Section 1: Basic Math & Foundations

Math as a Language: Math has its own symbols and rules, just like any language. Understanding how to interpret and manipulate these symbols is crucial to solving problems. Mathematical expressions are essentially "sentences" that convey relationships between numbers and variables. The concept of "math as a language" is an essential foundation in understanding how mathematics functions not just as a set of rules and formulas but as a means of expressing and solving problems.

- **Mathematical Symbols as Words**: Just as words in a spoken language convey meaning, mathematical symbols represent quantities, operations, and relationships. For instance, symbols like +, -, ×, and ÷ are akin to verbs in a sentence, while numbers and variables act as nouns.
- **Equations as Sentences**: Equations and inequalities are like sentences in mathematical language. For example, the equation 2x + 3 = 7 is a mathematical sentence expressing a relationship between quantities. Just as sentences in English have a structure (subject, verb, object), mathematical sentences follow specific rules of syntax and semantics.
- **Mathematical Syntax and Grammar**: Just as language has grammar rules, mathematics has syntax rules. For example, in the expression 3 + 4 × 2, the order of operations (PEMDAS/BODMAS) dictates how to interpret the expression correctly. This ensures that mathematical statements are unambiguous and universally understood.
- **Functions as Mathematical Verbs**: Functions, such as $f(x) = x^2$, describe actions or operations performed on variables, like how verbs describe actions in sentences. Functions map inputs to outputs and help describe relationships between different quantities.
- **Graphs as Visual Sentences**: Graphs and charts are visual representations of mathematical relationships, like how illustrations can complement written text. A graph of a linear function, for instance, visually represents how the output changes with the input.

"Three more than twice a number is equal to seven." → $2x + 3 = 7$

"The area of a circle is calculated by multiplying the radius squared by pi." → $A(r) = \pi r^2$

"Three times a number plus two equals eleven." → $3x + 2 = 7$

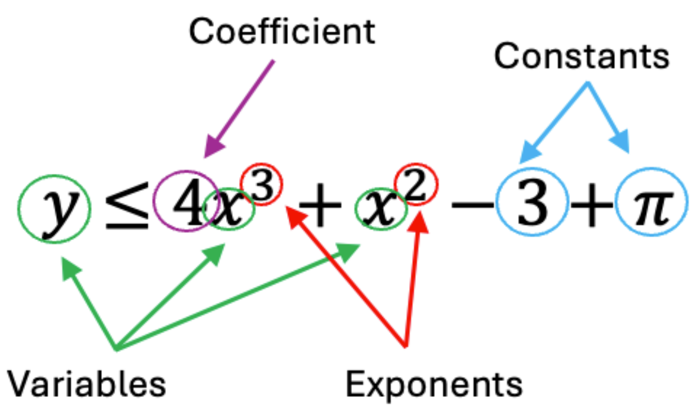
- The key take away is that mathematics is a quantitative language that enables us to describe real-world situations and solve problems.

Example: Convert the following phrase into a mathematical expression:

Two times a number minus five is equal to seven.

Mathematical Symbols: They are shorthand notations used to represent numbers, operations, relationships, and concepts in mathematics. They are essential for efficiently expressing and solving mathematical problems. Understanding these symbols is crucial for solving equations, interpreting functions and graphs, and working with quantitative expressions. A firm grasp of this foundational knowledge is necessary to support further study in algebra, calculus, and other advanced mathematical topics.

- Basic arithmetic symbols include addition (+), subtraction (-), multiplication (× or ·), and division (÷ or /).
- Equality and inequality symbols are used to compare values. The equal symbol (=) indicates that two expressions have the same value. The not equal symbol (≠) means that two expressions do not have the same value.
- The greater than symbol (>) shows that one number is larger than another. Conversely, the less than symbol (<) indicates that one number is smaller than another.
- The greater than or equal to symbol (≥) means that one number is greater than or equal to another. The less than or equal to symbol (≤) means one number is smaller than or equal to another.
- Algebraic symbols include variables, constants, coefficients, and exponents. A variable, such as x or y, represents an unknown or changing quantity. A constant denotes a fixed value that does not change. A coefficient is a number that multiplies a variable. An exponent indicates how many times a number or variable is multiplied by itself.



Types of Numbers: There are different types of numbers in mathematics, each serving a specific purpose. Each type of number plays a unique role, helping to solve different kinds of problems and represent various quantities. Understanding these different types of numbers will provide a strong foundation for studying more advanced concepts. In college level mathematics, there are several other types of numbers beyond those typically introduced in high school. These numbers are used in various specialized fields and help to solve more complex problems and as such, are beyond the scope of the SAT. The basic types of numbers that you should be familiar with are:

- **Natural Numbers:** The natural numbers are the numbers you first learned to count with. They start from 1 and go on indefinitely: 1, 2, 3, 4, 5, and so on. These numbers are used for counting objects, ordering them, and other basic tasks. For example, if you have 3 apples, you're dealing with a natural number, 3. Notice that natural numbers do not include zero or any negative numbers.
- **Whole Numbers:** Whole numbers include all natural numbers plus zero. So, the set of whole numbers is 0, 1, 2, 3, 4, and so on. Whole numbers are used in situations where you need to count objects, but you also need to represent "nothing" or "zero." For example, if you have no apples, you have 0 apples. This concept is crucial in many areas, such as computer science and counting systems.
- **Integers:** Integers include all whole numbers as well as their negative counterparts. So, the set of integers is ..., -3, -2, -1, 0, 1, 2, 3, and so on. Integers are useful when you need to

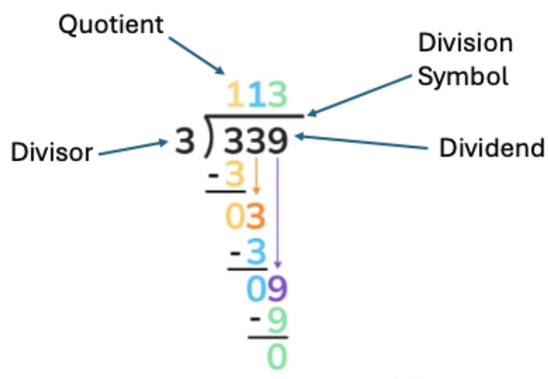
represent quantities that can be positive, negative, or zero. For example, if you owe someone 5 dollars, you can represent this debt as -5. Integers are widely used in everyday situations, like measuring temperature (e.g., -10 degrees) or calculating profit and loss.

- **Rational Numbers:** Rational numbers are numbers that can be expressed as a fraction of two integers, where the numerator is an integer, and the denominator is a non-zero integer. For example, $\frac{1}{2}, \frac{3}{4}, and\ -\frac{7}{3}$ are all rational numbers. Even whole numbers and integers are rational because they can be written as fractions with a denominator of 1 (e.g., 4 can be written as 4/1). Rational numbers are important because they allow us to represent parts of a whole or divisions of quantities. They are used in measurements, probabilities, and many other areas.
- **Irrational Numbers:** Irrational numbers cannot be expressed as a simple fraction of two integers. Their decimal expansions are non-terminating and non-repeating. Examples of irrational numbers include the square root of 2 (√2), pi (π), and the number e (used in calculus). Irrational numbers are crucial in mathematics because they fill in the "gaps" on the number line that rational numbers leave out. For example, π is essential in geometry for calculating the circumference and area of circles.
- **Real Numbers:** Real numbers include all rational and irrational numbers. In other words, any number that can be found on the number line is a real number. This includes natural numbers, whole numbers, integers, rational numbers, and irrational numbers. Real numbers are used to measure continuous quantities, such as length, area, volume, and time. For example, the number 3.5 is a real number because it represents a point on the number line between 3 and 4.
- **Complex Numbers:** Complex numbers extend the concept of real numbers by including imaginary numbers. A complex number is written in the form $a + bi$, where a is the real part and bi is the imaginary part. The imaginary unit i is defined as the square root of -1, or $\sqrt{-1}$. For example, $2 + 3i$ is a complex number. Complex numbers are used in advanced mathematics, engineering, and physics, particularly in situations involving waves, electrical circuits, and quantum mechanics.
- **Prime Numbers:** Prime numbers are natural numbers greater than 1 that have no positive divisors other than 1 and themselves. In other words, a prime number cannot be divided evenly by any number other than 1 and itself. Examples of prime numbers include 2, 3, 5, 7, 11, and 13. Prime numbers are fundamental in number theory because they are the "building blocks" of all natural numbers, as any natural number greater than 1 can be factored into prime numbers.
- **Composite Numbers:** Composite numbers are natural numbers greater than 1 that are not prime. This means they can be divided evenly by numbers other than 1 and themselves. For example, 4, 6, 8, 9, and 12 are composite numbers because they can be factored into smaller natural numbers (e.g., 4 = 2 × 2, 6 = 2 × 3). Understanding composite numbers is important for factorization and simplifying fractions.

Mathematical Operations: Mathematical operations are the basic actions you can perform on numbers to manipulate and combine them. Understanding these operations is essential for solving problems and building more complex mathematical skills. Each mathematical operation serves a different purpose and provides a foundation for solving more complex problems. Mastering these operations is crucial for success in mathematics and for applying math to real-world situations. The four basic mathematical operations are:

- **Addition:** Addition is the process of combining two or more numbers to find their total or sum. When you add numbers, you're essentially combining their values. For example, if you have 3 apples and someone gives you 2 more, you now have 5 apples, so 3 + 2 = 5. Addition is commutative, meaning the order in which you add numbers doesn't change the result: 3 + 2 is the same as 2 + 3. Addition is also associative, meaning that when adding three or more numbers, the way you group them doesn't affect the result: (1 + 2) + 3 = 1 + (2 + 3).
- **Subtraction:** Subtraction is the process of finding the difference between two numbers by removing the value of one number from another. It's the opposite of addition. For example, if you have 5 apples and eat 2, you have 3 left, so 5 - 2 = 3. Subtraction is not commutative, meaning the order of the numbers does matter: 5 - 2 is not the same as 2 - 5. Subtraction is also not associative: (5 - 2) - 1 is different from 5 - (2 - 1). Subtraction is often used to compare quantities, measure differences, and calculate change.
- **Multiplication:** Multiplication is a shortcut for repeated addition. When you multiply two numbers, you're adding one of the numbers to itself repeatedly based on the value of the other number. For example, 4 × 3 means adding 4 three times: 4 + 4 + 4 = 12. Multiplication is commutative, so 4 × 3 is the same as 3 × 4. It's also associative, meaning the grouping doesn't matter: (2 × 3) × 4 = 2 × (3 × 4). Multiplication is used for many applications, such as calculating area, scaling quantities, and working with ratios.
- **Division:** Division is the process of splitting a number into equal parts. It's the opposite of multiplication. For example, if you have 12 apples and want to divide them equally among 4 people, each person gets 3 apples, so 12 ÷ 4 = 3. Division is not commutative, so 12 ÷ 4 is not the same as 4 ÷ 12. Division is also not associative: (12 ÷ 4) ÷ 2 is different from 12 ÷ (4 ÷ 2). Division is crucial for distributing quantities, finding rates, and working with fractions.
    - Long division is a method used to divide larger numbers that cannot be easily divided in your head. It breaks down a division problem into a series of simpler steps, allowing you to find the quotient (the result of the division) and sometimes the remainder. The steps for long division are:

        1. Setup: Start by writing the dividend (the number you want to divide) inside the long division symbol (which looks like an extended L), and the divisor (the number you are dividing by) outside, to the left of the symbol.
        2. Divide the first digit: Look at the first digit of the dividend. Ask yourself, "How many times does the divisor go into this digit?" If the divisor is larger than the first digit, consider the first two digits instead.
        3. Multiply and subtract: Multiply the divisor by the number you just wrote above the division symbol. Subtract this number from the first digit (or second digit if the divisor was too large).
        4. Bring down the next digit: Bring down the next digit of the dividend, next to the remainder you just calculated.
        5. Repeat the process: Continue until there are no digits left to divide by. If you have no remainder, then whatever number is on top of the division symbol at this point is your answer. If there is a remainder, then that number is set up as a fraction with the divisor as the denominator, and it is added to the number above the long division symbol. Whatever form, the answer you get from division is the quotient.



- **Absolute Value:** The absolute value of a number is its distance from zero on the number line, regardless of direction. For example, the absolute value of both 3 and -3 is 3, written as |3| = 3 and |-3| = 3. Absolute value is used to measure magnitude, determine error margins, and solve equations where only the size of the difference matters, not the direction.

Example: Use long hand division for the following:

a. What is 136 divided by 4?
b. What is 1,225 divided by 5?

Complex Operations: In addition, there are other operations that will be covered in greater detail in later sections:

- **Exponentiation:** Exponentiation is the process of raising a number to a power, which means multiplying that number by itself a certain number of times. For example, 3^2 (read as "three squared") means 3 × 3, which equals 9. Here, 2 is the exponent, and 3 is the base. Exponentiation is <u>not commutative</u>, so 3^2 is not the same as 2^3. It's used to represent large numbers, model exponential growth, and calculate areas and volumes of geometric shapes.
- **Roots:** Taking the root of a number is the inverse operation of exponentiation. The square root of a number, for example, is a value that, when multiplied by itself, gives the original number. For example, the square root of 16 is 4 because 4 × 4 = 16. Roots are used to simplify expressions, solve quadratic equations, and work with areas and lengths in geometry.

Advanced Operations: There are several other more advanced operations that are beyond the scope of the SAT, but a short description for some of them is provided below:

- **Factorials:** The factorial of a number is the product of all positive integers up to that number. It's denoted by an exclamation mark. For example, 5! (read as "five-factorial") is 5 × 4 × 3 × 2 × 1 = 120. Factorials are used in combinatorics to calculate permutations and combinations, in probability theory, and in series expansions.
- **Logarithms:** A logarithm is the inverse of exponentiation. The logarithm of a number is the exponent to which a base must be raised to produce that number. For example, $\log_2(8) = 3$ because 2^3 = 8. Logarithms are used in many fields, including engineering, economics, and information theory, to solve exponential equations, analyze growth patterns, and work with large-scale data.

PEMDAS & the Order of Operations: PEMDAS is an acronym for the order of operations. It dictates the order in which operations should be performed in an expression to get the correct result. The order of operations is a rule used to clarify which procedures should be performed first in a mathematical expression. Some places teach BODMAS. They are essentially the same just different terminology. The important aspect is the order of operations. A standardized order of operations ensures that everyone solves expressions in the same way, preventing confusion and ensuring conformity when reporting mathematical values and expressions.

