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# The Conservation of Light's Energy, Mass and Momentum

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## ABSTRACT

An advance in the foundations of quantum mechanics was presented at the previous “Nature of Light” meeting which brings new insights to the conservation of light’s energy, mass and momentum.<sup>1,2</sup> Those discoveries suggest that the “photon” is a time-based collection of sub-photonic elementary light particles. Incorporation of this new understanding into quantum mechanics has allowed the determination of universal constants for the energy, mass, and momentum of light.<sup>3-7</sup> The energy constant for light is  $6.626 \times 10^{-34}$  J/osc, meaning the energy of a single oscillation of light is constant irrespective of the light’s frequency or wavelength. Likewise, the mass and momentum of a single oscillation of light are constant, regardless of changes to either time or space. A realistic understanding of the conservation of energy, mass and momentum for both matter and light in a single conservation law is now possible. When a body with mass absorbs or emits light, its energy, mass and momentum change in quantized amounts according to the relationship:

$$\Delta E = N\hbar\omega = Nm_0c^2 = N\rho_0c$$

where “N” is the number of oscillations absorbed or emitted by the body and  $\hbar$ ,  $m_0$ , and  $\rho_0$  are the constant energy, mass and momentum of an oscillation. Implications extend from general relativity and gravity to space sails and light driven nanomotors.

**Keywords:** Light, oscillation, energy, mass, momentum, force, conservation, symmetry, fractal

## 1. INTRODUCTION

The idea that physical properties of light are conserved has been around for a long time. In 1837, the chemist Karl Mohr wrote, “*Apart from the known chemical elements, there exists in nature only one agent, and that is Kraft [force or energy]; it can show itself in appropriate relationships as motion, chemical affinity, cohesion, electricity, **light**, heat or magnetism. And out of each of these kinds of phenomena all the others can be produced*”.<sup>8</sup> (Emphasis added)

Five (5) years later, Judge William Grove gave the inaugural lecture at the London Institution, speaking on “*the correlation of physical forces*”, in which he treated light as a manifestation of energy in the same way that mechanics, heat, electricity and magnetism were treated.<sup>9</sup> “*Physical Science treats of Matter, and what I shall to-night term its Affections; namely, Attraction, Motion, Heat, **Light**, Electricity, Magnetism, Chemical-Affinity. When these re-act upon matter, they constitute Forces. The present tendency of theory seems to lead to the opinion that all these Affections are resolvable into one, namely, Motion ... It appears to me that heat and **light** may be considered as affections; or, according to the Undulatory theory, vibrations of matter itself ... **Light**, Heat, Electricity, Magnetism, Motion, and Chemical affinity, are all convertible material affections*”. (Emphasis added)

These conservation concepts for matter and light were developed into a mathematical framework a few years later by the young army surgeon, Hermann von Helmholtz, M.D. In his treatise on the conservation of energy he showed that the many different forms of energy are interconvertible, “*...heat, electricity, magnetism, **light**, and chemical affinity ... from each of these different manifestations of [energy] we can set every other [manifestation] in motion*”.<sup>10</sup> Helmholtz was later appointed Chair of the physics department at the University of Berlin, where he taught the young Max Planck.

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After some junior professorships, Planck also found his way to the University of Berlin as a professor. It was there in 1900, that he developed his famous black-body radiation equation and quantum hypothesis.<sup>11</sup> Planck's quantum formula,

$$E = h \nu \quad (1)$$

(where "E" is energy, "h" Planck's action\* constant, and "ν" frequency) was foundational to the development of quantum mechanics and other 20<sup>th</sup> century technology concepts.

Albert Einstein's 1905 paper on the emission and transformation of light, for which he later won the Nobel prize in physics, was inspired by Planck's quantum hypothesis and formula. Einstein also suggested – contrary to the prevailing undulatory wave theory of light - that light was physically quantized into small discrete packets of energy.<sup>12</sup> Initially there was great resistance to these concepts, and reports from the subsequent Solvay Conferences included fiery exchanges between the participants. Robert Millikan<sup>13</sup> referred to Einstein's theories at one point as "reckless" and Niels Bohr did not relent in his opposition until the great Arthur Compton declared in a 1923 paper that the scattering of X-rays was a quantum phenomenon.<sup>14</sup>

In the meantime, Einstein developed his theories of relativity, and in 1916 published a short book on them. He described relativity concepts related to the energy and mass of light, as well as his energy-mass equivalence principle, "E = mc<sup>2</sup>" (where "m" is mass and "c" the constant speed of light).<sup>15</sup>

*"The most important result of a general character to which the special theory of relativity has led is concerned with the conception of mass. Before the advent of relativity, physics recognized two conservation laws of fundamental importance, namely, the law of the conservation of energy and the law of the conservation of mass, these two fundamental laws appeared to be quite independent of each other. By means of the theory of relativity they have been united into one law ..."*

*A body moving with the velocity v, which absorbs an amount of energy E<sub>0</sub> in the form of radiation without suffering an alteration in velocity in the process, has, as a consequence, its energy increased by an amount:"*

$$\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

*... Hence we can say: If a body takes up an amount of energy E<sub>0</sub>, then its inertial mass increases by an amount*

$$\frac{E_0}{c^2}$$

*...The inertial mass of a system of bodies can even be regarded as a measure of its energy. The law of the conservation of the mass of a system becomes identical with the law of the conservation of energy...Writing the expression for the energy in the form*

$$\frac{mc^2 + E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

*we see that the term mc<sup>2</sup>, which has hitherto attracted our attention, is nothing else than the energy possessed by the body before it absorbed the energy E<sub>0</sub>."*

There were certain unresolved problems with Einstein's relativity theory, however, and they became very apparent in his general relativity work. In classical Newtonian/Helmholtz physics, energy is conserved locally, e.g., for a volume of matter in space, the energy flowing out across its boundary is equal to the decrease of energy inside the volume and the energy is

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\* "Action" in the scientific sense, and as used in Planck's constant "h", is the product of energy X time. Hence Planck's action constant is defined as 6.626 X10<sup>-34</sup> Joule seconds.

thus “conserved”. “[T]he principle of energy conservation follows from symmetry under time translations. This applies to theories having a **finite** continuous symmetry group; theories that are Galilean”.<sup>16†</sup> (emphasis added) Finite continuous symmetry defines an actual (scalar) amount of energy which is conserved for the system in question. Energy flowing out of the system or being converted to another form can be totaled and accounted for regardless of the time periods involved. Thus the scalar, real amount of energy is constant even though it is transformed over a period of time from one form to another. This was the Galilean – Newtonian – Helmholtz theory.

