ONLINE APPENDIX

I. Data Sources

Our underlying data sources include:

AQR data library (https://www.aqr.com/Insights/Datasets)

Bloomberg (https://www.bloomberg.com/)

Cambridge Associates (https://www.cambridgeassociates.com/private-investment-benchmarks)

Fama-French data (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

Global Financial Data (https://globalfinancialdata.com/)

HFRI (https://www.hfr.com/family-indices/hfri)

ICF at Yale (https://som.yale.edu/centers/international-center-for-finance)

NACUBO (https://www.nacubo.org/research)

NAREIT (https://www.reit.com/)

Robeco (https://www.robeco.com/en/insights/2019/01/data-sets-volatility-sorted-portfolios.html)

Two Centuries Investments (https://www.twocenturies.com/)

II. Asset Class and Portfolio Extension Details

Traditional Sub-Asset Classes Extensions

For the traditional sub-asset classes of stocks and bonds, we obtain index data not only from the data providers specified in Table 1 and Appendix I, but also by constructing the following three custom timeseries.

Asset Class	Actual Inception	Proxy Created	Comparison of Proxy and Actual During Overlapping Period
S&P500 U.S. Growth Stocks index	1974	Equally weighted combination of the Fama-French Low Book-to-Price and High Investment thirds portfolios	97.5% correlation between proxy and actual Annualized returns of 12.09% for actual vs 12.01% for proxy, volatilities of ~17% for both
NAREIT index	12/31/1971	Capitalization-weighted composite of individual REITs and real estate sector stocks from the GFD database GFD includes return data for 664 stocks categorized as either REIT or real estate prior to 1971	From 12/31/1971 to 12/31/2020, the NAREIT Index returned 0.88% monthly while our custom bottom-up cap weighted index returned 0.85% monthly
U.S. Aggregate Bond (U.S. Agg) index	01/31/1976	Weighted combination of USA 5-year Government Total Return Index (60%) and GFD Indices USA Total Return AAA Corporate Bond Index (40%), which roughly corresponds to the current composition of the U.S. Agg	Correlation between the proxy and the actual is 96%, with annualized returns of 5.2% actual and 5.3% proxy respectively

While REIT tax laws and the first official REITs were established in 1960, plenty of real estate management companies existed prior to that. For example, as of 12/31/1959, there were 49 real estate development companies, according to GFD. Although the 1960 legislation affected how REITs were taxed and governed, for the purposes of high-level asset allocation, we believe that exposure to the returns and risks of real estate cycles remains similar before and after the 1960s. Real estate is a significant allocation in the *Endowment* portfolio, thus this simulation of REITs returns is important an important contribution not only for this paper, but also for the future research on the topic.

Hedge Fund Extensions

For the allocation to alternatives utilized only in our *Endowment* portfolio, specifically hedge funds and private equity, we relied on synthetic extension methods. While we sacrifice precision in estimating returns during the exact months of the added history, we believe we add to the overall accuracy of the crash risk estimates by adding the previously unseen history of the *Endowment* asset allocation methodology that heavily relies on exposure to alternative assets i.e., hedge funds.

Asset Class	Actual Inception	Proxy Created	Comparison of Proxy and Actual
HFRI Fund Weighted Composite	12/31/1989	Equally weighted basket of all 15 factor premia: U.S. Equity Value, Momentum, Size, Low Volatility, International Equity Value and Momentum, Country Equity, Currency and Fixed Income Value and Momentum, Commodity Futures Value, Momentum, and Basis	Actual since inception (1989) return 9.6% per year, and proxy 9.9% per year (1926), though since 1990 proxy return was 6.9% year. Proxy volatility is 3.5% per year, while Actual is 6.8% since 1989. Correlation during overlapping period is 3%.

While a basket of factor premia behaves very differently from a composite of hedge funds with a very low correlation, there are important similarities that make it a candidate proxy. Specifically, both have lower volatility and lower beta compared to equity and fixed income asset classes, hence providing a critical source of diversification. Both have also enjoyed a positive overall average return, albeit with a noticeable degradation in the proxy's returns, over the past 10 years. Of course, we realize that quant-heavy investment styles would not have been feasible to implement during those prior decades. Hence this extension is by far the most approximate in our research. Nevertheless, we believe that it represents the effect of adding hedge funds to a portfolio, by providing the strong diversification benefits that come from an asset class that exhibits low correlation with traditional asset classes and a historically positive yet recently deteriorating mean.

Private Assets Extensions

Private and real assets play a large role in many institutional portfolios and are often not adequately addressed in research that focuses only on liquid investments (Goetzmann, et al., 2021).

Asset Class	Actual Inception	Proxy Created	Correlation Between Proxy and Actual During Overlapping Period
Private equity	01/31/1987	Cambridge Associates' quarterly returns for U.S. private equity, U.S. venture capital, and private real estate indices These are unsmoothed to dampen the smoothing effect on returns (Ilmanen, 2020), and interpolated to the monthly frequency by using proxy indices of S&P500, U.S. Small Cap, and REITs respectively	Our process ensures that the resulting proxy's monthly returns have identical compounded total return as the actual unsmoothed quarterly PE indices. By unsmoothing and extrapolating to monthly, we arrive closer to "mark-to-market" higher volatilities for proxy than for actual. For example, U.S. private equity actual volatility 9.5% per year

			vs. 16.1% for the proxy, correlation 93%.
Private Energy and Agriculture (~1-3% in Endowment portfolios per NACUBO data)		Fama-French Energy and Agriculture industry returns supply most of the history except the first seven months of 1926, which come from the Cowles return for the Oil and Agricultural Equipment industries	

Our interpolation was conducted in three steps, in the spirit of Couts et al. (2019). First, we removed two quarters of autocorrelation inside the original quarterly returns. Second, we ran a regression to estimate the beta between the quarterly private equity return and the proxy liquid index. Third, we took the beta-adjusted comparative index and applied two monthly "plugs."

The first "plug" adjusts each quarter to match the quarterly returns of our new synthetic index with that of the un-autocorrelated Cambridge Associates index (created from Step 1). The second "plug" is a full history adjustment applied to all months to match the final cumulative return value of our synthetic index to that of the original Cambridge Associates index. This generates our final synthetic index for the same date range as Cambridge Associates (1987 onwards). This approach ensures that our compounded proxy return is identical to actual, while creating a monthly proxy with a more realistic volatility, than exhibited by the smoothed private equity returns.

Additionally, the combination of the Extended Commodity index, the Extended Private Real Estate index, and the Energy and Agriculture exposure, creates a rather balanced representation of the "Real Asset" exposure in *Endowment* Allocations. Arguably, the only other popular sub-asset class missing from this category is TIPs (Inflation-protected bonds). The *Endowment* allocation to this asset class is less than a half of a percent, according to the 2019 NACUBO report, thus its financial impact on the strategy is immaterial.

Factor Premia Extensions

In our study, factor premia are utilized in two ways. First, we use it to extend and proxy the hedge fund allocation used in the *Endowment Portfolio* prior to the 1989 HFRI index inception, as described above. Second, we use it in the construction of the *Factor-Based* portfolio that allocates 30% to an equally weighted combination of those 15 premia and 70% to the U.S. 60/40. The 15 factors (see Table 1) include cross-sectional U.S. Equity Value, Momentum, Size, Low Volatility, International Equity Value and Momentum, Country Equity, Currency and Fixed Income Value and Momentum, as well as Commodity Futures Value, Momentum, and Basis.

We use a combination of data from Kenneth French's website (Fama and French, 2015), AQR factor data library (Asness et al., 2013), and additional input from Robeco, for the Low-Volatility factor returns (Blitz et al., 2019) to represent and extend the returns of the selected factor premia long-short portfolios. We rely primarily on Geczy and Samonov (2016, 2017, 2019) to extend those premia back to the beginning of 1926. Several items come from (Samonov, 2020) and others are specifically built for this paper. (labeled "custom" in Table 1). For example, momentum factor returns for U.S. Stocks come from a Geczy and Samonov (2016) paper that extend the price momentum effect in U.S. stock level history back to 1801. We also extend the Low Volatility factor based on the stock-level data from Geczy and Samonov (2016) back to 1926, before its 1929 start date in the Robeco database.

The long-run extension of the U.S. Value factor was documented in Samonov (2020), using a spliced combination of the International Center of Finance Stock level dividend data (see Goetzmann, et al.,2000) and the resulting cross-sectional Dividend Yield factor (1825-1871), Cowles Industry PE ratios (1871-1926), and the Fama-French HML factor after 1926. Though we

now have a proxy of the U.S. Value factor extended back to 1825, for this study, we only needed the first six months to fill in the Fama-French HML factor returns during the first half of 1926.

We use the UK value and momentum to proxy for international value and momentum both because of data availability and because the UK was a dominant international market during those decades. The long-run extension for the UK Value factor was also created in Samonov (2021), which uses the individual U.K. stock-level dividend yield and total return data from GFD to create a long-short monthly rebalanced value factor. Long-run extension for the UK momentum factor came from the same sources and uses the standard 10-month definition of momentum. Recent factor data for the UK Value factor comes from Fama-French, starting in 1975, and the data for Momentum comes from the AQR data library, starting in 1972. Hence our extensions add a significant amount of history to both of those factors.

We rely on the return spread between U.S. Small Cap and Large Cap Indices to extrapolate the U.S. Size factor from the Fama-French start date of June of 1926 back to January 1926. Although this is a rather crude approach, it only accounts for six months of the history where the Fama-French size factor returns do not exist. While the difference between the two indices is not exactly the same as the Fama-French Size factor, we believe it reasonably proxies for the exposure to the size factor during that short period.

For the non-stock level momentum factor in the country equities, currencies, bonds, and commodities, we start with AQR's famous "Value Momentum Everywhere" data (Asness et al., 2013) and extrapolate it using the results from Geczy and Samonov (2016) for equities, currencies, and bonds and Geczy and Samonov (2019) for commodity futures factors.

For the non-stock level value factors, we also start with AQR's data and extrapolate it using the longest available data for those four asset classes. Our definition of value varies from AQR's in that AQR relies on mean reversion to approximate the value effect in these asset classes, while

we rely on relative yield differences. Specifically, we use country dividend yields to sort country equities into value portfolios, fixed income yields for country bond value portfolios, and short-term yields for currencies. We rely on the five-year spot price mean reversion definition from AQR for the commodity futures value factor. Our definitions of value are similar to those used by Baltussen et al. (2019).

For completeness, we also include the Commodity Futures Basis factor, which we can extend using CFTC futures data, hand-collected for the Geczy and Samonov (2019) study, that goes back to 1877. For the three years of recent history between 2017 and 2020, we rely on the recently created Bloomberg-GSAM commodity carry index, which derives a large portion of its return from the basis effect, going long commodities that are in backwardation and shorting those in contango.

For the factor premia basket, we take an equally weighted average of the 15 long-short factor portfolios and add the monthly risk-free return to arrive at the total return. For the purpose of this study, we make typical assumptions such as absence of management fees, transaction costs, leverage costs, and any advanced portfolio construction.

Endowment Portfolio Extension Details

Endowment proxy uses high-level and sub-asset class weights from the 2020 NACUBO report to create allocations. Specifically, we include 13% Public U.S. Equities, 13% Public Non-U.S. Equities, 7.3% Global Equities, 13.5% Private Equity, 9.3% Venture Capital, 20% Marketable Alternatives (i.e., Hedge Funds), 12.3% Fixed Income, 11.1% Real Estate. Within the Non-U.S. Equities, we break down further between 5.9% Emerging, and 7.3% Developed markets. Within the Fixed Income, we include 6.6% Investment Grade, 0.6% Non-Investment Grade, 1.3% Private Debt, and 3.8% Cash. Among Real Assets, we use 1.16% Real Assets Marketable, 5.52%

Private Real Estate, 3.53% Private Energy, and 1.13% in Other Private Assets (which based on 2019 NACUBO report can be estimated by Agriculture, Farmland and Timber). Although allocation of endowment portfolios varied throughout their history, (Chambers et al., 2020), we believe that using the latest weights is a better proxy for the expected drawdown thresholds for those portfolios going forward.

III. Simulated Portfolio Example.

Consider an investor who invested in the *US 60/40* portfolio for the long term (a 40-year investment horizon) starting in 1970. The investment period ends at the end of 2010. In this test, we consider four potential periods of prior history that will inform the investor of the portfolio's maximum drawdown (10-year, 25-year, 50-year, and the full period starting in 1926). If the full history is considered, the maximum observed drawdown of the *US 60/40* portfolio as evident in 1970 is 62%. However, the 10-year look back would only show an 18.5% drawdown. As mentioned above, depending on the investor's risk tolerance, we posit three categories of ex-post risk response behavior: risk-tolerant investors sell when the drawdown approaches or surpasses the maximum observed drawdown of 62% (18.5%); risk-neutral investors will sell at 75% of the maximum, 46.9% (13.8%), and risk-averse investors will sell at 50% of the observed maximum, 31.5% (9.02%).

In the case of the *U.S. 60/40* portfolio invested at the beginning of 1970, the informed, risk-tolerant and risk-neutral investors stay invested through until the end of 2010 and earn a 3651.97% cumulative return; however, risk-averse informed investors sell in February 2009, and earn a 2398.57% return. The outcome for risk-averse informed investors is still positive, although significantly less than their risk-tolerant peers.

Furthermore, uninformed investors who only rely on the most recent 10-year period of data, sell in March, June, and August of 1974 depending on their risk tolerance. Their outcomes are a cumulative 11.48%, 6% and -4.24% respectively, with annualized returns of 3.4%, 1.68%, and -1.17% as opposed to the 9.5% they could have earned annually holding the portfolio till the end of the investment horizon.

Table A1. Summary performance of the simulated Global 60/40 portfolios.

This table presents summary of the performance of the portfolios across different dimensions, including ranges of the length of the history considered by investors when committing to the strategy (from 10 years to the maximum available, nearly 100 years), investment horizon 5-40 years and risk tolerance (averse, neutral and tolerant). We show maximum drawdown observed in the historical data and full investment period returns as a benchmark for analysis of the actual investor returns realized based on the level of selected drawdown threshold that may trigger a decision to liquidate the portfolio early.

History Length	Max Observed Drawdown	Full Period Cumulative	Full Period Annual	Averse			Neutral			Tolerant		
				Threshold Drawdown	Investor Cumulative	Investor Annual	Threshold Drawdown	Investor Cumulative	Investor Annual	Threshold Drawdown	Investor Cumulative	Investor Annual
Horizon 5 years												
10	-0.21	0.59	0.09	-0.10	0.38	0.06	-0.15	0.43	0.05	-0.21	0.44	0.04
25	-0.27	0.59	0.09	-0.14	0.46	0.05	-0.20	0.46	0.05	-0.27	0.51	0.06
50	-0.37	0.59	0.09	-0.18	0.46	0.05	-0.28	0.48	0.06	-0.37	0.54	0.08
Max	-0.52	0.59	0.09	-0.26	0.50	0.06	-0.39	0.59	0.09	-0.52	0.59	0.09
Horizon 10 years												
10	-0.19	1.63	0.09	-0.09	0.92	0.06	-0.14	1.02	0.05	-0.19	1.04	0.03
25	-0.26	1.63	0.09	-0.13	1.20	0.05	-0.20	1.22	0.04	-0.26	1.43	0.06
50	-0.37	1.63	0.09	-0.18	1.22	0.04	-0.28	1.31	0.06	-0.37	1.45	0.07
Max	-0.52	1.63	0.09	-0.26	1.33	0.05	-0.39	1.63	0.09	-0.52	1.63	0.09
Horizon 20 years												
10	-0.17	6.07	0.10	-0.08	3.17	0.08	-0.13	3.53	0.07	-0.17	3.71	0.06
25	-0.27	6.07	0.10	-0.13	3.70	0.07	-0.20	3.89	0.05	-0.27	5.56	0.09
50	-0.40	6.07	0.10	-0.20	3.91	0.06	-0.30	4.86	0.08	-0.40	5.52	0.09
Max	-0.52	6.07	0.10	-0.26	4.51	0.06	-0.39	6.07	0.10	-0.52	6.07	0.10
Horizon 40 years												
10	-0.22	35.89	0.09	-0.11	6.43	0.06	-0.16	7.59	0.05	-0.22	10.75	0.03
25	-0.29	35.89	0.09	-0.14	6.39	0.03	-0.21	7.53	0.02	-0.29	19.35	0.09
50	-0.52	35.89	0.09	-0.26	10.71	0.01	-0.39	35.89	0.09	-0.52	35.89	0.09
Max	-0.52	35.89	0.09	-0.26	10.71	0.01	-0.39	35.89	0.09	-0.52	35.89	0.09

Table A2. Summary performance of the simulated Diversified portfolios.

This table presents summary of the performance of the portfolios across different dimensions, including ranges of the length of the history considered by investors when committing to the strategy (from 10 years to the maximum available, nearly 100 years), investment horizon 5-40 years and risk tolerance (averse, neutral and tolerant). We show maximum drawdown observed in the historical data and full investment period returns as a benchmark for analysis of the actual investor returns realized based on the level of selected drawdown threshold that may trigger a decision to liquidate the portfolio early.

History Length	Max Observed Drawdown	Full Period Cumulative	Full Period Annual	Averse			Neutral			Tolerant		
				Threshold Drawdown	Investor Cumulative	Investor Annual	Threshold Drawdown	Investor Cumulative	Investor Annual	Threshold Drawdown	Investor Cumulative	Investor Annual
Horizon 5 years												
10	-0.21	0.61	0.10	-0.10	0.34	0.05	-0.15	0.42	0.06	-0.21	0.44	0.05
25	-0.24	0.61	0.10	-0.12	0.38	0.06	-0.18	0.50	0.07	-0.24	0.49	0.06
50	-0.37	0.61	0.10	-0.19	0.44	0.07	-0.28	0.55	0.08	-0.37	0.55	0.08
Max	-0.61	0.61	0.10	-0.31	0.55	0.08	-0.46	0.61	0.10	-0.61	0.61	0.10
Horizon 10 years												
10	-0.18	1.71	0.10	-0.09	0.57	0.05	-0.14	0.95	0.05	-0.18	1.03	0.05
25	-0.22	1.71	0.10	-0.11	0.69	0.06	-0.17	1.33	0.06	-0.22	1.32	0.06
50	-0.37	1.71	0.10	-0.19	1.04	0.07	-0.28	1.48	0.08	-0.37	1.48	0.08
Max	-0.61	1.71	0.10	-0.31	1.47	0.07	-0.46	1.71	0.10	-0.61	1.71	0.10
Horizon 20 years												
10	-0.18	6.67	0.10	-0.09	0.73	0.07	-0.13	3.51	0.07	-0.18	3.61	0.06
25	-0.23	6.67	0.10	-0.12	0.88	0.07	-0.17	4.84	0.07	-0.23	4.83	0.07
50	-0.41	6.67	0.10	-0.20	4.33	0.09	-0.30	5.91	0.10	-0.41	5.91	0.10
Max	-0.61	6.67	0.10	-0.31	5.86	0.10	-0.46	6.67	0.10	-0.61	6.67	0.10
Horizon 40 years												
10	-0.22	47.47	0.10	-0.11	1.26	0.06	-0.16	12.59	0.04	-0.22	12.56	0.02
25	-0.22	47.47	0.10	-0.11	1.26	0.06	-0.16	12.59	0.04	-0.22	12.56	0.02
50	-0.61	47.47	0.10	-0.31	23.03	0.10	-0.46	47.47	0.10	-0.61	47.47	0.10
Max	-0.61	47.47	0.10	-0.31	23.03	0.10	-0.46	47.47	0.10	-0.61	47.47	0.10

Figure A1. Effect of Prior Information on the Outcomes of Investors: US 60/40.

Top-row charts show outcomes for the 5-year horizon while the bottom for the 40-year horizon. Left-column charts show risk-averse investor while right-column chart show results for risk-tolerant investor. Each chart shows three data points over time: dark bar shows least informed investors total return; light bar shows the most informed investors total return, and dark line shows the full period available total return.

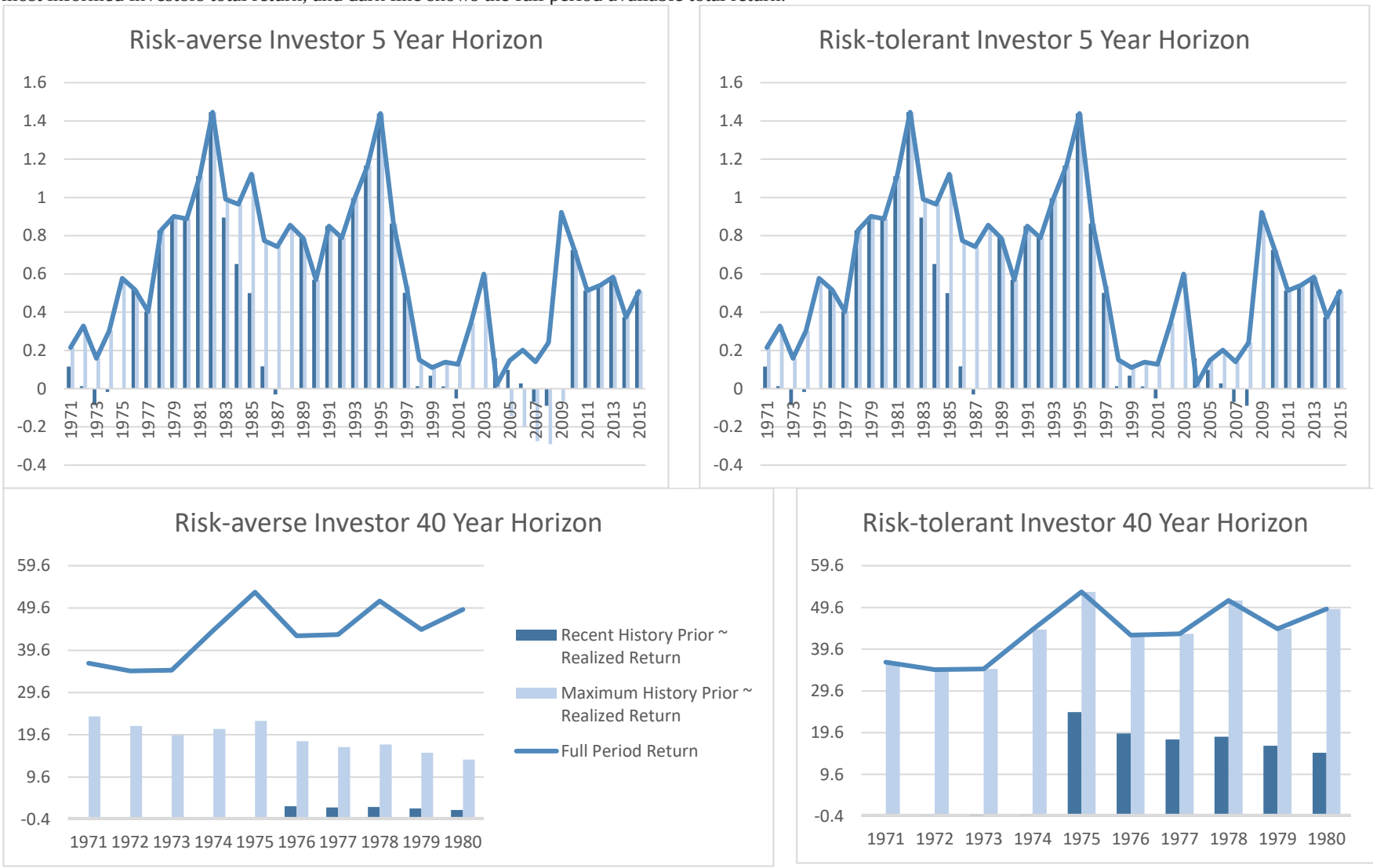

Figure A2. Effect of Information on the Outcomes of Investors: Global 60/40.

Top-row charts show outcomes for the 5-year horizon while the bottom for the 40-year horizon. Left-column charts show risk-averse investor while right-column chart show results for risk-tolerant investor. Each chart shows three data points over time: dark bar shows least informed investors total return; light bar shows the most informed investors total return, and dark line shows the full period available total return.

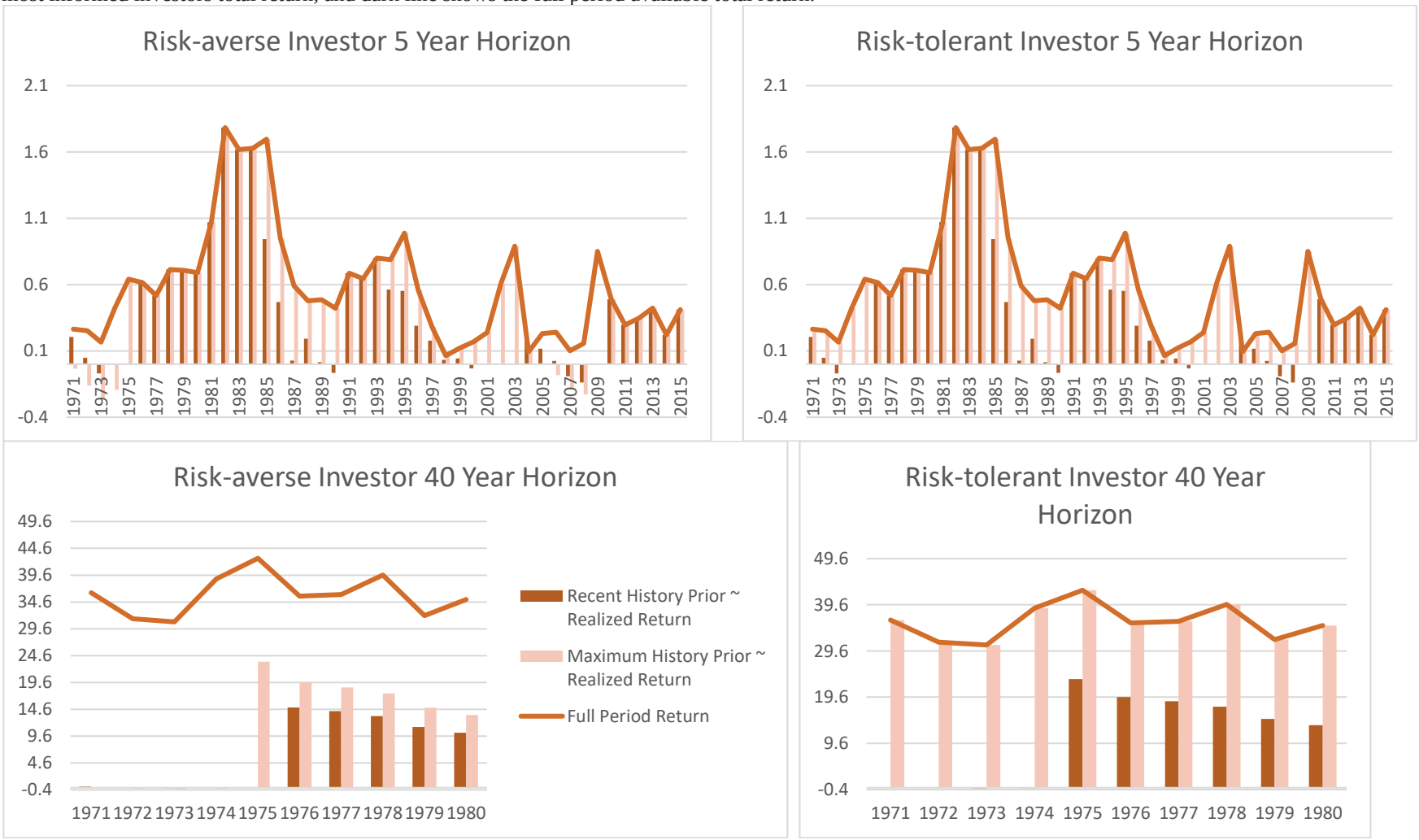

Figure A3. Effect of Information on the Outcomes of Investors: Risk Parity.

Top-row charts show outcomes for the 5-year horizon while the bottom for the 40-year horizon. Left-column charts show risk-averse investor while right-column chart show results for risk-tolerant investor. Each chart shows three data points over time: dark bar shows least informed investors total return; light bar shows the most informed investors total return, and dark line shows the full period available total return.

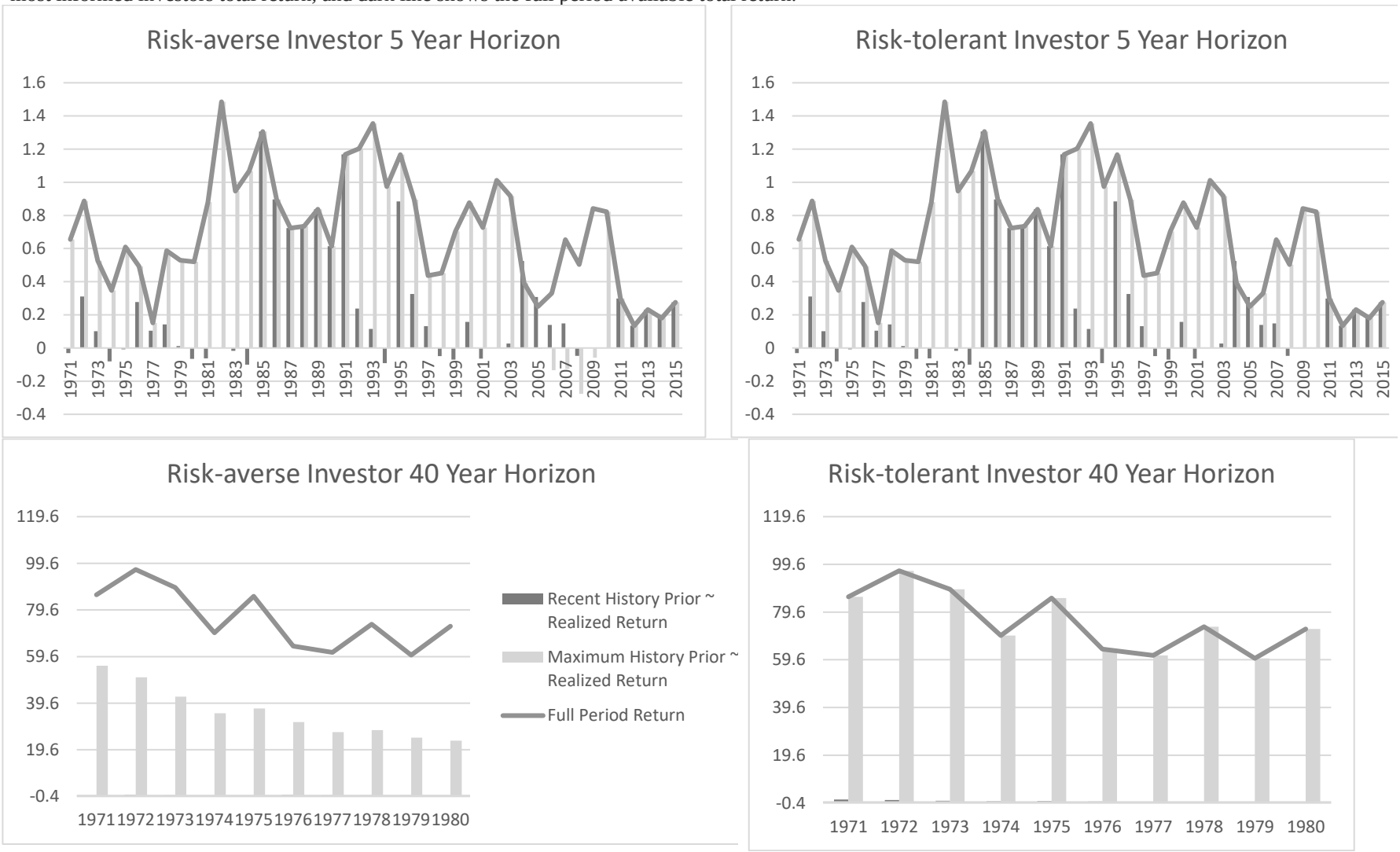

Figure A4. Effect of Information on the Outcomes of Investors: Diversified.

Top-row charts show outcomes for the 5-year horizon while the bottom for the 40-year horizon. Left-column charts show risk-averse investor while right-column chart show results for risk-tolerant investor. Each chart shows three data points over time: dark bar shows least informed investors total return; light bar shows the most informed investors total return, and dark line shows the full period available total return.

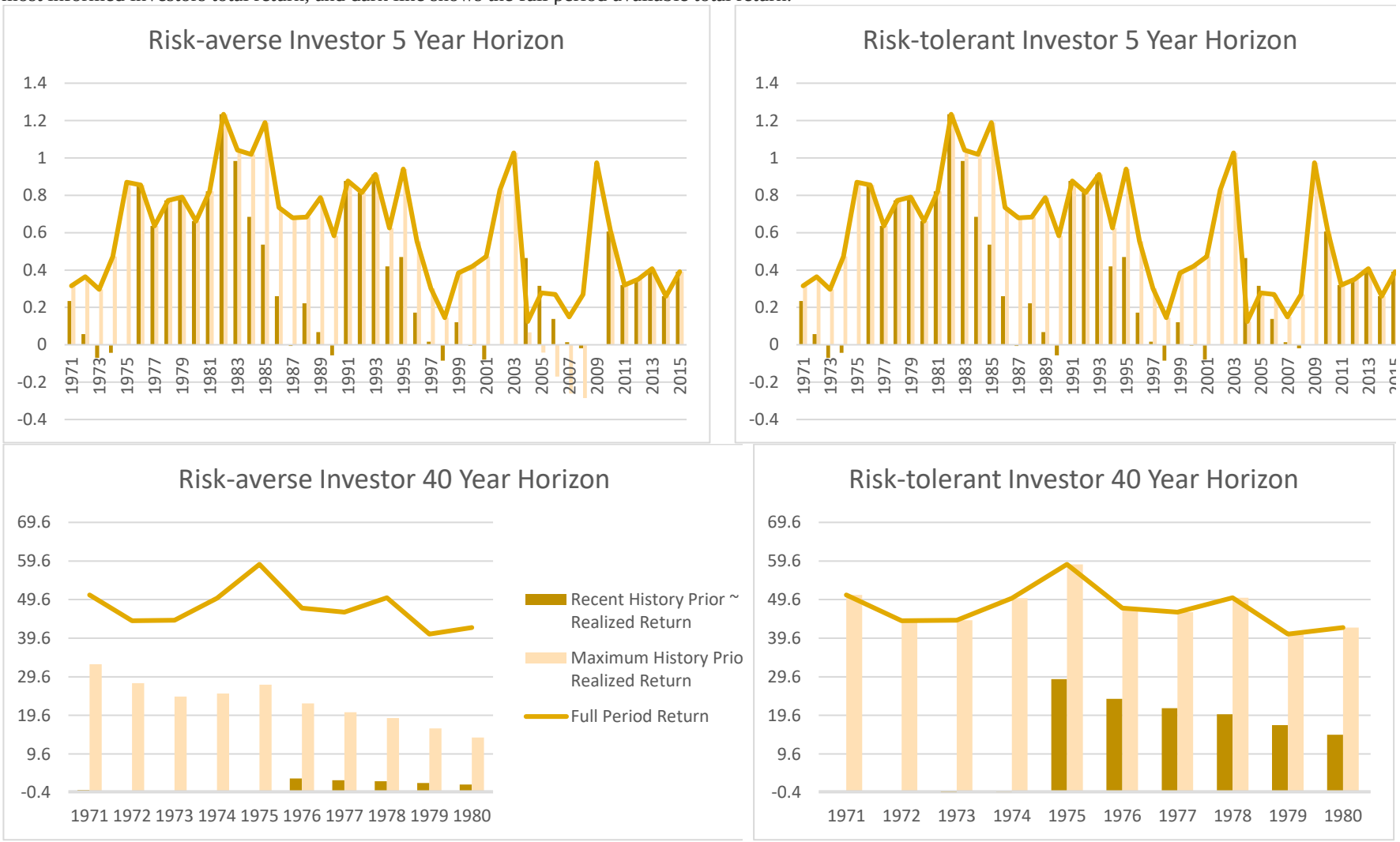

Figure A5. Effect of Information on the Outcomes of Investors: Endowments.

Top-row charts show outcomes for the 5-year horizon while the bottom for the 40-year horizon. Left-column charts show risk-averse investor while right-column chart show results for risk-tolerant investor. Each chart shows three data points over time: dark bar shows least informed investors total return; light bar shows the most informed investors total return, and dark line shows the full period available total return.

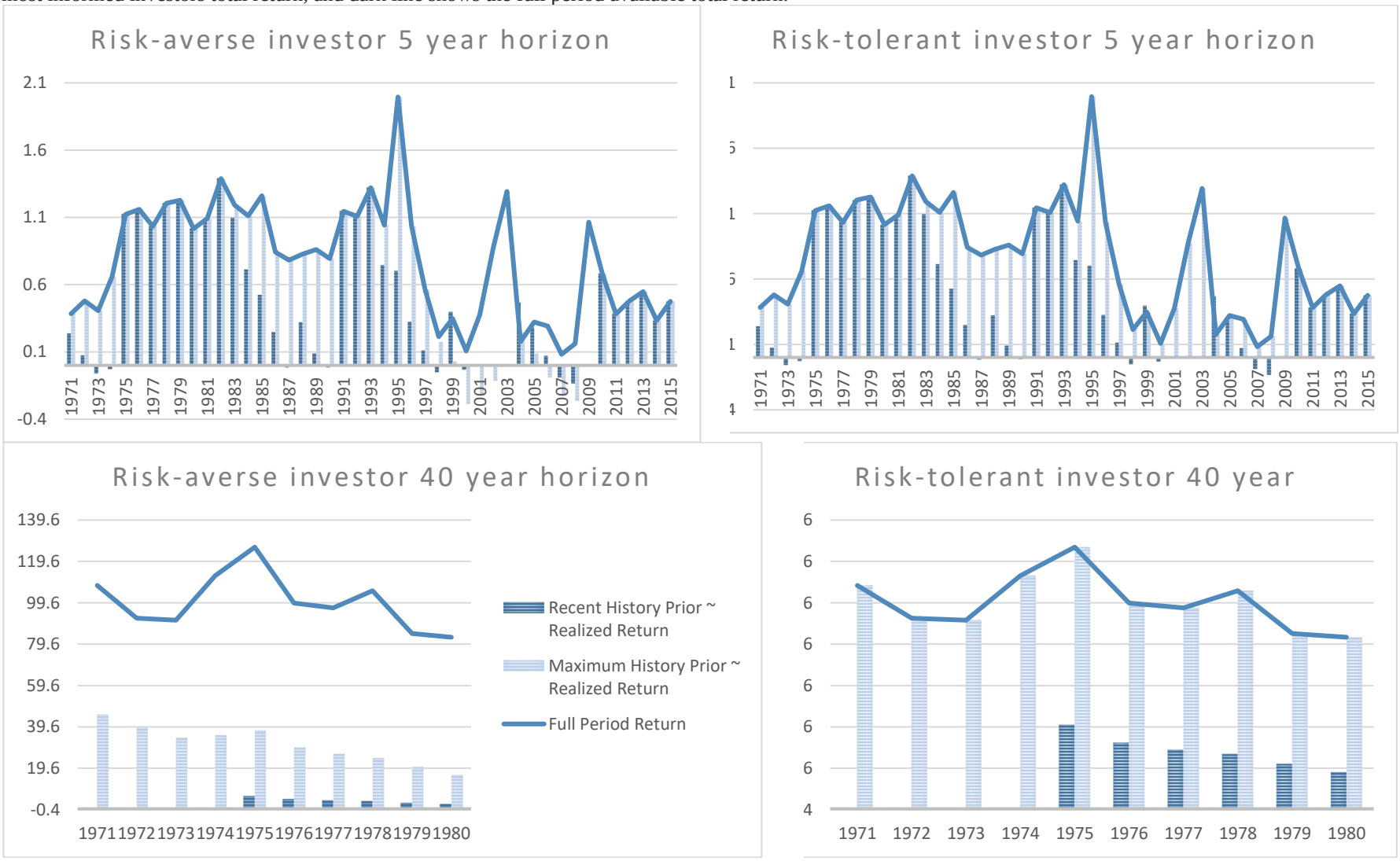

Figure A6. Effect of Information on the Outcomes of Investors: Factor-Based.

Top-row charts show outcomes for the 5-year horizon while the bottom for the 40-year horizon. Left-column charts show risk-averse investor while right-column chart show results for risk-tolerant investor. Each chart shows three data points over time: dark bar shows least informed investors total return; light bar shows the most informed investors total return, and dark line shows the full period available total return.