Mechanical Universe Notes: *by John Wsol*

Derivatives: Rate of change at any given point. Steepness is ratio of rise/distance. In other words, the slope of a tangent line at any point on a curve is its derivative.

$$\lim_{\Delta x \to \infty} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{d}{dx} f(x) \qquad (1)$$

Derivative of a line is a constant –m.

$$y = mx + b; \frac{dy}{dx} = m \qquad (2a)$$

Derivative of a line is a constant –m.

$$\frac{d}{dx}(mx+b) = m \qquad (2b)$$

Derivative of sine of x"

$$\frac{d}{dx}\sin(x) = \cos(x) \qquad (3a)$$

Derivative of cosine of x"

$$\frac{d}{dx}\cos(x) = -\sin(x) \qquad (3b)$$

The Sum Rule:

$$\frac{d}{dx}(y+z) = \frac{dy}{dx} + \frac{dz}{dx} \qquad (4)$$

The Product Rule: for example Area=length x width

$$\frac{d}{dx}(yz) = y\frac{dz}{dx} + z\frac{dy}{dx} \qquad (5a)$$

The Product Rule: for example the derivative of x^2

$$\frac{d}{dx}(x^2) = x\frac{dx}{dx} + x\frac{dx}{dx} = 2x\frac{dx}{dx} = 2x \qquad (5b)$$

The Product Rule: for example the derivative of x^n

$$\frac{d}{dx}(x^n) = nx^{(n-1)} \qquad (5c)$$

The Chain Rule: for example 34 mi/hr = 17 mi/gal x 2 gal/hr

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} \qquad (6)$$

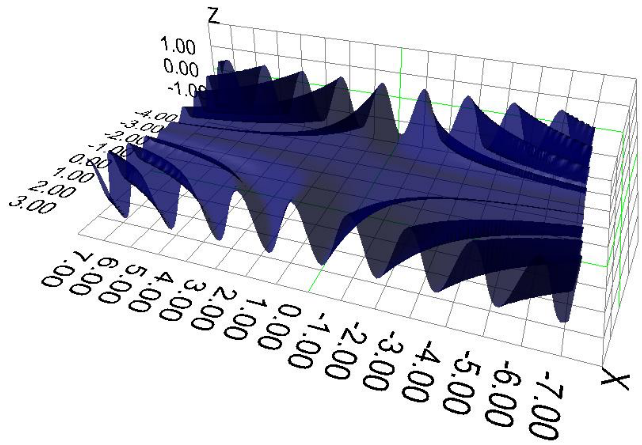
The derivative of a derivative: derivative of displacement = velocity; derivative of velocity = acceleration; derivative of acceleration = jerk

$$\frac{d\tau}{dt}\ln\tau = \frac{1}{\tau} \qquad (7a)$$

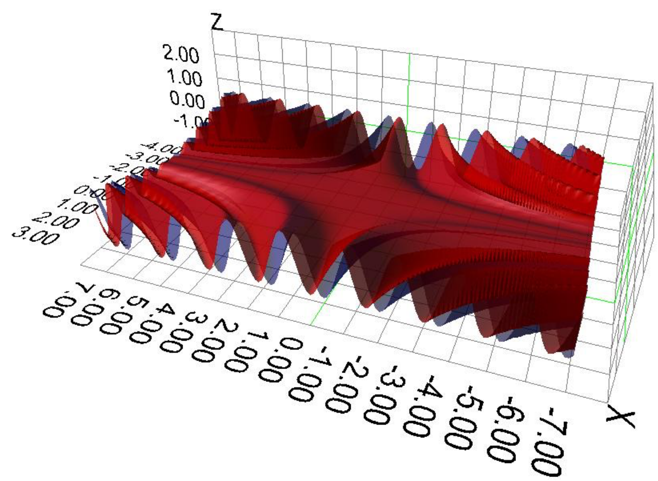
$$\int_1^\tau \frac{1}{t}dt = \ln\tau \qquad (7b)$$

The derivative of harmonic functions:

$$x = C\omega_0 \sin(\omega_0 t)$$
$$\frac{dx}{dt} = -C\omega_0 \cos(\omega_0 t) \qquad (8)$$



Its derivative in **red**:



Derivative of a discontinuous function at a point.

$$\frac{d@}{dx} = \lim_{x \to @} f(x) + \lim_{@ \leftarrow x} f(x) \qquad (9)$$

Einstein in a letter to Arnold Somerfield said – "I have a deep respect for mathematics."

1683: Isaac Newton and his three Laws:

1. A mass at rest remains at rest. (inertia)
2. A mass in motion continues so. (momentum)

Basic force-mass relationship: **F**=*m***a**

$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	(10a)
$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{s}}}{dt}\frac{d\vec{\mathbf{s}}}{dt} = \frac{d^2}{dt^2}\vec{\mathbf{s}}$	(10b)

The acceleration attributed to gravity

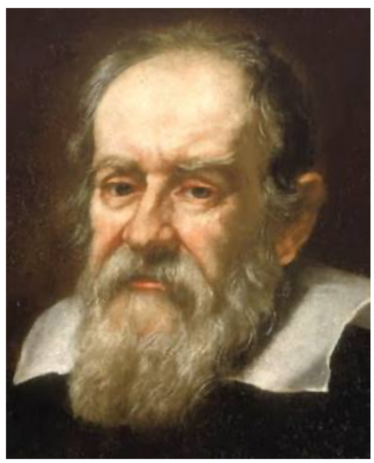
$\vec{\mathbf{F}} = -mg\hat{\mathbf{z}}$	(10c)
$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{s}}}{dt}\frac{d\vec{\mathbf{s}}}{dt} = \frac{d^2}{dt^2}\vec{\mathbf{s}}$	(10b)

Integral Calculus: area under a curve

Archimedes calculated the quadriture of a parabola.

Pythagoras discovered the formula for a right triangle: $r^2 = x^2 + y^2$.

Galileo Galilei (1564-1642) natural philosopher / astronomer asserted that bodies fall at the same rate regardless of mass; at least, whenever air resistance is negligible.



Nicolaus Copernicus
Culver Pictures, Inc.

Nicolai Copernicus (1473-1543) Polish astronomer: Sun centered solar system. Yet, perfect circular orbits with epicycles. Authored "Astronomia"

Johannes Kepler a German astronomer / mathematician (1571-1630) formulated 3 famous laws of planetary motion and computed the areas and volumes of 92 mathematical shapes.

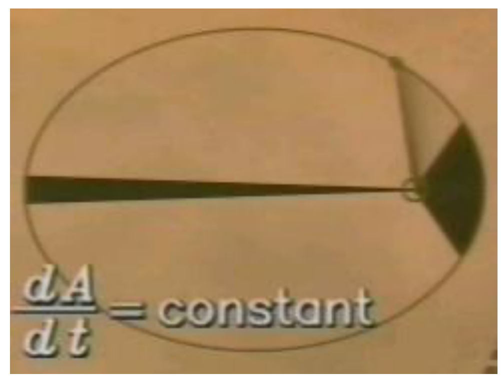
His ambitions book "Harmony of the Worlds" was a treatise of planetary motion, music, geometry, and math. The general formula for conic sections: for e=0 we have a circle, for e<1 an ellipse, e=1 a parabola, e>1 a hyperbola.

Law 1: Planetary orbits are ellipses.

$r = \frac{ed}{1 + e\cos\theta}$	(21a)

Law 2: Planets sweep out equal areas in equal times. L is angular momentum, M mass, A area, and t is time.

$\frac{dA}{dt} = \frac{L}{2M}$	(21b)



Law 3: T is the period of a planet is proportional to "a" its semi-major axis.