In Planck’s quantum hypothesis, however, the energy of light which we now call “photon” energy was highly variable, and in fact infinitely variable in its amount. Einstein used Planck’s idea of infinitely variable photon energy in his general and special relativity theories. Photon energy was thus part of “an **infinite** continuous group of symmetries”.<sup>†</sup> In other words, properties related to light were not constant over a shift in time or space, and these properties carried over into relativistic and gravitational properties of matter and light. This infinite variability of photon energy thus created some problems in attempting to apply the principles of classical energy conservation to light.

The mathematician Emmy Noether concluded in her 1918 analysis that, “The laws of conservation of energy of classical mechanics ... are “proper” since no infinite groups occur”.<sup>17</sup> Because an infinite groups **do** occur for photon properties, the “energy relationships become improper ... [and] the failure of proper laws of conservation of energy is a characteristic feature of the “general theory of relativity.”” She constructed a new energy transformation using energy-momentum pseudotensors, which made it possible to “formulate a principle of energy conservation by integrating over a large, asymptotically flat spacelike hypersurface”.

A few years later, in 1924, Louis de Broglie finished his doctoral thesis looking at the energy, mass and momentum of light from a slightly different perspective.<sup>18</sup> De Broglie was captivated by Einstein’s theories and quantum ideas, and they played heavily in his work. He began with a consideration of Einstein’s energy-mass equivalence principle, in which Einstein stated, “The mass of a body is a measure of its energy content; if the energy changes by [E], the mass changes in the same sense by  $E/c^2$  ... If a body emits the energy [E] in the form of radiation, its mass decreases by  $E/c^2$ .”<sup>14</sup> De Broglie explained that, “Following Einstein, energy may be considered as being equivalent to mass, and all mass represents energy ... we may regard material and energy as two terms for the same physical reality.” De Broglie puzzled over the importance of action in quantum formulations and on the absence of a constant (scalar) amount or “quantum” of energy, stating, “...we have returned to statements on energy as fundamental and ceased to question why action plays a large role... [yet it] is impossib[le] to consider an isolated quantity of energy”. De Broglie was looking for a unit of constant energy for light, in the same way that Millikan had provided the constant unit of charge for electricity. Without such a constant unit of energy, noted de Broglie, one was forced to always associate light energy with photon frequency (i.e., time).

De Broglie made the best of what was known, and taking Planck’s quantum energy formula, “ $E = h \nu$ ”, and Einstein’s energy-mass equivalence relationship, “ $E = mc^2$ ”, he set the two (2) products equal to each other,

$$h \nu = mc^2 \tag{2}$$

stating, “This hypothesis is the basis of our theory...”. Solving for the rest mass of light (“ $m_0$ ”), he obtained the expression,

$$m_0 = h \nu_0 / c^2. \tag{3}$$

De Broglie’s thesis advisor did not know if the young de Broglie was genius or insane! Einstein’s advice was sought and he defended the brilliance of the young doctoral student. In his Nobel prize speech a mere five (5) years later, de Broglie declared, “The general formulae ... may be applied to corpuscles of light on the assumption that here the rest mass  $m_0$  is infinitely small. ... the upper limit of  $m_0$  ... is approximately  $10^{-24}$  grams.”<sup>19</sup>

De Broglie’s calculation of rest mass relied on the frequency of the light and so he was faced with an infinitely variable number of values for the mass of light as well. This was the same quandary that was causing the laws of energy conservation and of general relativity to be “improper”. Because de Broglie did not possess an “isolated quantity of energy”, his calculations did not yield a constant value for the mass of light, in the same way that the proton and electron have constant mass. De Broglie’s photon mass was an infinite parameter and the existence of a conserved energy-mass equivalence for light could not be shown using classical mechanics.

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† Emmy Noether<sup>16</sup>

Just as Noether resorted to using momentum in her energy-momentum pseudotensors, De Broglie likewise calculated the momentum of photons, reasoning that if “ $h\nu$ ” could be set equal to “ $mc^2$ ”, then “ $h\nu/c$ ” could be set equal to “ $mc$ ” (i.e., photon momentum, “ $p$ ”). This yielded, of course, another set of infinitely variable values, this time for the momentum of light,

$$\rho = h / \lambda \quad \text{or} \quad \rho = h\nu / c \quad (4)$$

where “ $\lambda$ ” is wavelength of the light. De Broglie was no closer to an “*isolated quantity*” of anything. None-the-less, the concept of momentum for light proved to be very popular. Photon momentum could seemingly be determined without knowing the mass of light, or even more strangely without light having any mass at all.

As concepts surrounding light’s properties continued to evolve, the idea of the “mass-less” photon took hold. The constancy of light’s speed over a wide range of frequencies persuaded many to conclude that the “photon” is mass-less.<sup>20</sup> QED theory, which requires a zero-mass photon, also led to widespread acceptance of the concept of a mass-less photon. According to QED advocates, the gauge invariance of QED and Maxwell’s laws would be broken if the photon were found to have mass, and it must thus be a mass-less entity. Never-the-less, significant and repeated efforts have been made to experimentally determine whether or not photons have mass. No definitive photon mass has been detected yet and so, in case the mass is so small it simply cannot be measured with current technology, efforts have been focused on determining the upper limits of photon mass instead.

## 2. NEW DISCOVERIES IN QUANTUM MECHANICS

*“The possibility of a finite photon rest mass remains one of the most important issues in physics.”<sup>20</sup>*

The new discoveries in quantum mechanics suggest that the energy, mass and momentum of light are finite scalar values that are represented by universal constants. These discoveries have their origins in the quantum concepts and mathematical formulations of Max Planck’s famous paper on black-body radiation published in early 1901.<sup>11</sup> Planck had been pursuing two parallel lines of research in tandem: theoretical work on “resonant Hertzian radiation” (electromagnetic waves)<sup>26</sup> and theoretical work on black-body (thermal) radiation. In late 1900 Planck combined those two (2) lines of research into his derivation of the long sought after black-body equation, which described the thermal electromagnetic radiation emitted by an object based solely on its temperature. He borrowed some statistical methods from Boltzmann and the energy density method of Wien, while his quantum formula, “ $E = h\nu$ ”, was simply assumed as a mathematical given. This assumed formula was actually an abbreviated version of a mathematical relationship Planck used earlier in his electromagnetic theory,<sup>21</sup> namely:

$$\delta E \approx a \delta t_m \nu \quad (5)$$

where “ $a$ ” was a generic constant and “ $t_m$ ” was the measurement time for the electromagnetic (“EM”) radiation. Comparison of the two formulae, “ $E = at_m\nu$ ” versus “ $E = h\nu$ ” suggests that his later constant “ $h$ ” was actually the product of a generic constant “ $a$ ” and the measurement time “ $t_m$ ”. Investigation of the mathematical procedures used in Planck’s black-body derivation confirms that indeed, Planck’s famous action constant “ $h$ ”, is really the product of a time variable and another constant.<sup>‡</sup>

