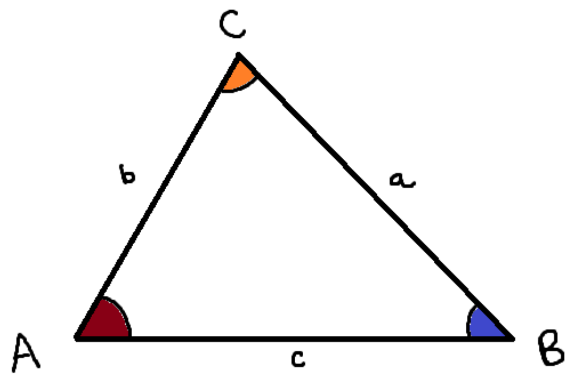
Non-Right Triangle Solutions

Triangle Figure:



A = Angle A

B = Angle B

C = Angle C

a = side a (opposite angle A)

b = side b (opposite angle B)

c = side c (opposite angle C)

Triangle Rules

Depending on the question. But generally, there are three different inputs:

- **Side, Side, Side (or SSS)**: 3 sides are given

Since all the sides are given, we apply The Law of Cosines to solve for the angles (A, B, and C)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Note: once the 2 angles are calculated, the third angle can be found by subtracting the first 2 angles from 180. This is because the sum of all angles in a triangle is 180 degrees

- **Side, Angle, Side (or SAS):** 1 angle and 2 sides are given

The given angle can be either less than 90 degrees or greater than or equal to 90 degrees. Let say angle A, side a and side c are given.

For **A ≥ 90** degrees:

- If **a ≤ c**, there are no possible triangles
- If **a > c** (as shown in the figure below), there is one possible solution. Then:
    - Use The Law of Sines to solve for angle C

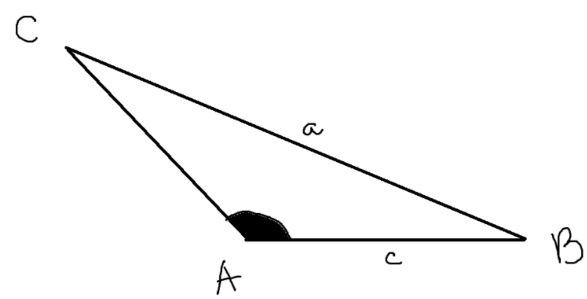
$$\boldsymbol{Angle\ C = sin^{-1}\left(\frac{c.\,sinA}{a}\right)}$$

    - The remaining angle B can be calculated by subtracting angle A and angle C from 180

Angle B = 180 – (angle A + angle C)

    - Use The Law of Sines to solve for side b

$$\boldsymbol{b = \frac{a.\,sinB}{sinA}}$$



For **A < 90** degrees:

- If **a ≥ c** (as shown below), there is one possible solution. Then:
    - Use The Law of Sines to solve for angle C

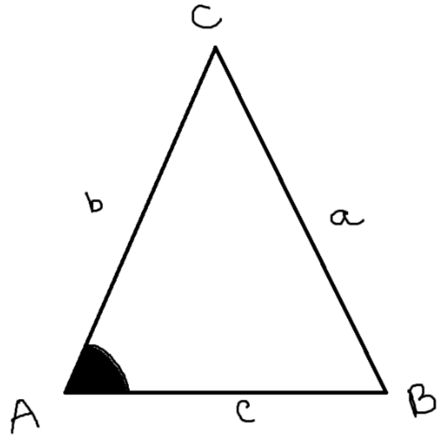
$$\boldsymbol{Angle\ C = sin^{-1}\left(\frac{c.\,sinA}{a}\right)}$$

    - The remaining angle B can be calculated by subtracting angle A and angle C from 180

Angle B = 180 – (angle A + angle C)

    - Use The Law of Sines to solve for side b

$$b = \frac{a.sinB}{sinA}$$



- If **a < c**, we will then need to find sinA and compare that to a/c ratio. If:
  - sinA $< \frac{a}{c}$: there are two possible triangles
    - Use the formula below to find two possible values for the 3rd side

      $$b = c.cosA \pm \sqrt{a^2 - c^2.sin^2A}$$

    - Now that, there are two solutions for side b, we will then have two solutions for angle B and two solutions for angle C. For each of the 3rd side, use The Law of Cosines to solve for each of angle B.

      $$\textbf{Angle } B = cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$$

    - The remaining angle C can be calculated by subtracting angle A and angle B from 180

      Angle C = 180 – (angle A + angle B)

  - sinA $= \frac{a}{c}$: there is one possible triangle
    - Use The Law of Sines to solve for angle C

      $$Angle\ C = sin^{-1}\left(\frac{c.sinA}{a}\right)$$

    - The remaining angle B can be calculated by subtracting angle A and angle C from 180

Angle B = 180 – (angle A + angle C)

- Use The Law of Sines to solve for side b

$$b = \frac{a.\,sinB}{sinA}$$

- sinA > $\frac{a}{c}$: there are no possible triangles

- **Angle, Side, Angle (or ASA):** 2 angles and 1 side are given
  - The remaining angle B is calculated by subtracting angle A and angle C from

Angle B = 180 – (angle A + angle C)

  - Use The Law of Sines to solve for each of the two sides

$$a = \frac{b.\,sinA}{sinB}$$

$$b = \frac{c.\,sinB}{sinC}$$

$$c = \frac{b.\,sinC}{sinB}$$