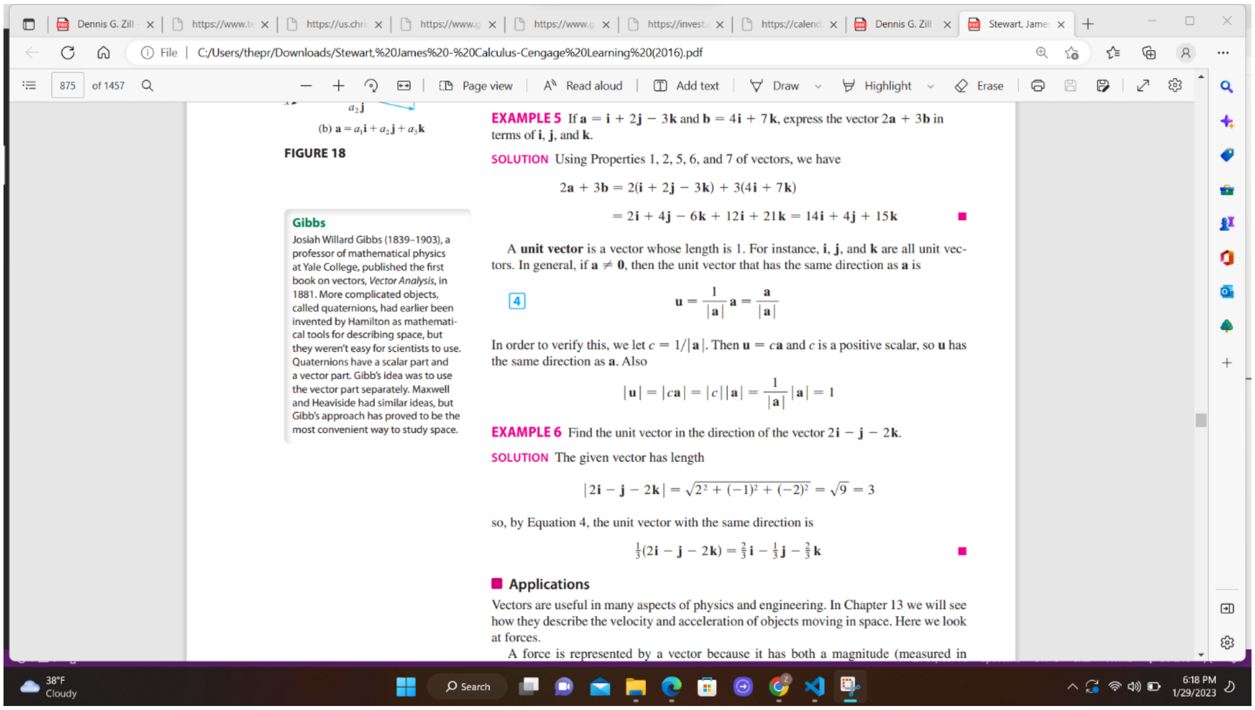
For Loop Proof Documentation

KM²



(b) $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$

FIGURE 18

Gibbs

Josiah Willard Gibbs (1839–1903), a professor of mathematical physics at Yale College, published the first book on vectors, *Vector Analysis*, in 1881. More complicated objects, called quaternions, had earlier been invented by Hamilton as mathematical tools for describing space, but they weren't easy for scientists to use. Quaternions have a scalar part and a vector part. Gibb's idea was to use the vector part separately. Maxwell and Heaviside had similar ideas, but Gibb's approach has proved to be the most convenient way to study space.

EXAMPLE 5 If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + 7\mathbf{k}$, express the vector $2\mathbf{a} + 3\mathbf{b}$ in terms of **i**, **j**, and **k**.

SOLUTION Using Properties 1, 2, 5, 6, and 7 of vectors, we have

$$2\mathbf{a} + 3\mathbf{b} = 2(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) + 3(4\mathbf{i} + 7\mathbf{k})$$
$$= 2\mathbf{i} + 4\mathbf{j} - 6\mathbf{k} + 12\mathbf{i} + 21\mathbf{k} = 14\mathbf{i} + 4\mathbf{j} + 15\mathbf{k}$$