$T^2 = \frac{4\pi^2}{GM}a^3$	(21c)
$\left(\frac{T}{2\pi}\right)^2 = \frac{a^3}{GM}$	(21d)

Earth to Mars Opportunity: [1]

Planet	Mean longitude of Ascending node °	′	″	Mean longitude of perihelion °	′	″	Inclination of Orbit to ecliptic °	′	″	AUs[2]	Eccentricity of orbit	Mean long. At the epoch °	′	″
Mercury	48	17	49	77	24	43	7	0	18	0.387097	0.205638	215	54	59
Venus	76	39	18	131	45	36	3	23	41	0.723332	0.006792	276	28	22
Earth	0	0	0	102	50	2	0	0	0	1.000002	0.016740	131	14	33
Mars	49	32	10	336	2	28	1	50	59	1.523705	0.093338	157	34	18
Jupiter	100	26	46	15	41	35	1	18	17	5.20237	0.048432	305	53	51
Saturn	113	36	11	89	19	52	2	29	7	9.56627	0.052749	14	13	29
Uranus	74	5	17	176	17	42	0	46	24	19.3045	0.043329	300	44	57
Neptune	131	45	22	3	49	12	1	46	8	30.2760	0.009733	298	33	15
Pluto	110	21	7	224	47	2	17	7	9	39.6595	0.251642	234	54	21

The Planets: Motion, Distance, and Brightness

Planet	Mean daily motion	Orbital velocity (mi/sec.)	Sidereal revolution	Synodic revolution days	Distance from Sun in millions of mi		Distance from Earth in millions of mi		1 Light at perihelion	aphelion
Mercury	14,732	29.75	88.0	115.9	43.4	28.6	136	50	10.58	4.59
Venus	5,768	21.76	224.7	583.9	67.7	66.8	161	25	1.94	1.89
Earth	3,548	18.51	365.3	-------	94.6	91.4	-----	---	1.03	0.97
Mars	1,886	14.99	687.0	779.9	155.0	128.5	248	35	0.524	0.360
Jupiter	299	8.12	4,331.8	398.9	507.0	460.6	600	368	0.0408	0.0333
Saturn	120	5.99	10,760.0	378.1	937.5	838.4	1,031	745	0.01230	0.00984
Uranus	42	4.23	30,684.0	369.7	1,859.7	1,669.3	1,953	1,606	0.00300	0.00250
Neptune	21	3.38	60,188.3	367.5	2,821.7	2,760.4	2,915	2,667	0.00114	0.00109
Pluto	14	2.95	90,466.8	366.7	4,551.4	2,756.4	4,644	2,663	0.00114	0.00042

1

2

René Descartes
PAR

Rene Descartes (1596-1650) mathematician/philosopher who combined algebra with geometry to create analytic geometry, and he also invented the Cartesian coordinate system. The total quantity of motion is a constant. A body will move in a straight line until acted upon.

Pierre de Fermat (1601-1665) who formulated number theory and probability theory.

Sir Isaac Newton
PAR

Isaac Newton (1642-(Black Plague 1665-1666)-1727) wrote the book "The Method of Fluxions" later to be called "derivatives". Developed differential and integral calculus, the universal laws of gravitation and consider the father of modern physics. Baron Gottfried Wilhelm von Leibnitz also created "The Calculus" and his notation won.

$$\int_{0}^{t} x^2 dx = \frac{1}{3}t^3 \tag{11}$$

1st fundamental theorem of calculus:

$$\frac{d}{dt}\int_{0}^{t} f(x)dx = f(x) \tag{12}$$

2nd fundamental theorem of calculus:

$$A(t) = \int_{a}^{t} \frac{dA}{dx}dx + A(a) \tag{13}$$

Applying this to motion: v=velocity; a=acceleration.

$$v(t) = \int_{a}^{t} a(\tau)d\tau + v(0) \tag{14a}$$

$$s(t) = \int \frac{ds}{d\tau}d\tau + s(0) \tag{14b}$$

$$s(t) = \int_{0}^{t} v(\tau)d\tau + s(0) \tag{14c}$$

In Principia: "The change in motion is proportional to the force impressed in the direction of the force." Newton's 2nd Law is expressed as **p** for momentum, **F** for force.

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \tag{15a}$$

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} \tag{15b}$$

Two interacting bodies each with momentum $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively:

$$\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0 \tag{15c}$$

$$\frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = 0 \tag{15d}$$

Since the derivative is 0 then $\mathbf{p}_1 + \mathbf{p}_2$ expresses a constant.

U denotes potential energy, **W**, work, or changing to Kinetic Energy **K** or dissipating into heat, **Q**. From 15a we derive velocity as:

$$\mathbf{v} = \frac{\mathbf{p}}{m} \tag{15d}$$

$$\mathbf{K} = \frac{1}{2}m\mathbf{v}^2 \tag{15e}$$

$$\mathbf{K} = \frac{1}{2}m\left(\frac{\mathbf{p}}{m}\right)^2 = \frac{\mathbf{p}^2}{2m} \tag{15f}$$

[Question: does the moon really pull on the earth with an equal and opposite force on the Earth?]

$$\int_{0}^{t} x^2 dx = \frac{1}{3}t^3 \tag{11}$$

Newton's Universal Gravitation equation:

$$\mathbf{F} = -G\frac{m_1 m_2}{r^2}\hat{r} \tag{13}$$

Every massive object inhibits the expansion of the space-time manifold – slowing the aging of the universe in an around itself.

---

[2] *The World Almanac® and Book of Facts 1997* is licensed from K-III Reference Corporation. Copyright © 1996 by K-III Reference Corporation. All rights reserved.

Earth's gravity: 32 ft/s^2

$$g = G\frac{M_E}{R_E^2} \quad (13a)$$

Earth-moon gravity: 1/20th inch/second^2.

$$\frac{a_m}{g} = \left(\frac{R_E}{r_m}\right)^2 \quad (13b)$$

Circular functions (trigonometry):

$$x = r\cos(\theta)$$
$$y = r\sin(\theta) \quad (14a)$$

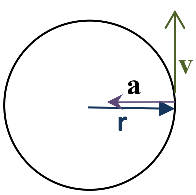
Constant speed circular motion – ω is angular speed.

$$\frac{d\theta}{dt} = \omega \quad (14a)$$

$$\int \frac{d\theta}{dt}; \theta = \omega t \quad (14b)$$

One revolution: $2\pi = \omega T$ (14c)

$$\therefore \frac{2\pi}{T} = \omega \quad (14d)$$



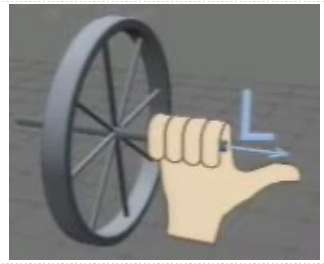
Circular motion expressed in vector notation (i,j,k)::(x,y,z)-components. $\mathbf{r}$ is the radius, $\mathbf{v}$ is velocity (rate of change), $\mathbf{a}$ is acceleration (rate of velocity change). Note velocity is at right angle to $\mathbf{r}$ and acceleration is opposite direction of $\mathbf{r}$. Note equations (3a) and (3b).

$$\mathbf{r} = r\cos\omega t\hat{\mathbf{i}} + r\sin\omega t\hat{\mathbf{j}} \quad (14e)$$

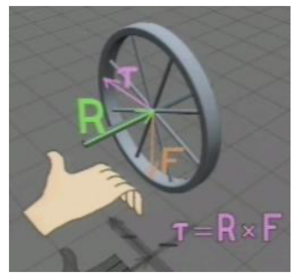
$$\mathbf{v} = \omega\mathbf{r} = 2\pi\mathbf{r} \quad (14f)$$
$$\mathbf{a} = \omega\mathbf{v} = 2\pi\mathbf{v} \quad (14g)$$

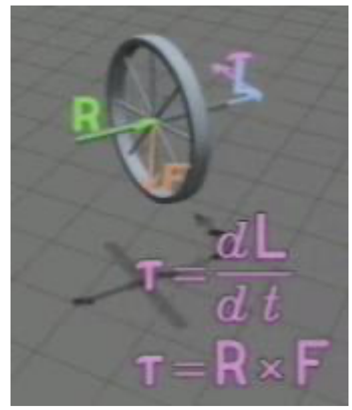
$$\mathbf{a} = \frac{\mathbf{v}^2}{\mathbf{r}} \quad (14h)$$?

Torque: $\mathbf{L}$ is angular momentum, τ is torque;



$$\tau = \frac{d\mathbf{L}}{dt} = \mathbf{R} \times \mathbf{F} \quad (20a)$$





R is radius offset; F is the force due to gravity.

