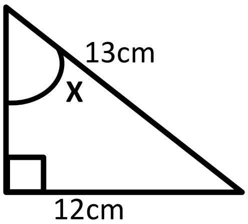
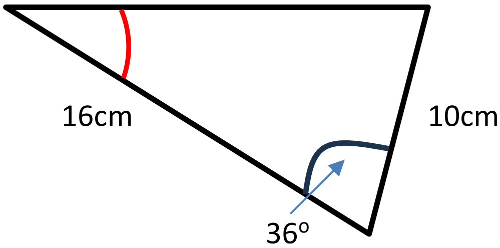
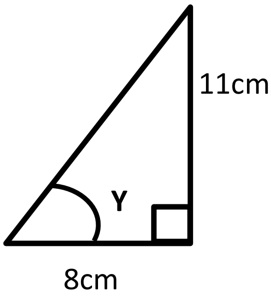
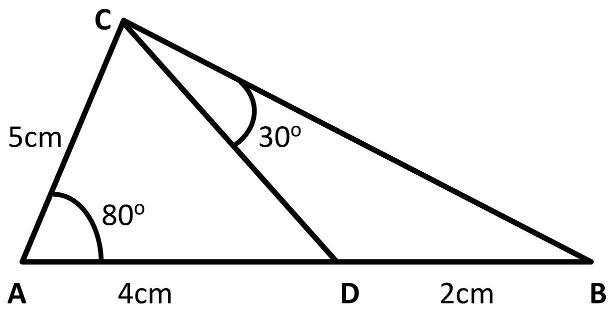
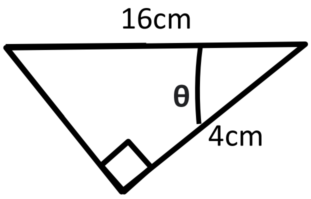
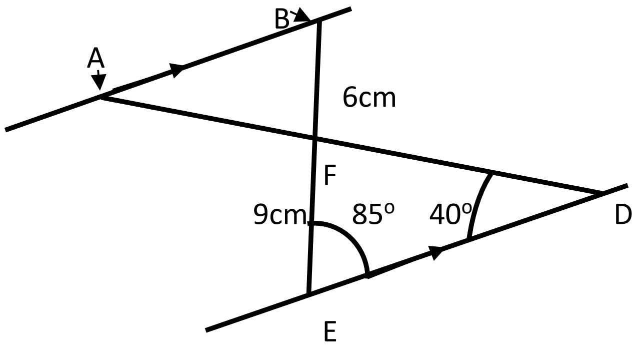
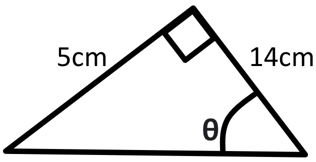
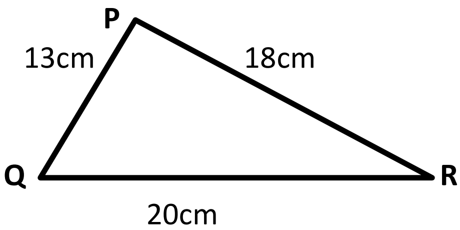
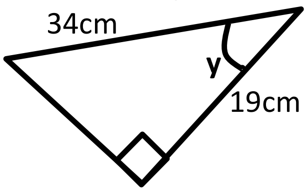
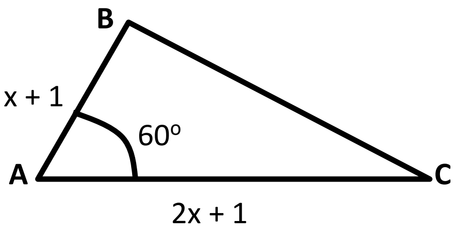
Name:

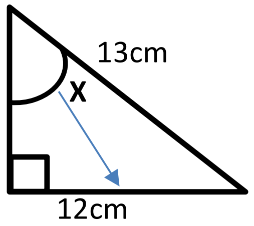
Test yourself: Trigonometry!

Foundation	Higher (Q5 is very hard!)
Calculate the size of angle X 	**Calculate the size of the red marked angle.** 
Calculate the size of angle Y 	**Work out the area of triangle BCD.** 
What is the value of θ? 	**Work out the length of AB** 
What is the value of θ? 	**Calculate angle QPR.** 
What is the value of y? 	**Write an algebraic expression for the length of BC** 

Solutions

Foundation

Calculate the size of angle X



Start off with S^{OH}C^{AH}T^{OA}

$$\sin\theta = \frac{Opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

We need to use Sin(X) since the 12cm is the opposite (because it is the side that is opposite the angle X) and the 13cm length is the Hypotenuse since it's the longest side!

Therefore:

$$\sin X = \frac{12}{13}$$

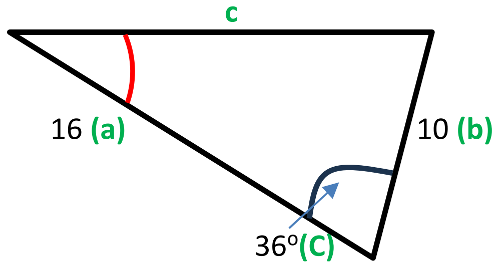
We need so get rid of sin, by doing the inverse sin to both sides. (You can't just divide by sin to get rid of it since it's a trig function!)

$$X = \sin^{-1}\left(\frac{12}{13}\right)$$

$$X = \mathbf{67.38^o}$$

Higher (Q5 is very hard!)

Calculate the size of the red marked angle.



We are now no longer dealing with right angled triangles so **S^{OH}C^{AH}T^{OA}** will no longer work. Instead, we need to use new trigonometry rules. For this question we must use 2 rules. The cosine rule and sine rule.

Let's use the Cosine rule (for sides) first to get the top length of the triangle because we need this length to use the sine rule (for angles) which will give us the answer:

$$c = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

Label the triangle in a similar fashion to below:

c = side we want (Top side)

a = 16cm

b = 10cm

θ = 36° (angle between 2 known sides)

Plug in the values and we obtain **C = 9.85cm**

Now we can use the sin rule for angles:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Lower case letters represent values for sides, and Upper-case letters are for the value of angles **opposite the sides.**

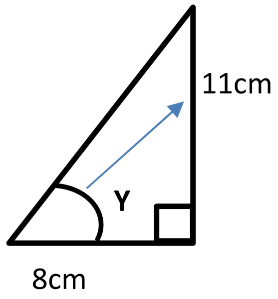
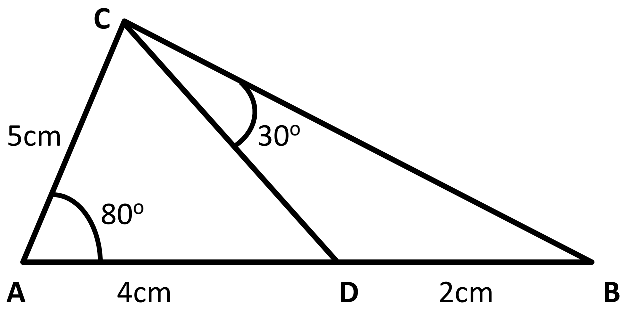
In this example:

b = 10cm, c = 9.85cm, C = 36°, B **= angle we want**

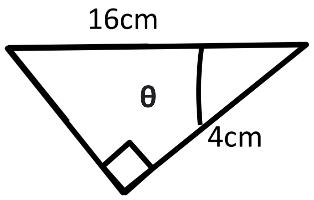
$$\frac{\sin B}{10} = \frac{\sin 36}{9.85}$$

$$\sin B = \frac{10 \times \sin 36}{9.85}$$

$$\boldsymbol{B} = \mathbf{36.64^o}$$

Calculate the size of angle Y	**Work out the area of triangle BCD.**
 We need to use tan(Y) since the 11cm is the opposite (because it is the side that is opposite the angle X) and the 8cm length is the Adjacent (because it's the side that's right next to it. Adjacent means next to it!) Therefore: $$\tan Y = \frac{11}{8}$$ We need so get rid of tan, by doing the inverse sin to both sides. (You can't just divide by tan to get rid of it since it's a trig function!) $$Y = tan^{-1}\left(\frac{11}{9}\right)$$ $$Y = \mathbf{53.97^o}$$	 REMEMBER: labelling triangles is helpful but remember that whenever you're substituting it's okay to name something the same so long as you inform the examiner or make it very clear! To make things easier, the solution below will use the letters in the formula and the labels on the triangles. First let's find the length of CD using the cosine rule: $$c = \sqrt{a^2 + b^2 - 2ab\, cos\, \theta}$$ c = CD? a = AC = 5cm b = AD = 4cm θ = 80º Substituting the values, CD = 5.84cm (2 d.p) Next, Find CB using the same rule: HINT (forget about the line CD for now) $$c = \sqrt{a^2 + b^2 - 2ab\, cos\, \theta}$$ Let: c = CB? a = AC = 5cm b = AB = (4+2) = 6 cm θ = 80º Substituting the values, CB = 7.11cm (2 d.p) Finally, use the area formula on the triangle BCD $$Area = \frac{1}{2} ab\, sin\, C$$ Where: a = CD = 5.84cm b = CB = 7.11 cm C = 30º Substituting the values, **Area = 10.38 cm²**

What is the value of θ?



We need to use cos(θ) since the 16cm is the Hypotenuse (since it's the longest side!) and the 4cm length is the Adjacent (because it's the side that's right next to it. Adjacent means next to it!)

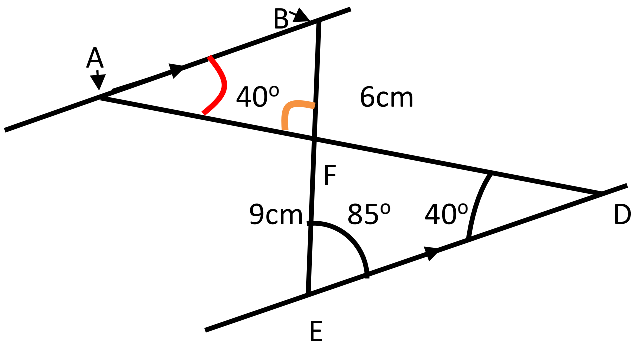
Therefore:

$$\cos\theta = \frac{4}{16}$$

We need so get rid of cos, by doing the inverse sin to both sides. (You can't just divide by cos to get rid of it since it's a trig function!)

$$\theta = cos^{-1}\left(\frac{4}{16}\right)$$
$$\theta = \mathbf{75.52^o}$$

Work out the length of AB



First, we need to use angle facts.
"Alternative angles are always equal" – The angle marked in red is also 40° by alternative angles.

Next, we need to find the value of the orange angle with 2 easy facts:

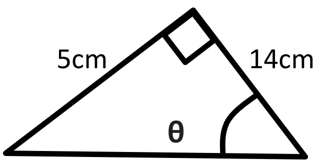
- Angles in a triangle sum to 180
- Angles that are opposite each other when two lines cross are equal.

So,
Angle EFD = 180 – 85 – 40 = 55°
Therefore, Angle ABF is also 55°.

Now, we use the sine rule for angles to obtain the answer.

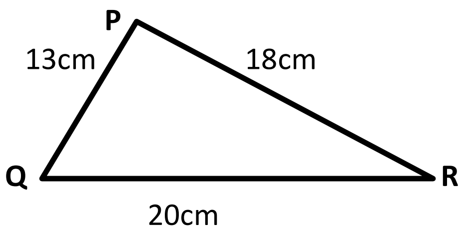
$$\frac{sin\,A}{a} = \frac{sin\,B}{b} = \frac{sin\,C}{c}$$
$$\frac{sin\,40}{6} = \frac{sin\,55}{AB}$$
$$AB = \frac{6 \times sin\,55}{sin\,40} = \mathbf{7.65cm}\ (2\ d.p)$$

What is the value of θ?



We need to use tan(θ) since the 5cm is the opposite (because it is the side that is opposite the angle X) and the 14cm length is the Adjacent (because it's the side that's right next to it. Adjacent means next to!)

Calculate angle QPR.



This is a direct use of the Cosine rule for angles:
Use this formula:

$$cos\,A = \frac{a^2 + b^2 - c^2}{2ab}$$

Let:
a = 13cm
b = 18cm

Therefore:

$$\tan\theta = \frac{5}{14}$$

We need so get rid of tan, by doing the inverse sin to both sides. (You can't just divide by tan to get rid of it since it's a trig function!)

$$\theta = tan^{-1}\left(\frac{5}{14}\right)$$

$$\boldsymbol{\theta = 19.65^o}$$

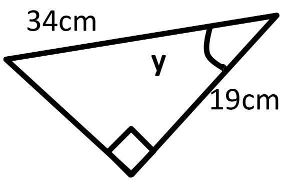
c = 20cm (This is the side that is opposite to the angle we want)

A = "Angle QPR"

$$cos(A) = \frac{13^2 + 18^2 - 20^2}{2(13)(18)}$$

$$A = Q\hat{P}R = cos^{-1}\left(\frac{13^2 + 18^2 - 20^2}{2(13)(18)}\right) = \mathbf{78.54^o}$$

What is the value of y?



We need to use cos(y) since the 34cm is the Hypotenuse (since it's the longest side!) and the 19cm length is the Adjacent (because it's the side that's right next to it. Adjacent means next to it!)

Therefore:

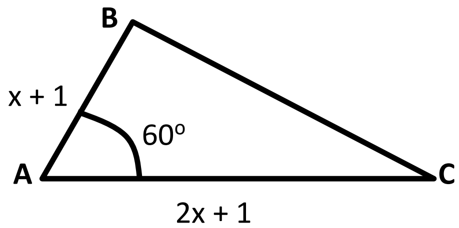
$$\cos y = \frac{19}{34}$$

We need so get rid of cos, by doing the inverse sin to both sides. (You can't just divide by cos to get rid of it since it's a trig function!)

$$y = cos^{-1}\left(\frac{19}{34}\right)$$

$$y = \mathbf{56.03^o}$$

Write an algebraic expression for the length of BC



This is a hard algebra question, but it is still just a direct use of the cosine rule for sides.

Use this formula:

$$c = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

Let:

a = x + 1

b = 2x + 1

θ = 60°

c = BC?

Let's substitute! (This solution will not go through how to expand brackets and collect like terms, see other worksheets to practise this skill):

$$BC = \overline{(x+1)^2 + (2x+1)^2 - 2(x+1)(2x+1)\,cos\,60}$$

$$= \sqrt{(x^2 + 2x + 1) + (4x^2 + 4x + 1) - 2(2x^2 + 3x + 1)\,cos\,60}$$

$$\cos(60) = \frac{1}{2}$$

$$BC = \sqrt{(5x^2 + 6x + 2) - (4x^2 + 6x + 2)\left(\frac{1}{2}\right)}$$

$$= \sqrt{(5x^2 + 6x + 2) - (2x^2 + 3x + 1)}$$

$$= \sqrt{\mathbf{3x^2 + 3x + 1}}$$