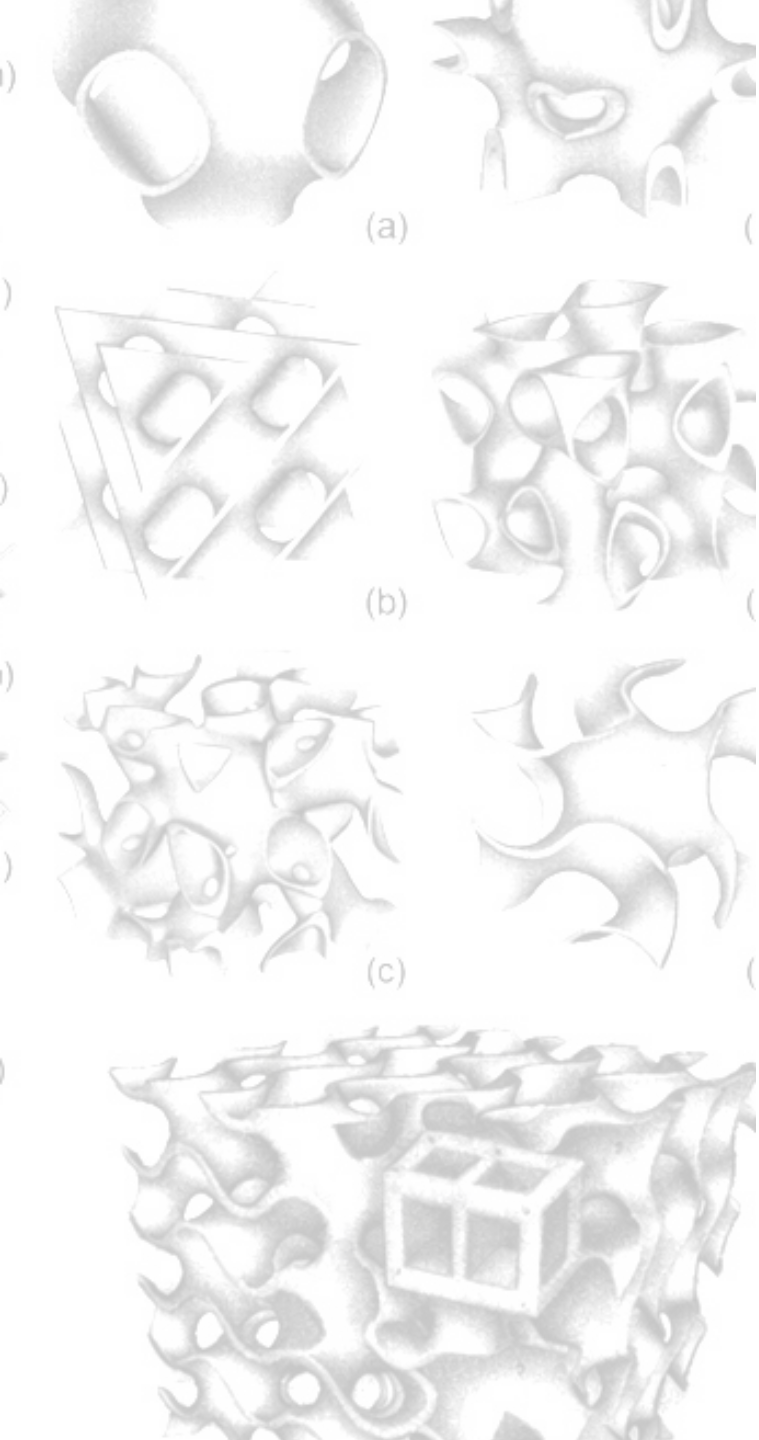




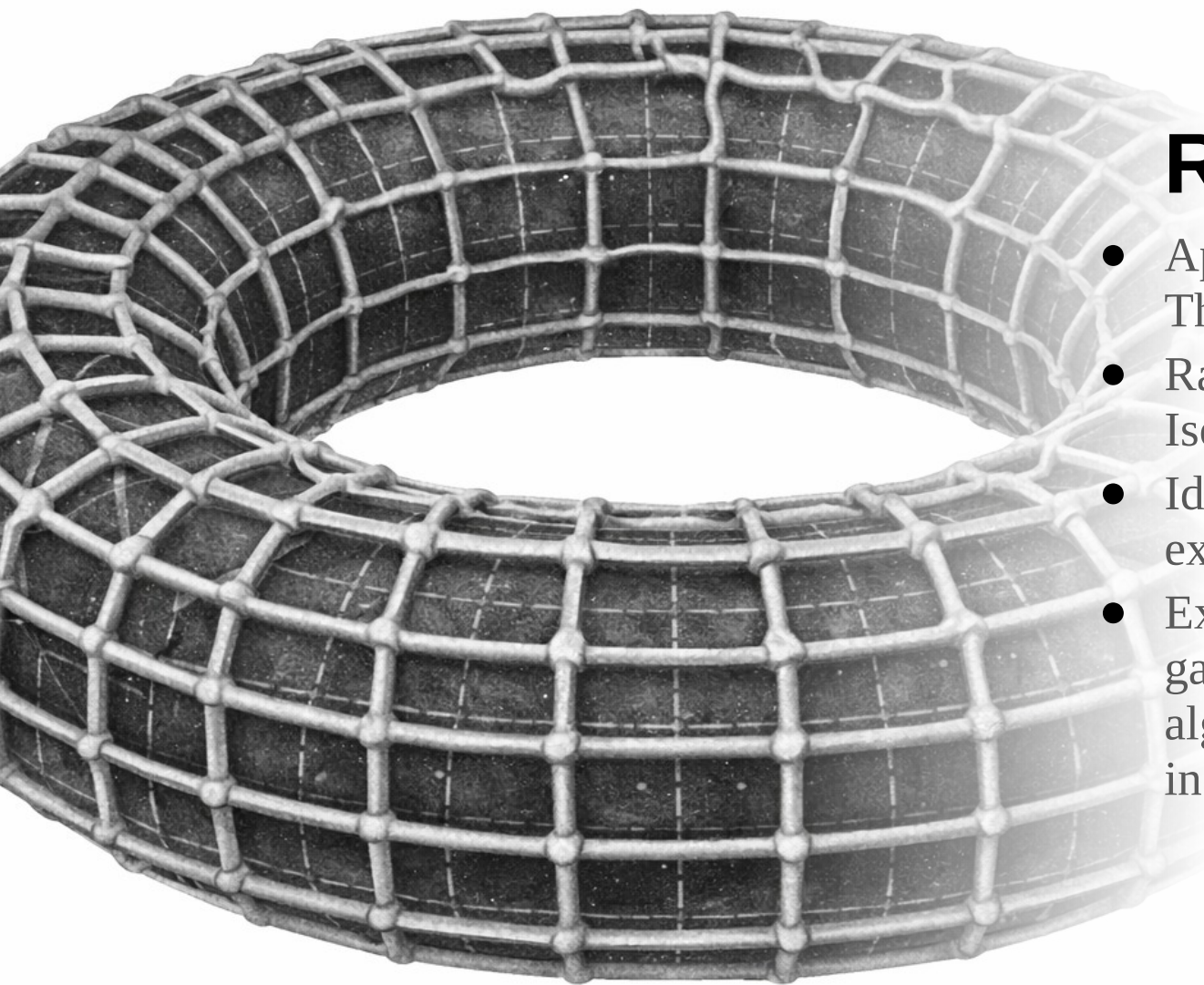
Why This Matters

- Useful for invariant development and computation
- Relevant to topology, group theory, quantum computing, Flat-band Photonics
- Educationally accessible to early researchers

Structural Observations



- Cyclic and lattice-like motifs in shadow spaces
- Bracelets reveal hidden ambient isotopies
- Basis transitions correspond to motion on a torus



Future Research

- Apply Pólya Enumeration Theory
- Ramsey's Theorem for Isotopy
- Identify and recuperate explicit bases, not just size
- Explore homology, quantum gates, and automation algorithms and applications in graduate school

Thank
You!