- P = Parentheses
- E = Exponents
- M & D = Multiplication and Division (left to right)
- A & S = Addition and Subtraction (left to right)

Example: Use PEMDAS to find the value of the expressions below:

$$7 + 7 \div 7 + 7 x 7 - 7 =$$

$$-2(1 \times 4 - 2 \div 2) + (6 + 2 - 3) =$$

$$(48 \div 22)2 - 4 + 23 \times 2 =$$

$$\left((23 + 4 \times 2)2 - (4 \div 22 + 1) \times (4 - 3 + 1)\right) \div (32 - 3 + 2) =$$

$$4 \times 4 - 3 + 1 + 2 \div 2 + 5 =$$

Rules for Fractions: Fractions represent a part of a whole and follow specific rules for addition, subtraction, multiplication, and division. The key to working with fractions is understanding how to find a common denominator for addition and subtraction. For multiplication and division, you work with numerators and denominators directly. Mastery of fractions is a fundamental mathematical skill required for solving equations, interpreting graphs, working with rational functions, handling conversions, and dealing with exponents—skills commonly tested on the SAT. Fractions are often preferred over decimals in certain contexts because they can be manipulated exactly without rounding errors, which can occur with decimals. Decimals may be rounded off, sometimes leading to discrepancies when comparing or combining numbers. Later sections will introduce strategies and concepts for simplifying expressions and equations involving fractions, which will help make these operations more manageable and precise.

- **Adding & Subtracting Fractions:** To add or subtract fractions, they must have the same denominator. Once the denominators are the same, you can add or subtract the numerators while keeping the denominator unchanged.
  - A helpful shortcut to get to a common denominator is to multiply one fraction by the denominator of the other, and vice versa. For example:

$$\frac{3}{4} + \frac{5}{6} = ?$$

$$\left(\frac{6}{6}\right)\frac{3}{4} + \frac{5}{6}\left(\frac{4}{4}\right) = \frac{18}{24} + \frac{20}{24}$$

$$\frac{18 + 20}{24} = \frac{38}{24}$$

$$\frac{38}{24} = \frac{19}{12} = 1\frac{7}{12}$$

  - However, if the two numerators have a common multiple, you can also determine what number to multiply each fraction by to get the same denominator. In the fractions above, 4 and 6 have a common multiple of 12, so:

$$\frac{3}{4} + \frac{5}{6} = ?$$

$$\left(\frac{3}{3}\right)\frac{3}{4} + \frac{5}{6}\left(\frac{2}{2}\right) = \frac{9}{12} + \frac{10}{12}$$

$$\frac{9 + 10}{12} = \frac{19}{12}$$

- **Multiplying Fractions:** Multiplying fractions is a straightforward process that involves two simple steps: Multiply the Numerators and then Multiply the Denominators.
- **Dividing Fractions:** Dividing fractions is slightly different from multiplying, but it can be done easily using the following method (Keep → Change → Flip):
  - **Step 1: Invert the Second Fraction (Reciprocal) -** When dividing fractions, the first step is to take the reciprocal of the second fraction (the one you're dividing by). To find the reciprocal, swap the numerator and denominator. For example, if you're dividing 3/4 by 2/5, the reciprocal of 2/5 is 5/2.
  - **Step 2: Multiply the Fractions -** Once you have the reciprocal of the second fraction, multiply it by the first fraction. This turns the division problem into a multiplication problem. Using the same example, 3/4 ÷ 2/5 becomes 3/4 × 5/2.
  - **Step 3: Multiply the Numerators -** Multiply the numerators together. For our example, multiply 3 by 5, which equals 15.
  - **Step 4: Multiply the Denominators -** Next, multiply the denominators together. In our example, multiply 4 by 2, which equals 8.
- **Simplify the Fraction (if necessary):** After multiplying the numerators and denominators, you might need to simplify the fraction. Simplifying involves dividing both the numerator and denominator by their greatest common factor (GCF). For instance, if the fraction was 8/16, you would simplify it to 1/2 because both 8 and 16 can be divided by 8.

Example: Find the answers to the expressions below:

a. $\frac{5}{4}+\frac{3}{5}=$

b. $\frac{11}{16}-\frac{7}{3}=$

c. $9\frac{3}{4} + 11\frac{13}{16} =$

d. $\frac{7}{3} \times \frac{9}{28} =$

e. $\frac{9}{13} \div \frac{27}{26} =$

f. $12\frac{2}{3} \times 6\frac{3}{4} =$

g. $8\frac{3}{4} \div 3\frac{1}{2} =$

h. $\dfrac{\frac{4}{7}}{\frac{3}{2}} =$

Reciprocals: The reciprocal of a number is what you multiply that number by to get 1. For any nonzero number x, its reciprocal is $\frac{1}{x}$. For example, the reciprocal of 5 is $\frac{1}{5}$. Reciprocals are crucial in division problems and solving equations, particularly when dealing with fractions.

Variables: In mathematics, a variable is a symbol, usually a letter, that represents an unknown or changeable value. Variables are essential because they allow us to write general equations and expressions that can apply to many different situations. By using variables, we can solve problems where the exact numbers may not be known or where the numbers can change.