A **unit vector** is a vector whose length is 1. For instance, **i**, **j**, and **k** are all unit vectors. In general, if $\mathbf{a} \neq \mathbf{0}$, then the unit vector that has the same direction as **a** is

$$\mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|} \qquad (4)$$

In order to verify this, we let $c = 1/|\mathbf{a}|$. Then $\mathbf{u} = c\mathbf{a}$ and c is a positive scalar, so **u** has the same direction as **a**. Also

$$|\mathbf{u}| = |c\mathbf{a}| = |c||\mathbf{a}| = \frac{1}{|\mathbf{a}|}|\mathbf{a}| = 1$$

EXAMPLE 6 Find the unit vector in the direction of the vector $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

SOLUTION The given vector has length

$$|2\mathbf{i} - \mathbf{j} - 2\mathbf{k}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3$$

so, by Equation 4, the unit vector with the same direction is

$$\tfrac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = \tfrac{2}{3}\mathbf{i} - \tfrac{1}{3}\mathbf{j} - \tfrac{2}{3}\mathbf{k}$$

Applications

Vectors are useful in many aspects of physics and engineering. In Chapter 13 we will see how they describe the velocity and acceleration of objects moving in space. Here we look at forces.

A force is represented by a vector because it has both a magnitude (measured in

- The reason why I am doing this is because, I just discovered we can map a vector field container with i, j, k for loop values, especially for these values that I have commented out in green, I am not sure if they already know this, because I'm just learning this language, I actually failed JavaScript twice but now I understand it. Look at this professor, with our for-loop values and counting I, j, k –n—as parameter we can produce a special kind of work!

//look at this though, the pyramid already resembles <i,j,k>= vector field param
//they could in fact set the param in the for loop to <isin + jcos + k>--or any other value-- please see Extra Credit Documentation

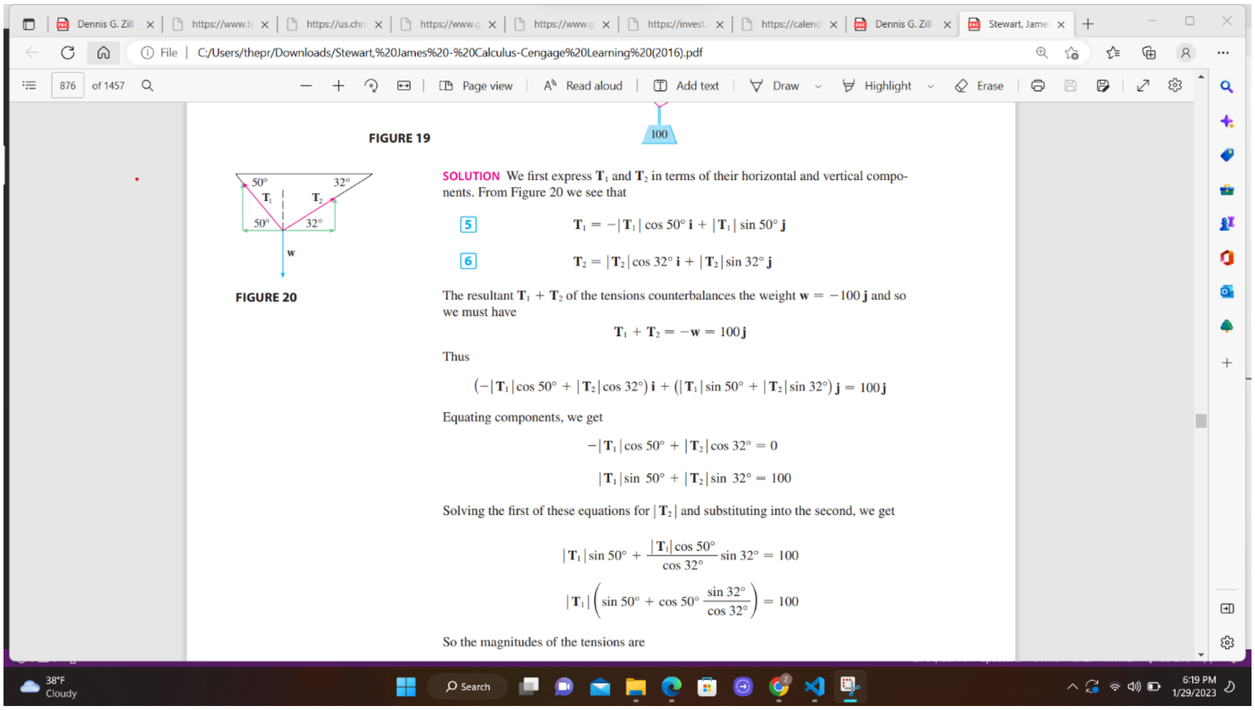


FIGURE 19

FIGURE 20

SOLUTION We first express $\mathbf{T}_1$ and $\mathbf{T}_2$ in terms of their horizontal and vertical components. From Figure 20 we see that

