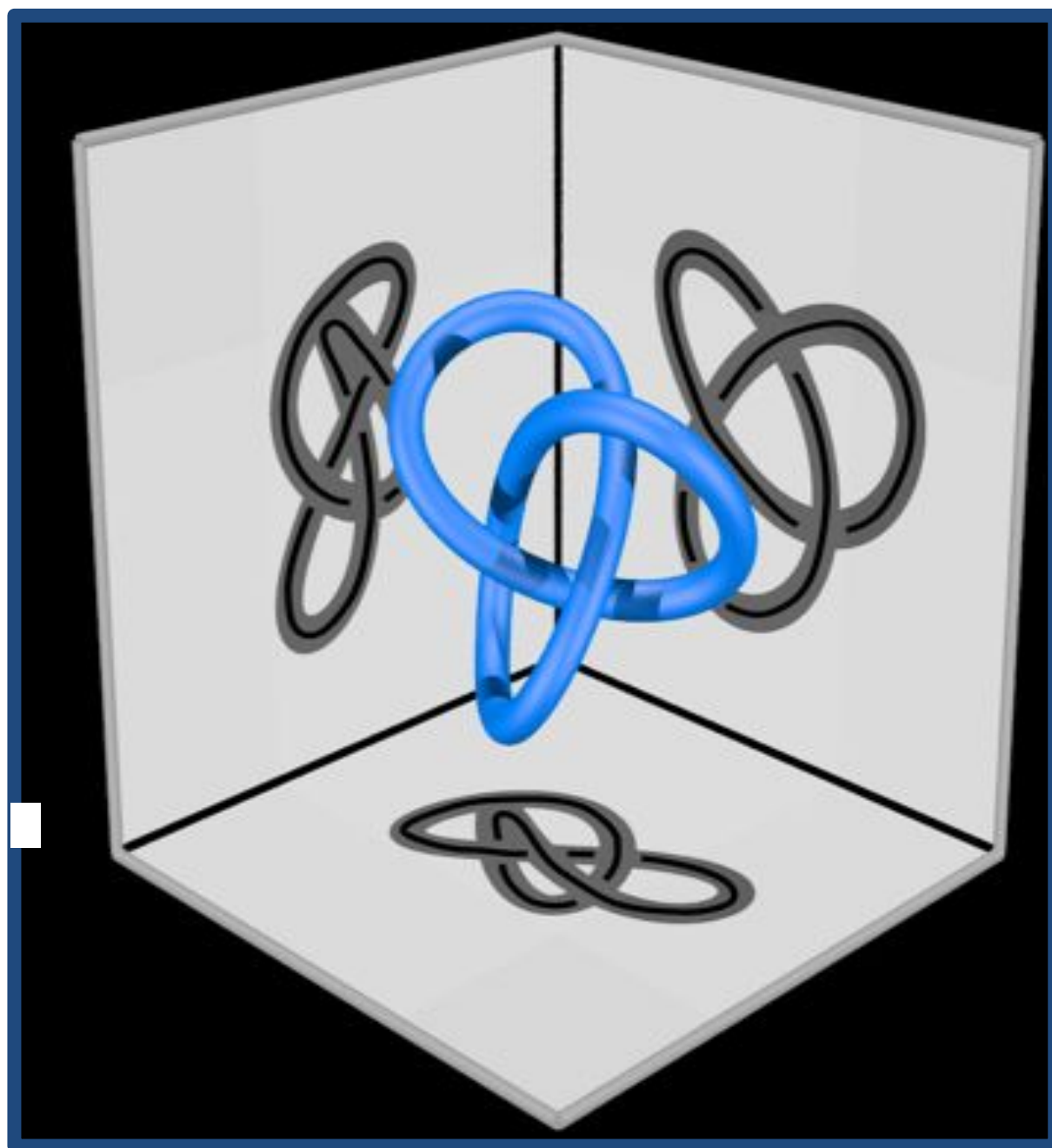