14f expresses circular motion, 20b Ω expresses precession rate.

$$\frac{\tau}{\mathbf{L}} = \Omega \quad (20b)$$

$$\Omega = \frac{\mathbf{R}mg}{mrv} = \frac{Rg}{rv} = \frac{Rg}{\omega r^2}$$

Acceleration due to gravity:

$$\frac{\mathbf{v}^2}{\mathbf{r}} = a = G\frac{m_e}{r^2} \quad (15a)$$

$$v = \sqrt{G\frac{m_e}{r^2}} \quad (15b)$$

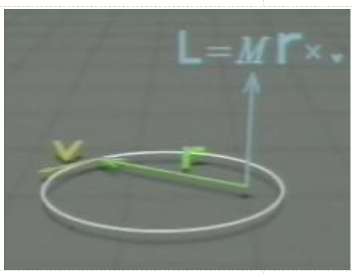
Newton explains Kepler's Laws:

$$\mathbf{F}_g = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}; G\left[\frac{Nm^2}{kg^2}\right] \quad (21a)$$

$$F = M\mathbf{a}$$
$$\mathbf{F}_g = \frac{-D}{r^2}\hat{\mathbf{r}}; \quad (22b)$$

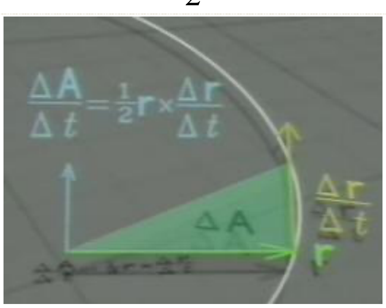
Equate the two: $$M\mathbf{a} = \frac{-D}{r^2}\hat{\mathbf{r}} \quad (22c)$$

Solve for acceleration: $$\mathbf{a} = \frac{-D}{Mr^2}\hat{\mathbf{r}} \quad (22d)$$

Express as diff(velocity):	$\frac{d\mathbf{v}}{dt} = \frac{-D}{Mr^2}\hat{\mathbf{r}}$	(22e)
Express as dif² (position):	$\frac{d^2\mathbf{r}}{dt^2} = \frac{-D}{Mr^2}\hat{\mathbf{r}}$	(22f)
Uniform circular motion:	$\hat{\mathbf{r}} \times F = 0$	(22g)
From (22b):	$\hat{\mathbf{r}} \times M\mathbf{a} = 0$	(22h)
Diff(velocity):	$M\hat{\mathbf{r}} \times \frac{d\mathbf{v}}{dt} = 0$	(22i)
Product Rule:	$\frac{d}{dt}(\mathbf{r}\times\mathbf{v}) = \frac{d\mathbf{r}}{dt}\times\mathbf{v} + \mathbf{r}\times\frac{d\mathbf{v}}{dt}$	(22j)
dr/dt=**v**; **v**x**v**=0:	$\frac{d}{dt}(\mathbf{r}\times\mathbf{v}) = \mathbf{r}\times\frac{d\mathbf{v}}{dt}$	(22k)
Zero torque:	$M\frac{d}{dt}(\mathbf{r}\times\mathbf{v}) = 0$	22k2
Integral form:	$\int 0 = \int (M\mathbf{r}\times\mathbf{v})\frac{d}{dt}$	
L is the Angular Momentum: $\mathbf{L} = M\mathbf{r}\times\mathbf{v}$		
It's constant:	$\mathbf{L} = Mr^2\frac{d\theta}{dt}\hat{\mathbf{k}}$	(22l)

Area changes at a constant rate:

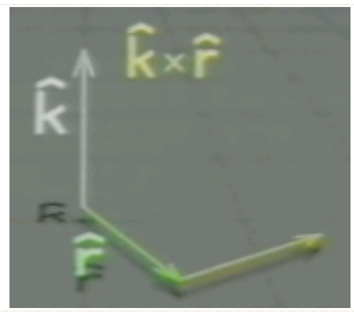
$$\Delta\mathbf{A} = \frac{1}{2}\mathbf{r}\times\Delta\mathbf{r}$$



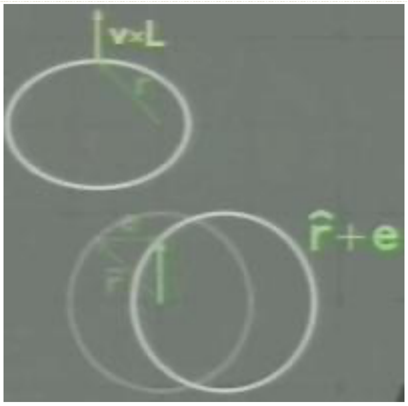
$$\lim_{\Delta t\to 0}\left(\frac{\Delta\mathbf{A}}{\Delta t}\right) = \lim_{\Delta t\to 0}\left(\frac{1}{2}\mathbf{r}\times\frac{\Delta\mathbf{r}}{\Delta t}\right)$$

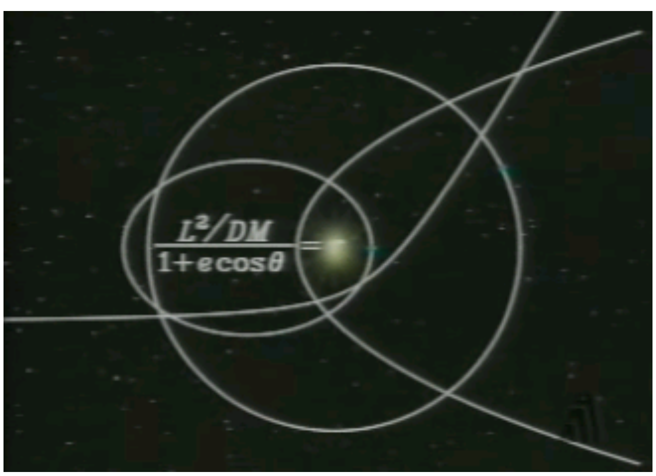
$$= \lim_{\Delta t\to 0}\left(\frac{1}{2}\mathbf{r}\times\mathbf{v}\right) = \frac{1}{2}\frac{L}{M}$$

(22e)x(22l):	$\mathbf{a}\times\mathbf{L} = \frac{-D}{Mr^2}\hat{\mathbf{r}}\times Mr^2\frac{d\theta}{dt}\hat{\mathbf{k}}$	(22m)
Group like terms:	$\mathbf{a}\times\mathbf{L} = -D\frac{Mr^2}{Mr^2}\frac{d\theta}{dt}\hat{\mathbf{r}}\times\hat{\mathbf{k}}$	(22n)
Simplify:	$\mathbf{a}\times\mathbf{L} = -D\frac{d\theta}{dt}\hat{\mathbf{r}}\times\hat{\mathbf{k}}$	(22o)
	$\mathbf{a}\times\mathbf{L} = D\frac{d\theta}{dt}\hat{\mathbf{k}}\times\hat{\mathbf{r}}$	(22p)



	$\mathbf{a}\times\mathbf{L} = D\frac{d\theta}{dt}\frac{d\hat{\mathbf{r}}}{d\theta}$	(22q)
	$\mathbf{a}\times\mathbf{L} = D\frac{d\hat{\mathbf{r}}}{dt}$	(22r)
	$\frac{d\mathbf{v}}{dt}\times\mathbf{L} = D\frac{d\hat{\mathbf{r}}}{dt}$	(22s)
(L is constant)	$\frac{d}{dt}(\mathbf{v}\times\mathbf{L}) = D\frac{d\hat{\mathbf{r}}}{dt}$	(22t)
	$\int(\mathbf{v}\times\mathbf{L})\frac{d}{dt} = \int D\frac{d\hat{\mathbf{r}}}{dt}$	(22u)
	$\mathbf{v}\times\mathbf{L} = D(\hat{\mathbf{r}}+e)$	(22v)

	
r dot: $r \cdot \mathbf{v} \times \mathbf{L} = rDr \cdot (\hat{\mathbf{r}} + e)$	
Exchange order: $\mathbf{r} \times \mathbf{v} \cdot \mathbf{L} = D\mathbf{r} \cdot (\hat{\mathbf{r}} + e)$	
Introduce Mass: $\frac{M\mathbf{r} \times \mathbf{v} \cdot \mathbf{L}}{M} = D\mathbf{r} \cdot (\hat{\mathbf{r}} + e)$	
L=Mrv: $\frac{\mathbf{L}^2}{M} = D\mathbf{r} \cdot (\hat{\mathbf{r}} + e)$	
$\frac{\mathbf{L}^2}{DM} = (\mathbf{r} \cdot \hat{\mathbf{r}} + \mathbf{r} \cdot e)$	
$\frac{\mathbf{L}^2}{DM} = r(1 + e\cos\theta)$	
Any Conic Section: $r = \frac{L^2 / DM}{1 + e\cos\theta}$	(22w)