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<sup>‡</sup> Reproduced from Chapter 1., The Fundamental Physics of Electromagnetic Waves, in *Electromagnetic Waves*:<sup>22</sup>

“Proof of these facts are found in Planck’s 1901 blackbody paper, in which he described the experimental data and mathematical methods he used:

*“§11. The values of both universal constants  $h$  and  $k$  may be calculated rather precisely with the aid of available measurement. F. Kurlbaum, designating the total energy radiating into air from 1 sq cm of a black body at temperature  $t^\circ$  C in 1 sec, by  $S_t$  found that:*

$$S_{100} - S_0 = 0.0731 \text{ watt/cm}^2 = 7.31 \times 10^5 \text{ erg/cm}^2 \text{ sec} \quad (\text{Underlines added})$$

Instead of multiplying Kurlbaum’s time-based power measurement by the measurement time to obtain total energy (as Planck had done in his earlier work), he converted the power measurement to energy density by dividing by the speed of light “ $c$ ” ( $3 \times 10^{10}$  cm/sec), according to Wien’s method:

(Continued at bottom of next page .....)

As to the identity of that constant, various mathematical methods and dimensional analyses have all lead to the same un-refuted conclusion – the previously hidden constant is an **energy** constant.<sup>1-7, 22</sup> To be precise, balancing the units reveals that the energy constant is the energy of a single wave or oscillation of EM radiation, i.e.,  $6.626 \times 10^{-34}$  J/osc.<sup>§</sup> Likewise, the measurement time variable was fixed at a value of one (1) second and simultaneously hidden, all because of Planck’s energy density calculation. So when Planck arrived at the point in his derivation where he calculated his constant “h”, it was inadvertently calculated as the **product** of an energy constant, which will be denoted as “ $\tilde{h}$ ”, and the hidden and fixed time variable “ $t_m$ ”:

$$h = \tilde{h} t_m, \quad \text{where } t_m = 1 \text{ second}, \quad (6)$$

hence 
$$h = 6.626 \times 10^{-34} \text{ J sec} = (6.626 \times 10^{-34} \text{ J}) (1 \text{ sec}). \quad (7)$$

Unwinding the energy constant and time variable from the action “constant” product provides the complete quantum formula:

$$E = \tilde{h} t_m \nu \quad (8)$$

Analysis of the oscillation energy constant over time and space (e.g., frequency and wavelength) demonstrates that the energy of a single wave (oscillation) of electromagnetic energy is constant regardless of its frequency or wavelength. The energy of a single oscillation of light is a property of light which is constant over a shift in time or space. Oscillation energy is thus a conserved parameter of light, and the energy constant is universal.<sup>30</sup>

The discovery of the hidden time variable and energy constant has many implications for practical applications of quantum mechanics. Chief among these are answering the ever practical question – What are photons? Einstein and others based their quantum “photon” concepts on the energy calculated by Planck’s abbreviated (and incomplete) quantum formula. They were unaware that the action “constant” was really the product of light’s energy constant and a fixed, hidden time variable.<sup>12, 24</sup> Our quantum forefathers did not realize that if one changes the measurement time to half a second, the “photon” energy is halved. Likewise, if one doubles the measurement time to two (2) seconds, the

(Continued from preceding page ...)

*“From this one can obtain the energy density of the total radiation energy in air at the absolute temperature*

$$\frac{4 \cdot 7.31 \times 10^5}{3 \times 10^{10} (373^4 - 273^4)} = 7.061 \times 10^{-15} \text{ erg/cm}^3 \text{ deg}^4$$

The time variables in the numerator and denominator cancelled out and Planck was seemingly able to address energy independent of time. Dividing by the constant speed of light however, is the same as multiplying by time:

$$\frac{E/t \text{ s}^2}{c} = \frac{E}{t \text{ s}^2} \times \frac{t}{s} = \frac{E}{\text{s}^3} \quad 12.$$

where “s” is distance. In this case the time value by which the power measurement was multiplied was the constant “one second” unit time of the constant speed of light. Planck seems to have been unaware that by using Wien’s energy density calculation he was actually causing the infinitely variable measurement time to be fixed at a constant value of one second. He also seems to have been unaware that the fixed time variable was subsequently hidden in the final calculations of his action constant “h”:

$$h = 6.626 \times 10^{-34} \text{ Joules seconds} \quad 13.$$

His action constant is actually the **product** of a true universal constant - “ $\tilde{h}$ ” - and the fixed, hidden measurement time variable, “ $t_m$ ”.

$$h = \tilde{h} t_m, \quad \text{where } t_m = 1 \text{ second} \quad 14.”$$

<sup>§</sup> Notably, the incomplete mathematical notation adopted by Planck - which omitted units for oscillations - is no longer necessary and the energy constant is given in proper and complete mathematical notation as energy per oscillation. See reference No. 22 for further discussion of units and dimensionality.

“photon” energy doubles. To paraphrase Gilbert Lewis who coined the term “photon”, “*The energy of an isolated photon, divided by the Planck constant, gives the frequency of photons [in a fixed one second measurement time]*.”<sup>24</sup>

An energy quantity such as “photon” energy, which depends on an arbitrarily fixed time period, cannot represent the fundamental particle of light. Such a construct produces a zoo of particles, each with differing energies. This is in stark contrast to other universal parameters such as the charge of the electron or the mass of the proton. This variability in “photon” energy and the lack of an “*isolated quantity of energy*” was the very problem that plagued quantum mechanics.<sup>18,19</sup> The same problem plagued general relativity and prevented it from displaying “proper” energy conservation. There was no finite and scalar amount of energy which could be conserved across time and space. The recently discovered energy of a single oscillation of light,  $6.626 \times 10^{-34}$  J/osc, is just such an “*isolated quantity*” of energy. It is the fundamental and elementary particle of light. The photon is not the elementary particle of light, but rather is a *collection* of sub-photonic individual light oscillations.

### 3. THE CONSTANTS FOR LIGHT

Two (2) universal constants related to light, the speed of light and the energy constant for light, have been discussed thus far. Together, these constants allow the calculation of two (2) more constants for light: constants for the invariant mass and invariant momentum of light. Further, classical Lagrangian mechanics can be used to derive and calculate light’s variable force and invariant energy constant.