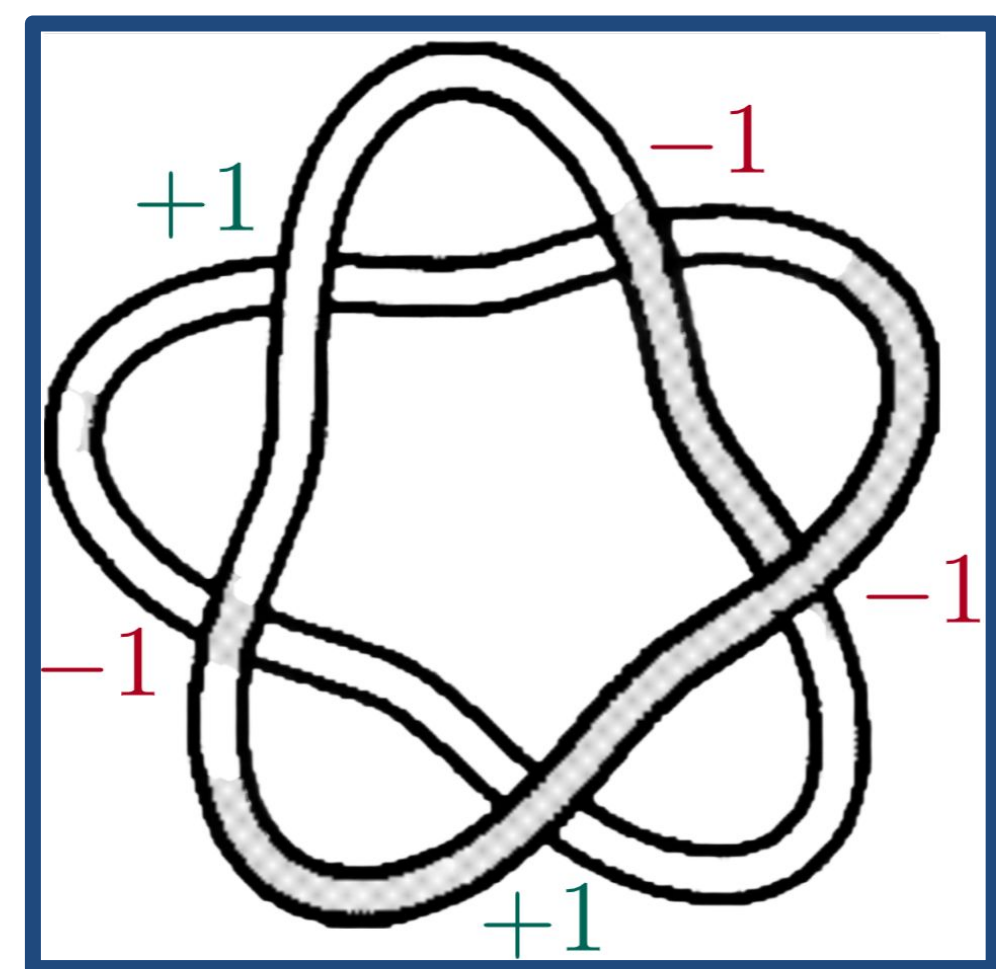