$D = GMM_0$	(26a)
$r = \frac{a(1-e^2)}{1 + e\cos\theta}$	(26b)
$L^2 / DM = a(1-e^2)$	(26c)

Substitute (26a): $\frac{L^2}{GM_0M^2} = a(1-e^2)$	(26d)
Solve for L/M: $\frac{L}{M} = \sqrt{GM_0a(1-e^2)}$	(26e)
Equal areas in equal times: $\frac{dA}{dt} = \frac{L}{2M}$	(26e)
Solve for T: $T = \frac{2A}{L/M}$	(26f)
Substitute (26e) for L/M: $T = \frac{2A}{\sqrt{GM_0a(1-e^2)}}$	(26g)
$\frac{\mathbf{v}^2}{\mathbf{r}} = a = G\frac{m_e}{r^2}$	(15a)

O. Roemer (-) a Danish Astronomer in 1675 computed the speed of light by timing the delayed observation of eclipses of the moons of Jupiter.

Arono Fezo (-) 1849 measured the speed of light in air and water.

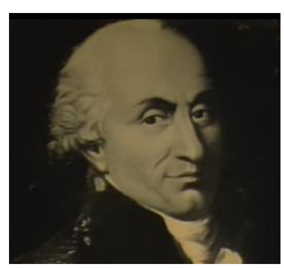
Someone *(13-Jun-1931 – 5-Nov-1979) Scottish mathematician*

John Foucault (-) 1852 measured the speed of light with a rotating mirror.

The 3 out of 4 "Fundamental Forces" of nature.

Gravity: *force constant is 6.67444x10^{-11} kg-m/s.*

$\mathbf{F}_g = -G\frac{m_1m_2}{r^2}\hat{\mathbf{r}}; G\left[\frac{Nm^2}{kg^2}\right]$	*(16a)*

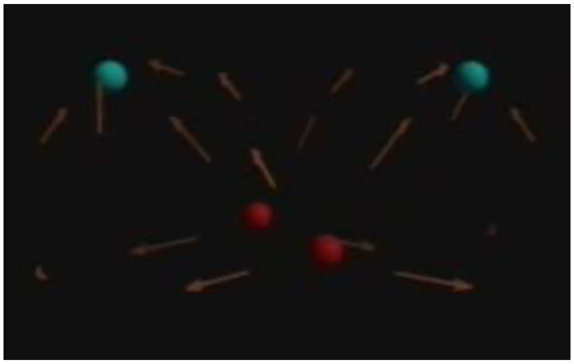


Charles Augustin de Coulomb (1736-1806) French physicist who pioneered electrical theory. In 1777 he invented the torsion balance to measure magnetic and electrical forces. He helped devise the metric system. Electrical charge is measured in the units of Coulombs.

Electricity: *Coulomb's Law - the electric charge force constant, K_e, is 8.987551788x10^9 Nm^2/C^2. r is the distance between the charges.*

$$\mathbf{F}_e = K_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}; K_e\left[\frac{Nm^2}{C^2}\right] \qquad (16b)$$

$$\mathbf{F}_{Total} = \sum_i K_e \frac{q_1 q_2}{r_i^2}\hat{\mathbf{r}}_i = \frac{q_1 q_i}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}_i; K_e\left[\frac{Nm^2}{C^2}\right] \qquad (16c)$$



Magnetism: *is unique in that it always has two poles. Note Amperes = Coulombs/second. The units* $Nm^2/(mC/s)^2$ *simplifies to* $1\times10^{-7}\ Ns^2/C^2$*:*

$$\mathbf{F}_m = K_m \frac{p_1 p_2}{r^2}\hat{\mathbf{r}} \qquad (16c)$$

Hans-Christian Oersted *(1777-1851) Danish scientist in 1820, was professor at Copenhagen, discovered that electric currents created a magnetic field perpendicular to the current. The magnetic field strength is proportional to the current.*

James Clerk Maxwell *(1831-1979) British physicist's great contribution to science is electromagnetic theory including his famous Maxwell's equations. He noticed the significance of the ratio between the electric and magnetic force constants = speed of light squared.*

$$c^2: \frac{K_e}{K_m} = 9\times10^{16}\frac{m^2}{s^2} \qquad (17a)$$

$$c = \sqrt{9\times10^{16}\frac{m^2}{s^2}} = 3\times10^{8}\frac{m}{s} \qquad (17b)$$

Sym	SI Units	Meaning
E	$\frac{volts}{meter}$	electric field intensity
H	$\frac{amperes}{meter}$	magnetic field intensity
D	$\frac{colombs}{meter^2}$	electric flux density (electric displacement field)
B	$\frac{weber}{meter}$	Aka Tesla: magnetic flux density (magnetic induction)
ρ	$\frac{colomb}{meter^3}$	free electric charge density not including dipole charges bound in a material
J	$\frac{ampere}{meter^2}$	free current density (not including polarization or magnetization currents bound in a material)
dA	$meter^2$	differential vector element of surface area A, with infinitesimally small magnitude and direction normal to surface S
dV	$meter^3$	differential of volume V enclosed by surface S
dl	*meter*	differential vector element of path length tangential to contour C enclosing surface S
$\nabla\cdot$	$meter^{-1}$	the divergence operator
$\nabla\times$		the curl operator

Maxwell's Four Famous Equations

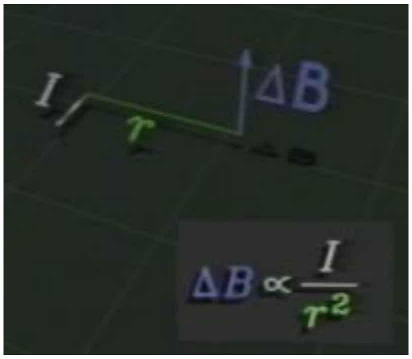
$$\oiint \mathbf{E}\cdot d\mathbf{A} = \frac{q}{\varepsilon_0} = 4\pi K_e q \qquad (27a)$$

$$\oiint \mathbf{B}\cdot d\mathbf{A} = 0 \qquad (27b)$$

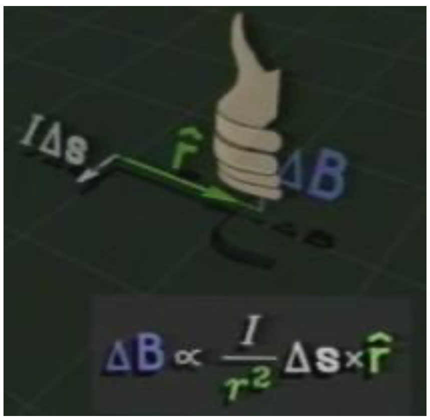
$$\oint \mathbf{E}\cdot d\mathbf{r} = -\frac{d}{dt}\iint \mathbf{B}\cdot d\mathbf{A} \qquad (27c)$$

$\oint \mathbf{B}\cdot d\mathbf{r} = \mu_0\left(1+\varepsilon_o \frac{d}{dt}\iint \mathbf{E}\cdot d\mathbf{A}\right)$	(27d)

Magnetic Field created by a current:

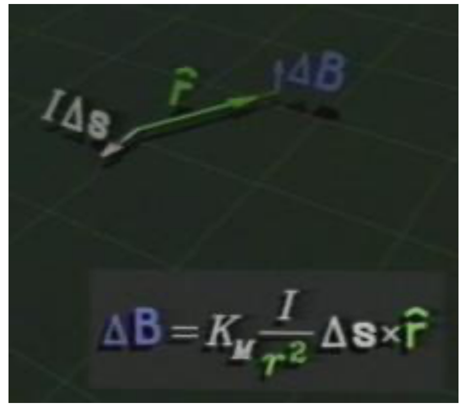


$\Delta B \alpha \frac{I}{r^2}$	*(35a)*

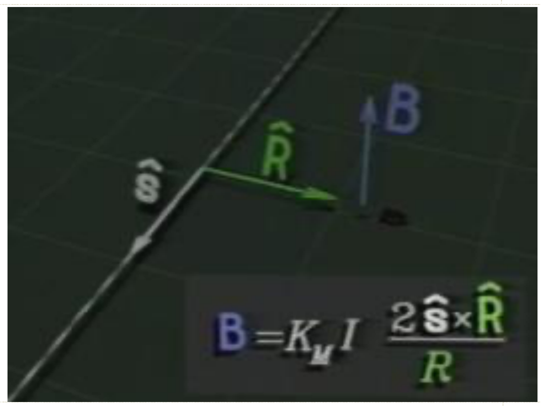


$\Delta \mathbf{B} = K_m \frac{I}{r^2}\Delta \mathbf{s}\times\hat{\mathbf{r}}$	*(35b)*

It is biggest when the distance vector r is at right angles to the current segment – its proportional to the sin(angle).



$\lim_{\Delta \mathbf{B}, \Delta \mathbf{s} \to 0}\left(\Delta \mathbf{B} = K_m \frac{I}{r^2}\Delta \mathbf{s}\times\hat{\mathbf{r}}\right)$	*(35c)*
$\int \Delta \mathbf{B} = \mathbf{B} = K_m I \int \frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$	*(35d)*



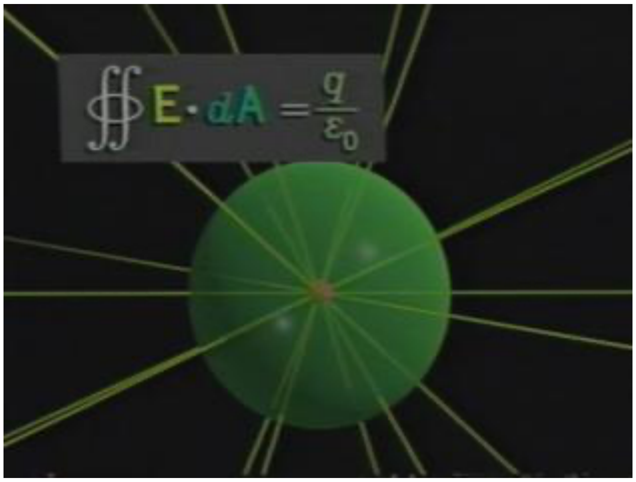
$\mathbf{B} = K_m I \frac{2\hat{\mathbf{s}}\times\hat{R}}{R}$	*(35e)*

$c^2 = \frac{K_e}{K_m} = 8.98755\times10^{16}\frac{m^2}{s^2}$	*(17a)*
$c = \sqrt{8.98755\times10^{16}\frac{m^2}{s^2}} = 299{,}792{,}458\frac{m}{s}$	*(17b)*

Maxwell's Four Famous Equations

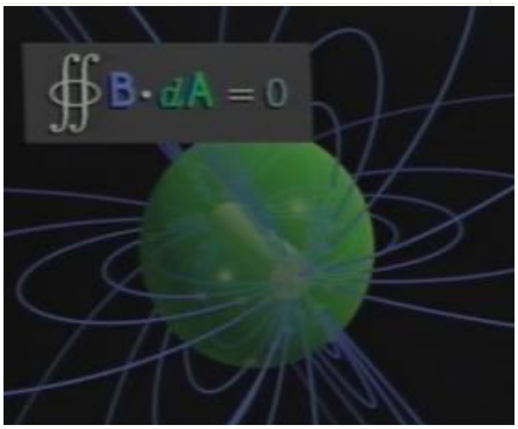
1st 2 also known as Gausses' Laws for electricity & magnetism.

Note $\varepsilon_0 = 8.854\times10^{-12}$ F/m []



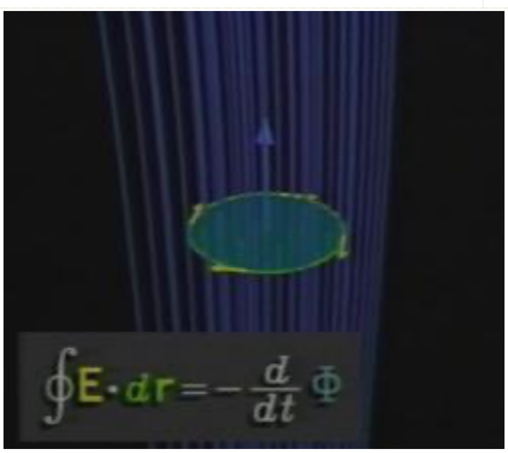
Electric Flux: Total charge enclosed by a surface.

$$\oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} = 4\pi K_e q \qquad (27a)$$



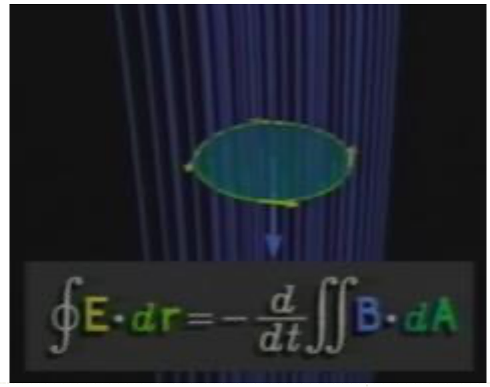
The magnetic (flux) through any closed surface=0.

$$ \oiint \mathbf{B} \cdot d\mathbf{A} = 0 \qquad (27b)$$



Circulation of Electric Field is the rate of change

$$\oint \mathbf{E} \cdot d\mathbf{r} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A} \qquad (27c)$$



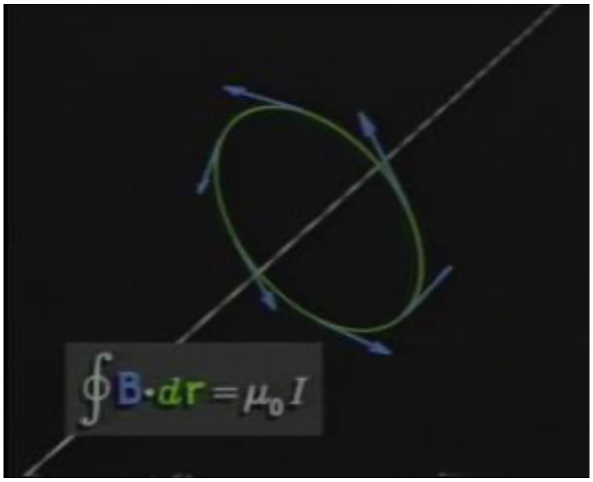
$$\oint \mathbf{B} \cdot d\mathbf{r} = \mu_0 \left(I + \varepsilon_o \frac{d}{dt} \iint \mathbf{E} \cdot d\mathbf{A} \right) \qquad (27d)$$

Quantum	Space	=	Time	$*c^2$
$h * f =$ [Js] [/s]	Energy	=	Matter	$*c^2$
EM	Electric Charge $K_e = 8.987551e^9$ $N(m/C)^2$	=	Magnetic Flux $K_m = 1e^{-7}$ $N(s/C)^2$	$*c^2$
Electro-magnetism	$F_e = K_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$		$F_m = K_m \frac{p_1 p_2}{r^2} \hat{\mathbf{r}}$	$*c^2$
G =	Gravity $6.6744447698e^{-11}$ $N(m/kg)^2$	=	Matter slows time $K_\Theta = 7.42632465e^{-28}$ $N(s/kg)^2$	$*c^2$
Gravity	$F_g = -G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}$			
Permittivity Permeability	$(1/\varepsilon_0)$ [F/m]	=	$\mu_0 = 4\pi \cdot K_m$	$*c^2$

Space-Time Matter-Energy Symmetries

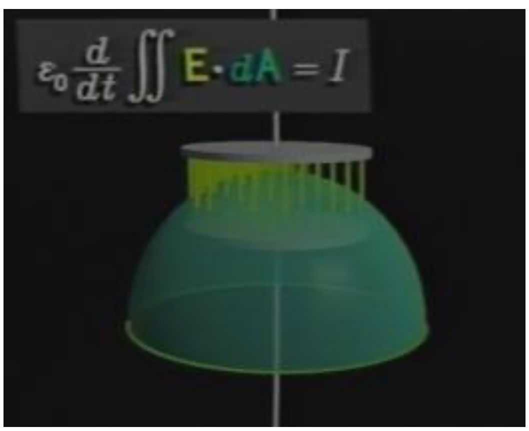
1 Newton is the force needed to accelerate 1 kg a distance of 1 meter per second per second. Or 1 gram a distance of 1 mm per millisecond per second.

Circulation of magnetic field around a current.



Ampere's Law: $\oint \mathbf{B}\cdot d\mathbf{r} = \mu_0 I$	(39a)

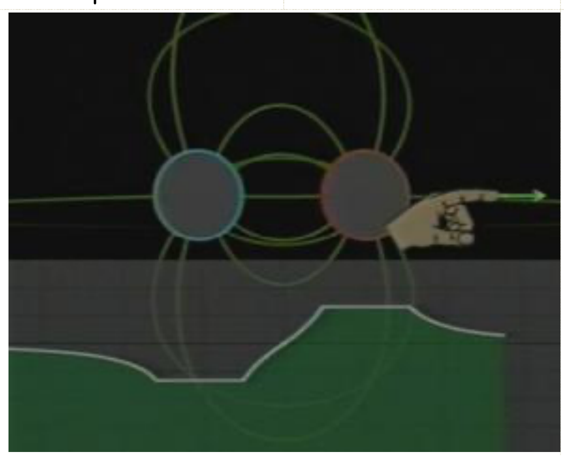
So what happens in a capacitor?



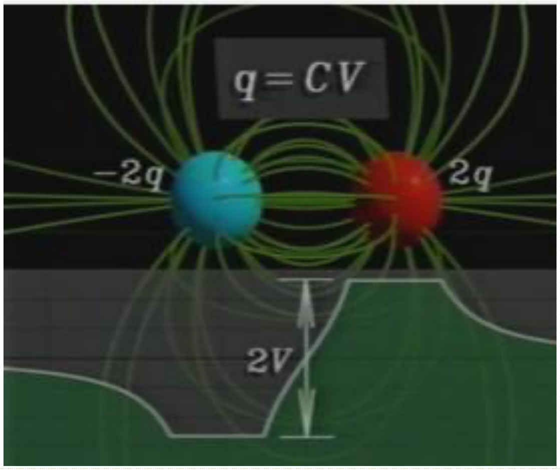
$\varepsilon_0 \iint \mathbf{E}\cdot d\mathbf{A} = q$	(39b)
$\varepsilon_0 \frac{d}{dt}\iint \mathbf{E}\cdot d\mathbf{A} = \frac{dq}{dt} = I$	(39c)

Force on a charge in an electric field.

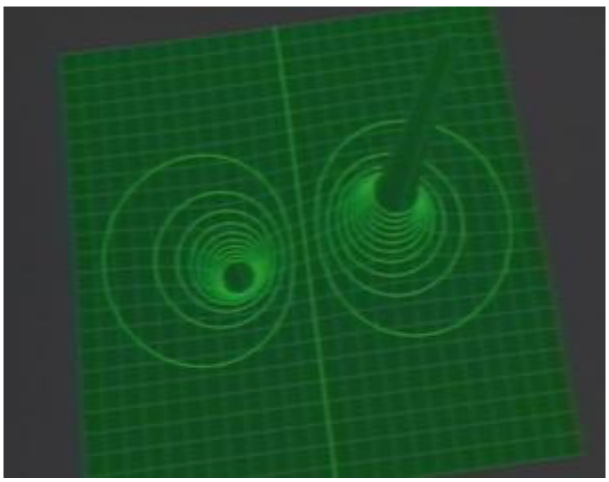
	Note: red particles are positively charged; green negative. $dW = -\mathbf{F}\cdot d\mathbf{r}$	(30a)
	Net work: $\Delta W = -\int \mathbf{F}\cdot d\mathbf{r}$ $\Delta U = \Delta W$ ΔU is the change in potential energy.	(30b)
The Force is the charge times the Electric field strength.	$\mathbf{F} = q\mathbf{E}$	(30c)
Delta potential is:	$\Delta U = -q\int \mathbf{E}\cdot d\mathbf{r}$	(30d)
Voltage potential difference is expressed as:	$\Delta V = -\int \mathbf{E}\cdot d\mathbf{r}$	(30e)



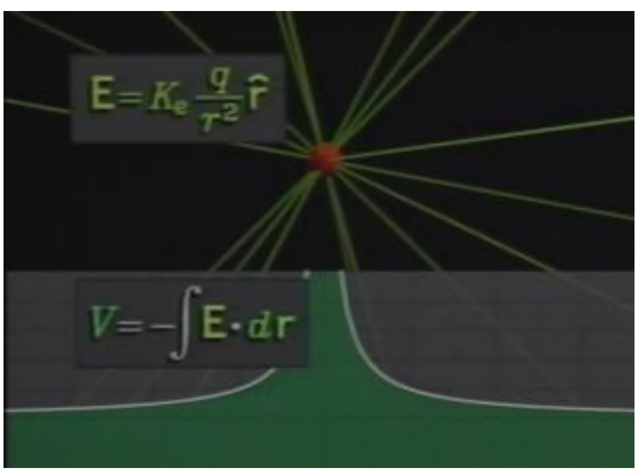
This potential difference is what happens when a battery is connected to two metal objects. The charge transferred, q, is capacitance times Voltage.



Charge increases when a capacitor's plates are closer or when the surface area increases.

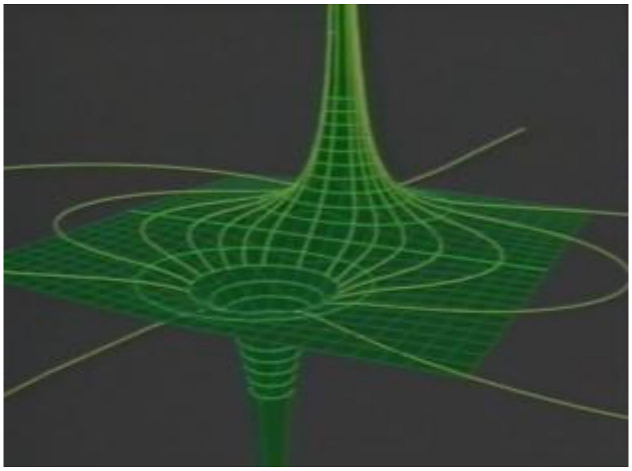


Electric Field is the vicinity of a point charge is:



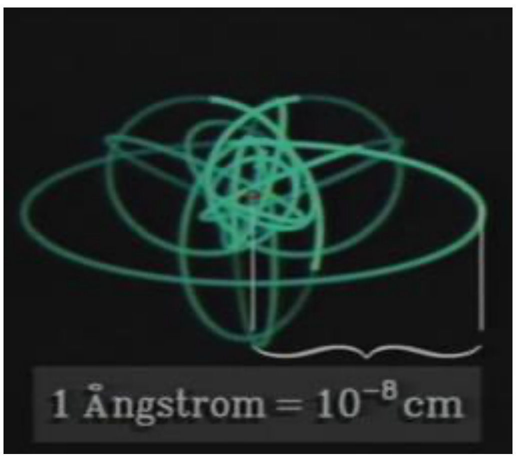
$$V = -K_e \int \frac{q}{r^2} \hat{\mathbf{r}} \cdot dr \quad (31a)$$

$$V = -K_e \int \frac{q}{r^2} dr = -K_e \frac{-q}{r} \quad (31b)$$

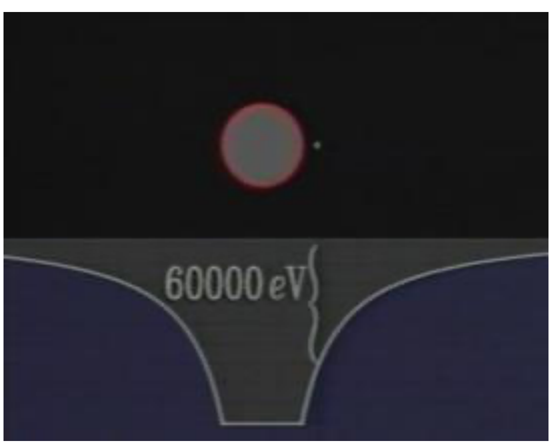


So the field is perpendicular to every equal potential contour line.

The distance from the center of an atomic nucleus out to its outer most electron is about 1 Angstrom which is 10^{-8} cm. Its electric potential is about 14.4 electron Volts.

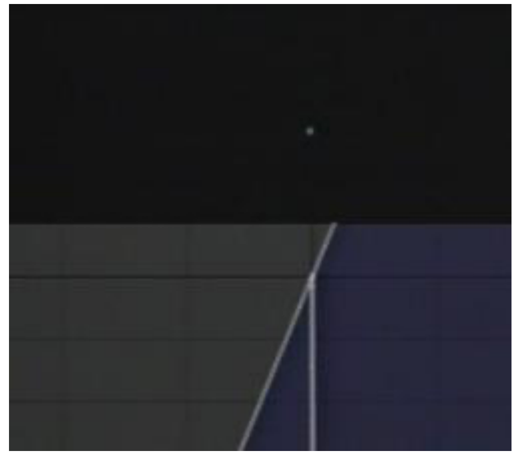


Van deGraaff generator vs an electron.

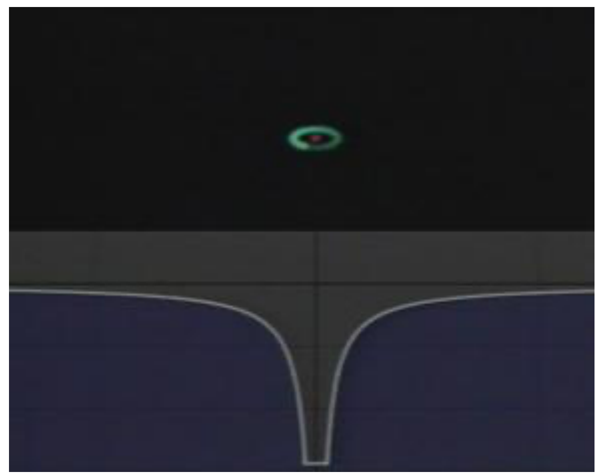


Note the slope at a given distance. Next the electric field strength at 1 Angstrom has a much greater slope.

The spark is a plasma of stray electrons and ionized molecules in the air.



Expanding the horizontal scale by 100,000 times we can see this spike as a negative plateau.



Albert A. Michelson (1852-1931) 1849 measured the speed of light between Mt. Wilson and Mt. San Antonio about 30 kilometers apart taking almost one ten-*thousandths of a second.*

The speed of light = 2.99792458×10^8 meters/second:

Waves:

Wavelength is the product of the period, T and the speed of the wave, **v**. The inverse of the period is called the frequency, f. From the perspective of the speed of light:

Waves:

Wavelength is the product of the period, T and the speed of the wave, $\mathbf{v}$. The inverse of the period is called the frequency, f.

$$v \approx a\sqrt{\frac{k}{m}} \quad (17c)$$

From the perspective of the speed of light:

$$c = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} = \frac{Z_0}{\mu_0} = \frac{1}{\varepsilon_o Z_0} = \frac{\lambda_P}{t_P} \quad (17c)$$

= 2.99792458e8 m/s (uncertainty 1e-9)

Although somewhat redundant c^2 occurs frequently in physics formulae; for example $E=mc^2$.

$$c^2 = \left(\frac{\lambda_P}{t_P}\right)^2 = \frac{1}{\varepsilon_0^2 Z_0^2} = \left(\frac{Z_0}{\mu_0}\right)^2 = \frac{1}{\varepsilon_0 \mu_0} \quad (17d)$$

= 8.987551787e16 m2/s2 (uncertainty 1e-9)

Planck's constant, h, 6.62606896x10-34 J s. h/c^2 yields the following equation-set for the mass-quanta:

$$\frac{h}{c^2} = h\left(\frac{t_P}{\lambda_P}\right)^2 = h\varepsilon_0\mu_0 = h\varepsilon_0^2 Z_0^2 = h\left(\frac{\mu_0}{Z_0}\right)^2 \quad (11c)$$

= 1.105109e-42 kg (uncertainty 1e-9?)

John Wsol's 1st physics constant $1.8984625(46)\times10^{20}$ m^7kg^2/s^5 which represents this following equation set:

$$\frac{Z_o}{\mu_o^3} = \frac{c}{\mu_0^2} = \frac{c^3}{Z_0^2} = c^5\varepsilon_0^2 = \frac{1}{\mu_0^2\varepsilon_0 Z_0} = \frac{1}{\sqrt{\mu_0^5\varepsilon_0}}$$

Solving for c we get the following equation-set:

$$c = \frac{Z_o}{\mu_0} = \frac{1}{\varepsilon_0 Z_0} = \frac{4\pi K_e}{Z_0} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Water wave's speed is a function of gravity:

$$v \approx \sqrt{\frac{g\lambda}{2\pi}}$$

Pendulums: The period of a pendulum (short motion) can be described by:

$$T = 2\pi\sqrt{\frac{L_P}{g_{Planet}}}$$

Robert A. Milliken (-) 187?, measured size of oil droplets and all the forces acting upon the. R is the radius of the droplet; η is the viscosity of the air; v is the velocity of the sphere. (18b) was worked out by George Stokes (19th century). Once constant velocity is achieved we have (18c).

$m\mathbf{a} = \sum \mathbf{F}_{all} = \mathbf{F}_g + \mathbf{F}_v + \mathbf{F}_e$	(18a)
$\mathbf{F}_v = 6\pi R\eta v$	(18b)
$mg = 6\pi R\eta v$	(18c)
$\frac{mg}{6\pi R\eta} = v$	(18d)

The known density, ρ, can be used to determine the mass of the droplet.

$\rho = \frac{mass}{volume} = \frac{m}{\frac{4}{3}\pi R^3}$	(19a)
$\left(\frac{4}{3}\right)\rho R^2 = \frac{m}{\pi R}$	(19b)
$\frac{m}{\pi R} = \frac{6v\eta}{g}$	(19c)
$\left(\frac{4}{3}\right)\rho R^2 = \frac{6v\eta}{g}$	(19d)
$R = \sqrt{\frac{6v\eta}{\left(\frac{4}{3}\right)g\rho}}$	(19e)

Charge with charged plates on or off.

$$v_{on} = \frac{qE - mg}{6\pi R\eta}$$	(20a)
$$v_{off} = \frac{mg}{6\pi R\eta}$$	(20b)

Add (20a) to (20b):

$$v_{on} + v_{off} = \frac{qE}{6\pi R\eta}$$	(20c)

Isolate q: final published value: 4.77×10^{-10}

$$q = \frac{6\pi R\eta(v_{on} + v_{off})}{E}$$	(20d)

In 1923 Milliken became the 1st American born scientist to be awarded the Nobel Prize in Physics. An formed CalTech.

---

13. Conservation of Energy

Work of a force applied to raising a mass a certain height within a gravitational field of strength g. (The slope of the space-time manifold.) This puts a gravitational potential energy, U, into the mass with respect to the field. Work is the transfer of energy from one form to another.

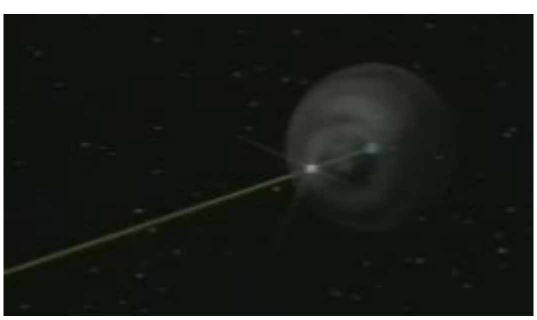
$$W = \mathbf{F}h$$	(21a)
$$W = mgh = U$$	(21b)

Work against the force of gravity.

$$W = \int_{\alpha}^{\omega} \mathbf{F}dx$$	(21c)

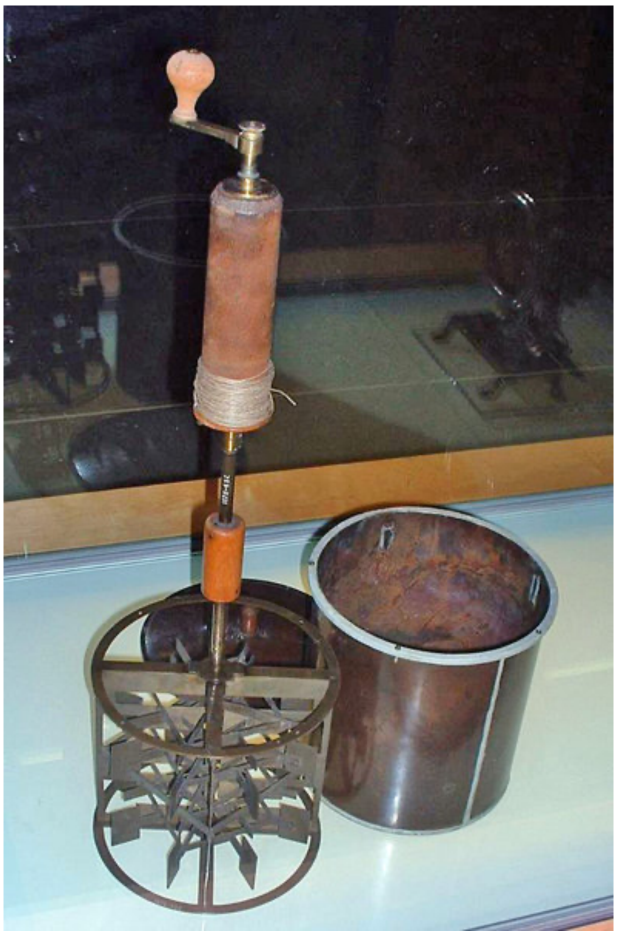
13. Kepler to Einstein

The Earth-Moon system has a center of mass about ¾ of the way from the Earth's center.



So the earth wobbles around the center of mass of the Earth-moon system. My theory is that on the side of the Earth facing the moon the slope in the space-time manifold the moon's mass creates causes the tides to bulge there. The opposite side of the Earth exhibits centrifugal force which causes the tides to bulge.

James Prescott Joule[3] (1818-1891) a British physicist is best known for his research in thermodynamics and electricity. Joules Law says the amount of heat produced in a conductor by the amount of electric current is and the conductor's resistance. He also created an apparatus that demonstrated how potential & mechanical energy gets converted into heat energy.



14. Potential Energy

Roger Boscovish (1711-) born to his Yugoslavian dad and an Italian mom. Trained as a Jesuit; he became many things: architect, archeologist, diplomat to N. Africa. He postulated that atoms forces might be attractive or repulsive at certain distances. Creating a distance of equilibrium at certain distances – an idea that intrigued scientists for years.

"Energy is conserved, without exception, strictly and absolutely."

Work against the force of gravity.

$$W = \int_{R_E}^{R_f} G\frac{mM_E}{r^2}dr$$	(14a)
$$W = GmM_E\int_{R_E}^{R_f}\frac{dr}{r^2}$$	(14b)
$$W = \left[\frac{-GmM_E}{r}\right]_{R_E}^{R_f}$$	(14d)

Escape Velocity can be defined as:

$$\frac{1}{2}mv^2 = \frac{GmM_E}{R_E}$$	(14e)
$$v = \sqrt{\frac{2GM_E}{R_E}}$$	(14f)

For Earth the escape velocity is about 11 km/s.

Potential Energy, U has units of kilograms-meters-per second squared also called Joules:

$$U = mgh\left[kg\frac{m^2}{s^2}\right] \equiv [J]$$	(14g)

4.2 Joules is equivalent to 1 calorie of heat. A food Calorie = 1000 calories.

16. Harmonic Motion

Force in the most general sense is the product of mass and the acceleration vector. In a harmonic system it is a measure of the displacement from the equilibrium position.

$$\mathbf{F} = m\mathbf{a} = m\frac{d^2x}{dt^2}$$	(16a)
$$\mathbf{F} = -kx$$	(16b)

This gives us the harmonic equality:

[3] http://en.wikipedia.org/wiki/James_Prescott_Joule

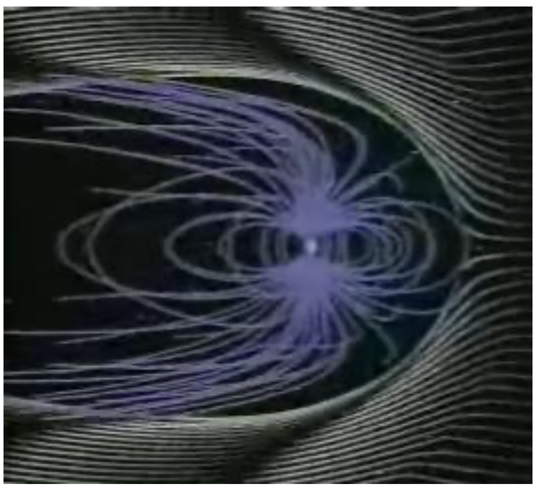
$$m\frac{d^2x}{dt^2} = -kx$$	(16c)

The equation that describes simple harmonic (angle of a pendulum, displacement of a spring or string.)motion is:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$	(16c)

34. <u>Magnetism</u>

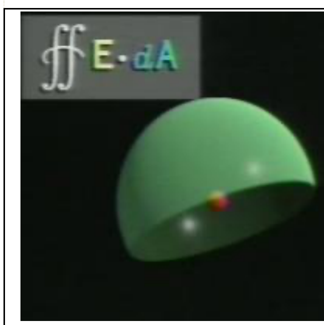
William Gilbert (1544-1603) who wrote "De Magnete" published in 1600. In 1600 he became personal physician to Queen Elizabeth I. He bored her to death talking about magnetism. The Earth, itself, is a gigantic magnet.

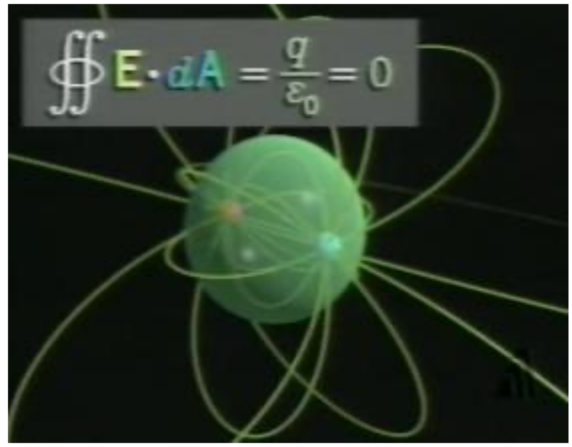


Some materials are magnetic: iron, steal. Some materials are not magnetic: copper, aluminum. Each magnetic material has its own temperature limit that it can maintain its magnetic properties. Gadolinium is not magnetic at room temperature, but well below room temperature it is magnetic.

Gauss' Law

$$\oiint \mathbf{E}\cdot d\mathbf{A} = 4\pi K_e q = \frac{q}{\varepsilon_0}$$	*(27a)* *(34b)*





Magnetic Dipole net flux is the sum of the outward plus inward yielding zero.

The beginnings of my unified field equations:

$$K_e = \frac{1}{4\pi\varepsilon_0}$$	(A1)

$$c = \frac{Z_o}{\mu_0} = \frac{1}{\varepsilon_0 Z_0} = \frac{4\pi K_e}{Z_0} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$	(A2)

Electricity: Coulomb's Law - the electric charge force constant, K_e, is 8.987551788x10^9 Nm^2/C^2. *r* is the distance between the charges.

$$\mathbf{F}_e = K_e\frac{q_1q_2}{r^2}\hat{\mathbf{r}} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}; K_e\left[\frac{Nm^2}{C^2}\right]$$	*(16b)* *(A3)*

Magnetism: *Magnetic Force equation.*

$$\mathbf{F}_m = K_m\frac{p_1p_2}{r^2}\hat{\mathbf{r}}$$	*(16c)* *(34a)*

Now to derived K_m from (A2):

$$cZ_o\mu_0 = \frac{Z_o^2}{1} = \frac{4\pi K_e\mu_0}{1} = Z_o\sqrt{\frac{\mu_0}{\varepsilon_0}}$$	*(16c)* *(34a)*