$$\mathbf{T}_1 = -|\mathbf{T}_1|\cos 50^\circ\,\mathbf{i} + |\mathbf{T}_1|\sin 50^\circ\,\mathbf{j} \quad (5)$$

$$\mathbf{T}_2 = |\mathbf{T}_2|\cos 32^\circ\,\mathbf{i} + |\mathbf{T}_2|\sin 32^\circ\,\mathbf{j} \quad (6)$$

The resultant $\mathbf{T}_1 + \mathbf{T}_2$ of the tensions counterbalances the weight $\mathbf{w} = -100\,\mathbf{j}$ and so we must have

$$\mathbf{T}_1 + \mathbf{T}_2 = -\mathbf{w} = 100\,\mathbf{j}$$

Thus

$$(-|\mathbf{T}_1|\cos 50^\circ + |\mathbf{T}_2|\cos 32^\circ)\,\mathbf{i} + (|\mathbf{T}_1|\sin 50^\circ + |\mathbf{T}_2|\sin 32^\circ)\,\mathbf{j} = 100\,\mathbf{j}$$

Equating components, we get

$$-|\mathbf{T}_1|\cos 50^\circ + |\mathbf{T}_2|\cos 32^\circ = 0$$

$$|\mathbf{T}_1|\sin 50^\circ + |\mathbf{T}_2|\sin 32^\circ = 100$$

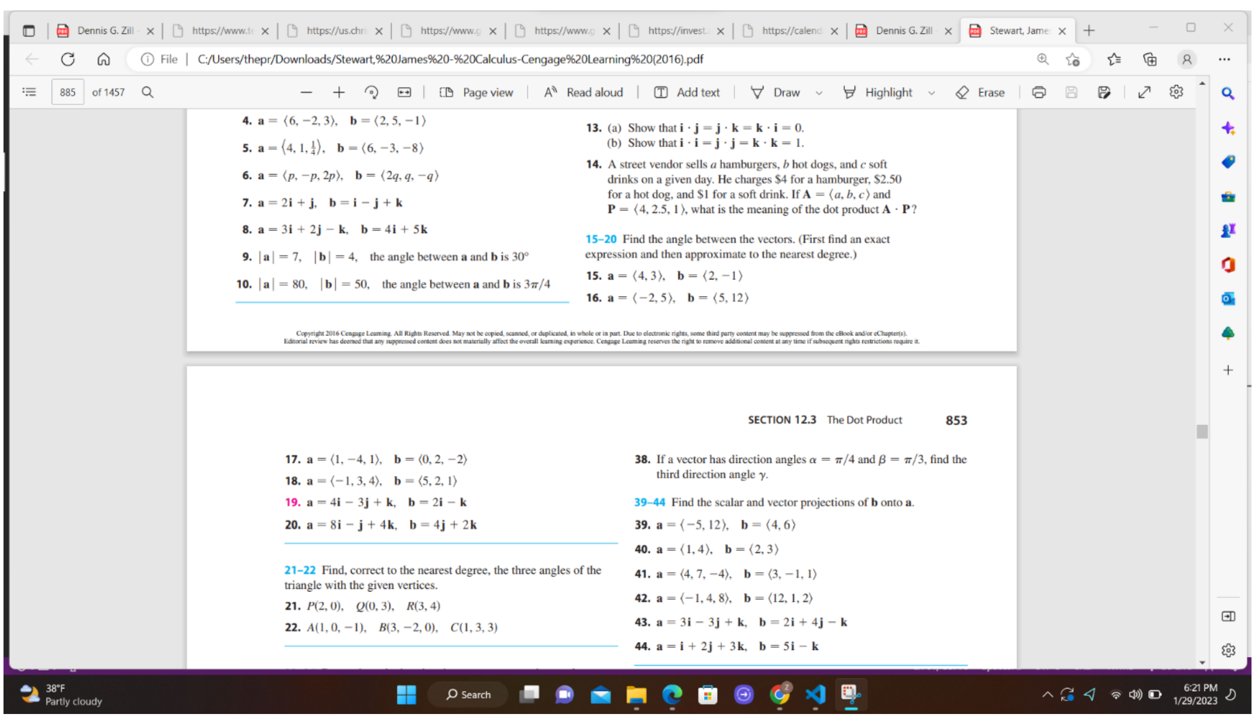
Solving the first of these equations for $|\mathbf{T}_2|$ and substituting into the second, we get