- A variable can be thought of as a placeholder for a number. For example, in the equation $x + 3 = 7$, the letter x is a variable. The equation is asking: "What number can you add to 3 to get 7?" In this case, x would be 4.
- Variables are often letters such as $x, y, or\ z$ but they can be any symbol that represents a value. In algebra, variables are typically used to represent numbers, but in more advanced mathematics, they can represent functions, vectors, or even entire expressions.

Using Variables in Mathematical Operations: Just like numbers, variables can be added, subtracted, multiplied, divided, and used in other mathematical operations. Here's how different operations work with variables:

- Addition and Subtraction: You can add or subtract variables just like numbers. For example, $if\ x = 5\ and\ y = 3, then\ x + y = 8$. If you don't know the values of the variables, you can still work with them. For example, $x + y$ simply represents the sum of $x\ and\ y$, whatever those values might be. However, when working with algebraic expressions, you can only add or subtract like terms. Like terms are terms that have the same variable(s) raised to the same exponent(s). This concept will be explored in greater detail throughout the program, highlighting how to combine like terms and the principles of adding and subtracting algebraic expressions.
- Multiplication: Multiplication with variables is often written without the multiplication sign. For example, $3 \times x$ is usually written as $3x$. When multiplying variables, the result is usually written as a single term. For example, $x \times y$ is written as $xy. If\ x = 2\ and\ y = 4, then\ xy = 8$.
- Division: Division with variables is written as a fraction. For example, $x \div y$ is written as $\frac{x}{y}$. $If\ x = 6\ and\ y = 2, then \frac{x}{y} = 3$.
- Exponents: Variables can also be raised to powers. For example, x^2 means that you take the value x, whatever that value is, multiplied by itself two times. $If\ x = 3, then\ x^2 = 9$.

Rules for Working with Variables: When using variables in mathematical expressions, there are a few important rules to keep in mind:

- Like Terms: When adding or subtracting variables, you can only combine like terms. Like terms are terms that have the same variable raised to the same power.
- Distributive Property: When you multiply a number or variable by a sum of variables, or a sum of numbers and a variable(s), you distribute the multiplication across each term.

- For example:

$$3(x + y) = 3x + 3y$$

$$x(y + z + 4) = xy + xz + 4x$$

- <u>Substituting Values:</u> If you know the value of a variable, you can substitute it into an expression or equation to find the result.
- <u>Equations with Variables:</u> When you have an equation with a variable, your goal is often to solve for the variable. This means finding the value of the variable that makes the equation true.

Why Variables are Important: Variables are a fundamental concept in mathematics because they allow us to generalize problems and create formulas that can be used in many different situations. They are the building blocks for algebra, and understanding how to work with them is essential for more advanced topics in mathematics and science. By mastering the use of variables, you can manipulate equations to solve complex problems and find quantifiable answers to important questions. This skill is not only critical for success in math but also in fields like physics, engineering, and economics, where variables are used to model and solve real-world problems.

PEMDAS in Reverse for Isolating Variables: When solving equations for a variable, you often use PEMDAS in reverse. This means you start by undoing any addition or subtraction, then move on to multiplication or division, and finally, handle exponents and parentheses to isolate the variable. To solve equations with one variable, you need to isolate the variable on one side of the equation. This often involves using inverse operations to cancel out other terms. However, this can also be done with equations that contain multiple variables, in which case you would find the value of a variable in terms of the other variable(s). This concept will be exercised more throughout the program.

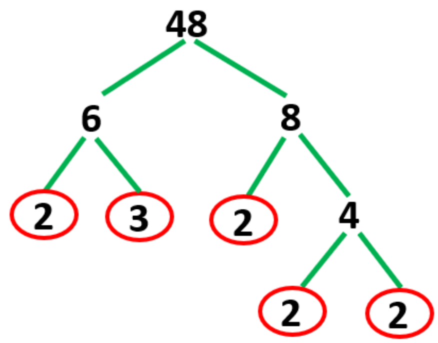
Roots of Numbers: Taking the root of a number is a mathematical operation that is essentially the reverse of raising a number to a power (exponentiation). When you take the root of a number, you are trying to find a value that, when multiplied by itself a certain number of times, gives you the original number. The most common root is the square root, but there are other types, such as cube roots and fourth roots. Roots will be covered in greater detail in the sections on functions and polynomials. Understanding roots is crucial because they are used in a variety of mathematical contexts, including algebra, geometry, and calculus. Roots are often involved in solving equations, particularly quadratic equations, and they are also important in understanding functions and graphs. Taking roots also comes up in real-world applications, such as finding the side length of a square when you know the area or determining the magnitude of a force or vector in physics.

- The root is denoted by a radical symbol (√) with a small number written to the upper left to indicate which root you are taking (this small number is called the "index"). If no number is written, it is assumed to be 2, which means the square root. The square root of 25 is 5 because 5 × 5 = 25. The cube root of 27 is 3 because 3 × 3 × 3 = 27.
- **Square Root (√):** The square root of a number is a value that, when multiplied by itself, equals the original number. For example, the square root of 9 is 3 because 3 × 3 = 9. The square root of 16 is 4 because 4 × 4 = 16. The square root is the most common root and is denoted simply by the radical symbol √.

