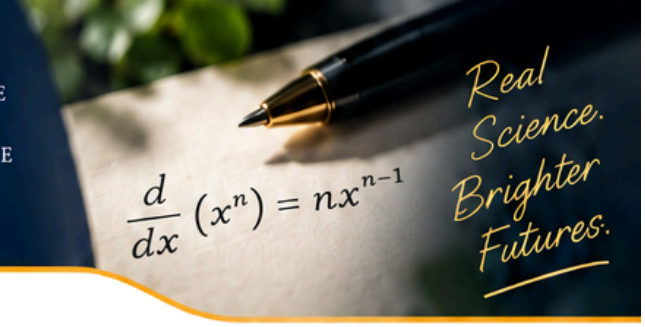


Mathematics 6.6
Differentiating Exponential and Logarithmic Functions
WORKSHEET

Show clear working in all questions. Give exact answers where appropriate and use correct notation.

You will need to:
- use differentiation techniques from 6.1–6.5
- apply the chain, product and quotient rules
- solve problems in context
- give answers in their simplest form where possible

A Basic differentiation

Differentiate the following with respect to x.

1	$y = e^x$	[1 mark]
2	$y = 3e^x$	[1 mark]
3	$y = e^{2x}$	[2 marks]
4	$y = \ln x$	[1 mark]
5	$y = 4\ln x$	[1 mark]
6	$y = \ln(3x)$	[2 marks]
7	$y = \ln(x^2)$	[2 marks]
8	$y = 2^x$	[2 marks]
9	$y = 5^x$	[2 marks]
10	$y = e^{-3x}$	[2 marks]

B Using the chain rule

Differentiate the following with respect to x.

11	$y = e^{3x+1}$	[2 marks]
12	$y = \ln(2x+5)$	[2 marks]
13	$y = e^{x^2}$	[3 marks]
14	$y = \ln(x^2+1)$	[3 marks]
15	$y = e^{\sin x}$	[3 marks]
16	$y = \ln(\cos x)$	[3 marks]
17	$y = 3^{2x}$	[2 marks]
18	$y = 5^{x^2}$	[3 marks]
19	$y = \ln(4 - x^2)$	[3 marks]
20	$y = e^{2x-1}$	[2 marks]

C Products, quotients and mixed functions

Differentiate the following with respect to x.

21	$y = xe^x$	[3 marks]
22	$y = x\ln x$	[3 marks]
23	$y = \dfrac{e^x}{x}$	[3 marks]
24	$y = \dfrac{\ln x}{x}$	[3 marks]
25	$y = (x^2+1)e^x$	[4 marks]
26	$y = \dfrac{e^{2x}+1}{e^x}$	[4 marks]
27	$y = x^2\ln(3x)$	[4 marks]
28	$y = \dfrac{\ln(x^2+1)}{x}$	[4 marks]

D Modelling and problem solving

29 A population is modelled by $P(t) = 200e^{0.3t}$, where t is in years.
(a) Find $\dfrac{dP}{dt}$.
(b) Find the rate of increase when $t = 5$. [3 marks]

30 The value of an asset is modelled by $V(t) = 5000\ln(t+1)$, where t is in years.
(a) Find $\dfrac{dV}{dt}$.
(b) Find the rate of change when $t = 4$. [3 marks]

31 A curve is given by $y = e^x + \ln x$.
Find the equation of the tangent to the curve at the point where $x = 1$. [4 marks]

32 The function $f(x) = xe^x$ has a stationary point.
(a) Find the x-coordinate of the stationary point.
(b) Determine whether it is a maximum or a minimum. [5 marks]

33 The function $g(x) = \ln(x^2)$ is defined for $x > 0$.
(a) Find $g'(x)$.
(b) Explain why $g(x)$ is an increasing function for $x > 0$. [4 marks]

You should now be able to:
- ☐ differentiate exponential and logarithmic functions
- ☐ use the chain, product and quotient rules where appropriate
- ☐ solve problems in context, including modelling and optimisation
- ☐ check your answers for sensible form and domain restrictions

"New skills today, greater possibilities tomorrow."