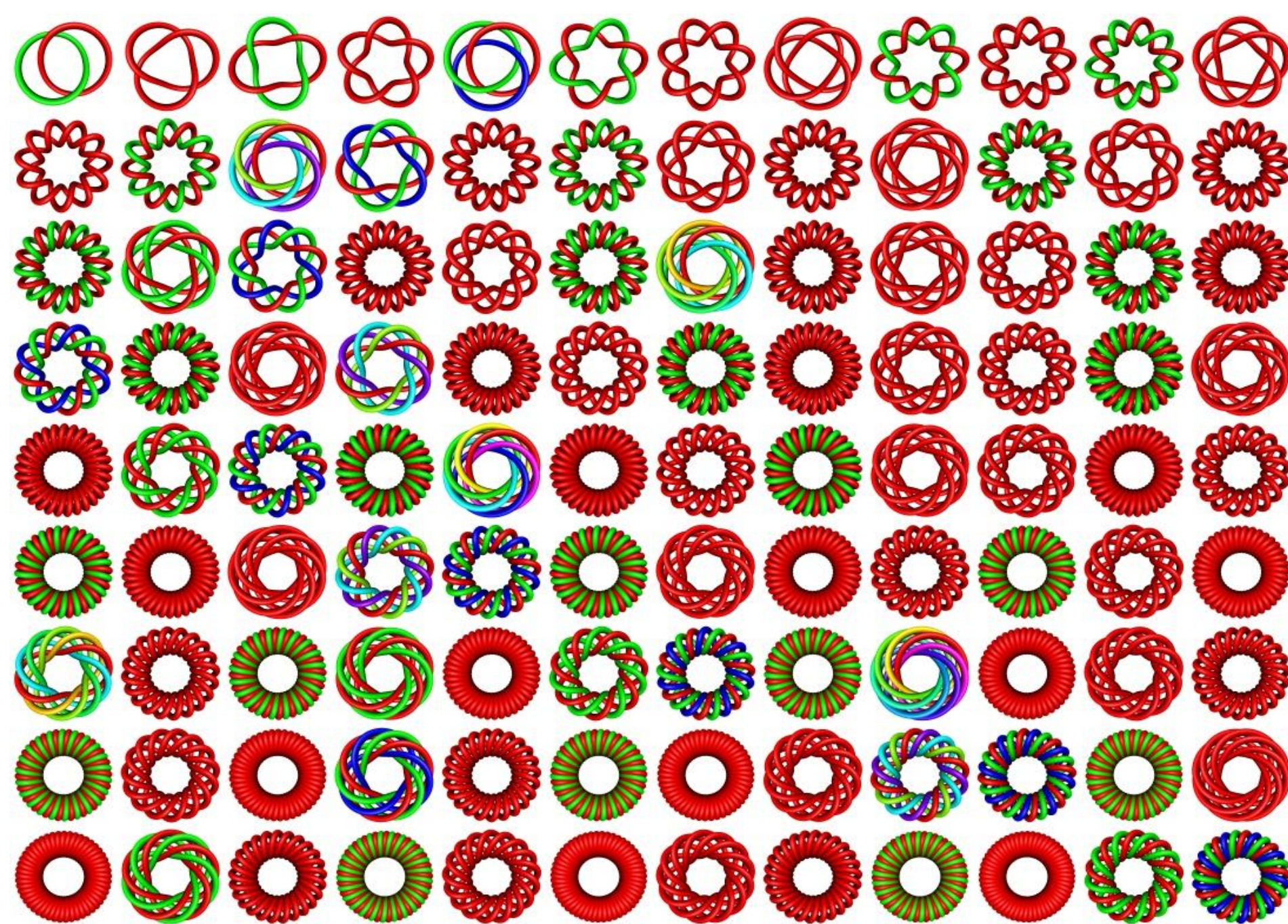
What is a Shadow Space?