#### 3.1 Calculation of the Mass Constant for Light

Using de Broglie’s equation for the mass of light, the frequency “ $\nu$ ” is set equal to one (1) oscillation per second. One obtains for the mass of a single oscillation of light,

$$m_0 = h \nu / c^2, \text{ where } \nu = 1 \text{ osc/sec} \quad [5]$$

$$m_0 = 7.372 \times 10^{-51} \text{ kg/osc} \quad [6]$$

Using the complete quantum formula, the constant mass of a single oscillation is given by “ $m_0 = \hbar t_m \nu / c^2$ ”, using  $\nu = 1$  osc/sec and  $t_m = 1$  sec. This yields the same constant value for oscillation mass as above. To test the hypothesis of invariance of oscillation mass over a change in time, the frequency “ $\nu$ ” is set equal to a value other than one, and the measurement time is set equal to the time period for the oscillation (to determine the mass of only one oscillation at that frequency). For example, where  $\nu = 5$  osc/sec, then  $t_m = 0.2$  sec. One obtains the same constant value for the mass of a single oscillation of light,  $m_0 = 7.372 \times 10^{-51}$  kg/osc, regardless of frequency. The same holds true for a change in wavelength.

Light’s oscillation mass is invariant and conserved over a change in time (frequency) or space (wavelength).

#### 3.2 Calculation of the Momentum Constant for Light

Using de Broglie’s equation for the momentum of light, and setting the frequency “ $\nu$ ” to one (1) oscillation per second, one obtains for the momentum of a single oscillation of light,

$$\rho = m_0 c = h \nu / c, \text{ where } \nu = 1 \text{ osc/sec} \quad [7]$$

$$\rho = 2.21 \times 10^{-42} \text{ kg m / sec osc} \quad [8]$$

To test the hypothesis of invariance of momentum over a change in time, one proceeds as above using the complete quantum formula, and obtains the same constant value for the momentum of a single oscillation of light,  $2.21 \times 10^{-42}$  kg m / sec osc, regardless of frequency or wavelength. Light’s oscillation momentum is invariant and conserved over a change in time (frequency) or space (wavelength).

#### 3.3 Derivation of the Variable Force Exerted by Light

Force is the product of mass and the change in velocity “*during the instant dt*” when the velocity changes, as Louis Lagrange explained in his famous book on analytical mechanics,<sup>26</sup>

$$F = m \nu / dt \quad [9]$$

where “ $\nu$ ” is velocity. Lagrange was careful to explain that his work contained an important assumption, namely that the unit time for velocity and the time interval in which an acceleration took place were identical, “*assuming that for each*

*accelerating force ...the basic unit of time...[is] the same time if it moved uniformly*". All accelerations described in Lagrange's book thus took place in time intervals at least as large as the velocity unit time of one second, allowing him to formulate acceleration using the square of the single time variable "t", i.e., "t<sup>2</sup>". Lagrange never intended his mechanical equations describe "instantaneous" accelerations or decelerations, unless modified to include a 2<sup>nd</sup> time variable for "the instant dt" when the acceleration takes place. Instantaneous accelerations, according to Lagrange, must be accounted for using modifications to the simple mechanical framework he laid out.<sup>6</sup>

Abiding by Lagrange's caveats, the variable force exerted by light can be derived using classical Lagrangian mechanics. Consider what occurs when a single oscillation of light encounters a massive body and is absorbed. The forward translational movement of the light oscillation halts and the oscillation experiences an "instantaneous" change in its velocity vector, as its energy and mass is added to the energy and mass of the absorbing body. The "instant dt" during which this velocity change takes place is the time period of the oscillation, during which the oscillation's energy is absorbed. Hence the force of a single oscillation of light is written:

$$F_0 = m_0 c / \tau \quad [10]$$

using "c" for velocity and time period "τ" for "dt". This can also be written using frequency since time period and frequency are reciprocals (i.e., 1/τ = ν):

$$F_0 = m_0 c \nu \quad [11]$$

As noted above, frequency and wavelength are infinitely variable, and hence the force exerted by oscillations of different frequencies or wavelengths is variable.\*\* The force of light is not constant. The shorter the time period or wavelength over which the mass of a single oscillation of light is absorbed or emitted, the greater the acceleration/deceleration and hence force that the light oscillation mass exerts on an absorbing or emitting object.

### 3.4 Derivation of Energy Constant for Light

According to Lagrange, the energy associated with an accelerating (or decelerating) entity is the product of the force exerted over the distance "s",

$$E = F s \quad [12]$$

Using the wavelength of an oscillation of light as the distance over which its associated force is exerted,

$$E_0 = F_0 \lambda \quad [13]$$

Written using frequency this becomes:

$$E_0 = m_0 c \nu \lambda \quad [14]$$

The product of frequency and wavelength, "ν λ", is the constant speed of light, "c". Thus

$$E_0 = m_0 c^2 \quad [15]$$

And,

$$m_0 c^2 = 6.626 \times 10^{-34} \text{ J/osc} \quad [16]$$

yields the energy constant for light. The energy of a single oscillation of light is constant and invariant over a change in time or space.

### 3.5 General Oscillation Conservation

Light's constants for energy, mass and momentum can be combined into a single law based on the number, "N", of light oscillations absorbed or emitted by a body with mass,

$$\Delta E = N \tilde{h} = N m_0 c^2 = N p_0 c \quad [17]$$

The change in energy, mass and momentum is proportional to the number of oscillations absorbed or emitted.

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\*\* Substituting "λ / c" for time interval "τ" reduces to F = mc<sup>2</sup> / λ.



## 4. DISCUSSION

*“The most beautiful discoveries since the Renaissance, indeed since the beginnings of all science, are the laws governing light ... these laws are the basis of an admirable science ... A science that extends our vision into the furthest reaches of space and into the smallest parts of matter, allowing us to discover many things that had been concealed”. Maupertuis, 1744.<sup>27</sup>*

Recent discoveries in the foundations of quantum mechanics suggest a startling interpretation quite different from that adopted over the last century. If correct, this interpretation may significantly simplify current understandings of the quantum world and allow a more realistic approach to the laws of nature.

### 4.1 Light’s Action versus Energy

Planck formulated his proportionality value “h” as an action constant, however recent discoveries suggest that dimensionally, the value he calculated was an *energy* constant. The arithmetic in Planck’s black-body derivation is clear. By dividing physical time-based power measurements by the speed of light, Planck multiplied power by a constant fixed value of one (1) second. This maneuver fixed the measurement time at a value of one (1) second, and it subsequently reappeared, embedded in Planck’s action constant “h”,  $6.626 \times 10^{-34}$  J sec. Deconstructed, Planck’s action constant is the product of the energy constant for a single oscillation of light, and the fixed, one (1) second measurement time.