- **Cube Root ($\sqrt[3]{}$):** The cube root of a number is a value that, when multiplied by itself three times, equals the original number. For example, the cube root of 8 is 2 because 2 × 2 × 2 = 8. Cube roots are useful when dealing with volumes, as they tell you the side length of a cube that has a given volume.
- **Higher-Order Roots:** You can also take the fourth root ($\sqrt[4]{}$), fifth root, and so on. These are less common but follow the same principle: the $nth\ root$ of a number is a value that, when raised to the $nth\ power$, equals the original number.
- **Rules for Taking Roots:**
  - **Positive and Negative Roots:** For any positive number, the square root has two answers: a positive and a negative value. For example, the square roots of 9 are 3 and -3, because 3 × 3 = 9 and -3 × -3 = 9. However, when we refer to "the square root," we usually mean the positive root.
  - **Zero Root:** The square root of 0 is 0. There is only one answer because 0 × 0 = 0.
  - **Imaginary Numbers:** The square root of a negative number does not have a real solution because no real number, when squared, equals a negative number. Instead, the square root of a negative number is defined as an imaginary number. These will be discussed in the final section.
  - **Product Property of Square Roots:** The square root of a product is equal to the product of the square roots. For example, $\sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$. This property helps simplify expressions involving square roots.
  - **Quotient Property of Square Roots:** The square root of a quotient is equal to the quotient of the square roots. For example, $\sqrt{\left(\frac{25}{4}\right)} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2} = 2.5$. This property is useful when simplifying fractions inside a square root.
  - **Simplifying Roots:** Sometimes, the number under the root symbol (called the radicand) can be simplified. For example, $\sqrt{18}$ can be simplified because 18 = 9 × 2, and $\sqrt{9} = 3$. So, $\sqrt{18} = 3\sqrt{2}$. This will become more apparent when we discuss factor trees.

Factor Trees for Radicals: Factor trees help break down a number into its prime factors, which can simplify radicals. This method can be done for determining the factors of a number as well as solving for the roots of a number, especially when the number itself does not have a perfect root for the root being taken. Factor trees show how to solve for the root of a number by decomposing it step by step.

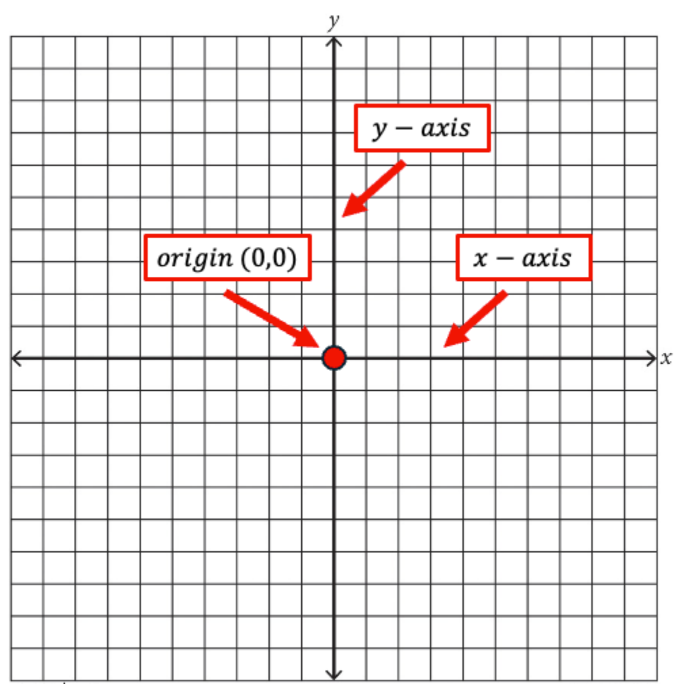
- The objective is to identify the factors of a number in a series of branching segments, until you have found a single number that appears n-times when you are taking the nth root of a number.
- That number can then be "taken out" of the radical. If more than one number qualifies to be taken out, then they are multiplied together. If any numbers do not qualify to be taken out, they remain in the radical. Again, if multiple numbers do not qualify to be taken out, they are multiplied, and the result remains in the radical.
- Below is an example using the number 48 to help demonstrate:



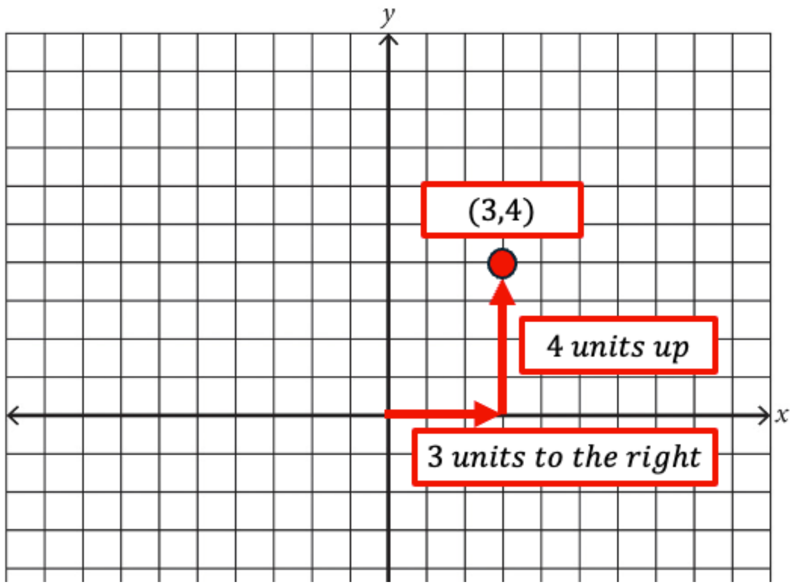
- If we were looking for $\sqrt{48}$, then the answer would be $4\sqrt{3}$
- If we were looking for $\sqrt[3]{48}$, then the answer would be $2\sqrt[3]{6}$
- If we were looking for $\sqrt[4]{48}$, then the answer would be $2\sqrt[4]{3}$

Cartesian Coordinates: The Cartesian coordinate system is a way to visually represent mathematical relationships and functions on a two-dimensional plane. It forms the foundation for studying graphs, understanding functions, analyzing quadratics, and eventually working with concepts like derivatives. Named after the French mathematician René Descartes, the Cartesian coordinate system allows us to plot points, lines, curves, and shapes in a clear and precise manner.

- The Cartesian coordinate system consists of two perpendicular lines called axes: the horizontal axis, known as the x-axis, and the vertical axis, known as the y-axis. These axes intersect at a point called the origin, which has coordinates (0, 0). The plane created by these axes is called the coordinate plane or the Cartesian plane.
- Each point on the plane is identified by an ordered pair of numbers (x, y), where the first number, x, represents the position along the x-axis (horizontal direction) and the second number, y, represents the position along the y-axis (vertical direction). The implication of this is that YOU CANNOT EVER refer to a point on the graph with just one number, you must provide an ordered pair to specify the position of a point in the Cartesian Coordinate system.



- For example, the point (3, 4) is located 3 units to the right of the origin on the x-axis and 4 units up on the y-axis.



- The Cartesian plane is divided into four quadrants by the x-axis and y-axis. Understanding these quadrants helps in identifying the location of points and interpreting graphs.
  1. First Quadrant: Both x and y are positive (e.g., (2, 3)).
  2. Second Quadrant: x is negative, and y is positive (e.g., (-2, 3)).
  3. Third Quadrant: Both x and y are negative (e.g., (-2, -3)).
  4. Fourth Quadrant: x is positive, and y is negative (e.g., (2, -3)).

- To plot a point on the Cartesian plane:
  1. Start at the origin (0, 0).
  2. Move horizontally to the x-coordinate. If the x-coordinate is positive, move to the right; if negative, move to the left.
  3. From this new position, move vertically to the y-coordinate. If the y-coordinate is positive, move up; if negative, move down.
  4. Mark the point where you stop.

- The Cartesian coordinate system is not just a theoretical tool; it has practical applications in various fields such as physics, engineering, economics, and computer graphics. For example, engineers use Cartesian coordinates to design structures, while computer graphics rely on this system to render images and animations. A solid grasp of working with the Cartesian coordinate system is necessary to build a strong foundation for more advanced mathematical concepts that enable you to solve real-world problems involving graphs, functions, and rates of change. This understanding is crucial for success in higher-level math courses and standardized tests like the SAT.

Introduction to Graphs: Graphing is a powerful tool in mathematics that allows us to visually represent relationships between variables. It helps us understand the behavior of functions, identify patterns, and solve equations. Graphing involves plotting points, lines, curves, and shapes on the Cartesian coordinate plane to visually represent mathematical equations and functions. Each point on the graph corresponds to an ordered pair (x, y), where:

- x is the input (independent variable) along the horizontal axis (x-axis).
- y is the output (dependent variable) along the vertical axis (y-axis).
- By connecting these points, we can see how one variable depends on another, which is crucial for understanding and analyzing quantitative relationships.
- The scale on the axes determines how units are represented. The scale must be consistent to accurately represent the relationship between x and y.
- Lines and Curves: A line or curve is created by connecting multiple points that represent a function or equation. For linear equations, the graph is a straight line, while for quadratic equations, it is a parabola.
- **To graph a function, follow these steps:**

1. **Identify the Function:** Determine the equation you want to graph. For example, $y = 2x + 1$ is a linear function, while $y = x^2 - 4$ is a quadratic function.
2. **Create a Table of Values:** Choose several x-values and calculate the corresponding y-values using the function. This table will give you specific points to plot on the graph.

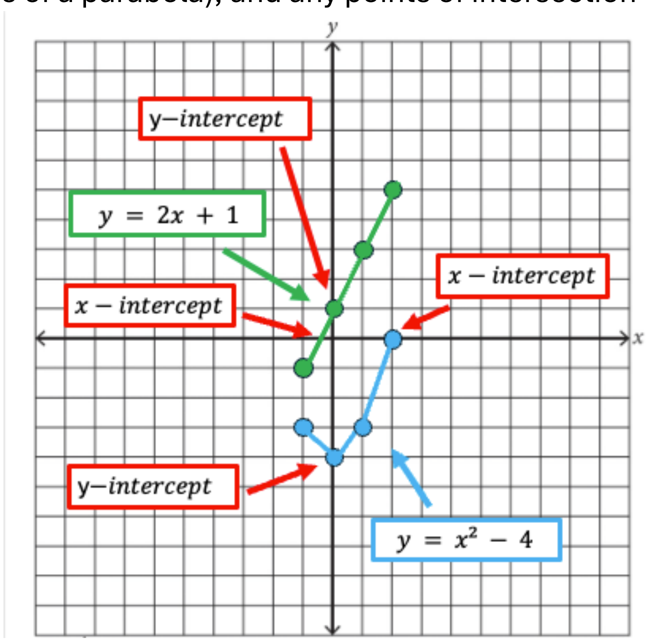
$y = 2x + 1$

x	y
-1	-1
0	1
1	3
2	5

$y = x^2 - 4$

x	y
-1	-3
0	-4
1	-3
2	0

3. **Plot the Points:** Using the table of values, plot each point on the Cartesian plane by moving horizontally to the x-coordinate and then vertically to the y-coordinate.
4. **Draw the Line or Curve:** Connect the points with a straight line (for linear functions) or a smooth curve (for nonlinear functions) to complete the graph.
5. **Label Important Features:** For more complex problems or functions, you may be asked to identify and label important features such as the x-intercept (where the graph crosses the x-axis), y-intercept (where the graph crosses the y-axis), vertex (in the case of a parabola), and any points of intersection with other lines or curves.



Graphing is a crucial skill for understanding and analyzing mathematical concepts such as:

- **Visualizing Relationships:** Graphs allow you to see how variables are related. This visual representation makes it easier to understand the behavior of functions and to identify trends and patterns.
- **Solving Equations:** By graphing an equation, you can find its solutions visually. For example, the points where a graph crosses the x-axis represent the solutions to the equation.
- **Analyzing Functions:** Understanding the shape and features of different types of graphs helps you analyze and interpret functions. This is essential for working with quadratics, exponential functions, and other types of equations.
- **Foundation for Advanced Math:** Mastery of graphing provides a solid foundation for studying more advanced topics, such as derivatives in calculus. The concept of the slope of a line on a graph is directly related to the derivative, which measures the rate of change of a function.
- **Real-World Applications:** Graphing is used in a wide range of fields, from science and engineering to economics and finance. For example, scientists use graphs to model physical phenomena, such as the trajectory of a projectile or the growth of a population. In economics, graphs are used to analyze trends in data, such as supply and demand curves.

Introduction to Equations: Equations are a cornerstone of algebra and mathematics, representing a balance between two expressions. They are used to describe relationships between quantities, solve problems, and model real-world situations. Understanding equations is essential for studying more complex mathematical concepts, including functions, graphs, and calculus.

- An equation is a mathematical statement that asserts the equality of two expressions. It is written in the form of A=B where A and B are expressions that can include numbers, variables, and operations. The goal in solving an equation is to find the value of the variable(s) that make the equation true.

Types of Equations: The primary equations you will need to be able to work with on the SAT are linear, quadratic, and rational equations. There are many others, but they are not on the SAT. Nonetheless, additional descriptions are provided for reference.

- Linear Equations: Linear equations are equations of the first degree, meaning they involve variables raised to the power of 1. They can be written in the form $ax + b = c, where\ a, b, and\ c$ are constants. The graph of a linear equation is a straight line.
- Quadratic Equations: Quadratic equations involve variables raised to the power of 2 and are typically written in the form $ax^2 + bx + c = 0$. The graph of a quadratic equation is a parabola.
- Exponential Equations: Exponential equations involve variables in the exponent. They can be written in the form $a^x = b$.
- Rational Equations: Rational equations involve fractions where the variable is in the numerator or denominator. They are solved by finding a common denominator and solving the resulting equation.
- Radical Equations: Radical equations involve square roots or other roots. To solve, isolate the radical expression and then raise both sides to the same power as the value of the index of the radical to eliminate the radical.

Solving Equations: To solve an equation, you must be able to isolate the variable on one side of the equation using the reverse PEMDAS method described above. This often involves performing inverse operations to undo addition, subtraction, multiplication, or division. For instance, use subtraction to undo addition, or division to undo multiplication. Always check your solutions by substituting them back into the original equation to ensure they satisfy the expression.

Solving Systems of Equations: Systems of equations involve finding values that satisfy multiple equations simultaneously. Methods include substitution, combination, and graphical solutions. These will be covered in greater detail in Section 3.

Importance of Equations: Equations are essential in mathematics because they allow us to describe and solve problems involving relationships between variables. They are used to model real-world scenarios, solve practical problems, and understand complex mathematical concepts.

Notes