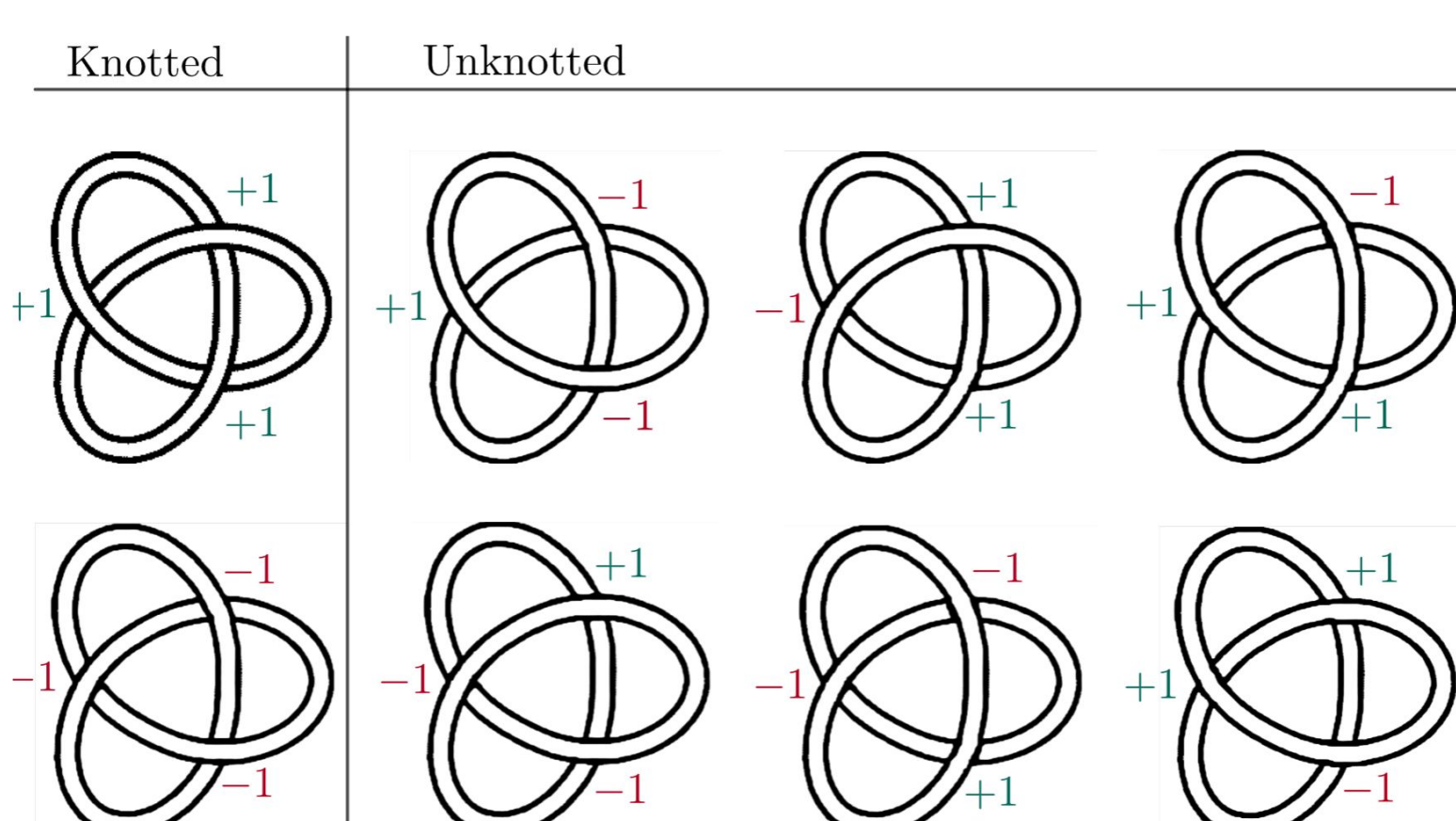
On the left is a trefoil knot represented in three-dimensional space, along with its **shadow**, or projection, onto a two-dimensional plane. When the 3D knot is projected onto 2D, we lose important spatial information, especially about which strands pass over or under at the crossings. This can make it harder to distinguish the true structure of the knot just by looking at its flat image.