This case of mistaken identity had far reaching implications for the core foundational quantum concepts which were adopted by subsequent generations of scientists. Planck cherished the Principle of Least Action (natural motions proceed along the path of shortest distance, briefest time, and minimal energy) and referred to his constant as the ultimate “*quantum of action*”. His action constant was the ideal fit with Sir William Hamilton’s *quantity of action* equation,<sup>28</sup>

$$S = \int (K-U) \delta t \tag{17}$$

where “S” is action, “K” is kinetic energy, and “U” is potential energy. In the Hamilton formulation, energy was variable, and distance and time were treated as fixed (Table 1, below). Likewise, in the Planck formulation, photon energy was variable and the distance per time traveled by photons was constant (speed of light, “c”).

Parameter	Hamilton Action	Planck Photon	Oscillation Theory
<b>Time</b>	Constant	Constant	Variable & Symmetric
<b>Distance</b>	Constant	Constant	Variable & Symmetric
<b>Energy</b>	Variable	Variable	Constant

Table 1. Comparison of physical parameters for light under the Hamilton action construct, the Planck photon model, and the Oscillation Energy interpretation. Under the Hamiltonian, photon energy is variable, while the time and distance through which photons travel is constant. Using the recently identified oscillation energy constant, the contrary set of parameters is obtained. The energy for a single oscillation of light is constant, and although the time periods and wavelengths of light oscillations are infinitely variable, they vary symmetrically pursuant to “c” the coupling constant for light between time and space.

Planck’s constant “h” was deemed Hamilton’s minimum action for matter and light, and thus all *action* of light or matter was thought to take place in integer multiples of Planck’s constant. Heisenberg used Hamiltonian mechanics in his matrices,<sup>29</sup> and Schrödinger used Hamiltonian mechanics in his wave equations. Use of Hamilton’s equation created a problem, however, because conservation of energy was not required under Hamilton’s principle. As a result, Noether’s complicated new energy transformations using energy-momentum pseudotensors and integrating over large, asymptotically flat spacelike hypersurfaces became necessary. Under Feynman and QED, light came to be viewed as

following all possible paths simultaneously based on action-determined probability amplitudes, instead of light following a single path whose action was minimized as in classical mechanics.

Had correct dimensionality for the energy constant been recognized from the beginning, the lack of congruity between oscillation parameters and the Hamilton equation would have been apparent. (Table 1., above) Time-based “photons” would not have been mistaken as elementary particles of light. The Hamiltonian action equation would not have been used so extensively, if at all. Heisenberg would not have encountered uncertainties and the inability to reconcile his matrices with gravitational concepts. Likewise, Schrödinger would not have found that his wave equations produced a plethora of values for energy not specified by Planck’s quantum formula (reflecting variable measurement times).<sup>30</sup> He would not have found it necessary to introduce a normalization factor of “one” to limit energy values his equation gave to only those provided by Planck’s quantum formula. Conservation of energy based on an invariant energy constant would have been “*proper*” from the beginning, and Einstein would not have encountered “*improper*” conservation constructs in his development of general relativity. De Broglie would have succeeded in calculating the mass and momentum constants for light. Finally, the suggestion that light quanta travel all different physical paths simultaneously would never have arisen.<sup>31</sup>

## 4.2 Conservation of Energy

The energy of light is conserved, as suggested so long ago by Mohr, Grove and Helmholtz. The principle of energy conservation for a single oscillation of light follows from the invariance of light’s energy constant and its symmetry under time translations. The energy constant represents a finite continuous symmetry group and its conservation is proper in a Galilean framework. Truly, *Light, Heat, Electricity, Magnetism, Motion, and Chemical affinity, are all convertible ... manifestations of energy*”.

## 4.3 Symmetry and Conservation of Energy and Time-Space for Light

Noether's theorem indicates that for every continuous symmetry in a physical situation there corresponds a conservation law. The constant oscillation energy of EM radiation demonstrates not only conservation of light’s energy in a proper sense, but also the continuously constant *symmetry* of both time and space for light (Table 1., above). The constant energy of an EM oscillation occupies variable times and spaces, i.e., variable time periods and wavelengths. The time period and wavelength of a light oscillation do not vary randomly or asymmetrically, however. The variations in time and space are symmetrical. As the time occupied by an oscillation energy quantum increases, so too does its space. If the time period decreases, the wavelength decreases as well. These changes in space that accompany a change in time are constantly and continuously symmetric for a light oscillation. Space and time are coupled for light. This symmetric coupling of time and space for a light oscillation is given by the time-space coupling constant, “*c*” (also known as the “speed of light”).

Thus, not only is the energy of a light oscillation conserved over changes in time or space, light’s relationship between time and space *themselves* is constant and continuously symmetric as well. The existence of this additional continuous symmetry, and the coupling of time and space for oscillation energy, suggests an extension of the energy conservation law for light. **The energy and time-space of light are both conserved properties.**

When a property exhibits constant and continuous symmetry in spite of changes in time or space, the property is said to be symmetrically scaled in a self-similar manner. Light oscillations are thus symmetrically scaled in time and space, in a self-similar manner. In other words, an oscillation of light exhibits energy/time-space scale invariance.<sup>††</sup> No matter what the symmetric time-space scale, the energy of a light oscillation is the same. Symmetrical scaling and invariance in a self-similar manner are properties of fractals. Light oscillations exhibit fractal properties, and light is absorbed and emitted by matter. These fractal-like properties<sup>‡‡</sup> for light and matter may form the basis for many of the fractal shapes and structures seen in nature (Figure 3., below). In other words, fractals may be exhibited at the macroscale,<sup>32</sup> because light and matter themselves possess fractal structure at the microscale.

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<sup>††</sup> Scale invariance is not obtained in the “photon” interpretation.

<sup>‡‡</sup> Light’s time-space symmetric single oscillation appears to fit the criteria for a fractal, having a Hausdorff dimension of 4 (one time and 3 space dimensions), with a topology of 1 (one invariant energy).

