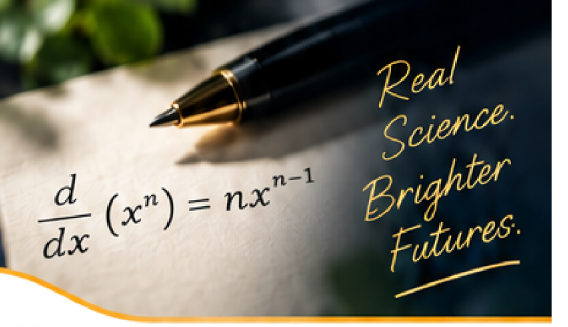


Mathematics 6.2

Differentiation

Further Applications

Build confidence with more advanced differentiation techniques and their applications, from tangents and normals to optimisation problems.

You will need to:

- use differentiation techniques from 6.1
- apply second derivaiues
- solve problems and interpret your answers in context
- give answers in their simplest form where possible

Show clear working in all questions. Give exact answers where appropriate and use correct notation.

Challenge yourself further! ☺

A Derivatives, tangents and normals

1 Given $f(x) = 4x^3 - 3x + 5$, find $f'(x)$ and $f''(x)$. [3 marks]

2 Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 4$ at the point where $x = 2$. [4 marks]

3 Find the equation of the normal to the curve $y = x^3 - 6x$ at the point where $x = 1$. [4 marks]

4 The curve $y = 3x^4 - 4x^2 + 1$ has a stationary point at $x = 0$. Find the equation of the tangent at this point and state whether it is a maximum, minimum or point of inflection. [5 marks]

B Stationary points and behaviour

5 Find the stationary points of $f(x) = x^3 - 6x^2 + 9x$. [4 marks]

6 For the function $f(x) = 2x^3 - 3x^2 - 12x + 4$, use the second derivative to classify each stationary point as a maximum, minimum or point of inflection. [5 marks]

7 Determine the intervals where the function $f(x) = x^3 - 4x^2 + 4x + 1$ is increasing and decreasing. [4 marks]

8 Given $f(x) = x^4 - 5x^2 + 4$, sketch the graph, showing the coordinates and nature of any stationary points, and state the intervals where the function is increasing and decreasing. [6 marks]

C Maximum, minimum and optimisation

9 A rectangle has length x cm and width $(10 - 2x)$ cm. Find the value of x that gives the maximum area, and find the maximum area. [4 marks]

10 A rectangular box with a square base and open top has volume 108 cm^3. Show that the surface area S is given by $S = x^2 + \frac{432}{x}$, where x is the side length of the base. Find the value of x that minimises S, and find this minimum surface area. [6 marks]

11 A rectangular box with no lid is to be made from a rectangular piece of card of length 30 cm and width 20 cm by cutting out squares of side x cm from each corner and folding up the sides. Find the value of x that gives the maximum volume, and find this maximum volume. [6 marks]

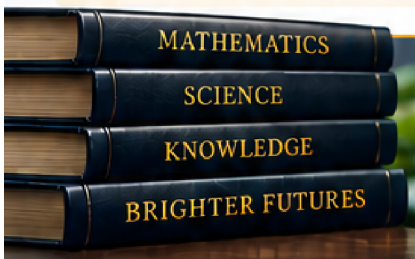
12 Find the stationary points of $f(x) = x^3 - 3x$ and use the second derivative to show that one is a maximum and the other is a minimum. Find the corresponding maximum and minimum values. [5 marks]

D Rates of change

13 The distance travelled by a particle is given by $s(t) = 2t^3 - 5t^2 + 4t$, where t is in seconds and s is in metres.
(a) Find the velocity $v(t)$ and acceleration $a(t)$.
(b) Find the velocity and acceleration when $t = 3$. [5 marks]

14 The height of a projectile above the ground is given by $h(t) = -5t^2 + 20t$, where t is in seconds and h is in metres.
(a) Find the velocity and acceleration.
(b) Determine the time when the particle is instantaneously at rest.
(c) Find the greatest height attained and the time at which it occurs. [6 marks]

15 The volume V of water in a tank is modelled by $V(t) = t^3 - 9t^2 + 24t + 50$, where t is in minutes and V is in litres.
The rate of change of volume with respect to time is given by $\frac{dV}{dt}$.
(a) Find $\frac{dV}{dt}$ when $t = 2$.
(b) Find the times when the volume is neither increasing nor decreasing.
(c) Determine the intervals for which the volume is increasing and decreasing. [6 marks]



"Curiosity today, a deeper understanding tomorrow."