Given the shadow (or projection) of a knot onto a plane, we can assign crossing information, indicating which strand goes over and which goes under, at each crossing point. Each such assignment produces a possible reconstruction of the original knot, called a **resolution** of the shadow. The collection of all possible such resolutions is known as the **shadow space** of the knot.

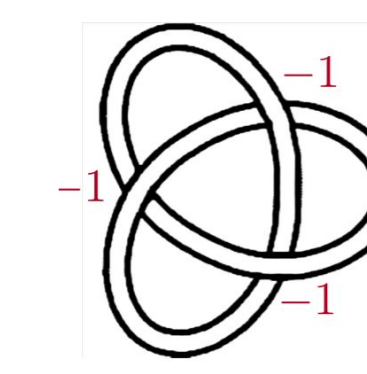
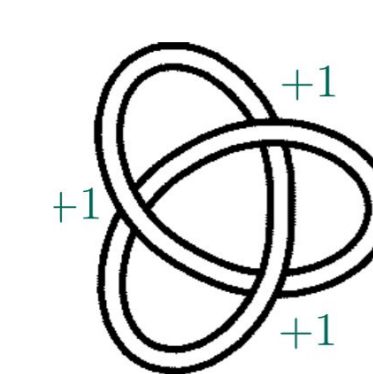
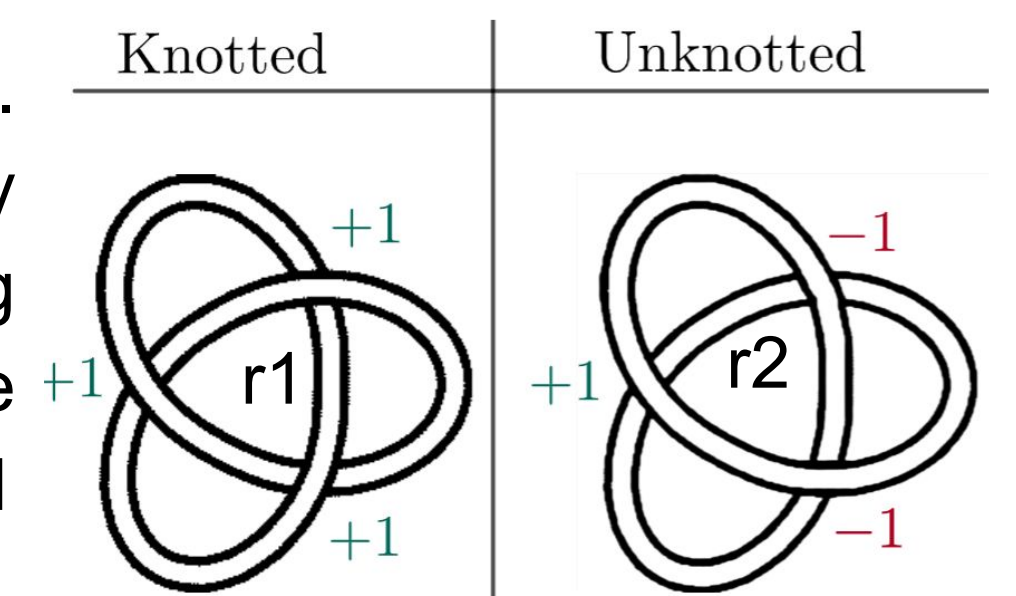


The Shadow Space of the Trefoil

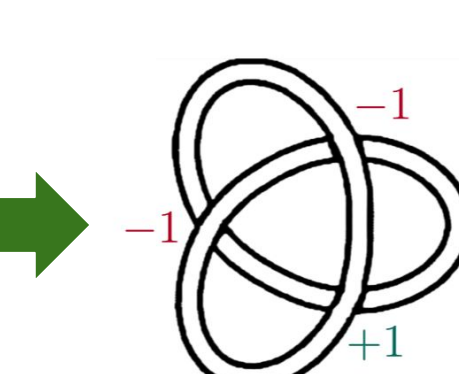
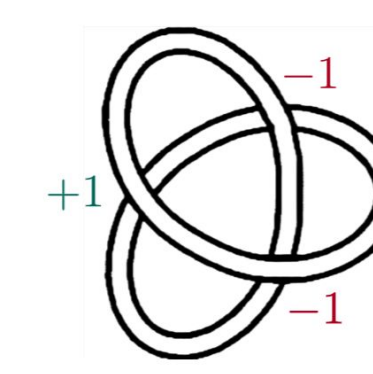


On the left is the shadow space of the trefoil knot with $2^3 = 8$ resolutions. We split the set of resolutions into two categories: *knotted* (left) and *unknotted* (right). To classify them this way, we would need to examine all 8 possible resolutions individually. For more complex knot shadows, the number of resolutions grows exponentially, making this process computationally expensive.