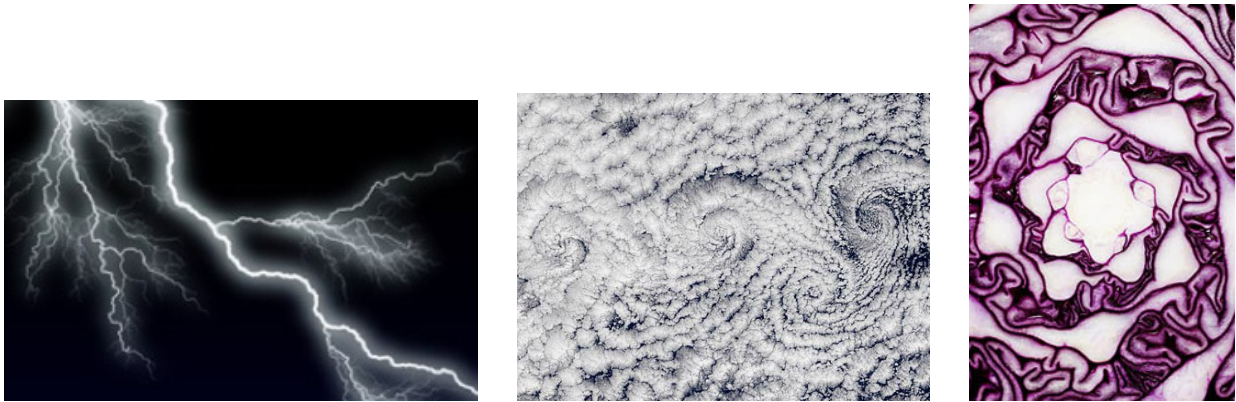


Figure 3. Examples of Fractal shapes and structures in nature: from left to right lightening, clouds, and cabbage.

#### 4.4 Conservation of Light's Mass

The prevailing belief in the scientific community is that light possesses no mass, in spite of possessing energy and momentum, both of which require mass in a classical context. This belief has been bolstered by experimental limitations and theoretical considerations. Experimentally, attempts to determine the mass of a “photon” have not been possible technologically. Thus, efforts have focused instead on trying to identify at least the upper limits of possible photon mass.

Theoretical considerations are discussed in a recent review on the mass of the photon, a “*basic implication of Maxwell's [and Lorentz's] electromagnetism is the constant speed, in vacuum, of all electromagnetic radiation. Experimental studies have indeed confirmed to a high degree of accuracy that all electromagnetic radiation travels at the speed of light,  $c$ , over a wide range of frequencies. In turn, this implies that the quantum of light, or photon, appears to be massless. ... The photon conveys energy and momentum through space-time and propagates in vacuum at the constant velocity  $c$ , independent of the frame of reference, as per the second postulate of Einstein's theory of special relativity. A corollary of this is that a particle with finite mass can never attain the speed of light,  $c$ , or in other words, such a particle cannot exist in the frame of rest of a photon. ... The enormous successes of [Feynman's] quantum electrodynamics (QED) have led to an almost total acceptance of this concept of the massless photon. ... The fact that light could not be brought to a stand-still made this point of view reasonable and it is theoretically difficult to find any kind of contradictory counter-example.*”<sup>20</sup> Thus, theoretical considerations related to the work of Maxwell, Lorentz, Einstein and Feynman convinced many that light is mass-less, and QED theory and its requirement of gauge invariance led to almost total acceptance of the mass-less photon.

Light's oscillation mass has been calculated, however. The mass of a single oscillation of light is  $7.372 \times 10^{-51}$  kg/osc. This value is in close agreement and within the same order of magnitude as calculations by *Luo et al* for the upper limit of “photon” mass.<sup>25§§</sup> Clearly, it is no longer theoretically difficult to find a contradictory counter-example. The work of Maxwell, Lorentz, Einstein, and Feynman was performed without knowledge of various constants and variables relevant to the emission and absorbance of light, and theories must needs be amended when new information comes to light.

One of the major objections to the idea of light possessing mass is the gauge invariance of QED theory. QED formulations are based on artifactual “photon” concepts however, and do not take the elementary properties of light oscillations into consideration. Under the QED construct, if “photons” have mass, the mass would be infinitely variable, as shown by de Broglie. This infinite variability would indeed break gauge invariance. The “photon” is not the elementary particle of light however. It is, in reality, a time-based collection of sub-photonic elementary particles, oscillations of light.

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§§ When *Luo et al's* upper limit of “photon” mass is converted to mass per oscillation, an upper limit of  $4.32 \times 10^{-51}$  kg/osc is obtained, well within the same order of magnitude as the constant for invariant mass of a single light oscillation.

Oscillation properties differ significantly from “photon” properties. The energy and mass of “photons” are construed as being infinitely variable, while the energy and mass of light’s oscillations are constant and invariant. Thus it is not necessary that non-zero mass for light results in gauge invariance. To the contrary, the constant energy and mass of light’s oscillations demands and supports gauge invariance. Gauge invariance is not broken by virtue of light possessing mass.

The mass associated with a single EM oscillation does not vary over changes in time or space, and is in fact continuously constant and symmetrical with changes in time and space. Hence, the mass of light is a gauge invariant and conserved property. **The energy, time-space and mass of light are all conserved properties.**

#### 4.5 Light’s Mass and General Relativity

Einstein’s general relativity concepts related to the relativity of accelerations in different frames of reference. The acceleration of massive bodies due to the gravitational influences of other massive bodies thus played a large part in his theoretical framework. Einstein similarly considered the effects of gravity on light, “... [A]ccording to the general theory of relativity, a ray of light will experience a curvature of its path when passing through a gravitational field, this curvature being similar to that experienced by the path of a body which is projected through a gravitational field. As a result of this theory, we should expect that a ray of light which is passing close to a heavenly body [such as the sun] would be deviated towards the [sun]”.<sup>15</sup>

Because Einstein could not effectively attribute mass to photons, he was unable to model the effects of gravitational fields on light in a completely classical manner. Instead, he accounted for half of the light deviation in a gravitational field based on the classical gravitational forces of a “heavenly body” such as the sun. For the other half, he developed the concept of curved space-time, “half of this deflection is produced by the Newtonian field of attraction of the sun, and the other half by the geometrical modification (“curvature”) of space caused by the sun.”

With a mass constant for light oscillations, it may no longer be necessary to model curvature of space to explain the bending or deflection of light by large massive objects. Any such deflection can potentially be explained by the Newtonian field of attraction between the mass of both the sun and the light oscillations. Space may indeed be curved, however its curvature may no longer be necessary to explain the gravitational effects of massive objects on light.

A Newtonian field of attraction between the mass of the sun and the mass of individual light oscillations is also quite consistent with Einstein’s predicted red-shift of light escaping the surface of a large massive body such as the sun, and its “displacement towards the red ... for spectral lines produced at the surface of stars as compared with the spectral lines of the same element produced at the surface of the earth”. This gravitational red-shift may be explained on the basis of the gravitational attraction between the mass of the sun and the mass of the light oscillation, causing a physical lengthening of the oscillation in space and time, as it escapes the gravitational field of the sun.

