Mathematical Proof of the Audio Distance Method

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Abstract

This document presents a formal proof that the Audio Distance Method (ADM), based on the L2-norm of discrete audio signals, defines a valid metric. The ADM is foundational in applications such as radio stream comparison and quality detection in GNOME Radio.

1 Introduction

Let $x(t), y(t) \in \mathbb{R}^T$ be discrete-time audio signals of equal length T, sampled at a fixed rate. Define the Audio Distance Method (ADM) between two signals x and y as:

$$D(x,y) = \sqrt{\sum_{t=0}^{T-1} (x(t) - y(t))^2} \tag{1}$$

We will show that $D(x,y)$ satisfies the axioms of a metric space.

2 Metric Space Axioms

Let $\mathcal{A} = \mathbb{R}^T$ be the space of all real-valued audio signals of length T. We define $D : \mathcal{A} \times \mathcal{A} \to \mathbb{R}_{\geq 0}$ by Equation 1. We now verify the four metric axioms:

Axiom 1: Non-negativity

$$D(x,y) = \sqrt{\sum_{t=0}^{T-1} (x(t) - y(t))^2} \geq 0$$

because the square of a real number is always non-negative, and the sum of non-negative numbers is non-negative.

Axiom 2: Identity of Indiscernibles

$$D(x,y) = 0 \iff \sum_{t=0}^{T-1} (x(t) - y(t))^2 = 0 \iff \forall t,\ x(t) = y(t)$$

Hence, $D(x,y) = 0 \iff x = y$.

Axiom 3: Symmetry

$$D(x,y) = \sqrt{\sum_{t=0}^{T-1}(x(t)-y(t))^2} = \sqrt{\sum_{t=0}^{T-1}(y(t)-x(t))^2} = D(y,x)$$

since $(x(t)-y(t))^2 = (y(t)-x(t))^2$.

Axiom 4: Triangle Inequality

We want to show:

$$D(x,z) \leq D(x,y) + D(y,z)$$

This follows from the Minkowski inequality for $p = 2$, which states:

$$\left(\sum_{t=0}^{T-1} |x(t)-z(t)|^2\right)^{1/2} \leq \left(\sum_{t=0}^{T-1} |x(t)-y(t)|^2\right)^{1/2} + \left(\sum_{t=0}^{T-1} |y(t)-z(t)|^2\right)^{1/2}$$

Hence,

$$D(x,z) \leq D(x,y) + D(y,z)$$

3 Conclusion

Since the Audio Distance Method satisfies all four axioms, it defines a metric on the space of audio signals of equal length. This validates its use in comparing discrete-time signals in GNOME Radio and related audio applications.