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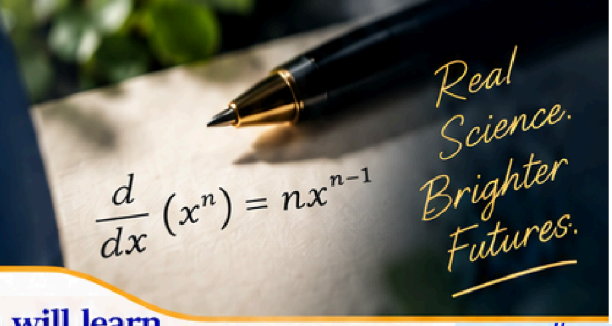

Mathematics 6.4
The Product Rule

REVISION NOTES

What you will learn

How to differentiate products of functions using the product rule, with the substitution notation u and v. You will also see how to apply the product rule to polynomials, trigonometric functions and mixed expressions, using knowledge from 6.1–6.3.

Small steps. Big progress.

1 The product rule

If $y = uv$, where u and v are functions of x, then

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Differentiate each function in turn, multiply as shown, then add the two terms.

2 What are u and v?

Choose u and v to be the two functions being multiplied. They can be any differentiable functions (e.g. polynomials, trigonometric functions, or a mixture).

Example

If $y = (2x+1)\sin x$, take $v = 2x+1$ and $v = \sin x$.

3 Simple example (polynomial × polynomial)

Differentiate $y = x^2(3x-1)$.

Let $u = x^2, \quad v = 3x - 1$

$$\frac{du}{dx} = 2x, \qquad \frac{dv}{dx} = 3$$

$$\frac{dy}{dx} = x^2 \times 3 + (3x-1) \times 2x$$

$$= 3x^2 + 6x^2 - 2x = 9x^2 - 2x$$

4 Product of a polynomial and a trig function

Differentiate $y = x\sin x$.

Let $u = x, \quad v = \sin x$

$$\frac{du}{dx} = 1, \qquad \frac{dv}{dx} = \cos x$$

$$\frac{dy}{dx} = x\cos x + \sin x$$

Another example

Differentiate $y = (2x+1)\cos x$.

$u = 2x+1, \quad v = \cos x$

$$\frac{dy}{dx} = (2x+1)(-\sin x) + 2\cos x$$

$$= -(2x+1)\sin x + 2\cos x$$

5 Product of two trigonometric functions

Differentiate $y = \sin x\cos x$.

Let $u = \sin x, \quad v = \cos x$

$$\frac{du}{dx} = \cos x, \qquad \frac{dv}{dx} = -\sin x$$

$$\frac{dy}{dx} = \sin x(-\sin x) + \cos x\cos x$$

$$= \cos^2 x - \sin^2 x$$

Another example

Differentiate $y = \sin(2x)\cos x$.

$u = \sin(2x), \quad v = \cos x$

$$\frac{du}{dx} = 2\cos(2x), \qquad \frac{dv}{dx} = -\sin x$$

$$\frac{dy}{dx} = 2\cos(2x)\cos x - \sin(2x)\sin x$$

6 More mixed examples

Differentiate $y = (x^2+1)\sin(3x)$.

$u = x^2 + 1, \quad v = \sin(3x)$

$$\frac{du}{dx} = 2x, \qquad \frac{dv}{dx} = 3\cos(3x)$$

$$\frac{dy}{dx} = (x^2+1) \times 3\cos(3x) + 2x\sin(3x)$$

Differentiate $y = \cos(x^2)(2x+1)$.

$u = \cos(x^2), \quad v = 2x + 1$

$$\frac{du}{dx} = -\sin(x^2) \times 2x, \qquad \frac{dv}{dx} = 2$$

$$\frac{dy}{dx} = -2x\sin(x^2)(2x+1) + 2\cos(x^2)$$

7 Choosing u and v

- You can choose either function to be u or v – the product rule is symmetric.
- Look for a sensible choice that makes the differentiation straightforward.
- Apply the chain rule first when finding $\frac{du}{dx}$ or $\frac{dv}{dx}$ if the function is composite (e.g. $\sin(3x)$).
- Simplify your answer where possible.

8 Common forms

The product rule can be used with many types of functions, for example:

- polynomial × polynomial
- polynomial × trigonometric
- trigonometric × trigonometric
- trigonometric × composite function
- combinations such as $(x^2+1)\sin(3x)$ or $\cos(x^2)(2x+1)$

As long as the function is a product of two differentiable functions, the product rule applies.

9 Key points to remember

- If $y = uv$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$.
- Use the substitution notation u and v.
- Differentiate each function in turn, then multiply and add.
- You can use the chain rule (from 6.3) when finding $\frac{du}{dx}$ or $\frac{dv}{dx}$.
- The product rule can be used with polynomials, trigonometric functions and other functions.
- Simplify your answer where possible.
- Check your answer for sensible form and sign.

You should now be able to:

- ☐ recall the product rule and use the u, v notation
- ☐ differentiate products of polynomials
- ☐ differentiate products involving trigonometric functions
- ☐ apply the chain rule where needed within the product rule
- ☐ simplify your answers and check they make sense

"Build the methods today, for bigger problems tomorrow."

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