#### 4.6 Conservation of Light’s Momentum

The momentum of light oscillations traveling at constant velocity “c” *in vacuo*, is  $2.21 \times 10^{-42}$  kg m / sec osc, regardless of their frequency or wavelength. Using Planck’s incomplete quantum formula and bereft of the “isolated quantity of energy” (light’s energy constant), de Broglie calculated the momentum of light based not on constant properties of light, but instead on the variable wavelength and frequency associated with “photons”. Hence he arrived at his momentum formula, “ $p = h / \lambda = h \nu / c$ ”, which yielded an infinitely continuous range of values for photon momentum.

The additive nature of momenta is a well recognized concept from classical mechanics. The “photon” is a time-based collection of sub-photon elementary particles, whose frequency provides the oscillation number “N” as shown in equation No. 17., (above) for general oscillation conservation. Thus, de Broglie was simply adding together all of the individual momenta from individual oscillations in a “photon”, based on the number of oscillations in a one second measurement interval, as given by the frequency, “N osc/sec”. The greater the number of “N” oscillations per second denoted by frequency, the greater the number of constant oscillation momenta de Broglie summed to obtain his variable photon momentum values.

The constant momentum possessed by a light oscillation of constant mass traveling at the constant speed of light, is invariant. The momentum of a light oscillation is independent of the frame of reference, i.e., its center of mass travels at the same velocity *in vacuo*, regardless of reference frame, wavelength or frequency. The momentum of light is thus a conserved property under standard tensor formats, and consideration of its conservation does not require the use of pseudotensors.

The principle of momentum conservation for a single oscillation of light follows from the invariance of light's momentum constant and its symmetry under time and space translations. The momentum constant represents a finite continuous symmetry group and its conservation is proper in a Galilean framework. **The energy, mass, momentum, and time-space of light are all conserved properties.**

#### 4.7 One Conservation Law

The most important result of a general character to which the oscillation theory has led is concerned with the conception of light. Before the advent of oscillation theory, quantum physics recognized the fundamental conservation of mass for ponderable bodies, but not for light. By means of the oscillation theory it is now possible to form a law of conservation of mass for light, and to unite the laws of conservation of energy, mass and momentum into one law for both light and matter,

*When a body with mass absorbs or emits light, its energy, mass and momentum change in quantized amounts according to the relationship:*

$$\Delta E = N\hbar = Nm_0c^2 = N\rho_0c$$

*where "N" is the number of oscillations absorbed or emitted by the body and  $\hbar$ ,  $m_0$ , and  $\rho_0$  are the constant energy, mass and momentum of an oscillation..*

#### 4.8 The Variable Force of Light

Although the energy, mass, and momentum of light's oscillations are constant, in stark contradiction to previous quantum concepts about light, the old quantum mechanics and oscillation theory are in solid agreement on one point: the **force** exerted by light is variable, depending on light's wavelength and frequency (Table 2., below).

PROPERTY	PHOTON MODEL	OSCILLATION THEORY
<b>Energy</b>	variable	constant
<b>Mass</b>	variable	constant
<b>Momentum</b>	variable	constant
<b>Force</b>	variable	variable

Table 2. Comparison of physical parameters for light under the photon model and oscillation theory. Under the photon model energy, mass, momentum and force are all variable. Using the recently discovered time variable and energy constant, a quite different set of parameters is obtained. The energy, mass and momentum of a light oscillation are constant, and only the force of an oscillation is variable.

Einstein discussed both the momentum and force of light in his 1916 paper on the quantum theory of EM radiation. *"When a body emits an energy E, it has a **recoil (momentum)** ...If a radiation bundle in a given direction does work on a [molecule], the corresponding energy is removed from the radiation bundle [and] ...there also corresponds a **momentum** transfer from the radiation bundle to resonator...The resonator is thus acted upon by a **force**... If the energy transfer is negative, then the **force** acts on the resonator in the opposite direction. If the quantum hypothesis holds... a **momentum**  $(E_m - E_n)/c$  is transferred to the molecule ...we can easily calculate the average **momentum** transferred to the molecule **per unit time i.e., [force]**."* Under Einstein's concepts, the force exerted by or upon light was variable because photon momentum was variable.

The momentum of a light oscillation is constant, which might lead one to suppose that its force would be constant also. Application of Lagrange's mechanical principles show that this is not the case, however. The *"instant dt"* (the oscillation's time period) during which the oscillation accelerates or decelerates by virtue of being emitted or absorbed, respectively, is variable. This variability in the oscillation's instant of acceleration is what provides for variability in the force associated with light. The constancy of light's momentum does not alter the variability of light's force.

The force exerted by light both upon its absorption and emission have been demonstrated experimentally. She et al<sup>34</sup> showed in 2008 that light emitted by a body produces a recoil as predicted by Max Abraham<sup>35</sup> and Einstein. A burst of light leaving the free end face of a nanometer silica filament exerted an "inward push force", or recoil, perpendicular to the end face of the filament. This push force causes the filament to recoil in response.

More recently, Lin et al demonstrated not only the force exerted by light, but also its dependence on the resonant absorption and deceleration of light.<sup>36</sup> A nanoscale structure with differing resonant plasmon frequencies in different portions of the structure, was illuminated with light of various frequencies. When resonant light was absorbed on one set of the structure's surfaces, the force exerted by the absorbed light caused the structure to rotate in one direction. When resonant light of a different frequency was absorbed on surfaces which produced the opposite torque on the spinning motor, the force of the light caused the nanomotor to rotate in the opposite direction. The variable acceleration or deceleration experienced by a light oscillation upon its emission or absorption, respectively, determines the force exerted by an oscillation of light.

## CONCLUSION

The initial discoveries in the foundations of quantum mechanics presented in 2009 have undergone additional development. The identification of light's energy constant and the measurement time variable, which had been hidden in Planck's action constant "h", has allowed the determination of additional constants for light. Energy, mass and momentum are all constant and conserved for an oscillation of light. Not only are these properties constant across shifts and transformations in time and space, time and space themselves are symmetric and self-similar for these properties. The light oscillation – light's elementary particle – appears to be a fractal structure. This fractal structure inherent to EM energy may be responsible for the fractal structures seen in nature. The new constants and variables suggest a more realistic interpretation of quantum mechanics in which energy, mass and momentum are all quantized.

By means of the oscillation theory it is now possible to unite the energy, mass and momentum laws of conservation for matter and light into one conservation law,

*When a body with mass absorbs or emits light, its energy, mass and momentum change in quantized amounts according to the relationship:*

$$\Delta E = N\tilde{h} = Nm_0c^2 = N\rho_0c$$

*where "N" is the number of oscillations absorbed or emitted by the body and the energy, mass and momentum of an oscillation are represented by  $\tilde{h}$ ,  $m_0$ , and  $\rho_0$ .*

The implications of these new theoretical discoveries are highly significant for optical applications ranging from nanotechnology to space sails to quantum computing.

## REFERENCES

- [1] Brooks (Mortenson), J. "Hidden Variables: The Elementary Quantum of Light", Proceedings of SPIE, The Nature of Light: What are Photons? III, 7421, 74210T-3 (2009).
- [2] Brooks (Mortenson), J., "Hidden Variables: The Resonance Factor", Proc. of SPIE, The Nature of Light: What are Photons? III, 2009, 7421, 74210C (2009).
- [3] Brooks (Mortenson), J., "Advance in the Foundations of Quantum Mechanics", Bull. Am. Phys. Soc., 55(1), <http://meetings.aps.org/link/BAPS.2010.APR.X14.9> (2010).
- [4] Brooks (Mortenson), J., "Planck's Energy Constant", Bulletin of the American Physical Society, 55 (1), <http://meetings.aps.org/link/BAPS.2010.APR.R1.42> (2010).
- [5] Brooks (Mortenson), J., "De Broglie's Rest Mass of Light", Bulletin of the American Physical Society, 55 (1), <http://meetings.aps.org/link/BAPS.2010.APR.R1.34> (2010).
- [6] Mortenson, J., "Double Your Kinetic Energy: Disappearance of the Classical Limit in Quantized Classical Energy Mechanics," Proceedings of Materials Science & Technology 2010, 2907 - 2918 (2010).
- [7] (Brooks) Mortenson, J., "Quantum Treatment in GHz-THz Electromagnetic Waves", Invited Talk, Japan Physical Society Autumn Meeting, Osaka, Japan (2010).
- [8] Mohr, K. F., "Über die Natur der Wärme, Zeitschrift für Physik", Mathematik und verwandte Wissenschaften", 5, 419 (1837).
- [9] Grove, W., [Correlation of Physical Forces], Spottiswoode & Co., London (1874).

- [10] Helmholtz, H., [Ostwald's Kalssiker der Exacten wissenschaften]. "Über die Erhaltung der Kraft", Verlag von Wilhem Engelmann, Leipzig (1889).
- [11] Planck, M., "On the Law of Distribution of Energy in the Normal Spectrum", *Annalen der Physik*, 4, 553 (1901).
- [12] Einstein, A., "On a Heuristic Point of View Concerning the Production and Transformation of Light", *Annalen der Physik*, 17, 132-148 (1905).
- [13] Millikan, R. A., "A Direct Photoelectric Determination of Planck's 'h' ", *Phys Rev*, 7(3), 355 – 388 (1916).
- [14] Compton, A., "Quantum Theory of the Scattering of X-rays by Light Elements", *The Physical Review*, 21(5), 483-502 (1923).
- [15] Einstein, A., [Relativity: The Special and General Theory], Methuen & Co Ltd, London (1920).
- [16] Byers, N., E. "Noether's Discovery of the Deep Connection Between Symmetries and Conservation Laws", *Israel Mathematical Conference Proceedings*, 12, (1999)
- [17] Noether, E., "Invariant Variation Problems", *Nachr. d. König. Gesellsch. d. Wiss. zu Göttingen, Math-phys. Klasse*, 235–257 (1918).
- [18] De Broglie, L., "Researches on the theory of quanta", Thesis, Paris, France (1924).
- [19] De Broglie, L., "The Wave Nature of the Electron", Nobel Lecture, Stockholm, Sweden, (1929).
- [20] Tu, L. C., et al, "The mass of the photon", *Rep. Prog. Phys.* 68, 77-130 (2005).
- [21] Planck, M., "Ueber irreversible Strahlungsvorgänge", *Ann. Phys.*, 306(1), 69-122 (1900).
- [22] Mortenson, J., [Electromagnetic Waves], InTech, Croatia, 3-26 (2011).
- [23] Brooks (Mortenson), J., "Einstein's Hidden Variables: Part A & Part B", *Proceedings of Materials Science & Technology 2009*, 573 – 597, (2009).
- [24] Lewis, G., "The Conservation of Photons", *Nature*, 118(2), 874-875 (1926).
- [25] Luo, J., et al, "New Experimental Limit on the Photon Rest Mass with a Rotating Torsion Balance", *Physical Review Letters*, 90(8), 81801 (2003).
- [26] Lagrange, J. L., [Mécanique Analytique, nouvelle édition d' 1811], Trans. Boissonnade A. and Vagliente, V. N., Kluwer Academic Publishers, Boston (1997).
- [27] Maupertuis, P.L., "Accord Between Different Laws of Nature that Seemed Incompatiable." *Ouevres de Maupertuis*, Acts of the Royal Academy of Sciences of Paris, Translated by Willow, W., (1744).
- [28] Hamilton, W. R., "On a General Method of Expressing the Paths of Light, and of the Planets, by the Coefficients of a Characteristic Function", *Dublin University Review and Quarterly Magazine*, 1, 795-826, (1833).
- [29] Heisenberg, W., "Ueber den anschaulichen Inhalt der quantentheoretischen Kinematik and Mechanik", *Zeitschrift für Physik*, 43, 172-198 (1927).
- [30] Schrödinger, E., [Collected Papers on Wave Mechanics], "Quantization as a Problem of Proper Values (Part I)", Chelsea Publ. Co., New York, 1-12 (1982).
- [31] Feynman, R., [QED. The Strange Theory of Light and Matter], Princeton Univ. Press, Princeton (1988).
- [32] Mandelbrot, B.B., [The Fractal Geometry Of Nature], WH Freeman, San Francisco (1982).
- [33] Einstein, A., "Zur Quantentheorie der Strahlung", *Mitteilungen Der Physikalishcen Ges. Z*, 18, {English translation in *Sources of Quantum Mechanics*, Ed. Van der Waerden, B.L., Dover Publ., New York, 1967} (1916).
- [34] She, W., et al, "Observation of a Push Force on the End Face of a Nanometer Silica Filament Exerted by Outgoing Light", *Physical Review Letters*, 101(24), (2008).
- [35] Abraham, M., "Sull'elettrodinamica di Minkowski", *Rend. Circ. Mat. Palermo*, 28, 1 (1909).
- [36] Liu, M. et al, "Light-driven nanoscale plasmonic motors", *Nature Nanotechnology*, 5, 1-4, (2010).