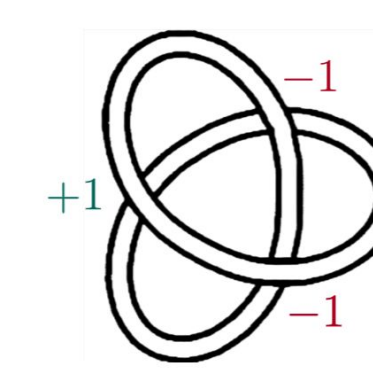
Consider the two resolutions r1 and r2 shown to the right. Determining the states of these two resolved knots uniquely determines the states of all knots within the corresponding shadow space. In other words, the state of every knot in the shadow space can be inferred directly from the states of r1 and r2, without additional computation.



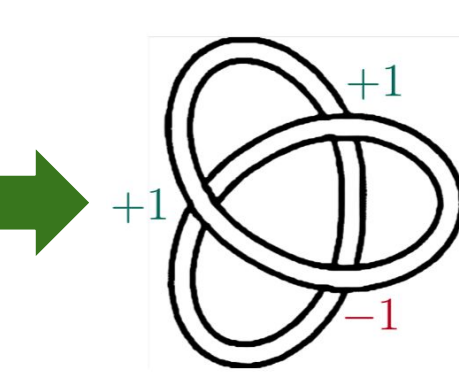
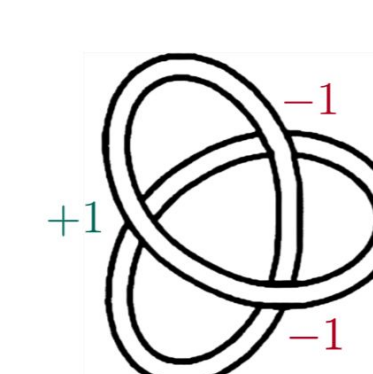
r1 can obtain the information of another resolution through reflection.



r2 can obtain the information of two other resolutions through rotation.



r2 can obtain the information of a third resolutions through reflection.



r2 can finally obtain the information of a fourth and fifth resolution through reflection and rotation.

Since the state of every knot in the shadow space can be inferred directly from the states of r1 and r2, we say that they form a basis for the shadow space

Results and remaining questions:

There are three main questions we wish to explore

- Given an arbitrary (p,q)-torus knot, how many resolutions are in its shadow space's basis?
- Once we know how large the basis is, how do we find the elements of that basis and how many resolutions are in the span of each element of the basis?
- For all (p,q)-torus knots, can we find an explicit count of how many resolutions in its shadow space yield the unknot and how many yield each specific knot/link type?

Bonus: Explore these three questions for pretzel knots, rational knots, and all knots/links in general.

The following result is a conjecture to answer question (I) above:

CONJECTURE 2.1. Let K be a torus knot, then the number of resolutions in the basis, B , of the shadow space $\mathcal{S}(K)$ is given by

$$|B| = \frac{1}{2n} \sum_{k=0}^{n-1} \left[2^{\gcd(n,k)} + \text{Fix}(\rho_k \circ c) \right]$$

where:

$$\text{Fix}(\rho_k \circ c) = \begin{cases} 2^{\gcd(n,k)} & \text{if } \frac{n}{\gcd(n,k)} \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

Acknowledgments
and Resources:

