Name:

Simultaneous equations!

Foundation Level
$x + y = 7$ $3x - 2y = 4$
$4x - y = 3$ $x + 2y = 5$
$3x + 2y = 10$ $5x - 4y = 6$
$x - 2y = -3$ $3x + y = 7$
$2x + 3y = 8$ $x - y = 2$

Higher Level (Q4 & Q5 are a challenge but not expected at GCSE)
$x^2 + y^2 = 10$ $2x - 3y = 5$
$x^2 + xy = 10$ $2x - y = 3$
$x^2 + y^2 = 29$ $y - x = 3$
$2x^2 - 3xy + y^2 = 4$ $3x^2 + 2xy - 4y^2 = 5$
$x^3 + y^3 = 28$ $x + y = 4$

Solutions!

Foundation Level

$2x + y = 7$ (1)

$3x - 2y = 4$ (2)

For Substitution:
We need to be able to plug in an equation in terms of x and y into the other equation so there is only one unknown. In this question we can change equation 1 to $y = 7 – 2x$.
We can then plug this into equation 2 and solve for x. Plug the x value into one of the equations to find y.

For Elimination:
We need to cancel out either the x or y so that we only have one unknown in the equations.
We need the x or y to be of a common multiple in both equations. In this case we can make both equations have 2y or -2y in it.
To cancel the y we then need to add 2y and -2y to make 0 [$2y + (–2y) = 0$]

We can now solve!

By Substitution:	
$2x + y = 7$ (-2x to both sides)	Label the equations 1 and 2.
$y = 7 – 2x$	Make y the subject of equation 1.
$3x – 2(7 – 2x) = 4$	Substitute the equation for y into equation 2.
$3x – 14 +4x=4$	Expand the equation.
$7x – 14 = 4$ (+14 to both sides)	Collect like terms.
$7x = 18$ (÷ both sides by 7)	
$x = 18/7$	Solve for x.
Substitute x into equation 1:	Plug the x value into one of the 2 equations. (Tip: Choose the easiest to substitute x into. For this one, choose equation 1).
$2(18/7) + y = 7$	
$36/7 + y = 7$ (-36/7 from both sides)	Expand the brackets.
$y = 13/7$	Solve for y.
$x = 18/7$ when $y = 13/7$	Finally write the solution.
By Elimination:	
$2x + y = 7$ (Multiply equation 1 by 2)	Change one of the equations so there is a common multiple of (in this case) y.
$4x + 2y = 14$ [(1) x 2]	
$3x – 2y = 4$ (2)	
$4x + 2y + 3x – 2y = 14 + 4$ (Add the two equations above)	Cancel out the y (in this case we do this by adding the equations together).
$7x = 18$ (÷ by 7 to both sides)	Collect like terms.
$x = 18/7$	Solve for x.
Substitute x into equation 1:	Plug the x value into one of the 2 equations. (Tip: Choose the easiest to substitute x into. For this one, choose equation 1).
$2(18/7) + y = 7$	
$36/7 + y = 7$ (-36/7 from both sides)	Expand the brackets.
$y = 13/7$	
$x = 18/7$ when $y = 13/7$	Finally write the solution.

4x - y = 3 (1) x + 2y = 5 (2)	By Substitution: x + 2y = 5 (-2y to both sides) x = 5 – 2y 4(5 – 2y) – y = 3 20 – 8y – y = 3 20 – 9y = 3 (-3 and +9y to both sides) 17 = 9y (÷ both sides by 9) y = 17/9 Substitute y into equation 2: x + 2(17/9) = 5 x + 34/9 = 5 (-34/9 from both sides) x = 11/9 x = 11/9 when y = 17/9 By Elimination: 4x – y = 3(Multiply equation 1 by 2) 8x – 2y = 6 [(1) x 2] x + 2y = 5 (2) 8x – 2y + x + 2y = 6 + 5(Add the two equations above) 9x = 11 (÷ both sides by 7) x = 11/9 Substitute x into equation 1: 4(11/9) – y =3 44/9 - y = 3 (+y and -44/9 to both sides). y = 44/9 – 3 y = 17/9 x = 11/9 when y = 17/9	Label the equations 1 and 2. Make x the subject of equation 2. Substitute the equation for x into equation 1. Expand the equation. Collect like terms. Solve for y. Plug the y value into one of the 2 equations. (Tip: Choose the easiest to substitute y into. This example plugs y into equation 2). Expand the brackets. Solve for x. Finally write the solution. Change one of the equations so there is a common multiple of either x or y (in this case you can do either, but the example finds a common multiple of y). Cancel out the y (add the equations together). Collect like terms. Solve for x. Plug the x value into one of the 2 equations. (This example plugs y into equation 1). Expand the brackets. Solve for y. Finally write the solution.

$3x + 2y = 10$ (1) $5x - 4y = 6$ (2)	By Elimination: 3x + 2y = 10 (Multiply equation 1 by 2) 6x + 4y = 20 [(1) x 2] 5x – 4y = 6 (2) 6x + 4y + 5x – 4y= 20 + 6 (Add the two equations above) 11x = 26 (÷ both sides by 11) x = 26/11 Substitute x into equation 1: 3(26/11) + 2y = 10 78/11 + 2y = 10 (-78/11 from both sides) 2y = 32/11 (÷ both sides by 2) y = 16/11 x = 26/11 when y = 16/11	Change one of the equations so there is a common multiple of either x or y (in this case you find a common multiple of y). Cancel out the y (add the equations together). Collect like terms. Solve for x. Plug the x value into one of the 2 equations. (This example plugs y into equation 1). Expand the brackets. Solve for y. Finally write the solution.
$x - 2y = -3$ (1) $3x + y = 7$ (2)	By Substitution: x – 2y = -3 (+2y to both sides) x = 2y - 3 3(2y – 3) + y = 7 6y – 9 + y =7 7y - 9 = 7 (+9 to both sides) 7y = 16 (÷ both sides by 9) y = 16/7 Substitute y into equation 1: x - 2(16/7) = -3 x – 32/7 = -3 (+24/7 to both sides) x = 11/7 x = 11/7 when y = 16/7	Label the equations 1 and 2. Make x the subject of equation 1. Substitute the equation for x into equation 2. Expand the equation. Collect like terms. Solve for y. Plug the y value into one of the 2 equations. This example plugs y into equation 1. Expand the brackets. Solve for x. Finally write the solution.

2x + 3y = 8 (1) x - y = 2 (2)	By Substitution: x – y = 2 (+y to both sides) x = 2 + y 2(2 + y) + 3y= 8 4 + 2y + 3y = 8 5y + 4 = 8 (-4 from both sides) 5y = 4 (÷ by 5 from both sides). y = 4/5 Substitute y into equation 2: x – 4/5 = 2 (+4/5 from both sides) x = 2 + 4/5 x = 14/5 x = 14/5 when y = 4/5 By Elimination: x – y = 2 (Multiply equation 2 by 2) 2x – 2y = 4 [(2) x 2] 2x + 3y = 8 (1) 2x – 2y – (2x + 3y) = 4 - 8 (Subtract the equations above) -5y = -4 y = 4/5 Substitute y into equation 2: x – 4/5 = 2 (+4/5 to both sides) x = 2 + 4/5 x = 14/5 x = 14/5 when y = 4/5	Label the equations 1 and 2. Make x the subject of equation 2. Substitute the equation for x into equation 1. Expand the equation. Collect like terms. Solve for y. Plug the x value into one of the 2 equations. (Tip: Choose the easiest to substitute x into. For this one, choose equation 2). Expand the brackets. Solve for x. Finally write the solution. Change one of the equations so there is a common multiple of (in this case) x. Cancel out the x (in this case we do this by **subtracting** the equations from each other). Collect like terms. Solve for y. Plug the y value into one of the 2 equations. (Tip: Choose the easiest to substitute y into. For this one, choose equation 2). Expand the brackets. Finally write the solution.	

Solutions!

Higher Level (Q4 & Q5 are a challenge but not expected at GCSE)

$x^2 + y^2 = 10$ (1)

$2x - 3y = 5$ (2)

$2x - 3y = 5$ (+y to both sides)

$2x = 5 + 3y$ (÷2 to both sides)

$x = \frac{5+3y}{2}$

$(\frac{5+3y}{2})^2 + y^2 = 10$

$\frac{25+30y+9y^2}{4} + y^2 = 10$

$25 + 30y + 9y^2 + 4y^2 = 40$

$13y^2 + 30y - 15 = 0$

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{-(30) \pm \sqrt{(30)^2 - 4(13)(-15)}}{2(13)}$

$y = \frac{-30 \pm \sqrt{1680}}{26}$

$y = \frac{-30 \pm 4\sqrt{105}}{26}$

$y = \frac{-15 + 2\sqrt{105}}{13}$ (y1)

$y = \frac{-15 - 2\sqrt{105}}{13}$ (y2)

Substitute y1 into equation 2:

$2x - 3(\frac{-15 + 2\sqrt{105}}{13}) = 5$

$(\frac{-45 + 6\sqrt{105}}{13}) + 5 = 2x$

$\frac{20 + 6\sqrt{105}}{13} = 2x$

$x = \frac{10 + 3\sqrt{105}}{13}$

Substitute y2 into equation 2:

$2x - 3(\frac{-15 - 2\sqrt{105}}{13}) = 5$

$(\frac{-45 - 6\sqrt{105}}{13}) + 5 = 2x$

$\frac{20 - 6\sqrt{105}}{13} = 2x$

$x = \frac{10 - 3\sqrt{105}}{13}$

$x = \frac{10 + 3\sqrt{105}}{13}$ when $y = \frac{-15 + 2\sqrt{105}}{13}$

$x = \frac{10 - 3\sqrt{105}}{13}$ when $y = \frac{-15 - 2\sqrt{105}}{13}$

Label the equations 1 and 2.

Make x the subject of equation 2.

Substitute the equation for x into equation 1.

Expand the equation.

Move everything to one side.

Solve for y. In this case we would have to use the quadratic formula.

Plug each y into equation 2

Expand the brackets.

+3y to both sides.

÷both sides by 2

Expand the brackets.

+3y to both sides.

÷ both sides by 2

Finally write the two solutions.

$x^2 + xy = 10$ (1) $2x - y = 3$ (2)	$2x – y = 3$ (+y to both sides)	Label the equations 1 and 2.
	$2x = 3 + y$ (-3 from both sides)	Make y the subject of equation 2.
	$2x – 3 = y$	
	$x^2 + x(2x - 3) = 10$	Substitute the equation for y into equation 1.
	$x^2 + 2x^2 - 3x = 10$	Expand the equation.
	$3x^2 - 3x = 10$	
	$3x^2 - 3x - 10 = 0$	Move everything to one side.
	$x = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$	Solve for x. In this case we would have to use the quadratic formula.
	$x = \frac{-(-3)\pm\sqrt{(-3)^2 - 4(3)(-10)}}{2(3)}$	
	$x = \frac{3 + \sqrt{129}}{6}$ (x1)	
	$x = \frac{3 - \sqrt{129}}{6}$ (x2)	
	Substitute x1 into equation 2: $2(\frac{3 + \sqrt{129}}{6})- y = 3$	Plug each x into one of the 2 equations. (Tip: Choose the easiest to substitute x into. For this one choose equation 2).
	$(\frac{3 + \sqrt{129}}{3})- y = 3$	Expand the brackets.
	$\frac{3 + \sqrt{129}}{3} - 3 = y$	-3 and -y to both sides.
	$y = \frac{-6 + \sqrt{129}}{3}$	
	Substitute x2 into equation 2: $2(\frac{3 - \sqrt{129}}{6}) – y = 3$	
	$(\frac{3 - \sqrt{129}}{3}) – y = 3$	Expand the brackets.
	$\frac{3 - \sqrt{129}}{3} – 3 = y$	-3 and -y to both sides.
	$y = \frac{-6 - \sqrt{129}}{3}$	
	$x = \frac{3 + \sqrt{129}}{6}$ when $y = \frac{-6 + \sqrt{129}}{3}$	
	$x = \frac{3 - \sqrt{129}}{6}$ when $y = \frac{-6 - \sqrt{129}}{3}$	Finally write the two solutions.

$x^2 + y^2 = 29$ (1) $y - x = 3$ (2)	$y - x = 3$ (+x to both sides) $y = x + 3$ $x^2 + (x + 3)^2 = 29$ $x^2 + 6x + 9 + x^2 = 29$ $2x^2 + 6x - 20 = 0$ $x^2 + 3x - 10 = 0$ $x^2 - 2x + 5x - 10 = 0$ $x(x - 2) + 5(x - 2) = 0$ $(x - 2)(x + 5) = 0$ $x - 2 = 0$ or $x + 5 = 0$ $x = 2$ (x1) or $x = -5$ (x2) Substitute x1 into equation 2: $y - x = 3$ $y - 2 = 3$ (+2 to both sides) $y = 5$ Substitute x2 into equation 2: $y - x = 3$ $y - (-5) = 3$ $y + 5 = 3$ (-5 from both sides) $y = -2$ $x = 2$ when $y = 5$ $x = -2$ when $y = -5$	Label the equations 1 and 2. Make y the subject of equation 2. Substitute the equation for y into equation 1. Expand the equation. Move everything to one side. Solve for x. In this case we would have to use factorising. Plug each x into equation 2. Finally write the two solutions.
$2x^2 - 3xy + y^2 = 4$ (1) $x - y = 2$ (2)	$x - y = 2$ (+y to both sides) $x = y + 2$ $2(y + 2)^2 - 3y(y + 2) + y^2 = 4$ $2(y^2 + 4y + 4) - 3y^2 - 6y + y^2 = 4$ $2y^2 + 8y + 8 - 3y^2 - 6y + y^2 = 4$ $2y + 8 = 4$ $2y = -4$ $y = -2$ Substitute y into equation 2: $x - (-2) = 2$ $x + 2 = 2$ (+2 to both sides) $x = 0$ $x = 0$ when $y = -2$	Label the equations 1 and 2. Make x the subject of equation 2. Substitute the equation for x into equation 1. Expand the equation. Solve for y. Plug y into equation 2. Solve for x. Finally write the solution.

$x^3 + y^3 = 28$ (1) $x = 3y$ (2)	$x = 3y$ $(3y)^3 + y^3 = 28$ $27y^3 + y^3 = 28$ $28y^3 = 28$ (÷28 to both sides) $y^3 = 1$ (cube root) $y = 1$ $x = 3(1)$ $x = 3$ $x = 3$ when $y = 1$	Label the equations 1 and 2. Substitute equation 2 into equation 1. Solve for y. Plug y into equation 2 to find x. Finally write the two solutions.