Mathematical Framework for Visualizing Audio Data Through Particle Motion

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Abstract

This paper presents a theorem for visualizing sinusoidal audio data as particle motion in a 2D canvas. The formulation integrates sinusoidal functions, scaling factors, and random jitter to simulate dynamic visualization.

1 Theorem and Proof

Theorem 1. *Let n be the total number of particles, and let each particle's horizontal and vertical positions be defined by x_i and y_i, respectively. For sinusoidal audio data $A(t) = \sin(2\pi t)$, scaled amplitude S, jitter J_y, canvas width W, height H, and gap g, the position of particle i at time t is given by:*

$$x_i = (i + 0.5) \cdot g, \quad y_i = \frac{H}{2} - S \cdot \sin\left(2\pi \frac{i}{n}\right) + J_y,$$

where $J_y \sim$ Uniform$(-k, k)$, and k is the jitter scale factor.

Proof. The horizontal position x_i is defined as:

$$x_i = (i + 0.5) \cdot g,$$

ensuring uniform spacing between particles with a gap of g.

The vertical position y_i is determined by the sinusoidal function:

$$y_i = \frac{H}{2} - S \cdot \sin\left(2\pi \frac{i}{n}\right),$$

where:

- $\sin(2\pi t)$ represents the audio amplitude normalized over one period.
- S is the amplitude scale factor to fit the sinusoid within the canvas dimensions.
- $H/2$ centers the sinusoid vertically.

To simulate natural randomness in the visualization, jitter J_y is added to the vertical position:

$$J_y \sim \text{Uniform}(-k, k),$$

where k is the jitter scale factor.

Thus, the complete formula for y_i is:

$$y_i = \frac{H}{2} - S \cdot \sin\left(2\pi \frac{i}{n}\right) + J_y.$$

By combining these results, the particle positions are fully defined, completing the proof. □

2 Discussion

The theorem provides a scalable framework for mapping audio waveforms to particle motion. The sinusoidal representation ensures periodicity, while jitter introduces dynamic realism.

3 Source Code and Presentation

The complete implementation and an interactive presentation of the framework can be found at:

- Interactive Presentation and Source Code

The JavaScript source code and additional resources are available on the same page.