$$|\mathbf{T}_1|\sin 50^\circ + \frac{|\mathbf{T}_1|\cos 50^\circ}{\cos 32^\circ}\sin 32^\circ = 100$$

$$|\mathbf{T}_1|\left(\sin 50^\circ + \cos 50^\circ\,\frac{\sin 32^\circ}{\cos 32^\circ}\right) = 100$$

So the magnitudes of the tensions are

- Looping also respects this value, but you might have to use 'in-loop' for such a case.



4. $\mathbf{a} = \langle 6, -2, 3\rangle$, $\mathbf{b} = \langle 2, 5, -1\rangle$
5. $\mathbf{a} = \langle 4, 1, \frac{1}{4}\rangle$, $\mathbf{b} = \langle 6, -3, -8\rangle$
6. $\mathbf{a} = \langle p, -p, 2p\rangle$, $\mathbf{b} = \langle 2q, q, -q\rangle$
7. $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$
8. $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = 4\mathbf{i} + 5\mathbf{k}$
9. $|\mathbf{a}| = 7$, $|\mathbf{b}| = 4$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is 30°
10. $|\mathbf{a}| = 80$, $|\mathbf{b}| = 50$, the angle between $\mathbf{a}$ and $\mathbf{b}$ is $3\pi/4$

13. (a) Show that $\mathbf{i}\cdot\mathbf{j} = \mathbf{j}\cdot\mathbf{k} = \mathbf{k}\cdot\mathbf{i} = 0$.
(b) Show that $\mathbf{i}\cdot\mathbf{i} = \mathbf{j}\cdot\mathbf{j} = \mathbf{k}\cdot\mathbf{k} = 1$.
14. A street vendor sells a hamburgers, b hot dogs, and c soft drinks on a given day. He charges \$4 for a hamburger, \$2.50 for a hot dog, and \$1 for a soft drink. If $\mathbf{A} = \langle a, b, c\rangle$ and $\mathbf{P} = \langle 4, 2.5, 1\rangle$, what is the meaning of the dot product $\mathbf{A}\cdot\mathbf{P}$?

15–20 Find the angle between the vectors. (First find an exact expression and then approximate to the nearest degree.)

15. $\mathbf{a} = \langle 4, 3\rangle$, $\mathbf{b} = \langle 2, -1\rangle$
16. $\mathbf{a} = \langle -2, 5\rangle$, $\mathbf{b} = \langle 5, 12\rangle$

SECTION 12.3 The Dot Product 853

17. $\mathbf{a} = \langle 1, -4, 1\rangle$, $\mathbf{b} = \langle 0, 2, -2\rangle$
18. $\mathbf{a} = \langle -1, 3, 4\rangle$, $\mathbf{b} = \langle 5, 2, 1\rangle$
19. $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$
20. $\mathbf{a} = 8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = 4\mathbf{j} + 2\mathbf{k}$

21–22 Find, correct to the nearest degree, the three angles of the triangle with the given vertices.

21. $P(2, 0)$, $Q(0, 3)$, $R(3, 4)$
22. $A(1, 0, -1)$, $B(3, -2, 0)$, $C(1, 3, 3)$

38. If a vector has direction angles $\alpha = \pi/4$ and $\beta = \pi/3$, find the third direction angle γ.

39–44 Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

39. $\mathbf{a} = \langle -5, 12\rangle$, $\mathbf{b} = \langle 4, 6\rangle$
40. $\mathbf{a} = \langle 1, 4\rangle$, $\mathbf{b} = \langle 2, 3\rangle$
41. $\mathbf{a} = \langle 4, 7, -4\rangle$, $\mathbf{b} = \langle 3, -1, 1\rangle$
42. $\mathbf{a} = \langle -1, 4, 8\rangle$, $\mathbf{b} = \langle 12, 1, 2\rangle$
43. $\mathbf{a} = 3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$
44. $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 5\mathbf{i} - \mathbf{k}$

This value respects a hashing order of a dot product, but using the do/while and while at such a limit I would have to embed the math codes etc. I just need more practice and I can prove it, and I must get familiar codes, I should be up to speed about time I take –R—